量子场论 相互作用、格林函数与路径积分

本章建立 量子场论 中量子“相互作用场”的正则和路径积分描述

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S矩阵与正规乘积

S矩阵与场的产生湮灭算符展开

$$\Phi_\alpha = a^\dagger \cdots b^\dagger \Phi_0 \qquad \Phi_\beta = d^\dagger \cdots f^\dagger \Phi_0$$

$$S_{\beta\alpha} = (\Phi_\beta, S\Phi_\alpha) = (\Phi_\beta, \mathbf{T}\, e^{-i\int_{-\infty}^{\infty} dt\, V(\psi,\pi)}\Phi_\alpha) = (\Phi_\beta, U(\infty,-\infty)\Phi_\alpha)$$

$$\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [\underbrace{e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)}_{\psi^+} + \underbrace{e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)}_{\psi^-}] \qquad \pi(x) = \pi^+(x) + \pi^-(x)$$

需要处理很多个产生湮灭算符的真空期望值的方法！

正规乘积： $\mathbf{N}(UVW\cdots Z) = \delta_P U'V'W'\cdots Z'$ 上章用的符号是 $:\cdots\cdots:$

其中$U'V'W'\cdots Z'$中的算符排列顺序是在原来$UVW\cdots Z$的算符排列顺序基础之上，将所有的消灭算符排在所有产生算符的左边面对大家，也就是纸面上的右边，而消灭算符之间的相对顺序和产生算符之间的相对顺序则保持不变。

从$UVW\cdots Z$调换到$U'V'W'\cdots Z'$，费米子交换偶数次$\delta_P = 1$,否则$\delta_P = -1$。

算符的收缩：$\overbrace{\psi_l(x)\psi_{l'}}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

$$\overbrace{\psi_l(x)\psi_{l'}}(x') = (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)$$

证明： 不妨设$t > t'$

$$\begin{aligned}
\mathbf{T}(\psi_l(x)\psi_{l'}(x')) &= \mathbf{T}(\psi_l(x))\psi_{l'}(x') = \mathbf{N}(\psi_l(x))(\psi_{l'}^+(x') + \psi_{l'}^-(x')) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) + \psi_l^+(x)\psi_{l'}^-(x') \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) \pm \psi_{l'}^-(x')\psi_l^+(x) + [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp} \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) + \mathbf{N}(\psi_l^+(x)\psi_{l'}^-(x')) + [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp} \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l(x)\psi_{l'}^-(x')) + (\Phi_0, [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp}\Phi_0) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \psi_l^+(x)\psi_{l'}^-(x')\Phi_0) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \psi_l(x)\psi_{l'}(x')\Phi_0) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)
\end{aligned}$$

算符的收缩：$\overbrace{\psi_l(x)\psi_{l'}(x')} \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

$$\overbrace{\psi_l(x)\psi_{l'}(x')} = (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)$$

证明： 不妨设$t > t'$

$$\begin{aligned}\mathbf{T}(\psi_l(x)\psi_{l'}(x')) &= \mathbf{T}(\psi_l(x))\psi_{l'}(x') = \mathbf{N}(\psi_l(x))(\psi_{l'}^{+}(x') + \psi_{l'}^{-}(x')) \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + [\psi_l^{+}(x), \psi_{l'}^{-}(x')]_{\mp} \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)\end{aligned}$$

推论：相互之间的对易或反对易性质导致

- 相邻的不同场的场算符之间的收缩等于零
- 相邻的消灭算符之间,产生算符之间的收缩等于零

算符的收缩: $\overbracket{\psi_l(x)\psi_{l'}(x')} \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值
相邻的不同场之间的收缩等于零; 消灭算符之间,产生算符之间的收缩等于零

定义不相邻的场算符的收缩: $\mathbf{N}(U\ \overbracket{VW\cdots YZ}) \ \equiv\ \delta_P\ \overbracket{VZ}\,\mathbf{N}(UW\cdots Y)$

其中的上划线是指将两端的算符V和Z进行收缩操作,对上划线中间的算符$W\cdots Y$不发生作用,只是考虑因子$\delta_P = (-1)^n$, n是将V和Z调到一起所交换的费米子算符的数目.

推论: $\mathbf{N}(U\ \overbracket{V\overbracket{W\cdots XY}\,Z}) \ \equiv\ \delta_P\ \overbracket{VZ}\ \overbracket{WY}\,\mathbf{N}(U\cdots X)$

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(\overbracket{UV\cdots XYZ}) + \mathbf{N}(U\overbracket{V\cdots XYZ}) + \cdots + \mathbf{N}(UV\cdots X\overbracket{YZ})$$

证明: 如果Z是湮灭算符

$\overbracket{UZ} = \mathbf{T}(UZ) - \mathbf{N}(UZ) = UZ - UZ = 0 \ \Rightarrow$ 预备定理恒成立, 只需讨论Z是产生算符的情况!

Wick定理

算符的收缩：$\psi_l(x)\psi_{l'}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

定义不相邻的场算符的收缩: $\mathbf{N}(U\ VW\cdots YZ) \equiv \delta_P\ VZ\ \mathbf{N}(UW\cdots Y)$

$\delta_P = (-1)^n$, n是将V和Z调到一起所交换的费米子算符的数目.

推论: $\mathbf{N}(U\ V\ W\cdots XY\ Z) \equiv \delta_P\ VZ\ WY\ \mathbf{N}(U\cdots X)$

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ)$$

Z是产生算符; 若$UV\cdots XY$是产生算符，则因它们的收缩为零，结果恒成立

证明: Z是产生算符; 若$UV\cdots XY$是湮灭算符的情形成立，则对一产生算符P

$$P\mathbf{N}(UV\cdots XY)Z = P\mathbf{N}(UV\cdots XYZ) + P\mathbf{N}(UV\cdots XYZ) + P\mathbf{N}(UV\cdots XYZ) + \cdots + P\mathbf{N}(UV\cdots XYZ)$$

利用$PZ = 0 \quad \Rightarrow \quad \mathbf{N}(PUV\cdots XYZ) = 0$ 上式等号左右两边的P都可以移入正规乘积，加上本行式子得到下面结果

$$\mathbf{N}(PUV\cdots XY)Z = \mathbf{N}(PUV\cdots XYZ) + \mathbf{N}(PUV\cdots XYZ) + \mathbf{N}(PUV\cdots XYZ) + \cdots + \mathbf{N}(PUV\cdots XYZ)$$

$\Rightarrow$ 进一步可以假设U是产生算符，$V\cdots XY$是湮灭算符 重复一遍上面证明，结果仍然成立

$\Rightarrow$ 预备定理成立,这时$UV\cdots XY$产生湮灭算符都有 只需讨论$UV\cdots XY$是湮灭算符的情况！

算符的收缩: $\overbracket{\psi_l(x)\psi_{l'}}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

相邻的不同场之间的收缩等于零; 消灭算符之间,产生算符之间的收缩等于零

定义不相邻的场算符的收缩: $\mathbf{N}(U\,VW\cdots YZ) \equiv \delta_P\, VZ\, \mathbf{N}(UW\cdots Y)$

$\delta_P = (-1)^n$, n是将V和Z调到一起所交换的费米子算符的数目.

推论: $\mathbf{N}(U\,V\,W\cdots XY\,Z) \equiv \delta_P\, VZ\; WY\, \mathbf{N}(U\cdots X)$

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ)$$

证明: Z是产生算符; $UV\cdots XY$湮灭算符: 归纳法

$$n=1: \quad \mathbf{N}(Y)Z = YZ = \mathbf{T}(YZ) = \mathbf{N}(YZ) + \overbracket{YZ}$$

$\Rightarrow$ 预备定理成立, 假设对n成立, 只需推出对$n+1$也成立! 对湮灭算符R:

$$R\mathbf{N}(UV\cdots XY)Z = \underbrace{R\mathbf{N}(UV\cdots XYZ)}_{\text{计算见下行}} + R\mathbf{N}(UV\cdots XYZ) + R\mathbf{N}(UV\cdots XYZ) + \cdots + R\mathbf{N}(UV\cdots XYZ)$$

$$R\mathbf{N}(UV\cdots XYZ) = \delta_P \mathbf{T}(RZ)UV\cdots XY = \delta_P(\mathbf{N}(RZ) + \overbracket{RZ})UV\cdots XY = \mathbf{N}(RUV\cdots XYZ) + \mathbf{N}(RUV\cdots XYZ)$$

$$\mathbf{N}(RUV\cdots XY)Z = \mathbf{N}(RUV\cdots XYZ) + \mathbf{N}(RUV\cdots XYZ) + \mathbf{N}(RUV\cdots XYZ) + \cdots + \mathbf{N}(RUV\cdots XYZ)$$

Wick定理：

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ)$$

定理1:

$$\begin{aligned}
\mathbf{T}(UV\cdots XYZ) \quad = \quad & \mathbf{N}(UV\cdots XYZ) && \text{无收缩项}\\
& + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ) + && \text{一次收缩项}\\
& + \mathbf{N}(UV\cdots XYZ) + \cdots && \text{二次收缩项}\\
& + \cdots \\
& + \mathbf{N}(UVW\cdots XYZ) + \cdots && \text{全部收缩项}
\end{aligned}$$

证明: 归纳法

$n=2$阶成立! 假设对n阶已经成立. 右乘时间早于所有其它算符时间的算符R

$$\mathbf{T}(UV\cdots XYZ)R = \mathbf{T}(UV\cdots XYZR)$$

$$\mathbf{N}(UV\cdots XYZ)R = \mathbf{N}(UV\cdots XYZR) + \mathbf{N}(UV\cdots XYZR) + \mathbf{N}(UV\cdots XYZR) + \cdots$$

$$\mathbf{N}(UV\cdots XYZ)R = \delta_P UV\mathbf{N}(W\cdots XYZ)R = \mathbf{N}(UV\cdots XYZR) + \mathbf{N}(UV\ W\cdots XYZR) + \cdots$$

$\cdots\cdots \quad \Rightarrow \quad n+1$阶也成立!

Wick定理：

定理1:

$$\mathbf{T}(UV\cdots XYZ) = \mathbf{N}(UV\cdots XYZ) \qquad \text{无收缩项}$$

$$+\mathbf{N}(\underbrace{UV}\cdots XYZ)+\cdots+\mathbf{N}(UV\cdots \underbrace{XY}Z)+ \qquad \text{一次收缩项}$$

$$+\mathbf{N}(\underbrace{UV}\cdots \underbrace{XY}Z)+\cdots \qquad \text{二次收缩项}$$

$$+\cdots$$

$$+\mathbf{N}(UVW\cdots XYZ)+\cdots \qquad \text{全部收缩项}$$

定理2:

若T乘积中含N乘积,展开仍成立,但需将各N乘积内部各算符之间的收缩略去.

此时若N乘积内部是纯湮灭或纯产生算符，本来就不存在内部的收缩。

若N乘积是一对湮灭产生算符，N乘积就等于T乘积减去收缩。再多用归纳法。

$$\Phi_\alpha = a^\dagger\cdots b^\dagger\Phi_0 \qquad \Phi_\beta = d^\dagger\cdots f^\dagger\Phi_0$$

$$S_{\beta\alpha} = (\Phi_\beta, S\Phi_\alpha) = (\Phi_\beta, \mathbf{T}\, e^{-i\int_{-\infty}^{\infty} dt\ V(\psi,\pi)}\Phi_\alpha)$$

$$\psi(x) = \sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[\underbrace{e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)}_{\psi^+}+\underbrace{e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)}_{\psi^-}] \qquad \pi(x) = \pi^+(x)+\pi^-(x)$$

场的收缩：

$$\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec p,\sigma)a(\vec p,\sigma)+e^{ip\cdot x}v(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]$$

$$\overbracket{\psi_l(x)\psi_{l'}(x')}=(\Phi_0,\mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)=\theta(t-t')[\psi_l^+(x),\psi_{l'}^-(x')]_\mp\pm\theta(t'-t)[\psi_{l'}^+(x'),\psi_l^-(x)]_\mp$$

标量场: $u(\vec p)=v(\vec p)=\frac{1}{\sqrt{2}(\vec p^2+M^2)^{1/4}}$　　$f(z)=\frac{1}{2\pi i}\oint_{逆时针}dw\frac{f(w)}{w-z}$

$$\overbracket{\phi(x)\phi^\dagger(y)}=\int\frac{d^3pd^3p'\ \{[e^{-ip\cdot x}a(\vec p),\ e^{ip'\cdot y}a^\dagger(\vec p')]\theta(x^0-y^0)+[e^{-ip'\cdot y}a^c(\vec p'),\ e^{ip\cdot x}a^{c\dagger}(\vec p)]\theta(y^0-x^0)\}}{(2\pi)^3 2(\vec p^2+M^2)^{1/4}(\vec p'^2+M^2)^{1/4}}$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{2(2\pi)^3\sqrt{\vec p^2+M^2}}[e^{-i\sqrt{\vec p^2+M^2}(x^0-y^0)}\theta(x^0-y^0)+e^{i\sqrt{\vec p^2+M^2}(x^0-y^0)}\theta(y^0-x^0)]$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{2(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0\quad e^{-ip^0(x^0-y^0)}}{2i\pi\sqrt{\vec p^2+M^2}}[-\frac{1}{p^0-\sqrt{\vec p^2+M^2}+i0^\dagger}+\frac{1}{p^0+\sqrt{\vec p^2+M^2}-i0^\dagger}]$$

$$=\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\ \frac{i}{p^2-M^2+i0^\dagger}\ =$$

Wick定理

场的收缩: $$\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec p,\sigma)a(\vec p,\sigma)+e^{ip\cdot x}v(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]$$

$$\psi_l(x)\psi_{l'}(x')=(\Phi_0,\mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)=\theta(t-t')[\psi_l^+(x),\psi_{l'}^-(x')]_\mp\pm\theta(t'-t)[\psi_{l'}^+(x'),\psi_l^-(x)]_\mp$$

旋量场: $u(\vec p,\sigma)=\sqrt{M/p^0}D(L(p))u(0,\sigma)\qquad v(\vec p,\sigma)=\sqrt{M/p^0}D(L(p))v(0,\sigma)$

$$\psi(x)_l\overline{\psi}_{l'}(y)=\int\frac{d^3pd^3p'}{(2\pi)^3}\sum_{\sigma\sigma'}\{[e^{-ip\cdot x}u_l(\vec p,\sigma)a(\vec p,\sigma),\ e^{ip'\cdot y}\overline{u}_{l'}(\vec p',\sigma')a^\dagger(\vec p',\sigma')]_+\theta(x^0-y^0)$$

$$-[e^{-ip'\cdot y}\overline{v}_{l'}(\vec p',\sigma')a^c(\vec p',\sigma'),\ e^{ip\cdot x}v_l(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]_+\theta(y^0-x^0)\}$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{(2\pi)^3}\sum_\sigma[e^{-i\sqrt{\vec p^2+M^2}(x^0-y^0)}u_l(\vec p,\sigma)\overline{u}_{l'}(\vec p,\sigma)\theta(x^0-y^0)-e^{i\sqrt{\vec p^2+M^2}(x^0-y^0)}\overline{v}_{l'}(-\vec p,\sigma)v_l(-\vec p,\sigma)\theta(y^0-x^0)]$$

$N_{l\bar l}(\vec p)\equiv\sum_\sigma u_l(\vec p,\sigma)u_{\bar l}^*(\vec p,\sigma)=\sum_\sigma u_l(\vec p,\sigma)\overline{u}_{\bar l}(\vec p,\sigma)\beta=\frac{1}{2p^0}[p^\mu\gamma_\mu+M]\beta\qquad M_{l\bar l}(\vec p)\equiv\sum_\sigma v_l(\vec p,\sigma)v_{\bar l}^*(\vec p,\sigma)=\sum_\sigma v_l(\vec p,\sigma)\overline{v}_{\bar l}(\vec p,\sigma)\beta=\frac{1}{2p^0}[p^\mu\gamma_\mu-M]\beta$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0\quad e^{-ip^0(x^0-y^0)}}{2i\pi2\sqrt{\vec p^2+M^2}}\left[-\frac{p^0\gamma^0-\vec p\cdot\vec\gamma+M}{p^0-\sqrt{\vec p^2+M^2}+i0^\dagger}-\frac{-p^0\gamma^0+\vec p\cdot\vec\gamma-M}{p^0+\sqrt{\vec p^2+M^2}-i0^\dagger}\right]_{ll'}$$

$$=\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\left[\frac{i(p_\mu\gamma^\mu+M)}{p^2-M^2+i0^\dagger}\right]_{ll'}=\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\left[\frac{i}{p_\mu\gamma^\mu-M+i0^\dagger}\right]_{ll'}=$$

场的收缩：

$$\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)+e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]$$

$$\overbrace{\psi_l(x)\psi_{l'}(x')}=(\Phi_0,\mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)=\theta(t-t')[\psi_l^+(x),\psi_{l'}^-(x')]_\mp\pm\theta(t'-t)[\psi_{l'}^+(x'),\psi_l^-(x)]_\mp$$

有质矢量场: $u^\mu(\vec{p},\sigma)=v^{\mu*}(\vec{p},\sigma)=(2p^0)^{-1/2}e^\mu(\vec{p},\sigma)\qquad e^\mu(\vec{p},\sigma)\equiv L^\mu_{\ \nu}(\vec{p})e^\nu(0,\sigma)$

$$\overbrace{v^\mu(x)v^{\nu\dagger}}(y)=\int\frac{d^3pd^3p'}{2(2\pi)^3(\vec{p}^2+M^2)^{1/4}(\vec{p}'^2+M^2)^{1/4}}\sum_{\sigma\sigma'}\{[e^{-ip\cdot x}e^\mu(\vec{p},\sigma)a(\vec{p},\sigma),\ e^{ip'\cdot y}e^{\nu*}(\vec{p}',\sigma')a^\dagger(\vec{p}',\sigma')]$$

$$+[e^{-ip'\cdot y}e^\nu(\vec{p}',\sigma')a^c(\vec{p}',\sigma'),\ e^{ip\cdot x}e^\mu(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]\theta(y^0-x^0)\}$$

$$=\int\frac{d^3p\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}}{2(2\pi)^3\sqrt{\vec{p}^2+M^2}}\sum_\sigma[e^{-i\sqrt{\vec{p}^2+M^2}(x^0-y^0)}e^\mu(\vec{p},\sigma)e^{\nu*}(\vec{p},\sigma)\theta(x^0-y^0)+e^{i\sqrt{\vec{p}^2+M^2}(x^0-y^0)}e^\nu(-\vec{p},\sigma)e^{\mu*}(-\vec{p},\sigma)\theta(y^0-x^0)]$$

$$=\int\frac{d^3p\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0\quad e^{-ip^0(x^0-y^0)}}{2i\pi2\sqrt{\vec{p}^2+M^2}}[-\frac{-g^{\mu\nu}+p^\mu p^\nu/M^2}{p^0-\sqrt{\vec{p}^2+M^2}+i0^\dagger}+\frac{-g^{\mu\nu}+p^\mu p^\nu/M^2}{p^0+\sqrt{\vec{p}^2+M^2}-i0^\dagger}]$$

$$=\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\ \frac{i(-g^{\mu\nu}+p^\mu p^\nu/M^2)}{p^2-M^2+i0^\dagger}\quad=$$

约化公式

态之间的编时乘积的格林函数 $(\Psi^-_\beta, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$

$U(t,t') = \Omega^\dagger(t)\Omega(t') \quad \psi_H(x) = \Omega(t)\psi(x)\Omega^\dagger(t) \quad \Psi^\pm_\alpha = \Omega(\mp\infty)\Phi_\alpha$

$$S_{\beta\alpha} = (\Phi_\beta, U(\infty,-\infty)\Phi_\alpha) = (\Phi_\beta, \mathbf{T}\, e^{-i\int d^4x\, V(\psi,\pi)}\Phi_\alpha)$$

$$S_{\beta\alpha}[J] \equiv (\Phi_\beta, \mathbf{T}\, e^{-i\int d^4x\, V_J(\psi,\pi)}\Phi_\alpha) \Leftarrow V_J(\psi,\pi) \equiv V(\psi,\pi) + \psi_l(x)J_l(x)$$

$$\left.\frac{\delta S_{\beta\alpha}[J]}{\delta J_{l_1}(x_1)\delta J_{l_2}(x_2)\cdots\delta J_{l_n}(x_n)}\right|_{J_l(x)=0} = (-i)^n(\Phi_\beta, \mathbf{T}\, \psi_{l_1}(x_1)\psi_{l_2}(x_2)\cdots\psi_{l_n}(x_n)\, e^{-i\int d^4x\, V(\psi,\pi)}\Phi_\alpha)$$

$= (-i)^n(\Phi_\beta, \mathbf{T}\, \psi_{l_1}(x_1)\psi_{l_2}(x_2)\cdots\psi_{l_n}(x_n)\, U(\infty,-\infty)\Phi_\alpha)$ **Wick定理** 所有可能的全部收缩项之和

$$\overset{t_1\geq t_2\geq\cdots\geq t_n}{=\!=\!=\!=\!=\!=} (-i)^n(\Phi_\beta, U(\infty,t_1)\psi_{l_1}(x_1)U(t_1,t_2)\psi_{l_2}(x_2)U(t_2,t_3)\cdots U(t_{n-1},t_n)\psi_{l_n}(x_n)U(t_n,-\infty)\Phi_\alpha$$

$$= (-i)^n(\Phi_\beta, \Omega^\dagger(\infty)\Omega(t_1)\psi_{l_1}(x_1)\Omega^\dagger(t_1)\Omega(t_2)\psi_{l_2}(x_2)\Omega^\dagger(t_2)\cdots\Omega(t_n)\psi_{l_n}(x_n)\Omega^\dagger(t_n)\Omega(-\infty)\Phi_\alpha)$$

$$= (-i)^n(\Psi^-_\beta, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

$$= (-i)^n(\Psi^-_\beta, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

编时乘积的格林函数$(\Psi_0^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$

以后会证明: 若已知所有的格林函数,则就可以得到体系的作用量!

对前n个粒子在无穷将来$t_1,\ldots,t_n \to \infty$， 后$l$个粒子在无穷过去$t_{n+1},\ldots,t_{n+l} \to -\infty$的情况:

$$
\begin{aligned}
&(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}\\
&= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)\\
&\quad\times \sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l} \theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)\\
&\quad\times(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)
\end{aligned}
$$

- 编时因子$\theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$是 不可以 被去掉 的!
- $(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+) =$ $(\Psi_0^-, \psi_{H,l_1}(x_1+a)\cdots\psi_{H,l_n}(x_n+a)\psi_{H,l_{n+1}}(x_{n+1}+a)\cdots\psi_{H,l_{n+l}}(x_{n+l}+a)\Psi_0^+)$

编时乘积的格林函数$(\Psi_0^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$

讨论前n个粒子在无穷将来，后l个粒子在无穷过去的情况:

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times\sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l}\theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)$$

$$\times(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times\sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l}\theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)$$

$$\times\int d\gamma d\gamma'(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\gamma^-)(\Psi_\gamma^-, \Psi_{\gamma'}^+)(\Psi_{\gamma'}^+, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$ 前n个粒子在无穷将来，后l个粒子在无穷过去的情况

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times \sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l} \theta(x_1^0 - x_2^0)\cdots\theta(x_{n-1}^0 - x_n^0)\theta(x_{n+1}^0 - x_{n+2}^0)\cdots\theta(x_{n+l-1}^0 - x_{n+l}^0)$$

$$\times \sum_{\sigma_1\cdots\sigma_{n+l}} \int d\vec{p}_1\cdots d\vec{p}_{n+l}\,(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$\Psi_0^\pm = \Psi_0 \quad \Psi^\pm_{\vec{p},\sigma} = \Psi_{\vec{p},\sigma}$ 进态和出态的粒子无穷分离 $\psi_{H,l_i}(x_i) = e^{iP\cdot x_i}\psi_H(0)e^{-iP\cdot x_i}$

$$(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}) = (\Psi_0, \psi_{H,l_1}(x_1)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(x_n)\Psi_{\vec{p}_n,\sigma_n})$$

$$= e^{-ip_1\cdot x_1 - \cdots - ip_n\cdot x_n}(\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= (\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(x_{n+1})\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(x_{n+l})\Psi_0)$$

$$= e^{ip_{n+1}\cdot x_{n+1} + \cdots + ip_{n+l}\cdot x_{n+l}}(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(0)\Psi_0)$$

为什么只考虑n和l粒子的中间态？

$$\int d\gamma d\gamma' (\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\gamma^-)(\Psi_\gamma^-, \Psi_{\gamma'}^+)(\Psi_{\gamma'}^+, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= \sum_{\sigma_1\cdots\sigma_{n+l}} \int d\vec{p}_1\cdots d\vec{p}_{n+l}\,(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}) = (\Psi_0, \psi_{H,l_1}(x_1)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(x_n)\Psi_{\vec{p}_n,\sigma_n})$$

$$(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= (\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(x_{n+1})\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(x_{n+l})\Psi_0)$$

- ♣ 如果考虑$m > n$粒子的中间态： 因子化后会出现真空态与单粒子态的内积,因而为零
- ♠ 如果考虑$m < n$粒子的中间态： 无穷分离时因子化后的会出现单个场的真空期望值，因而为零

多粒子态的归一化：记$\Phi_{p_1,\sigma_1,n;p_2,\sigma_2,n_2;\cdots} = \Phi_{p_1,p_2,\cdots}$

真空态Φ_0和单粒子态Φ_q

$$(\Phi_0,\Phi_0) = 1 \qquad (\Phi_{q'},\Phi_q) = \delta(q'-q) \equiv \delta^3(\vec{q}\,'-\vec{q})\delta_{\sigma'\sigma}\delta_{n'n}$$

对两粒子态$\Phi_{q_1q_2}$ $(\Phi_{q_1'q_2'},\Phi_{q_1q_2}) = \frac{1}{2!}[\delta(q_1'-q_1)\delta(q_2'-q_2) \pm \delta(q_2'-q_1)\delta(q_1'-q_2)]$

负号对两个粒子都是费米子的情形,正号对其它情形(两个粒子都是玻色子或一个是玻色子一个是费米子).

一般情况: $$(\Phi_{q_1'q_2'\cdots q_M'},\Phi_{q_1q_2\cdots q_N}) = \frac{\delta_{MN}}{N!}\sum_{\mathcal{P}}\delta_{\mathcal{P}}\prod_i \delta(q_i - q_{\mathcal{P}i}')$$

求和对所有可能的对指标$1,2,\cdots,N$的交换排序$\mathcal{P}$实行.对交换排序中涉及奇数次费米子交换时,$\delta_{\mathcal{P}}=-1$,其它情况的交换排序$\delta_{\mathcal{P}}=1$.

约定: $$(\Phi_{\alpha'},\Phi_\alpha) = \delta(\alpha'-\alpha) \qquad \int d\alpha\cdots \equiv \sum_{n_1\sigma_1n_2\sigma_2\cdots}\int d\vec{p}_1\int d\vec{p}_2\cdots$$

$$\Phi = \int d\alpha\,\Phi_\alpha(\Phi_\alpha,\Phi) \qquad 1 = \int d\alpha\,\Phi_\alpha)(\Phi_\alpha$$

多粒子态构成完备集!

约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times \sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l} \theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)$$

$$\times \sum_{\sigma_1\cdots\sigma_{n+l}}\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1x_1-\cdots-ip_nx_n+ip_{n+1}x_{n+1}+\cdots+ip_{n+l}x_{n+l}}(\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\sum_{\mathcal{P}\ 1,\cdots,n}\theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)=1 \qquad \sum_{\mathcal{P}\ n+1,\cdots,n+l}\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)=1$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times \sum_{\sigma_1\cdots\sigma_{n+l}}\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1x_1-\cdots-ip_nx_n+ip_{n+1}x_{n+1}+\cdots+ip_{n+l}x_{n+l}}(\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

约化公式

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$$ 前n个粒子在无穷将来，后l个粒子在无穷过去的情况

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times\sum_{\sigma_1\cdots\sigma_{n+l}}\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1x_1-\cdots-ip_nx_n+ip_{n+1}x_{n+1}+\cdots+ip_{n+l}x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\theta(x_i^0-x_{i+1}^0) = -\frac{1}{2i\pi}\int_{-\infty}^{\infty}d\omega_i\frac{e^{-i\omega_i(x_i^0-x_{i+1}^0)}}{\omega_i+i0+}$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}d\omega_1\cdots d\omega_{n+l}\frac{e^{-i\omega_1x_1^0-\cdots-i\omega_nx_n^0+i\omega_{n+1}x_{n+1}^0+\cdots+i\omega_{n+l}x_{n+l}^0}}{(\omega_1+i0+)\cdots(\omega_{n+l}+i0+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}\frac{d\omega_1\cdots d\omega_{n+l}e^{-i\omega_1x_1^0\cdots-i\omega_nx_n^0+i\omega_{n+1}x_{n+1}^0\cdots+i\omega_{n+l}x_{n+l}^0}}{(\omega_1+i0^+)\cdots(\omega_{n+l}+i0^+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}\frac{d\omega_1\cdots d\omega_{n+l}}{(\omega_1+i0^+)\cdots(\omega_{n+l}+i0^+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip'_1\cdot x_1-\cdots-ip'_n\cdot x_n+ip'_{n+1}\cdot x_{n+1}+\cdots+ip'_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\vec{p}_i' = \vec{p}_i \quad i = 1,2,\cdots,n+l$$

$$p_i^{0\prime} = \sqrt{\vec{p}_i^2+M_i^2}+\omega_i \quad i=1,2,\cdots,n+l$$

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}\frac{d\omega_1\cdots d\omega_{n+l}}{(\omega_1+i0^+)\cdots(\omega_{n+l}+i0^+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1'\cdot x_1-\cdots-ip_n'\cdot x_n+ip_{n+1}'\cdot x_{n+1}+\cdots+ip_{n+l}'\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\vec{p}_i' = \vec{p}_i \quad i=1,2,\cdots,n+l$$

$$p_i^{0\prime} = \sqrt{\vec{p}_i^2+M_i^2}+\omega_i \quad i=1,2,\cdots,n+l$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int d^4p_1\cdots d^4p_{n+l}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})$$

$$\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)$$

$$\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)(p_1^0-\sqrt{\vec{p}_1^2+M_1^2}+i0^+)^{-1}\cdots(p_{n+l}^0-\sqrt{\vec{p}_{n+l}^2+M_{n+l}^2}+i0^+)^{-1}$$

约化公式

$\left(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+\right)\Big|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int d^4p_1\cdots d^4p_{n+l}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})$$

$$\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)$$

$$\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)\left(p_1^0-\sqrt{\vec{p}_1^2+M_1^2}+i0^+\right)^{-1}\cdots\left(p_{n+l}^0-\sqrt{\vec{p}_{n+l}^2+M_{n+l}^2}+i0^+\right)^{-1}$$

$$\frac{1}{(p^0-\sqrt{\vec{p}^2+M^2}+i0^+)}=\frac{(p^0+\sqrt{\vec{p}^2+M^2}-i0^+)}{(p^0)^2-(\vec{p}^2+M^2-i0^+)}=\frac{2\sqrt{\vec{p}^2+M^2}}{p^2-M^2+i0^+}+\text{非极点项}$$

$$=\int\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\left\{\frac{2i(2\pi)^3p_1^0(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2-M_1^2+i0^+}\right.$$

$$\cdots\frac{2i(2\pi)^3p_n^0(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2-M_n^2+i0^+}(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})$$

$$\left.\times\frac{2i(2\pi)^3p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{H,l_{n+1}}(0)\Psi_0)}{p_{n+1}^2-M_{n+1}^2+i0^+}\cdots\frac{2i(2\pi)^3p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{H,l_{n+l}}(0)\Psi_0)}{p_{n+l}^2-M_{n+l}^2+i0^+}+\text{非物理极点项}\right\}$$

约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$
=\int\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\left\{\frac{2i(2\pi)^3p_1^0(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2-M_1^2+i0^+}\right.
$$

$$
\cdots\frac{2i(2\pi)^3p_n^0(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2-M_n^2+i0^+}(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})
$$

$$
\left.\times\frac{2i(2\pi)^3p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{H,l_{n+1}}(0)\Psi_0)}{p_{n+1}^2-M_{n+1}^2+i0^+}\cdots\frac{2i(2\pi)^3p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{H,l_{n+l}}(0)\Psi_0)}{p_{n+l}^2-M_{n+l}^2+i0^+}+\text{非物理极点项}\right\}
$$

$$
=\int\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\left\{\frac{2i(2\pi)^3p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2-M_1^2+i0^+}\right.
$$

$$
\cdots\frac{2i(2\pi)^3p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2-M_n^2+i0^+}(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})
$$

去掉了下标H！

$$
\left.\times\frac{2i(2\pi)^3p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)}{p_{n+1}^2-M_{n+1}^2+i0^+}\cdots\frac{2i(2\pi)^3p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{n+l}(0)\Psi_0)}{p_{l_{n+l}}^2-M_{n+l}^2+i0^+}+\text{非物理极点项}\right\}
$$

约化公式

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}$$

$$=\int\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\left\{\frac{2i(2\pi)^3p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2-M_1^2+i0^+}\right.$$

$$\cdots\frac{2i(2\pi)^3p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2-M_n^2+i0^+}(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})$$

$$\left.\times\frac{2i(2\pi)^3p_{n+1}^0(\Psi_{\vec{p}_{n+1}\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)}{p_{n+1}^2-M_{n+1}^2+i0^+}\cdots\frac{2i(2\pi)^3p_{n+l}^0(\Psi_{\vec{p}_{n+l}\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)}{p_{n+l}^2-M_{n+l}^2+i0^+}+\text{非物理极点项}\right\}$$

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来,后}l\text{个粒子在无穷过去的情况}}$$

$$=\int\frac{d^4p_1\cdots d^4p_{n+l}}{(i\pi)^{n+l}}e^{-ip_1\cdot x_1\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\left\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n})\right.$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots\vec{p}_{n+l},\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec{p}_{n+1}\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l}\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

$$\left.+\text{物理零点项}\right\}$$

约化公式

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$$ 前n个粒子在无穷将来,后l个粒子在无穷过去的情况

$$=\int\frac{d^4p_1\cdots d^4p_{n+l}}{(i\pi)^{n+l}}e^{-ip_1\cdot x_1\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\Big\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots\vec{p}_{n+l},\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

$$+\text{物理零点项}\Big\}$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$ 不再要求 前n个粒子在无穷将来，后l个粒子在无穷过去

$$\equiv\int\frac{d^4p_1\cdots d^4p_{n+l}}{(2\pi)^{4n+4l}}e^{-ip_1\cdot x_1\cdots-ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}G_{00}(p_1,\cdots,p_n,p_{n+1},\cdots,p_{n+l})$$

$$(-p_1^2+M_1^2)\cdots(-p_{n+l}^2+M_{n+l}^2)G_{00}(p_1,\cdots,p_n,-p_{n+1},\cdots,-p_{n+l})$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}\Big\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n,\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots\vec{p}_{n+l},\sigma_{n+l}})$$

$$\times p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)+\text{物理零点项}\Big\}$$

约化公式

$$(-p_1^2+M_1^2)\cdots(-p_{n+l}^2+M_{n+l}^2)G_{00}(p_1,\cdots,p_n,-p_{n+1},\cdots,-p_{n+l})$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}\Big\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1}\sigma_{n+1},\cdots\vec{p}_{n+l}\sigma_{n+l}})$$

$$\times p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)+\text{物理零点项}\Big\}$$

$$=\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$\lim_{p_i^2\to M_i^2}\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1}\sigma_{n+1},\cdots\vec{p}_{n+l}\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec p_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec p_n\sigma_n})$$

$$\times(\Psi^-_{\vec p_1,\sigma_1,\cdots,\vec p_n\sigma_n},\Psi^+_{\vec p_{n+1}\sigma_{n+1},\cdots\vec p_{n+l}\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec p_{n+1}\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec p_{n+l}\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

实标量场: $\phi(x)=\int\frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec p)+e^{ip\cdot x}a^\dagger(\vec p)]$ $\quad a(\vec p)\Phi_{\vec q}=\delta(p-q)\Phi_0$

$a(\vec p)\Psi_{\vec q}=Z_\phi\delta(p-q)\Psi_0$ $\quad (2\pi)^{3/2}\sqrt{2q^0}(\Psi_0,\phi(0)\Psi_{\vec q})=Z_\phi$ $\quad Z_\phi=0$ 束缚态

$$(\Psi^-_{\vec q_1,\cdots,\vec q_n},\Psi^+_{\vec q_{n+1},\cdots,\vec q_{n+l}})$$

$$=\frac{i^{n+l}}{(2\pi)^{3(n+l)/2}Z_\phi^{n+l}\sqrt{2q_1^0\cdots2q_{n+l}^0}}\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n\cdot x_n-iq_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_{n+l})\Psi_0^+)$$

约化公式

复标量场: $\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{c\dagger}(\vec{p})]$

$$\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n^c\cdot x_n-ip_{n+1}^c\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots$$

$$\phi_H(x_{n'})\phi_H^\dagger(x_{n'+1})\cdots\phi_H^\dagger(x_n)\phi_H(x_{n+1})\cdots\phi_H(x_{n+l'})\phi_H^\dagger(x_{n+l'+1})\cdots\phi_H^\dagger(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}p_1^0(\Psi_0,\phi(0)\Psi_{\vec{p}_1})\cdots p_{n'}^0(\Psi_0,\phi(0)\Psi_{\vec{p}_{n'}})p_{n'+1}^{c0}(\Psi_0,\phi^\dagger(0)\Psi_{\vec{p}^c_{n'+1}})\cdots p_n^{c0}(\Psi_0,\phi^\dagger(0)\Psi_{\vec{p}^c_n})$$

$$\times(\Psi^-_{\vec{p}_1,\cdots,\vec{p}_{n'},\vec{p}^c_{n'+1},\cdots,\vec{p}^c_n},\Psi^+_{\vec{p}^c_{n+1},\cdots,\vec{p}^c_{n+l'},\vec{p}_{n+l'+1}\cdots,\vec{p}_{n+l}})p^{c0}_{n+1}(\Psi_{\vec{p}^c_{n+1}},\phi(0)\Psi_0)\cdots p^{c0}_{n+l'}(\Psi_{\vec{p}^c_{n+l'}},\phi(0)\Psi_0)$$

$$\times p^0_{n+l'+1}(\Psi_{\vec{p}_{n+l'+1}},\phi^\dagger(0)\Psi_0)\cdots p^0_{n+l}(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\phi^\dagger(0)\Psi_0)$$

$$a(\vec{p})\Psi_{\vec{q}} = Z_\phi\delta(p-q)\Psi_0 \qquad (2\pi)^{3/2}\sqrt{2q^0}(\Psi_0,\phi(0)\Psi_{\vec{q}}) = (2\pi)^{3/2}\sqrt{2q^0}(\Psi_{\vec{q}},\phi^\dagger(0)\Psi_0) = Z_\phi$$

$$a^c(\vec{p})\Psi_{\vec{q}^c} = Z_\phi\delta(p-q^c)\Psi_0 \qquad (2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_0,\phi^\dagger(0)\Psi_{\vec{q}^c}) = (2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_{\vec{q}^c},\phi(0)\Psi_0) = Z_\phi$$

$$(\Psi^-_{\vec{q}_1,\cdots,\vec{q}_{n'},\vec{q}^c_{n'+1}\cdots,\vec{q}^c_n},\Psi^+_{\vec{q}^c_{n+1},\cdots,\vec{q}^c_{n+l'},\vec{q}_{n+l'+1},\cdots,\vec{q}_{n+l}}) = \frac{[i/(2\pi)^{3/2}\sqrt{2}]^{n+l}}{Z_\phi^{n+l}\sqrt{q_1^0\cdots q_n^{c0}q_{n+1}^{c0}\cdots q_{n+l}^0}}\int d^4x_1\cdots d^4x_{n+l}$$

$$\times e^{iq_1\cdot x_1\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H^\dagger(x_n)\phi_H(x_{n+1})\cdots\phi_H^\dagger(x_{n+l})\Psi_0^+)$$

$$\int d^4x_1\cdots d^4x_{n+l}\; e^{iq_1\cdot x_1+\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots$$

$$\psi_{H,l_{n'}}(x_{n'})\overline{\psi}_{H,l_{n'+1}}(x_{n'+1})\cdots\overline{\psi}_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l'}}(x_{n+l'})\overline{\psi}_{H,l_{n+l'+1}}(x_{n+l'+1})\cdots\overline{\psi}_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_n^c\sigma_{n+1}^c\cdots\sigma_{n+l}} q_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{q}_1,\sigma_1})\cdots q_n^{c0}(\Psi_0,\overline{\psi}_{l_n}(0)\Psi_{\vec{q}_n^c,\sigma_n^c})$$

$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n^c,\sigma_n^c},\Psi^+_{\vec{q}_{n+1}^c,\sigma_{n+1}^c,\cdots,\vec{q}_{n+l},\sigma_{n+l}})q_{n+1}^{c0}(\Psi_{\vec{q}_{n+1}^c,\sigma_{n+1}^c},\psi_{n+1}(0)\Psi_0)\cdots q_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}},\overline{\psi}_{l_{n+l}}(0)\Psi_0)$$

旋量场: $\psi_l(x)=(2\pi)^{-3/2}\sum_\sigma\int d\vec{p}\,[u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma)+v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)]$

$$\overline{\psi}_l(x)=(2\pi)^{-3/2}\sum_\sigma\int d\vec{p}\,[\overline{u}_l(\vec{p},\sigma)e^{ip\cdot x}a^\dagger(\vec{p},\sigma)+\overline{v}_l(\vec{p},\sigma)e^{-ip\cdot x}a^c(\vec{p},\sigma)]$$

$a(\vec{p})\Psi_{\vec{q}}=Z_\psi\delta(p-q)\Psi_0$ $\quad$ $a^c(\vec{p})\Psi_{\vec{q}^c}=Z_\psi\delta(p-q^c)\Psi_0$

$(2\pi)^{3/2}(\Psi_0,\psi_l(0)\Psi_{\vec{q},\sigma})=Z_\psi u_l(\vec{q},\sigma)$ $\quad$ $(2\pi)^{3/2}(\Psi_{\vec{q},\sigma},\overline{\psi}_l(0)\Psi_0)=Z_\psi\overline{u}_l(\vec{q},\sigma)$

$(2\pi)^{3/2}(\Psi_0,\overline{\psi}_l(0)\Psi_{\vec{q}^c,\sigma^c})=Z_\psi\overline{v}_l(\vec{q}^c,\sigma^c)$ $\quad$ $(2\pi)^{3/2}(\Psi_{\vec{q}^c,\sigma^c},\psi_l(0)\Psi_0)=Z_\psi v_l(\vec{q}^c,\sigma^c)$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\overline{\psi}_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-2^{5/2}i\pi^{3/2})^{n+l}Z_\psi^{n+l}q_1^0\cdots q_n^{c0}q_{n+1}^{c0}\cdots q_{n+l}^0\sum_{\sigma_1\cdots\sigma_n^c\sigma_{n+1}^c\cdots\sigma_{n+l}}u_{l_1}(\vec{q}_1,\sigma_1)\cdots\bar{v}_{l_n}(\vec{q}_n^c,\sigma_n^c)$$

$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n^c,\sigma_n^c},\Psi^+_{\vec{q}_{n+1}^c,\sigma_{n+1}^c\cdots,\vec{q}_{n+l},\sigma_{n+l}})v_{l_{n+1}}(\vec{q}_{n+1}^c,\sigma_{n+1}^c)\cdots\bar{u}_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})$$

$$u(\vec{p},\sigma)=\sqrt{M/p^0}D(L(p))u(0,\sigma)\qquad v(\vec{p},\sigma)=\sqrt{M/p^0}D(L(p))v(0,\sigma)\qquad D(L(p))\beta D^\dagger(L(p))=\beta$$

$$\beta=\gamma^0=\begin{pmatrix}0&-I\\-I&0\end{pmatrix}\quad u(0,\tfrac{1}{2})=\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{-1}{\sqrt{2}}\\0\end{bmatrix}\quad u(0,-\tfrac{1}{2})=\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\\0\\\frac{-1}{\sqrt{2}}\end{bmatrix}\quad v(0,\tfrac{1}{2})=\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix}\quad v(0,-\tfrac{1}{2})=\begin{bmatrix}\frac{-1}{\sqrt{2}}\\0\\\frac{-1}{\sqrt{2}}\\0\end{bmatrix}$$

$$\bar{u}(\vec{p},\sigma)u(\vec{p},\sigma')=\frac{M}{p^0}\bar{u}(0,\sigma)\beta D^\dagger(L(p))\beta D(L(p))u(0,\sigma')=\frac{M}{p^0}\bar{u}(0,\sigma)u(0,\sigma')=\frac{M}{p^0}\delta_{\sigma\sigma'}$$

$$\bar{v}(\vec{p},\sigma)v(\vec{p},\sigma')=\frac{M}{p^0}\bar{v}(0,\sigma)\beta D^\dagger(L(p))\beta D(L(p))v(0,\sigma')=\frac{M}{p^0}\bar{v}(0,\sigma)v(0,\sigma')=-\frac{M}{p^0}\delta_{\sigma\sigma'}$$

约化公式

$$
\left(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}^c_n,\sigma^c_n},\ \Psi^+_{\vec{q}^c_{n+1},\sigma^c_{n+1},\cdots,\vec{q}_{n+l},\sigma_{n+l}}\right)
$$

$$
\begin{aligned}
&= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots v_{l_n}(\vec{q}^c_n,\sigma^c_n)\\
&\quad\times\bar{v}_{l_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\\
&\quad\times(\partial_1^2+M_1^2)\cdots(\partial^2_{n+l}+M^2_{n+l})\left(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\overline{\psi}_{H,l_{n+l}}(x_{n+l})\Psi_0^+\right)
\end{aligned}
$$

$$
\begin{aligned}
&= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots v_{l_n}(\vec{q}^c_n,\sigma^c_n)\\
&\quad\times\bar{v}_{l_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\\
&\quad\times[(-i\partial\!\!\!/_{x_1}-M_1)(i\partial\!\!\!/_{x_1}-M_1)]_{l_1\bar{l}_1}\cdots[(-i\partial\!\!\!/_{x_n}-M_n)(i\partial\!\!\!/_{x_n}-M_n)]_{\bar{l}_nl_n}[(-i\partial\!\!\!/_{x_{n+1}}-M_{n+1})(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})]_{l_{n+1}\bar{l}_{n+1}}\\
&\quad\cdots[(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})(i\partial\!\!\!/_{x_{n+l}}-M_{n+l})]_{\bar{l}_{n+l}l_{n+l}}\left(\Psi_0^-,\mathbf{T}\psi_{H,\bar{l}_1}(x_1)\cdots\overline{\psi}_{H,\bar{l}_n}(x_n)\psi_{H,\bar{l}_{n+1}}(x_{n+1})\cdots\psi_{H,\bar{l}_{n+l}}(x_{n+l})\Psi_0^+\right)
\end{aligned}
$$

约化公式

$$\left(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}^c_n,\sigma^c_n},\ \Psi^+_{\vec{q}^c_{n+1},\sigma^c_{n+1},\cdots,\vec{q}_{n+l},\sigma_{n+l}}\right)$$

$$= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots v_{l_n}(\vec{q}^c_n,\sigma^c_n)$$

$$\times\bar{v}_{l_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times[(-i\partial\!\!\!/_{x_1}-M_1)(i\partial\!\!\!/_{x_1}-M_1)]_{l_1\bar{l}_1}\cdots[(-i\partial\!\!\!/_{x_n}-M_n)(i\partial\!\!\!/_{x_n}-M_n)]_{\bar{l}_nl_n}[(-i\partial\!\!\!/_{x_{n+1}}-M_{n+1})(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})]_{l_{n+1}\bar{l}_{n+1}}$$

$$\cdots[(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})(i\partial\!\!\!/_{x_{n+l}}-M_{n+l})]_{\bar{l}_{n+l}l_{n+l}}\left(\Psi^-_0,\mathbf{T}\psi_{H,\bar{l}_1}(x_1)\cdots\bar{\psi}_{H,\bar{l}_n}(x_n)\psi_{H,\bar{l}_{n+1}}(x_{n+1})\cdots\psi_{H,\bar{l}_{n+l}}(x_{n+l})\Psi^+_0\right)$$

$$= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots v_{l_n}(\vec{q}^c_n,\sigma^c_n)$$

$$\times\bar{v}_{l_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times[(-q\!\!\!/_1-M_1)(i\partial\!\!\!/_{x_1}-M_1)]_{l_1\bar{l}_1}\cdots[(-i\partial\!\!\!/_{x_n}-M_n)(q\!\!\!/^c_n-M_n)]_{\bar{l}_nl_n}[(q\!\!\!/^c_{n+1}-M_{n+1})(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})]_{l_{n+1}\bar{l}_{n+1}}$$

$$\cdots[(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})(-q\!\!\!/_{n+l}-M_{n+l})]_{\bar{l}_{n+l}l_{n+l}}\left(\Psi^-_0,\mathbf{T}\psi_{H,\bar{l}_1}(x_1)\cdots\bar{\psi}_{H,\bar{l}_n}(x_n)\psi_{H,\bar{l}_{n+1}}(x_{n+1})\cdots\psi_{H,\bar{l}_{n+l}}(x_{n+l})\Psi^+_0\right)$$

$$(q\!\!\!/_{n+l}-M_{n+l})u(\vec{q}_{n+l},\sigma_{n+l})=0\quad \bar{u}(\vec{q}_1,\sigma)(q\!\!\!/_1-M_1)=0\quad (q\!\!\!/^c_n+M_n)v(\vec{q}^c_n,\sigma^c_n)=0\quad \bar{v}(\vec{q}^c_{n+1},\sigma^c_{n+1})(q\!\!\!/^c_{n+1}+M_{n+1})=0$$

约化公式

$$\left(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_{n'},\sigma_{n'},\vec{q}^c_{n'+1},\sigma^c_{n'+1},\cdots,\vec{q}^c_n,\sigma^c_n},\ \Psi^+_{\vec{q}^c_{n+1},\sigma^c_{n+1},\cdots,\vec{q}^c_{n+l'},\sigma^c_{n+l'},\vec{q}_{n+l'+1},\sigma_{n+l'+1},\cdots,\vec{q}_{n+l},\sigma_{n+l}}\right)$$

$$=(-1)^{-n'+l'-l}[(2\pi)^{\frac{3}{2}}i]^{-n-l}Z_\psi^{-n-l}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots\bar{u}_{l_{n'}}(\vec{q}_{n'},\sigma_{n'})v_{l_{n'+1}}(\vec{q}^c_{n'+1},\sigma^c_{n'+1})\cdots v_{l_n}(\vec{q}^c_n,\sigma^c_n)$$

$$\times\bar{v}_{l_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots\bar{v}_{l_{n+l'}}(\vec{q}^c_{n+l'},\sigma^c_{n+l'})u_{l_{n+l'+1}}(\vec{q}_{n+l'+1},\sigma_{n+l'+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})$$

$$\times\int d^4x_1\cdots d^4x_{n+l}\,e^{iq_1\cdot x_1+\cdots+iq_{n'}\cdot x_{n'}+iq^c_{n'+1}\cdot x_{n'+1}+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}-\cdots-iq^c_{n+l'}\cdot x_{n+l'}-iq_{n+l'+1}\cdot x_{n+l'+1}-\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times(i\partial\!\!\!/_{x_1}-M_1)_{l_1\bar{l}_1}\cdots(i\partial\!\!\!/_{x_{n'}}-M_{n'})_{l_{n'}\bar{l}_{n'}}(-i\partial\!\!\!/_{x_{n'+1}}-M_{n'+1})_{\bar{l}_{n'+1}l_{n'+1}}\cdots(-i\partial\!\!\!/_{x_n}-M_n)_{\bar{l}_nl_n}$$

$$\times(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})_{l_{n+1}\bar{l}_{n+1}}\cdots(i\partial\!\!\!/_{x_{n+l'}}-M_{n+l'})_{l_{n+l'}\bar{l}_{n+l'}}(-i\partial\!\!\!/_{x_{n+l'+1}}-M_{n+l'+1})_{\bar{l}_{n+l'+1}l_{n+l'+1}}\cdots(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})_{\bar{l}_{n+l}l_{n+l}}$$

$$\times\left(\Psi_0^-,\mathbf{T}\psi_{H,\bar{l}_1}(x_1)\cdots\overline{\psi}_{H,\bar{l}_n}(x_n)\psi_{H,\bar{l}_{n+1}}(x_{n+1})\cdots\overline{\psi}_{H,\bar{l}_{n+l}}(x_{n+l})\Psi_0^+\right)$$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}^\dagger(x_n)v_{H,\mu_{n+1}}(x_{n+1})\cdots v_{H,\mu_{n+l}}^\dagger(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_n^c\sigma_{n+1}\cdots\sigma_{n+l}^c}q_1^0(\Psi_0,v_{\mu_1}(0)\Psi_{\vec q_1,\sigma_1})\cdots q_n^{c0}(\Psi_0,v_{\mu_n}(0)\Psi_{\vec q_n^c,\sigma_n^c})$$

$$\times(\Psi^-_{\vec q_1,\sigma_1,\cdots,\vec q_n^c,\sigma_n^c},\Psi^+_{\vec q_{n+1}^c\sigma_{n+1}^c,\cdots\vec q_{n+l}\sigma_{n+l}})q_{n+1}^{c0}(\Psi_{\vec q_{n+1}^c,\sigma_{n+1}^c},v_{\mu_{n+1}}(0)\Psi_0)\cdots q_{n+l}^0(\Psi_{\vec q_{n+l},\sigma_{n+l}},v_{\mu_{n+l}}(0)\Psi_0)$$

矢量场: $v^\mu(x)=(2\pi)^{-3/2}\sum_{\sigma=\pm1}\int\frac{d\vec p}{\sqrt{2p^0}}[e^\mu(\vec p,\sigma)a(\vec p,\sigma)e^{-ip\cdot x}+e^{\mu*}(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)e^{ip\cdot x}]$

$(2\pi)^{3/2}\sqrt{2q^0}(\Psi_0,v^\mu(0)\Psi_{\vec q,\sigma})=Z_ve^\mu(\vec q,\sigma)$ $\quad(2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_{\vec q^c,\sigma^c},v^\mu(0)\Psi_0)=Z_ve^{\mu*}(\vec q^c,\sigma^c)$

$(2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_0,v^{\mu\dagger}(0)\Psi_{\vec q^c,\sigma^c})=Z_ve^\mu(\vec q^c,\sigma^c)$ $\quad(2\pi)^{3/2}\sqrt{2q^0}(\Psi_{\vec q,\sigma},v^{\mu\dagger}(0)\Psi_0)=Z_ve^{\mu*}(\vec q,\sigma)$

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}v_{H\mu_1}(x_1)\cdots v^\dagger_{H\mu_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-4i\pi^{3/2})^{n+l}Z_v^{n+l}\sqrt{q_1^0\cdots q_{n+l}^0}\sum_{\sigma_1\cdots\sigma_{n+l}}e_{\mu_1}(\vec q_1,\sigma_1)\cdots e_{\mu_n}(\vec q_n^c,\sigma_n^c)(\Psi^-_{\vec q_1,\sigma_1,\cdots,\vec q_n^c,\sigma_n^c},\Psi^+_{\vec q_{n+1}^c\sigma_{n+1}^c,\cdots\vec q_{n+l}\sigma_{n+l}})$$

$$\times e^*_{\mu_{n+1}}(\vec q_{n+1}^c,\sigma_{n+1}^c)\cdots e^*_{\mu_{n+l}}(\vec q_{n+l},\sigma_{n+l})$$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)\left(\Psi_0^-,\mathbf{T}v_{H\mu_1}(x_1)\cdots v^\dagger_{H\mu_{n+l}}(x_{n+l})\Psi_0^+\right)$$

$$=(-4i\pi^{3/2})^{n+l}Z_v^{n+l}\sum_{\sigma_1\cdots\sigma^c_{n+l}}\sqrt{q_1^0}e_{\mu_1}(\vec q_1,\sigma_1)\cdots\sqrt{q_n^{c0}}e_{\mu_n}(\vec q_n^c,\sigma_n^c)\left(\Psi^-_{\vec q_1,\sigma_1,\cdots,\vec q_n^c,\sigma_n^c},\Psi^+_{\vec q^c_{n+1},\sigma^c_{n+1},\cdots\vec q_{n+l},\sigma_{n+l}}\right)$$

$$\times\sqrt{q_{n+1}^{c0}}e^*_{\mu_{n+1}}(\vec q^c_{n+1},\sigma^c_{n+1})\cdots\sqrt{q^0_{n+l}}e^*_{\mu_{n+l}}(\vec q_{n+l},\sigma_{n+l})$$

$$\left(\Psi^-_{\vec q_1,\sigma_1,\cdots,\vec q_n^c,\sigma_n^c},\Psi^+_{\vec q^c_{n+1},\sigma^c_{n+1},\cdots,\vec q_{n+l},\sigma_{n+l}}\right)\qquad e^\mu(\vec p,\sigma)e_\mu(\vec p,\sigma')=-\delta_{\sigma\sigma'}$$

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$$=\frac{e^{\mu_1}(\vec q_1,\sigma_1)\cdots e^{\mu_n}(\vec q_n^c,\sigma_n^c)e^{*\mu_{n+1}}(\vec q^c_{n+1},\sigma^c_{n+1})\cdots e^{*\mu_{n+l}}(\vec q_{n+l},\sigma_{n+l})}{(4i\pi^{3/2})^{n+l}Z_v^{n+l}\sqrt{q_1^0}\cdots\sqrt{q_n^{c0}}\sqrt{q^{c0}_{n+1}}\cdots\sqrt{q^0_{n+l}}}\int d^4x_1\cdots d^4x_{n+l}$$

$$\times e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial^2_{n+l}+M^2_{n+l})\left(\Psi_0^-,\mathbf{T}v_{H,\mu_1}(x_1)\cdots v^\dagger_{H,\mu_{n+l}}(x_{n+l})\Psi_0^+\right)$$

态之间的编时乘积的格林函数$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,1}(x_1)\psi_{H,2}(x_2)\cdots\psi_{H,n}(x_n)\Psi_\alpha^+)$

Wick定理:

$$\mathbf{T}(UV\cdots XYZ) = \mathbf{N}(UV\cdots XYZ) \qquad \text{无收缩项}$$

$$+\mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ) + \qquad \text{一次收缩项}$$

$$+\mathbf{N}(UV\cdots XYZ) + \cdots \qquad \text{二次收缩项}$$

$$+\ \cdots$$

$$+\mathbf{N}(UVW\cdots XYZ) + \cdots \qquad \text{全部收缩项}$$

若T乘积中含N乘积,展开仍成立,但需将各N乘积内部各算符之间的收缩略去.

应用Wick定理: $U(t,t') = \Omega^\dagger(t)\Omega(t')$　$\Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha$

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$$= (\Omega(\infty)\Phi_\beta, \mathbf{T}\ \Omega(t_1)\psi_{l_1}(x_1)U(t_1,t_2)\psi_2(x_2)U(t_2,t_3)\cdots U(t_{n-1},t_n)\psi_{l_n}(x_n)\Omega^\dagger(t_n)\Omega(-\infty)\Phi_\alpha)$$

$$= (\Phi_\beta, \mathbf{T}\ \psi_{l_1}(x_1)\psi_2(x_2)\cdots\psi_{l_n}(x_n)U(\infty,-\infty)\Phi_\alpha) = \underline{\text{所有可能的特别收缩项之和}}$$

保留下来的正规乘积中的湮灭算符必须正好湮灭Φ_α态到某个态，产生算符的厄米共扼必须正好湮灭Φ_β态到同样的态! 特别地，

$$(\Psi_\beta^-, \Psi_\alpha^+) = (\Phi_\beta, U(\infty,-\infty)\Phi_\alpha)$$

$$(\Psi_0^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = (\Phi_0, \mathbf{T}\ \psi_{l_1}(x_1)\psi_2(x_2)\cdots\psi_{l_n}(x_n)U(\infty,-\infty)\Phi_0)$$

编时乘积的格林函数$(\Psi^-_\beta, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$

考虑 $x^0_1,\cdots,x^0_r > 0,\quad x^0_{r+1},\cdots,x^0_n < 0$

$$(\Psi^-_\beta, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

$$= \theta(\min[x^0_1\cdots x^0_r] - \max[x^0_{r+1}\cdots x^0_n]) \quad \Leftarrow \text{这里已把前r个场晚于后n-r个场的要求通过}\theta\text{函数施加进去了}$$

$$\times(\Psi^-_\beta, \mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi^+_\alpha)$$

$$= \theta(\min[x^0_1\cdots x^0_r] - \max[x^0_{r+1}\cdots x^0_n])$$

$$\times\int d\gamma(\Psi^-_\beta, \mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\Psi_\gamma)(\Psi_\gamma, \mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi^+_\alpha)$$

$$= \text{单粒子中间态项} + \text{多粒子中间态项}$$

Ψ_γ 是一组多粒子态完备集；是 H 的本征态

单粒子中间态项： $x_1^0,\cdots,x_r^0>0$, $x_{r+1}^0,\cdots,x_n^0<0$

$$\theta(\min[x_1^0\cdots x_r^0]-\max[x_{r+1}^0\cdots x_n^0])$$

$$\times\sum_\sigma\int d\vec{p}\,(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi_\alpha^+)$$

$$\psi_{H,l_i}(x_i)=e^{iP\cdot x_i}\psi_{H,l_i}(0)e^{-iP\cdot x_i}\qquad\psi_{H,l_i}(x_i-x)=e^{-iP\cdot x}\psi_{H,l_i}(x_i)e^{iP\cdot x}$$

$$\psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_r}(x_r)=e^{iP\cdot x_1}\psi_{H,l_1}(0)\psi_{H,l_2}(x_2-x_1)\cdots\psi_{H,l_r}(x_r-x_1)e^{-iP\cdot x_1}$$

$$\psi_{H,l_{r+1}}(x_{r+1})\psi_{H,l_{r+2}}(x_{r+2})\cdots\psi_{H,l_n}(x_n)=e^{iP\cdot x_{r+1}}\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(x_{r+2}-x_{r+1})\cdots\psi_{H,l_n}(x_n-x_{r+1})e^{-iP\cdot x_{r+1}}$$

$$e^{-iP\cdot x_1}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}e^{iP\cdot x_{r+1}}=e^{-iP\cdot x_1}\Psi_{\vec{p},\sigma})(e^{-iP\cdot x_{r+1}}\Psi_{\vec{p},\sigma}=e^{-ip\cdot(x_1-x_{r+1})}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}$$

$$=\theta(\min[x_1^0\cdots x_r^0]-\max[x_{r+1}^0\cdots x_n^0])$$

$$\times\sum_\sigma\int d\vec{p}\,e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}}(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(x_2-x_1)\cdots\psi_{H,l_r}(x_r-x_1)\}\Psi_{\vec{p},\sigma})$$

$$\times(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(x_{r+2}-x_{r+1})\cdots\psi_{H,l_n}(x_n-x_{r+1})\}\Psi_\alpha^+)$$

单粒子中间态项：$x_i = x_1 + y_i \ (i=2,3,\cdots,r) \quad x_i = x_{r+1} + y_i \ (i = r+2,\cdots,n)$

$$\theta(\min[x_1^0\cdots x_r^0] - \max[x_{r+1}^0\cdots x_n^0])$$

$$\times\sum_\sigma\int d\vec{p}\,(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi_\alpha^+)$$

$$= \theta(\min[x_1^0\cdots x_r^0] - \max[x_{r+1}^0\cdots x_n^0])$$

$$\times\sum_\sigma\int d\vec{p}\; e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}}(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(x_2-x_1)\cdots\psi_{H,l_r}(x_r-x_1)\}\Psi_{\vec{p},\sigma})$$

$$\times(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(x_{r+2}-x_{r+1})\cdots\psi_{H,l_n}(x_n-x_{r+1})\}\Psi_\alpha^+)$$

$$\min[x_1^0,\cdots,x_r^0]-\max[x_{r+1}^0,\cdots,x_n^0]=x_1^0-x_{r+1}^0+\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0]$$

$$= \frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])} \qquad \theta(\tau)=-\int_{-\infty}^{\infty}\frac{d\omega}{2i\pi}\frac{e^{-i\omega\tau}}{\omega+i0^+}$$

$$\times\sum_\sigma\int d\vec{p}(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$\times e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

格林函数的极点

$$(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) \qquad x_1^0,\cdots,x_r^0>0,\quad x_{r+1}^0,\cdots,x_n^0<0$$

$$= \frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

$$\times\sum_\sigma\int d\vec{p}(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p}\sigma})(\Psi_{\vec{p}\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$x_i = x_1 + y_i\ (i=2,3,\cdots,r)\quad x_i = x_{r+1}+y_i\ (i=r+2,\cdots,n)$$

$$(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) = \int\frac{d^4q_1\cdots d^4q_n}{(2\pi)^{4n}}\,e^{-iq_1\cdot x_1-\cdots-iq_n\cdot x_n}G_{\beta\alpha}(q_1,\cdots,q_n)$$

$$-i(q_1\cdot x_1+\cdots+q_n\cdot x_n)$$

$$= -i[(q_1+\cdots+q_r)\cdot x_1 + (q_{r+1}+\cdots+q_n)\cdot x_{r+1} + q_2\cdot y_2+\cdots+q_r\cdot y_r + q_{r+2}\cdot y_{r+2}+\cdots+q_n\cdot y_n]$$

$$(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) \qquad \tilde{q}_1\equiv q_1+\cdots+q_r\quad \tilde{q}_{r+1}\equiv q_{r+1}+\cdots+q_n$$

$$= \int\frac{d^4\tilde{q}_1 d^4\tilde{q}_{r+1}d^4q_2\cdots d^4q_r d^4q_{r+2}\cdots d^4q_n}{(2\pi)^{4n}}\,e^{-i\tilde{q}_1\cdot x_1-i\tilde{q}_{r+1}\cdot x_{r+1}-iq_2\cdot y_2-\cdots-iq_r\cdot y_r-iq_{r+2}\cdot y_{r+2}-\cdots-iq_n\cdot y_n}$$

$$\times G_{\beta\alpha}(q_1,\cdots,q_n)$$

格林函数的极点

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)|_{\text{单粒子中间态}} \qquad x_1^0,\cdots,x_r^0>0,\quad x_{r+1}^0,\cdots,x_n^0<0$$

$$= \frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}\int d\vec{p}\ e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

$$\times\sum_\sigma(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) \qquad \tilde{q}_1\equiv q_1+\cdots+q_r \quad \tilde{q}_{r+1}\equiv q_{r+1}+\cdots+q_n$$

$$=\int\frac{d^4\tilde{q}_1 d^4\tilde{q}_{r+1}d^4q_2\cdots d^4q_r d^4q_{r+2}\cdots d^4q_n}{(2\pi)^{4n}}e^{-i\tilde{q}_1\cdot x_1-i\tilde{q}_{r+1}\cdot x_{r+1}-iq_2\cdot y_2-\cdots-iq_r\cdot y_r-iq_{r+2}\cdot y_{r+2}-\cdots-iq_n\cdot y_n}$$

$$\times G_{\beta\alpha}(q_1,\cdots,q_n) \qquad y_i=x_i-y_1\ (i=2,3,\cdots,r) \qquad y_i=x_i-x_{r+1}\ (i=r+2,\cdots,n)$$

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\ e^{i\tilde{q}_1\cdot x_1+i\tilde{q}_{r+1}\cdot x_{r+1}+iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

$$\times(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$$=\int d^4x_1 d^4x_{r+1}d^4y_2\cdots d^4y_r d^4y_{r+2}\cdots d^4y_n\ (\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+$$

$$\times\ e^{i\tilde{q}_1\cdot x_1+i\tilde{q}_{r+1}\cdot x_{r+1}+iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

格林函数的极点

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)|_{\text{单粒子中间态}} \qquad x_1^0,\cdots,x_r^0>0,\quad x_{r+1}^0,\cdots,x_n<0$$

$$= \frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}\int d\vec{p}\ e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

$$\times\sum_\sigma(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$\tilde{q}_1\equiv q_1+\cdots+q_r\quad \tilde{q}_{r+1}\equiv q_{r+1}+\cdots+q_n\quad y_i=x_i-y_1\ (i=2,\cdots,r)\quad y_i=x_i-x_{r+1}\ (i=r+2,\cdots,n)$$

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1d^4x_{r+1}d^4y_2\cdots d^4y_rd^4y_{r+2}\cdots d^4y_n(\Psi_\beta^-,\mathbf{T}\psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+$$

$$\times\ e^{i\tilde{q}_1\cdot x_1+i\tilde{q}_{r+1}\cdot x_{r+1}+iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

$\Downarrow$ 下面只考虑了前**r**个场晚于后**n-r**个场的情形 $\Downarrow$

$$G_{\beta\alpha}(q_1,\cdots,q_n)|_{\text{单粒子中间态}}=\int d^4y_2\cdots d^4y_rd^4y_{r+2}\cdots d^4y_n\ e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

$$\times\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}\int d\vec{p}\ (2\pi)^8\delta(\vec{p}-\vec{p}_\beta-\vec{\tilde{q}}_1)$$

$$\times\delta(-\sqrt{\vec{p}^2+M^2}-\omega+p_\beta^0+\tilde{q}_1^0)\delta(-\vec{\tilde{q}}_{r+1}+\vec{p}_\alpha-\vec{p})\delta(\tilde{q}_{r+1}^0-p_\alpha^0+\sqrt{\vec{p}^2+M^2}+\omega$$

$$\times\sum_\sigma(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$G_{\alpha\beta}(q_1,\cdots,q_n)|$单粒子中间态

$$=\int d^4y_2\cdots d^4y_r d^4y_{r+2}\cdots d^4y_n e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}$$

$$\times\sum_\sigma\int d\vec{p}(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi$$

$$\times(2\pi)^8\delta(\vec{p}-\vec{p}_\beta-\vec{q}_1-\cdots-\vec{q}_r)\delta(-\sqrt{\vec{p}^2+M^2}-\omega+p_\beta^0+q_1^0+\cdots+q_r^0)$$

$$\times\delta(-\vec{q}_{r+1}-\cdots-\vec{q}_n+\vec{p}_\alpha-\vec{p})\delta(q_{r+1}^0+\cdots+q_n^0-p_\alpha^0+\sqrt{\vec{p}^2+M^2}+\omega)$$

$q\equiv q_1+\cdots+q_r+p_\beta=-q_{r+1}-\cdots-q_n+p_\alpha$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)\frac{\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)}{q^0-\sqrt{\vec{q}^2+M^2}+i0^+}$$

$$\frac{1}{q^0-\sqrt{\vec{q}^2+M^2}+i0^+}=\frac{q^0+\sqrt{\vec{q}^2+M^2}-i0^+}{(q^0)^2-(\vec{q}^2+M^2-i0^+)}=\frac{2\sqrt{\vec{q}^2+M^2}}{q^2-M^2+i0^+}+\text{非极点项}$$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)\frac{2q^0\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)}{q^2-M^2+i0^+}$$

格林函数的极点

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\, e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}(\Psi_\beta^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$G_{\beta\alpha}(q_1,\cdots,q_n)|_{单粒子中间态}$

$$=\int d^4y_2\cdots d^4y_r d^4y_{r+2}\cdots d^4y_n e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}$$

$$\times\sum_\sigma\int d\vec{p}\,(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi$$

$$\times(2\pi)^8\delta(\vec{p}-\vec{p}_\beta-\vec{q}_1-\cdots-\vec{q}_r)\delta(-\sqrt{\vec{p}^2+M^2}-\omega+p_\beta^0+q_1^0+\cdots+q_r^0)$$

$$\times\delta(-\vec{q}_{r+1}-\cdots-\vec{q}_n+\vec{p}_\alpha-\vec{p})\delta(q_{r+1}^0+\cdots+q_n^0-p_\alpha^0+\sqrt{\vec{p}^2+M^2}+\omega)$$

$q\equiv q_1+\cdots+q_r+p_\beta=-q_{r+1}-\cdots-q_n+p_\alpha$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)\frac{2q^0\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)}{q^2-M^2+i0^+}+\text{非极点项}$$

$$M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)=\int d^4y_2\cdots d^4y_r e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r}(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{q},\sigma})$$

$$M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)=\int d^4y_{r+2}\cdots d^4y_n e^{iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}(\Psi_{\vec{q},\sigma},\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

格林函数的极点

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\; e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}(\Psi^-_\beta,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

$$M_{\beta|\vec q,\sigma}(q_2,\cdots,q_r)=\int d^4y_2\cdots d^4y_r e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r}(\Psi^-_\beta,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec q,\sigma})$$

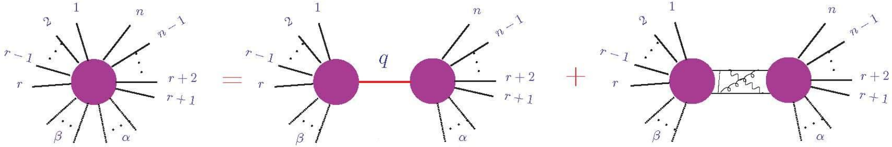
$$M_{-\vec q,\sigma|\alpha}(q_{r+2},\cdots,q_n)=\int d^4y_{r+2}\cdots d^4y_n e^{iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}(\Psi_{\vec q,\sigma},\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi^+_\alpha)$$

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\sum_\sigma\int\frac{d^4q}{(2\pi)^4}\left[(2\pi)^4\delta(q_1+\cdots+q_r+p_\beta-q)(2\pi)^{\frac{3}{2}}(2q^0)^{\frac{1}{2}}M_{\beta|\vec q,\sigma}(q_2,\cdots,q_r)\right]$$

$$\times\frac{i}{q^2-M^2+i0^+}$$ 通常单粒子态的贡献是最主要的！ $\Rightarrow$ 单粒子为主 有效拉氏量

$$\times\left[(2\pi)^4\delta(q+q_{r+1}+\cdots+q_n-p_\alpha)(2\pi)^{\frac{3}{2}}(2q^0)^{\frac{1}{2}}M_{-\vec q,\sigma|\alpha}(q_{r+2},\cdots,q_n)\right]$$

$+$多粒子中间态项 $+$ 非极点项



格林函数的极点

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\; e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}\left(\Psi_\beta^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+\right)$$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)(q^2-M^2+i0^+)^{-1}2q^0\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)$$

$$+\text{多粒子中间态项}+\text{非极点项}$$

$$M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)=\int d^4y_2\cdots d^4y_r e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r}\left(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{q},\sigma}\right)$$

$$M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)=\int d^4y_{r+2}\cdots d^4y_n e^{iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}\left(\Psi_{\vec{q},\sigma},\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$\int d^4x d^4y e^{iq\cdot x+iq'\cdot y}\left(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x)\psi_{H,l_2}(y)\Psi_0^+\right)=\int d^4x d^4y e^{iq\cdot(x-y)+i(q+q')\cdot y}\left(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x)\psi_{H,l_2}(y)\Psi_0\right.$$

$$=i(2\pi)^7\delta^4(q+q')\frac{2q^0\sum_\sigma\left(\Psi_0^-,\psi_{H,l_1}(0)\Psi_{\vec{q},\sigma}\right)\left(\Psi_{\vec{q},\sigma},\psi_{H,l_2}(0)\Psi_0^+\right)}{q^2-M^2+i0^+}+\text{其它项}$$

$$\left(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x)\psi_{H,l_2}(y)\Psi_0^+\right)=\int\frac{d^4q}{2\pi}\,e^{-iq\cdot(x-y)}\frac{2iq^0\sum_\sigma\left(\Psi_0^-,\psi_{H,l_1}(0)\Psi_{\vec{q},\sigma}\right)\left(\Psi_{\vec{q},\sigma},\psi_{H,l_2}(0)\Psi_0^+\right)}{q^2-M^2+i0^+}+\cdot$$

两点函数的谱函数: X:标量 $\psi(x)=e^{iP\cdot x}\psi(0)e^{-iP\cdot x}$ Ψ_0完全可替换为Ψ_α

$$(\Psi_0,X(x)X^\dagger(y)\Psi_0)=\sum_n(\Psi_0,X(x)\Psi_n)(\Psi_n,X^\dagger(y)\Psi_0)=\sum_n e^{-ip_n\cdot(x-y)}|(\Psi_0,X(0)\Psi_n)|^2$$

$$=\int d^4p\, e^{-ip\cdot(x-y)}\underbrace{\sum_n\delta^4(p-p_n)|(\Psi_0,X(0)\Psi_n)|^2}_{(2\pi)^{-3}\theta(p^0)\rho(p^2)\qquad p_n^0\ge 0}=\int_0^\infty d\mu^2\rho(\mu^2)\underbrace{\int\frac{d^4p}{(2\pi)^3}\,e^{-ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)}_{\Delta_+(x-y,\mu^2)}$$

$$(\Psi_0,X^\dagger(y)X(x)\Psi_0)=\sum_n(\Psi_0,X^\dagger(y)\Psi_n)(\Psi_n,X(x)\Psi_0)=\sum_n e^{ip_n\cdot(x-y)}|(\Psi_n,X(0)\Psi_0)|^2$$

$$=\int d^4p\, e^{ip\cdot(x-y)}\underbrace{\sum_n\delta^4(p-p_n)|(\Psi_n,X(0)\Psi_0)|^2}_{(2\pi)^{-3}\theta(p^0)\tilde\rho(p^2)\qquad p_n^0\ge 0}=\int_0^\infty d\mu^2\tilde\rho(\mu^2)\underbrace{\int\frac{d^4p}{(2\pi)^3}\,e^{ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)}_{\Delta_+(y-x,\mu^2)}$$

$$(\Psi_0,[X(x),X^\dagger(y)]\Psi_0)=\int_0^\infty d\mu^2[\rho(\mu^2)\Delta_+(x-y,\mu^2)-\tilde\rho(\mu^2)\Delta_+(y-x,\mu^2)]\xRightarrow{\text{类空;求导}}\rho(\mu^2)=\tilde\rho(\mu^2)$$

$$(\Psi_0,\mathbf{T}\,X(x)X^\dagger(y)\Psi_0)=\int_0^\infty d\mu^2\rho(\mu^2)\,\underbrace{[\theta(x^0-y^0)\Delta_+(x-y,\mu^2)+\theta(y^0-x^0)\Delta_+(y-x,\mu^2)]}_{\Delta_F(x-y,\mu^2)}$$

格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2)\Delta_F(x-y,\mu^2) \quad \Delta_+(x-y,\mu^2) \equiv \int \frac{d^4p}{(2\pi)^3} e^{-ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)$$

$$(\Psi_0, [X(x), X^\dagger(y)]\Psi_0) = \int_0^\infty d\mu^2\ \rho(\mu^2)[\Delta_+(x-y,\mu^2) - \Delta_+(y-x,\mu^2)]$$

$$\frac{\partial}{\partial x^0}\Delta_+(x-y,\mu^2)\Big|_{x^0=y^0} = \int \frac{d^4p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\theta(p^0)(-ip^0)\delta((p^0)^2 - \vec{p}^2 - \mu^2) = -\frac{i}{2}\delta(\vec{x}-\vec{y})$$

$$\frac{\partial}{\partial x^0}\Delta_+(y-x,\mu^2)\Big|_{x^0=y^0} = \int \frac{d^4p}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\theta(p^0)(ip^0)\delta((p^0)^2 - \vec{p}^2 - \mu^2) = \frac{i}{2}\delta(\vec{x}-\vec{y})$$

$$(\Psi_0, [\dot{X}(x), X^\dagger(y)]\Big|_{x^0=y^0}\Psi_0) = -i\delta(\vec{x}-\vec{y}) \int_0^\infty d\mu^2\ \rho(\mu^2)$$

如果 $\dot{X}\dot{X}^\dagger = -\ddot{X}X^\dagger$ 是动能项，$\dot{X}$对应的广义动量就是 $-X^\dagger$。 $\dot{\phi} = \frac{\delta H}{\delta\pi}$

$$[\dot{X}(x), X^\dagger(y)]\Big|_{x^0=y^0} = -i\delta(\vec{x}-\vec{y}) \Rightarrow \int_0^\infty d\mu^2\ \rho(\mu^2) = 1$$

只对基本场成立！

格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3\sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3\sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2)\Delta_F(x-y,\mu^2) \qquad \Delta_+(x-y,\mu^2) \equiv \int \frac{d^4p}{(2\pi)^3} e^{-ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)$$

$$\Delta_F(x-y,\mu^2) \equiv [\theta(x^0-y^0)\Delta_+(x-y,\mu^2) + \theta(y^0-x^0)\Delta_+(y-x,\mu^2)]$$

$$= \int \frac{d^4p}{(2\pi)^3}\ \delta((p^0)^2-(\vec{p})^2 - \mu^2)\theta(p^0)[e^{-ip\cdot(x-y)}\theta(x^0-y^0) + e^{ip\cdot(x-y)}\theta(y^0-x^0)]$$

$$= \int \frac{d^4p}{2p^0(2\pi)^3}\ \delta(p^0-\sqrt{\vec{p}^2+\mu^2})\theta(p^0)[e^{-ip\cdot(x-y)}\theta(x^0-y^0) + e^{ip\cdot(x-y)}\theta(y^0-x^0)]$$

$$= \int \frac{d\vec{p}}{2\sqrt{(\vec{p})^2+\mu^2}(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}[e^{-i\sqrt{\vec{p}^2+\mu^2}(x^0-y^0)}\theta(x^0-y^0) + e^{i\sqrt{\vec{p}^2+\mu^2}(x^0-y^0)}\theta(y^0-x^0)]$$

$$= \int \frac{d\vec{p}}{2\sqrt{(\vec{p})^2+\mu^2}(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0}{2i\pi}\ e^{-ip^0(x^0-y^0)+i\vec{p}\cdot(\vec{x}-\vec{y})}\left[-\frac{1}{p^0-\sqrt{\vec{p}^2+\mu^2}+i0^+} + \frac{1}{p^0+\sqrt{\vec{p}^2+\mu^2}-i0^+}\right]$$

$$= \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\frac{i}{p^2-\mu^2+i0^+} \qquad (\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\int_0^\infty d\mu^2\frac{i\rho(\mu^2)}{p^2-\mu^2+i0^+}$$

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, \Delta_X(p^2+i0^+) \qquad \Delta_X(z) = \int_0^\infty d\mu^2\, \frac{i\rho(\mu^2)}{z-\mu^2}$$

若中间态谱中最低质量M的单粒子态$\Psi_{\vec{p}_\alpha}$ $(\Psi_0, X(0)\Psi_{\vec{p}_\alpha}) \neq 0$，其对谱函数贡献：

$$(2\pi)^3 \int d\vec{p}_\alpha \delta(p^0 - \sqrt{\vec{p}_\alpha^2 + M^2})\delta(\vec{p}-\vec{p}_\alpha)|(\Psi_0, X(0)\Psi_{\vec{p}_\alpha})|^2$$

$$= (2\pi)^3\delta(p^0 - \sqrt{\vec{p}^2+M^2})|(\Psi_0, X(0)\Psi_{\vec{p}})|^2 = (2\pi)^3\theta(p^0)2p^0\delta((p^0)^2 - \vec{p}^2 - M^2)|(\Psi_0, X(0)\Psi_{\vec{p}})|^2$$

$$= Z_\alpha\delta(p^2 - M^2) \qquad Z_\alpha \equiv (2\pi)^3\theta(p^0)2p^0|(\Psi_0, X(0)\Psi_{\vec{p}})|^2 \geq 0$$

$$\rho(\mu^2) = Z_\alpha\delta(\mu^2 - M^2) + \sigma(\mu^2) \qquad 1 = Z_\alpha + \int_{M_0^2}^\infty d\mu^2\sigma(\mu^2)$$ $M_0 \geq M$为其它粒子或连续谱出现的下限 $Z_\alpha \leq 1$

$$\Delta_X(z) = \frac{iZ_\alpha}{z-M^2} + \int_{M_0^2}^\infty d\mu^2\, \frac{i\rho(\mu^2)}{z-\mu^2}$$ 可能有割线！ $-i\Delta_X(M^2 \pm 0^+) = \pm\infty \quad -i\Delta_X(M^2 \pm i0^+) = \mp i\infty$