Gauge boson mass:

1.

- Revolution identity

$$\int |\Psi\rangle\langle\Psi| e^{-\bar{\Psi}\Psi} d\bar{\Psi}\, d\Psi = 1$$

Proof:

$$\int |\Psi\rangle\langle\Psi| e^{-\bar{\Psi}\Psi} d\bar{\Psi}\, d\Psi = 1$$

$$= \int (|0\rangle - \Psi|1\rangle)(\langle 0| - \langle 1|\bar{\Psi})(1 - \bar{\Psi}\Psi)\, d\bar{\Psi}\, d\Psi$$

$$= \int \Psi \langle 1|1\rangle\langle 1|\bar{\Psi}\, d\bar{\Psi}\, d\Psi + |0\rangle\langle 0|\bar{\Psi}\Psi\, d\bar{\Psi}\, d\Psi$$

$$= |0\rangle\langle 0| + |1\rangle\langle 1|$$

- trace identity:

$$tr(\hat{O}) = \int \langle -\bar{\Psi}| O |\Psi\rangle e^{-\bar{\Psi}\Psi} d\bar{\Psi}\, d\Psi$$

$$= \int \langle 0|\hat{O}|0\rangle - \bar{\Psi}\Psi\, d\bar{\Psi}\, d\Psi + -\int \bar{\Psi}\langle 1|\hat{O}|1\rangle \Psi\, d\bar{\Psi}\, d\Psi$$

$$= \langle 0|\hat{O}|0\rangle + \langle 1|\hat{O}|1\rangle$$

Fermion many body:

single particle:

$$Z = tr(e^{-\beta H}) = \int \langle -\Psi_N | e^{-H\beta\Delta\tau} | \Psi_N \rangle$$

$e^{-\bar{\Psi}_N \Psi_N} \, d\bar{\Psi}_N \, d\Psi_N$ $\qquad e^{-\bar{\Psi}_{N-1} \Psi_{N-1}}$

$$= \int \langle -\Psi_N | e^{-H\beta\Delta\tau} | \Psi_{N-1} \rangle \langle \Psi_{N-1} | e^{-H\beta\Delta\tau} | \Psi_{N-2} \rangle$$

$$\cdots\cdots \langle \Psi_0 | e^{-H\beta\cdot\Delta\tau} | \Psi_N \rangle$$

$$\langle \Psi_n | 1 - \beta H \cdot \Delta\tau | \Psi_{n-1} \rangle = e^{-\bar{\Psi}_n \Psi_{n-1}}$$

$(1 - H(\bar{\Psi}_i, \Psi_i))$

$$= \int e^{-\bar{\Psi}_n (\Psi_n - \Psi_{n-1}) - H(\bar{\Psi}_i - \Psi_i)}$$

$d\bar{\Psi}_i \, d\Psi_i$ - - - - - - - - - - -

$$= \int D[\Psi] \, e^{iS}$$

$$S = \bar{\Psi} \partial_\tau \Psi - H = \bar{\Psi} (\partial_\tau + H) \Psi$$

Fermion many body:

2.

$$\Psi(0) = \Psi(\beta) \quad : \quad \Psi(\beta) = -\Psi(0)$$

- Fourier transformation:

$$\begin{cases} \Psi(\tau) = \dfrac{1}{\sqrt{\beta}} \sum_n e^{-i\omega_n \tau} \Psi(i\omega_n) \\ \omega_n = \dfrac{(2n+1)\pi}{\beta} \end{cases}$$

$$\Psi(\tau) \xrightarrow{\Downarrow \text{空间中}} \Psi(x(\tau))$$

$$Z = \int D[\bar{\Psi}(x,\tau)\,\Psi(x,\tau)]\, e^{-S[\bar{\Psi},\Psi]}$$

$$S[\bar{\Psi},\Psi] = \int_0^\beta d\tau \Big(\bar{\Psi}(x,\tau)\,\partial_\tau \Psi(x,\tau) + H(x,\tau)$$
$$- \mu \bar{\Psi}\Psi \Big)$$

- Bosonic system:

$$\hat{a}|a\rangle = a|\hat{a}\rangle$$

expand

$$|a\rangle = e^{-\frac{|a|^2}{2} + a a^+} \sum_n |0\rangle$$

金属的蒸发效应

软物质不是 Quantum

- $$\int \frac{da\,da^*}{\pi} |a\rangle\langle a| = 1$$

- $$\langle a | a' \rangle = e^{-\frac{1}{2}(|a|^2 + |a'|^2) + a^* a}$$

- trace/identity:

$$\mathrm{tr}(\hat{O}) = \int \langle a | \hat{O} | a \rangle \frac{da\,da^*}{\pi}$$

- Partition function for bosonic system

$$Z = \mathrm{tr}(e^{-\beta H}) = \int \langle a_N | e^{-\beta H} | a_{N-1} \rangle$$

$$\cdots\cdots \langle a_1 | e^{-\beta H} | a_N \rangle$$

$$=$$

- Source field:

$$\int d\bar{z}_1 \, dz_1 \cdots d\bar{z}_n \, dz_n \; e^{-\bar{z}_i A_{ij} z_j + \bar{\omega}_i z_i + \bar{z}_i \omega_i}$$

$$= \pi^N (\det A^{-1}) \, e^{\bar{\omega}_i A_{ij} \omega_j}$$

$$\Rightarrow \langle \bar{z}^*_{i_1} z_{j_1} \rangle = Q(A^{-1})_{j_1 i_1}$$

Wick theorem:

complex Number

$$\langle \bar{z}^*_{i_1} \cdots \cdots \bar{z}_{i_n} \; z_{j_1} \cdots \cdots z_{j_n} \rangle$$

$$= \sum_P Q(A^{-1})_{i_1 j_{P(1)}} (A^{-1})_{i_2 j_{P(2)}} \cdots \cdots (A^{-1})_{i_n j_{P(n)}}$$

- $$\int e^{-\bar{\Psi}_i M_{ij} \Psi_j} \cdots\cdots d\bar{\Psi}_1 d\Psi_1 \cdots d\bar{\Psi}_n d\Psi_n$$

$$= \det(M)$$

- Supersymmetry:

$$(\det M)^{-1} \det M = 1$$

• two particle Hilbert space:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2$$

$$\Downarrow \qquad\qquad \Downarrow$$

左边的 i 乘上右边的 i 等于一吗？

左边 有 i, j, k ; 右边 i, j, k

• $$\int e^{-\bar{\Psi} M \Psi + \bar{\Psi}_i \nu_i + \bar{\nu}_i \Psi_i} \, d\bar{\Psi}_1 d\Psi_1 \cdots \cdots d\bar{\Psi}_n d\Psi_n = \det(M) \, e^{\bar{\nu}_i M_{ij} \nu_j}$$

• $$\langle \Psi_{j_1} \cdots \cdots \Psi_{j_n} \bar{\Psi}_{i_n} \cdots \cdots \bar{\Psi}_{i_1} \rangle$$

$$= \sum_P \underline{\mathrm{sgn}(P)} \, A^{-1}_{j_1 i_{P_1}} \cdots \cdots \cdots A^{-1}_{j_n i_{P_n}}$$

费米子交换   Grassmann Number

$\left\{ \begin{array}{l} \text{Direction} \\ \text{muscle} \end{array} \right.$

格林函数 / 重武器主要是技术上