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Competing Orders in One-Dimensional Spin-3/2 Fermionic Systems

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Novel competing orders are found in spin-3/2 cold atomic systems in one-dimensional optical traps and lattices. In particular, the quartetting phase, a four-fermion counterpart of Cooper pairing, exists in a large portion of the phase diagram. The transition between the quartetting and singlet Cooper pairing phases is controlled by an Ising symmetry breaking in one of the spin channels. The singlet Cooper pairing phase also survives in the purely repulsive interaction regime. In addition, various charge and bond ordered phases are identified at commensurate fillings in lattice systems.

Optical traps and lattices open up a whole new direction in the study of strongly correlated systems in cold atomic physics. In particular, they provide a controllable way to investigate high spin physics by using atoms with hyperfine spin multiplets. For example, polar and ferromagnetic condensations in spin-1 bosonic systems (e.g., ^{23}Na and ^{87}Rb) have been extensively studied both experimentally [1,2] and theoretically [3]. Spin-nematic orders and the exotic spin liquid behavior are arousing much interest [4–7] in Mott-insulating phases. On the other hand, high spin fermionic systems also exhibit many properties different from those in the usual spin-1/2 systems. For example, Cooper pairing shows new structures with total spin $S \geq 2$ in the *s*-wave channel [8,9].

High spin systems also provide a wonderful opportunity to investigate the multiparticle clustering (MPC) instability and its competition with Cooper pairing in cold atomic physics. Taking into account the recent exciting achievement of fermionic superfluids by using Feshbach resonances [10], the MPC instability will be one of the next focus directions. For example, the three-body Efimov bound states in bosonic systems have attracted wide attention [11]. In fermionic systems, the MPC instability is forbidden in two-component spin-1/2 systems due to Pauli's exclusion principle, but is allowed in high spin systems with multiple components. Knowledge learned from the MPC phase in cold atomic systems will also shed light on the *α*-particle formation [12] in nuclear physics, which is a four-fermion bound state. Some previous theoretical works have studied the MPC phase in the $SU(N)$ symmetric fermionic models by the Bethe ansatz at 1D [13] and a variational method at high dimensions [14]. However, the stability of the MPC phase remains an open problem when systems do not possess the $SU(N)$ symmetry, and the nature of the transition between the MPC and Cooper pairing phases has not been clarified before.

Spin-3/2 systems provide an ideal starting point to study the simplest MPC phase in fermionic systems, i.e., the quartetting phase, which can be realized by using ^{132}Cs,

^{9}Be, ^{135}Ba, ^{137}Ba atoms. In this article, we investigate the quartetting phase in one-dimensional (1D) spin-3/2 systems where its strong coupling nature can be handled by applying the methods of bosonization and renormalization group (RG). The investigation is greatly facilitated by the generic $SO(5)$ symmetry recently identified in spin-3/2 systems [15,16]. We find rich phase structures, including a gapless Luttinger liquid phase and two distinct spin gap phases at incommensurate fillings. One of these spin gap phases is characterized by the quartet formation, while the other is dominated by Cooper pairing. The transition between them is controlled by an Ising symmetry breaking in one of the spin channels. Both quartets and Cooper pairs can further undergo either the quasilong range ordered (QLRO) superfluidity or charge density wave (CDW) instabilities. In contrast, the $SU(4)$ symmetry in the previous study [13] requires four particles to form an $SU(4)$ singlet, thus only the quartetting phase is allowed and Cooper paring is excluded. Furthermore, various charge and valence bond orders are identified at commensurate fillings.

We begin with phase structures at incommensurate fillings. After linearizing the spectra around the Fermi wave vector k_f, we decompose fermion operators into right and left moving parts as $\psi_\alpha = \psi_{R,\alpha} e^{ik_f x} + \psi_{L,\alpha} e^{-ik_f x}$ ($\alpha = \pm\frac{3}{2}$, $\pm\frac{1}{2}$). The right (left) moving currents are classified into $SO(5)$'s scalar, vector, and tensor currents as $J_{R(L)}(z) = \psi^\dagger_{R(L),\alpha}(z)\psi_{R(L),\alpha}(z)$, $J^a_{R(L)}(z) = \frac{1}{2}\psi^\dagger_{R(L),\alpha}(z)\Gamma^a_{\alpha\beta}\psi_{R(L),\beta}(z) \times (1 \leq a \leq 5)$, and $J^{ab}_{R(L)}(z) = \frac{1}{2}\psi^\dagger_{R(L),\alpha}(z)\Gamma^{ab}_{\alpha\beta}\psi_{R(L),\beta}(z)(1 \leq a < b \leq 5)$, where Γ^a, Γ^{ab} are the 4×4 Dirac matrices defined in Ref. [15]. Classified in terms of the usual spin $SU(2)$ group, the scalars $J_{R(L)}$ are charge currents, the 5-vectors $J^a_{R(L)}$ are spin-nematic currents with spin $S = 2$, and the 10-tensors $J^{ab}_{R(L)}$ contain two degenerate parts with spin $S = 1, 3$.

Spin-3/2 systems are characterized by two independent *s*-wave scattering parameters g_0 and g_2 in the total spin singlet ($S_T = 0$) and quintet ($S_T = 2$) channels, respectively. Taking these into account, the low energy effective

Hamiltonian density reads

$$\mathcal{H}_0 = v_f \left\{ \frac{\pi}{4} J_R J_R + \frac{\pi}{5} (J_R^a J_R^a + J_R^{ab} J_R^{ab}) + (R \to L) \right\},$$

$$\mathcal{H}_{\text{int}} = \frac{g_c}{4} J_R J_L + g_v J_R^a J_L^a + g_t J_R^{ab} J_L^{ab}, \quad (1)$$

where $g_{c,v,t}$ are effective dimensionless coupling constants with their bare values given by

$$2g_c = g_0 + 5g_2, \qquad 2g_v = g_0 - 3g_2,$$
$$2g_t = -(g_0 + g_2). \quad (2)$$

We neglect the chiral interaction terms. Equation (1) is closely related to the extensively studied two-coupled spin-1/2 Luttinger liquids [17–19]. In the latter case, Fermi wave vectors in two bands are usually different, while they are the same for all of the four components in the former case, thus much more low energy interband interactions are allowed. An even larger symmetry $SU(4)$ can be obtained by fine tuning $g_0 = g_2$, i.e., $g_v = g_t$, which means that the four spin components are equivalent. In this case, $J_{R(L)}^a$ and $J_{R(L)}^{ab}$ form the generators of the $SU(4)$ group. Then Eq. (1) reduces to the single chain $SU(4)$ spin-orbit model [20].

The RG equations can be derived through the current algebra and operator product expansion techniques [21]. The charge current satisfies the $U(1)$ Kac-Moody algebra, and thus g_c is not renormalized at the one-loop level due to the absence of Umklapp terms. The vector and tensor currents in spin channels form the $SU(4)$ Kac-Moody algebra, which gives rise to

$$\frac{dg_v}{d\ln(L/a)} = \frac{4}{2\pi} g_v g_t, \qquad \frac{dg_t}{d\ln(L/a)} = \frac{1}{2\pi}(3g_t^2 + g_v^2), \quad (3)$$

where L is the length scale and a is the short distance cutoff. The $SU(4)$ symmetry is preserved in the RG process along the line $g_v = g_t$. Equation (3) can be integrated as $|g_t^2 - g_v^2| = c|g_v|^{3/2}$ (c: constant) with the RG flows as shown in Fig. 1. The parameter space (g_v, g_t) is divided into three phases as (A) the gapless Luttinger liquid, (B) and (C) two different spin gap phases with the formations of quartets and singlet pairs, respectively, which will be clarified below. Phase (A) lies between $g_t = g_v < 0$ (line 1) and $g_t = -g_v < 0$ (line 2), where RG flows evolve to the fixed point (FP) $g_v = g_t = 0$. Phase (B) is bounded by line 2 and $g_v = 0$ (line 3) where RG flows evolve to the $SU(4)$ marginally relevant FP of $g_v = g_s \to +\infty$ (line 4). Phase (C) is symmetric to (B) under the reflection $g_v \to -g_v$. RG flows evolve to the line 5 with $-g_v = g_t \to +\infty$. The boundary between (B) and (C), i.e., line 3, is controlled by the unstable FP of $g_v = 0$, $g_t \to +\infty$. From the relations in Eq. (2), we replot the phase diagram in terms of the s-wave scattering lengths g_0 and g_2 as shown in Fig. 2. The boundary between phases (A) and (C), i.e., $g_0 = g_2 > 0$, is exact from the $SU(4)$ symmetry regardless of the one-
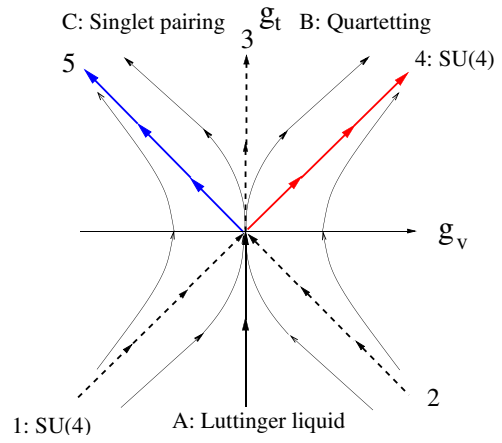

FIG. 1 (color online).   RG flows in the spin channels. Various phases are determined as: (A) the gapless Luttinger liquid phase; (B) the quartetting phase with QLRO superfluidity at $K_c > 2$ or $2k_f$ CDW at $K_c < 2$; (C) the singlet pairing phase with QLRO superfluidity at $K_c < 1/2$ or $4k_f$ CDW at $K_c > 1/2$. They are controlled by the FPs of $(0,0)$, $(+\infty, +\infty)$ (line 4), and $(-\infty, +\infty)$ (line 5), respectively. Phase boundaries (lines 1, 2, 3) are marked with dashed lines.

loop approximation. On the contrary, boundaries between (A) and (B), (B) and (C) will be modified in higher order perturbations.

To clarify the nature of each phase, we employ the Abelian bosonization. The bosonization identity reads $\psi_{R(L)\alpha}(x) = \eta_\alpha / \sqrt{2\pi a} \exp\{\pm i\sqrt{\pi}[\phi_\alpha(x) \pm \theta_\alpha(x)]\}(\alpha = \pm\frac{3}{2}, \pm\frac{1}{2})$ where ηs are the Klein factors. Boson fields ϕ_α and their dual fields θ_α are conveniently reorganized into $\phi_c(\theta_c)$ in the charge channel, and $\phi_v(\theta_v)$, $\phi_{t1}(\theta_{t1})$, $\phi_{t2}(\theta_{t2})$ in the spin channels via $\phi_{c,v} = (\phi_{3/2} \pm \phi_{1/2} \pm \phi_{-1/2} + \phi_{-3/2})/2$, $\phi_{t1,t2} = (\phi_{3/2} \mp \phi_{1/2} \pm \phi_{-1/2} - \phi_{-3/2})/2$. Similar expressions also hold for θ's. The
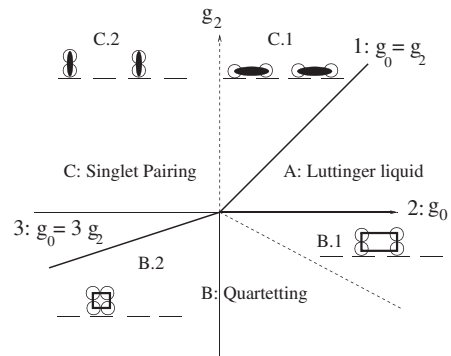

FIG. 2.   Phase diagram in terms of g_0 and g_2. We also show the charge and bond ordered states at quarter filling in lattice systems. Phase (A) is the $SU(4)$ gapless spin liquid. Both phase (B) and (C) split into two parts. With $g_u \to +\infty$, (B.1) represents quartets with both bond and charge orders; (C.1) is dimerization of spin Peierls order. With $g_u \to -\infty$, (B.2) is CDW phases of quartets and (C.2) is CDW of singlet Cooper pairs. Boundaries among (A), (B), and (C) are marked with solid lines, and those between (B.1) and (B.2), and (C.1) and (C.2) are sketched with dashed lines.

quadratic part of the Hamiltonian density is standard
($\nu = c, v, t_1, t_2$)

$$\mathcal{H}_0 = \frac{v_\nu}{2} \sum_\nu \left\{ K_\nu (\partial_x \theta_\nu)^2 + \frac{1}{K_\nu} (\partial_x \phi_\nu)^2 \right\}, \qquad (4)$$

with Luttinger parameters K_ν and velocities v_ν in each
channel. The nonquadratic terms are summarized as

$$\mathcal{H}_{\text{int}} = -\frac{1}{2(\pi a)^2} \{ \cos\sqrt{4\pi} \phi_{t1} + \cos\sqrt{4\pi} \phi_{t2} \}$$
$$\times \{ (g_t + g_v) \cos\sqrt{4\pi} \phi_v + (g_t - g_v) \cos\sqrt{4\pi} \theta_v \}$$
$$-\frac{g_t}{2(\pi a)^2} \cos\sqrt{4\pi} \phi_{t1} \cos\sqrt{4\pi} \phi_{t2}, \qquad (5)$$

with the convention of Klein factors as
$\eta_{3/2} \eta_{3/2} \eta_{-1/2} \eta_{-3/2} = 1$.

Various order parameters are needed to characterize
phase structures, including the $2k_f$-CDW operator N, the
spin density wave (SDW) operators in the SO(5) vector
channel N^a and their tensor channel version N^{ab}, the
singlet pairing operator η and its quintet counterpart χ^a,
the $4k_f$-CDW operator $O_{4k_f,\text{CDW}}$, as well as the quartetting
operator O_{qt}. They are defined as

$$N = \psi^\dagger_{R\alpha} \psi_{L\alpha}, \qquad N^a = \psi^\dagger_{R\alpha} \frac{\Gamma^a_{\alpha\beta}}{2} \psi_{L\beta},$$
$$N^{ab} = \psi^\dagger_{R\alpha} \frac{\Gamma^{ab}_{\alpha\beta}}{2} \psi_{L\beta}; \qquad \eta^\dagger = \psi^\dagger_{R,\alpha} R_{\alpha\beta} \psi^\dagger_{L,\beta},$$
$$\chi^{a,\dagger} = i \psi^\dagger_{R,\alpha} (R\Gamma^a)_{\alpha\beta} \psi^\dagger_{L,\beta}; \qquad (6)$$
$$O_{4k_f,\text{CDW}} = \psi^\dagger_{R\alpha} \psi^\dagger_{R\beta} \psi_{L\beta} \psi_{L\alpha},$$
$$O_{\text{quar}} = (\epsilon_{\alpha\beta\gamma\delta}/4) \psi^\dagger_{R\alpha} \psi^\dagger_{R\beta} \psi^\dagger_{L\gamma} \psi^\dagger_{L\delta},$$

where $R = \Gamma_1 \Gamma_3$ [19] is the charge conjugation matrix and
$\epsilon_{\alpha\beta\gamma\delta}$ is the rank-4 Levi-Civita symbol.

Now we are ready to discuss the nature in each phase. In
the Luttinger liquid phase (A), all the cosine terms are
marginally irrelevant. As shown in Fig. 2, the Luttinger
phase exists in the repulsive region with $g_0 \geq g_2 \geq 0$, thus
$K_c < 1$. As a result, only $2k_f$-CDW and SDW susceptibil-
ities diverge with scaling dimensions $\Delta_N = \Delta_{N^a} =
\Delta_{N^{ab}} = (K_c + 3)/4$.

Phase (B) is characterized by spin gaps in ϕ_v, ϕ_{t1}, and
ϕ_{t2} channels. Its $SU(4)$ FP of $g_v = g_t \to +\infty$ means
the formation of the $SU(4)$ invariant quartets as follows.
In the ground state, cosine operators acquire nonvanishing
expectation values as $\langle \cos\sqrt{4\pi} \phi_v \rangle = \langle \cos\sqrt{4\pi} \phi_{t1} \rangle =
\langle \cos\sqrt{4\pi} \phi_{t2} \rangle$, whose classical values are just 1 or -1 as
related by a Z_2 symmetry [22]. We fix the gauge by
choosing $\langle \phi_v \rangle = \langle \phi_{t1} \rangle = \langle \phi_{t2} \rangle = 0$. By checking scaling
dimensions of various order parameters in Eq. (6), we
conclude that the competing instabilities are the QLRO
superfluidity O_{quar} and CDW of quartets. Because the
average distance between two quartets is $d = \pi/k_f$, this
CDW is of $2k_f$ type. Their expressions are reduced to

$O_{\text{quar}} \propto e^{2i\sqrt{\pi} \theta_c}$ and $N \propto e^{i\sqrt{\pi} \phi_c}$ respectively, with scaling
dimensions $\Delta_{\text{quar}} = 1/K_c$ and $\Delta_N = \frac{1}{4} K_c$. The leading
instability is O_{quar} at $K_c > 2$, and N at $K_c < 2$ respectively.
We extend the quartets formation regime from the previous
Bethe-ansatz results along $SU(4)$ line 4 [13] to the entire
phase (B). On the other hand, correlations of pairing op-
erators $\eta^\dagger$ and $\chi^{a\dagger}$ decay exponentially.

Phase (C) is controlled by the FP of $-g_v = g_t \to +\infty$
which is characterized by the singlet Cooper pairing, and
possesses another $SU(4)$ symmetry denoted as $SU'(4)$. Its
right (left) generators $J'_{R(L)}$ belong to the $SU(4)$ fundamen-
tal (antifundamental) representations defined as

$$J^{l,ab}_R = J^{ab}_R, \qquad J^{l,a}_R = J^a_R;$$
$$J^{l,ab}_L = J^{ab}_L, \qquad J^{l,a}_L = -J^a_L. \qquad (7)$$

It is different from the $SU(4)$ symmetry in phase (B) where
both right and left generators are in the fundamental rep-
resentation. The singlet Cooper pair is invariant under this
$SU'(4)$ symmetry but not the $SU(4)$ symmetry in phase (B).
We choose the pinned bosonic fields to be $\langle \phi_{t1} \rangle = \langle \phi_{t2} \rangle =
\langle \theta_v \rangle = 0$, where the dual field θ_v instead of ϕ_v is pinned as
in phase (B). Again, the superfluidity and CDW orders of
pairs compete. Because the average distance between ad-
jacent pairs is $d = \pi/(2k_f)$, this CDW is of the $4k_f$ type.
These two orders are reduced to $\eta^\dagger \propto e^{-i\sqrt{\pi} \theta_c}$ and
$O_{4k_f,\text{CDW}} \propto e^{i\sqrt{4\pi} \phi_c}$, with scaling dimensions as $\Delta_\eta =
1/(4K_c)$ and $\Delta_{4k_f,\text{CDW}} = K_c$. The leading instability is
$\eta^\dagger$ at $K_c > 1/2$, and $O_{4k_f,\text{CDW}}$ at $K_c < 1/2$ respectively.
Remarkably, due to the presence of the spin gap, the singlet
pairing instability dominates even in the purely repulsive
region at $g_0 > g_2 > 0$ when $K_c > 1/2$ as shown in Fig. 2.
This is similar to the situation in high T_c superconductivity.

The transition between quartetting and pairing phases is
controlled by an Ising duality in the $\theta_v(\phi_v)$ channel. Near
the phase boundary, $\phi_{t1,2}$ are pined, thus the residual
interaction becomes

$$H_{\text{res}} = -\frac{\lambda}{2(\pi a)^2} \{ (g_t + g_v) \cos\sqrt{4\pi} \phi_v$$
$$+ (g_t - g_v) \cos\sqrt{4\pi} \theta_v \}, \qquad (8)$$

where $\lambda = -\frac{1}{\pi a} (\langle \cos\sqrt{4\pi} \phi_{t1} \rangle + \langle \cos\sqrt{4\pi} \phi_{t2} \rangle)$. The sin-
glet pairing operator $\eta^\dagger \propto \psi^\dagger_{3/2} \psi^\dagger_{-3/2} - \psi^\dagger_{1/2} \psi^\dagger_{-1/2} \propto
e^{-i\sqrt{\pi} \theta_c} (e^{-i\sqrt{\pi} \theta_v} + e^{i\sqrt{\pi} \theta_v})$, thus $\sqrt{4\pi} \theta_v$ is the relative
phase between the pairs of $\Delta^\dagger_1 = \psi^\dagger_{3/2} \psi^\dagger_{-3/2}$ and $\Delta^\dagger_2 =
\psi^\dagger_{1/2} \psi^\dagger_{-1/2}$, and ϕ_v is the vortex field dual to θ_v. The
transition can be viewed as the phase locking problem in
a two-component superfluids. In the pairing phase, θ_v can
be pinned at either 0 or $\sqrt{\pi}$ by the internal Josephson term
$\cos\sqrt{4\pi} \theta_v$. This symmetry is Z_2, and thus the pairing
phase with pinned θ_v is in the Ising ordered phase. In
contrast, the quartetting phase is the Ising disordered phase
where the dual field ϕ_v is locked instead. The Ising nature
of the transition is also clear by representing Eq. (8) in

terms of two Majorana fermions [23] as $H_{\text{res}} = \sum_{a=1}^{2}(\xi_R^a i\partial_x \xi_R^a - \xi_L^a i\partial_x \xi_L^a) + i\lambda(g_t \xi_R^1 \xi_L^1 + g_v \xi_R^2 \xi_L^2);$ ξ^1 is always off-critical as $g_t \to +\infty$, while g_v is the mass of ξ^2 which changes sign across the boundary. Thus the quartetting and pairing phases are Ising disordered and ordered phases for ξ^2, respectively.

Next we consider the charge and valence bond ordered state as depicted in Fig. 2 at quarter filling, i.e., one particle per site. The $8k_f$ Umklapp term appears, and decouples with spin channels, which can be bosonized as

$$\mathcal{H}'_{\text{um}} = \frac{g_u}{2(\pi a)^2} \cos(\sqrt{16\pi}\phi_c - 8k_f x), \qquad (9)$$

where g_u is at the order of $O(g_0^3, g_2^3)$ at the bare level. At $K_c < 1/2$, this term is relevant and opens the charge gap. The real parts of N and $O_{4k_f,\text{CDW}}$ describe the usual CDW orders, while their imaginary parts mean the bond orders, i.e., the $2k_f$ and $4k_f$ spin Peierls orders. Phase (A) remains gapless in spin channels as described by the $SU(4)$ Heisenberg model. The quartetting phase (B) splits into two parts (B.1) and (B.2) with $g_u \to +\infty(-\infty)$ respectively. In (B.1), Im $O_{4k_f,\text{CDW}}$, ReN and ImN are long range ordered. The $SU(4)$ singlet quartets exhibit both charge and spin Peierls orders. Instead, the $2k_f$ CDW of quartets becomes long range ordered in (B.2). Similarly, the singlet pairing phase (C) splits into (C.1) and (C.2) as $g_u \to +\infty(-\infty)$ respectively. In (C.1), Im$O_{4k_f,\text{CDW}}$, i.e., the spin Peierls order is stabilized. Instead, the CDW of singlet pairs becomes long range ordered in (C.2). The boundaries between phase (B.1) and (B.2), phase (C.1) and (C.2) are determined by the bare value of $g_{u0} = 0$ as sketched in Fig. 2. However, due to the nonuniversal relation between g_{u0} and $g_{0,2}$, the exact boundaries are hard to determine.

The model discussed above can be realized by loading spin-3/2 fermions into one-dimensional optical tubes. The quartetting phase can be probed by the radio-frequency spectroscopy to measure the excitation gaps of the successive quartet breaking process as $4 \to 1 + 3 \to 1 + 1 + 2 \to 1 + 1 + 1 + 1$. In contrast, in the pairing phase, only one pairing breaking process of $2 \to 1 + 1$ exits as measured experimentally [24]. The charge and bond ordered states break translational symmetry. The periodicity of phases (C.1) and (C.2) is two lattice constants, while that of (B.1) and (B.2) is doubled. These can be detected by using the elastic Bragg spectroscopy [25].

In summary, we have investigated the global phase diagram in 1D spin-3/2 systems, including the gapless Luttinger liquid, the spin gapped quartetting and Copper pairing phases. The competition between the quartetting and pairing phases is found to be controlled by an Ising duality. Both quartets and pairs can either undergo QLRO superfluidity or CDW instabilities, depending on the value of Luttinger parameter K_c. The QLRO singlet pair super-

fluidity can survive in the purely repulsive interaction regime. The Mott-insulating phases at commensurate fillings exhibit various charge and valence bond orders.

Note added.—After this Letter had been submitted for publication, the author became aware of the work by Lecheminant, Azaria, and Boulat [26], where the two spin gap phases and Mott phases at quarter filling are obtained independently.

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