Outline

- Long-range XY Model and Algorithm
- Results
  - Phase diagram
  - Evidence from Low-temperature Phase
  - Evidence from High-temperature Phase
  - Evidence from Critical Point
- Conclusion

Long-Range $O(n)$ Model

Each site interacts with all other sites:

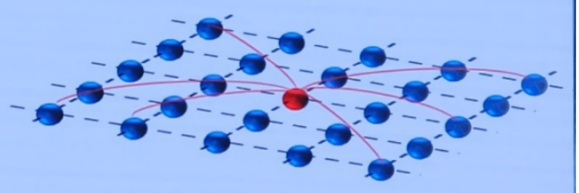
$$H = -\sum_i \sum_{j>i} \frac{J}{r^{2+\sigma}} \boldsymbol{s}_i \cdot \boldsymbol{s}_j$$

$\boldsymbol{s}$ refers to a unit n-vector.

- Long-range interacting systems exhibit intriguing physical phenomena and have been realized in various atomic, molecular, and optical (AMO) setups.

- The Mermin-Wagner theorem is no longer applicable to long-range models

2D long-range(LR) $O(n)$ model:



- Under different σ values, the system exhibits various phase transition properties and exotic critical behavior emerges, such as the double-power correlation function in the 2D LR Ising model [1].

- However, until now, there still exist some controversies!

[1] M. C. Angelini, G. Parisi, and F. Ricci-Tersenghi, Phys. Rev. E 89, 062120 (2014).

Consensuses and Controversies

Consensuses:

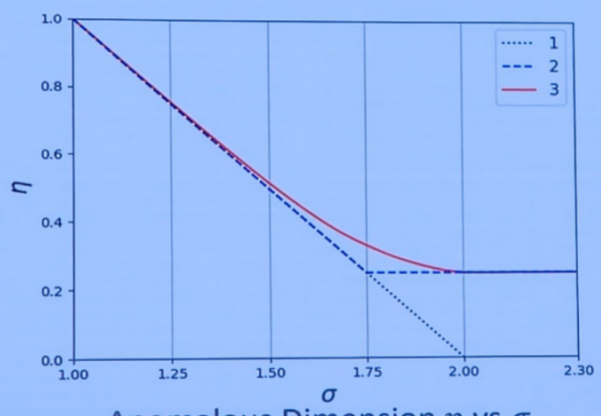
- $\sigma < \frac{d}{2}$ (mean-field region): system's behavior is controlled by Gaussian fixed point.
- $\frac{d}{2} < \sigma < \sigma_*$ (non-classical region): critical behavior is different from both mean-field and short-range(SR) one but depends on the parameter σ.
- $\sigma > \sigma_*$ (SR region): the SR behavior is recovered.
- $\sigma_* < 2$, that is, for $\sigma > 2$, the SR behavior is recovered.

Ref: Michael E. Fisher, Shang-keng Ma, and B. G. Nickel, ,Phys. Rev. Lett., 29:917–920, Oct 1972.

Consensuses and Controversies

Controversies on the value of σ_*:

1. Fisher: $\eta = 2 - \sigma$ for $\sigma \leq 2$; $\eta = \eta_{SR}$ for $\sigma > 2$. Then, $\sigma_* = 2$.
2. Sak "criterion": $\eta = \max(2 - \sigma, \eta_{SR})$, then $\sigma_* = 2 - \eta_{SR}$.
3. η interpolates smoothly between $\eta = 2 - \sigma$ for $\sigma \to 1$ and $\eta = \eta_{SR}$ for $\sigma = 2$. Then, $\sigma_* = 2$.



Anomalous Dimension η vs σ

Ref: J. Sak. ,Phys.Rev. B, 8:281–285, Jul 1973

Special Scenario of 2D LRXY Model

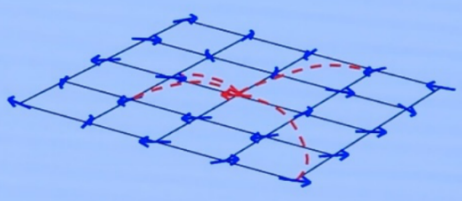
The LRXY model

$$\mathcal{H} = -\sum_{i<j} \frac{J}{r_{i,j}^{d+\sigma}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$\boldsymbol{S}_i$ Two component spin

d Spatial dimension

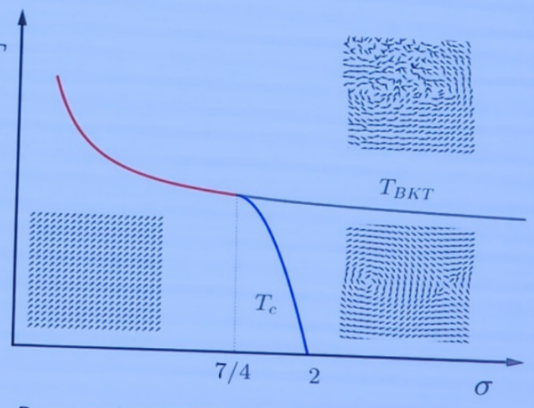
σ Decay exponent



- The SR XY model undergoes a Berezinskii-Kosterlitz-Thouless (BKT) transition.
- The interplay between BKT mechanism and long-range interaction is complicated.

Recent field-theoretical study:

- For $1.75 < \sigma < 2$, as the T decrease, the system first enter an intermediate quasi-long-range order (QLRO) phase, and then transition to a long-range order (LRO) phase



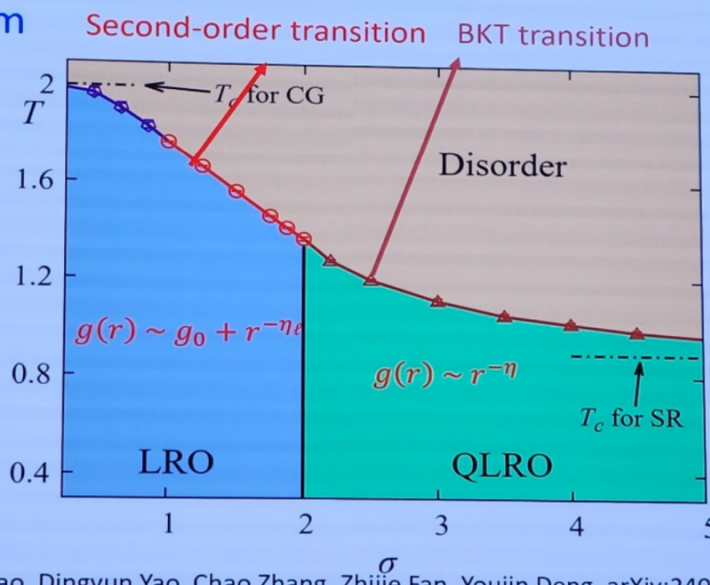
Proposed Phase Diagram of 2D LRXY Model

Guido Giachetti, Andrea Trombettoni etc., Phys. Rev. Lett., 127:156801, Oct 2021.

Simulation Results

- Large-scale simulation using clock factorized Monte Carlo method, up to a linear system size of $L = 8192$, without truncating interaction range.
- Instead of brute-forcing the critical exponents, we study the low-T, and high-T properties to identify the change of transition type and determine the σ_*.

➢ Phase Diagram



Ref: 1. Tianning Xiao, Dingyun Yao, Chao Zhang, Zhijie Fan, Youjin Deng. arXiv:2404.08498

Phase Transition at various σ

Behavior of Correlation Length Ratio

- Correlation length divided by system size: $\xi/L = \frac{1}{2L\sin(k/2)}\sqrt{\frac{<M^2>}{<M_k^2>}-1}$

where $M_k^2 = \left|\frac{1}{N}\sum_{i=1}^{N}\vec{s}_i \exp\left(i\vec{k}\cdot\vec{r}\right)\right|^2$ and $\vec{k} = (2\pi/L, 0)$