

This talk is based on discussion with many people, in particular collaborations with

Frank Pollmann (now at TU München) Erez Berg (now at Weizmann Institute) Ari M. Turner (now at Technion)

Shunsuke Furuya (now at RIKEN) Yuan Yao (ISSP) Chang-Tse Hsieh (Kavli IPMU & ISSP)

Hirosi and Me

PRL 108, 161803 (2012)

PHYSICAL REVIEW LETTERS

week ending 20 APRIL 2012



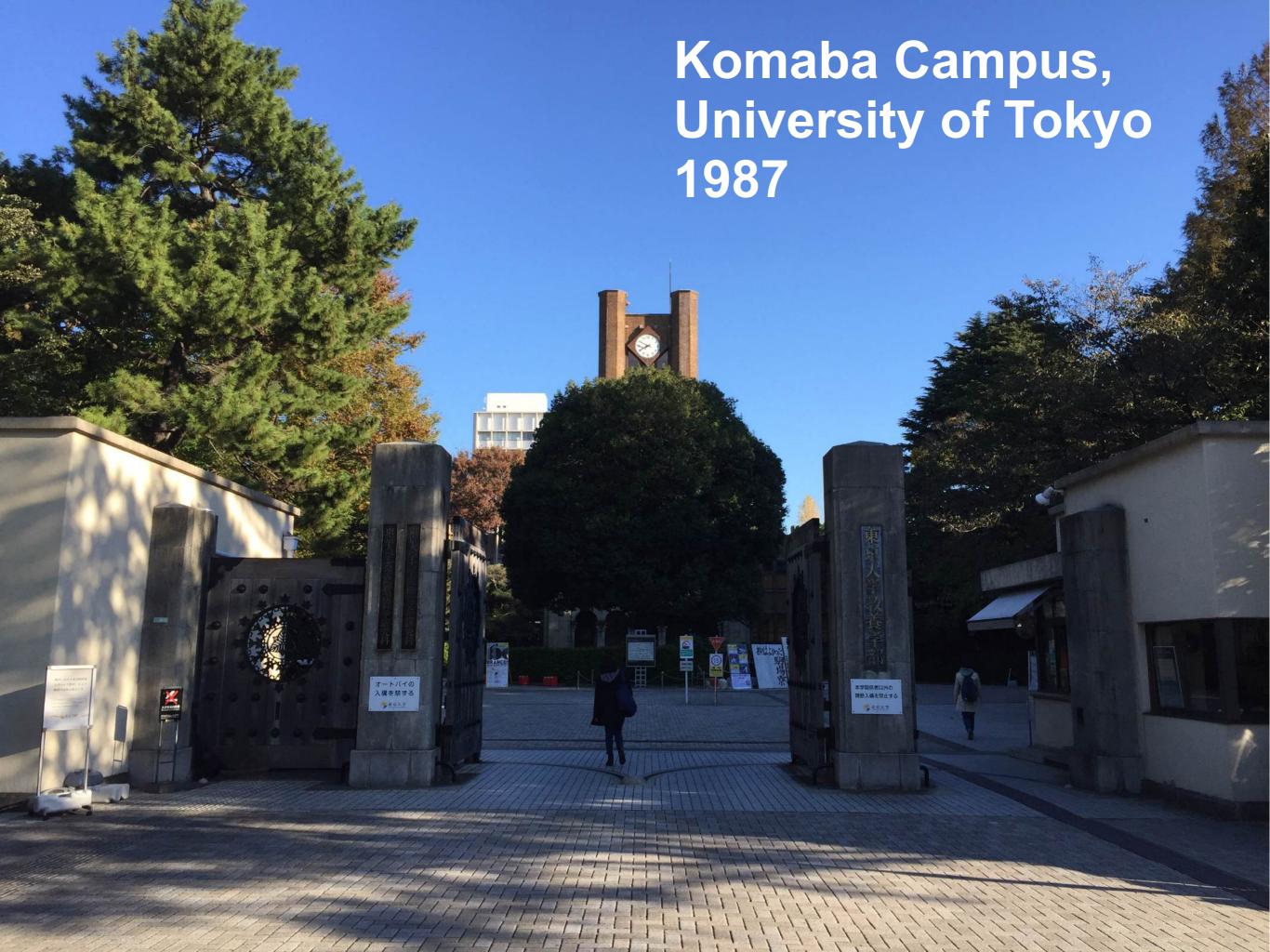
Instability in Magnetic Materials with a Dynamical Axion Field

Hirosi Ooguri^{1,2} and Masaki Oshikawa³

¹California Institute of Technology, 452-48, Pasadena, California 91125, USA
 ²Kavli IPMU, University of Tokyo (WPI), Kashiwa 277-8583, Japan
 ³Institute for Solid State Physics, University of Tokyo, Kashiwa 277-8581, Japan (Received 14 December 2011; published 20 April 2012)

It has been pointed out that axion electrodynamics exhibits instability in the presence of a background electric field. We show that the instability leads to a complete screening of an applied electric field above a certain critical value and the excess energy is converted into a magnetic field. We clarify the physical origin of the screening effect and discuss its possible experimental realization in magnetic materials where magnetic fluctuations play the role of the dynamical axion field.

DOI: 10.1103/PhysRevLett.108.161803 PACS numbers: 14.80.Va, 73.61.-r



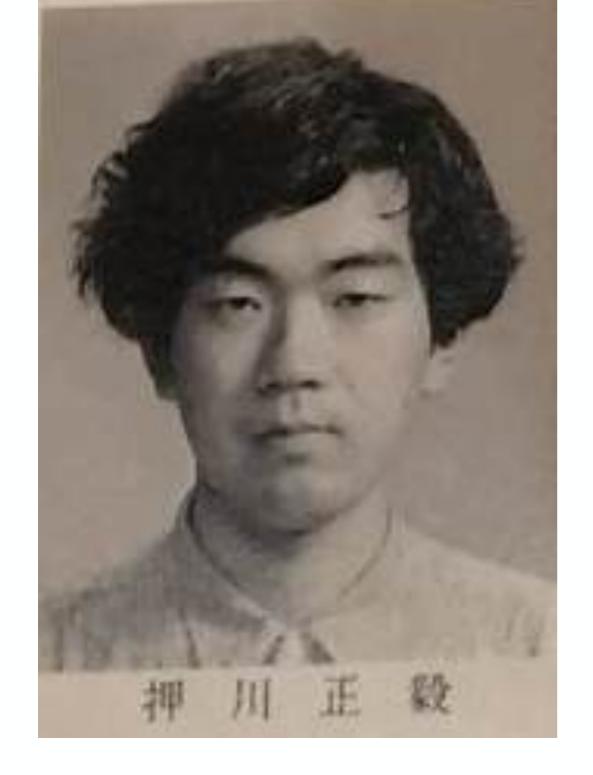
Hirosi: obtained Master's degree from Kyoto Univ.
and immediately appointed to "Joshu"
position at UTokyo in 1986, without Ph. D.
(very rare occurrence in high-energy theory)
"Joshu" = Research Associate / Assistant Prof. /
Wissenschaftlicher Assistent

I: entered UTokyo in April 1986, proceeded to Department of Physics in Summer 1987

Fall Semester 1987:

I was in Hirosi's class for "Seminar"

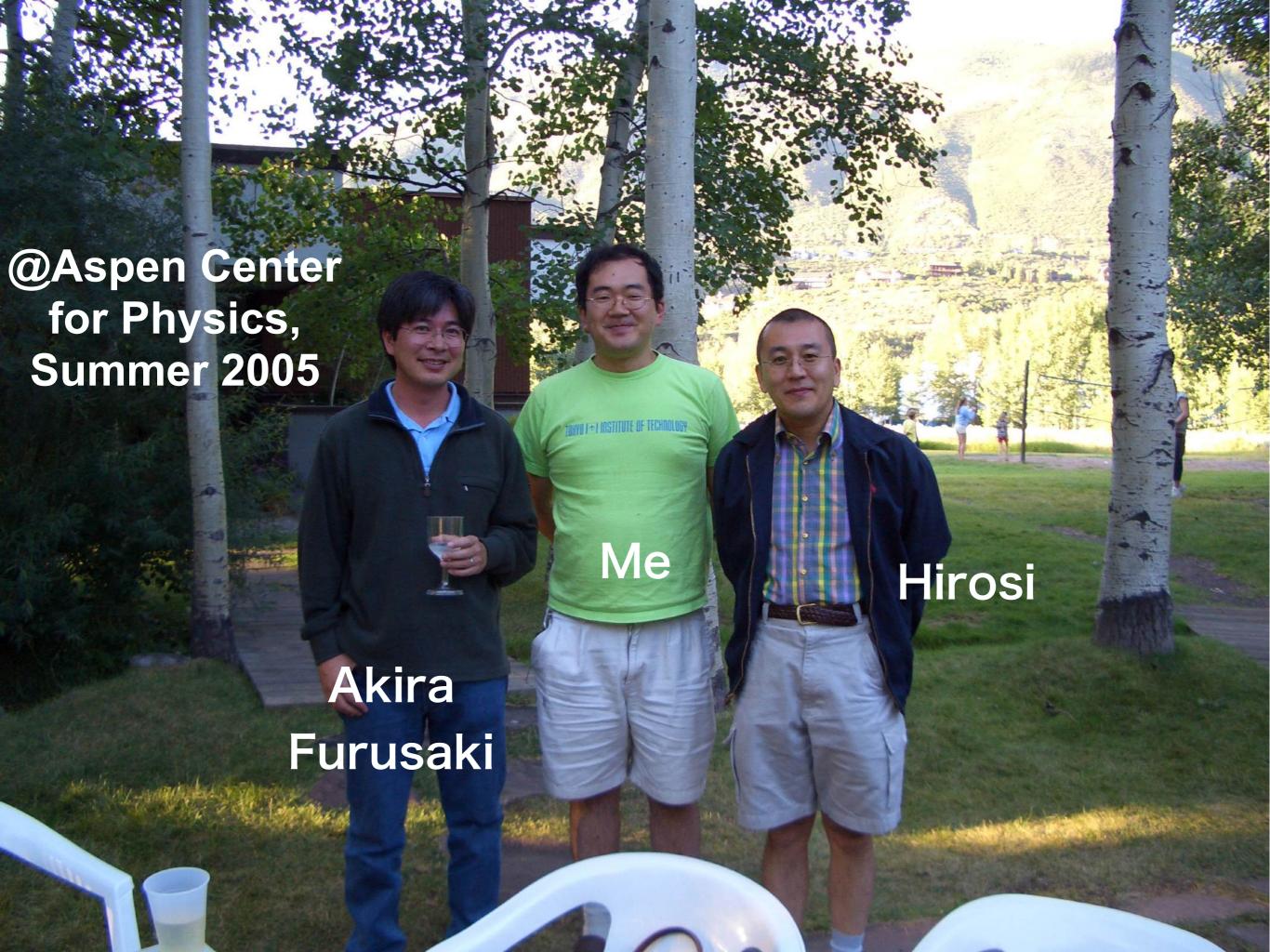
(problem-solving session) in Komaba Campus!



Me (1990)



Hirosi (1987?)





Instability in Magnetic Materials with a Dynamical Axion Field

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DOI: 10.1103/PhysRevLett.108.161803 PACS numbers: 14.80.Va, 73.61.-r

arXiv:1808.10466

Axion instability and non-linear electromagnetic effect

Tatsushi Imaeda, Yuki Kawaguchi, and Yukio Tanaka
Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan

Masatoshi Sato
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
(Dated: September 3, 2018)

We investigate the instability due to dynamical axion field near the topological phase transition of insulators. We first point out that the amplitude of dynamical axion field is bounded for magnetic insulators in general, which suppresses the axion instability. Near the topological phase transition, however, the axion field may have a large fluctuation, which decreases the critical electric field for the instability and increases the axion induced magnetic flux density. Using two different model Hamiltonians, we report the electromagnetic response of the axion field in details.

深い関係

無数の原子が配列してつくり出す空間は、物質の中にひろがる宇宙。 そこで起こる特異な現象の数々。その1つの超伝導にヒントを得た 南部理論が素粒子や宇宙の謎を解き明かす。一方、究極の素粒子理 論が超微細回路の新現象を予言する。

2014年9月28日日

14:30~16:00 (13:30 開場)

会場

柏の葉カンファレンスセンター (柏の葉キャンパス駅前)

事前申し込み

Fax または以下の URL から

定員に達した場合は申し込みを締切ります

Fax: 04-7136-3216

URL: http://www.issp.u-tokyo.ac.jp/

public/issplecture/



次元物質と 弦の理論 押川正毅

東京大学 物性研究所教授



宇宙は超伝導か

大栗博司

ISSP Public Lectures カリフォルニア工科大学東京大学国際高等研究所所長東京大学国際高等研究所

2014 カブリ数物連携宇宙研究機構 主任研究員

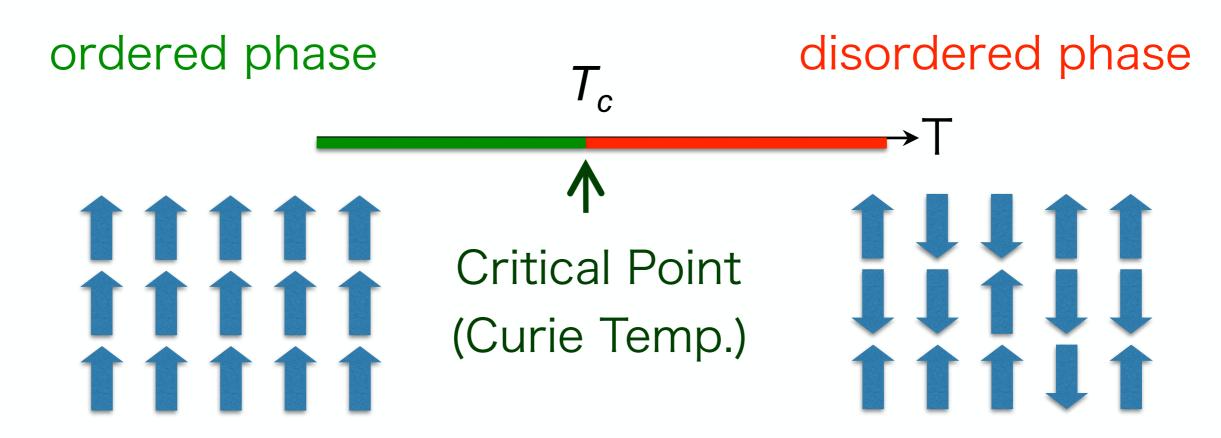




Classification of states of matter

= distinction of different phases

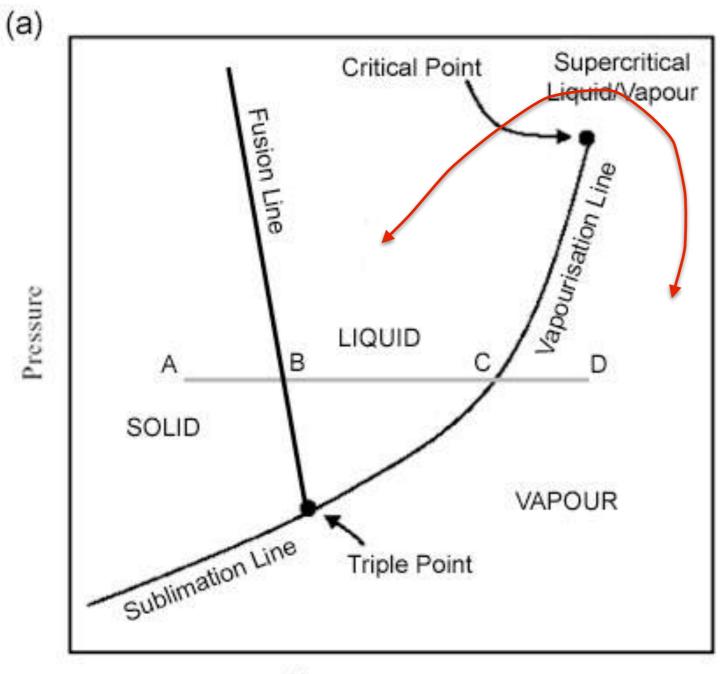
Phase diagram of a ferromagnet



simple model: (classical) Ising model

$$\mathcal{H} = J \sum_{\langle j,k \rangle} \sigma_j^z \sigma_k^z$$

Are liquid and gas different?



Phase transition can be "avoided" by going beyond the critical point

Liquid/gas are
"essentially
indistinguishable"

Temperature

Figure from Sonntag R E, Borgnakke C, Van Wylen G J, "Fundamentals of Thermodynamics" 12

What about solid?

Can we avoid the phase transition between solid/liquid at, e.g. higher pressures?

NO!

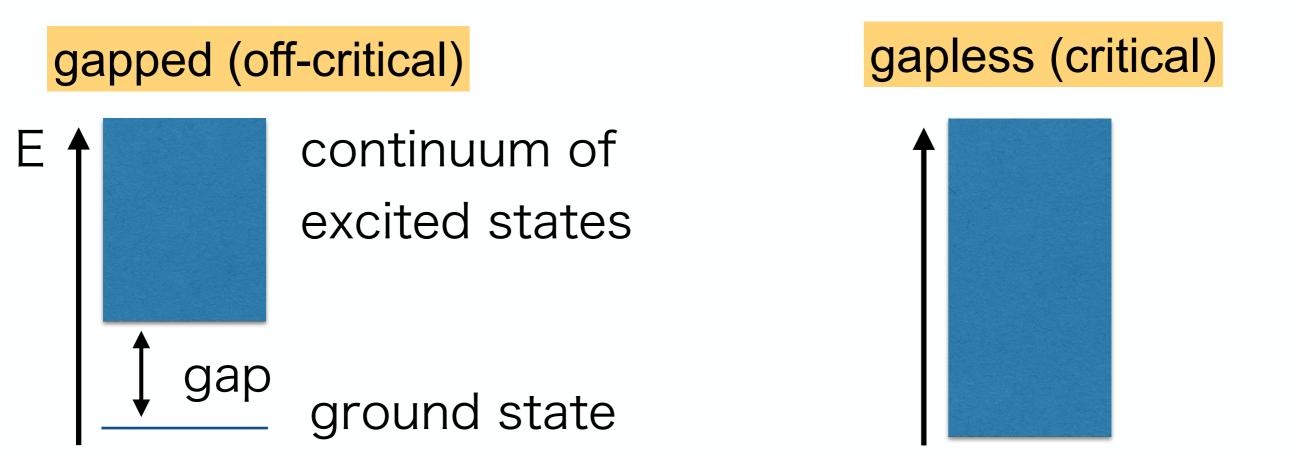
in solid, translation symmetry is spontaneously broken, while it is not in liquid/gas

SSB clearly distinguish different phases, implying existence of phase transitions

Quantum Phase Transitions

Quantum fluctuations can drive the system at T=0 into different quantum phases, and cause quantum phase transitions between quantum phases

Similarity (and in fact mathematical mapping in many cases) to classical phase transition driven by thermal fluctuations



What distinguishes different phases?

Different orders (or their absence) characterize each phase

Ferromagnet: magnetic order Superfluid (3D): off-diagonal long-range order (order of U(1) phase of wavefunctions)

etc.

"order" ⇔ Spontaneous Symmetry Breaking

??

However·····

Recently, it has been recognized that there are many quantum phases that are beyond understanding in terms of conventional orders/spontaneous symmetry breaking

"topological phases"

how to define them? how to distinguish different phases?

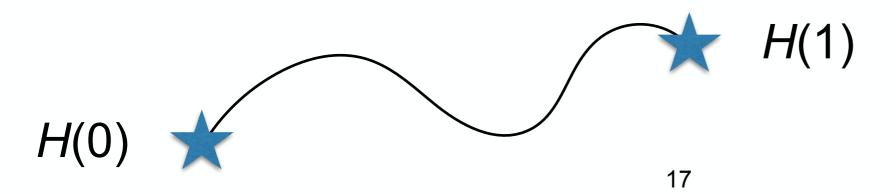
"Operational" definition of phases

A family of Hamiltonians H(g) parametrized by g

Singularity in the ground state of H(g), as a function of $g \Rightarrow$ quantum phase transition

If the two gapped ground states are connected adiabatically, i.e. if there exists a path H(g) the two Hamiltonians without a quantum phase transition **they belong to the same phase**

Otherwise (if there is no adiabatic path connecting the two) they belong to different phases, even if there is no distinction in terms of SSB



Topological Order in 1D

Any gapped ground state of a local 1D Hamiltonian is connected to a trivial state adiabatically

Chen-Gu-Wen (2011)

Absence of (genuine) "topologically ordered phase" in 1D!

i.e. there is only one, trivial phase in 1D (in the absence of symmetries)

However, there can be more variety of phases if some symmetries are imposed

Imposing Symmetries

For a gapped Hamiltonian with a symmetry

- 1) the ground state is in a trivial phase
- 2) the symmetry is spontaneously broken in the ground state (SSB phase)
- 3) the ground state cannot be adiabatically connected to a trivial (product) state, even if we break the symmetry (topological order, absent in 1D)
- 4) the symmetry is unbroken, but the ground state cannot be adiabatically connected to a trivial (product) state as long as the symmetry is kept

"SPT phase"

the symmetry is unbroken, but the gapped ground state can

NOT be adiabatically connected

to a trivial state,

if and only if the Hamiltonian respects the symmetry

then the ground state belongs to a Symmetry-Protected Topological Phase

(Generalization of "topological insulators" of free fermions to interacting systems)

Haldane gap

Heisenberg antiferromagnetic chain

$$\mathcal{H} = J \sum_{j} \vec{S}_{j} \cdot \vec{S}_{j+1}$$

S=1/2, 3/2, 5/2.....

"massless" = gapless, power-law decay of spin correlations

S=1, 2, 3,

"massive" = non-zero gap, exponential decay of spin correlations

Haldane conjecture (1981)

Lieb-Schultz-Mattis theorem

For **translation** & SU(2) invariant spin chains

```
if S is integer: no constraint
```

```
if S is half-odd-integer:the system must be gapless,OR the ground state is at least doubly degenerate
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Lieb-Schultz-Mattis 1961 (S=1/2 chain at zero magnetization) Affleck-Lieb 1986 (arbitrary S chain at zero magnetization) MO-Yamanaka-Affleck 1997, MO 2000, Hastings 2004, etc etc.

more generally, "filling-enforced constraints"

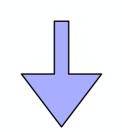
"Proof" by large gauge invariance

$$T_x|\Psi_0\rangle = e^{iP_x^0}|\Psi_0\rangle$$

LSM, Affleck-Lieb, M.O.....

$$|\Psi_0
angle \,$$
 adiabatic flux insertion

momentum unchanged



 $|\Psi_0'\rangle$

Large gauge transformation



$$T_x \left(U_x | \Psi_0' \right) = e^{2\pi i \nu} U_x T_x | \Psi_0' \right)$$

$$= e^{i(P_x^0 + 2\pi\nu)} \left(U_x | \Psi_0' \rangle \right)$$

 $U_x|\Psi_0'\rangle$

momentum shift by $2\pi\nu=2\pi(S-m)$

the "new" ground state has extra momentum π for half-odd-int S

A Proof of Part of Haldane's Conjecture on Spin Chains

Affleck-Lieb 1986 S: half-odd-integer

→ gapless or

2-fold g.s. degeneracy

IAN AFFLECK* and ELLIOTT H. LIEB**

Departments of Mathematics and Physics, Princeton University, P.O. Box 708, Princeton, NJ 08544, U.S.A.

(Received: 10 March 1986)

Abstract. It has been argued that the spectra of infinite length, translation and U(1) invariant, anisotropic, antiferromagnetic spin s chains differ according to whether s is integral or $\frac{1}{2}$ integral: There is a range of parameters for which there is a unique ground state with a gap above it in the integral case, but no such range exists for the $\frac{1}{2}$ integral case. We prove the above statement for $\frac{1}{2}$ integral spin. We also prove that for all s, finite length chains have a unique ground state for a wide range of parameters. The argument was extended to SU(n) chains, and we prove analogous results in that case as well.

ANNALS OF PHYSICS: 16, 407-466 (1961)

was a generalization of "Lieb-Schultz-Mattis Theorem"

Two Soluble Models of an Antiferromagnetic Chain

Elliott Lieb, Theodore Schultz, and Daniel Mattis

Thomas J. Watson Research Center, Yorktown, New York

II. THE XY MODEL

A. Formulation

The first model consists of N spin $\frac{1}{2}$'s (N even) arranged in a row and having only nearest neighbor interactions. It is

$$H_{\gamma} = \sum_{i} [(1 + \gamma) S_{i}^{x} S_{i+1}^{x} + (1 - \gamma) S_{i}^{y} S_{i+1}^{y}], \qquad (2.1)$$

a's and a''s do not preserve this mixed set of canonical rules. However, it is possible to transform to a new set of variables that are strictly Fermi operators and in terms of which the Hamiltonian is just as simple. Let

$$c_i \equiv \exp \left[\pi i \sum_{1}^{i-1} a_j^{\dagger} a_j \right] a_i$$

Main Result of "LSM" paper: S=1/2 XY chain is solvable by mapping to fermions

What about the LSM theorem?

APPENDIX B. NONDEGENERACY OF THE GROUND STATE AND ABSENCE OF AN ENERGY GAP IN THE HEISENBERG MODEL

We prove two exact theorems about the ground state and excitation spectrum for a Heisenberg model with nearest neighbor interactions in one dimension.

Appendix....

APPENDIX B. NONDEGENERACY OF THE GROUND STATE AND ABSENCE OF AN ENERGY GAP IN THE HEISENBERG MODEL

We prove two exact theorems about the ground state and excitation spectrum for a Heisenberg model with nearest neighbor interactions in one dimension. The generalization to longer range interactions and higher-dimensional lattices is indicated. A further generalization to particles of spin $\neq \frac{1}{2}$ and a discussion of the ordering of excited state energy levels has been submitted for publication in the Journal of Mathematical Physics by Lieb and Mattis.

Perhaps refers to this paper

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 3, NUMBER 4 JULY-AUGUST 1962

Ordering Energy Levels of Interacting Spin Systems

ELLIOTT LIEB AND DANIEL MATTIS

Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York
(Received October 6, 1961)

But no mention is actually made on the generalization of LSM theorem?!

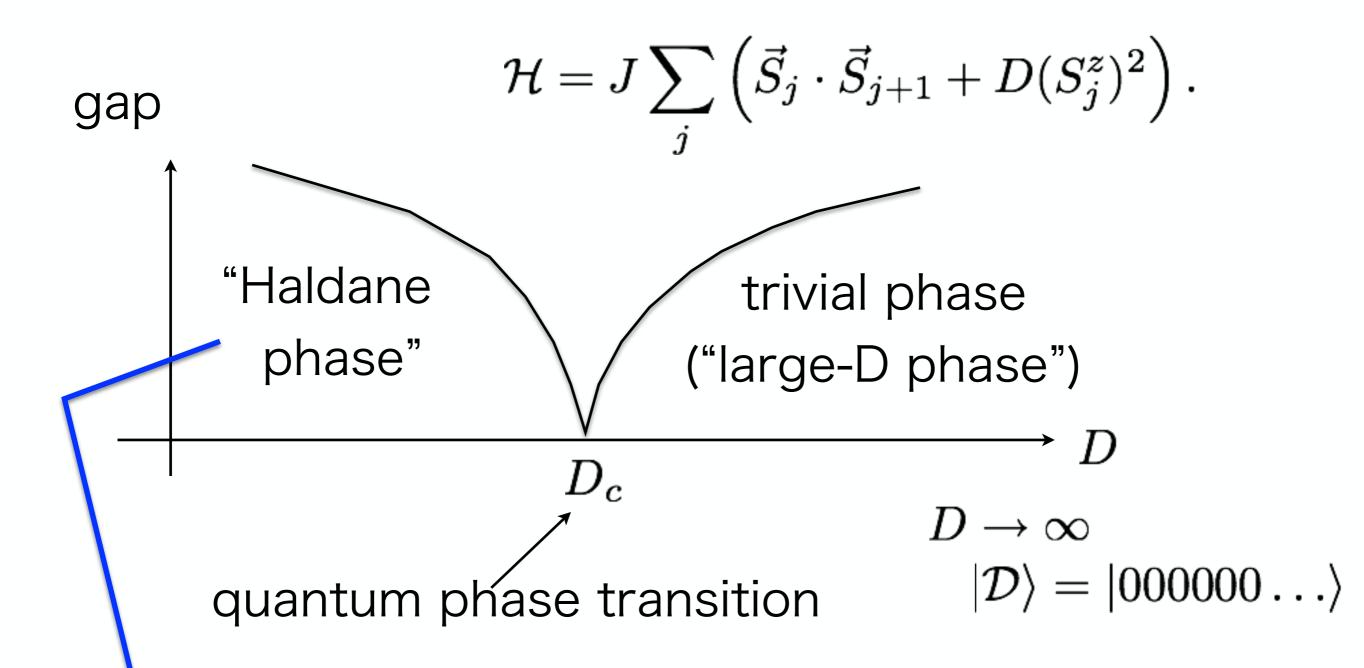
Maybe....

LSM tried to generalize their theorem to general S, but "failed" to prove it for integer S

So they scrapped the generalization and never published (until Affleck-Lieb paper 25 years ago)

.... maybe missing the evidence of the "Haldane gap"??

Haldane Phase and QPT



No local order parameter, but is a distinct "Symmetry-Protected Topological Phase"

Haldane Phase as a SPT

In the presence of *any one* of the following symmetries, the Haldane phase is separated from a trivial (product) state by a quantum phase transition:

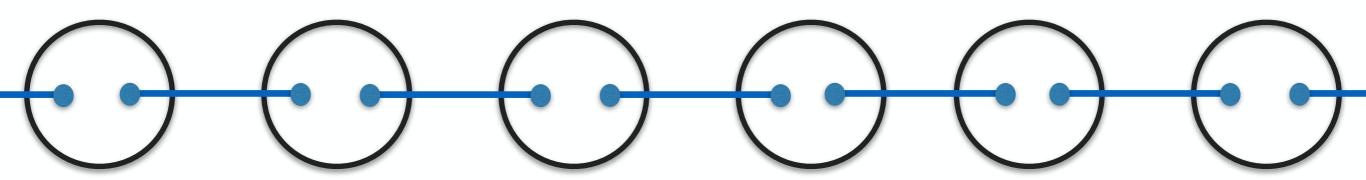
- i) time reversal symmetry
- ii) dihedral ($Z_2 \times Z_2$) symmetry (π -rotation about x, y, and z axes)
- iii) lattice inversion symmetry about a bond center

Gu-Wen (2009) Pollmann-Turner-Berg-MO (2010)

AKLT model/state

$$\mathcal{H} = J \sum_{j} \left[\vec{S}_{j} \cdot \vec{S}_{j+1} + \frac{1}{3} \left(\vec{S}_{j} \cdot \vec{S}_{j+1} \right)^{2} \right]$$

Exact groundstate: (Affleck-Kennedy-Lieb-Tasaki 1987)



•

Singlet pair of two S=1/2's -"valence bonds"

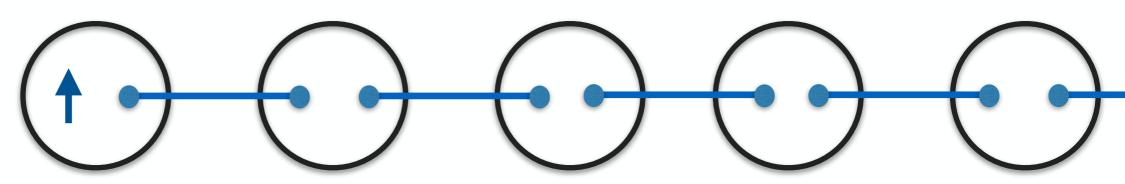


Symmetrization of two S=1/2's $\Rightarrow S=1$

Inon-zero gap, exponential decay of correlations (supporting the Haldang conjecture)

Why SPT?

Pollmann-Turner-Berg-MO (2009)



Open boundary condition: "edge state" of S=1/2

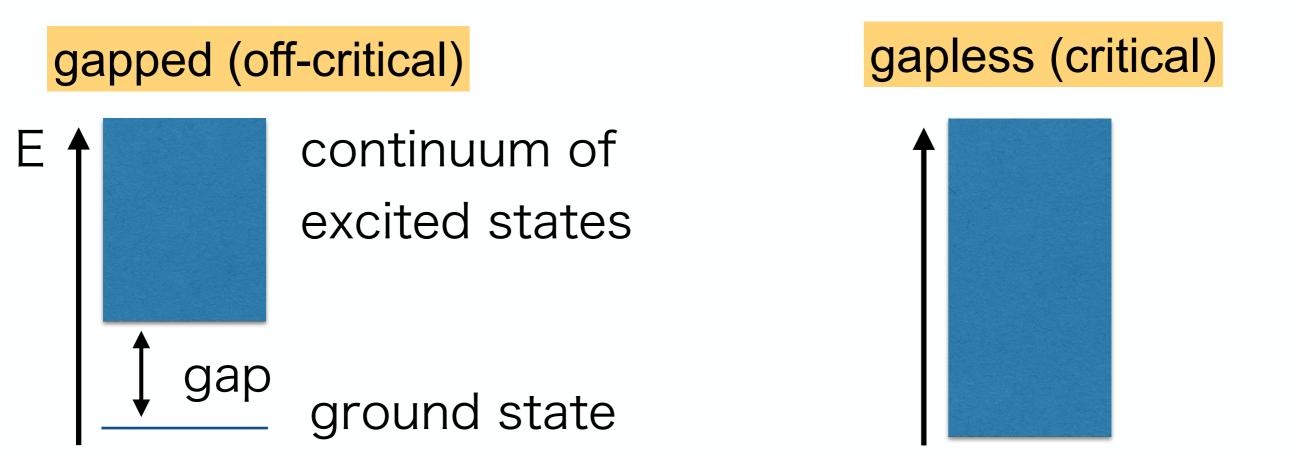
The ground state is doubly degenerate because of the edge spin (4-fold considering both ends)

- This degeneracy is exact under time reversal (Kramers degeneracy):
- ⇒ time reversal must be broken, or there must be a quantum phase transition to remove the degeneracy!

Quantum Phase Transitions

Quantum fluctuations can drive the system at T=0 into different quantum phases, and cause quantum phase transitions between quantum phases

Similarity (and in fact mathematical mapping in many cases) to classical phase transition driven by thermal fluctuations



Gapless Quantum Critical Point

Gapless excitations appear at quantum critical points

e.g. (quantum) transverse Ising model

$$\mathcal{H} = -\sum_{\langle j,k\rangle} \sigma_j^z \sigma_k^z - \Gamma \sum_j \sigma_j^x$$
 ordered phase
$$\Gamma = \Gamma_c \quad \text{disordered phase}$$

$$\Gamma = \Gamma_c \quad \text{disordered phase}$$

$$\Gamma$$
 Quantum Critical Point

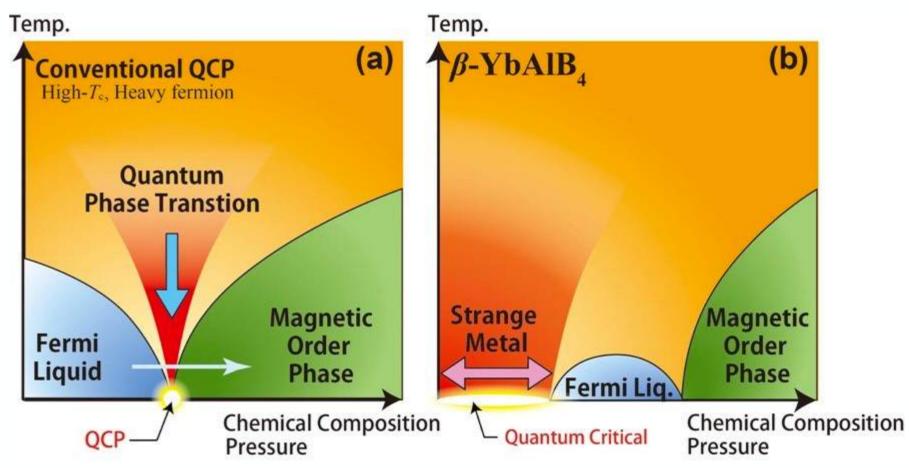
critical point = RG fixed point relevant perturbation → gap

Gapless Critical Phases

However, quantum critical phases often appear in cond-mat physics without any apparent fine-tuning

- metallic systems
- Dirac/Weyl semimetals

- β-YbAlB₄



[Nakatsuji Group, ISSP]

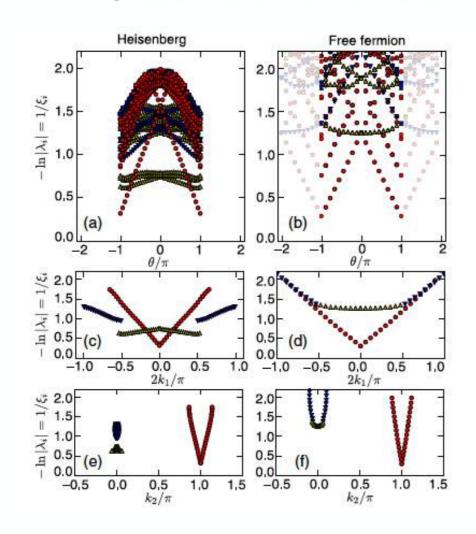
Gapless Critical Phases

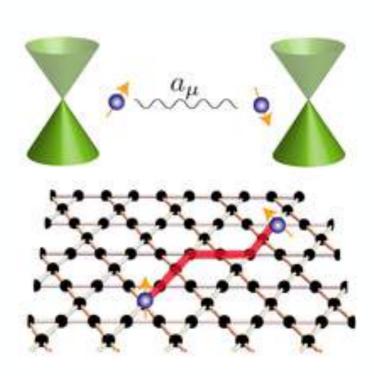
- Kagome spin liquid (S=1/2 antiferromagnet): Dirac spin liquid?

PHYSICAL REVIEW X 7, 031020 (2017)

Signatures of Dirac Cones in a DMRG Study of the Kagome Heisenberg Model

Yin-Chen He, 1,2,3 Michael P. Zaletel, 4,3,6 Masaki Oshikawa, 5,3 and Frank Pollmann 1,3,7





Gapless Critical Phases

- Why are they stable?
- Classification/characterization of these phases

We do have some understanding based on CFT etc. but we need more!

We can gain some insights from the recent developments in the classification of gapped topological phases...

Generalization of SPT phases?

I will attempt to extend the notion of "Symmetry-Protected (Topological) Phases" to gapless phases

I will discuss an example in I+I dimensions (spin chains, effective CFT) although the concept can be hopefully generalized to higher dimensions

S. C. Furuya & M. O. Phys. Rev. Lett. 118, 021601 (2017)
Y. Yao, C.-T. Hsieh, & M. O. arXiv:1805.06885

Our Model

Spin-S antiferromagnetic chain with the global SU(2) and lattice translation symmetries

$$\mathcal{H} = \sum_{j} \left[\vec{S}_{j} \cdot \vec{S}_{j+1} + J_{q} \left(\vec{S}_{j} \cdot \vec{S}_{j+1} \right)^{2} + J_{2} \vec{S}_{j} \cdot \vec{S}_{j+2} \cdots \right]$$

Lorentz invariance is expected;

when gapless, low-energy physics should be described by a SU(2) symmetric CFT

 $SU(2)_k$ Wess-Zumino-Witten theory characterized by "level" k = 1, 2, 3, ...

$$\langle ec{S}_0 \cdot ec{S}_r
angle \propto (-1)^r \left(rac{1}{r}
ight)^{3/(k+2)}$$

Our Claim

In the presence of the SU(2) and lattice translation (by one site) symmetries,

$$S = 1/2, 3/2, 5/2, \dots$$

- The system is gapped with a SSB of the translation symmetry (doubly degenerate GS)
- OR The system is gapless, described by

 $SU(2)_k$ WZW with an odd k

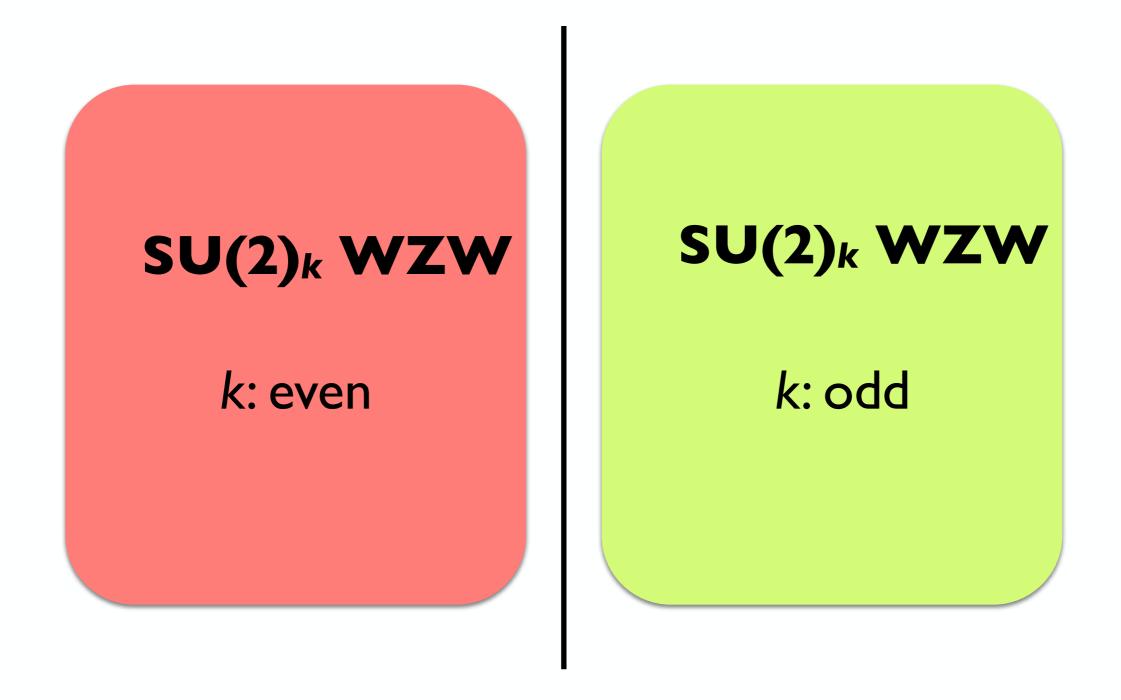
$$S = 1, 2, 3, \dots$$

- The system is gapped (can be without SSB)
- OR The system is gapless, described by

 $SU(2)_k$ WZW with an even k

"Symmetry Protected" gapless phases

SU(2) + Lorentz + lattice translation symmetries



SU(2) WZW Theory

$$S = S_0 + k\Gamma_{WZ}$$

g: SU(2) matrix-valued field

$$S_0 = \frac{1}{2\lambda^2} \int d^2x \operatorname{Tr}[(g^{-1}\partial_{\mu}g)^2]$$

$$\Gamma_{WZ} = \frac{1}{12\pi} \int_{\mathcal{B}} d^3x \, \epsilon^{ijk} \operatorname{Tr}[(g^{-1}\partial_i g)(g^{-1}\partial_j g)(g^{-1}\partial_k g)]$$

original space-time: surface of the sphere

uniqueness of $k\Gamma_{WZ}$ (modulo 2π)

 \Rightarrow k: integer

RG has a nontrivial fixed point if $k\neq 0 \rightarrow \text{gapless critical phase}$

B: (inside) sphere

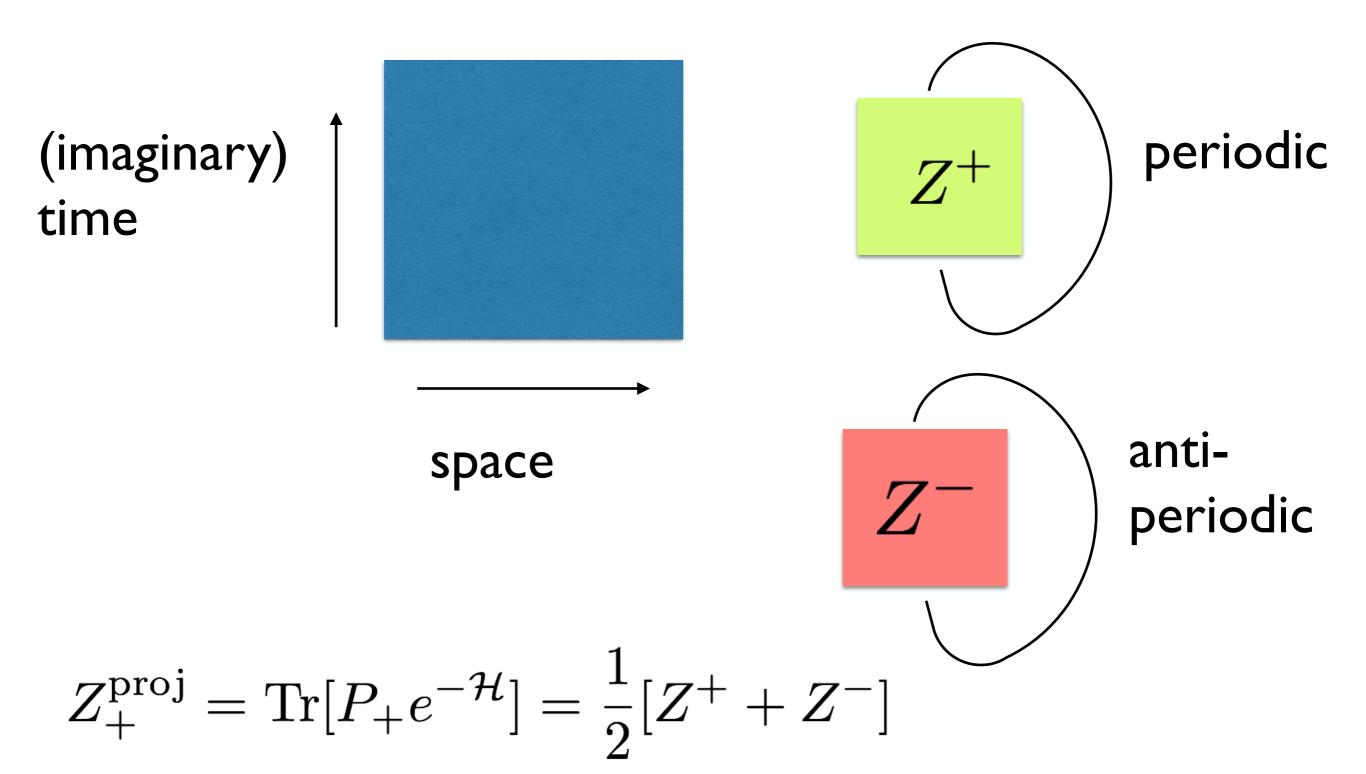
Spin chain and WZW

$$\vec{S}_i \sim \vec{J}_i + \text{const.}(-1)^i \text{tr}(g\vec{\sigma})$$

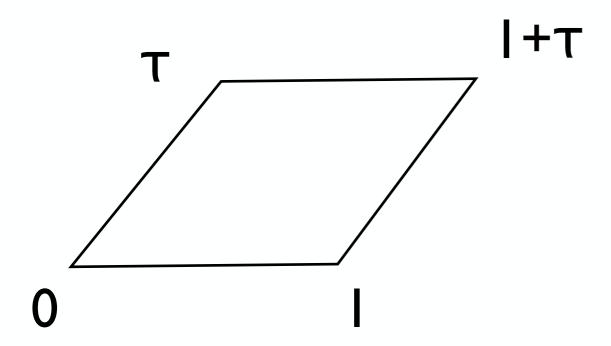
Lattice translation symmetry \Leftrightarrow discrete Z_2 symmetry $g \to -g$

If there is the Z_2 symmetry, we should be able to consider a projection to Z_2 -symmetric subspace?

Projection vs. Path Integral



Modular Invariance



Partition function of a consistent CFT must be invariant under modular transformations generated by

$$S: \tau \to -1/\tau$$

$$\mathcal{T}: \tau \to \tau + 1$$

Orbifold Construction

The "projected" partition function Z_{+}^{proj} is not modular invariant by itself — must be supplemented by twisted sectors

$$Z_{+} = (1 + \mathcal{S} + \mathcal{T}\mathcal{S})Z_{+}^{\text{proj}} - Z_{\text{WZW}}$$

The resulting partition function represents the " \mathbb{Z}_2 orbifold" of the original $SU(2)_k$ WZW theory

Global Anomaly

('t Hooft anomaly)

The Z_2 orbifold should be modular invariant by construction — but this is NOT always the case!

The Z_2 orbifold is **modular invariant if** k **is even**, but it is **modular NON-invariant if** k **is odd**

Gepner-Witten 1986

STRING THEORY ON GROUP MANIFOLDS

Doron GEPNER and Edward WITTEN

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Received 26 May 1986

What does this mean?

If the orbifold is modular invariant, we can consider projection onto the symmetric sector, and open a gap within that sector to obtain the unique ground state

However, if it is modular non-invariant (ie. k: odd), we cannot open the gap to obtain a unique ground state within the symmetric sector;

ground states in the symmetric/antisymmetric sectors must be degenerate!

"Lieb-Schultz-Mattis (LSM) constraint" in CFT!

```
Global anomaly =

"ingappability" in the presence of the symmetry

(S. Ryu et al. on edge theory)
```

Selection Rule

Perturb $SU(2)_k$ WZW with SU(2) and Z_2 -symmetric relevant operators; suppose the RG flow reaches $SU(2)_{k'}$ WZW

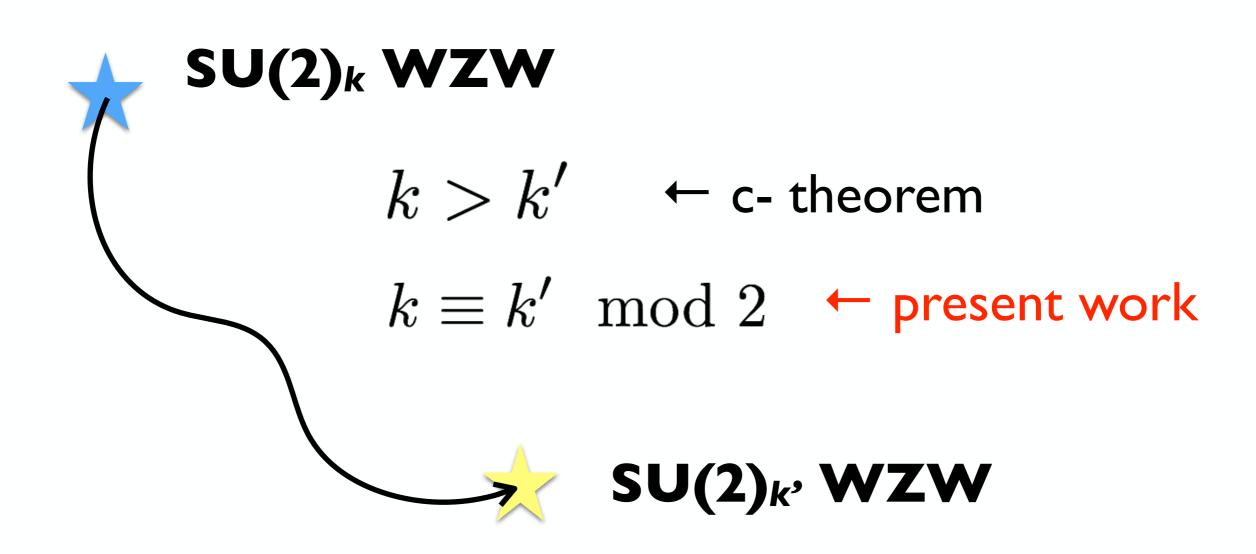
if k is even, we should be able to consider the projection onto Z_2 symmetric sector; the RG flow can be understood in terms of the Z_2 orbifold $\rightarrow k'$ is also even

if k is odd, the IR fixed point should also have the global anomaly (otherwise contradicts with LSM)

 $\rightarrow k'$ is also odd

"anomaly matching"

In terms of RG...



 $SU(2)_0$ WZW is identified with gapped phase with a unique ground state

Spin Chains and WZW

There is a special integrable (Bethe-ansatz solvable) spin chain model for any S, Takhtajan-Babujian (TB) model

e.g. for S=I:
$$\mathcal{H}_{TB} = \sum_j \left[\vec{S}_j \cdot \vec{S}_j - (\vec{S}_j \cdot \vec{S}_j)^2 \right]$$

Spin-STB model is described by $SU(2)_{2S}WZW$ (k=2S even if S is integer, k odd if S is half-odd integer)

Other models can be regarded as

TB model + perturbations, so

k: even if S is integer, k:odd if S is half-odd integer
if the one-site translation symmetry is kept

Our Claim

In the presence of the SU(2) and lattice translation (by one site) symmetries,

$$S = 1/2, 3/2, 5/2, \dots$$

- The system is gapped with a SSB of the translation symmetry (doubly degenerate GS)
- OR The system is gapless, described by

 $SU(2)_k$ WZW with an odd k

$$S = 1, 2, 3, \dots$$

- The system is gapped (can be without SSB)
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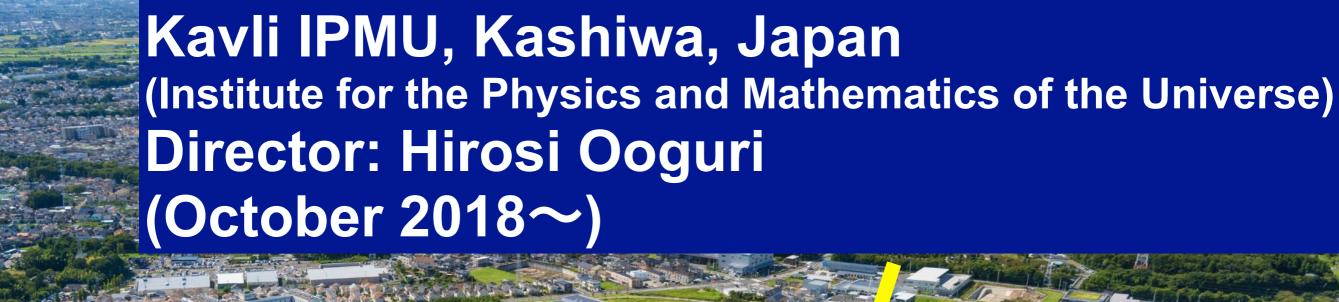
 $SU(2)_k$ WZW with an even k

Summary & Outlook

"Global **Z**₂ anomaly" ('t Hooft anomaly) discovered by Gepner and Witten in 1986 can be interpreted, in the condensed matter / lattice context, an inheritance of **Lieb-Schultz-Mattis constraint** in the microscopic model to CFT as low-energy effective field theories

⇒ "Symmetry-Protected Critical Phases"

Lieb-Schultz-Mattis type constraints in higher dimensions and for various symmetries: corresponding anomalies?





High-energy physics and mathematics are closer to condensed matter physics, than you might think...

We will see more

unexpected and fruitful encounters!