Phases of (2+1)D SO5 non-linear sigma model with WZW term on a sphere: Multi-critical point and disorder phase

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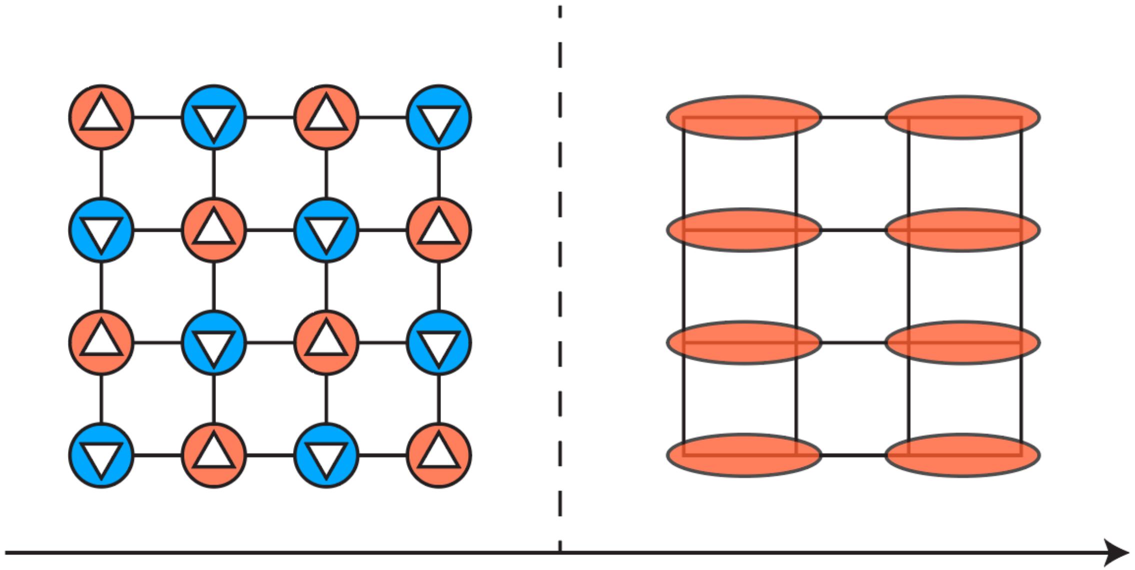
[**BC**, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL 132, 246503 (2024)]

[**BC**, X. Zhang, Z. Y. Meng, arXiv:2405.04470]

➢ Deconfined Quantum Critical Point (DQCP)

- **Transition between symmetry-breaking phases**

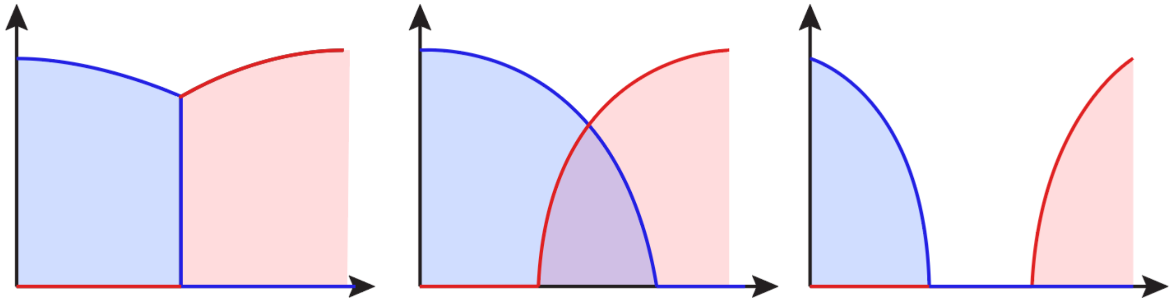
Example:



Néel | Valence Bond Solid (VBS)

O(3)-Breaking | Z4-Breaking

Landau-allowed scenario:



First-order | Co-exist | Disorder phase

Beyond Landau scenario:

A direct continuous transition? DQCP!

[T. Senthil, et al, PRB2004; A. Nahum, et al, PRX2015; C. Wang, et al, PRX2017; T. Senthil, arXiv:2306.12638]

➢ The Enigma of DQCP

- **Spin systems** J-Q model: $H = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - Q\sum_{\langle ijkl\rangle}(\mathbf{S}_i\cdot\mathbf{S}_j-\frac{1}{4})(\mathbf{S}_k\cdot\mathbf{S}_l-\frac{1}{4})$
  - ✓ Emergent continuous symmetry and fractional excitation
    [A. Sandvik, PRL2007]
    [A. Nahum, PRL2015, PRL2019]
    [N. Ma, PRL2019]
  - Incompatible scaling relation with conformal bootstrap
    [Y. Nakayama, T. Ohtsuki, PRL2016]
    [D. Poland, S. Rychkov, A. Vichi, RMP2019]
  - First-order transition/multicritical point
    [A. Kuklov, et al, PRL2008]
    [K. Chen, et al, PRL2013]
    [B. Zhao, et al, PRL2020]
  - ✓ DQCP in SU(N) systems
    [R. Kaul, A. Sandvik, PRL2012; M. Block, R. Melko, R. Kaul, PRL2013; M. Song, et al, arXiv:2307.02547]
- **Fermion systems**
  - Continuous QSH-SC / Neel-VBS transitions
    [Y. Liu, et al, Nat. Comm. 2019]
    [Z. H. Liu, et al, PRL2022, PRL2023]

- **Key challenge**
  - ✓ Symmery requirement

    Z_4 => U(1)

    U(1) x SU(2) => SO(5)
  - ✓ Extra length scales required for such emergent symmetries: Slow RG flow

➢ The Enigma of DQCP

○ **SO(5) NLSM model with WZW term**

[M. Ippoliti, R. Mong, F. Assaad, M. Zaletel, PRB2018]
[Z. Wang, M. Zaletel, R. Mong, F. Assaad, PRL2021]

$$H = \frac{1}{2}\int \mathrm{d}\mathbf{r}\left\{U_0[\psi^\dagger(\mathbf{r})\psi(\mathbf{r}) - 2]^2 - \sum_{i=1}^{5} u_i[\psi^\dagger(\mathbf{r})\Gamma^i\psi(\mathbf{r})]^2\right\}$$

4-component Dirac fermion: $\psi_{\tau\sigma}(\mathbf{r})$ With valley τ and spin σ.

5 Gamma matrices: $\Gamma^i \in \{\tau_x \otimes \mathbb{I}, \tau_y \otimes \mathbb{I}, \tau_z \otimes \sigma_x, \tau_z \otimes \sigma_y, \tau_z \otimes \sigma_z\}$

10 SO(5) group generators: $L^{ij} = -\frac{\mathrm{i}}{2}[\Gamma^i, \Gamma^j]$

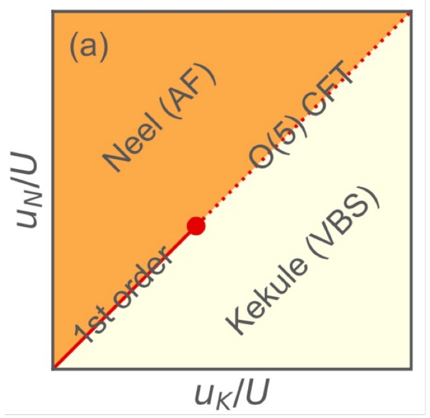
$u_K \equiv u_1 = u_2$ $\quad$ $u_N \equiv u_3 = u_4 = u_5$

- In the cases of $u_K \neq u_N$, the system has SO(3)⊗O(2) symmetry.

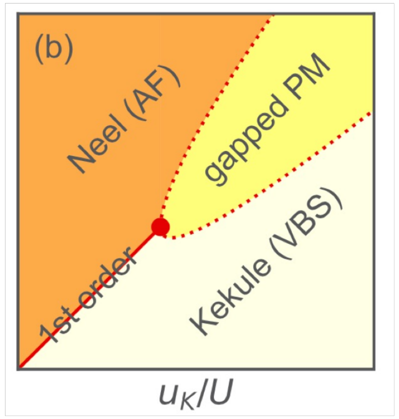
  if $u_K > u_N$, favour VBS order;
  otherwise, it favours Neel order.

When $u_K = u_N$, the system has exact SO(5) symmetry.

DQCP scenario? Or Landau-allowed scenario?



DQCP scenario



Landau-allowed scenario

➢ Spherical Landau Level regularization

○ Landau Level on Sphere

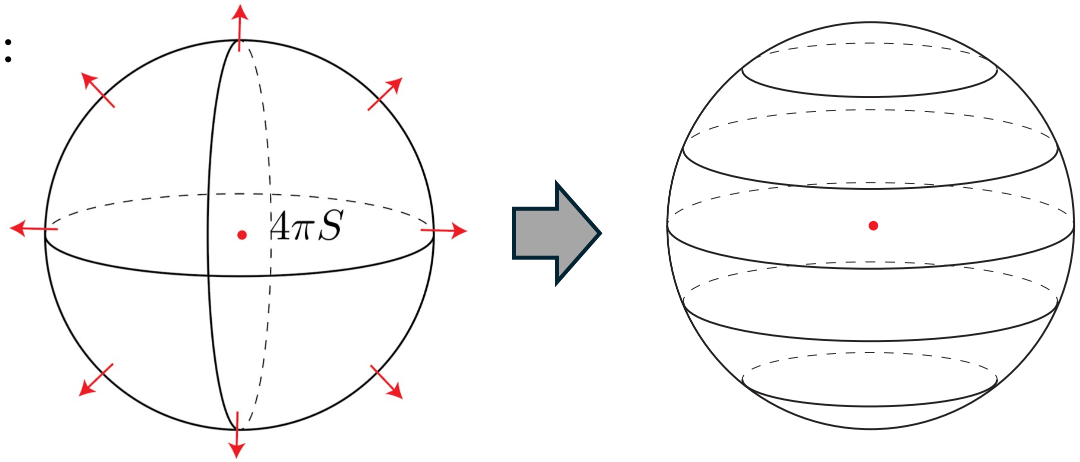
[D. Haldane, PRL1983; W. Zhu, et al, PRX2023]

✓ Single-particle Hamiltonian:

$$H_0 = \frac{1}{2M_e R^2}\Lambda_\mu^2$$

with

$$\Lambda_\mu = \partial_\mu + \mathrm{i}A_\mu$$



(2s+1)-Fold degenerated

Lowest Landau level (LLL)

$$\Phi_m(\theta,\phi) = N_m e^{im\phi}\cos^{s+m}\left(\frac{\theta}{2}\right)\sin^{s-m}\left(\frac{\theta}{2}\right)$$

$$m = -s, -s+1, \cdots, s$$

○ LLL projection of the SO(5) Model via $\psi_\alpha(\mathbf{r}) = \sum_m \Phi_m(\mathbf{r})c_{m,\alpha}$

$$H_0 = u_0 \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}(c^\dagger_{m_1,\alpha}\delta^{\alpha\beta}c_{m_1+m,\beta} - 2\delta_{m0})(c^\dagger_{m_2,\alpha}\delta^{\alpha\beta}c_{m_2-m,\beta} - 2\delta_{m0})$$

$$H_K = \sum_{i=1,2} u_i \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}(c^\dagger_{m_1,\alpha}(\Gamma^i)^{\alpha\beta}c_{m_1+m,\beta})(c^\dagger_{m_2,\alpha}(\Gamma^i)^{\alpha\beta}c_{m_2-m,\beta})$$

Favour VBS order

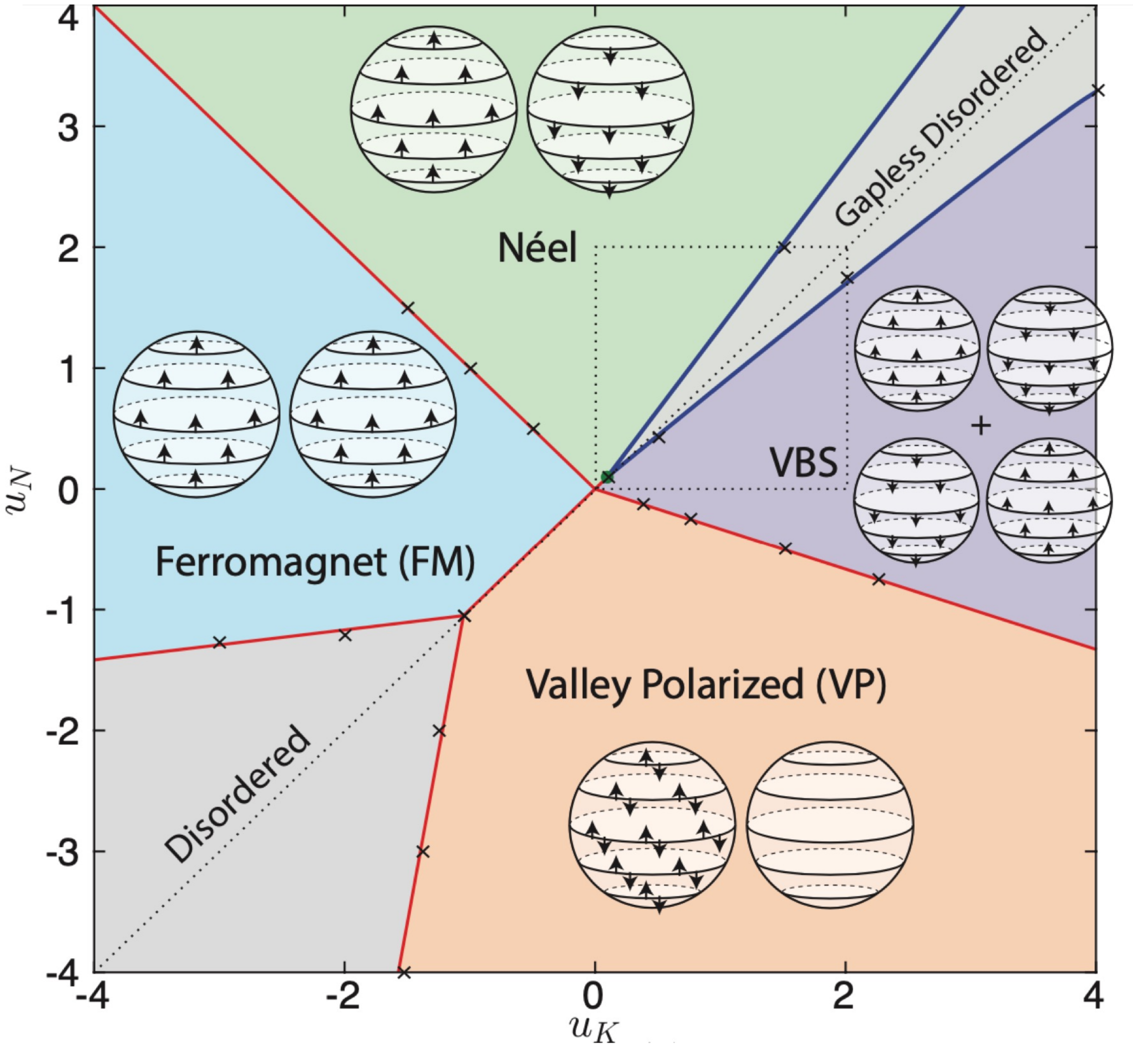
$$H_N = \sum_{i=3,4,5} u_i \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}(c^\dagger_{m_1,\alpha}(\Gamma^i)^{\alpha\beta}c_{m_1+m,\beta})(c^\dagger_{m_2,\alpha}(\Gamma^i)^{\alpha\beta}c_{m_2-m,\beta})$$

Favour Neel order

- System size: Number of orbitals N = 2s + 1

Numerical Results

- **Full Phase Diagram**



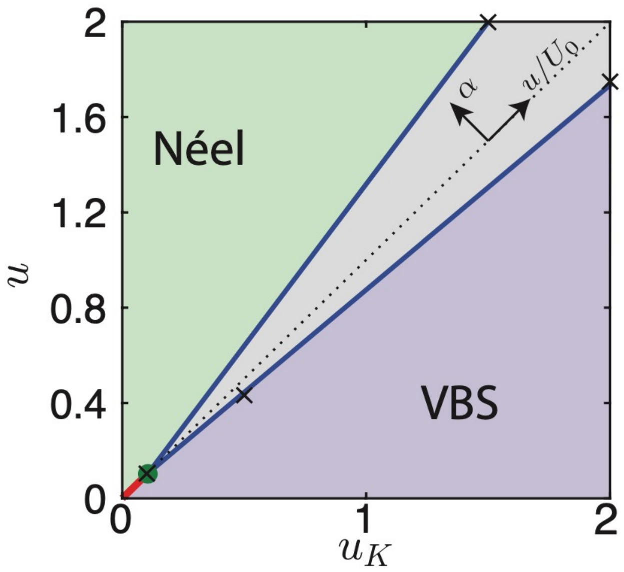
- **Methods**
  - ✓ DMRG

$SU(2)_{spin}$ x $U(1)_{charge}$ x $U(1)_{angular\text{-}momentum}$ symmetries

Up to 4096 SU(2) multiplets are kept (~12000 states)

Truncation error: 10^{-5} at N = 16.

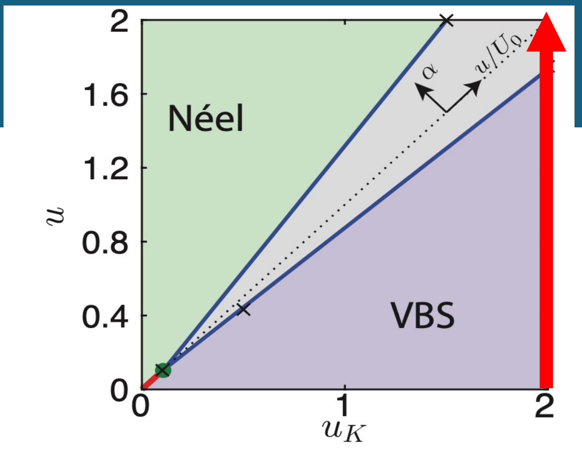
  - ✓ DQMC
  - ✓ ED



- ✓ Various ordered phases
- ✓ Intermediate disorder phase
- ✓ Non-Wilson-Fisher transition
- ✓ Multicritical point

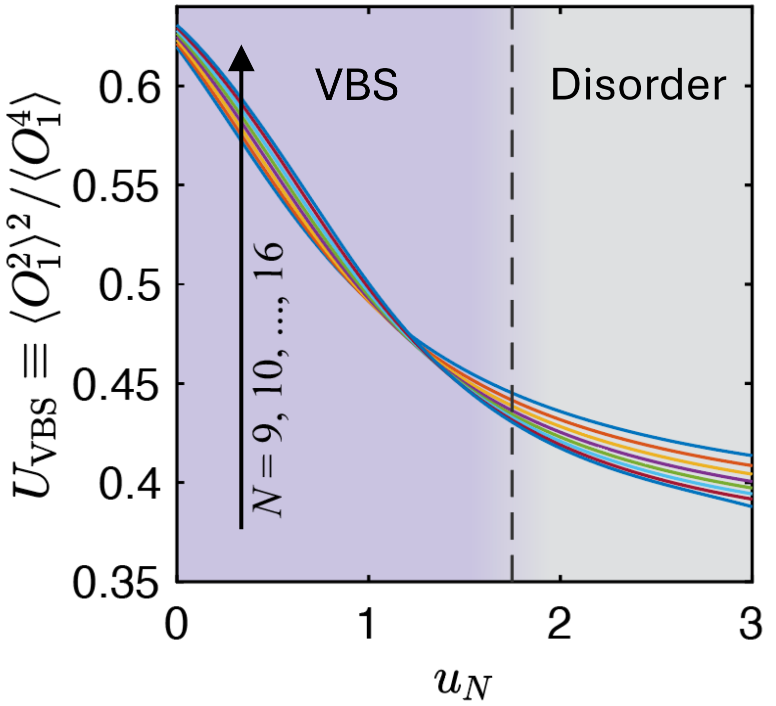
[**BC**, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL 132, 246503 (2024)]

Numerical Results

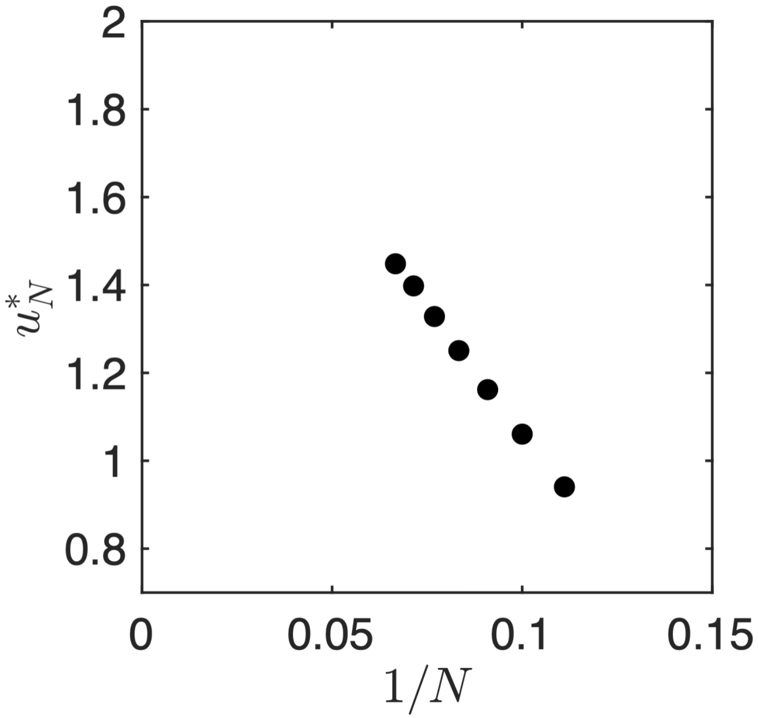


Non-Wilson-Fisher transition from VBS to disorder

- VBS order parameter: $$O_i = \int d\Omega \psi^\dagger(\Omega)\Gamma^i \psi(\Omega) = \sum_m c_m^\dagger \Gamma^i c_m$$ with *i* = 1,2
- VBS Binder ratio: $$U_{\rm VBS} \equiv \langle O_1^2 \rangle^2 / \langle O_1^4 \rangle$$



✓ Binder ratios nicely cross



✓ Crossing point dirfts due to finite sizes

➢ Numerical Results

○ Crossing point analysis

Consider the standard scaling form, $O(\delta, L) = L^{-\kappa/\nu} f(\delta L^{1/\nu}, \lambda L^{-\omega})$

(Here, $\delta = q - q_c$, and the leading irrelevant field λ and its corresponding exponent ω.)

We express it as function of N, $O(\delta, N) \; = \; N^{-\frac{\kappa}{2\nu}} f(\delta N^{\frac{1}{2\nu}}, \lambda N^{-\frac{\omega}{2}}) \; \simeq \; N^{-\frac{\kappa}{2\nu}} (a_0 + a_1 \delta N^{\frac{1}{2\nu}} + b_1 N^{-\frac{\omega}{2}} + \cdots)$

At the crossing point $\delta^*(N)$ between size pair $(N, N+x)$, $O(\delta^*, N) = O(\delta^*, N+x)$

We then have

$$\delta^*(N) = \frac{a_0}{a_1} \frac{(1+x/N)^{-\frac{\kappa}{2\nu}} - 1}{1 - (1+x/N)^{\frac{1-\kappa}{2\nu}}} N^{-\frac{1}{2\nu}} \quad + \quad \frac{b_1}{a_1} \frac{(1+x/N)^{-\frac{\omega}{2}-\frac{\kappa}{2\nu}} - 1}{1 - (1+x/N)^{\frac{1-\kappa}{2\nu}}} N^{-\frac{1}{2\nu}-\frac{\omega}{2}} \quad + \cdots$$

$$O(\delta^*, N) = N^{-\frac{\kappa}{2\nu}} \left[a_0 + a_1 \left[\frac{a_0}{a_1} \frac{(1+x/N)^{-\frac{\kappa}{2\nu}} - 1}{1 - (1+x/N)^{\frac{1-\kappa}{2\nu}}} N^{-\frac{1}{2\nu}} \; + \; \frac{b_1}{a_1} \frac{(1+x/N)^{-\frac{\omega}{2}-\frac{\kappa}{2\nu}} - 1}{1 - (1+x/N)^{\frac{1-\kappa}{2\nu}}} N^{-\frac{1}{2\nu}-\frac{\omega}{2}} \quad + \cdots \right] N^{\frac{1}{2\nu}} + b_1 N^{-\frac{\omega}{2}} + \cdots \right]$$

For Binder ratio, we have $\kappa = 0$, and by neglecting x/N, we then have

$$\delta^*(N) = c\, N^{-(\frac{1}{2\nu}+\frac{\omega}{2})} + \cdots$$

$$U(\delta^*, N) = a + bN^{-\frac{w}{2}} + \cdots$$

➢ Numerical Results

○ Crossing point analysis

To independently determine ν, we can consider

$$U(\delta, N) = a_0 + a_1 \delta N^{\frac{1}{2\nu}} + b_1 N^{-\frac{\omega}{2}} + c_1 \delta N^{\frac{1}{2\nu}-\frac{\omega}{2}} + \cdots$$

Its derivative w.r.t δ is

$$U'(\delta, N) = a_1 N^{\frac{1}{2\nu}} + c_1 N^{\frac{1}{2\nu}-\frac{\omega}{2}} + \cdots$$

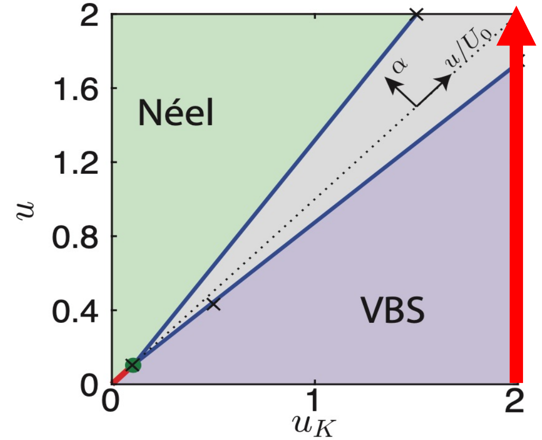
Then the difference of the logarithmic of the above equation between size N and $N + x$ will be

$$\frac{2N}{x} \ln \frac{U'(\delta^*, N + x)}{U'(\delta^*, N)} = \frac{1}{\nu} - dN^{-\frac{\omega}{2}}$$

When defining $\frac{1}{\nu^*}(\delta^*, N) = \frac{2N}{x} \ln \frac{U'(\delta^*, N + x)}{U'(\delta^*, N)}$, we have

$$\boxed{\frac{1}{\nu^*(\delta^*, N)} = \frac{1}{\nu} - dN^{-\frac{\omega}{2}}}$$

➢ Numerical Results



- Non-Wilson-Fisher transition from VBS to disorder
  - VBS order parameter: $O_i = \int d\Omega \psi^\dagger(\Omega)\Gamma^i\psi(\Omega) = \sum_m c_m^\dagger \Gamma^i c_m$ with $i = 1,2$
  - VBS Binder ratio: $U_{\mathrm{VBS}} \equiv \langle O_1^2\rangle^2 / \langle O_1^4\rangle$

- Crossing point analysis

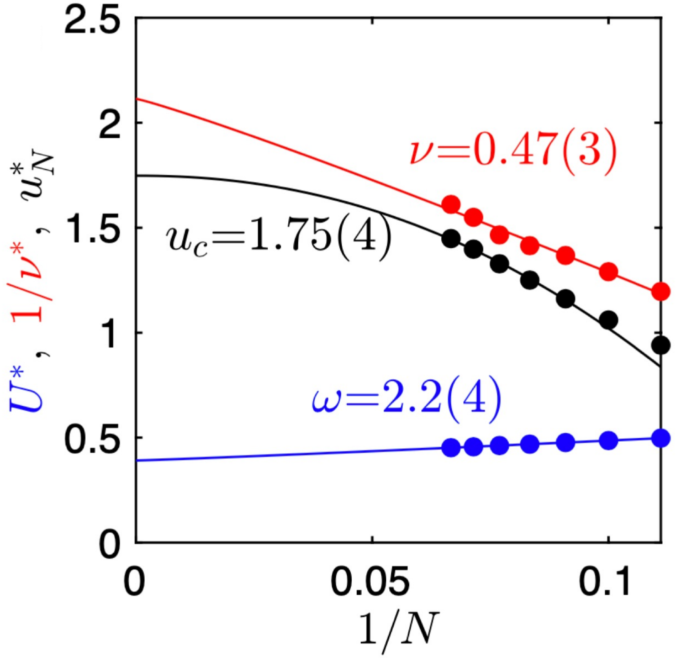
$$\delta^*(N) = c\,N^{-(\frac{1}{2\nu}+\frac{\omega}{2})} + \cdots$$

$$U(\delta^*, N) = a + bN^{-\frac{w}{2}} + \cdots$$

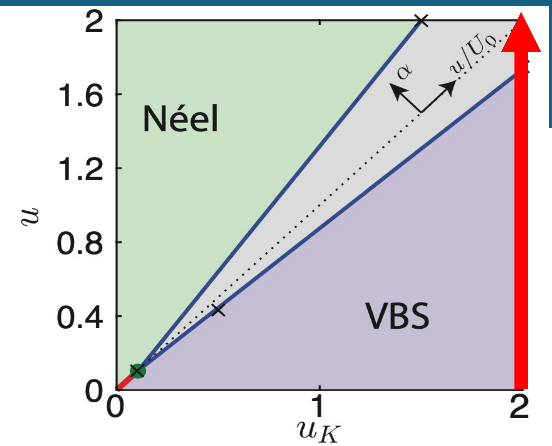
$$\delta^* = u_N^* - u_c$$

When defining $\frac{1}{\nu^*}(\delta^*, N) = \frac{2N}{x} \ln \frac{U'(\delta^*, N+x)}{U'(\delta^*, N)}$

$$\frac{1}{\nu^*(\delta^*, N)} = \frac{1}{\nu} - dN^{-\frac{\omega}{2}}$$



➢ Numerical Results



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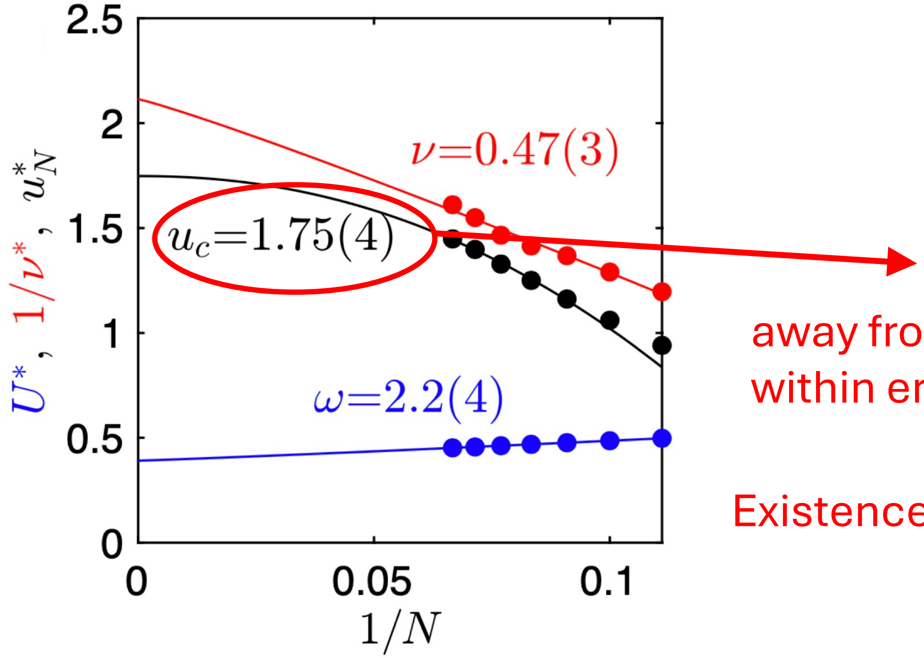
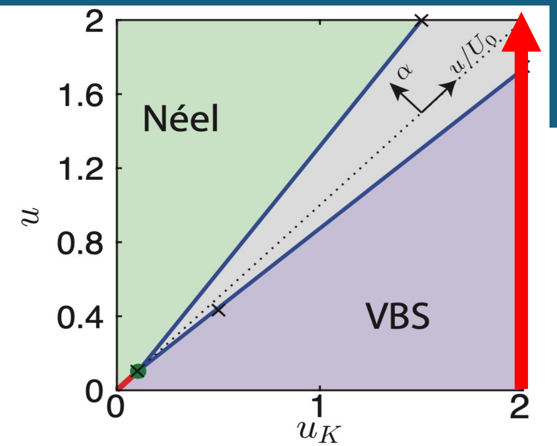


away from the SO(5) line within errorbar

Existence of Disorder Phase

Numerical Results



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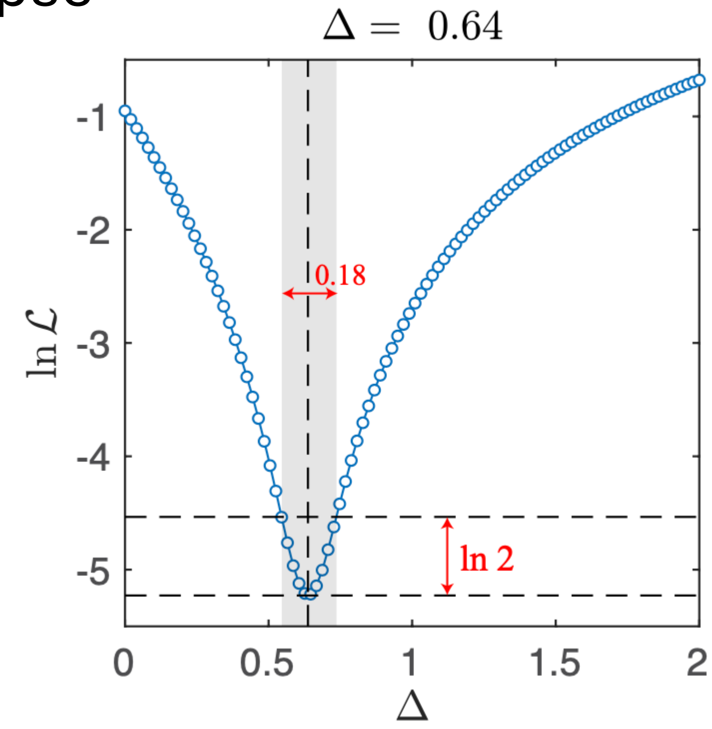
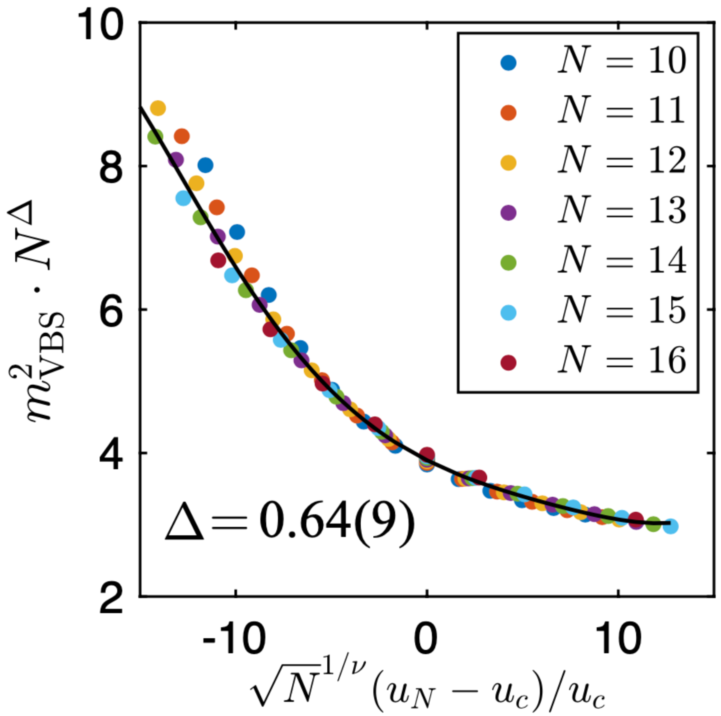
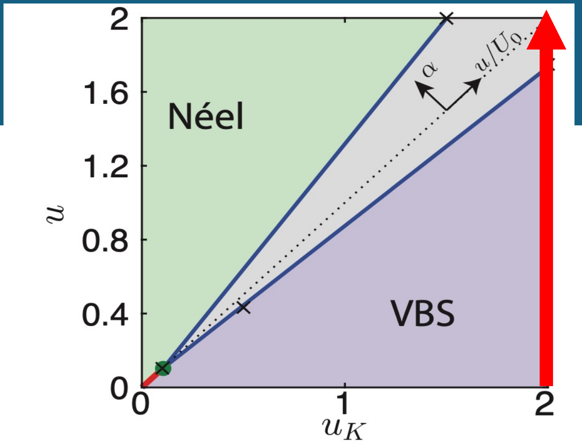
- Data Collapse





➢ Numerical Results



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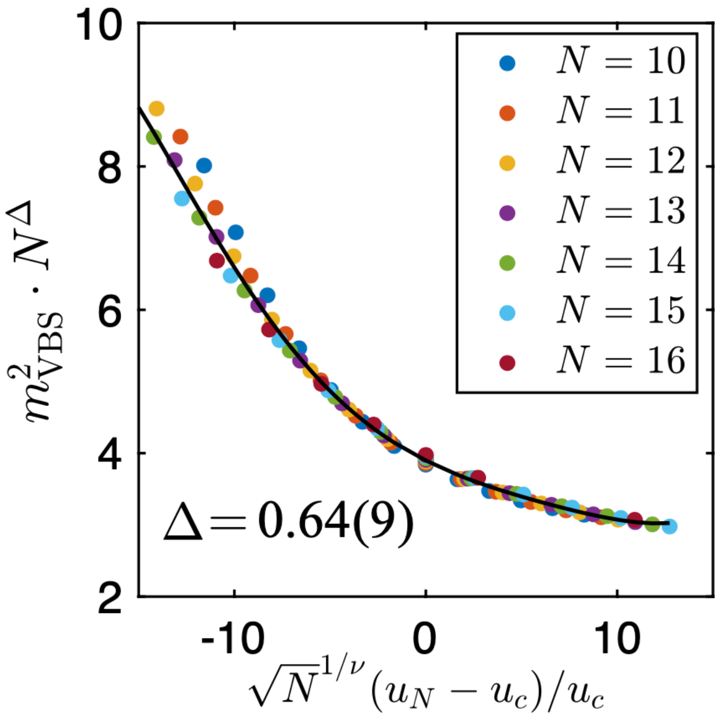


- Non-Wilson-Fisher O(2) transition

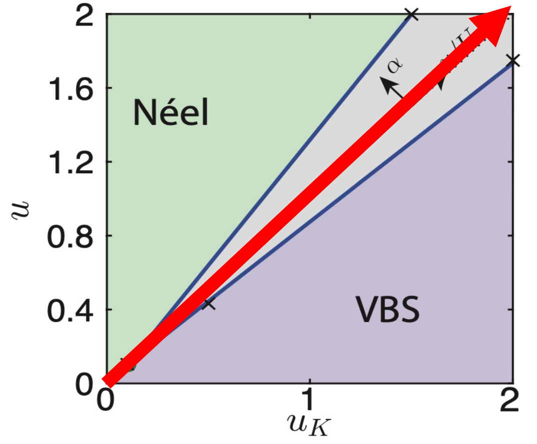
Correlation length exponent: $\nu = 0.47(3)$

Leading correction scaling exponent: $\omega = 2.2(4)$

Scaling dimension of order parameter: $\Delta_\phi = 0.64(9)$

(Scaling dimension of order parameter for Wilson-Fisher O(2) transition is 0.519.)

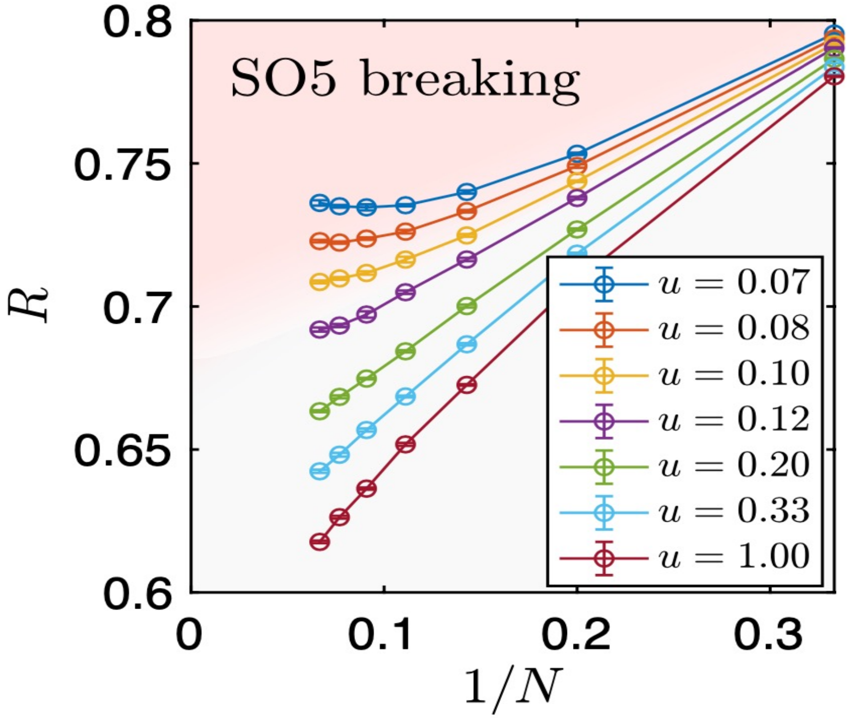
➢ Numerical Results



○ Multicritical point

- Susceptibility: $O_{i,l,m} = \int d\Omega\, Y^*_{lm}(\Omega)\, \psi^\dagger(\Omega)\, \Gamma^i \psi(\Omega)$ (ordered at *l*=0)
- SO5 correlation ratio: $R \equiv 1 - \langle \mathbf{O}^2_{l=1} \rangle / \langle \mathbf{O}^2_{l=0} \rangle$

$$\mathbf{O}_l \equiv (O_{1,l}, \cdots, O_{5,l})$$ with angular momentum shift *l*

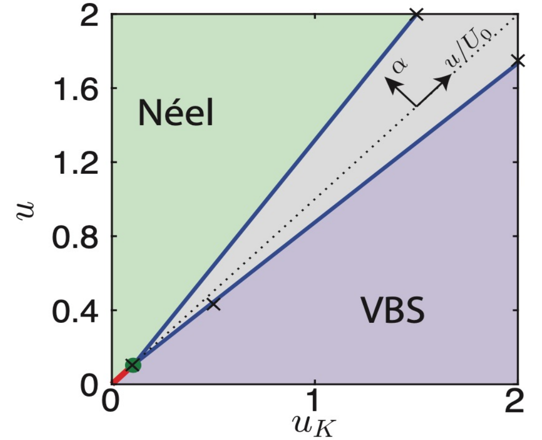


✓ SO5 symmetry breaking at small *u*

✓ SO5 disordered at large *u*

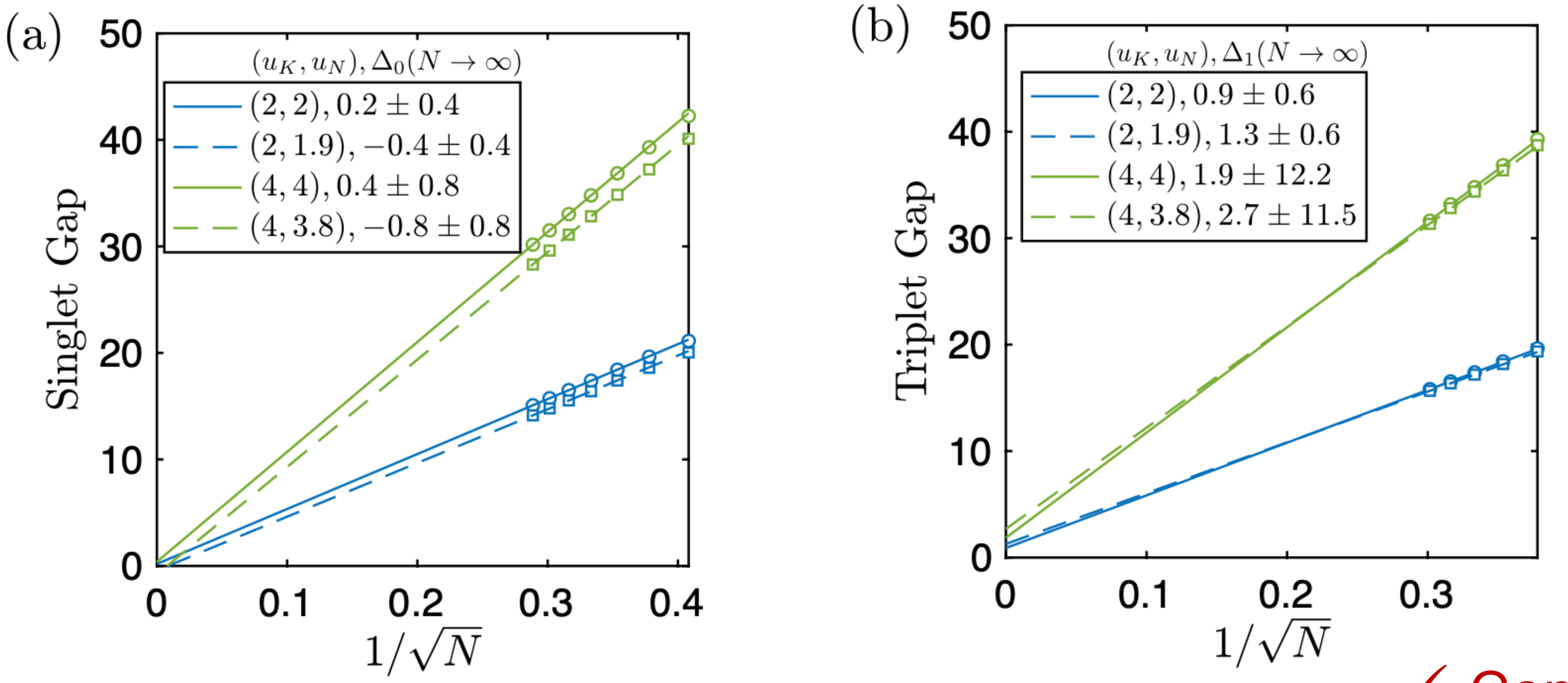
✓ Multicriticality at *u~0.12*

Numerical Results



Properties of the Disorder phase

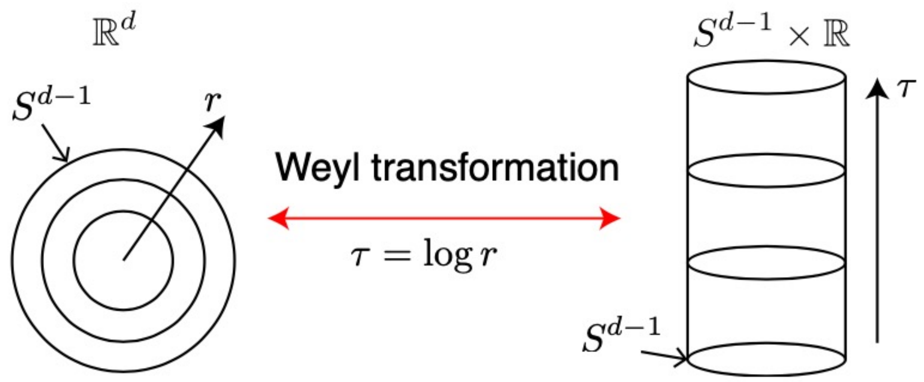
- Spin-singlet gap: $\Delta_0 = E_1(S=0) - E_0(S=0)$
- Spin-triplet gap: $\Delta_1 = E_0(S=1) - E_0(S=0)$



✓ Gapless disorder phase

➢ CFT Operator Spectrum: State-Operator Correspondence

○ Radial quantization



The Eigen-states of a quantum Hamiltonian on S^{d-1}

are on one-to-one correspondence with CFT's scaling operators.

$$E_k - E_0 = \frac{v}{R}\Delta_k$$

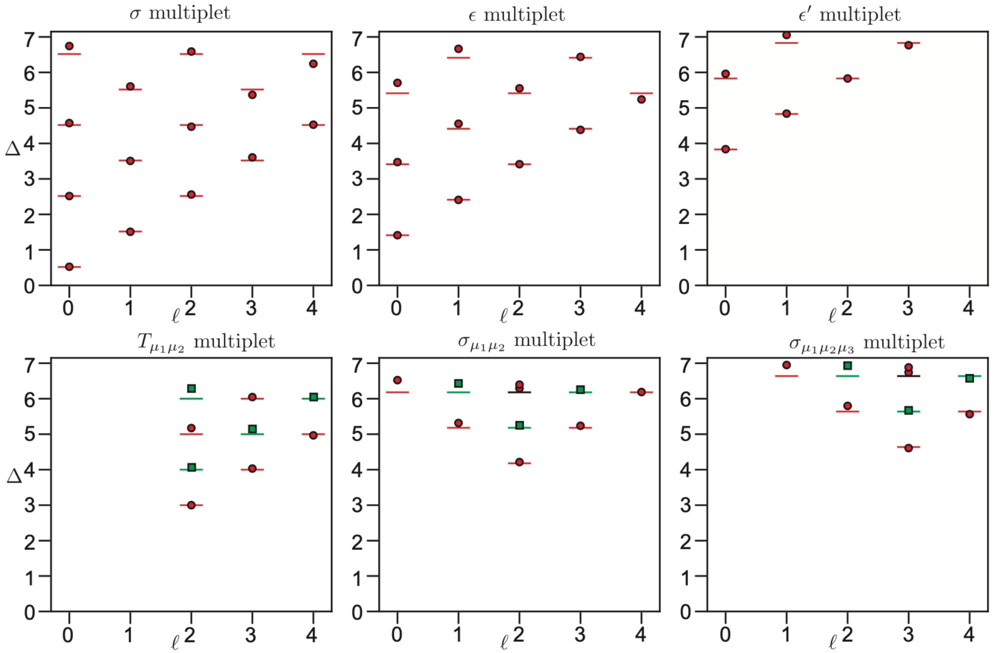
- CFT primaries and descendants

$$O \;\rightarrow\; \partial_\mu O \;\rightarrow\; \partial_\mu \partial_\nu O \;\rightarrow\; \cdots$$

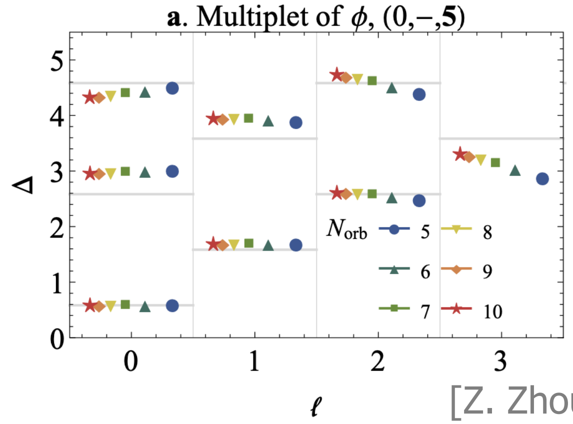
$$\Delta \;\rightarrow\; \Delta + 1 \;\rightarrow\; \Delta + 2 \;\rightarrow\; \cdots$$

○ 3D Ising CFT

[W. Zhu, et al, PRX2023]



○ Pseudo-criticality in SO(5) model



[Z. Zhou, et al, arXiv:2306.16435]

➢ CFT Operator Spectrum: State-Operator Correspondence

○ Identifying CFT operators in ED

TABLE S1. The Young diagrams of different SO(5) irreducible representations (denoted as IREP) and the corresponding state degeneracies in different (σ^z, τ^z) sectors.

SO(5) IREP	Young diagram	(0,0)	(0,2)	(0,4)	(0,6)	(2,0)	(2,2)	(2,4)	(4,0)	(4,2)	(6,0)
1		1									
5	□	1	1			1					
10	□ □	2	1			1	1				
14	□□	2	1	1		1	1		1		
30	□□□	2	2	1	1	2	1	1	1	1	1
35	□□ □	3	3	2		3	2	1	1	1	

○ Identifying CFT operators in DMRG

TABLE S2. The Young diagrams of different SO(5) irreducible representations (denoted as IREP) and the corresponding state degeneracies in different sectors with total spin S at half-filling case $q^z = 2(2s+1)$.

SO(5) IREP	Young diagram	0	1	2	3
1		1			
5	□	2	3		
10	□ □	1	9		
14	□□	3	6	5	
30	□□□	4	9	10	
35	□□ □	2	18	15	

➢ CFT Operator Spectrum: State-Operator Correspondence

- CFT spectrum in DMRG

SO(5) IREP	Young diagram	0	1	2
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5		2	3	
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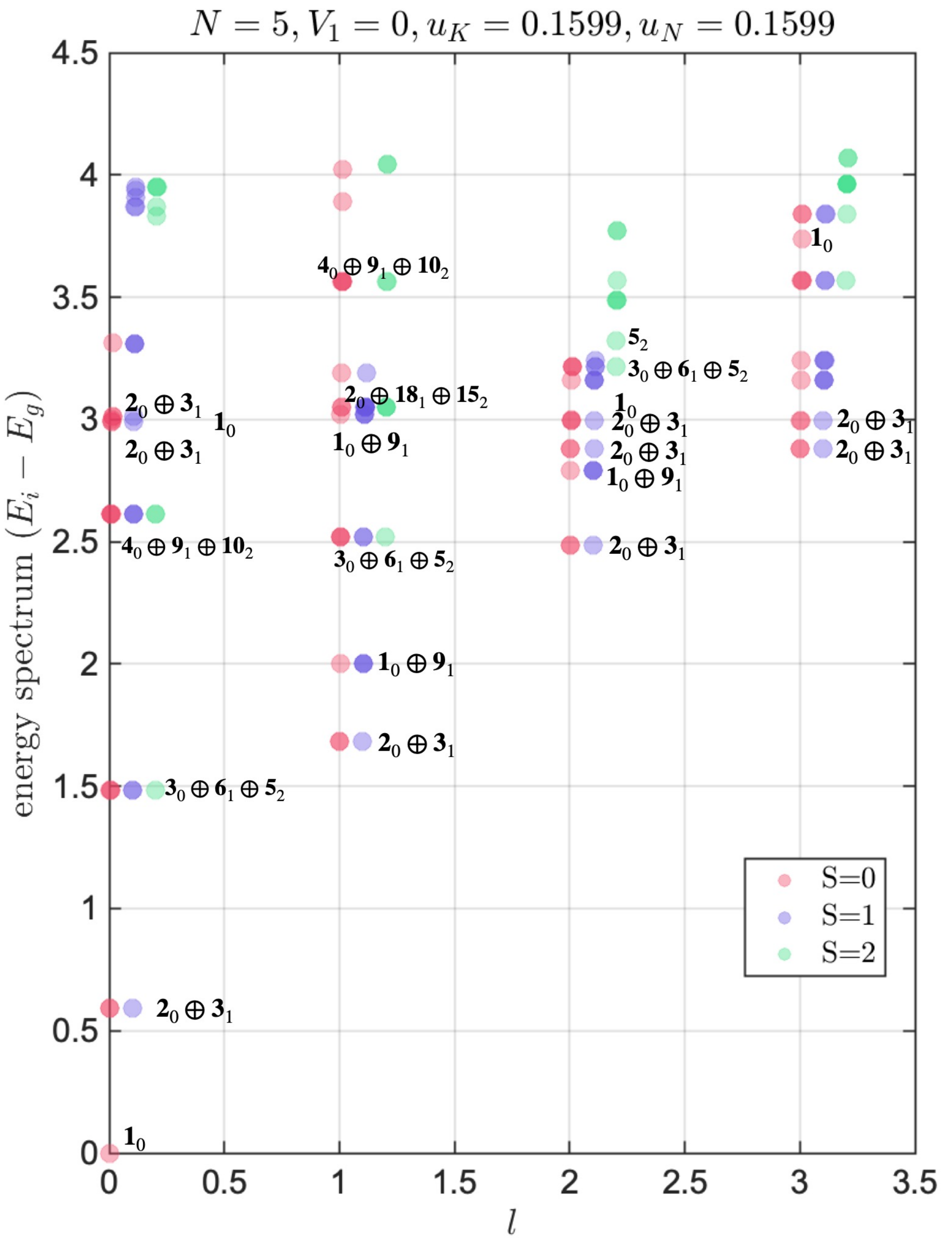

➢ CFT Operator Spectrum: State-Operator Correspondence

- CFT spectrum in DMRG

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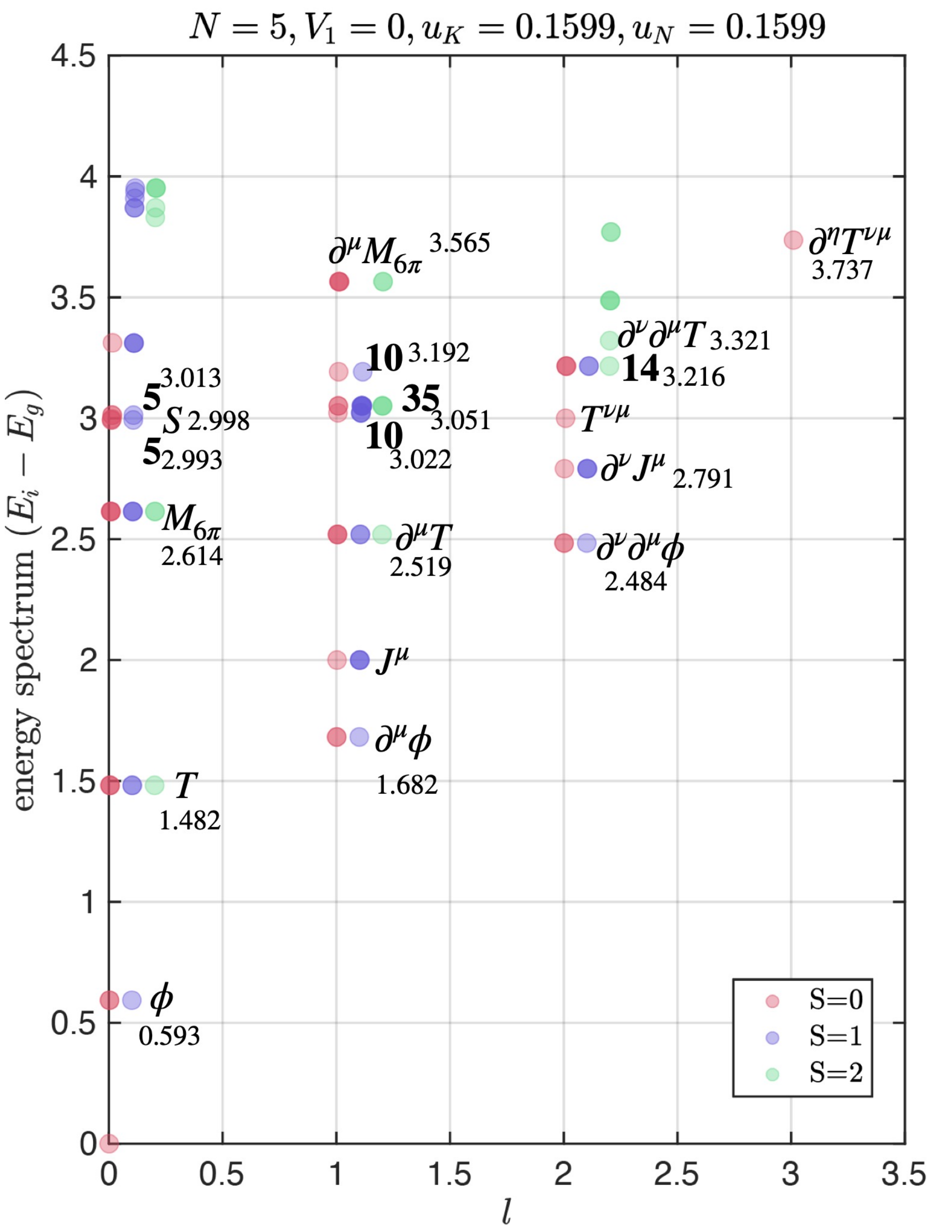


➢ Numerical Results: CFT Operator Spectrum

$$H_\Gamma = \frac{1}{2}\int d\Omega_1 d\Omega_2 \left(\frac{g_0}{R^2}\delta(|\mathbf{r}_1-\mathbf{r}_2|)+\frac{g_1}{R^4}\nabla^2\delta(|\mathbf{r}_1-\mathbf{r}_2|)\right)\sum_{i=0}^{5} U_i\left[\psi^\dagger(\Omega_1)\Gamma^i\psi(\Omega_1)-C(\Omega_1)\delta_{i0}\right]\left[\psi^\dagger(\Omega_2)\Gamma^i\psi(\Omega_2)-C(\Omega_2)\delta_{i0}\right]$$

$$=\sum_i U_i \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}\left(c^\dagger_{m_1,\alpha}\Gamma^i_{\alpha,\beta}c_{m_1+m,\beta}-2\delta_{i0}\delta_{m0}\right)\left(c^\dagger_{m_2,\alpha}\Gamma^i_{\alpha,\beta}c_{m_2-m,\beta}-2\delta_{i0}\delta_{m0}\right)$$

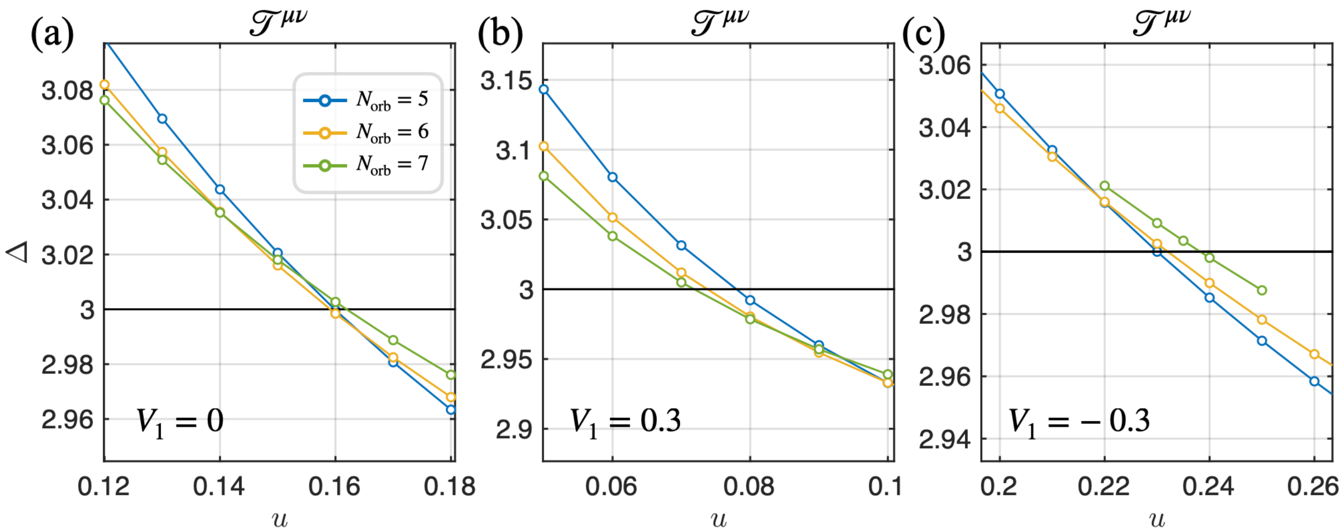
$$V_{m_1,m_2,m_3,m_4}=\sum_l V_l\,(4s-2l+1)\begin{pmatrix} s & s & 2s-l \\ m_1 & m_2 & -m_1-m_2\end{pmatrix}\begin{pmatrix} s & s & 2s-l \\ m_4 & m_3 & -m_3-m_4\end{pmatrix}$$

$$\begin{cases} V_0=\left(\frac{g_0}{R^2}-s\frac{g_1}{R^4}\right)\frac{(2s+1)^2}{(4s+1)}=\frac{g_0(2s+1)-g_1 s}{(4s+1)} \\ V_1=\left(s\frac{g_1}{R^4}\right)\frac{(2s+1)^2}{(4s-1)}=\frac{g_1 s}{(4s-1)}\end{cases}$$

- Scaling dimension of energy-momentum tensor

For CFT, we have

$$\Delta_{J^\mu}=2$$
$$\Delta_{T^{\mu\nu}}=3$$

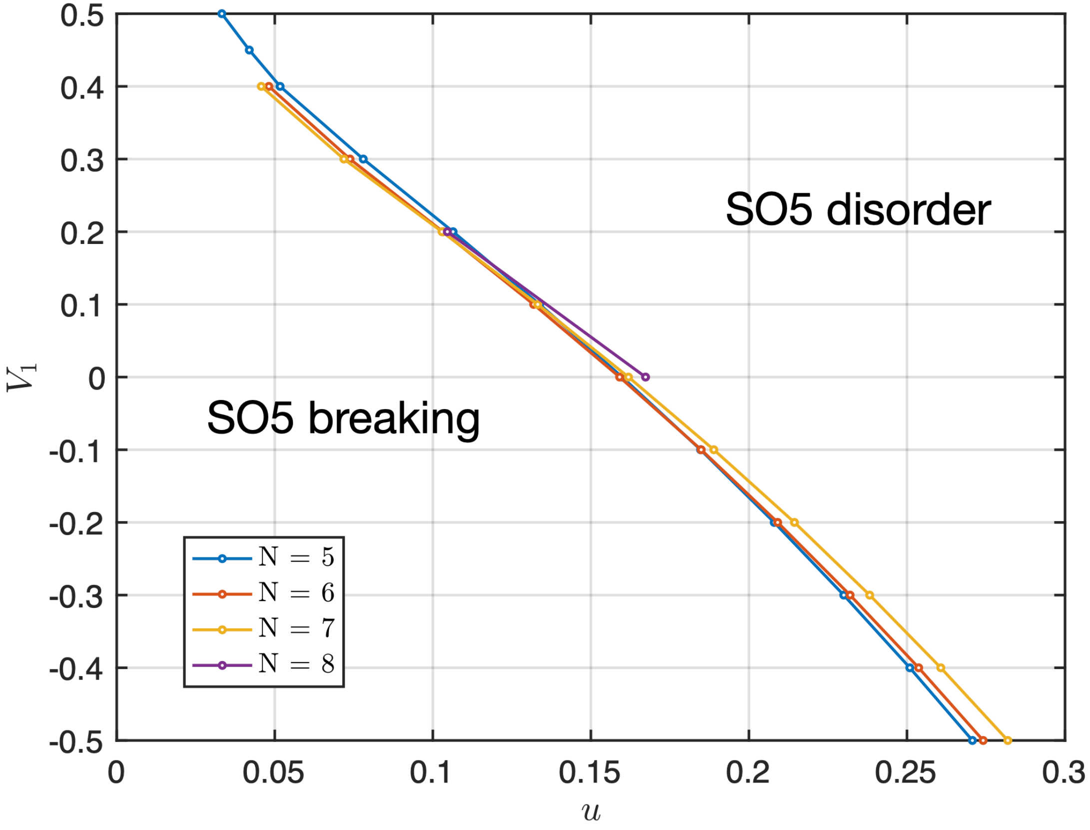


➢ Numerical Results: CFT Operator Spectrum

$$H_\Gamma = \frac{1}{2}\int d\Omega_1 d\Omega_2 \left(\frac{g_0}{R^2}\delta(|\mathbf{r}_1-\mathbf{r}_2|)+\frac{g_1}{R^4}\nabla^2\delta(|\mathbf{r}_1-\mathbf{r}_2|)\right)\sum_{i=0}^{5}U_i\left[\psi^\dagger(\Omega_1)\Gamma^i\psi(\Omega_1)-C(\Omega_1)\delta_{i0}\right]\left[\psi^\dagger(\Omega_2)\Gamma^i\psi(\Omega_2)-C(\Omega_2)\delta_{i0}\right]$$

$$=\sum_i U_i \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}\left(c^\dagger_{m_1,\alpha}\Gamma^i_{\alpha,\beta}c_{m_1+m,\beta}-2\delta_{i0}\delta_{m0}\right)\left(c^\dagger_{m_2,\alpha}\Gamma^i_{\alpha,\beta}c_{m_2-m,\beta}-2\delta_{i0}\delta_{m0}\right)$$

- Full Phase Diagram



✓ SO(5) transition line

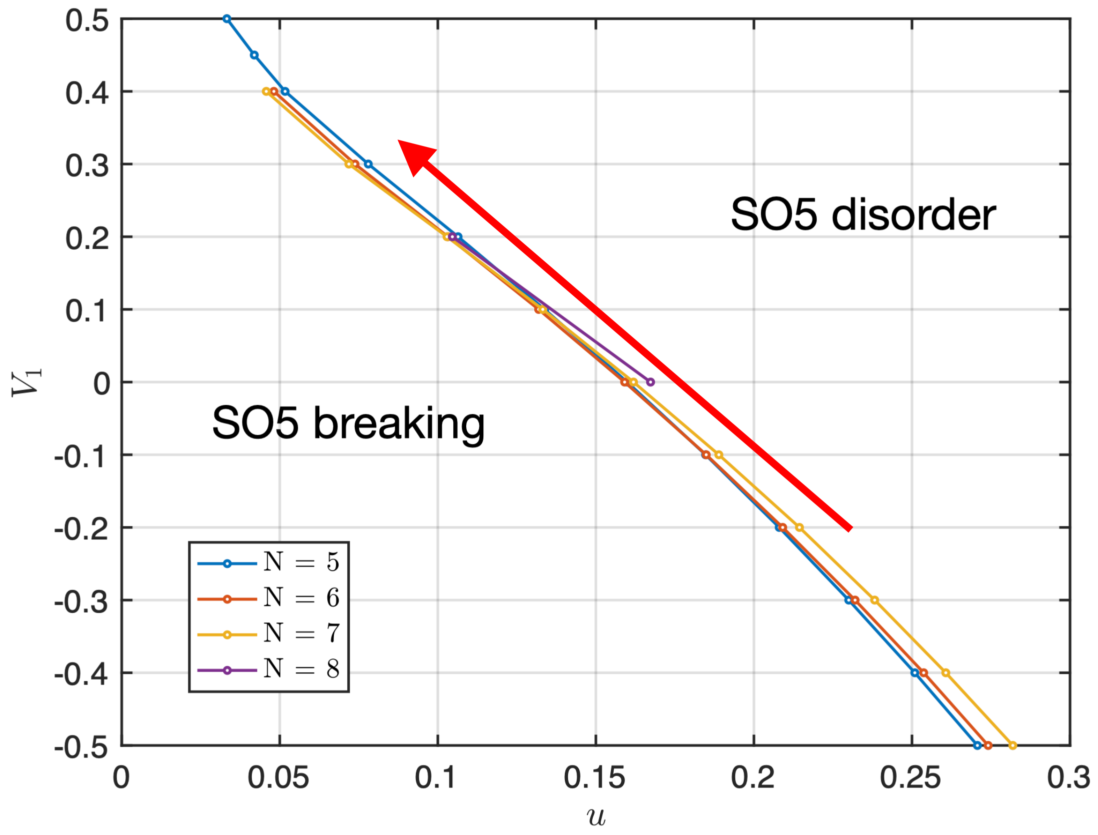
$$\Delta_{T^{\mu\nu}} = 3$$

➤ Numerical Results: CFT Operator Spectrum

$$H_\Gamma = \frac{1}{2}\int d\Omega_1 d\Omega_2 \left(\frac{g_0}{R^2}\delta(|\mathbf{r}_1-\mathbf{r}_2|)+\frac{g_1}{R^4}\nabla^2\delta(|\mathbf{r}_1-\mathbf{r}_2|)\right)\sum_{i=0}^{5} U_i \left[\psi^\dagger(\Omega_1)\Gamma^i\psi(\Omega_1) - C(\Omega_1)\delta_{i0}\right]\left[\psi^\dagger(\Omega_2)\Gamma^i\psi(\Omega_2) - C(\Omega_2)\delta_{i0}\right]$$

$$= \sum_i U_i \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}\left(c^\dagger_{m_1,\alpha}\Gamma^i_{\alpha,\beta}c_{m_1+m,\beta} - 2\delta_{i0}\delta_{m0}\right)\left(c^\dagger_{m_2,\alpha}\Gamma^i_{\alpha,\beta}c_{m_2-m,\beta} - 2\delta_{i0}\delta_{m0}\right)$$

- Full Phase Diagram

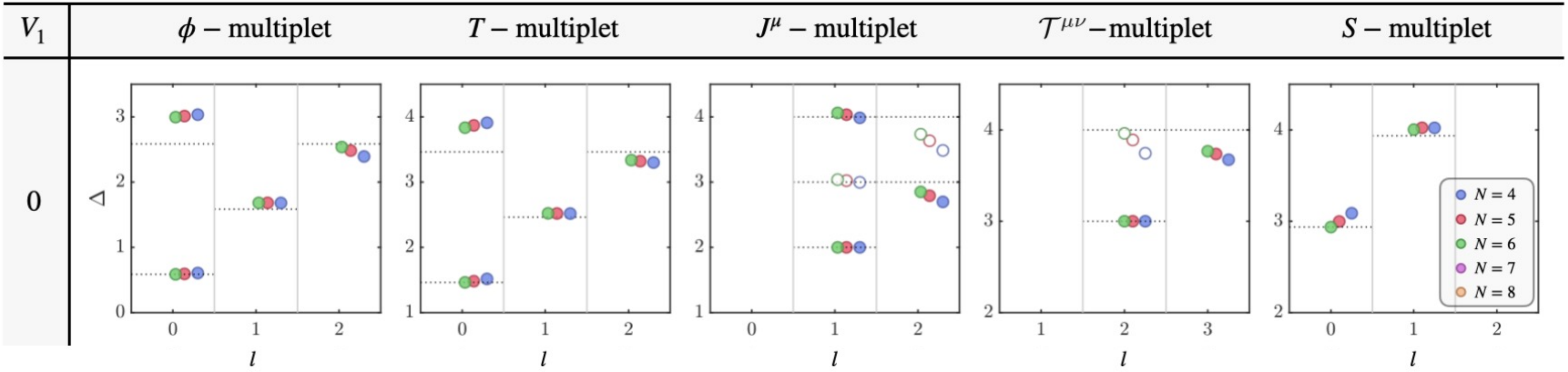


✓ SO(5) transition line

$$\Delta_{T^{\mu\nu}} = 3$$

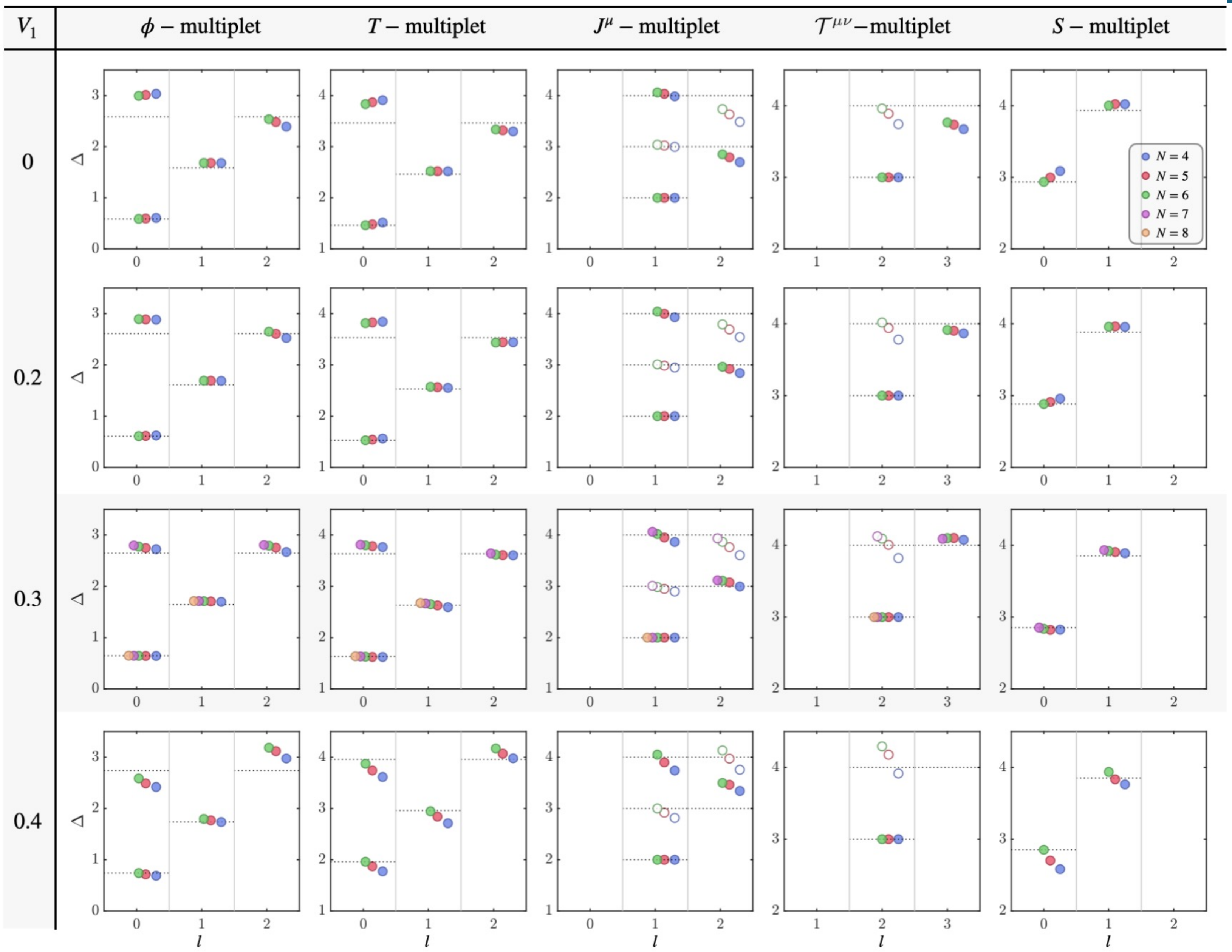
➢ Numerical Results: CFT Operator Spectrum

- CFT tower



➢ Numerical Results: CFT Operator Spectrum

- CFT tower



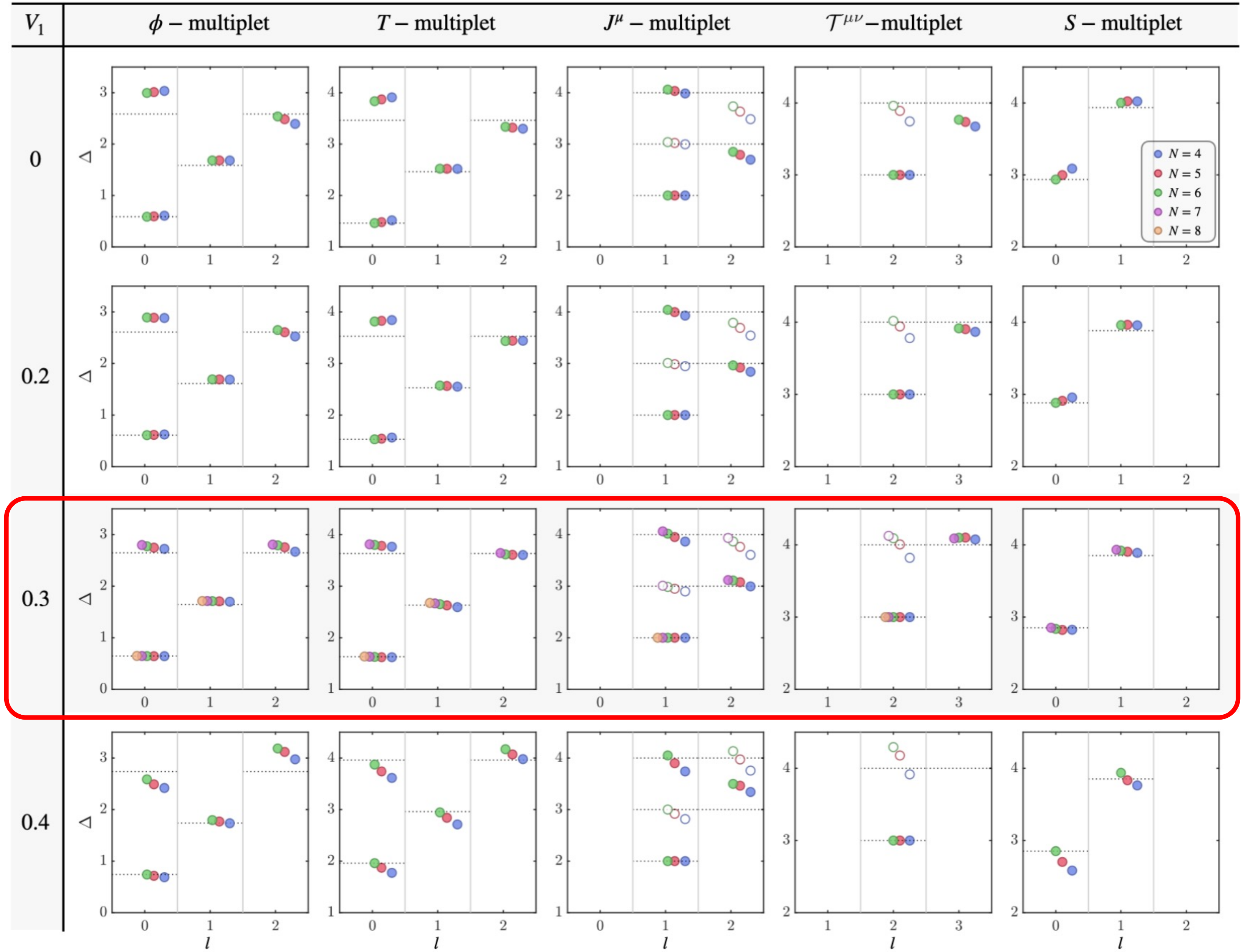
➢ Numerical Results: CFT Operator Spectrum

- CFT tower
- Relevant primary operators

	N				
Operators	4	5	6	7	8
ϕ	0.642	0.642	0.644	0.646	0.647
T	1.622	1.622	1.627	1.633	1.636
J^{μ}	2.000	2.000	2.000	2.000	2.000
S	2.853	2.823	2.873	2.884	—
$M_{6\pi}$	2.825	2.861	2.836	2.852	—
$\mathcal{T}^{\mu\nu}$	3.000	3.000	3.000	3.000	3.000

✓ S = 2.884 < 3

✓ *Relevant away from SO(5) line*
i.e. Multicritical point



➢ Numerical Results: Correlation ratio

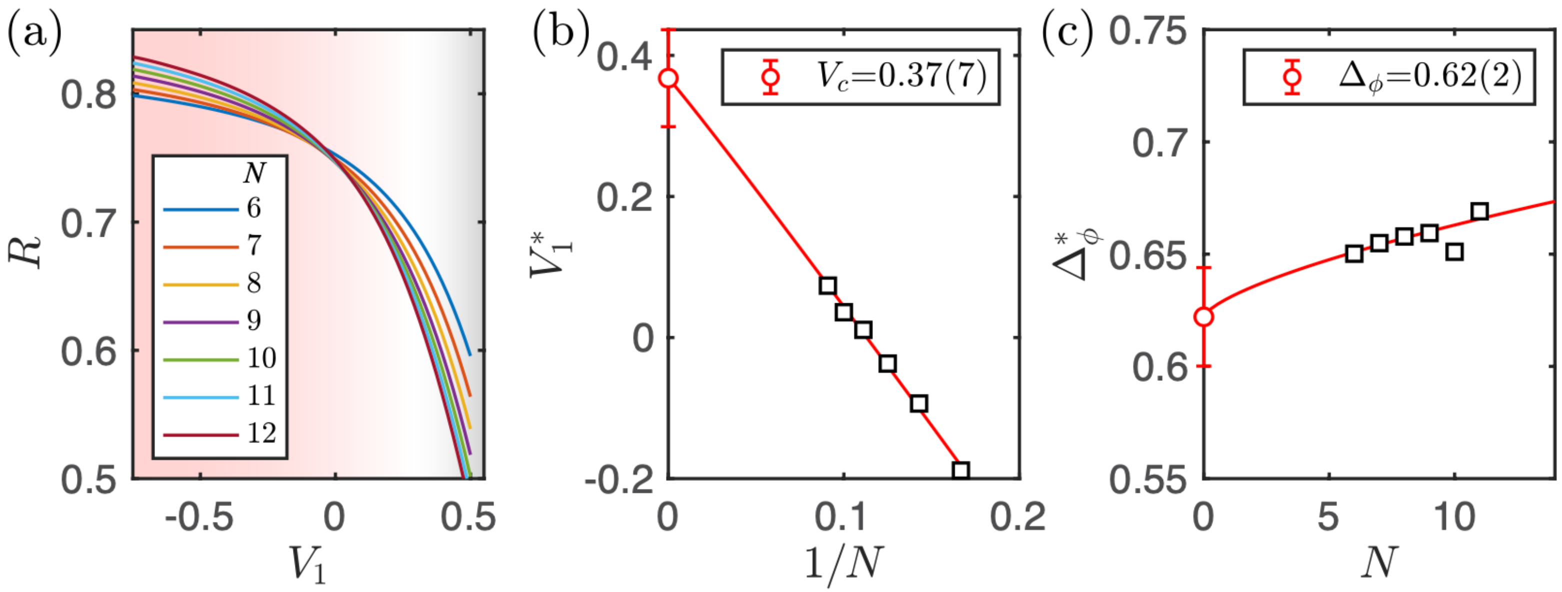
- SO5 correlation ratio: $R = 1 - m^2_{l=1}/m^2_{l=0}$

$$m_l^2 = \frac{1}{N^2}\sum_{i=1}^5 \langle O^2_{i=1,l}\rangle$$ with angular momentum shift l

- Crossing point analysis:

$$V_1^*(N, N+1) = V_c + N^{-\frac{1}{2\nu}-\frac{\omega}{2}}$$
$$\Delta_\phi^*(N) = \Delta_\phi + aN^{\frac{1}{2\nu}}$$

$$\Delta_\phi^*(N) = -N\log\frac{m^2(V_c,N+1)}{m^2(V_c,N)}$$



- We take

$$\nu = \frac{1}{3-\Delta_T} \simeq 0.733$$

obtaining

$$\Delta_\phi = 0.62(2)$$

Consistent with CFT spectrum

$$\Delta_\phi = 0.647$$

Conclusion

- **Full Phase Diagram of the SO5 model**

  [**BC**, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL 132, 246503 (2024)]

  - ✓ Various ordered phases: Néel, VBS, FM, VP
  - ✓ Intermediate disorder region between Néel and VBS phase
  - ✓ Non-Wilson-Fisher transition from both Néel and VBS phase to disorder phase
  - ✓ Multicritical point

- **SO5 transtion**

  [**BC**, X. Zhang, Z. Y. Meng, arXiv:2405.04470]

  - ✓ CFT Operator Spectrum of the Multicritical point
  - ✓ 4 relevant primary operators identified

Thank you!