Thoughts About Quantum Field Theory

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IAS

Thank Edward Witten for many relevant discussions

QFT is the language of physics
It is everywhere

- Particle physics: the language of the Standard Model
  - Enormous success, e.g. the electron magnetic dipole moment is theoretically *1.001 159 652 18 …* experimentally *1.001 159 652 180...*
- Condensed matter
  - Description of the long distance properties of materials: phases and the transitions between them
- Cosmology
  - Early Universe, inflation
- …

QFT is the language of physics
It is everywhere

- String theory/quantum gravity
    - On the string world-sheet
    - In the low-energy approximation (spacetime)
    - The whole theory (gauge/gravity duality)
- Applications in mathematics especially in geometry and topology

- Quantum field theory is the modern calculus
    - Natural language for describing diverse phenomena
- Enormous progress over the past decades, still continuing

2011 Solvay meeting

Comments on QFT

5 minutes, only one slide

Should quantum field theory be reformulated?

- Should we base the theory on a Lagrangian?
    - Examples with no semi-classical limit – no Lagrangian
    - Examples with several semi-classical limits – several Lagrangians
    - Many exact solutions of QFT do not rely on a Lagrangian formulation
    - Magic in amplitudes – beyond Feynman diagrams
- Not mathematically rigorous
- Extensions of traditional local QFT

How should we organize QFTs?

QFT in High Energy Theory

Start at high energies Λ with a scale invariant theory, e.g. a free theory described by Lagrangian.

Deform it with

- a *finite* set of coefficients of relevant (or marginally relevant) operators, e.g. masses m

- a *finite* set of coefficients of exactly marginal operators, e.g. τ in 4d $\mathcal{N} = 4$.

- an *infinite* set of infinitesimal coefficients of irrelevant (or marginally irrelevant) operators, e.g. α in QED

Study the system as a function of these parameters.

QFT in Condensed Matter Physics

Start at high energies Λ with a lattice model, e.g. spins with some interactions.

Deform it:

- add an arbitrary number of degrees of freedom at the lattice scale

- add arbitrary coupling constants

such that the global symmetries (including their global structure) and various selection rules (e.g. no spacetime spinors) are preserved.

Study the system as a function of these parameters.

QFT in Condensed Matter Physics

Lattice Hamiltonian or Lagrangian
Allow all possible coupling constants
Infinite dimensional space

Explore all possible phases at long distances

QFT in High Energy Physics

Scale invariant continuum theory deformed by a finite number of relevant coupling constants infinitesimal values of irrelevant couplings

Much smaller set of coupling constants
More limited set of outcomes

Explore all possible phases at long distances

Expressing the CM (lattice) view in HE (continuum) terms

Start at Λ with some continuum QFT.

Add arbitrary degrees of freedom preserving the symmetries (and selection rules) at energies $M \ll \Lambda$.

They do not affect the dynamics at lower energies $m \ll M$.

Next, deform the parameters of this larger theory such that the new degrees of freedom do affect the dynamics.

This is a much larger space of theories, which can exhibit new phases.

We can refer to it as a deformation class of the QFT.

Duality

- In HE: different theories at Λ lead to the same physics either at all energies or only at low energies.

- In CM: different theories connected in the large space of deformations. They are in the same deformation class.

  – Often stated: ``These theories live in the same Hilbert space.'' (I am not quite sure what this phrase means.)

Global symmetries and 't Hooft anomalies

Unlike gauge symmetries, global symmetries are important intrinsic properties of the system.

Couple them to classical background gauge fields.

't Hooft anomalies of the global symmetries

- The partition function is not gauge invariant, but can be made gauge invariant by extending the background fields to higher dimensions.

- The original system is consistent, but these background gauge fields cannot be made dynamical.

- Example: $U(1)_{B-L}$ in the Standard Model of particle physics.

Global symmetries and 't Hooft anomalies

- In lattice systems, most global symmetries do not have 't Hooft anomalies. The exceptions are
  - emergent (accidental) symmetries at long distances
  - symmetries that do not act ``on-site'', e.g. shifting the lattice by one lattice unit.
- Adding degrees of freedom that preserve the symmetries (and the selection rules) at high energies $M \ll \Lambda$ does not change the 't Hooft anomalies.

All the theories in the same deformation class (obtained by adding such degrees of freedom and varying the parameters) have the same symmetries and anomalies.

14

Different systems with the same symmetries and 't Hooft anomalies

Such systems are candidate theories to be dual to each other.

One of these systems can be a candidate low-energy description of the other.

Theory A   Theory B   Theory A

Same IR behavior   Theory B
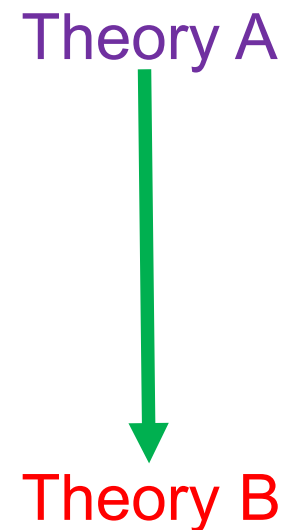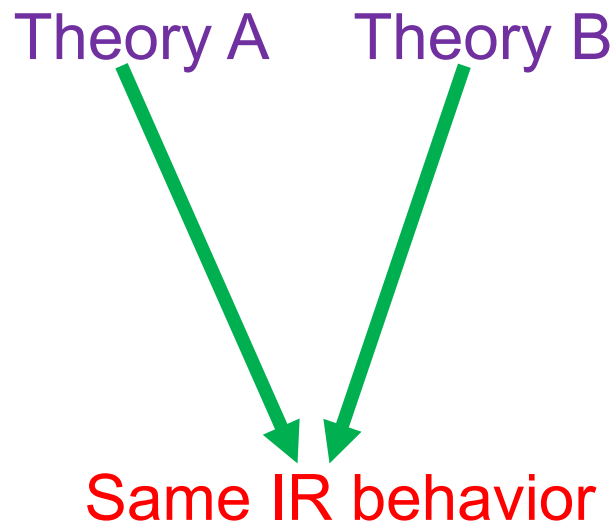
Finding theories with the same 't Hooft anomalies

4d $SU(5)$ with fermions in $\bar{\mathbf{5}} \oplus \mathbf{10}$

(motivated by [Raby, Dimopoulos, Susskind (1979)])

Add a scalar field in $\mathbf{5}$ with a Yukawa coupling to two $\mathbf{10}$'s. For an appropriate potential it condenses, Higgses $SU(5) \rightarrow SU(4)$ and gives masses to some of the fermions.

We are left with $SU(4)$ with fermions in $\mathbf{1} \oplus \mathbf{4} \oplus \bar{\mathbf{4}}$.

Standard 4d dynamics leaves a single massless fermion χ with the quantum numbers of a product of three microscopic fermions $\bar{\mathbf{5}} \cdot \bar{\mathbf{5}} \cdot \mathbf{10}$.

The low-energy theory of χ must have the same symmetry and 't Hooft anomaly as the UV gauge theory.

It could arise as the IR dynamics also in the theory without the added scalar.

Finding theories with the same 't Hooft anomalies

Connect left-moving 2d $E_8 \times E_8$ and $Spin(32)/\mathbb{Z}_2$ [Plamadeala, Mulligan, Nayak (2013)] (motivated by [Narain, Sarmadi, Witten; Ginsparg (1986)])

Different global symmetries. Need to break it.

Start with one of them.

Add a left and right-moving compact scalar ϕ with a potential, e.g. $-\cos\phi$. This does not change the IR behavior.

Tune the potential to zero to find $17 + 1$ free compact bosons.

Rotate their lattice.

Turn on such a scalar potential in another direction.

End up with the other theory.

Finding theories with the same 't Hooft anomalies

Characterize the full anomaly of 4d Maxwell theory
[Hsieh, Tachikawa, Yonekura (2019)]

Pure $SL(2,\mathbb{Z})$ anomaly and mixed $SL(2,\mathbb{Z})$-gravity anomaly. (Ignore higher-form global symmetries.)

Start with the 6d E-string.

- Coulomb branch: a tensor gauge field (with self-dual field strength) and some fermions

- Higgs branch: some scalars and fermions

Conclude: the gravitational anomaly of the tensor gauge field is the same as that of 28 6d fermions. (Possible interest in its two-form global symmetry.)

Finding theories with the same 't Hooft anomalies

Characterize the full anomaly of 4d Maxwell theory
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Conclude: the gravitational anomaly of the tensor gauge field is the same as that of 28 6d fermions. (Possible interest in its two-form global symmetry.)

Compactify this statement on a two-torus to learn that a 4d photon has the same anomalies as 56 4d Weyl fermions.

Since $SL(2, \mathbb{Z})$ of the photon (which acts also on the fermions) arises geometrically, these anomalies include pure $SL(2, \mathbb{Z})$ anomaly and mixed $SL(2, \mathbb{Z})$-gravity anomaly. (Possible interest in the one-form global symmetries.)

Going the opposite way

Given two theories

- in the same spacetime dimension
- with the same global symmetry (and selection rules)
- with the same 't Hooft anomalies

can we always add degrees of freedom at short distances so that we can interpolate between them? Are they in the same deformation class?

Conjecture that (suitably interpreted) the answer is YES.

This means that the deformation class can be defined by the symmetries and the 't Hooft anomalies.

Going the opposite way

Conjecture: the deformation class is uniquely specified by the symmetries and the 't Hooft anomalies, i.e. the space of theories with the same symmetries and 't Hooft anomalies is connected.

- Is it true with supersymmetry?
    - Does it fit the shortening anomaly of [Gomis, Komargodski, Ooguri, NS, Wang (2016)]?
    - Is [Gaiotto, Johnson-Freyd (2019)] a counter example?

Further topology in the space of coupling constants can be controlled by the anomalies in this space [Córdova, Freed, Lam, NS (2019)]

Final words

- We discussed the question of how to organize quantum field theories.

- One organizing principle is the global symmetries and their 't Hooft anomalies.

  – We demonstrated techniques to find different theories with the same global symmetries and anomalies.

  – We raised a question (and conjectured an answer) about connecting theories with the same symmetries and anomalies.

Final words

More generally

- QFT is the language of physics (particles, condensed matter, cosmology, string theory) with applications in mathematics

- It continues to surprise us with new phenomena, new connections between distinct problems, new insights, etc.

- We should definitely continue to explore it!