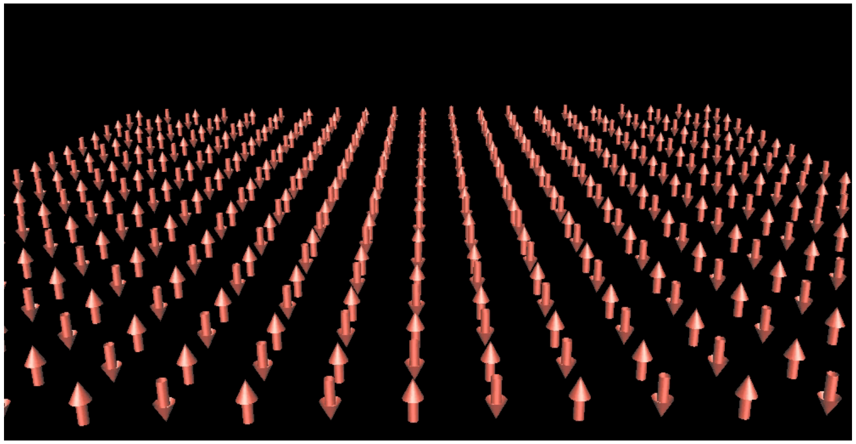
Deconfined Quantum Critical Points

with T. Senthil, Bangalore
A. Vishwanath, UCB
S. Sachdev, Yale
L. Balents, UCSB

Outline:

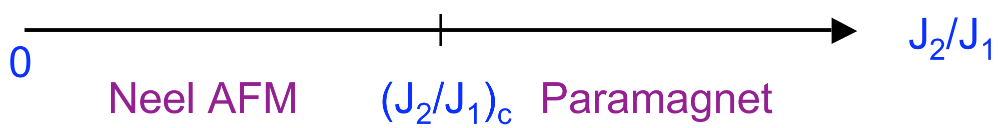
- conventional quantum critical points
- Landau paradigm
- Seeking a new paradigm - AF/VBS criticality
- "Deconfined" quantum criticality

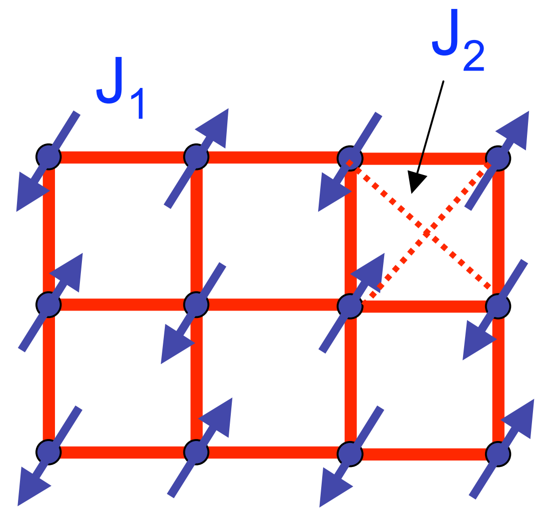


Quantum Phase Transitions

A T=0 phase transition between two distinct ground states as a function of a parameter in the Hamiltonian (eg. pressure, magnetic field…)

Example: Square lattice s=1/2 Antiferromagnet with nn exchange J_1 and nnn J_2





Landau-Ginzburg-Wilson approach

Identify "order parameter" $\mathbf{n_i} = (-1)^{\mathbf{x_i}+\mathbf{y_i}}\mathbf{S_i}$ (Neel vector non-zero in AFM)

Define coarse-grained field: $\mathbf{n_i} \rightarrow \mathbf{N}(\mathbf{r}, \tau)$ (space and imaginary time)

Construct free energy or Lagrangian as an expansion in powers of o.p. and gradients

$$\mathcal{L} = |\partial_\mu \mathbf{N}|^2 + \mathbf{r}|\mathbf{N}|^2 + \mathbf{u}|\mathbf{N}|^4 + \ldots$$

(space/time gradients)

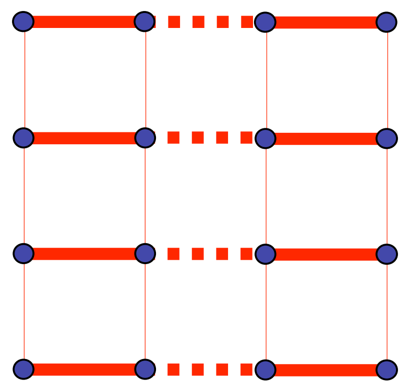
Problem with the Paramagnet!

For r<0 energy is minimized with $\langle \mathbf{N} \rangle \neq \mathbf{0}$ the Neel AFM

For r>0 LGW theory gives a featureless quantum paramagnet $\langle \mathbf{N} \rangle = \mathbf{0}$

But with s=1/2 per unit cell the simple paramagnets (eg. VBS) break symmetries - the only featureless QPM's have topological order

Valence Bond Solid phase cannot be described with LGW built from N-vector



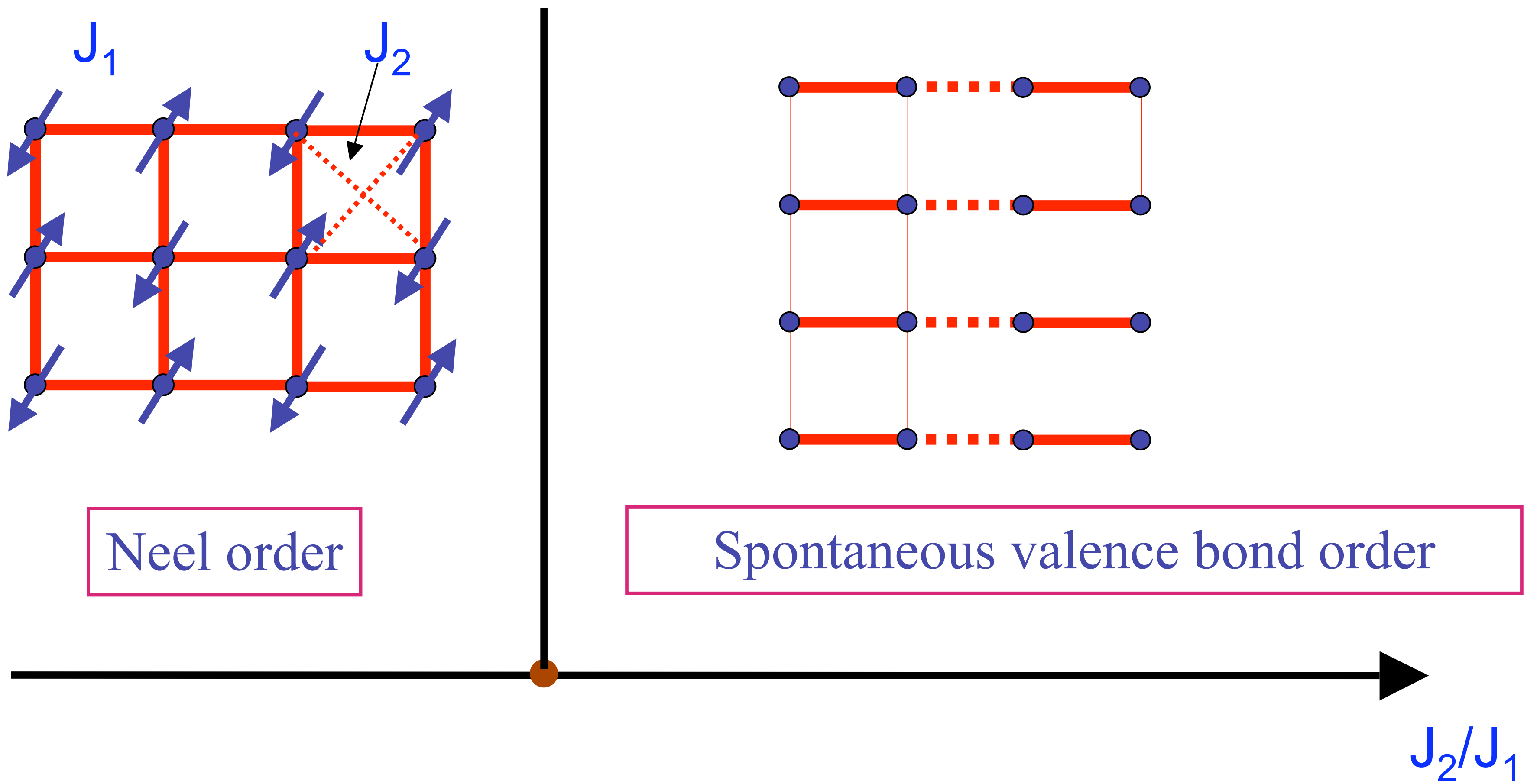
Soln: VBS order parameter

$$\psi_i = (-1)^{x_i} \mathbf{S_i} \cdot \mathbf{S_{i+x}} + \mathbf{i}(-\mathbf{1})^{\mathbf{y_i}} \mathbf{S_i} \cdot \mathbf{S_{i+y}} \rightarrow \psi(\mathbf{r}, \tau)$$

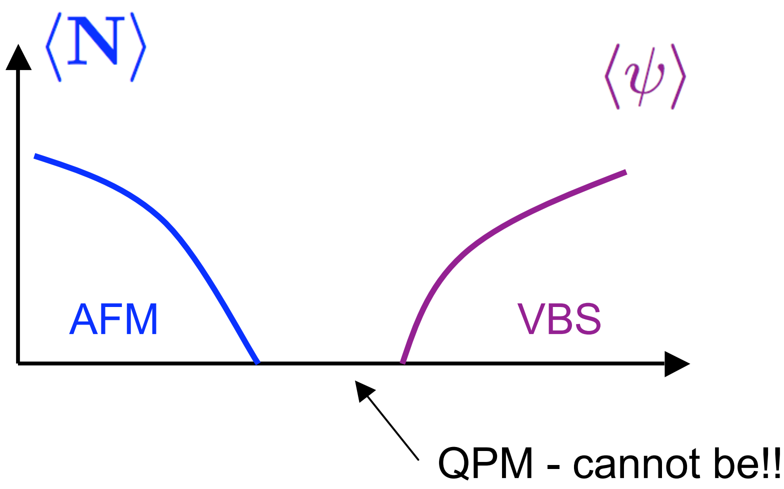
Construct LGW Lagrangian for both $\mathbf{N}(\mathbf{x}_\mu), \psi(\mathbf{x}_\mu)$

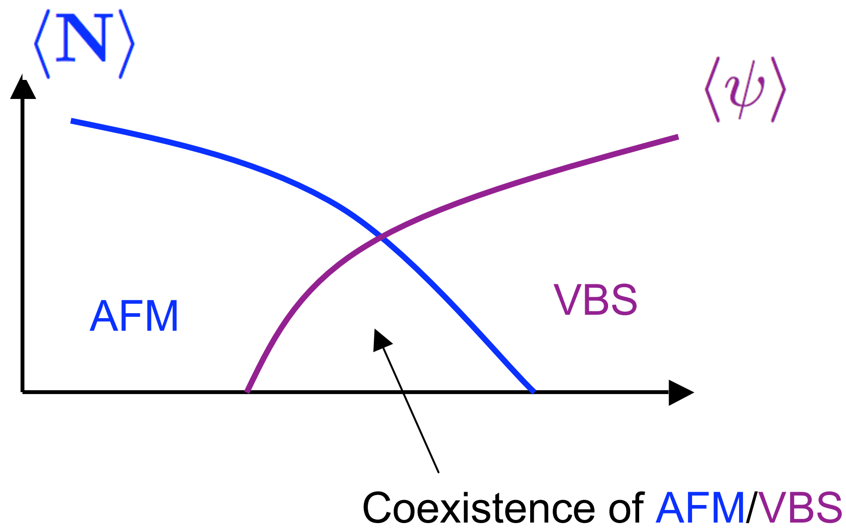
$$\mathcal{L}(\mathbf{N}, \psi) = \mathcal{L}_{\mathbf{N}}(\mathbf{N}) + |\partial_\mu \psi|^{\mathbf{2}} + \mathbf{r}_\psi |\psi|^{\mathbf{2}} + \mathbf{u}_\psi |\psi|^{\mathbf{4}} + \ldots$$

Can LGW describe a direct and continuous Neel-VBS Quantum Phase Transition?



Answer: No!! (not without fine tuning)



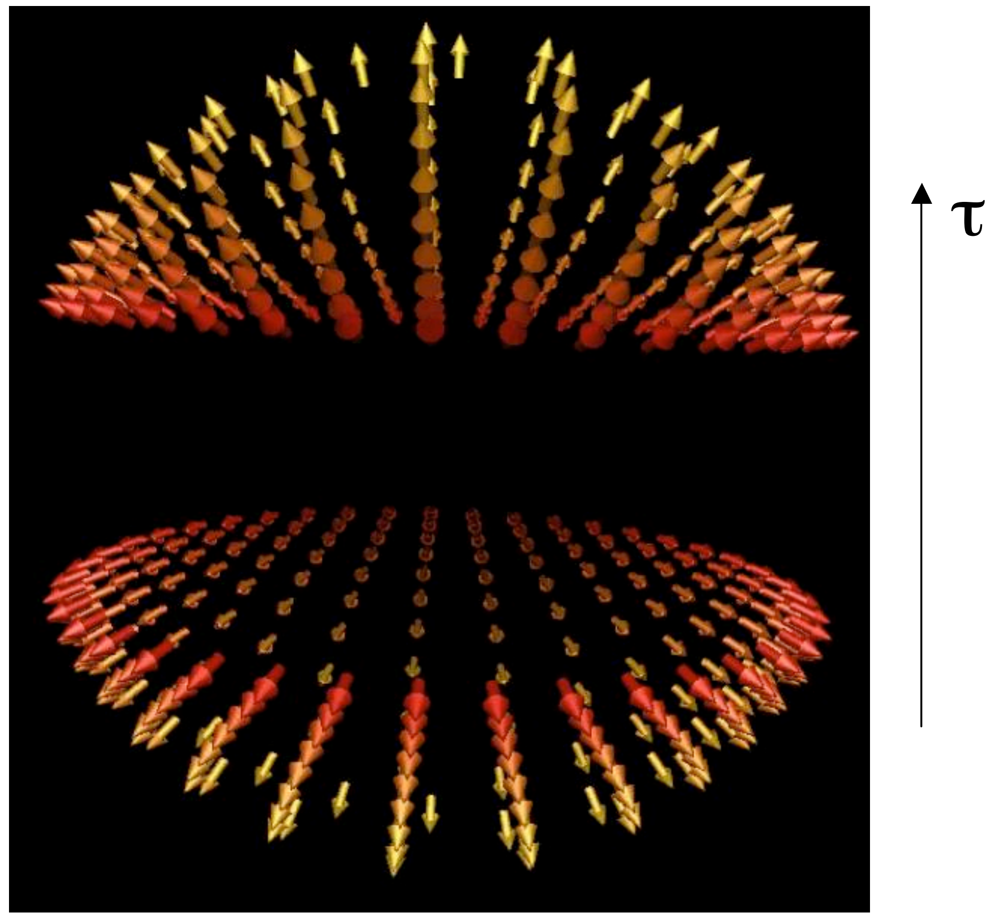


New Question: Is it POSSIBLE to have a generic direct and continuous quantum phase transition between Neel/VBS?

New Answer: Yes. Subtle quantum effects invalidate LGW, but beyond LGW a novel “Deconfined” Quantum critical transition is possible!

Hedgehogs

- quills and spins…



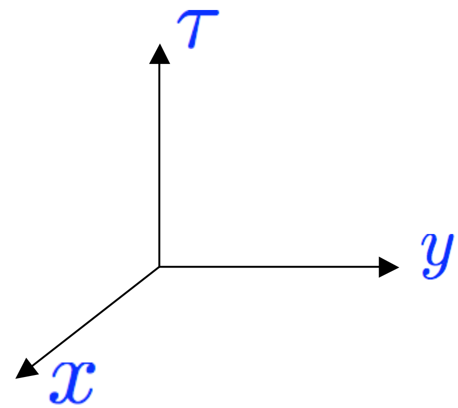
Hedgehogs in the O(3) non-linear Sigma model

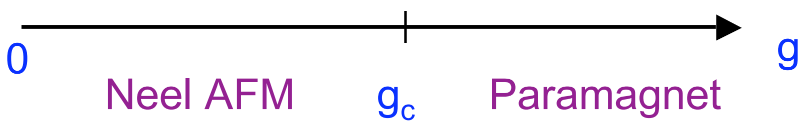
Define a unit length Neel vector: $\vec{n}(x_\mu) = \mathbf{N}(\mathbf{x}_\mu)/|\mathbf{N}(\mathbf{x}_\mu)|$ with $|\vec{n}(x_\mu)|^2 = 1$

$$x_\mu = (x, y, \tau)$$

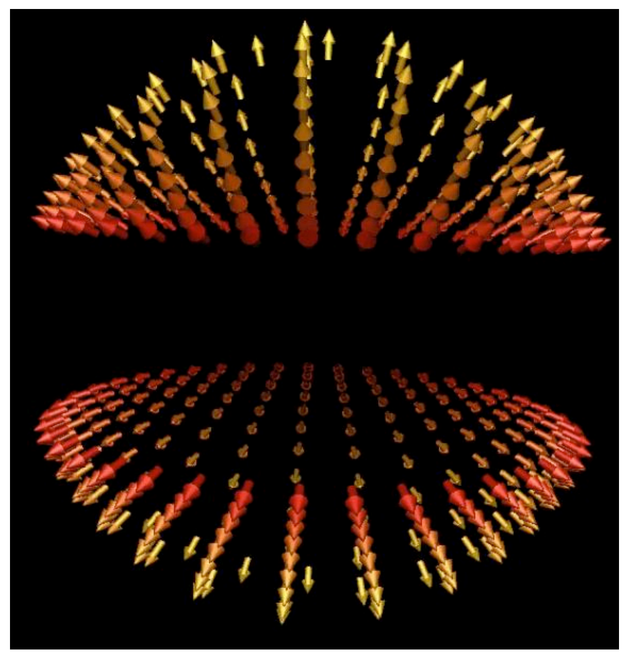
Consider space-time configurations of $\vec{n}(x_\mu)$
In the Neel state these will be slowly varying, described by a "non-linear sigma model" Lagrangian:

$$\mathcal{L}_\sigma = \frac{1}{2g}|\partial_\mu \vec{n}|^2$$





Hedgehog: Singular configuration of $\vec{n}(x_\mu)$ at one space-time point (smooth elsewhere)



In the Neel state: Hedgehogs are energetically costly, so absent.

In the Paramagnet Hedgehogs proliferate

Question: Hedgehogs at the QPT??

Fugacity Expansion

Consider the partition function of the non-linear Sigma model:

$$Z = \int \mathcal{D}\vec{n}\, \exp[-S_\sigma] \qquad\qquad S_\sigma = \int d^3x \mathcal{L}_\sigma$$

Idea: expand partition function in number of hedgehog events:

$$Z = Z_0 + \int_{r_1} \lambda(r_1) Z_1[r_1] + \frac{1}{2}\int_{r_1,r_2} \lambda(r_1)\lambda(r_2) Z_2[r_1, r_2] + \cdots$$

Z_0 describes “hedgehog-free O(3) model”

Due to Berry’s phase effects coming from the s=1/2 nature of the lattice model, the single hedgehog contribution $Z_1[r]$ is rapidly oscillating in r and can be dropped

First non-oscillatory contribution is from quadrupled hedgehogs, $\lambda_4(r) \approx [\lambda(r)]^4$

Numerous compelling arguments suggest λ_4 is *irrelevant* at QCP (quadrupling is crucial!)

Implement formally: U(1) gauge theory

CP[1] Representation of Neel vector in terms of "spinon" fields: $z_\alpha; \quad \alpha = \uparrow, \downarrow$

$$\vec{n} = z_\alpha^\dagger \vec{\sigma}_{\alpha\beta} z_\beta$$

with $z_\alpha^\dagger z_\alpha = 1$ required so that $|\vec{n}|^2 = 1$

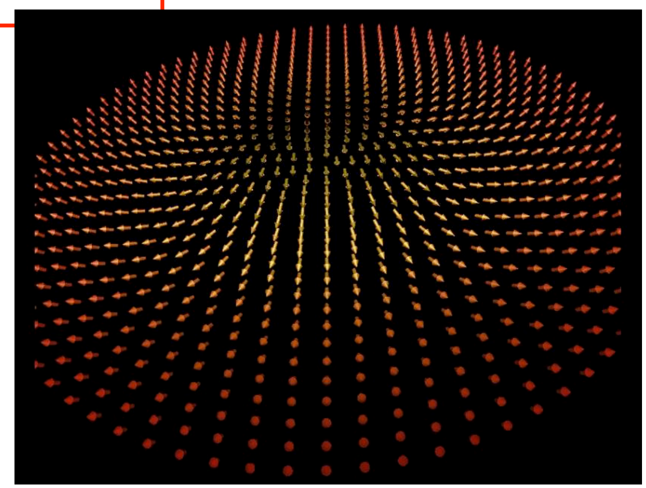
Now, we can rewrite non-linear sigma model, $\mathcal{L}_\sigma = \frac{1}{2g}|\partial_\mu \vec{n}|^2$ in terms of z's:

$$\mathcal{L}_\sigma = \frac{1}{g}|(\partial_\mu - A_\mu) z_\alpha|^2 \quad \text{with} \quad A_\mu = Im[z^\dagger \partial_\mu z]$$

Where are the hedgehogs?

Consider Topological Skyrmion excitations - integer "charge":

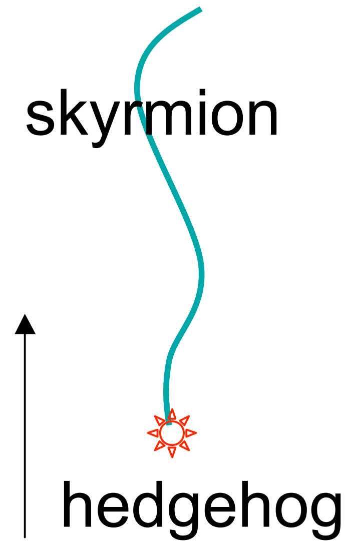
$$Q = \frac{1}{4\pi}\int d^2r \; \vec{n} \cdot \partial_x \vec{n} \times \partial_y \vec{n} \qquad \mathcal{R}^2 \rightarrow S^2$$



Remarkably U(1) gauge flux is bound to each Skyrmion:

$$Q = +1$$

Hedgehogs=Skyrmion Creation Events



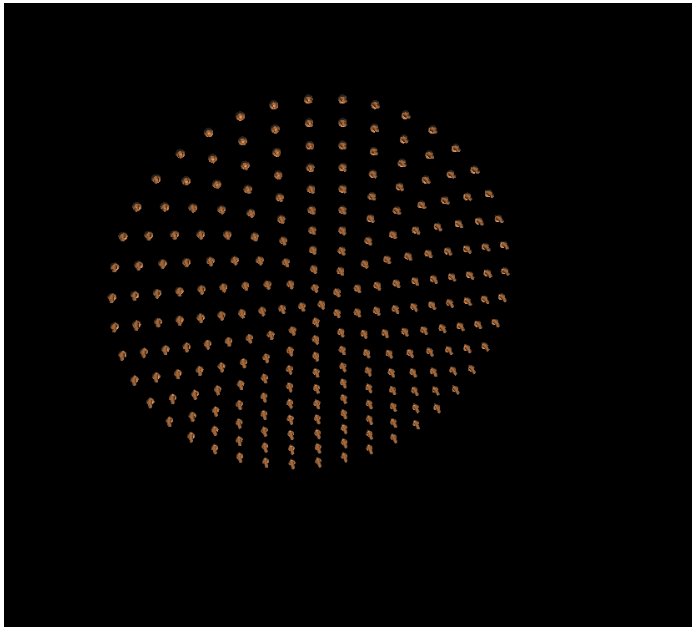
Define Skyrmion 3-current:

$$J_\mu = \frac{1}{8\pi}\epsilon_{\mu\nu\lambda}\vec{n}\cdot\partial_\nu\vec{n}\times\partial_\lambda\vec{n}$$

In CP^1 Representation this is the gauge flux (tube):

$$J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$$

Thus: Hedgehog is a Monopole
in the Gauge Flux!



Topological O(3) Transition

Studied previously in classical O(3) model with hedgehogs forbidden by hand (Kamal+Murthy. Motrunich+Vishwanath)

- Critical point has modified exponents

$$\langle \mathbf{N}(\mathbf{r}) \cdot \mathbf{N}(\mathbf{0}) \rangle \sim \frac{\mathbf{1}}{\mathbf{r}^{\mathbf{1}+\eta}} \qquad \eta_{O(3)} \approx 0.03; \quad \eta_{TO(3)} \approx 0.7$$

very broad spectral functions

Same critical behavior as monopole-free CP^1 model

$$\mathcal{L} = |(\partial_\mu - iA_\mu)z_\alpha|^2 + r|z|^2 + u[|z|^2]^2 + (\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$

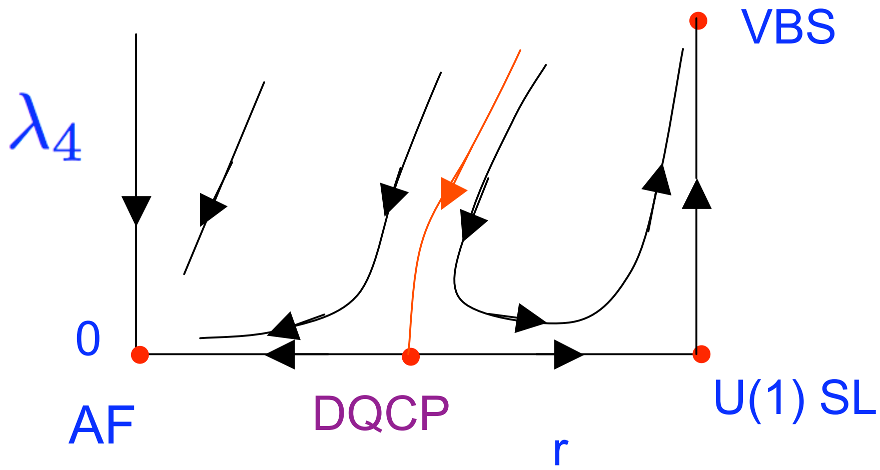
r<0 : Antiferromagnet $\langle z_\alpha \rangle \neq 0;$ ➡ $\langle \vec{n} \rangle = \langle z_\alpha^\dagger \rangle \vec{\sigma}_{\alpha\beta} \langle z_\beta \rangle \neq 0$

r>0 : Paramagnet - U(1) Spin liquid $\langle z_\alpha \rangle = 0; \quad \langle \vec{n} \rangle = 0$ (No hedgehogs)

r=0 : Novel Topological O(3) Transition "Deconfined Quantum Criticality" (DQCP)

Renormalization Group (RG) Picture:

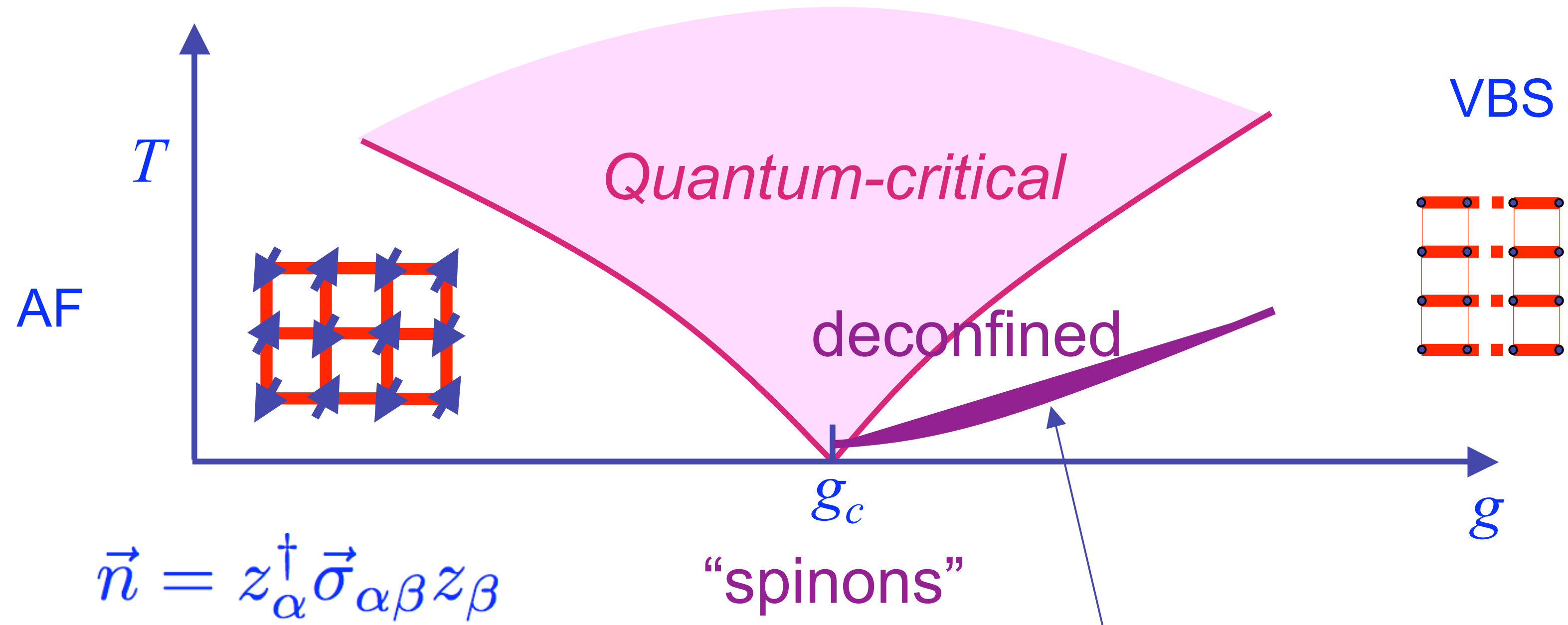
Quadrupled
monopole/hedgehog
fugacity



RG "Flow Diagram"

λ_4 is "dangerously irrelevant", ie. irrelevant at the critical point, but relevant in the ordered phase

Deconfined QCP: Direct AF-VBS Quantum Phase Transition

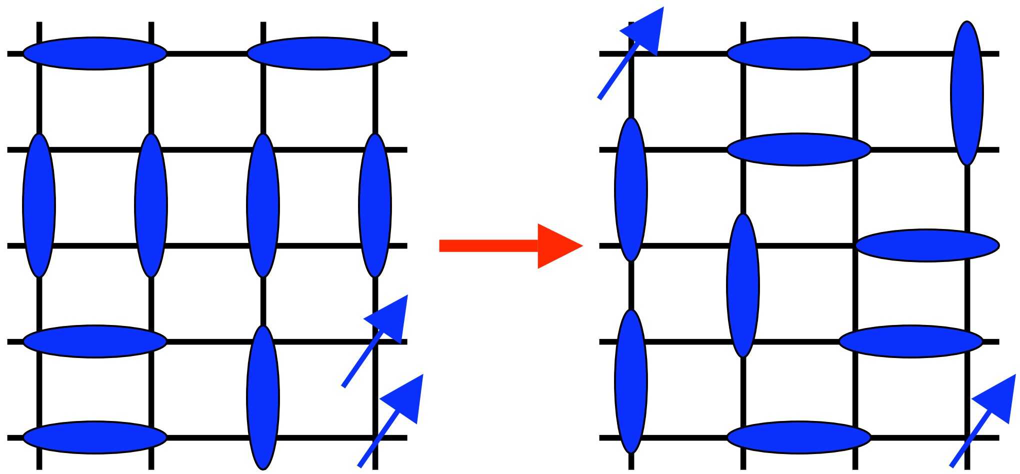


$$\vec{n} = z^{\dagger}_{\alpha} \vec{\sigma}_{\alpha\beta} z_{\beta}$$

small confinement scale since 4-monopole fugacity is "dangerously irrelevant "

Right at the critical point, the spinons are almost free/deconfined

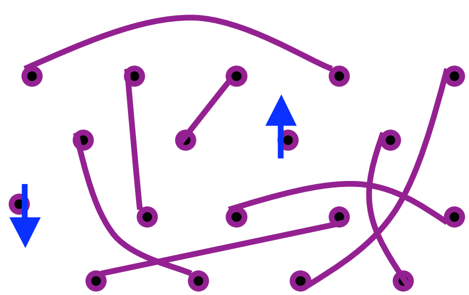
Compare to RVB Phase:



RVB Z_2 Spin liquid: Energy cost stays finite when spinons are separated.

Spinons are truly deconfined

Deconfined Quantum Critical point:



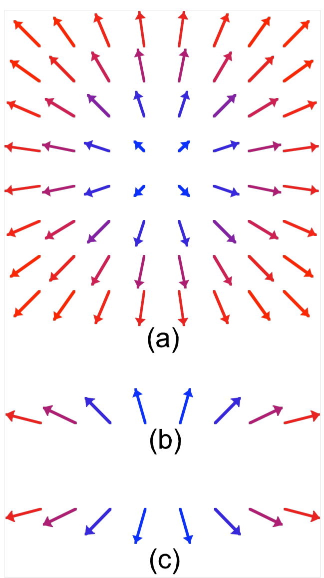
- Long-range valence bonds
- Gapless "spinons" interacting via U(1) gauge field

At AF-VBS criticality, the "spinons" are quasi-free

Easy-Plane Anisotropy

- Add term $\Delta\mathcal{L} = v n_z^2; \qquad n^\dagger \equiv n_x + i n_y \sim e^{i\phi}$

- Effect on Neel state
  - -Ordered moment lies in X-Y plane
  - -Skyrmions break up into *merons*

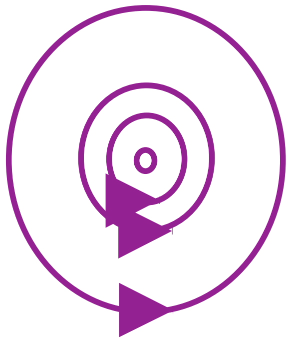


$$\int \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$

two “flavors” of vortices with “up” or “down” cores

$n^\dagger = z_1^* z_2$ vortex/antivortex in z_1/z_2

Focus on *vortex* excitations in easy-plane AF



vortex $\sim \Phi^\dagger$

anti-vortex $\sim \Phi$

Vortex creation operator

Antivortex creation operator

- Time-reversal exchanges vortices+antivortices
  - Expect *relativistic field theory for* Φ

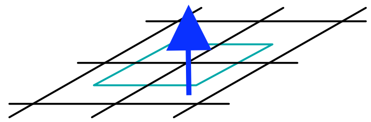
- Worry: vortex is a non-local object

Duality

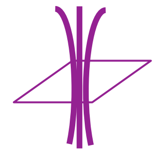
- Exact mapping from boson to vortex variables.

- Dual magnetic field $b = 2\pi n$

$$s_z + \frac{1}{2} = \frac{1}{2\pi}\vec{\nabla} \times \vec{a}$$



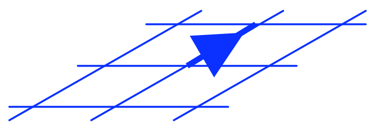
$s_z = 1/2$



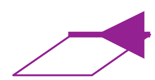
$$\int d^2x \;\; b = 2\pi$$

Dual "magnetic" field

$$\vec{\nabla}\phi = 2\pi \hat{z} \times \vec{e}$$



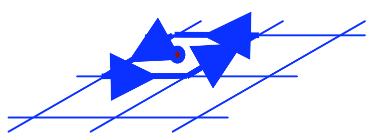
$\vec{\nabla}\phi$



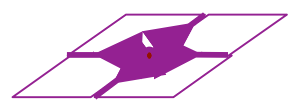
$\vec{e}$

Dual "electric" field

$$\vec{\nabla} \cdot \vec{e} = N$$



$$\int \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$



$N = 1$

Vortex number

Vortex carries dual gauge charge

- All non-locality is accounted for by dual U(1) gauge force

Vortices experience average dual magnetic field

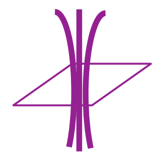
With:

$$s_z = +1/2 \Rightarrow b = 2\pi$$

$$s_z = -1/2 \Rightarrow b = 0$$

Thus, on average have non-zero dual "magnetic" flux:

$$\langle s_z \rangle = 0 \quad \rightarrow \langle b \rangle = \pi$$



$$\int d^2x \ \vec{\nabla} \times \vec{a}^0 = \pi$$

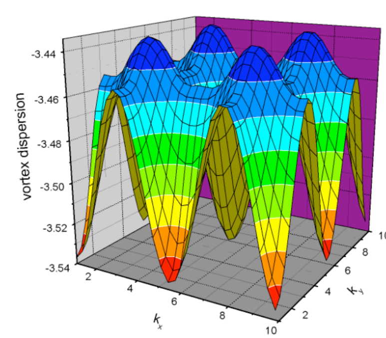
Vortex hopping Hamiltonian: pi flux per plaquette

$$H = -t_v \sum_{\langle ij \rangle} [\Phi_i^\dagger \Phi_j e^{i(a_{ij}+a^0_{ij})} + h.c.] + \sum [|\vec{e}|^2 + b^2]$$

Constraint:

$$(\nabla \cdot \vec{e} = \Phi^\dagger \Phi)$$

Vortex band structure: Two-fold degenerate minima



Define two vortex flavors: Φ_1, Φ_2 (merons)

Dual Vortex (meron) field theory

$$\mathcal{L}_v = |(\partial_\mu - ia_\mu)\Phi_\alpha|^2 + r_v|\Phi_\alpha|^2 + u|\Phi|^4 + w|\Phi_1|^2|\Phi_2|^2 + (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

$$\mathcal{L}_{\lambda_4} = -\lambda_4 Re[(\Phi_1^*\Phi_2)^4]$$

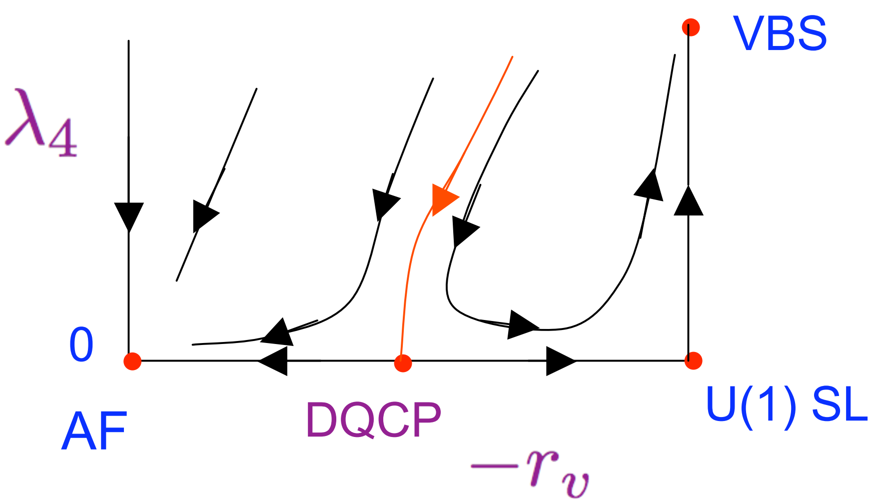
$$(|\Phi|^2 = |\Phi_1|^2 + |\Phi_2|^2)$$

AF Easy-plane: $\langle\Phi_\alpha\rangle = 0$

Vortex condensate

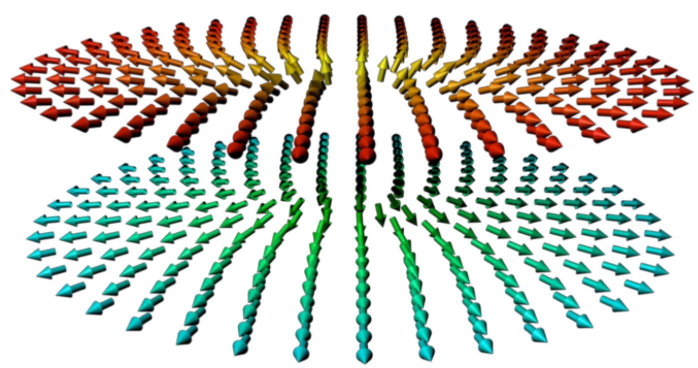
PM U(1) SL: $\langle\Phi_\alpha\rangle \neq 0; \quad \lambda_4 = 0$

VBS: $\langle\Phi_\alpha\rangle \neq 0; \quad \lambda_4 \neq 0$

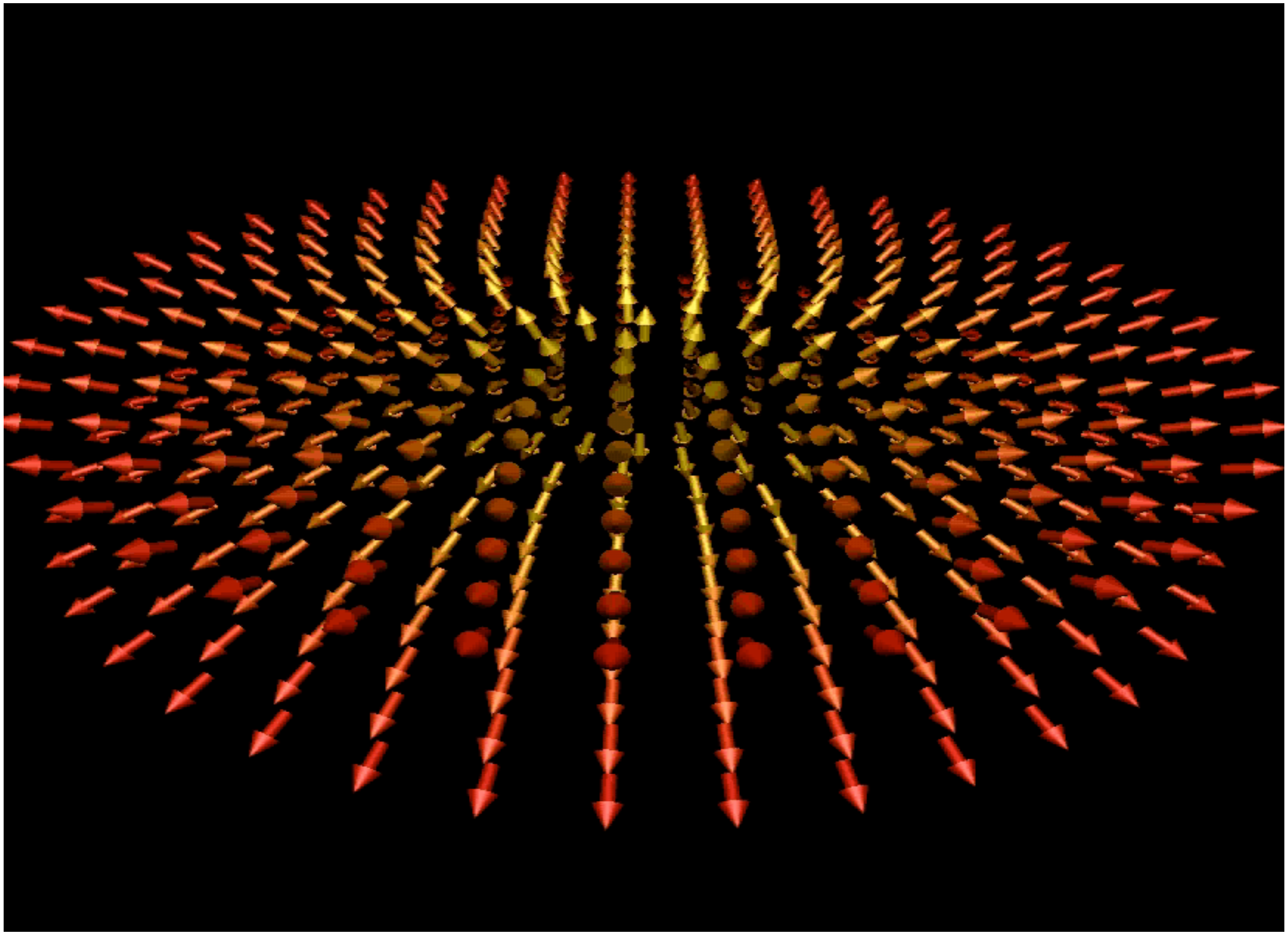


Skyrmion creation operator
(ie. hedgehog insertion):

$$\sim \Phi_1^*\Phi_2$$



Up-Down Meron Tunneling= Hedgehog



Summary & Conclusions

- In contrast to classical (finite temperature) phase transitions, quantum phase transitions can violate the Landau-Ginzburg-Wilson "order parameter" paradigm

- The violation is due to subtle Berry's phase effects reflecting the discreteness inherent in the quantum spins (ie. electron) degrees of freedom

- A direct AF/VBS "deconfined" quantum phase transition is possible, and "spinons", although absent in either phase, are "free" right at the critical point

- Much future work:
  - Numerical confirmation via Monte Carlo??
  - Other deconfined Quantum critical points?
  - DQCP with itinerant electrons?
  - Experimental candidates? (perhaps "heavy fermion" materials)