Matsubara Green's function

$$\tau>0 : \quad \mathcal{G}(\tau) = -\langle O_2(\tau)\, O_1(0)\rangle = \frac{1}{Z}$$

$$O_1(\tau) = e^{H\tau}\, O_1\, e^{-H_1\tau}$$

$$\log Z = e^{-\beta\Omega}$$

$$= -e^{+\beta\Omega}\sum_{n,m}\langle n|O_2(\tau)|m\rangle\langle m|O_1(\tau)|n\rangle$$

?

$$= -e^{\beta\Omega}\sum_{n,m} e^{-\beta E_n}\, e^{(E_n-E_m)\tau}\langle n|O_2|m\rangle\langle m|O_1|n\rangle$$

吸收　　傅立叶变换 n

$$\mathcal{G}(i\omega_n) = -e^{\beta\Omega}\int_0^{\beta} d\tau\, e^{i\omega_n\tau+(E_n-E_m)\tau}\langle n|O_2|m\rangle$$

$$\langle m|O_1|n\rangle\, e^{-\beta E_n}$$

$$= e^{\beta\Omega}\sum_{n,m}\frac{\langle n|O_2|m\rangle\langle m|O_1|n\rangle}{i\omega_n+E_n-E_m}\left(e^{-\beta E_n}\mp e^{-\beta E_m}\right)$$

$$= e^{\beta\Omega} \sum_{n,m} \frac{\langle n|O_2|m\rangle\langle m|O_1|n\rangle}{i\omega_n + E_n - E_m}$$

可以与物理可观测量建立联系

$$D_{ret}(t-t') = -i\,\Theta(t-t')\langle\, [O_2(t), O_1(t')]_{\mp} \rangle$$

$$= -i\,\Theta(t-t')\, e^{\beta\Omega} \sum_{n,m} \langle n|O_2|m\rangle\langle m|O_1|n\rangle\, e^{i(E_n-E_m)(t-t')} \left(e^{-\beta E_n} \mp e^{-\beta E_m}\right)$$

$\downarrow$

热平衡

$$D_{ret}(\omega) = \int_{-\infty}^{+\infty} d(t-t')\, e^{i\omega(t-t')}\, D_{ret}(t-t')$$

$$= e^{\beta\Omega} \sum_{n,m} \frac{\langle n|O_2|m\rangle\langle m|O_1|n\rangle}{\omega + (E_n - E_m) + i\eta} \left(e^{-\beta E_n} \mp e^{-\beta E_m}\right)$$

time-order Green — path integral

- 为什么不从 time-order Green 去做

因为 Lehman rep 能够计算可观测 Green

只需要: $i\omega_n$ — $\omega + i\eta$

Matsubara

$$G(t-t') = -i\Theta(t-t')\langle|O_2(t)O_1(t')|0\rangle$$

$$\mp i\Theta(t-t')\langle|O_1(t')O_2(t)|\rangle$$

$\downarrow$

零温可以用，有限温就不行了

$$= -i\Theta(t-t')e^{\beta\Omega}\sum_{n,m}\langle n|O_2|m\rangle\langle m|O_1|n\rangle e^{-\beta E_n} e^{i(E_n-E_m)(t-t')}$$

$$\mp(-i\Theta(t-t'))e^{\beta\Omega}\sum_{n,m}\langle m|O_2|m\rangle\langle n|O_1|n\rangle e^{-\beta E_m} e^{i(E_n-E_m)(t-t')}$$

$$= e^{\beta\Omega}\sum_{n,m}\langle n|O_2|m\rangle\langle m|O_1|n\rangle e^{i(E_n-E_m)(t-t')}$$

$$\left(-i\Theta(t-t')e^{-\beta E_n}e^{-\eta(t-t')} \pm -i\Theta(t-t')e^{-\beta E_m}e^{\eta(t-t')}\right)$$

$\downarrow$

$$G(\omega) = \sum_{n,m} e^{\beta\Omega}\langle n|O_2|m\rangle\langle m|O_1|n\rangle \left\{\frac{e^{-\beta E_n}}{\omega+E_n-E_m+i\eta} \mp \frac{e^{-\beta E_m}}{\omega+E_n-E_m-i\eta}\right\}$$

Matsubara

$$G(t-t') = -i\,\Theta(t-t')\langle|O_2(t)\,O_1(t')|0\rangle$$

$$\mp i\,\Theta(t-t')\langle|O_1(t')\,O_2(t)|\rangle$$

$\downarrow$

零温可以用，有限温就不行了

$$= -i\,\Theta(t-t')\,e^{\beta\Omega}\sum_{n,m}\langle n|O_2|m\rangle\langle m|O_1|n\rangle\,e^{-\beta E_n}$$

$$e^{i(E_n-E_m)(t-t')}$$

$$\mp(-i\,\Theta(t-t'))\,e^{\beta\Omega}\sum_{n,m}\langle n|O_2|m\rangle\langle m|O_1|n\rangle\,e^{-\beta E_m}$$

$$e^{i(E_n-E_m)(t-t')}$$

$$= e^{\beta\Omega}\sum_{n,m}\langle n|O_2|m\rangle\langle m|O_1|n\rangle\,e^{i(E_n-E_m)(t-t')}$$

$$\left(-i\,\Theta(t-t')\,e^{-\beta E_n}\,e^{-\eta(t-t')} \pm -i\,\Theta(t-t')\,e^{-\beta E_m}\,e^{\eta(t-t')}\right)$$

$\downarrow$

$$G(\omega) = \sum_{n,m} e^{\beta\Omega}\langle n|O_2|m\rangle\langle m|O_1|n\rangle$$

$$\left\{\frac{e^{-\beta E_n}}{\omega+E_n-E_m+i\eta} \mp \frac{e^{-\beta E_m}}{\omega+E_n-E_m-i\eta}\right\}$$

- Matsubara

$$\mathcal{G}(i\omega_n) = e^{\beta\Omega} \sum_{n,m} \frac{\langle n|O_2|m\rangle\langle m|O_1|n\rangle}{i\omega_n + E_n - E_m}$$

$$\left(e^{-\beta E_n} \mp e^{-\beta E_m}\right) \quad (\text{time order})$$

Retard Green function 的奇点都在实轴的下面

time-order 都在实轴上下面 (finite temp)

- Zero temperature : $\beta \to \infty$

$$G(\omega) = \sum \frac{\langle 0|O_2|m\rangle\langle m|O_1|0\rangle}{\omega + E_0 - E_m + i\eta} \mp \frac{\langle n|O_2|0\rangle\langle 0|O_1|n\rangle}{\omega + (E_0 - E_n) - i\eta}$$

$$G(\omega)\Big|_{at:\ \beta \to +\infty} : \quad \mathcal{G}(i\omega_n)\Big|_{i\omega_n \to \omega + i\,\mathrm{sign}(\omega)\,\eta} =$$

Matsubara Green func $\xrightarrow{\text{解析延拓}}$ Green func for $O_1 = C^\dagger,\ O_2 = C$

question: vector:

spin orbit coupling

上海交大: Tony Leggett

- Spectral function

$$A_+(\omega) = 2\pi \sum_{n,m} e^{\beta\Omega} \langle n|O_2|m\rangle\langle m|O_1|n\rangle$$
$$e^{-\beta E_n}\, \delta(\omega + E_n - E_m)$$

$$A_-(\omega) = 2\pi \sum_{n,m} e^{\beta\Omega} \langle n|O_2|m\rangle\langle m|O_1|n\rangle$$
$$e^{-\beta E_m}\, \delta(\omega + E_n - E_m)$$
$$= 2\pi \sum_{n,m} e^{\beta\Omega} \langle m|O_2|n\rangle\langle n|O_1|m\rangle\, e^{-\beta E_n}$$
$$\delta(\omega - (E_n - E_m))$$

$$A(\omega) = -2\,\mathrm{Im}\, D_{ret}(\omega) = A_+(\omega) \pm A_-(\omega)$$

$$A_+(\omega) = e^{\beta\omega} A_-(\omega)$$

if $O_1 = c^+$, $O_2 = c$

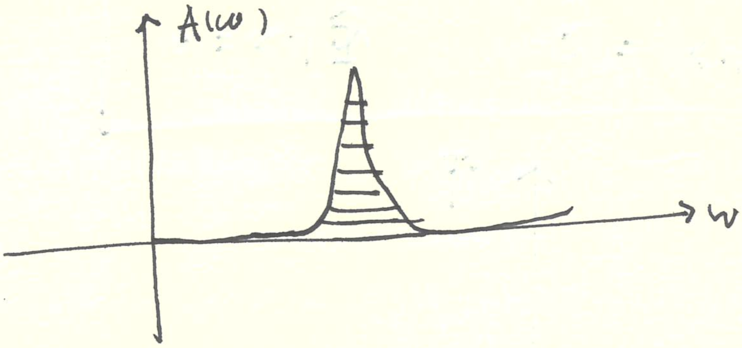
$$\int \frac{d\omega}{2\pi} A(\omega) \equiv \sum_{n,m} e^{\beta\Omega} \Big(|\langle n|c|m\rangle|^2 e^{-\beta E_n}$$

$$+ \cancel{e^{\beta\Omega}} \, |\langle n|c^+|m\rangle|^2 e^{-\beta E_n} \Big)$$

$$+ e^{-\beta E_n} \langle n|c|m\rangle\langle m|c^+|n\rangle + e^{-\beta E_n}\langle n|c^+|m\rangle$$

$$\langle m|c|n\rangle$$

$$= 1$$



食来矢地指间力

Bosonic 的谱函数没有这种形式

我们引入：

$$A_\pm(\omega) = -\frac{2\,\mathrm{Im}\,G(\omega)}{e^{-\beta\omega/2} \pm e^{-\beta\omega/2}} e^{\pm\beta\omega/2}$$

$$A_+(\omega) - A_-(\omega) = \cot\frac{\beta\omega}{2}\,\mathrm{Im}\,D_{ret}(\omega)$$

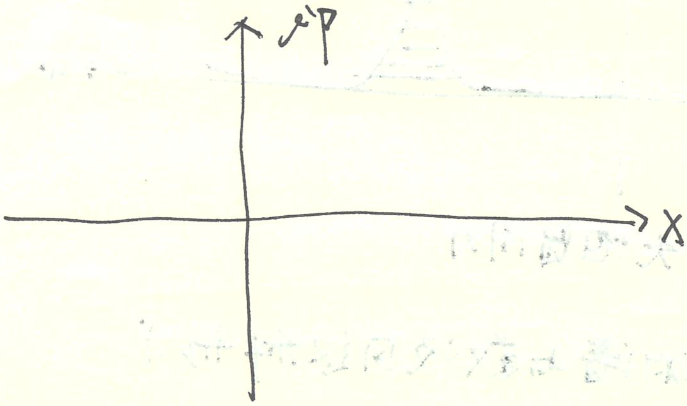
$\bar{\Phi}\Phi\,\partial = \bar{\Phi}\Phi + 1 = \langle \bar{\Phi} | \Phi \rangle$

Functional field path integral

你不要问我 Grassmann Number 是多少?

Operator: $\{\hat{\Phi}^{+}, \hat{\Phi}\} = 1 \quad \{\hat{\Phi}, \hat{\Phi}\} = \{\hat{\Phi}^{+}, \hat{\Phi}^{+}\}$

$$\boxed{\begin{array}{ll} \hat{\Phi}^{+}|0\rangle = |1\rangle & \hat{\Phi}|1\rangle = |0\rangle \\ \hat{\Phi}^{+}|1\rangle = 0 & \hat{\Phi}|0\rangle = 0 \end{array}}$$



$$|\Phi\rangle = |0\rangle - \Phi|1\rangle = e^{-\Phi C^{+}}|0\rangle$$

$$\hat{\Phi}|\Phi\rangle = \Phi(\hat{\Phi}|1\rangle)$$

$$\langle\bar{\Phi}| = \langle 0| - \langle 1|\bar{\Phi} = \langle 0| + \bar{\Phi}\langle 1|$$

$$\langle\bar{\Phi}|\hat{\Phi}^{+} = \langle\bar{\Phi}|\bar{\Phi}$$