Anomaly Matching and Symmetry-protected Criticality in 1d Quantum Many-body Systems

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Chiral Matter and Topology, NTU CTS

December 7, 2018

K A V L I

IPMU

discrete

chiral

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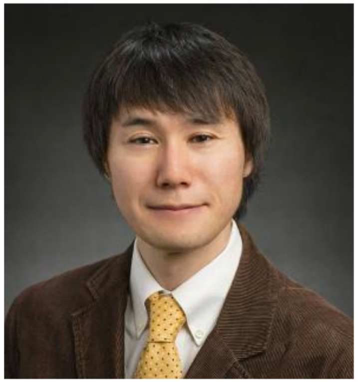

Gil Young Cho
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G. Y. Cho*, **C.-T. Hsieh***, and S. Ryu, PRB 96, 195105 (2017); arXiv:1705.03892

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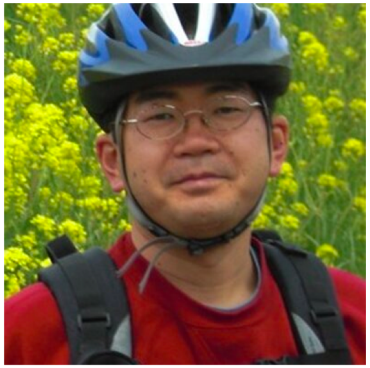

Yuan Yao
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Y. Yao*, **C.-T. Hsieh***, and M. Oshikawa, arXiv:1805.06885

*Equal contributions

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

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- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
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Introduction

- Identifying the "phase" of a generic many-body system is an important but, in general, difficult problem

- Quite often, symmetries play an essential role in such a problem

- Various classes of phases of matter:

➢ Conventional *Landau-Ginzburg-Wilson* symm-breaking paradigm

➢ Topological phases: Symm-protected top. (SPT) phases, etc

[Hasan-Kane 10; Qi-Zhang 11; Chiu-Teo-Schnyder-Ryu 16; Witten 16]
(all are Rev. Mod. Phys.)

Today's focus

- Phases associated with "symmetry-protected in-gap(p)-ability"

➢ Defined regarding "whether a system with symm *has* or *can be gapped into* a trivial – **unique** and **symmetric** – **gapped** ground state"

symmetry-respecting

$$H_0 + \alpha H_{\text{pert}}$$

trivial ⇔ gappable

nontrivial ⇔ ingappable

- Phases associated with "symmetry-protected in-gap(p)-ability"

➢ Ingappability (stability) of edge states of 2d SPT phases has been well studied, e.g. *helical edge states* of 2d QSHE

$$H_{\text{edge}} = \int dx \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$\Psi(x) = (\psi_R(x), \psi_L(x))^T$$

$$M = \psi_R^\dagger \psi_L + \text{h.c}$$

mass (magnetic impurity)

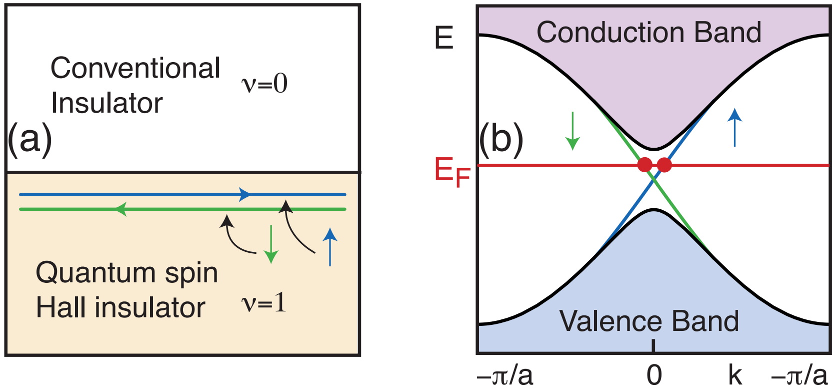

Figure from Hasan-Kane 10

- Phases associated with "symmetry-protected in-gap(p)-ability"

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$$H_{\text{edge}} = \int dx \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$\Psi(x) = (\psi_R(x), \psi_L(x))^T$$

breaks TR symm

$$\cancel{M = \psi_R^\dagger \psi_L + \text{h.c}}$$

mass (magnetic impurity)

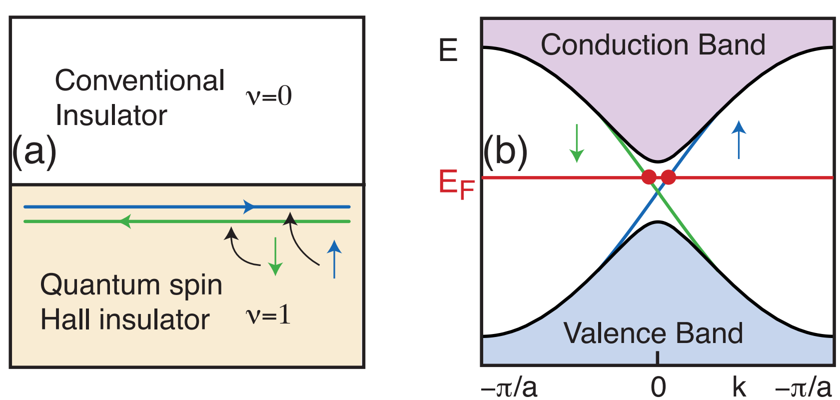

Figure from Hasan-Kane 10

- Phases associated with "symmetry-protected in-gap(p)-ability"

➢Ingappability (stability) of edge states of 2d SPT phases has been well studied, e.g. *helical edge states* of 2d QSHE

[Wu-Bernevig-Zhang 06; Xu-Moore 06]

$$H_{\text{edge}} = \int dx \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$\Psi(x) = (\psi_R(x), \psi_L(x))^T$$

$$I_{\text{fw}} = g_1 \psi_R^\dagger \psi_R \psi_L^\dagger \psi_L$$

gapless

or

$$I_{\text{Umkl}} = g_2\ e^{-i4k_F x}\psi_R^\dagger(x)\psi_R^\dagger(x+a)$$

SSB of TR $\times\ \psi_L(x+a)\psi_L(x) + \text{h.c}$

electron interactions

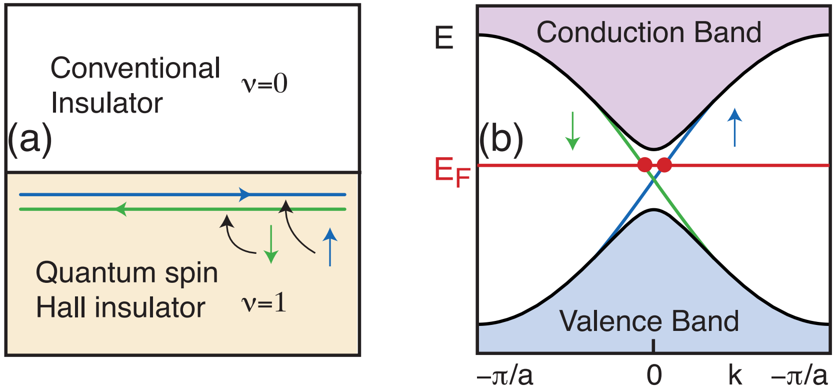

Figure from Hasan-Kane 10

- Phases associated with "symmetry-protected in-gap(p)-ability"

➢Ingappability (stability) of edge states of 2d SPT phases has been well studied, e.g. *helical edge states* of 2d QSHE [Wu-Bernevig-Zhang 06; Xu-Moore 06]

$$H_{\text{edge}} = \int dx \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$\Psi(x) = (\psi_R(x), \psi_L(x))^T$$

ingappable (stable) under TRS!

$$I_{\text{fw}} = g_1 \psi_R^\dagger \psi_R \psi_L^\dagger \psi_L$$

gapless

or

$$I_{\text{Umkl}} = g_2\ e^{-i4k_F x}\psi_R^\dagger(x)\psi_R^\dagger(x+a)$$

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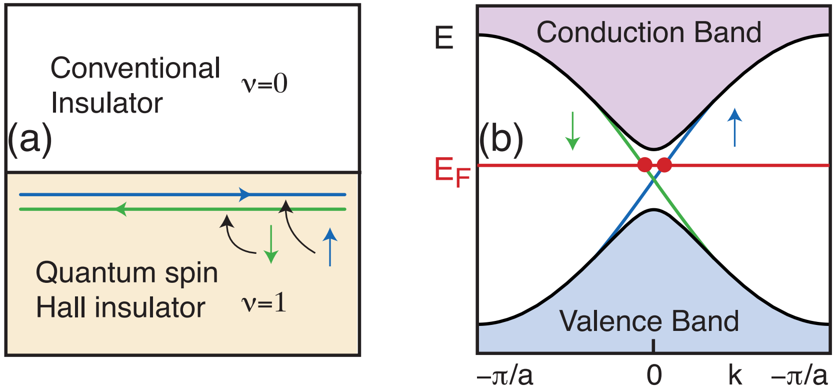


Figure from Hasan-Kane 10

- Phases associated with "symmetry-protected in-gap(p)-ability"

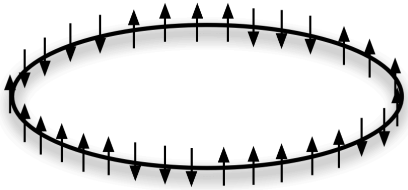
➢ Another example: purely 1d (lattice) spin systems w/ SO(3)× $\mathbb{Z}^{trans}$

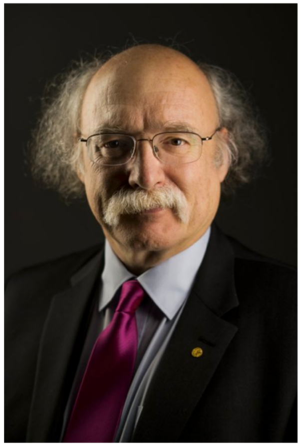
$$H_{\mathrm{HAF}}^{s=1} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapped

"Haldane's conjecture"

spin-1:





F. Duncan M. Haldane
(The Nobel Prize in Physics 2016)

- Phases associated with "symmetry-protected in-gap(p)-ability"

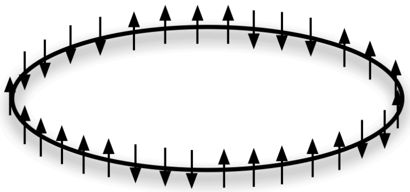
➢Another example: purely 1d (lattice) spin systems w/ SO(3)× $\mathbb{Z}^{trans}$

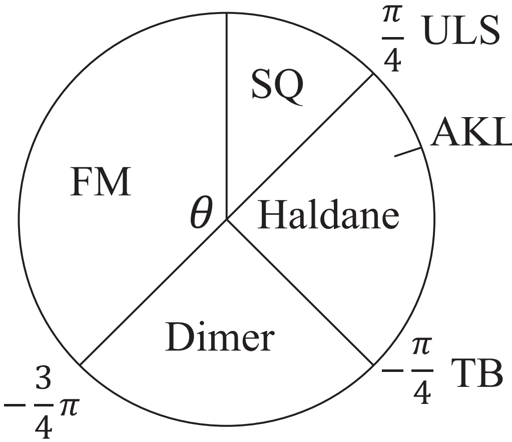
$$H^{s=1}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapped

$$H^{s=1}_{\mathrm{BB}} = \sum_i \left[\cos\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}) + \sin\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2\right]$$

spin-1: gappable





phase diagram (from Oh et al. 17)

- Phases associated with "symmetry-protected in-gap(p)-ability"

➢ Another example: purely 1d (lattice) spin systems w/ SO(3)× $\mathbb{Z}^{trans}$

$$H_{\mathrm{HAF}}^{s=1} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

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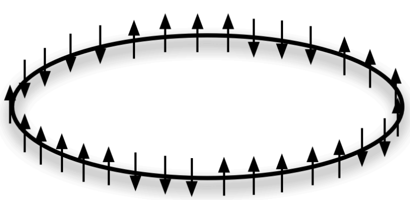
spin-1: gappable



$$H_{\mathrm{HAF}}^{s=1/2} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapless

$$H_{\mathrm{MG}}^{s=1/2} = \sum_i \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \frac{1}{2}\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+2} + \frac{3}{8}\right)$$

dimerized

spin-1/2: (seems) ingappable

- Phases associated with "symmetry-protected in-gap(p)-ability"

➢ Another example: purely 1d (lattice) spin systems w/ SO(3)$\times$ $\mathbb{Z}^{trans}$

$$H_{\mathrm{HAF}}^{s=1} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

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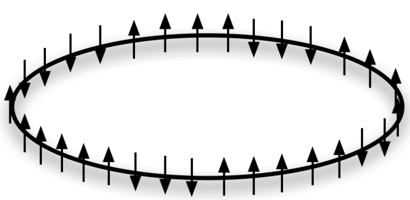
dimerized

spin-1/2: (seems) ingappable

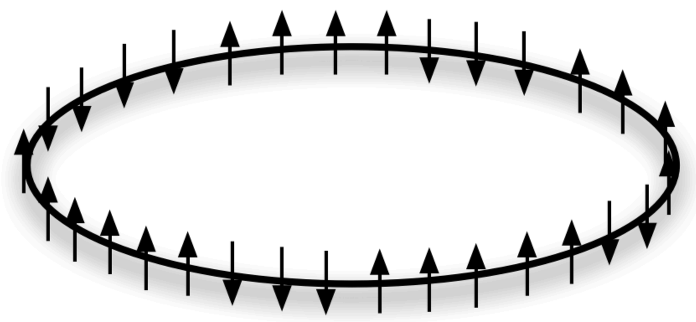


➢ Implied by the Lieb-Schultz-Mattis (LSM) theorem

- The *LSM theorem* for 1d SU(2) spin chains:

  [Lieb-Schultz-Mattis (61); Affleck-Lieb (86); etc]

A 1d SU(2) antiferromagnetic spin chain cannot have a unique gapped GS if the spin per site is half-integral and if the lattice-transl and spin-rotation symm are strictly imposed.



- **Q1**: Given any 1d lattice model w/ both **translation** and some **on-site symm** ($G^{site} \times \mathbb{Z}^{trans}$), e.g. Hubbard or Heisenberg models, could we determine, basing on the symm and microscopic d.o.f. of the model, whether the system is ingappable?

- **Q2**: If so, could we have further constraints on the (possible) low-energy phases of this model?

➢ In this talk, we will answer these questions from a field-theory aspect, focusing on the cases of **1d electron/spin systems**

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- The *LSM theorem* for 1d electron systems:

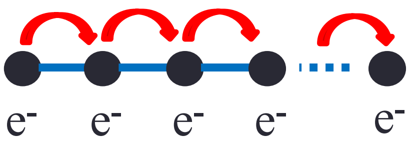
  [Oshikawa-Yamanaka-Affleck (96, 97); Oshikawa (00); etc]

A 1d electron-lattice system cannot be a trivial insulator if the filling per unit cell is fractional and if the lattice-transl symm and charge conservation are strictly imposed.

1d electron system with translation symmetry

- A simple model: tight-binding model of 1d spinless fermions

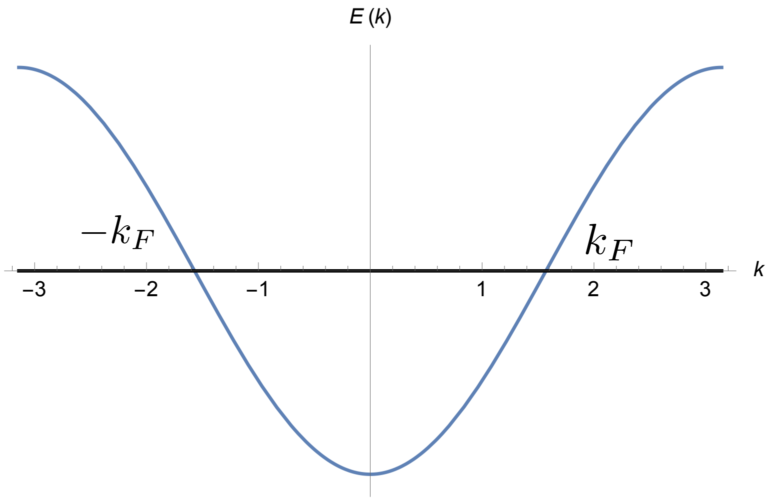
$$H = -t\sum_{x}^{L}(c_x^\dagger c_{x+1} + h.c.) - \mu \sum_{x}^{L} c_x^\dagger c_x$$



$$U(1)_Q: \quad c_x \to e^{i\phi} c_x,$$
$$\mathbb{Z}_{trans}: \quad c_x \to c_{x+1}.$$

$$|GS\rangle \propto \Big(\prod_{|k|\le k_F} c_k^\dagger\Big)|\mathrm{vac}\rangle$$

$$\text{filling } \nu = \frac{N_e}{L} = \frac{k_F}{\pi}$$



1d electron system with translation symmetry

- The continuum IR limit of the theory:

$$c_x \approx \psi_R(x) e^{ik_F x} + \psi_L(x) e^{-ik_F x}$$

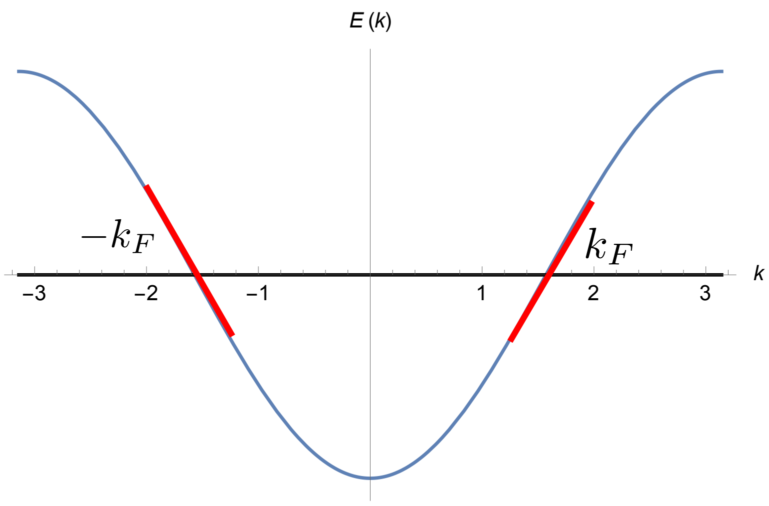
$$H = \int dx\ \Psi^\dagger(x)(-iv_F \partial_x)\sigma_z \Psi(x) \qquad \Psi(x) = (\psi_R(x), \psi_L(x))^T$$

Symmetry in the low-energy theory:

$$U(1)_Q: \quad \Psi(x) \to e^{i\phi}\Psi(x),$$

$$\mathbb{Z}_{trans}: \quad \Psi(x) \to e^{ik_F \sigma^z}\Psi(x)$$

Transl becomes (discrete) chiral symm!



- ➢ Taking ν = 1/2, we have $U(1)_Q \times Z_2$.
- ➢ For generic ν, we have $U(1)_Q \times Z$

Anomalies in the low-energy theory

- There is a "discrete" **chiral/axial anomaly** in the Dirac theory

$$H = \int dx\ \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$U(1): \Psi \to e^{i\phi}\Psi \qquad \text{vector}$$

$$\mathbb{Z}: \Psi \to e^{ik_F\sigma_z}\Psi = e^{i\pi\nu\sigma_z}\Psi \qquad \text{axial}$$

- That is, the part. func. in the presence of a $U(1)$ field is in general not invariant under the axial transf [Cho-Hsieh-Ryu 17]

$$Z(A_{U(1)}) = \int \mathcal{D}[\Psi^\dagger, \Psi]\ e^{iS[\Psi^\dagger,\Psi,A_{U(1)}]} \xrightarrow{\text{axial}} e^{i\nu \overbrace{\int F_A}^{2\pi\times\text{integer}}} Z(A_{U(1)})$$

=> discrete chiral anomaly is characterized by ν, the filling per unit cell!

Implication of the anomaly

- **IR**: If $\nu \neq$ integer, the low-energy theory is *anomalous*; it must be either gapless or, when perturbed by (symmetric) interactions, gapped with spontaneous symm breaking.

- **UV** (lattice): If the filling per unit cell is not integral, the system does not allow a unique gapped ground state; it must be in a gapless phase or a gapped phase breaking the transl symm.

    - This is nothing but the LSM theorem for an electron system!

Implication of the anomaly

- For example, a half-filled spinless ferm (ν = ½) is ingappable

e.g. $$-t\sum_x (c_x^\dagger c_{x+1} + h.c.) + V\sum_x n_x n_{x+1}$$

(V/t >>1, the system is gapped w/ **SSB** of transl)

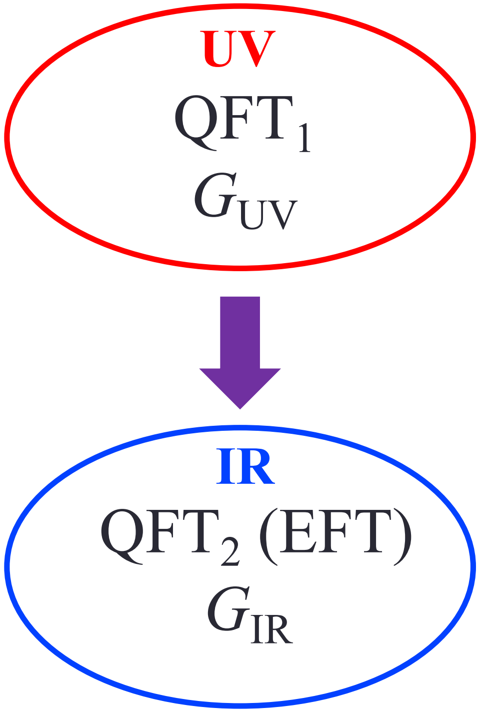
while two half-filled (spinful) ferm ($\nu_{\text{tot}} = 1$) is gappable

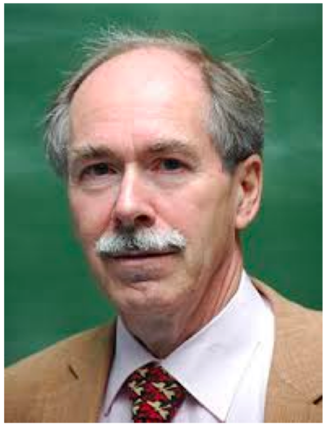
e.g. $$-t\sum_x (c_{\uparrow,x}^\dagger c_{\uparrow,x+1} + c_{\downarrow,x}^\dagger c_{\downarrow,x+1}) + U\sum_x c_{\uparrow,x}^\dagger c_{\downarrow,x} + h.c.$$

(U/t >>1, the system is **trivially** gapped)

Generality

- Our approach is based on the idea of ('t Hooft) anomaly matching ['t Hooft et al. 80], which enables us to obtain some fundamental constraints on the phase diagrams.





G. 't Hooft
(Nobel Prize in Physics 1999)

Generality

- Our approach is based on the idea of ('t Hooft) anomaly matching ['t Hooft et al. 80], which enables us to obtain some fundamental constraints on the phase diagrams.

It can be traced back to the *non-on-site* nature of (part of) the lattice symm

UV

H_{lattice}

$\underline{G^{\text{site}} \times \mathbb{Z}^{\text{trans}}}$

we identify a top. index, the *LSM index*, for any lattice system to characterize its phase

By "matching" the IR anomaly

IR

H_{EFT}

$\underline{G^{\text{vector}} \times \mathbb{Z}^{\text{axial}}}$

There is a potential *disc chiral anomaly* at IR

Such an anomaly can diagnose the *ingappability* of the system! [Hsieh et al. 14]

- The chiral anomaly – and thus the LSM index – is a topological quantity indep. of inter-particle interactions (at either UV or IR).
  - Because anomaly is "preserved" under RG ('t Hooft anomaly matching condition).
  - (1) Anomalous (IR) = nontrivial LSM index (UV) = ingappable
    (2) Anomaly-free (IR) = trivial LSM index (UV) = gappable

- At the lattice scale, the LSM index only depends on a quantity associated with G^{site} of the d.o.f. within a unit cell.

- Let's examine this approach with a more complicated example in the following discussion.

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1d spin chain with translation symmetry

- The *LSM theorem* for 1d *SU*(2) spin chains:
  [Lieb-Schultz-Mattis (61); Affleck-Lieb (86); etc]

A 1d SU(2) antiferromagnetic spin chain cannot have a unique gapped GS if the spin per site is half-integral and if the lattice transl symm and SO(3) symm are strictly imposed.

1d spin chain with translation symmetry

- A generalization to the case of *SU*(2*k*) spin chain, the *LSMA theorem*, was also known: [Affleck-Lieb (86)]

  A 1d SU(2k) antiferromagnetic spin chain cannot have a unique gapped GS if the ***Young tableau rep*** *per site has an odd number of boxes and if the lattice transl symm and PSU(2k) symm are strictly imposed.*

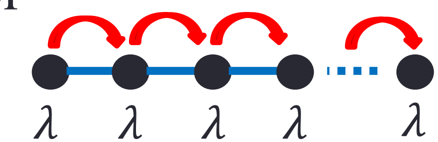
1d spin chain with translation symmetry

- How about for an $SU(N)$ chain with a generic Young-tableau (YT) rep λ per site and with both $PSU(N)$ and transl $\mathrm{Z}^{\mathrm{trans}}$ symm?

For N=2, $PSU(2)$ = SO(3)

1d spin chain with translation symmetry

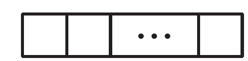
- How about for an $SU(N)$ chain with a generic Young-tableau (YT) rep λ per site and with both $PSU(N)$ and transl Z^{trans} symm?
  - ➢ A typical example is the (generalized) HAF model



$$\mathscr{H}_{\mathrm{HAF}} = \sum_{\langle i,j\rangle,\alpha,\beta} J S^{\alpha}_{i,\beta} S^{\beta}_{j,\alpha}, \quad J > 0$$

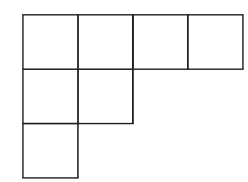
$$\left[S^{\alpha}_{i,\beta}, S^{\gamma}_{j,\delta}\right] = \delta_{i,j}\left(\delta^{\alpha}_{\delta} S^{\gamma}_{i,\beta} - \delta^{\gamma}_{\beta} S^{\alpha}_{i,\delta}\right)$$

$N = 2 : \lambda = \{2s\}$



$N > 2 : \lambda = \{4, 2, 1\}$

(for example)



- We will answer this question, again, by the anomaly matching argument

LSM indices of 1d *SU*(*N*) spin systems with translational symmetry

- Let's identify the LSM index for a 1d $SU(N)$ lattice model from the disc chiral anomaly at low-energy

  [Yao-Hsieh-Oshikawa 18]

$$\boxed{\text{LSM index } \mathcal{I}_N = (\#\text{ of YT boxes per unit cell}) \mod N}$$

UV

$\mathrm{H}_{\mathrm{lattice}}$

$PSU(N)^{\mathrm{site}} \times \mathbb{Z}^{\mathrm{trans}}$

IR

$\mathrm{H}_{\mathrm{EFT}}$

$PSU(N)^{\mathrm{vector}} \times \mathbb{Z}^{\mathrm{axial}}$

The chiral anomaly is represented by a mod N integer $\in \mathbb{Z}_N$

Math fact (cohomology theory):

$$H^3(PSU(N) \times \mathbb{Z}, U(1))/H^3(PSU(N), U(1)) \cong \mathbb{Z}_N$$

Check with simple examples:

- $N = 2$: For a system w/ spin s per unit cell we have

$$\mathcal{I}_2 = 2s \quad \mathrm{mod}\ 2$$

$$\underbrace{\boxed{}\boxed{}\boxed{\cdots}\boxed{}}_{2s \text{ YT boxes}}$$

$s \in \mathbb{Z} + 1/2$: nontrivial $\qquad$ $s \in \mathbb{Z}$: trivial

- $N =$ **even**: For a system w/ $SU(N)$ spin λ w/ an odd # of boxes

$$\mathcal{I}_N \neq 0 \quad \mathrm{mod}\ N$$

➢ Agree with the LSMA theorem!

- **Q1**: Given any 1d lattice model w/ both **translation** and some **on-site symm** ($G^{site} \times \mathbb{Z}^{trans}$), e.g. Hubbard or Heisenberg models, could we determine, basing on the symm and microscopic d.o.f. of the model, whether the system is ingappable?

- **Q2**: If so, could we have further constraints on the (possible) low-energy phases of this model?

- **Q1**: Given any 1d lattice model w/ both **translation** and some **on-site symm** ($G^{site} \times \mathbb{Z}^{trans}$), e.g. Hubbard or Heisenberg models, could we determine, basing on the symm and microscopic d.o.f. of the model, whether the system is ingappable?

- **Q2**: If so, could we have further constraints on the (possible) low-energy phases of this model?

Constraints on the low-energy phases

- The value of the LSM index I_N can be used to further **constrain** the *GSD* or the *possible universality class* when the system is in a gapped or a critical phase, respectively:

➢GSD of a gapped phase: $\mathrm{GSD} \in \frac{N}{\gcd(\mathcal{I}_N, N)}\mathbb{N}$ (1)

➢$SU(N)_k$ WZW CFT with transl symm $g \to e^{2\pi i m/N} g$:

$$\mathcal{I}_N = km \mod N \quad (2)$$

=> For $N = 2$, (2) agrees with Furuya-Oshikawa (PRL, 17)

Anomalies in *SU*(*N*) WZW theories

- The most natural univ classes of a critical $SU(N)$ spin model is the $SU(N)$ WZW theories

$$kI(g) = \frac{k}{8\pi}\int_{M_2} dtdx\,\mathrm{Tr}\left(\partial_\mu g^{-1}\partial^\mu g\right) + k\Gamma_{\mathrm{WZ}}$$

level

symm:

vector $PSU(N): g \to wgw^{-1},\ w \in SU(N)$

axial $\mathbb{Z}_n(\mathrm{trans}): g \to e^{2\pi i m/N} g,\ m \in \{0, 1, ..., N-1\}$

$n = N/\gcd(m, N)$

- Mixed anomaly of the $PSU(N) \times \mathbb{Z}_n$ symm [Yao-Hsieh-Oshikawa 18]:

$$Z(A_{PSU(N)}) \xrightarrow{\text{axial}} e^{2\pi i \frac{km}{N} \times \mathrm{integer}} Z(A_{PSU(N)})$$

characterized by km/N, or $km \bmod N$

- Our prediction agrees with known examples in previous studies of $SU(N)$ models.

	Model	YT	$\mathcal{I}_N$	GSD	IR CFT; m	Mixed anomaly
Greiter et al. 07	$SU(3)$ trimer model [43]	$\ydiagram{1}$	$1 \bmod 3$	$3 \in 3\mathbb{N}$	-	-
Greiter et al. 07	$SU(3)$ **10**-VBS model [43]	$\ydiagram{3}$	$0 \bmod 3$	$1 \in 1\mathbb{N}$	-	-
Greiter-Rachel 07	$SU(6)$ **70**-VBS model [44]	$\ydiagram{2,1}$	$3 \bmod 6$	$2 \in 2\mathbb{N}$	-	-
Takhtajan; Babujian 82	S-3/2 TB model[45, 46]	$\ydiagram{3}$	$1 \bmod 2$	-	$SU(2)_3$ WZW; 1	$1 \bmod 2$
Andrei-Johannesson 84 Johanness 86	$\mathscr{H}^{[3,2]}$ AJ model[47, 48]	$\ydiagram{2}$	$2 \bmod 3$	-	$SU(3)_2$ WZW; 1	$2 \bmod 3$
Rachel et al. 09	$SU(3)$ 1×2-YT HAF[49, 50]	$\ydiagram{2}$	$2 \bmod 3$	-	$SU(3)_1$ WZW; 2	$2 \bmod 3$
Dufour et al. 15	$SU(9)$ 2×1-YT HAF[51]	$\ydiagram{1,1}$	$2 \bmod 9$	-	$SU(9)_1$ WZW; 2	$2 \bmod 9$
Lecheminant 15	$SU(3)$ 2-leg ladder [52]	$\ydiagram{1} \otimes \ydiagram{1}$	$2 \bmod 3$	-	$SU(3)_1$ WZW; 2	$2 \bmod 3$

Y. Yao, C.-T. Hsieh, and M. Oshikawa, arXiv:1805.06885

In summary, if a spin model with an exact $SU(N)$ spin-rotation and transl symm has a **nontrivial LSM index**, i.e., the total umber of Young-tableau boxes per unit cell is not divisible by N, the system must have either

- degenerate gapped ground states, with the multiplicity (1), or
- gapless excitations / symm-protected critical states (SPC). If the low-energy SPC is given by an $SU(N)$ WZW theory, its level is constrained by (2).

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

Conclusion

- We apply the idea of ('t Hooft) anomaly matching to study 1d condensed matter systems – many-body systems in general – in the presence of both lattice transl and some on-site symm.

It can be traced back to the *non-on-site* nature of (part of) the lattice symm

UV

H_{lattice}

$\underline{G^{\text{site}} \times \mathbb{Z}^{\text{trans}}}$

we identify a top. index, the *LSM index*, for any lattice system to characterize its phase

By "matching" the IR anomaly

IR

H_{EFT}

$\underline{G^{\text{vector}} \times \mathbb{Z}^{\text{axial}}}$

There is a potential *disc chiral anomaly* at IR

Such an anomaly can diagnose the *ingappability* of the system! [Hsieh et al. 14]

Thank You!