Lecture 2: Deconfined quantum criticality

T. Senthil
(MIT)

General theoretical questions

- Fate of Landau-Ginzburg-Wilson ideas at quantum phase transitions?
- (More precise) Could Landau order parameters for the phases distract from the true critical behavior?

Study phase transitions in insulating quantum magnets

- Good theoretical laboratory for physics of phase transitions/competing orders.

(Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 2004)

Highlights

- Failure of Landau paradigm at (certain) quantum transitions

- Rough description: Emergence of `fractional' charge and gauge fields near quantum critical points between two CONVENTIONAL phases.

- ``Deconfined quantum criticality''

- Many lessons for competing order physics in correlated electron systems.

Phase transitions in quantum magnetism

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \ldots$$

- Spin-1/2 quantum antiferromagnets on a square lattice.

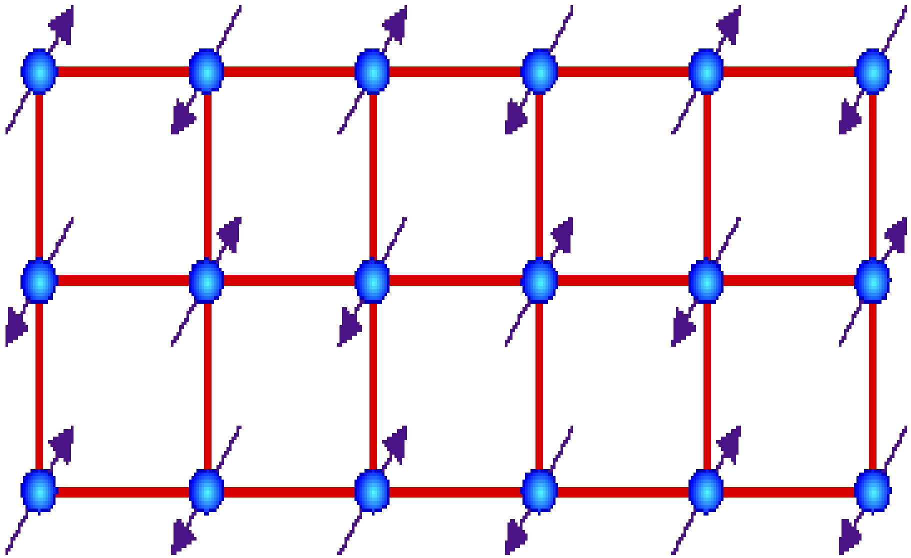
- ``……'' represent frustrating interactions that can be tuned to drive phase transitions.

(Eg: Next near neighbour exchange, ring exchange,…..).

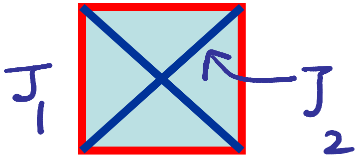
Possible quantum phases

- Neel ordered state

Order parameter – staggered magnetic moment vector



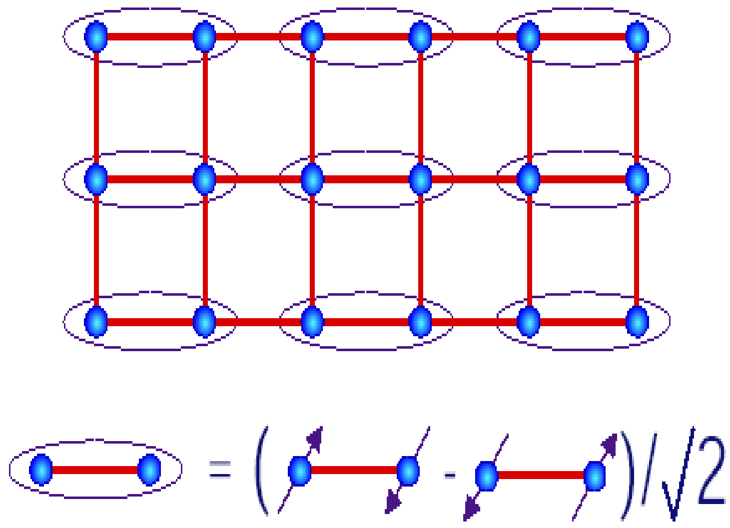
Neel ordering can be destroyed at zero temperature with suitable frustrating interactions



Example of a non-magnetic ground state - Valence Bond Solid State (VBS)

- Frozen arrangement of *singlet* bonds between pairs of local moments

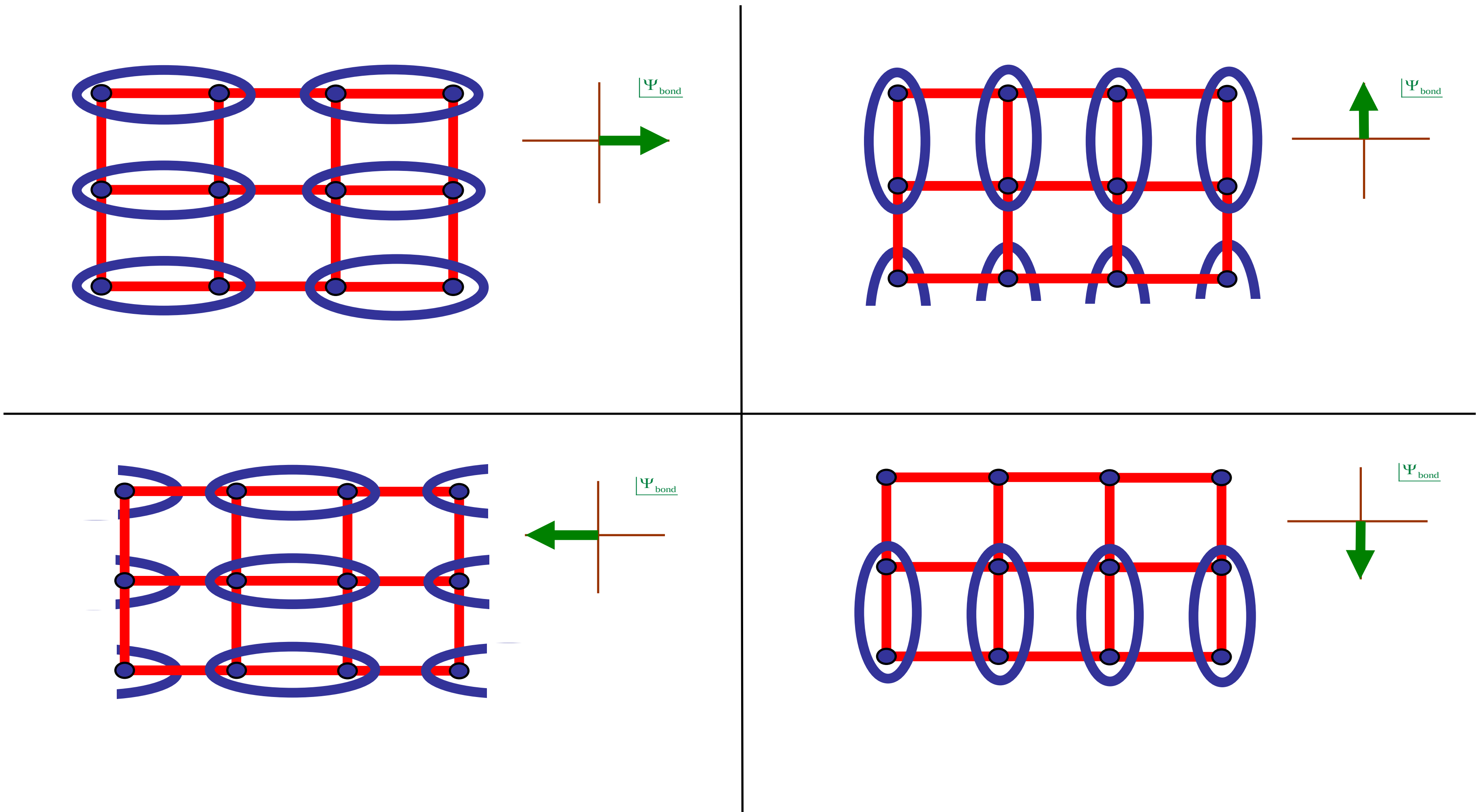
- Frozen singlet pattern breaks lattice translation and rotation symmetry



Discrete Z_4 Order Parameter Ψ_{bond}

VBS Order Parameter

- Associate a Complex Number Ψ_{bond}



Neel-valence bond solid(VBS) transition

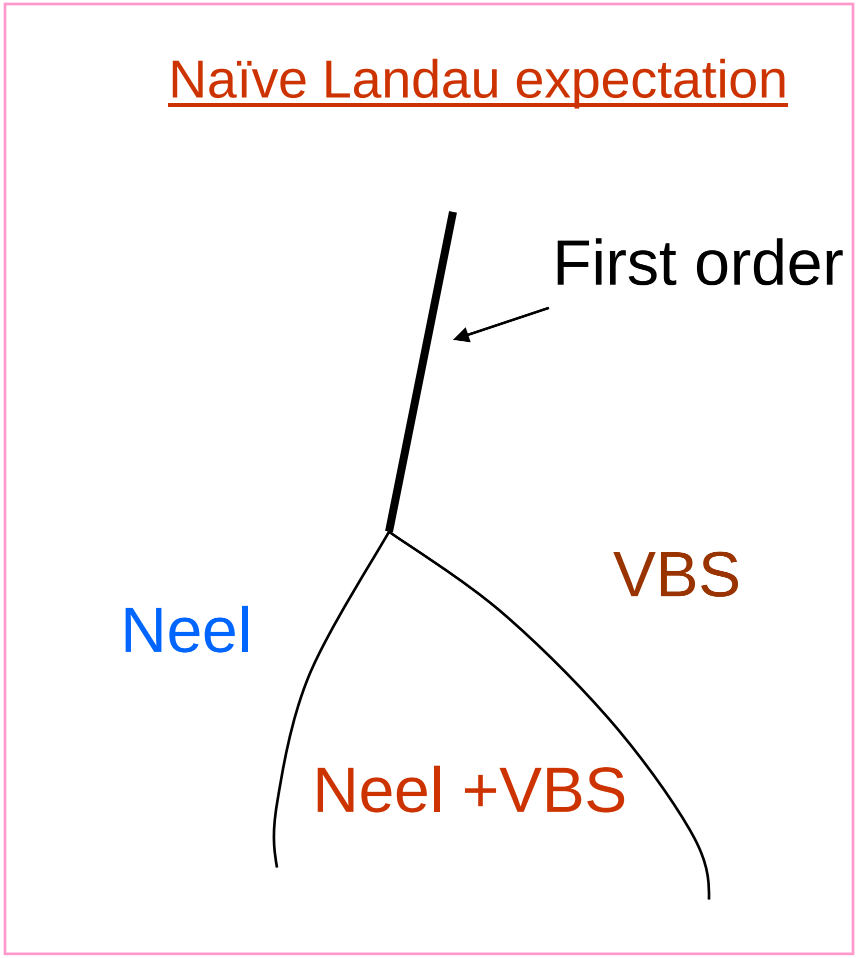
- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.

- Landau – Two independent order parameters.

- no generic direct second order transition.

- either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.



Neel-Valence Bond Solid transition

- Naïve approaches fail

Attack from Neel ≠Usual O(3) transition in D = 3

Attack from VBS ≠ Usual Z_4 transition in D = 3

(= XY universality class).

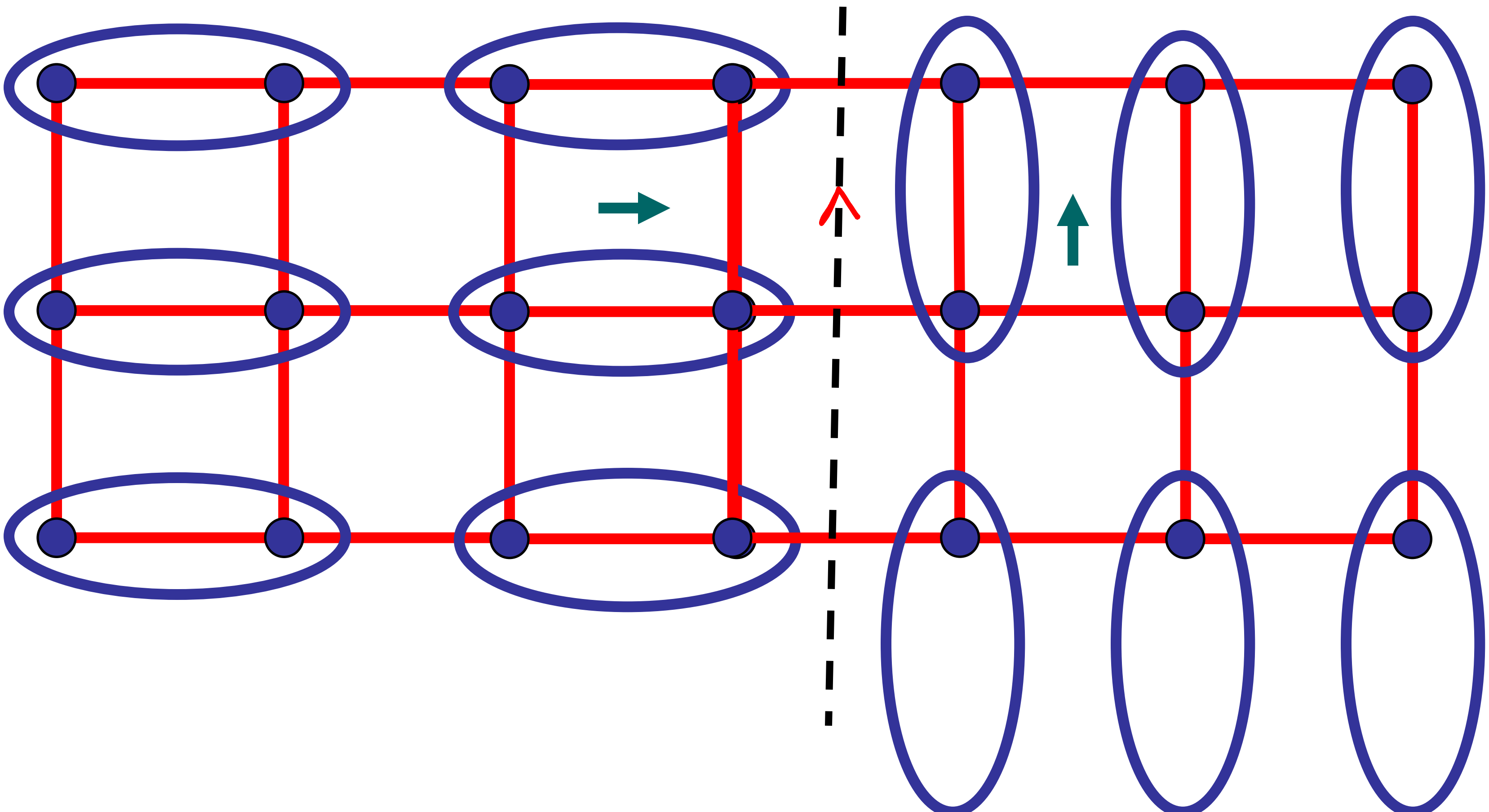
Why do these fail?

Topological defects carry non-trivial quantum numbers!

Attack from VBS (Levin, TS, '04)

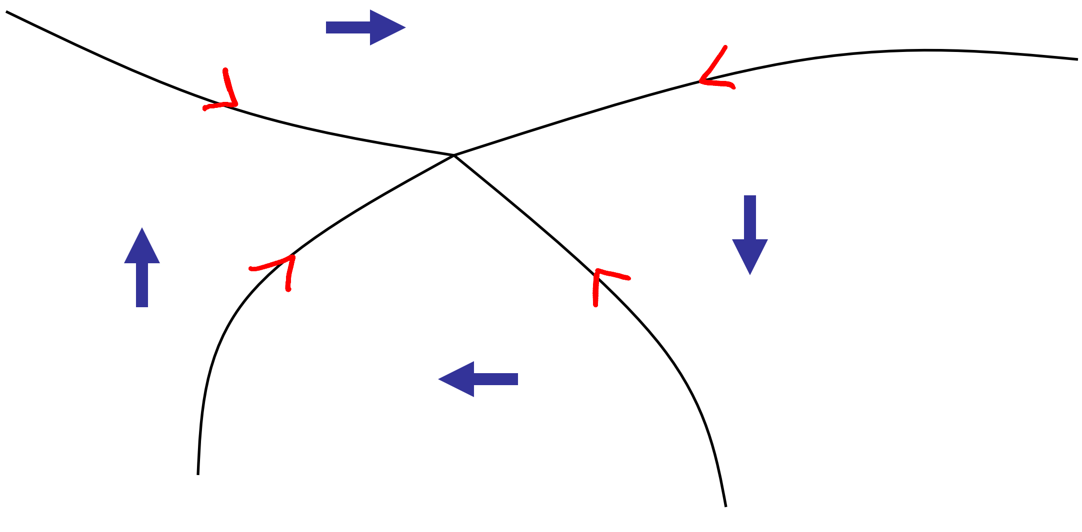
Topological defects in Z_4 order parameter

- Domain walls – elementary wall has π/2 shift of clock angle



Z_4 domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are Z_4 vortices.

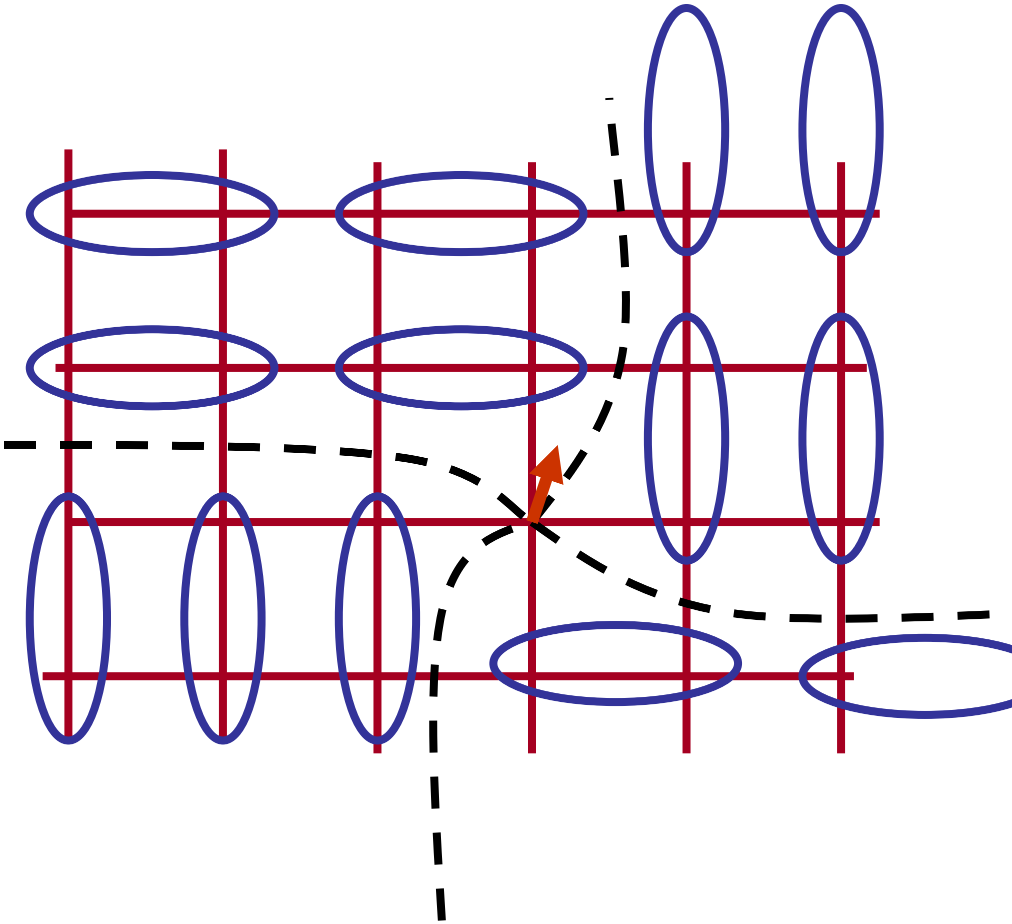


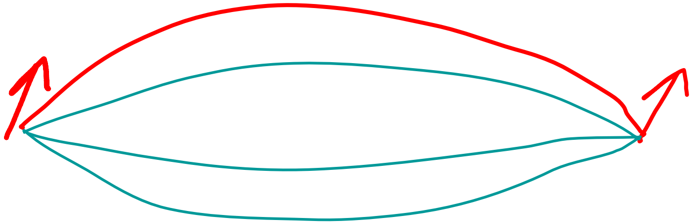
Z_4 vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

Z_4 vortices are spin-1/2 ``spinons".

Domain wall energy
$\Rightarrow$ linear confinement in VBS phase.





$E \sim L$

Z_4 disordering transition to Neel state

- As for usual (quantum) Z_4 transition, expect clock anisotropy is irrelevant.

(confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).

Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)

Phase mode - ``photon''

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons

=> Critical spinons minimally coupled to fluctuating U(1) gauge field*.

*non-compact

Critical theory
``Non-compact CP_1 model''

$$S = \int d^2 x d\tau \, |(\partial_\mu - i a_\mu) z|^2 + r |z|^2 + u |z|^4$$
$$+ (\varepsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

z = two-component spin-1/2 spinon field
a_{μ} = non-compact U(1) gauge field.
Distinct from usual O(3) or Z_4 critical theories*.

Theory not in terms of usual order parameter fields but involve fractional spin objects and gauge fields.

*Distinction with usual O(3) fixed point due to non-compact gauge field (Motrunich,Vishwanath, '03)

Attack from Neel

(TS, Vishwanath,Balents, Sachdev, Fisher ’04)

Field theory of quantum antiferromagnets

- Deep in Neel phase (or close to it) describe by quantum O(3) non-linear sigma model field theory.

- Succesful in describing experiments in cuprate Mott insulators with Neel ground states.

$$S = \int d\tau d^2x \frac{1}{2g}\left[\left(\partial_\tau \hat{n}\right)^2 + \left(\nabla \hat{n}\right)^2\right] + S_B$$

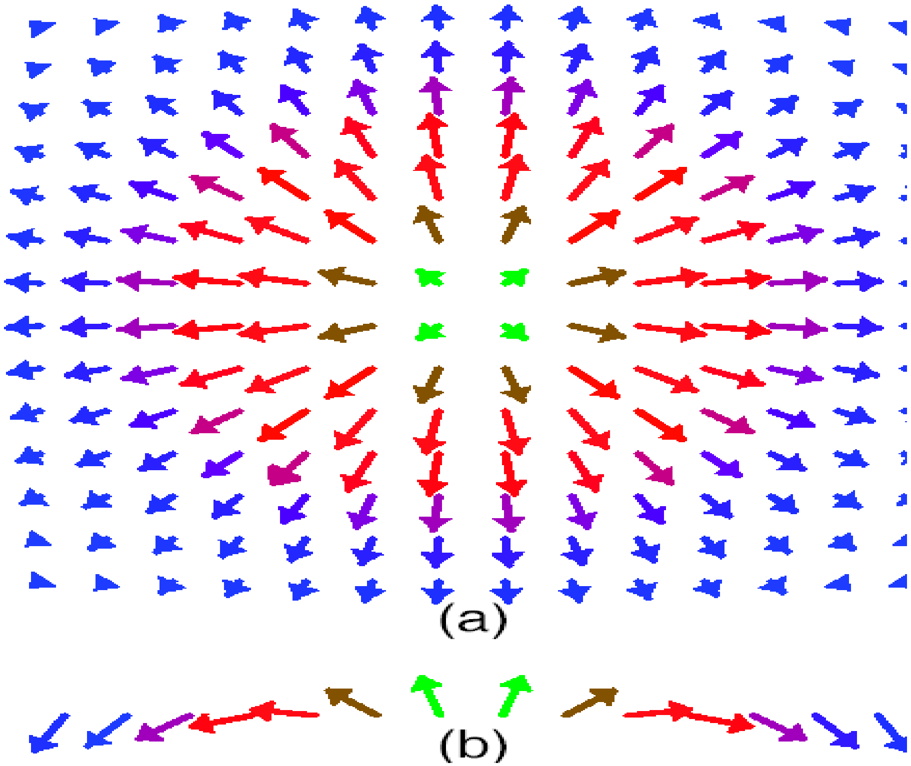
$\hat{n}$ = Neel order parameter field

- S_B = ``Berry phase'' sensitive to microscopic quantized spin at each site.

- Unimportant in Neel but crucial for VBS.

Topology of quantum antiferromagnetism

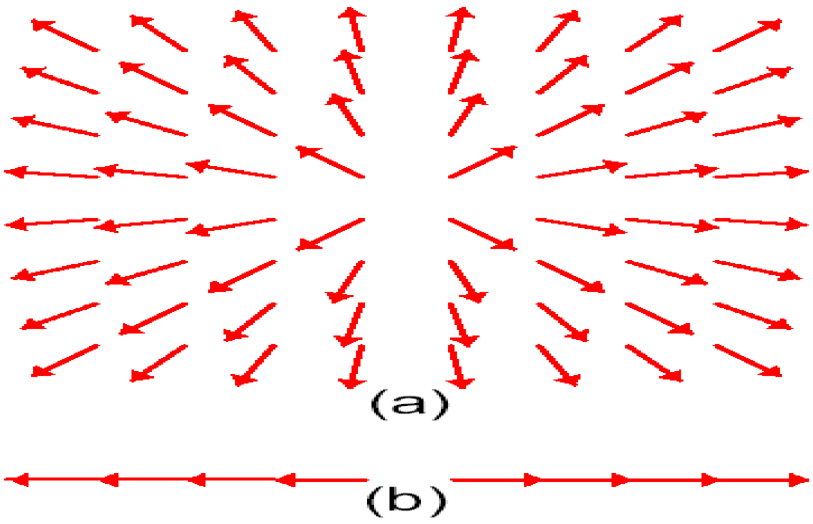
- In d= 2, there are skyrmions in the Neel field.



Topology (contd)

- For the original lattice model, skyrmion number can change (shrink it down to lattice scale and let it disappear).

- In D = 2 + 1, such an ``instanton'' or ``monopole'' event is a hedgehog configuration of the Neel field.



Berry phases and topology

For all smooth configurations of the Neel field, S_B vanishes.

However S_B finite in the presence of monopoles.

Precise calculation (Haldane ’88): Berry phase factor associated with each monopole event that oscillates from plaquette to plaquette.

Final result: Only quadrupled monopoles survive!

(Skyrmion number can change but in units of 4).

Field theory of VBS phases

- Sum over different skyrmion number changing events with Haldane phases (Read-Sachdev).
- Destruction of Neel order through monopole proliferation.
- Haldane phases of monopoles lead to VBS order.

Phase transition – tractable deformations

1. Easy plane anisotropy

2. Embed physical model in a family of models parametrized by integer N.

N = 2: model of interest.

N = 1 and N = ∞: solvable limits.

Same qualitative picture from all these tractable deformations

- Quadrupled monopoles irrelevant at transition!!

Critical theory

- Universality class of O(3) model with monopoles suppressed by hand

Early papers:

Dasgupta &Lau ‘89, Kamal & Murthy ‘94

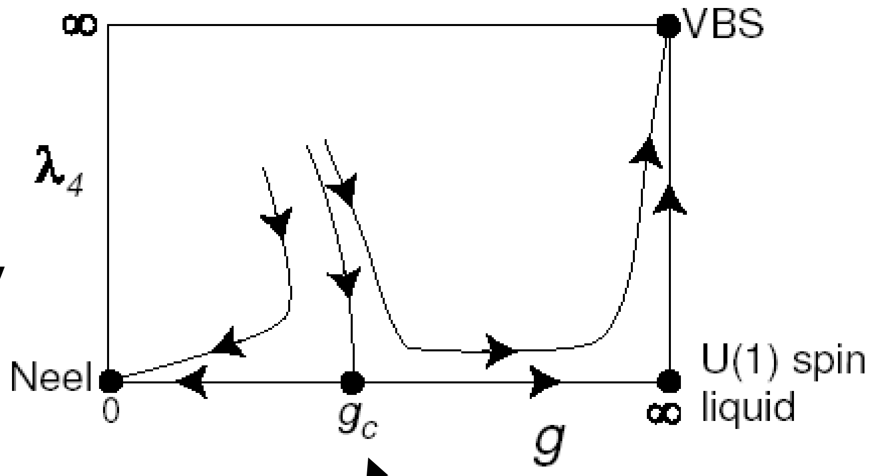
Important recent progress: Motrunich & Vishwanath ’04

Same result as that obtained by attack from VBS!

Quadrupled monopole fugacity = 4-fold clock anisotropy

Renormalization group flows

Clock anisotropy
= quadrupled
Instanton fugacity



Deconfined critical fixed point

Clock anisotropy is ``dangerously irrelevant''.

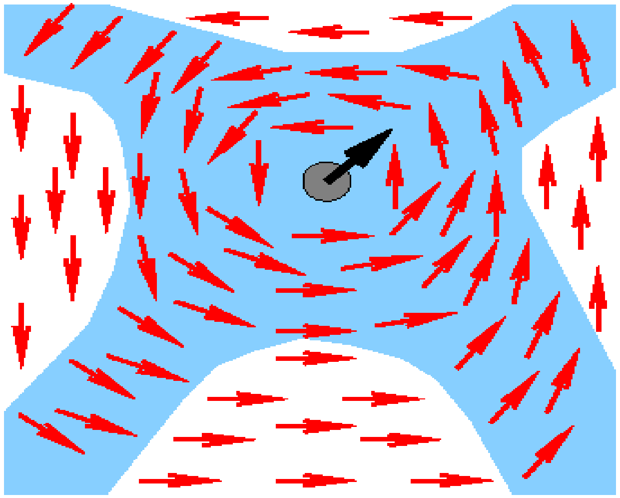
Precise meaning of deconfinement

- Z_4 symmetry gets enlarged to XY

$\Rightarrow$ Domain walls get very thick and very cheap near the transition.

=> Domain wall energy not effective in confining Z_4 vortices (= spinons)

.



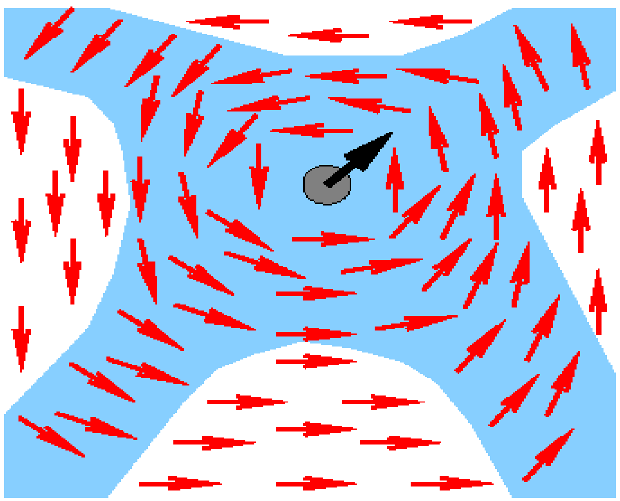
Formal: Extra global U(1) symmetry not present in microscopic model :

Two diverging length scales in paramagnet

ξ: spin correlation length

ξ_{VBS} : Domain wall thickness.

$\xi_{VBS} \sim \xi^{\kappa}$ diverges faster than ξ



Spinons confined in either phase but `confinement scale’ diverges at transition – hence `deconfined criticality’.

Analogies to heavy fermion physics

Insulating Magnet		Kondo lattice
Phases : Neel, VBS	⟷	Fermi liquid, magnetic metal
Mean field : Schwinger boson	⟷	Slave boson
Boson condensate : ("Higgs") Neel	⟷	Kondo fermi liquid
Confined : VBS	⟷	Magnetic metal
Phase transition : Deconfined critical	⟷	?? Similar ??

Other examples of deconfined critical points

1. VBS- spin liquid (Senthil, Balents, Sachdev, Vishwanath, Fisher, '04)
2. Neel –spin liquid (Ghaemi, Senthil, '06)
3. Certain VBS-VBS
(Fradkin, Huse, Moessner, Oganesyan, Sondhi, '04; Vishwanath, Balents,Senthil, '04)

4. Superfluid- Mott transitions of bosons at fractional filling on various lattices (Senthil et al, '04, Balents et al, '05,…….)
5. Spin quadrupole order –VBS on rectangular lattice
(Numerics: Harada et al, '07;Theory: Grover, Senthil, 07)

……..and many more!

Apparently fairly common

A `cool' new example

Grover, Senthil (forthcoming)

s-wave superconductor – "Quantum Spin Hall" insulator on honeycomb lattice

Spin Hall insulator $\vec{J}_{spin} = \sigma \, \hat{z} \times \vec{E}$

σ: quantized

$\vec{E}$: electric field

[Obtain "spin quantization axis" by spontaneously breaking spin symmetry ⇒ resulting phase has both conventional and topological order.]

Numerical/experimental sightings of Landau-forbidden quantum phase transitions

Weak first order/second order quantum transitions between two phases with very different broken symmetry surprisingly common….

Numerics

Antiferromagnet – superconductor (Assaad et al 1996)

Superfluid – density wave insulator on various lattices (Sandvik et al, 2002, Isakov et al, 2006, Damle et al, 2006))

Neel -VBS on square lattice (Sandvik, Singh, Sushkov,…………….)

Spin quadrupole order –dimer order on rectangular lattice (Harada et al, 2006)

Experiments:

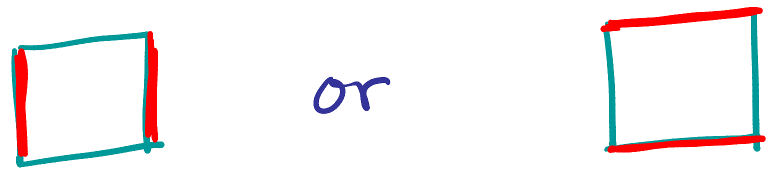
$UPt_{3-x}Pd_x$ SC – AF with increasing x. (Graf et al 2001)

Best numerical evidence: Neel-VBS on square lattice

(Sandvik '07, Melko & Kaul '07)

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - Q \sum_{\langle\langle (ij)(kl) \rangle\rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \right) \left(\vec{S}_k \cdot \vec{S}_l - \frac{1}{4} \right)$$

$(ij), (kl)$: parallel neighbor bonds

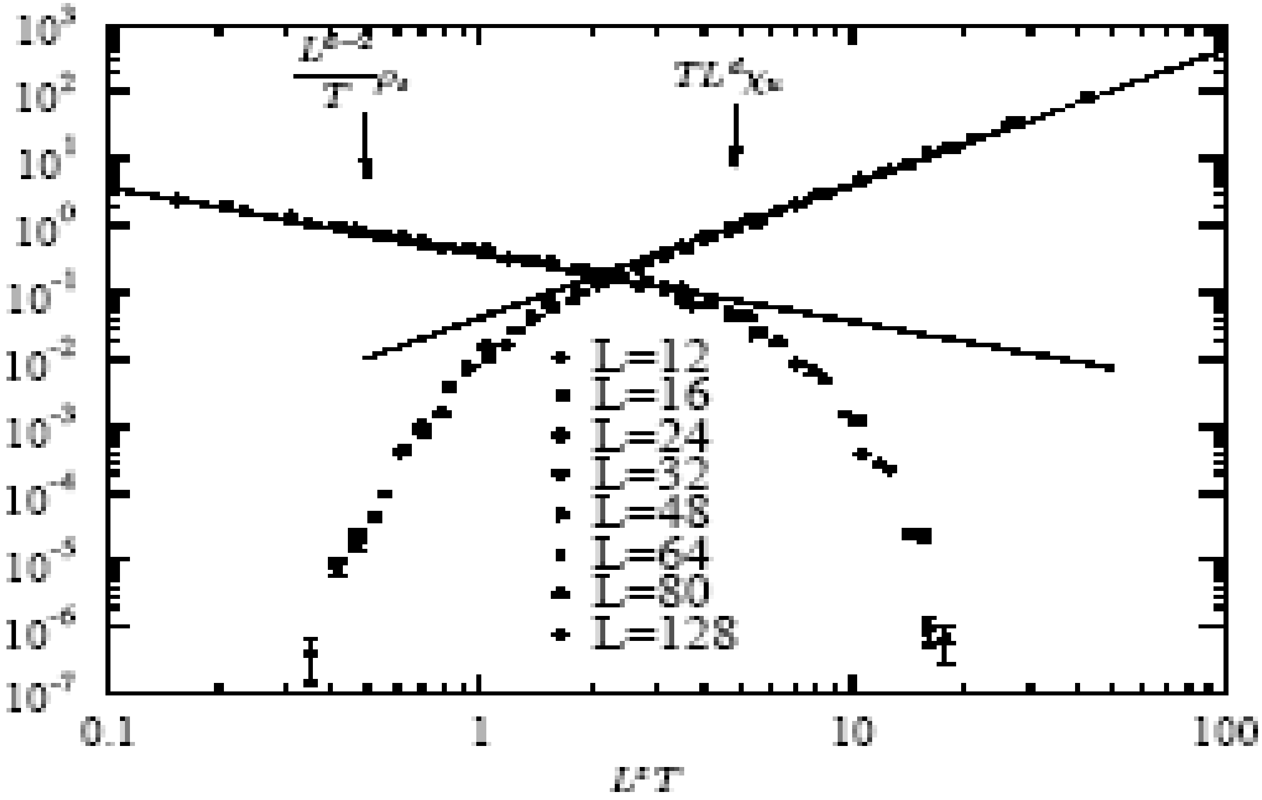


Q/J small : Neel order

Q/J large : VBS order

A sample scaling plot

Melko, Kaul '07

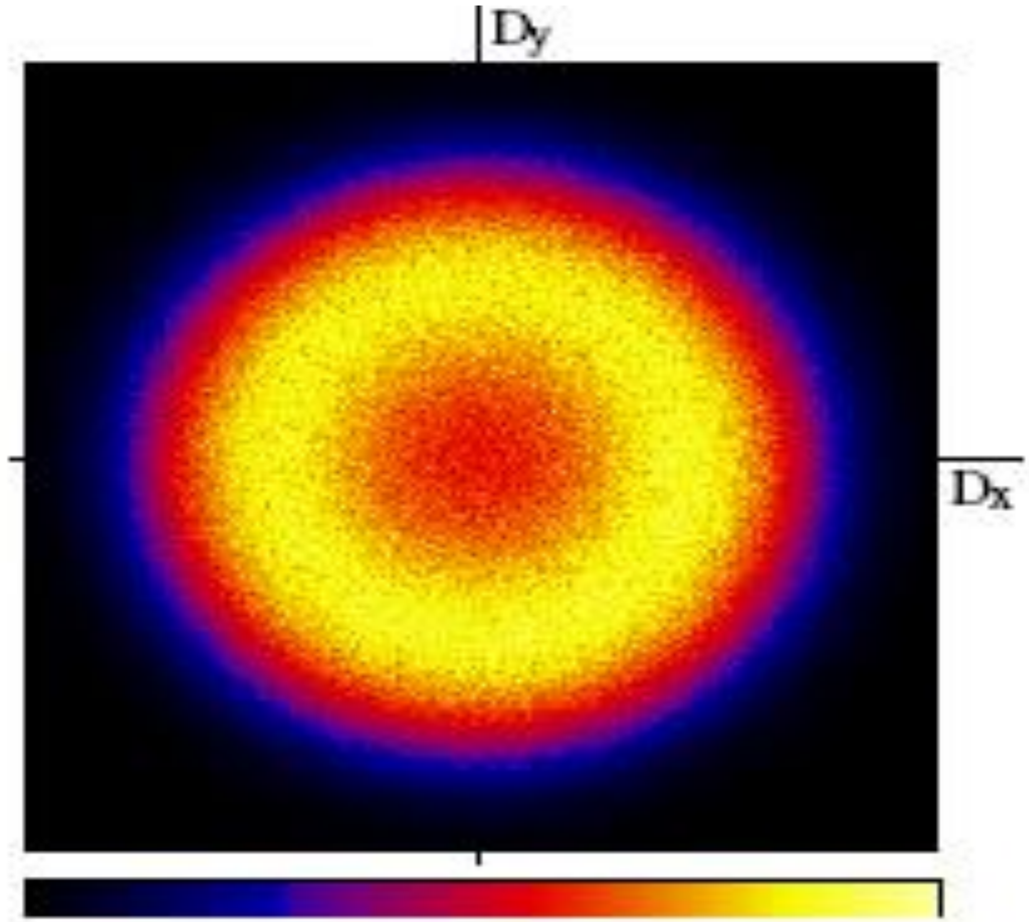


(Sandvik 2007)

Emergent XY symmetry for dimer order

Histogram of dimer order parameter shows full XY symmetry

⇒ irrelevance of Z_4 anisotropy!



Other results: critical quantum phases

(Self Organized Quantum Criticality)

- Stability of critical quantum liquid <u>phases</u> in two dimensions (Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, '04)

``Deconfined critical phases" or ``Algebraic spin/charge liquids"

No quasiparticle description of low energy physics!!

(Rantner, Wen, '02)

Huge emergent low energy symmetry/slow power law decay for many distinct orders (Hermele, Senthil, Fisher, '06)

Important implications for theories of underdoped cuprates

Algebraic spin liquids: Senthil, Lee, '05, Ghaemi, Senthil, '06

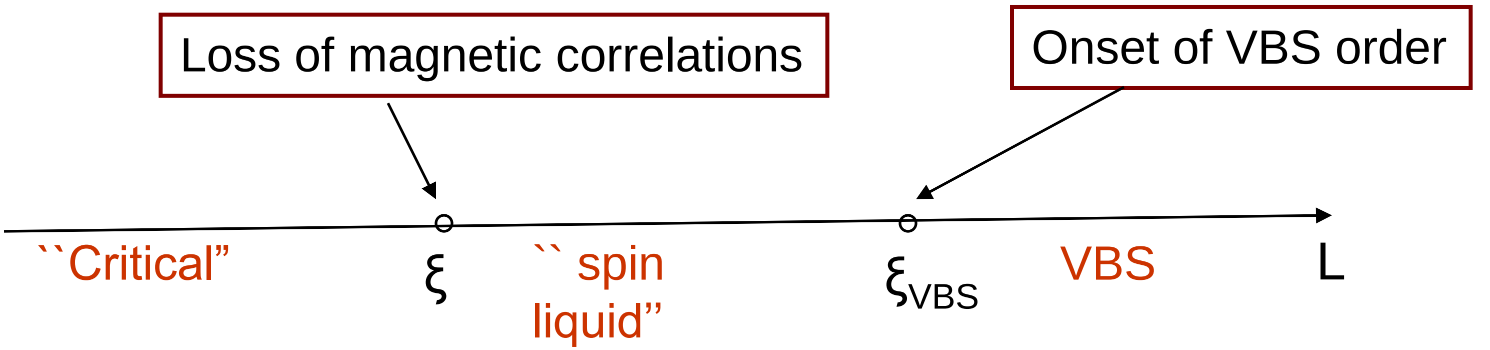
Algebraic charge liquids: Kaul, Kim, Sachdev, Senthil,'07

Some lessons-I

- Direct 2nd order quantum transition between two phases with different competing orders possible (eg: between different broken symmetries)

Separation between the two competing orders

not as a function of tuning parameter but as a function of (length or time) scale



Some lessons-II

- Striking ``non-fermi liquid'' (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, spin spectrum is anamolously broad - roughly due to decay into spinons- as compared to usual critical points.

Most important lesson:

Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics even if natural order parameters exist.

Strong impetus to radical approaches to non fermi liquid physics at

magnetic critical points in rare earth metals (and to optimally doped cuprates).

Outlook

- Theoretically important answer to 0^{th} order question posed by experiments:

Can Landau paradigms be violated at phases and phase transitions of strongly interacting electrons?

- But there still is far to go to seriously confront non-Fermi liquid metals in existing materials……….!

Can we go beyond the 0^{th} order answer?