

Domain wall Quasiparticles  $\leftrightarrow$  Flipped spin Quasiparticles.

$$H = -t \sum_{\langle j, k \rangle} (c_j^\dagger c_k + c_k^\dagger c_j) + V \sum_{\langle j, k \rangle} n_j n_k$$

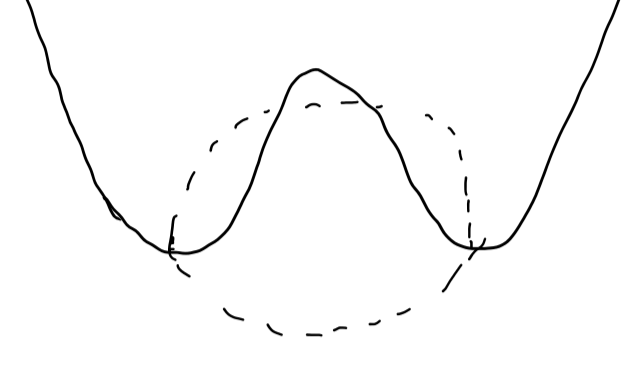
$$U(1) \text{ symmetry: } \begin{cases} c_j \rightarrow e^{i\theta} c_j \\ c_j^\dagger \rightarrow e^{-i\theta} c_j^\dagger \end{cases}$$

Nambu Goldstone theorem  $\rightarrow$  gapless excitation

There are many <sup>gapless</sup> systems without SSB.

However, any other mechanism for gaplessness?

Yes, if there is a lattice with translational invariance.

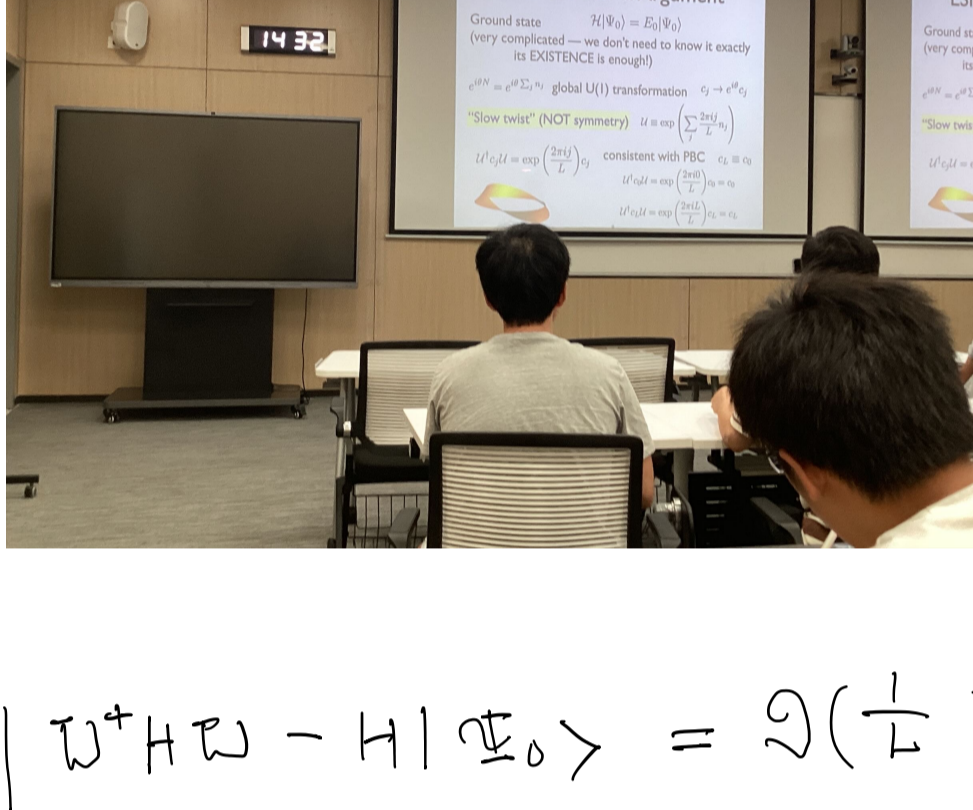


What is the difference between two system. One with gapped excitation spectrum, the other with gapless excitation spectrum.!

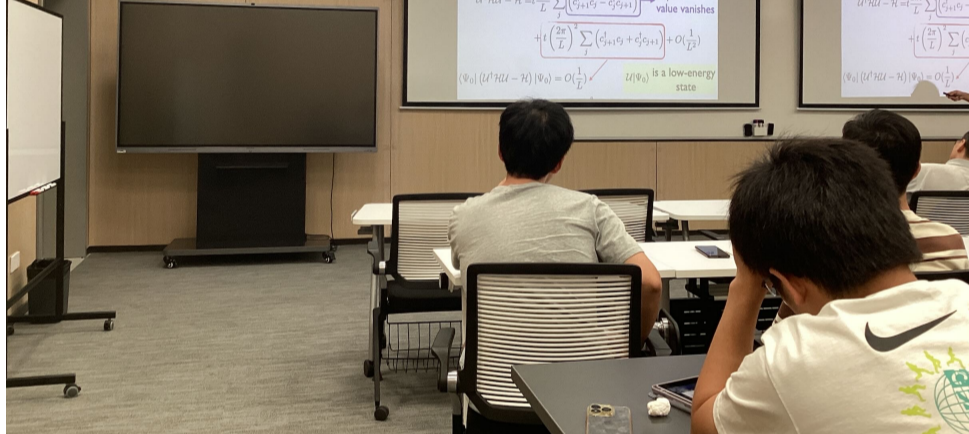
LSM: U(1) + Lattice translational symmetry + spatial inversion or time reversal

$$U(1) \text{ Generator: } e^{i\theta N} = e^{i\theta \sum_j c_j^\dagger c_j}$$

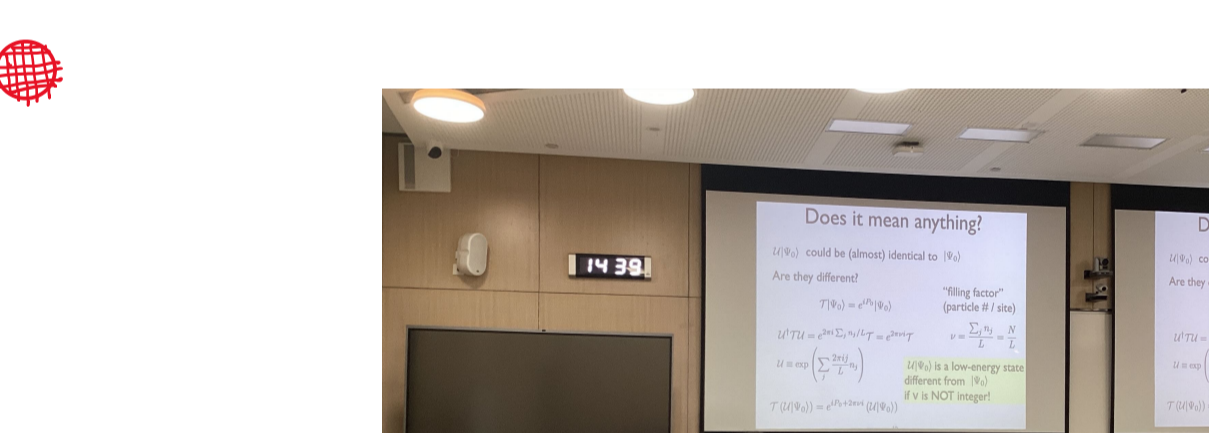
$$U^\dagger c_j U \rightarrow e^{i\theta_j} c_j$$



$$\langle \Phi_0 | U^\dagger H U - H | \Phi_0 \rangle = \mathcal{O}(1/L)$$



$$U^\dagger H U - H = t \frac{2\pi i}{L} \sum_j (c_{j+1}^\dagger c_j - c_j^\dagger c_{j+1}) + t \left(\frac{2\pi}{L}\right)^2 \sum_j (c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1}) + \mathcal{O}(1/L^3)$$



$U | \Phi_0 \rangle$  could be identical to  $| \Phi_0 \rangle$

$$T | \Phi_0 \rangle = e^{i p_0} | \Phi \rangle$$

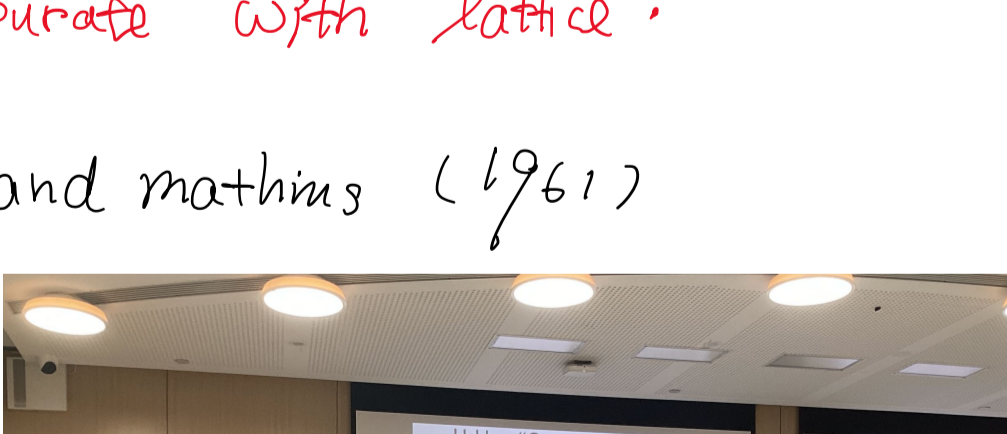
$$T U | \Phi_0 \rangle = T U T^\dagger T | \Phi_0 \rangle = e^{i p_0 + i v \theta}$$

$v \neq \text{integer}$



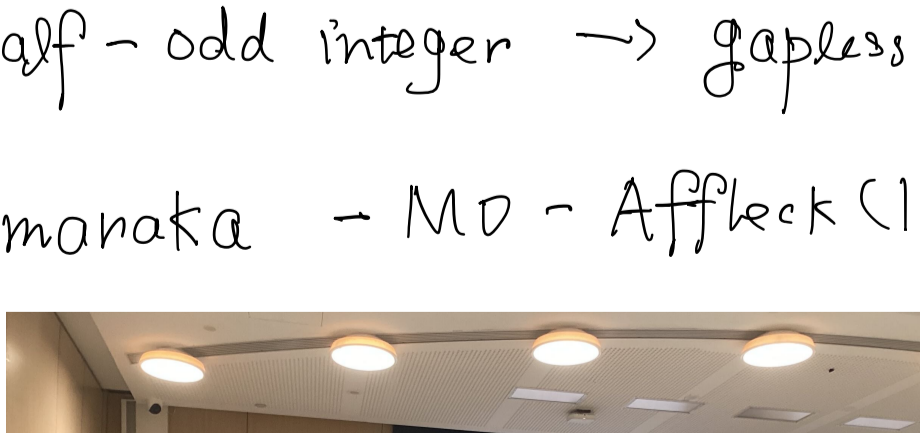
Gapless phase needs the particles to be locked, the density of particles need to be commensurate with lattice.

Lieb and mathias (1961)

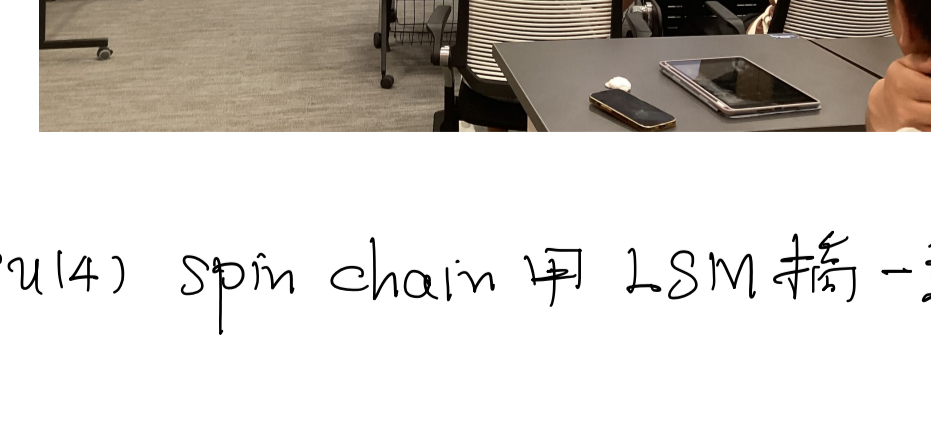


S: half-odd integer  $\rightarrow$  gapless

Yamanaka - MD - Affleck (1997)



把 SU(4) spin chain 用 LSM 推一推



Hooft anomaly.