Hidden Symmetry and Quantum Phases in Spin 3/2 Cold Atomic Systems

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Ref: C. Wu, Mod. Phys. Lett. B 20, 1707, (2006);

C. Wu, J. P. Hu, and S. C. Zhang, Phys. Rev. Lett. 91, 186402(2003);

C. Wu, Phys. Rev. Lett. 95, 266404 (2005);

S. Chen, C. Wu, S. C. Zhang and Y. P. Wang, Phys. Rev. B 72, 214428 (2005);

C. Wu, J. P. Hu, and S. C. Zhang, cond-mat/0512602.

Collaborators

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- S. C. Zhang, Stanford.

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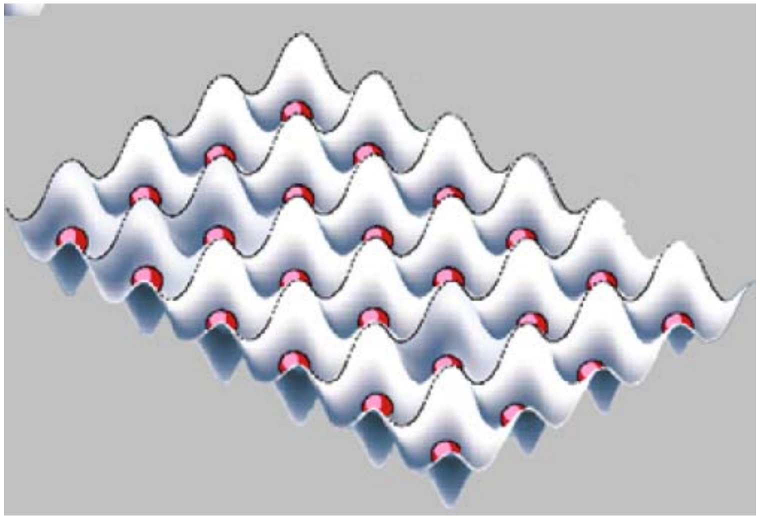
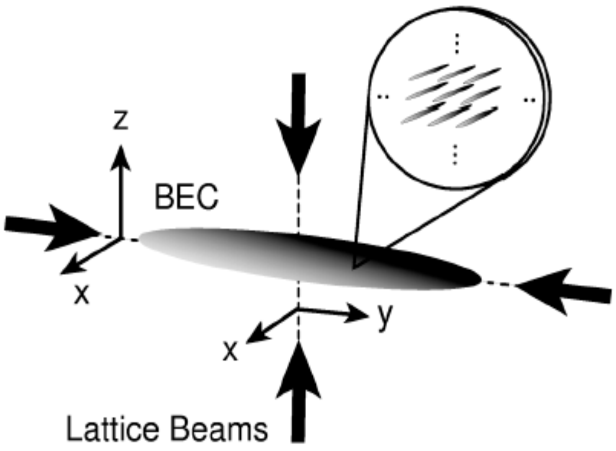
Rapid progress in cold atomic physics



M. Greiner et al., Nature 415, 39 (2002).



M. Greiner et. al., PRL, 2001.

• Magnetic traps: spin degrees of freedom are frozen.

• Optical traps and lattices: spin degrees of freedom are released; a controllable way to study high spin physics.

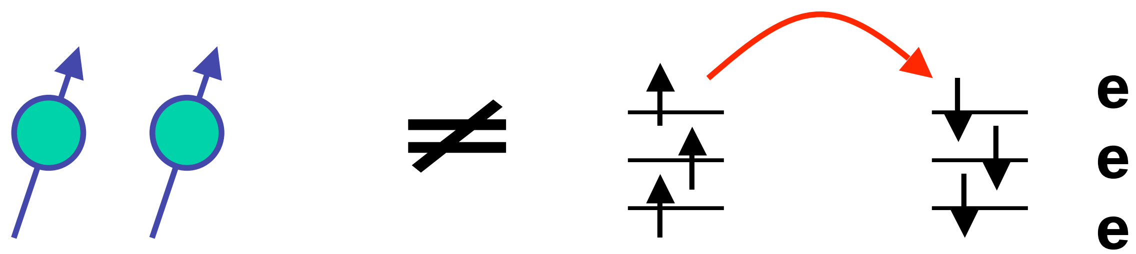
• In optical lattices, interaction effects are adjustable. New opportunity to study strongly correlated high spin systems.

High spin physics with cold atoms

- Most atoms have high hyperfine spin multiplets.

F= I (nuclear spin) + S (electron spin).

- Different from high spin transition metal compounds.



- Spin-1 bosons: ^{23}Na (antiferro), ^{87}Rb (ferromagnetic).

- High spin fermions: zero sounds and Cooper pairing structures.

D. M. Stamper-Kurn et al., PRL 80, 2027 (1998); T. L. Ho, PRL 81, 742 (1998);
F. Zhou, PRL 87, 80401 (2001); E. Demler and F. Zhou, PRL 88, 163001 (2002);
T. L. Ho and S. Yip, PRL 82, 247 (1999); S. Yip and T. L. Ho, PRA 59, 4653(1999).

Hidden symmetry: spin-3/2 atomic systems are special!

- Spin 3/2 atoms: ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba, ^{201}Hg.

- Hidden **SO(5)** symmetry without fine tuning!

Continuum model (s-wave scattering); the lattice-Hubbard model.

Exact symmetry regardless of the dimensionality, lattice geometry, impurity potentials.

SO(5) in spin 3/2 systems ←→ SU(2) in spin ½ systems

- This SO(5) symmetry is qualitatively **different** from the SO(5) theory of high T_c superconductivity.

C. Wu, J. P. Hu, and S. C. Zhang, PRL 91, 186402(2003).

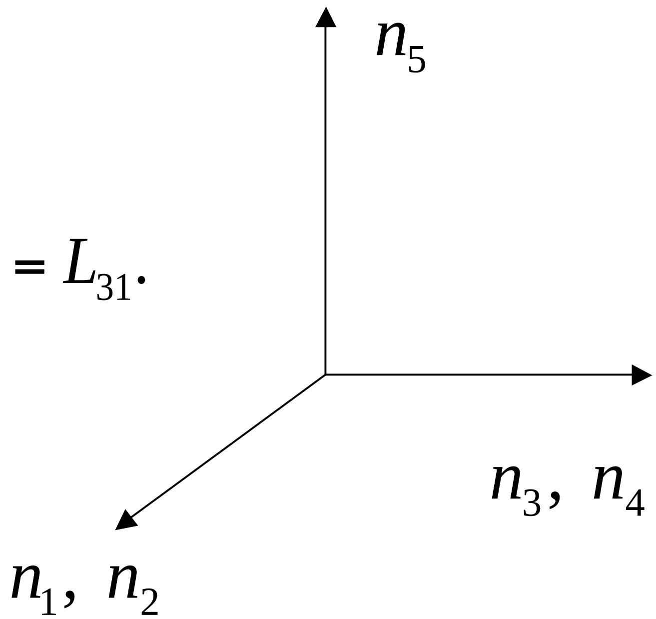
What is SO(5)/Sp(4) group?

- SO(3) /SU(2) group.

3-vector: x, y, z.

3-generator: $L_z = L_{12}, \quad L_x = L_{23}, \quad L_y = L_{31}.$

2-spinor: $|\uparrow\rangle, \quad |\downarrow\rangle$



- SO(5) /Sp(4) group.

5-vector: n_1, n_2, n_3, n_4, n_5

10-generator: $L_{ab} \ (1 \le a < b \le 5)$

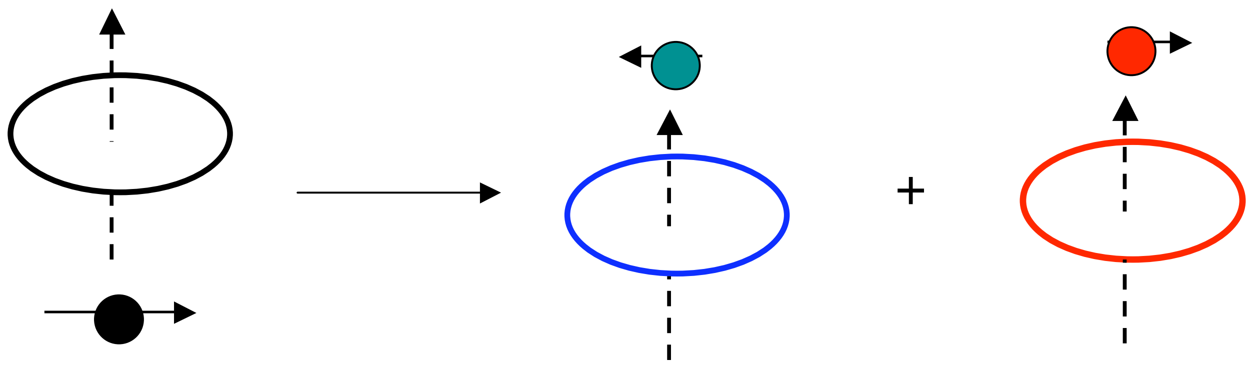
4-spinor: $\Uparrow \left|\frac{3}{2}\right\rangle \quad \uparrow \left|\frac{1}{2}\right\rangle \quad \downarrow \left|-\frac{1}{2}\right\rangle \quad \Downarrow \left|-\frac{3}{2}\right\rangle$

Quintet superfluidity and half-quantum vortex

- High symmetries do not occur frequently in nature, since every new symmetry brings with itself a possibility of new physics, they deserve attention.

---D. Controzzi and A. M. Tsvelik, cond-mat/0510505

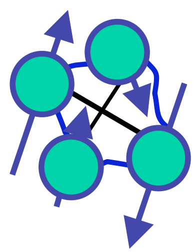
- Cooper pairing with $S_{pair}=2$.
- Half-quantum vortex (non-Abelian Alice string) and quantum entanglement.



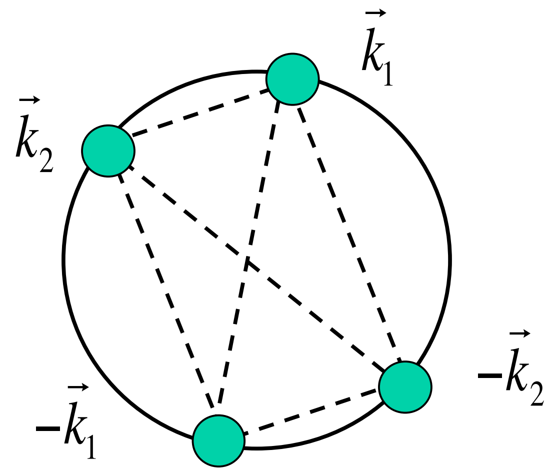
C. Wu, J. P. Hu, and S. C. Zhang, cond-mat/0512602.

Multi-particle clustering order

- Quartetting order in spin 3/2 systems.



4-fermion counter-part
of Cooper pairing.



- Feshbach resonances: Cooper pairing superfluidity.

- Driven by logic, it is natural to expect the quartetting order as a possible focus for the future research.

C. Wu, Phys. Rev. Lett. 95, 266404 (2005).

Strong quantum fluctuations in magnetic systems

- Intuitively, quantum fluctuations are weak in high spin systems.

$$[\frac{S_i}{S}, \frac{S_j}{S}] = i\varepsilon_{ijk} \frac{1}{S} \frac{S_k}{S}$$

Illustration by Dick Codor.

- However, due to the high SO(5) symmetry, quantum fluctuations here are even **stronger** than those in spin ½ systems.

large N (N=4) v.s. large S.

From Auerbach's book:

Outline

- **The proof of the exact SO(5) symmetry.**

  C. Wu, J. P. Hu, and S. C. Zhang, PRL 91, 186402(2003).

- Quintet superfluids and non-Abelian topological defects.

- Quartetting v.s pairing orders in 1D spin 3/2 systems.

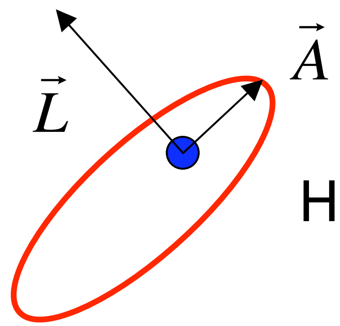
- SO(5) (Sp(4)) Magnetism in Mott-insulating phases.

The SO(4) symmetry in Hydrogen atoms

- An obvious SO(3) symmetry: $\vec{L}$ (angular momentum) .

- The energy level degeneracy n^2 is mysterious.

$\vec{A} \perp \vec{L}$



- Not accidental! 1/r Coulomb potential gives rise to a hidden conserved quantity.

> **Runge-Lentz** vector $\vec{A}$; SO(4) generators $\vec{A}, \vec{L}.$

- Q: What are the **hidden** conserved quantities in spin 3/2 systems?

Generic spin-3/2 Hamiltonian in the continuum

- The s-wave scattering interactions and spin SU(2) symmetry.

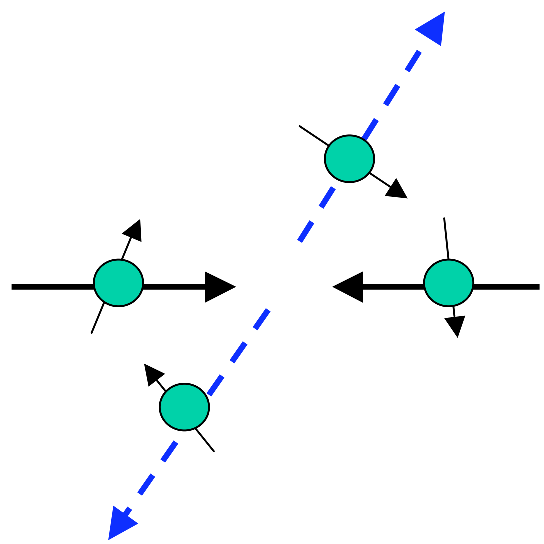
$$H = \int d^d \vec{r} \sum_{\alpha=\pm 3/2, \pm 1/2} \psi_\alpha^+(\vec{r})\left(\frac{-\hbar^2}{2m}\hat{\nabla}^2 - \mu\right)\psi_\alpha(\vec{r})$$
$$+ \frac{g_0}{2}\eta^+(\vec{r})\eta(\vec{r}) + \frac{g_2}{2}\sum_{a=1\sim 5}\chi_a^+(\vec{r})\chi_a(\vec{r})$$

$$\Uparrow \left|\tfrac{3}{2}\right\rangle \quad \uparrow \left|\tfrac{1}{2}\right\rangle \quad \downarrow \left|-\tfrac{1}{2}\right\rangle \quad \Downarrow \left|-\tfrac{3}{2}\right\rangle$$

- Pauli's exclusion principle: only $F_{tot}=0, 2$ are allowed; $F_{tot}=1, 3$ are forbidden.

singlet: $\eta^+(\vec{r}) = \sum_{\alpha\beta}\langle 00 | \tfrac{3}{2}\tfrac{3}{2};\alpha\beta\rangle \psi_\alpha^+(\vec{r})\psi_\beta^+(\vec{r})$

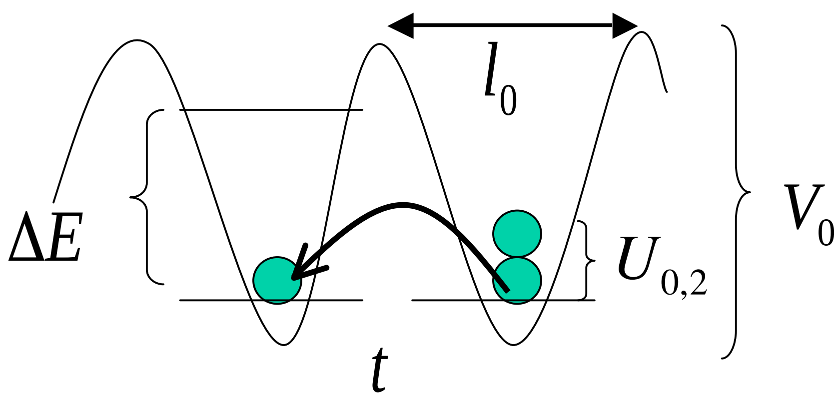
quintet: $\chi_a^+(\vec{r}) = \sum_{\alpha\beta}\langle 2a | \tfrac{3}{2}\tfrac{3}{2};\alpha\beta\rangle \psi_\alpha^+(\vec{r})\psi_\beta^+(\vec{r})$



Spin-3/2 Hubbard model in optical lattices

$$H = \sum_{\langle ij \rangle, \alpha} -t\{c^{+}_{i,\alpha} c_{j,\alpha} + h.c.\} - \mu \sum_{i} c^{+}_{i,\alpha} c_{i,\alpha}$$
$$+ U_0 \sum_{i} \eta^{+}(i)\eta(i) + U_2 \sum_{a=1\sim 5} \chi^{+}_{a}(i)\chi_{a}(i)$$

- The single band Hubbard model is valid away from resonances.



$$\frac{U_{0,2}}{\Delta E} < 0.1,$$

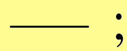
$$l_0 \sim 400\text{nm}, \; a_{s,0,2} \sim 100 a_B$$

$$\left(\frac{V_0}{E_r}\right)^{1/4} \approx 1 \sim 2$$

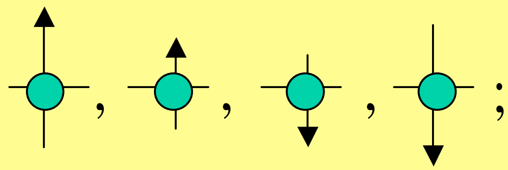
$a_{s:}$ scattering length, $E_{r:}$ recoil energy

SO(5) symmetry: the single site problem

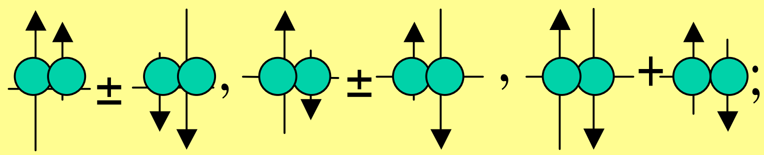
$E_0 = 0$



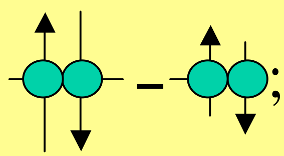
$E_1 = -\mu$



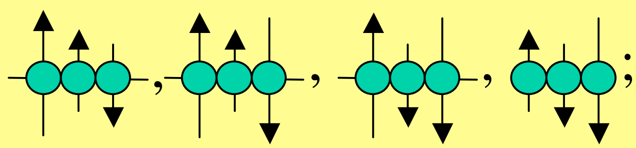
$E_2 = U_2 - 2\mu$



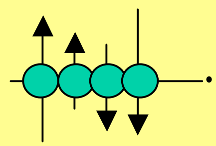
$E_3 = U_0 - 2\mu$



$E_4 = \frac{1}{2}U_0 + \frac{5}{2}U_2 - 3\mu$



$E_5 = U_0 + 5U_2 - 4\mu$



2^4=16 states.

	SU(2)	SO(5)	degen eracy
$E_{0,3,5}$	singlet	scalar	1
$E_{1,4}$	quartet	spinor	4
E_2	quintet	vector	5

- U_0=U_2=U, SU(4) symmetry.

$$H_{\text{int}} = \frac{U}{2} n(n-1)$$

Quintet channel (S=2) operators as SO(5) vectors

• Kinetic energy has an obvious SU(4) symmetry; interactions break it down to SO(5) (Sp(4)); SU(4) is restored at $U_0 = U_2$.

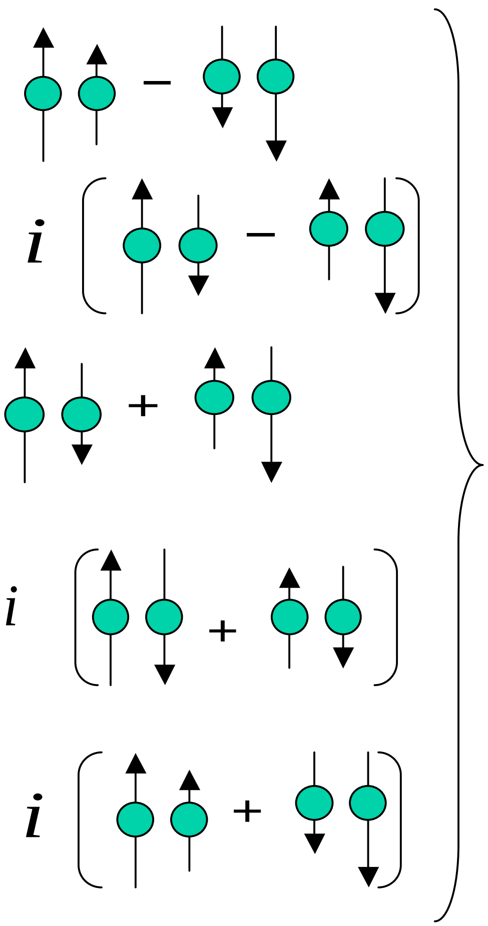
$d_{xy} : \chi_1^+(r) =$

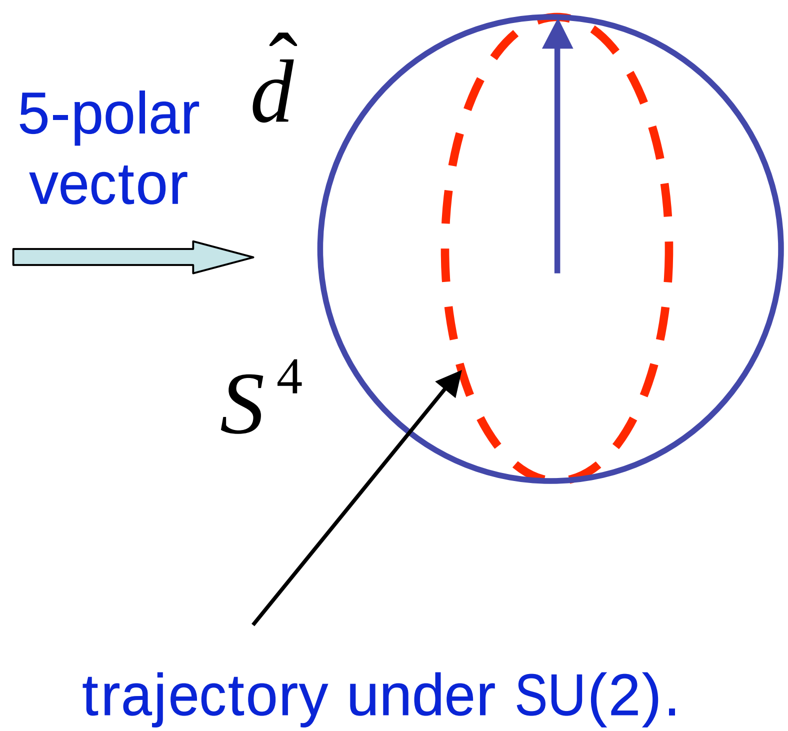
$d_{xz} : \chi_2^+(r) = i$

$d_{yz} : \chi_3^+(r) =$

$d_{3z^2-r^2} : \chi_4^+(r) = i$

$d_{x^2-y^2} : \chi_5^+(r) = i$





Particle-hole bi-linears $\psi^{+}_{\alpha} M_{\alpha\beta} \psi_{\beta}$ (I)

- Total degrees of freedom: $4^2=16=1+3+5+7$.

1 density operator and 3 spin operators are not complete.

$M_{\alpha\beta}$

rank: 0 $\quad 1,$

1 $\quad F_x, F_y, F_z$

2 $\quad \xi^a_{ij} F_i F_j \ (a = 1 \sim 5):$ ⟷ $F_x^2 - F_y^2, F_z^2 - \frac{5}{4},$ $\{F_x, F_y\}, \{F_y, F_z\}, \{F_z, F_x\}$

3 $\quad \xi^a_{ijk} F_i F_j F_k \ (a = 1 \sim 7)$

- **Spin-nematic matrices** (rank-2 tensors) form five-Γ matrices (SO(5) vector) .

$$\Gamma^a = \xi^a_{ij} F_i F_j, \quad \{\Gamma^a, \Gamma^b\} = 2\delta_{ab}, \quad (1 \le a, b \le 5)$$

Particle-hole channel algebra (II)

- Both $F_{x,y,z}$ and $\xi^{a}_{ijk} F_i F_j F_k$ commute with Hamiltonian.

10 generators of SO(5): 10=3+7.

7 spin cubic tensors are the hidden conserved quantities.

$$\Gamma^{ab} = \frac{i}{2}[\Gamma^a, \Gamma^b] \quad (1 \le a < b \le 5)$$

- **SO(5): 1 scalar + 5 vectors + 10 generators = 16**

		Time Reversal
1 density:	$n = \psi^{+}\psi;$	even
5 spin nematic:	$n_a = \frac{1}{2}\psi^{+}\Gamma^{a}\psi;$	even
3 spins + 7 spin cubic tensors:	$L_{ab} = \frac{1}{2}\psi^{+}\Gamma^{ab}\psi;$	odd

Outline

- The proof of the exact SO(5) symmetry**.**

- **Quintet superfluids and Half-quantum vortices.**

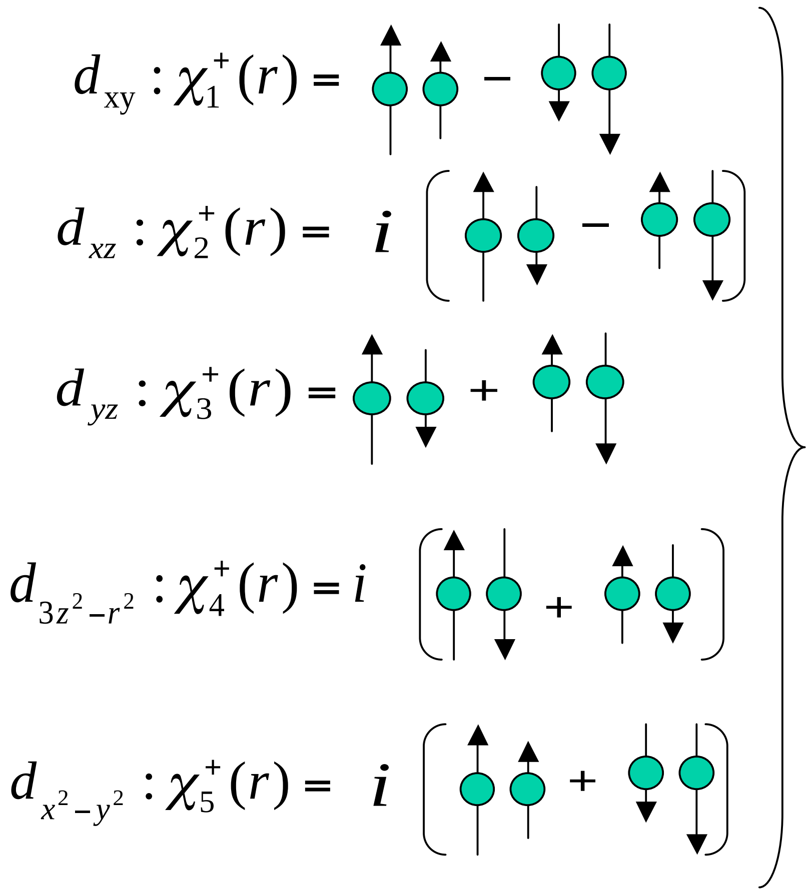
  C. Wu, J. P. Hu, and S. C. Zhang, cond-mat/0512602.

- Quartetting v.s pairing orders in 1D spin 3/2 systems.

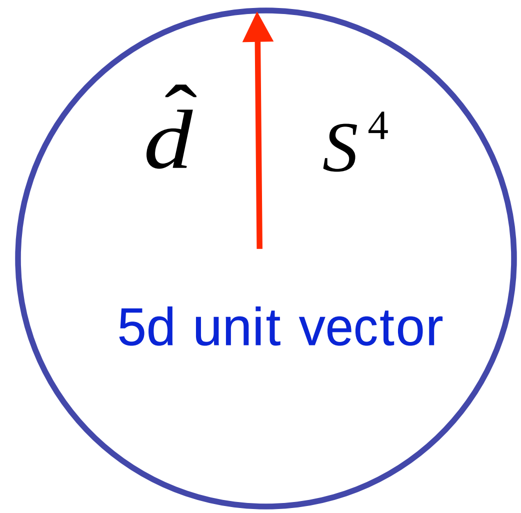
- SO(5) (Sp(4)) Magnetism in Mott-insulating phases.

$g_2<0$: s-wave quintet ($S_{pair}=2$) pairing

- BCS theory: polar condensation; order parameter forms an SO(5) vector.



$$\chi_a^+ = \sqrt{\rho}\, e^{i\theta}\, \hat{d}_a$$

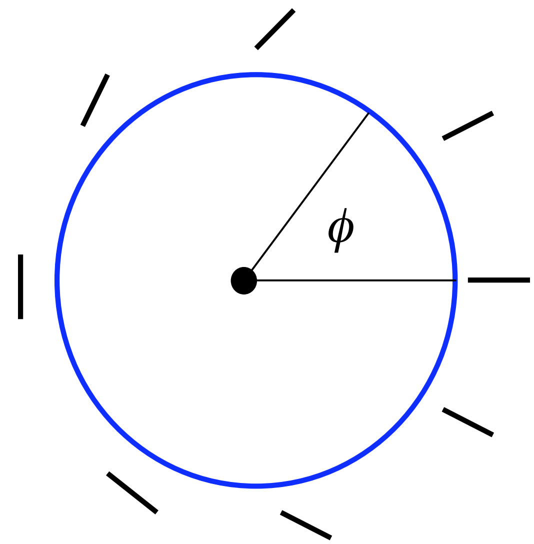


Ho and Yip, PRL 82, 247 (1999);

Wu, Hu and Zhang, cond-mat/0512602.

Superfluid with spin: half-quantum vortex (HQV)

- Z_2 gauge symmetry $\hat{d} \to -\hat{d}, \quad \theta \to \theta + \pi$

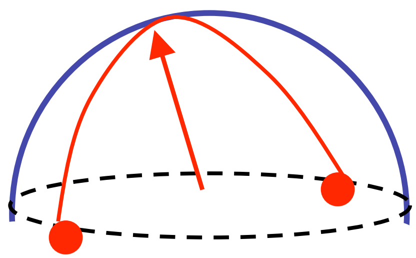


- π-disclination of $\hat{d}$ as a HQV.

$$\chi_a^+ = \sqrt{\rho} e^{i\theta} \hat{d}_a$$ remains single-valued.

- $\hat{d}$ is not a rigorous vector, but a directionless director.

$$\hat{d} : RP^4 = S^4 / Z_2$$



- Fundamental group of the manifold.

$$\pi_1(RP^4) = Z_2$$

Stability of half-quantum vortices

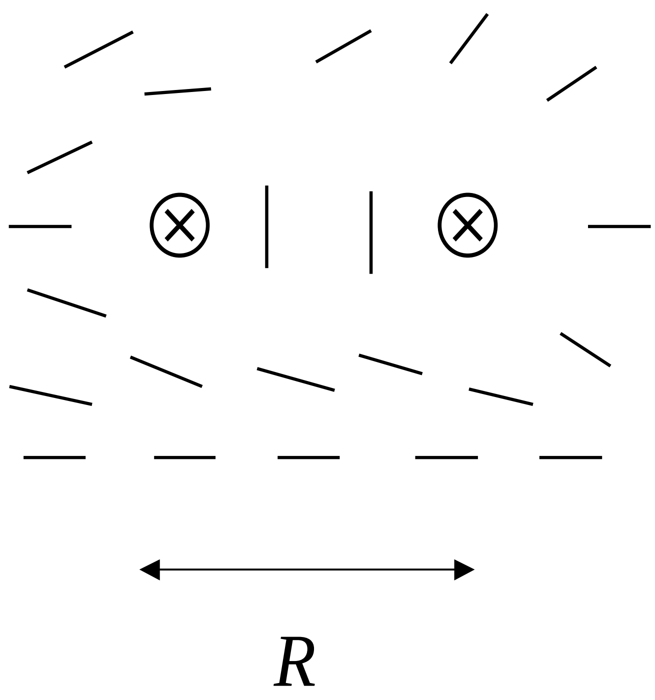
$$E = \int dr \frac{\hbar^2}{4M}\{\rho_{sf}(\nabla\theta)^2 + \rho_{sp}(\nabla\hat{d})^2\}$$

- Single quantum vortex:

$$E = \frac{h}{4M^2}\rho_{sf}\log\frac{L}{\xi}$$

- A pair of HQV:

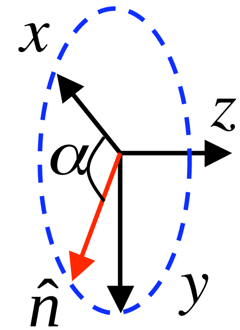
$$E = \frac{h}{4M^2}\left\{\frac{\rho_{sf}+\rho_{sp}}{2}\log\frac{L}{\xi} + \frac{\rho_{sf}-\rho_{sp}}{2}\log\frac{L}{R}\right\}$$



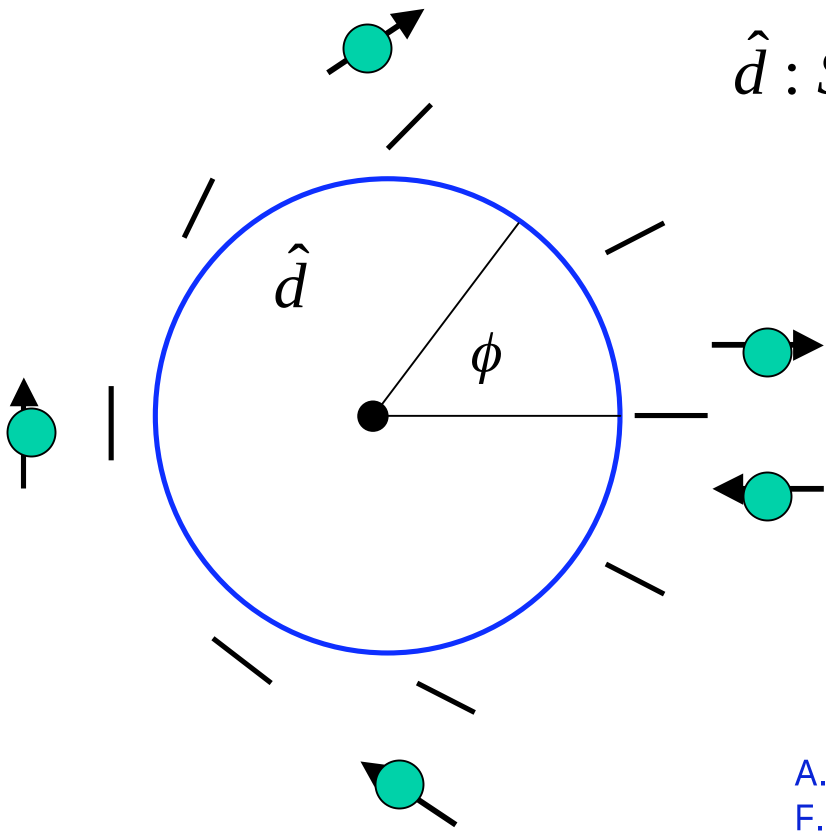
- Stability condition: $\rho_{sp} < \rho_{sf}$

Example: HQV as Alice string (^{3}He-A phase)

- A particle flips the sign of its spin after it encircles HQV.
- Example: ^{3}He-A, triplet Cooper pairing.



$$\hat{d} : S^2 / Z_2$$



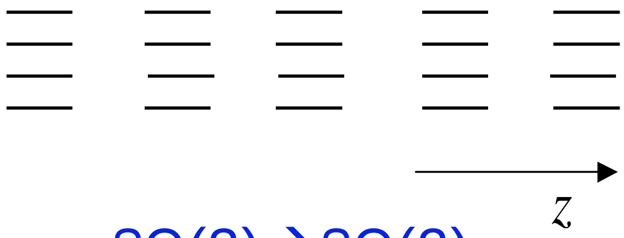
$$|\uparrow\rangle \to e^{i\alpha} |\downarrow\rangle, \quad |\downarrow\rangle \to e^{-i\alpha} |\uparrow\rangle$$

$$U(\hat{n}) = \begin{pmatrix} 0, & e^{i\alpha} \\ e^{-i\alpha}, & 0 \end{pmatrix}$$

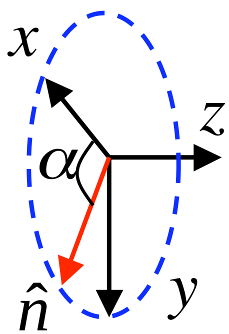
Configuration space U(1)

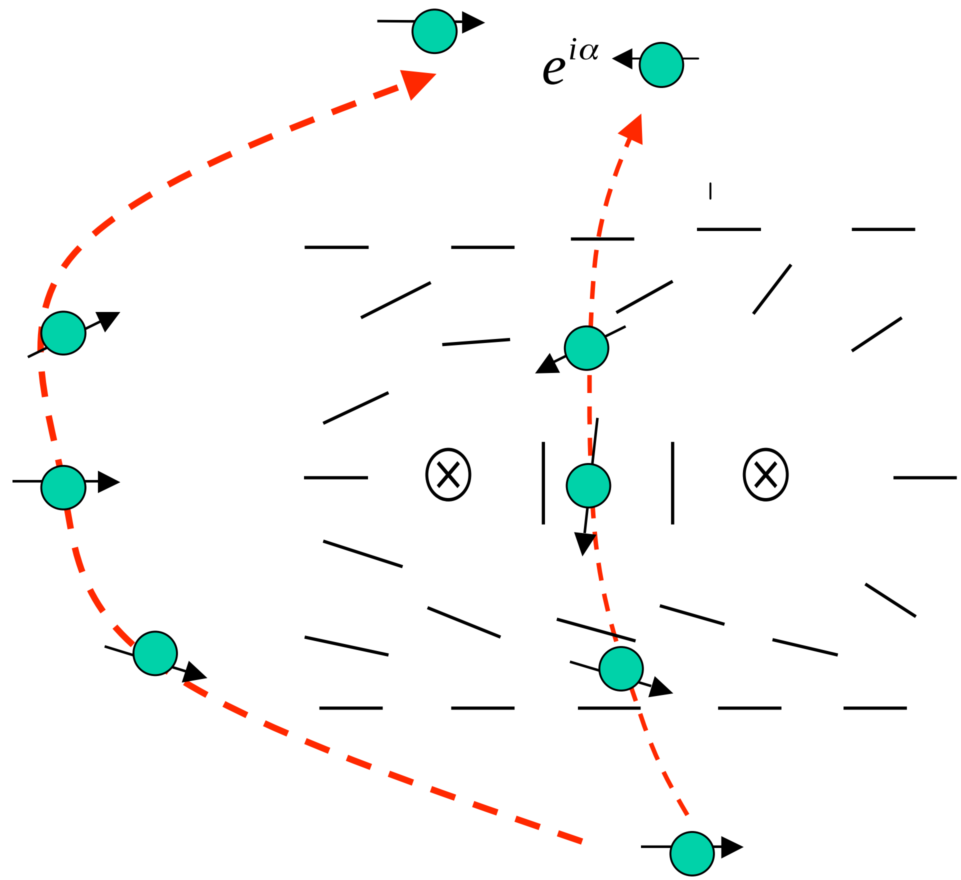
A. S. Schwarz et al., Nucl. Phys. B 208, 141(1982);
F. A. Bais et al., Nucl. Phys. B 666, 243 (2003);
M. G. Alford, et al. Nucl. Phys. B 384, 251 (1992).
P. McGraw, Phys. Rev. D 50, 952 (1994).

The HQV pair in 2D or HQV loop in 3D



SO(3)→SO(2)





- Phase state.

$$|\alpha\rangle_{vort} = \exp(iS_z\alpha)|\alpha = 0\rangle_{vort}$$

P. McGraw, Phys. Rev. D, 50, 952 (1994).

SO(2) Cheshire charge (^{3}He-A)

- HQV pair or loop can carry spin quantum number.

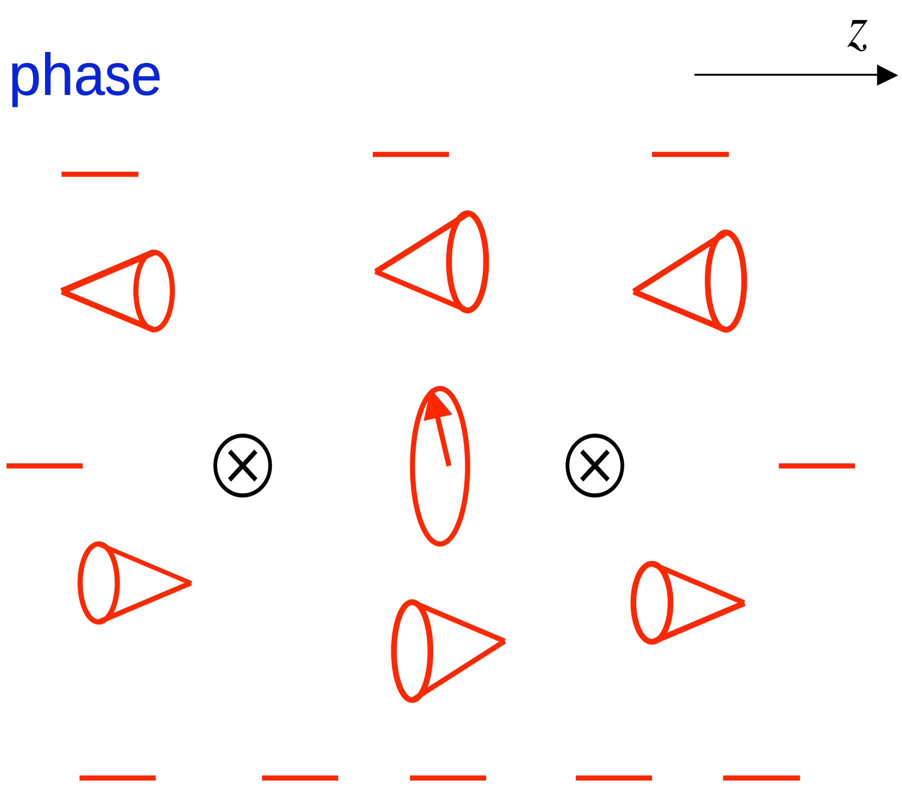
- For each phase state, SO(2) symmetry is only broken in a finite region, so it should be dynamically restored.

- Cheshire charge state (S_z) v.s phase state.

$$|m\rangle_{vort} = \int d\alpha \ \exp(im\alpha)\,|\alpha\rangle_{vort}$$

$$S_z|m\rangle_{vort} = m|m\rangle_{vort}$$

$$|m=0\rangle_{vort} = \int d\alpha\,|\alpha\rangle_{vort}$$



P. McGraw, Phys. Rev. D, 50, 952 (1994).

Spin conservation by exciting Cheshire charge

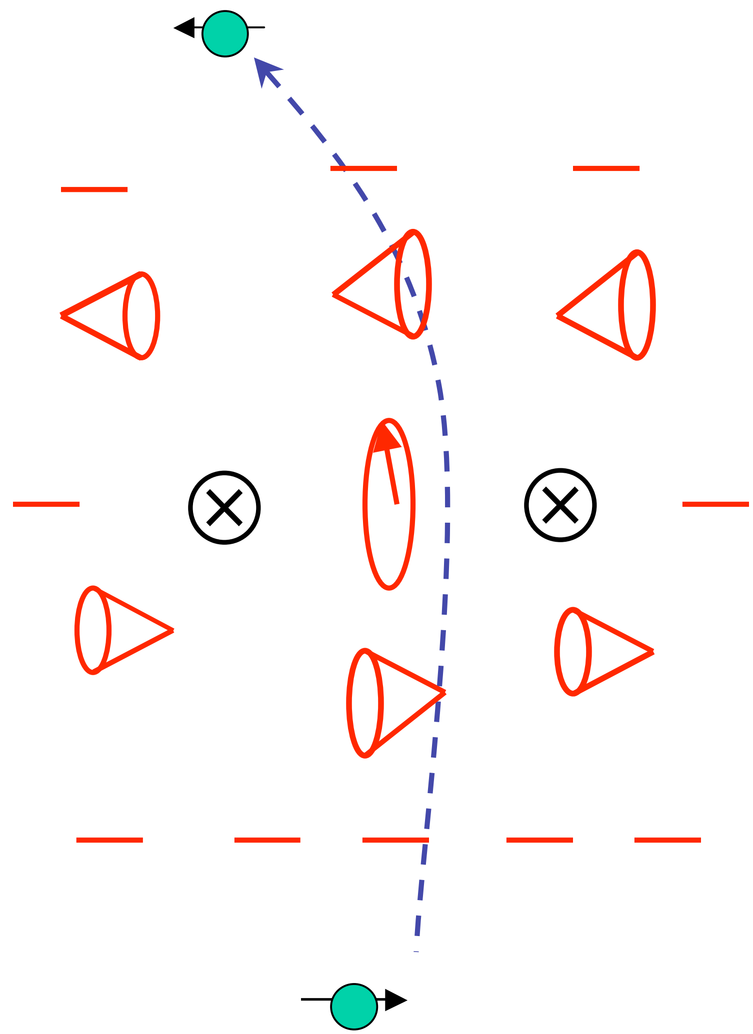
- Initial state: particle spin up and HQV pair (loop) zero charge.

- Final state: particle spin down and HQV pair (loop) charge 1.

$$|init\rangle = |\uparrow\rangle_p \otimes |m=0\rangle_{vort} = |\uparrow\rangle_p \otimes \int d\alpha \, |\alpha\rangle_{vort}$$

$$\longrightarrow$$

$$|final\rangle = |\downarrow\rangle_p \otimes \int d\alpha \, e^{i\alpha} |\alpha\rangle_{vort}$$
$$= |\downarrow\rangle_p \otimes |m=1\rangle_{vort}$$



- Both the initial and final states are product states. No entanglement!

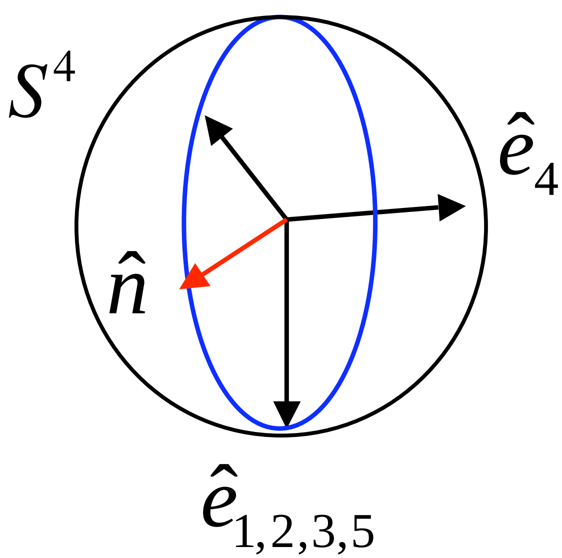
P. McGraw, Phys. Rev. D, 50, 952 (1994).

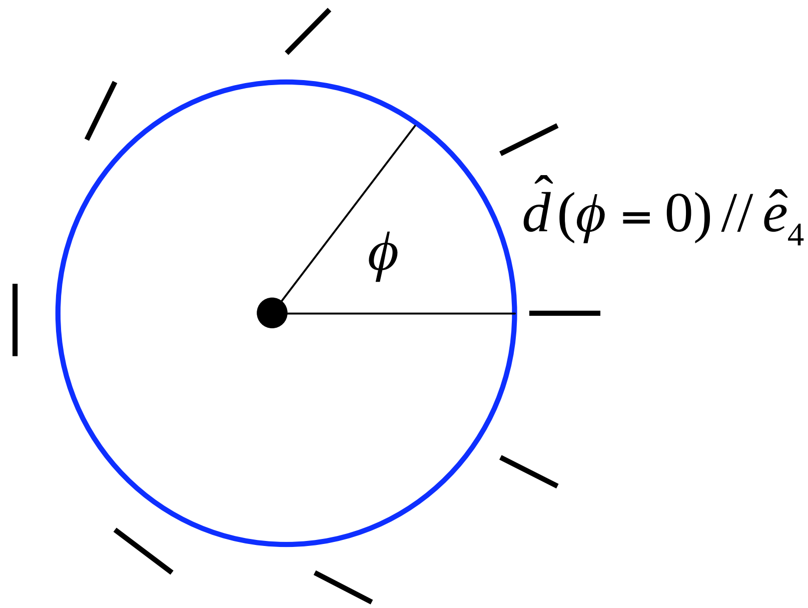
Quintet pairing as a non-Abelian generalization

- HQV configuration space $|\hat{n}\rangle_{vort}$, SU(2) instead of U(1).

equator: $\hat{n} \in S^3 = SU(2)$

$$\hat{d}(\phi, \hat{n}) = \cos\tfrac{\phi}{2}\hat{e}_4 - \sin\tfrac{\phi}{2}\hat{n}$$





π **-disclination**

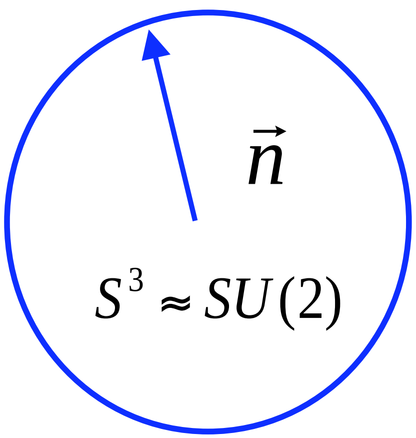
SU(2) Berry phase

- After a particle moves around HQV, or passes a HQV pair:

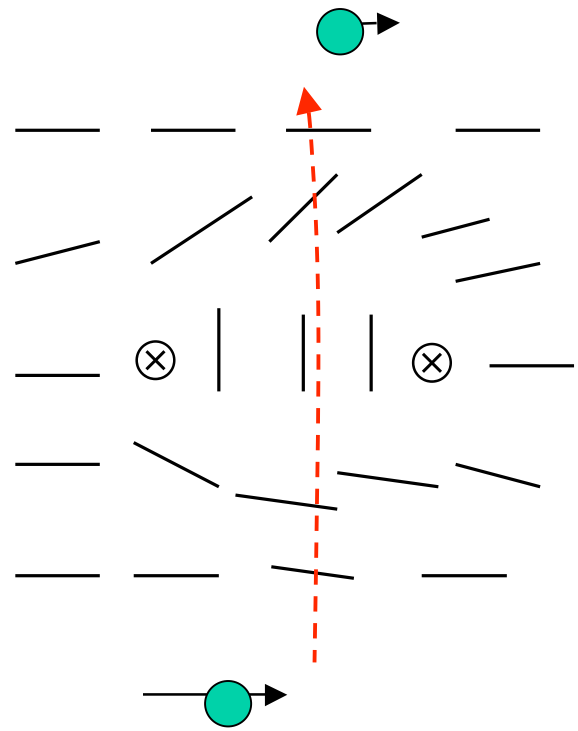
$$\psi = \begin{pmatrix} \left|\frac{3}{2}\right\rangle, \\ \left|-\frac{3}{2}\right\rangle, \\ \left|\frac{1}{2}\right\rangle, \\ \left|-\frac{1}{2}\right\rangle \end{pmatrix} \qquad \psi \rightarrow \begin{pmatrix} 0 & W \\ W^{+} & 0 \end{pmatrix} \psi$$

$$\left|\frac{3}{2}\right\rangle, \left|-\frac{3}{2}\right\rangle \Leftrightarrow \left|\frac{1}{2}\right\rangle, \left|-\frac{1}{2}\right\rangle$$

$$W(\hat{n}) = \begin{pmatrix} n_3 + in_2 & -n_1 + in_5 \\ n_1 - in_5 & n_3 - in_2 \end{pmatrix}$$

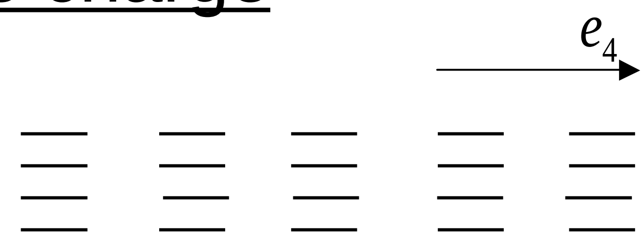


$$\hat{n} = n_1\hat{e}_1 + n_2\hat{e}_2 + n_3\hat{e}_3 + n_5\hat{e}_5$$



Non-Abelian Cheshire charge

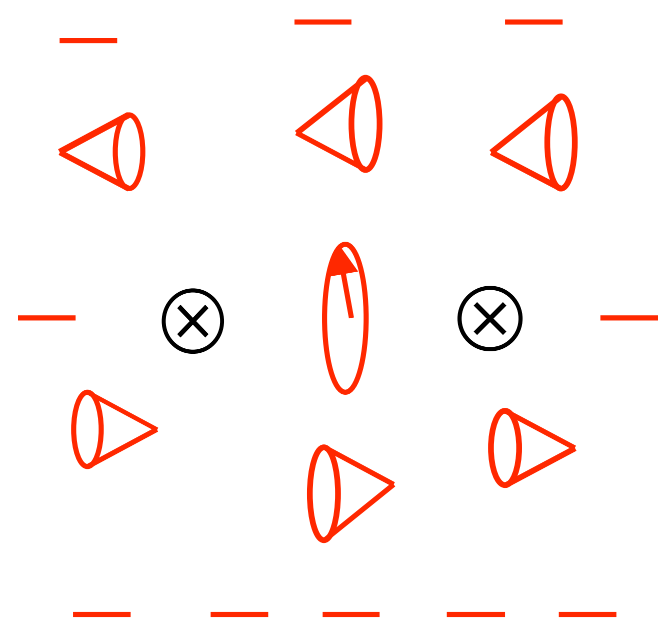
- Construct SO(4) Cheshire charge state for a HQV pair (loop) through S^3 harmonic functions.



SO(5) → SO(4)

$$\left|TT_3; T'T'_3\right\rangle_{vort} = \int_{n\in S^3} d\hat{n}\; Y_{TT_3;T'T'_3}(\hat{n})\, \left|\hat{n}\right\rangle_{vort}$$

$$\vec{T}\,(\vec{T}') = L_{12,} \pm L_{35}, L_{13,} \pm L_{25,}, L_{23} \pm L_{15}.$$



- Zero charge state.

$$\left|00;00\right\rangle_{vort} = \int_{n\in S^3} d\hat{n}\, \left|\hat{n}\right\rangle_{vort}$$

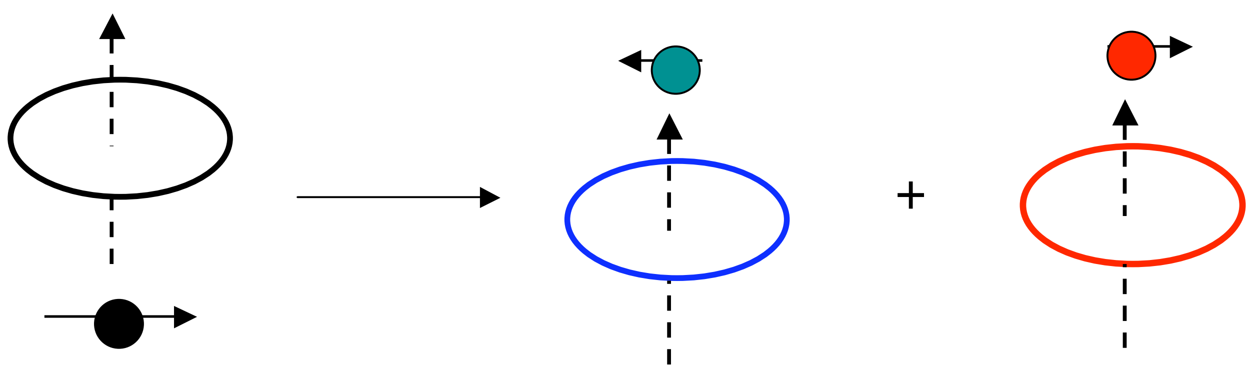
Entanglement through non-Abelian Cheshire charge!

- SO(4) spin conservation. For simplicity, only $S_z = T_3 + \frac{3}{2} T_3'$ is shown.

$$|init\rangle = \left|\tfrac{3}{2}\right\rangle_p \otimes |\text{zero charge}\rangle_{vort} \longrightarrow$$

$$|final\rangle = \left|\tfrac{1}{2}\right\rangle_p \otimes |S_z = 1\rangle_{vort} - \left|\tfrac{-1}{2}\right\rangle_p \otimes |S_z = 2\rangle_{vort}$$

- Generation of entanglement between the particle and HQV loop!



Outline

- The proof of the exact SO(5) symmetry.

- Alice string in quintet superfluids and non-Abelian Cheshire charge.

- **Quartetting and pairing orders in 1D systems.**

C. Wu, Phys. Rev. Lett. 95, 266404(2005).

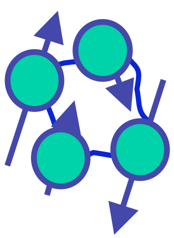
- SO(5) (Sp(4)) Magnetism in Mott-insulating phases.

Multiple-particle clustering (MPC) instability

• Pauli's exclusion principle allows MPC. More than two particles form bound states.

baryon (3-quark); alpha particle (2p+2n); bi-exciton (2e+2h)

• Spin 3/2 systems: quartetting order.



SU(4) singlet:

4-body maximally entangled states

$$O_{qt} = \psi^+_{3/2}(r)\psi^+_{1/2}(r)\psi^+_{-1/2}(r)\psi^+_{-3/2}(r)$$

• Difficulty: lack of a BCS type well-controlled mean field theory.

trial wavefunction in 3D: A. S. Stepanenko and J. M. F Gunn, cond-mat/9901317.

1D systems: strongly correlated but understandable

- Bethe ansatz results for 1D SU(2N) model:

2N particles form an SU(2N) singlet; Cooper pairing is not possible because 2 particles can not form an SU(2N) singlet.

P. Schlottmann, J. Phys. Cond. Matt 6, 1359(1994).

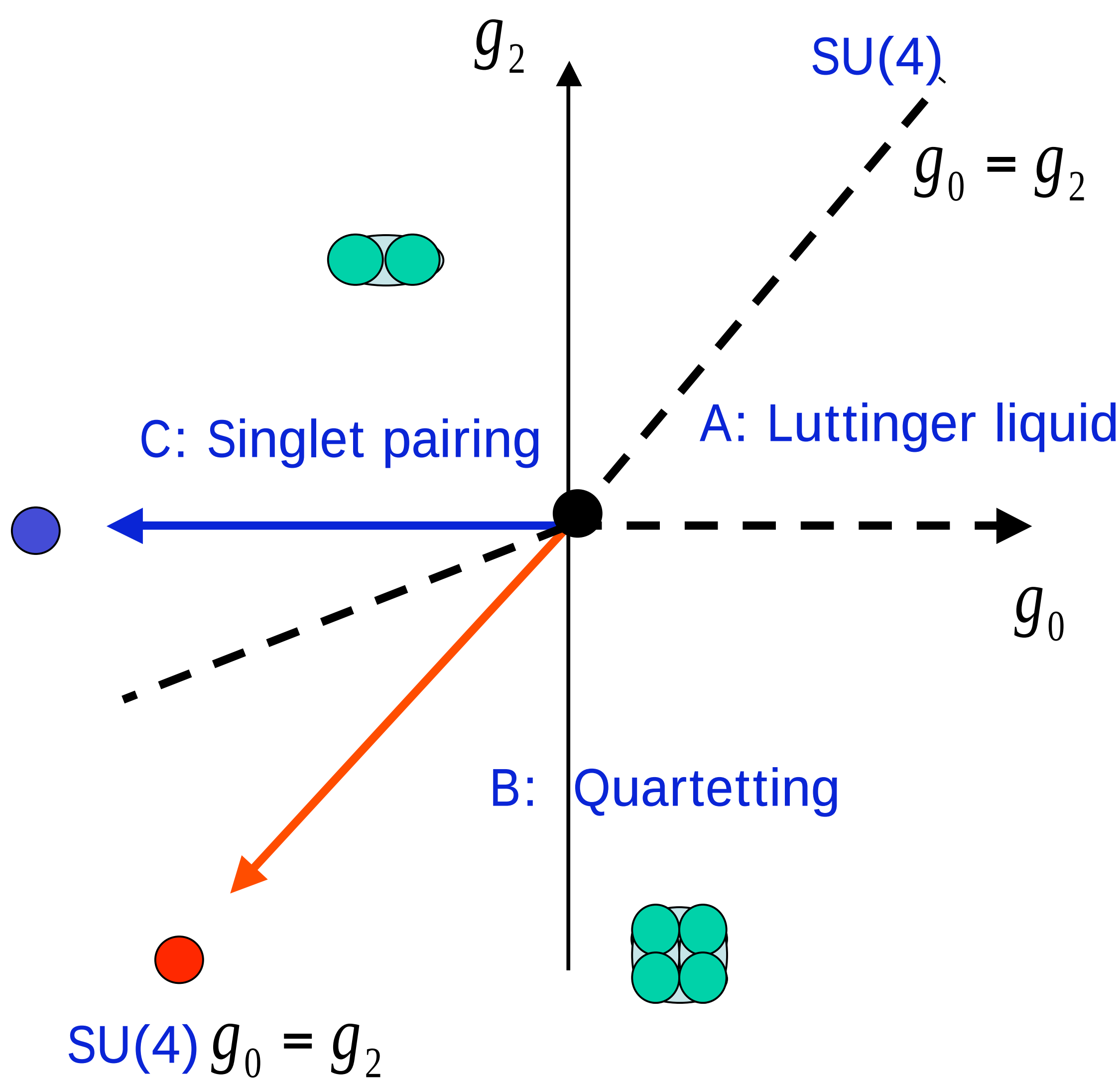
- Competing orders in 1D spin 3/2 systems with SO(5) symmetry.

Both quartetting and singlet Cooper pairing are allowed.

Transition between quartetting and Cooper pairing.

C. Wu, Phys. Rev. Lett. 95, 266404(2005).

Phase diagram at incommensurate fillings



- Gapless charge sector.
- Spin gap phases B and C: pairing v.s. quartetting.
- Ising transition between B and C.
- Singlet pairing in purely repulsive regime.
- Quintet pairing is not allowed.

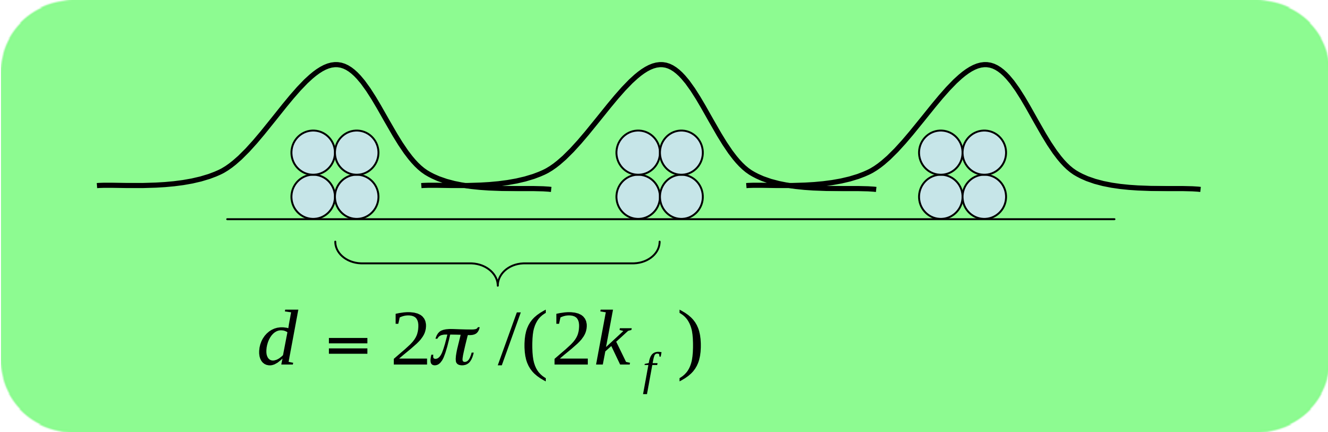
Phase B: the quartetting phase

- Quartetting superfluidity v.s. CDW of quartets ($2k_f$-CDW).

$$O_{qt} = \psi^+_{3/2}\psi^+_{1/2}\psi^+_{-1/2}\psi^+_{-3/2} \text{ wins at } K_c > 2;$$

$$N_{2k_f} = \psi^+_{R\alpha}\psi_{L\alpha} \text{ wins at } K_c < 2.$$

K_c: the Luttinger parameter in the charge channel.

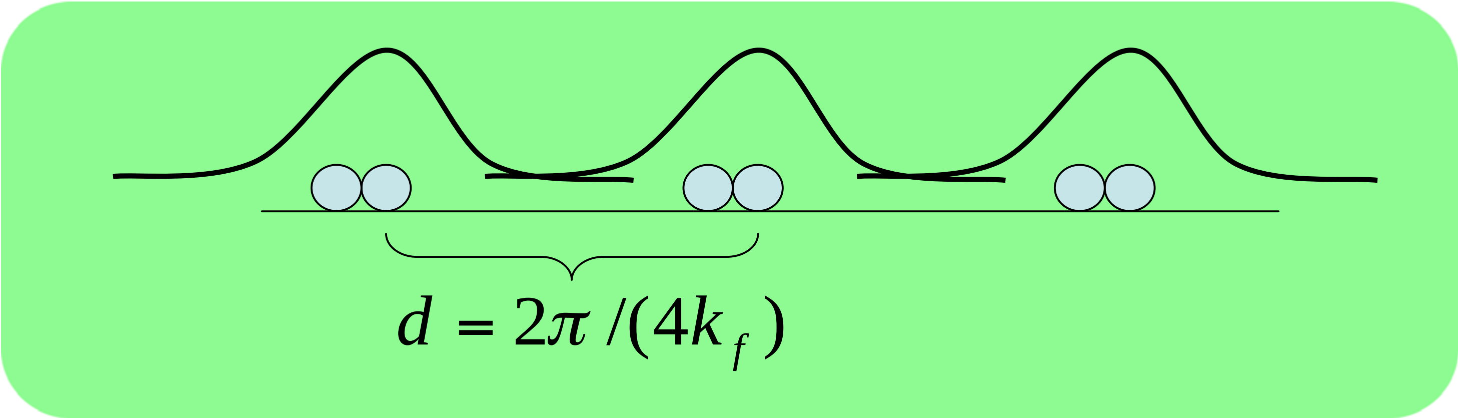


Phase C: the singlet pairing phase

- Singlet pairing superfluidity v.s CDW of pairs ($4k_f$-CDW).

$$\eta^+ = \psi^+_{3/2}\psi^+_{-3/2} - \psi^+_{1/2}\psi^+_{-1/2} \text{ wins at } K_c > \tfrac{1}{2};$$

$$O_{4k_f,cdw} = \psi^+_{R\alpha}\psi^+_{R\beta}\psi_{L\beta}\psi_{L\alpha} \text{ wins at } K_c < \tfrac{1}{2}.$$



Competition between quartetting and pairing phases

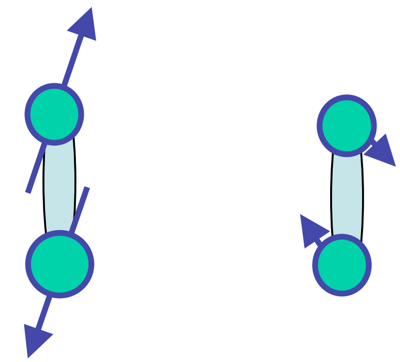
A. J. Leggett, Prog, Theo. Phys. 36, 901(1966); H. J. Schulz, PRB 53, R2959 (1996).

- Phase locking problem in the two-band model.

$$\eta^+ = \Delta_1^+ - \Delta_2^+ \propto e^{i\sqrt{\pi}\theta_c}\cos\sqrt{\pi}\theta_r;$$

$$O_{quar} = \Delta_1^+\Delta_2^+ \propto e^{i\sqrt{4\pi}\theta_c}\cos 2\sqrt{\pi}\varphi_r.$$

$$\Delta_1^+ = \psi_{3/2}^+\psi_{-3/2}^+ \qquad \Delta_2^+ = \psi_{1/2}^+\psi_{-1/2}^+$$



$\sqrt{\pi}\theta_c$ overall phase; $\sqrt{\pi}\theta_r$ relative phase.

$\sqrt{\pi}\varphi_r$ dual field of the relative phase

- No symmetry breaking in the overall phase (charge) channel in 1D.

$$\psi^+_{\pm\frac{3}{2},\pm\frac{1}{2}} \rightarrow \psi^+_{\pm\frac{3}{2},\pm\frac{1}{2}} e^{i\alpha} \quad i.e. \quad \sqrt{\pi}\theta_c \rightarrow \sqrt{\pi}\theta_c + 2\alpha$$

Ising transition in the **relative phase** channel

$$H_{eff}=\frac{1}{2}\{(\partial_x\theta_r)^2+(\partial_x\varphi_r)^2\}+\frac{1}{2\pi a}(\lambda_1\cos 2\sqrt{\pi}\theta_r+\lambda_2\cos 2\sqrt{\pi}\varphi_r)$$

- $\lambda_1>\lambda_2$ the relative phase is pinned: pairing order;
  $\lambda_1<\lambda_2$ the dual field is pinned: quartetting order.

Ising transition: two Majorana fermions with masses: $\lambda_1\pm\lambda_2$

- Ising symmetry: $\psi^{+}_{\pm\frac{3}{2}}\to i\psi^{+}_{\pm\frac{3}{2}},\quad \psi^{+}_{\pm\frac{1}{2}}\to -i\psi^{+}_{\pm\frac{1}{2}}$.

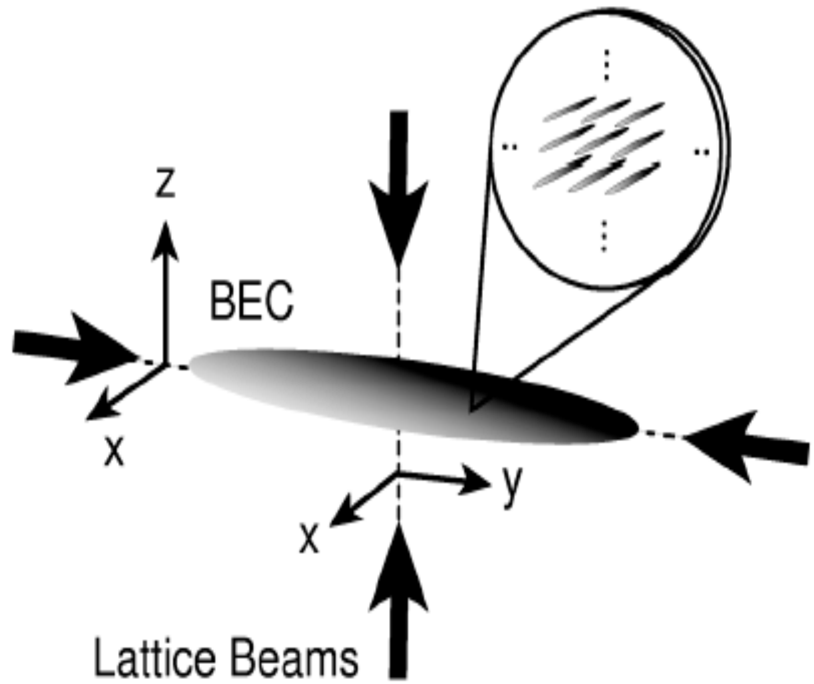
relative phase: $\sqrt{\pi}\theta_r\to\sqrt{\pi}\theta_r\pm\pi\qquad \sqrt{\pi}\varphi_r\to\sqrt{\pi}\varphi_r$

- Ising ordered phase: $\eta\to-\eta,$
  Ising disordered phase: $O_{quart}\to O_{quart}$

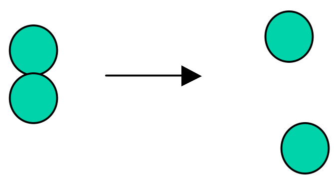
Experiment setup and detection

- Array of 1D optical tubes.

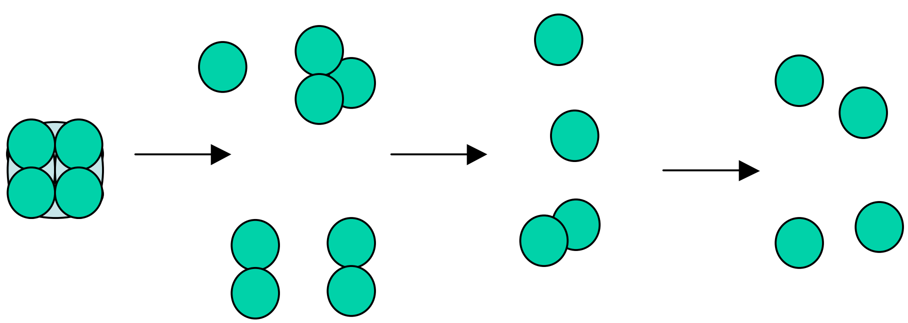
- RF spectroscopy to measure the excitation gap.



pair breaking:



quartet breaking:



M. Greiner et. al., PRL, 2001.

Outline

- The proof of the exact SO(5) symmetry.

- Alice string in quintet superfluids and non-Abelian Cheshire charge.

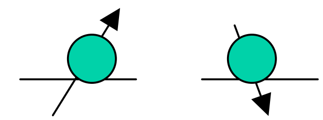
- Quartetting v.s pairing orders in 1D spin 3/2 systems.

- SO(5) (Sp(4)) **magnetism in Mott-insulating phases.**

S. Chen, C. Wu, S. C. Zhang and Y. P. Wang, Phys. Rev. B 72, 214428 (2005).

Spin exchange (one particle per site)

- Spin exchange: bond singlet (J_0), quintet (J_2). No exchange in the triplet and septet channels.



$$H_{ex} = \sum_{\langle ij \rangle} -J_0 Q_0(ij) - J_2 Q_2(ij)$$

$$J_0 = 4t^2/U_0, J_2 = 4t^2/U_2, J_1 = J_3 = 0$$

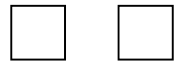
$$\frac{3}{2} \times \frac{3}{2} = 0 + 2 + 1 + 3$$

- Heisenberg model with bi-linear, bi-quadratic, bi-cubic terms.
- SO(5) or Sp(4) explicitly invariant form:

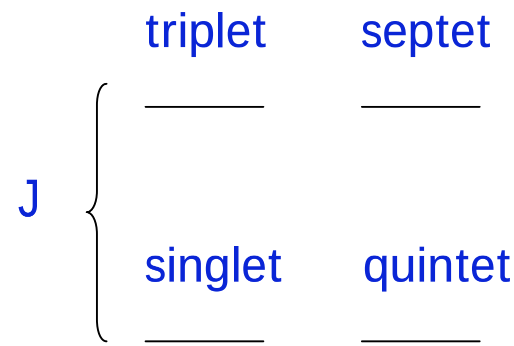
$$H_{ex} = \sum_{ij} \frac{J_0 + J_2}{4} L_{ab}(i) L_{ab}(j) + \frac{-J_0 + 3J_2}{4} n_a(i) n_b(j) \qquad a, b = 1 \sim 5$$

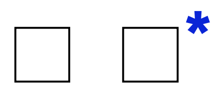
L_{ab}: 3 spins + 7 spin cubic tensors; n_a: spin nematic operators;
L_{ab} and n_a together form the15 SU(4) generators.

Two different SU(4) symmetries

- A: J_0=J_2=J, SU(4) point. 

$$H_{ex} = J\sum_{ij}\{L_{ab}(i)L_{ab}(j) + n_a(i)n_b(j)\}$$

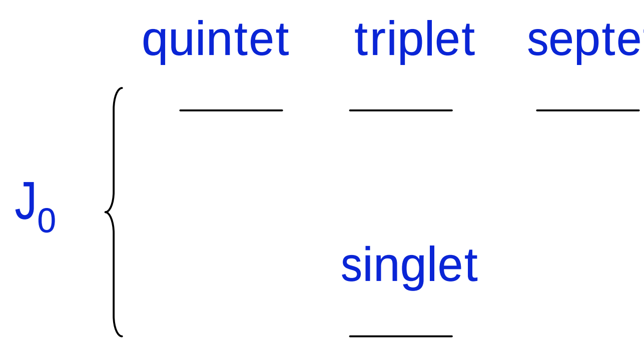


- B: J_2=0, the staggered SU'(4) point. 

In a bipartite lattice, a particle-hole transformation on odd sites:

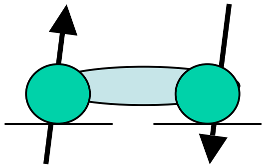
$$L_{ab}(j) = L'_{ab}(j) \qquad n_{ab}(j) = -n'_{ab}(j)$$

$$H_{ex} = \frac{J_0}{4}\sum_{ij}\{L_{ab}(i)L'_{ab}(j) + n_a(i)n'_a(j)\}$$



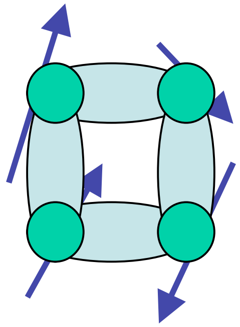
Construction of singlets

- The SU(2) singlet: 2 sites.



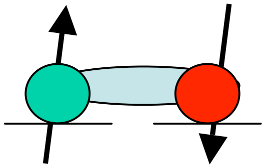
- The uniform SU(4) singlet: 4 sites.

baryon $\frac{\varepsilon_{\alpha\beta\gamma\delta}}{4!}\psi_{\alpha}^{+}(1)\psi_{\beta}^{+}(2)\psi_{\gamma}^{+}(3)\psi_{\delta}^{+}(4)\left|0\right\rangle$

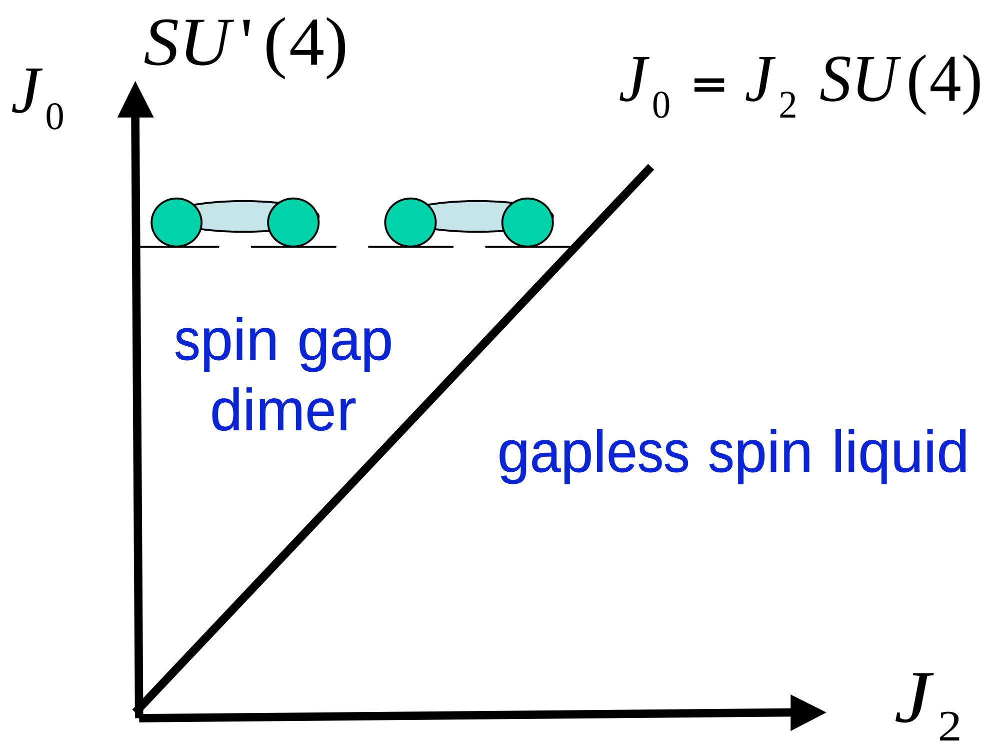


- The staggered SU'(4) singlet: 2 sites.

meson $\frac{1}{2}\psi_{\alpha}^{+}(1)R_{\alpha\beta}\psi_{\beta}^{+}(2)$



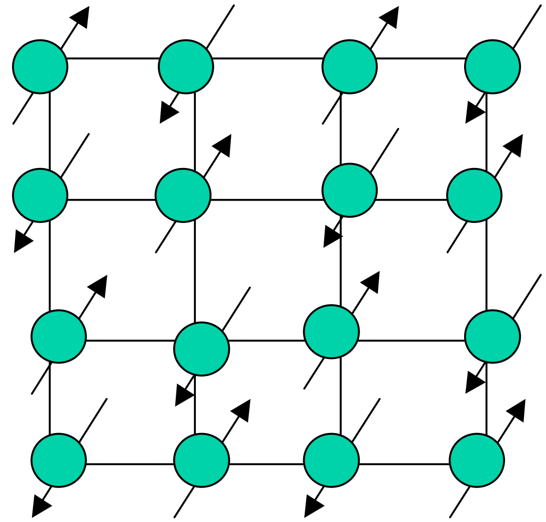
Phase diagram in 1D lattice (one particle per site)



- On the SU'(4) line, dimerized spin gap phase.
- On the SU(4) line, gapless spin liquid phase.

C. Wu, Phys. Rev. Lett. 95, 266404 (2005).

SU'(4) and SU(4) point at 2D

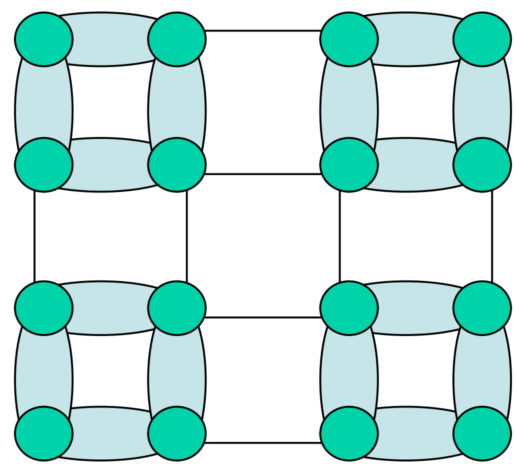


- At SU'(4) point (J_2=0), QMC and large N give the Neel order, but the moment is tiny.

  Read, and Sachdev, Nucl. Phys. B 316(1989).

  K. Harada et. al. PRL 90, 117203, (2003).

- J_2>0, no conclusive results!



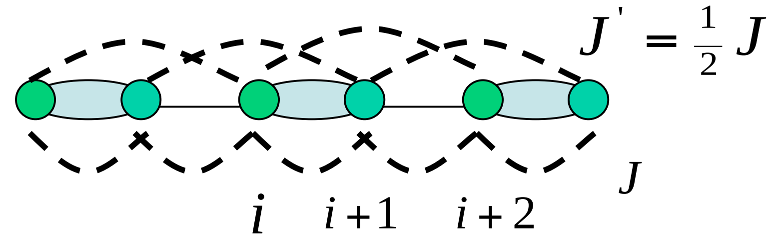
SU(4) point (J_0=J_2), 2D Plaquette order at the SU(4) point?

Exact diagonalization on a 4*4 lattice

Bossche et. al., Eur. Phys. J. B 17, 367 (2000).

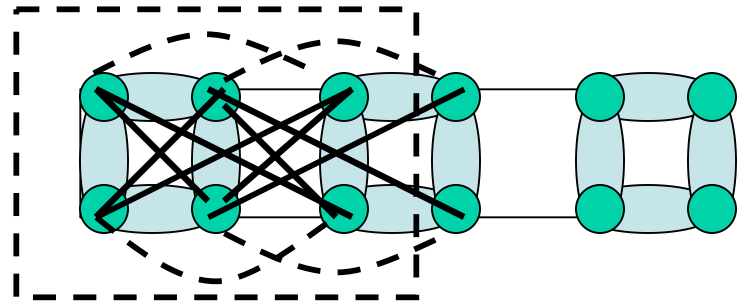
Exact result: SU(4) Majumdar-Ghosh ladder

- Exact dimer ground state in spin 1/2 M-G model.



$$H = \sum_i H_{i,i+1,i+2}, \quad H_{i,i+1,i+2} = \frac{J}{2} (\vec{S}_i + \vec{S}_{i+1} + \vec{S}_{i+2})^2$$

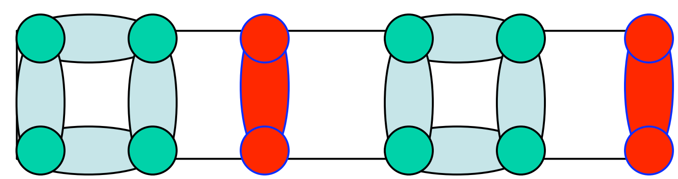
- SU(4) M-G: plaquette state.



$$H = \sum_{\text{every six-site cluster}} H_i$$

$$H_i = \Big(\sum_{\text{six sites}} L_{ab}\Big)^2 + \Big(\sum_{\text{six sites}} n_a\Big)^2$$

SU(4) Casimir of the six-site cluster



- Excitations as fractionalized domain walls.

S. Chen, C. Wu, S. C. Zhang and Y. P. Wang, Phys. Rev. B 72, 214428 (2005).

SU(4) plaquette state: a four-site problem

- Bond spin singlet:
- Plaquette SU(4) singlet:

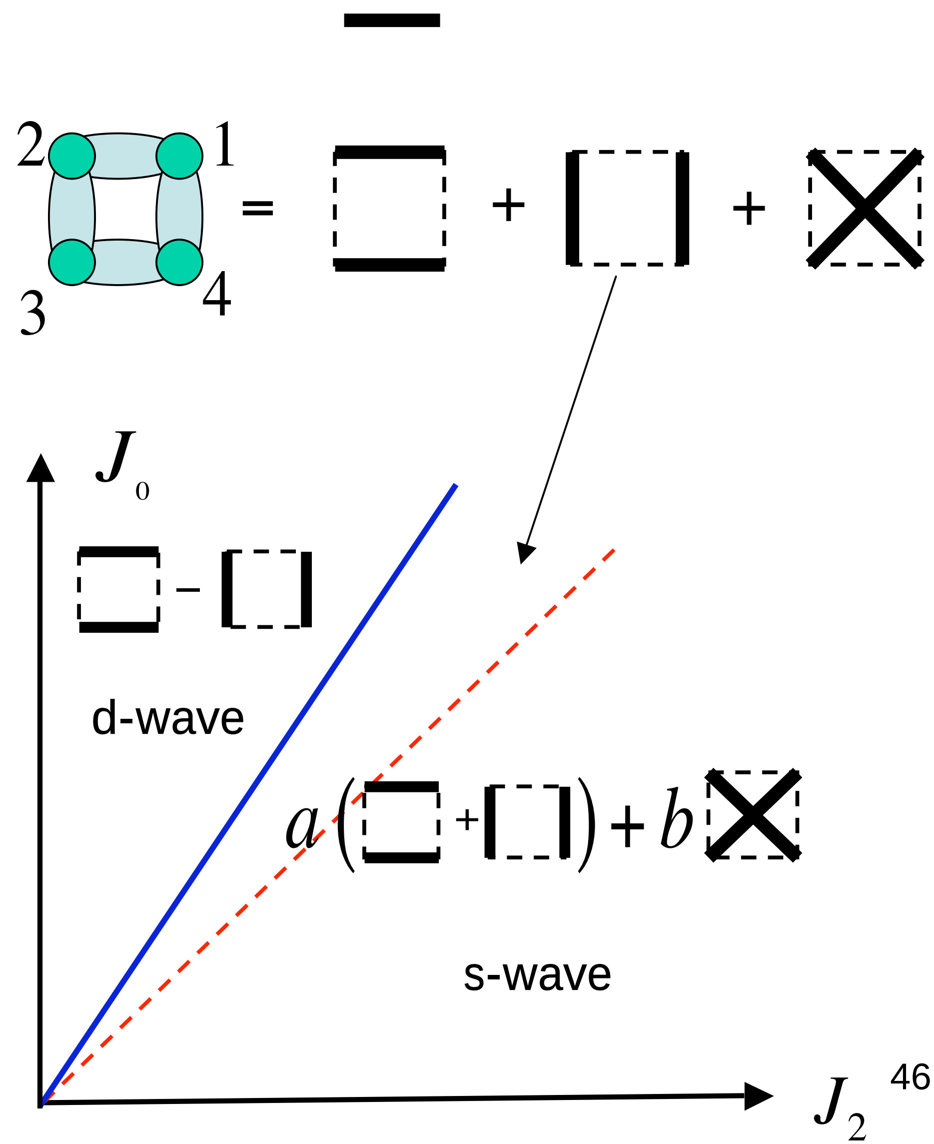
$$\frac{\varepsilon_{\alpha\beta\gamma\delta}}{4!}\psi^{+}_{\alpha}\psi^{+}_{\beta}\psi^{+}_{\gamma}\psi^{+}_{\delta}|0\rangle$$

4-body EPR state;
no bond orders

- Level crossing:

d-wave to s-wave

- Hint to 2D?



Speculations: 2D phase diagram ?

- $J_2=0$, Neel order at the SU'(4) point (QMC).

K. Harada et. al. PRL 90, 117203, (2003).

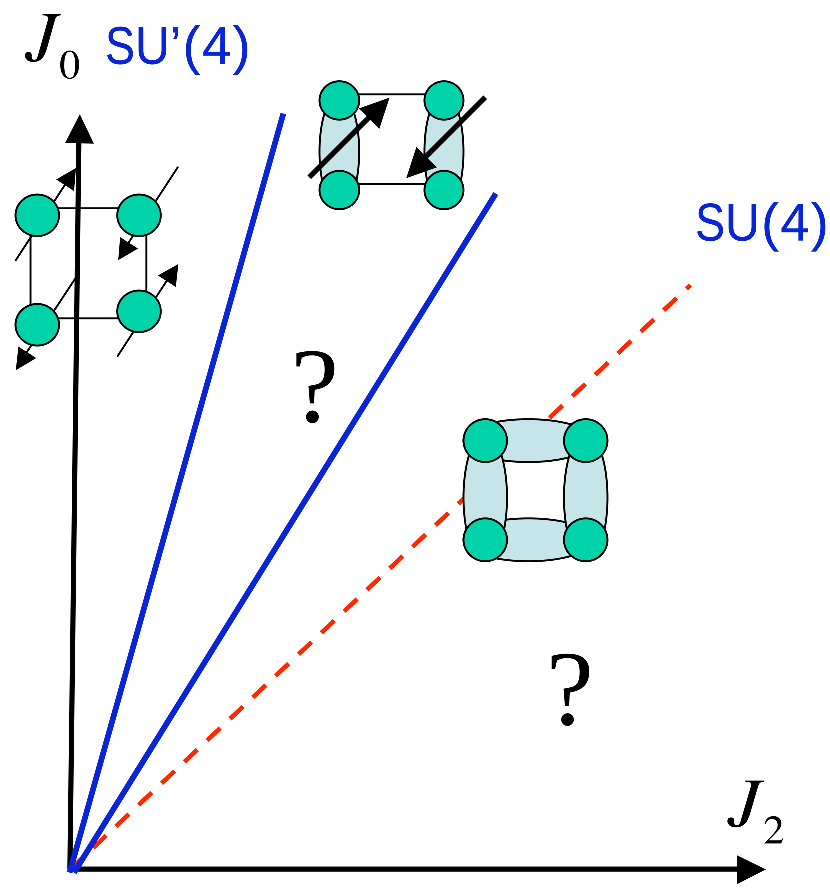
- $J_2>0$, no conclusive results!

2D Plaquette order at the SU(4) point?

Exact diagonalization on a 4*4 lattice

Bossche et. al., Eur. Phys. J. B 17, 367 (2000).

- Phase transitions as J_0/J_2? Dimer phases? Singlet or magnetic dimers?



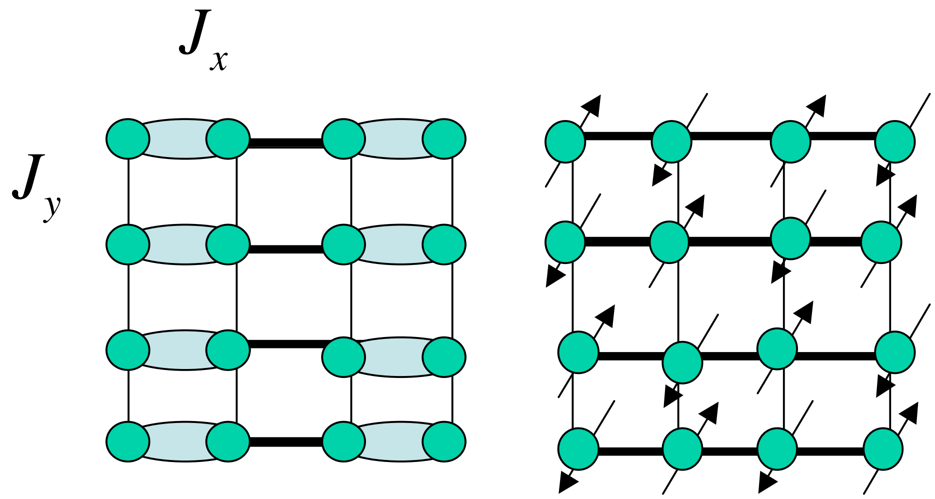
The SU'(4) model: dimensional crossover

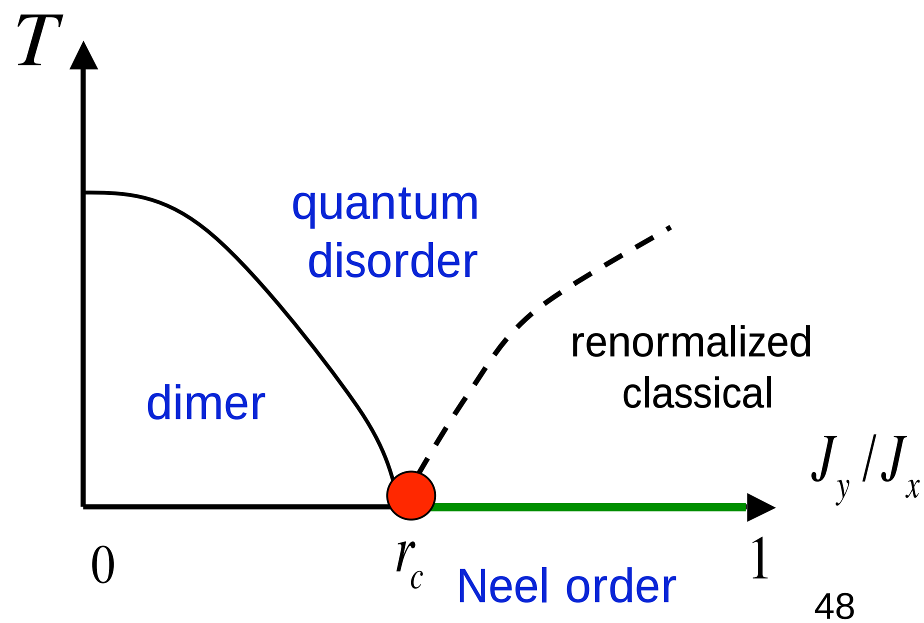
- SU'(4) model: 1D dimer order; 2D Neel order.

- SU'(4) model in a rectangular lattice; phase diagram as J_y/J_x.

- Competition between the dimer and Neel order.

- **No frustration**; transition accessible by QMC.





Conclusion

• Spin 3/2 cold atomic systems open up a new opportunity to study high symmetry and novel phases.

• Quintet Cooper pairing: the Alice string and topological generation of quantum entanglement.

• Quartetting order and its competition with the pairing order.

• Strong quantum fluctuations in spin 3/2 magnetic systems.