Intro to deconfined criticality & related ideas

Part 1: O(3) model and hedgehogs

Adam Nahum (Oxford)

Les Houches, 8 Sept 2019

Criticality outside the Wilson-Fisher world

Most symmetry-breaking transitions in CMT can be understood via Landau-Ginsburg / Wilson-Fisher.

$$\mathcal{L} = (\nabla\phi)^2 + m^2\phi^2 + \phi^4 + \ldots$$

Mean field + fluctuations, 4-epsilon, etc.

Weirder things well-known in 1+1D (e.g. sigma models with topological terms) but traditionally less discussed in higher D

This lecture: 'non-Landau' symmetry-breaking phase transitions in **3D** that require either topological terms or gauge theories (partons).

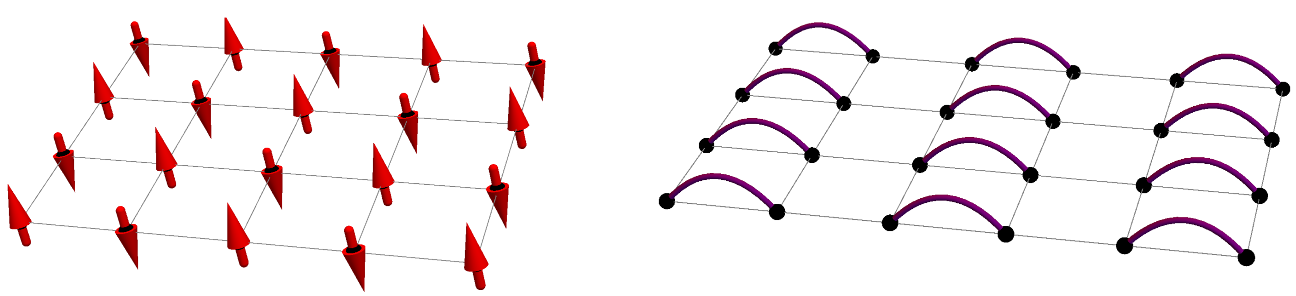
Senthil Vishwanath Balents Sachdev Fisher 04… Motrunich Vishwanath 04
Tanaka Hu 05, Senthil Fisher 06…. Sandvik 07 …

Part 1: 3D classical (focus on sigma model approach)

Part 2: 2+1D quantum (focus on gauge theory approach)

Part 1: The O(3) sigma model in 3D. What happens when you forbid topological defects (hedgehogs)?

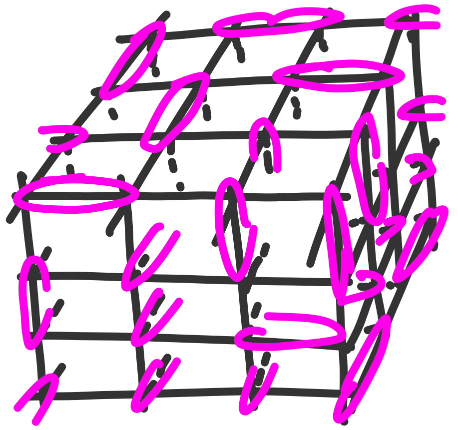
Part 2: The ‘deconfined’ Neel-VBS transition



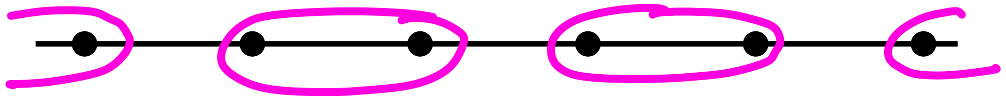
Plan: Part I

O(3) nonlinear sigma model with and without hedgehogs

U(1) symmetry from topological constraint; dimer model



Toy example: 1D dimer model



Five-component NLσM for the O(3) model without hedgehogs

Emergent symmetry

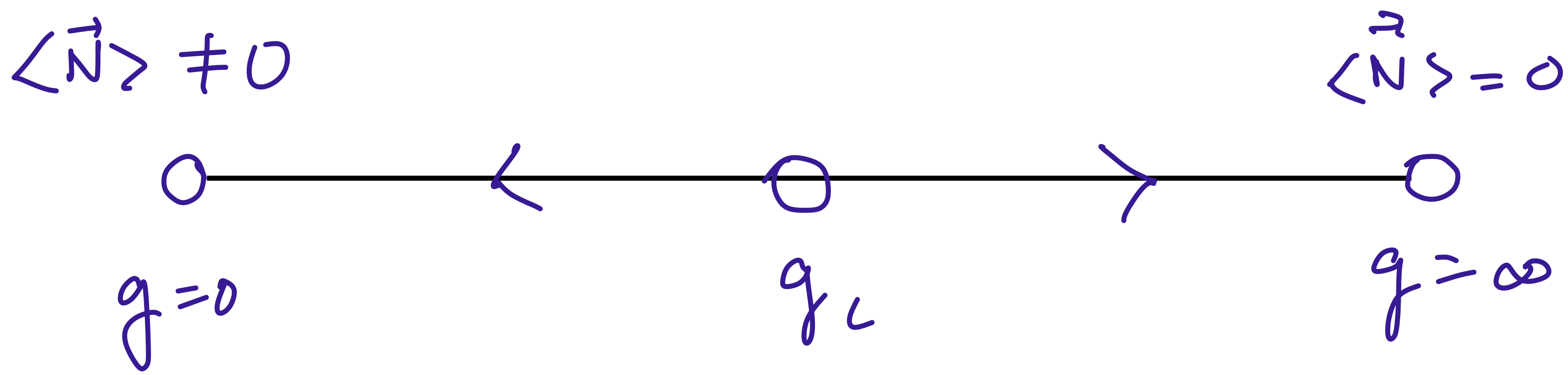
O(3) nonlinear sigma model

Stat mech of O(3) vector in 3D (3+0)

$$\vec{N} = (N_1, N_2, N_3)$$

$$Z = \int \mathcal{D}\vec{N}\, e^{-\frac{1}{g}\int d^3x\,(\nabla\vec{N})^2} \qquad |\vec{N}|^2 = 1$$

'Standard' RG flow diagram:



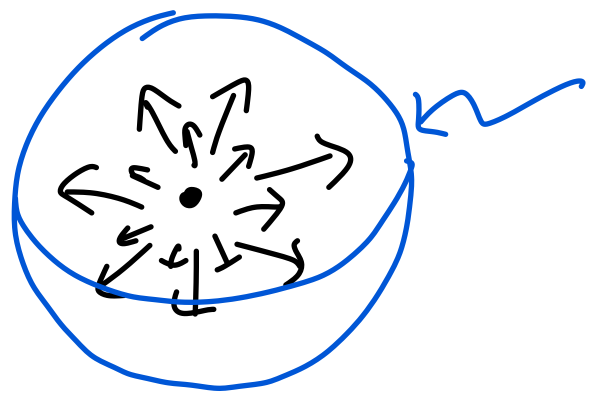
Usually say this is in universality class of O(3) Landau-Ginsburg theory (Wilson-Fisher).

However, let's consider topological defects more carefully.

Hedgehogs

$|\vec{N}|^2 = 1$

In three dimensions can have pointlike defects as $\pi_2(S^2) = \mathbb{Z}$



$\int_{\text{surface}} J^{\text{top}} \cdot dA = 1$

Point sources of topological flux: $J_\mu^{\text{top}} = \frac{1}{8\pi} \epsilon_{\mu\nu\lambda} \vec{N} \cdot (\partial_\nu \vec{N} \times \partial_\lambda \vec{N})$

Spacetime interpretation: skyrmion creation/annhilation events:

$$\int d^2x \, J_0^{\text{top}} = Q^{\text{top}}$$

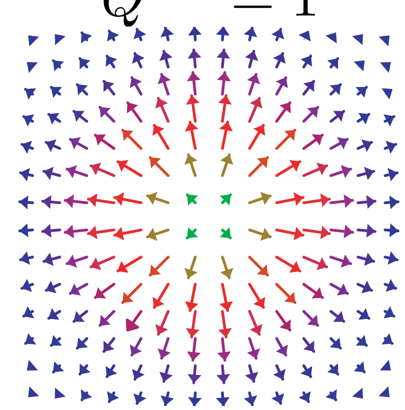


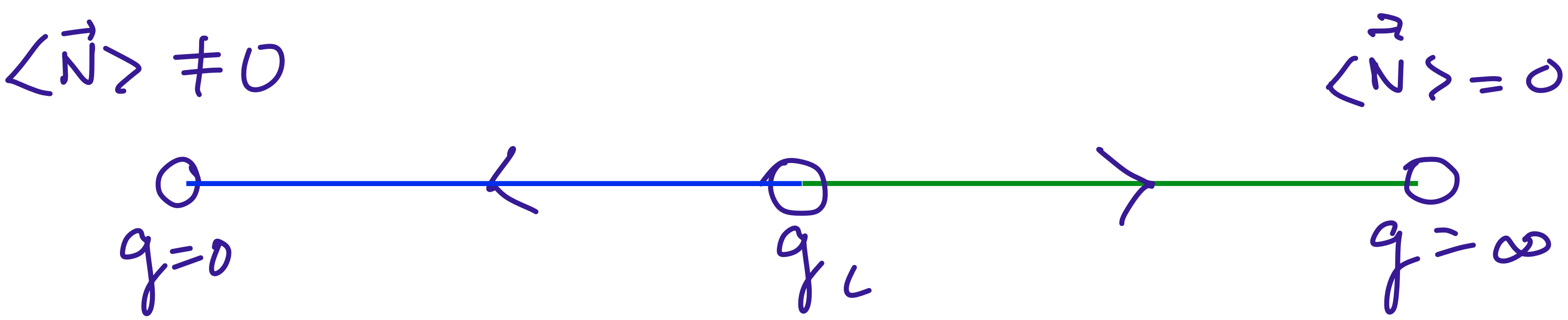
Fig: Senthil et al 03

Allow hedgehogs when we regularize NLσM path integral?

Two models

Allow hedgehogs when we regularize NLσM path integral?

Yes: Example: usual **lattice** O(3) model. Universal behaviour same as Landau-Ginsburg theory: $\mathcal{L} = (\nabla \vec{N})^2 + m^2 \vec{N}^2 + (\vec{N}^2)^2$

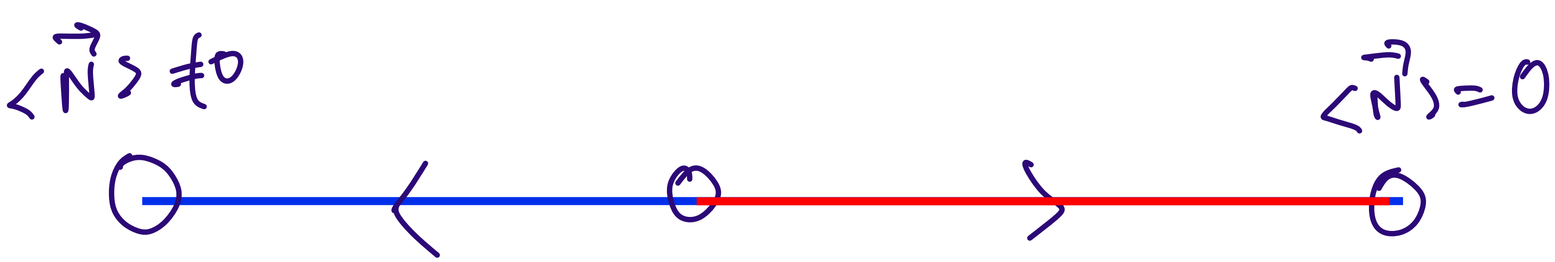


order

exponential decay of correlations

No: New phase diagram with new universal behaviour

Motrunich, Vishwanath 04
Kamal, Murthy 93, ….



order

power-law-correlated phase

Hedgehog-free O(3) model

80s: Question inspired by Kosterlitz & Thouless (2D XY):
are point defects are needed for the O(3) transition in 3D?

Cardy Hamber 80
Lau Dasgupta 87,88

Kamal & Murthy argued (numerics) for a disordering phase transition (contrary to a previous suggestion) but with new exponents

Lau Dasgupta 88
Kamal, Murthy 93

Motrunich & Vishwanath: 'disordered' phase is nontrivial.
'Photon' phase of a gauge theory (also appearing in deconfined quantum criticality)

Motrunich, Vishwanath 04

$$\mathcal{L}_{\mathrm{NCCP}^1} = |(\nabla - ia)\mathbf{z}|^2 + \kappa(\nabla \times a)^2 + m^2|\mathbf{z}|^2 + \lambda|\mathbf{z}|^4$$

This lecture: different (ahistorical) route

No gauge theory until part 2: Instead, use a nonlinear sigma model description introduced later in context of deconfined criticality

Tanaka Hu '05
Senthil Fisher '06

I will also rely on a (perhaps eccentric) lattice regularization using dimers

Freedman Hastings Nayak Qi 11
Sreejith Powell Nahum 19

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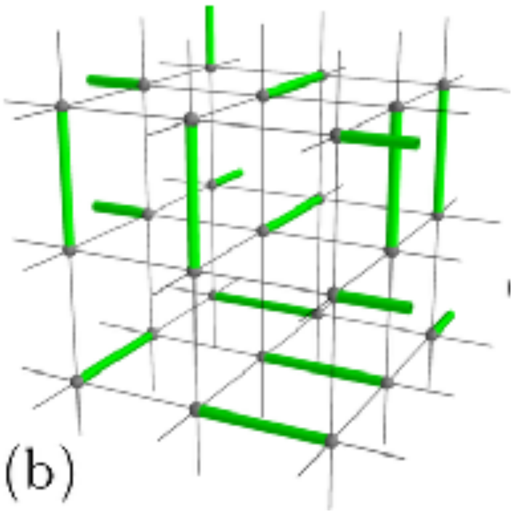
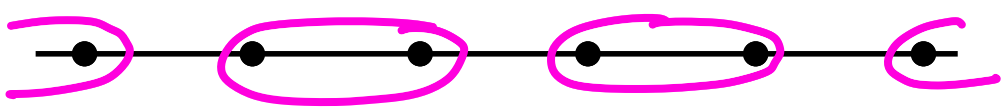
U(1) symmetry from topological constraint; dimer model



(b)

Toy example: 1D dimer model



Five-component NLσM for the O(3) model without hedgehogs

Emergent symmetry

Hedgehog-free O(3) model

Basic point: **topological constraint → extra U(1) symmetry**

whose conserved current is $J_\mu^{\rm top}$

Very heuristic (does not make sense as written!!):

$$Z \sim \int \mathcal{D}\vec{N}\, e^{-S[\vec{N}]}\, \delta\left(\nabla . J^{\rm top}\right)$$

$$\text{“} \int \mathcal{D}\theta e^{i \int d^3x\, \theta\, (\nabla . J^{\rm top})} \text{”}$$

U(1) symm: $\theta \to \theta + \text{const.}$ U(1) current: $J_\mu^{\rm top} \sim \partial_\mu \theta$

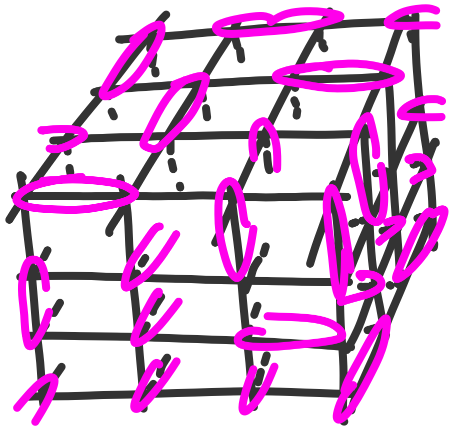
To make sense of this, let us consider a lattice version: **dimer model**

This will also allow numerical experimentation

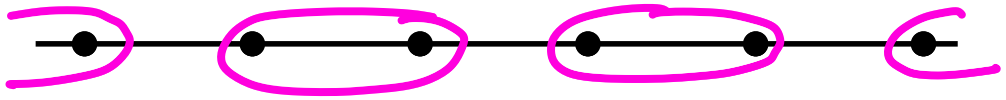
Plan: Part I

O(3) nonlinear sigma model with and without hedgehogs

U(1) symmetry from topological constraint; **dimer model**



Toy example: 1D dimer model



Five-component NLσM for the O(3) model without hedgehogs

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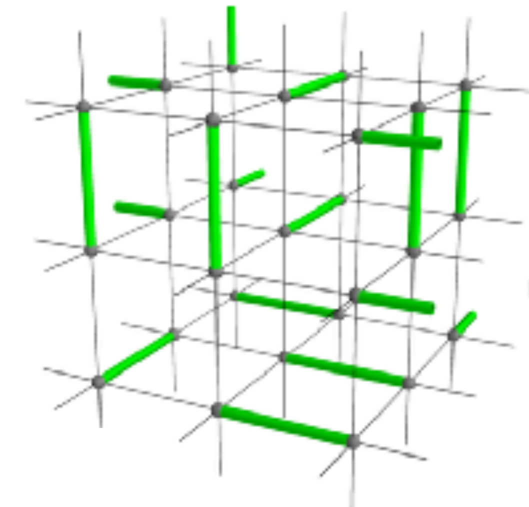
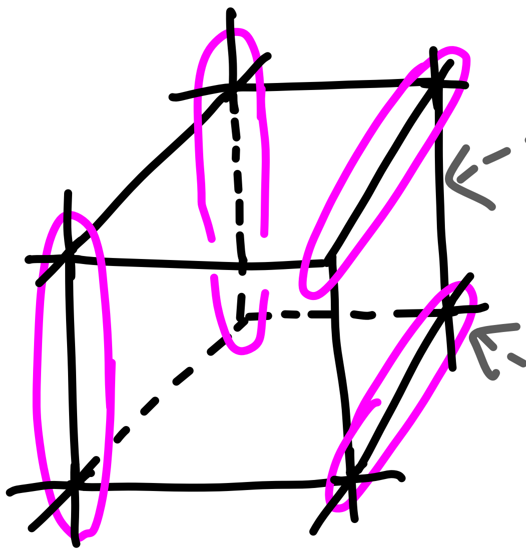
Classical dimer model on cubic lattice



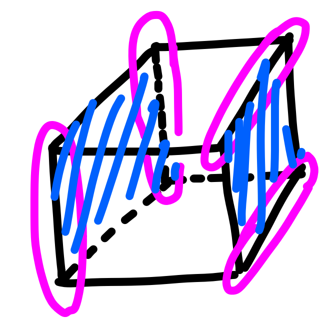


d.o.f.: link occupation #s: $n_l = 0, 1$

each site **r** touches 1 dimer: $\sum_{l \in \mathbf{r}} n_l = 1$

$$Z = \sum_{\{n\}} e^{-\beta E[n]} \left(\prod_{\mathbf{r}} \delta_{\sum n_l, 1} \right)$$

energy: aligning interactions:

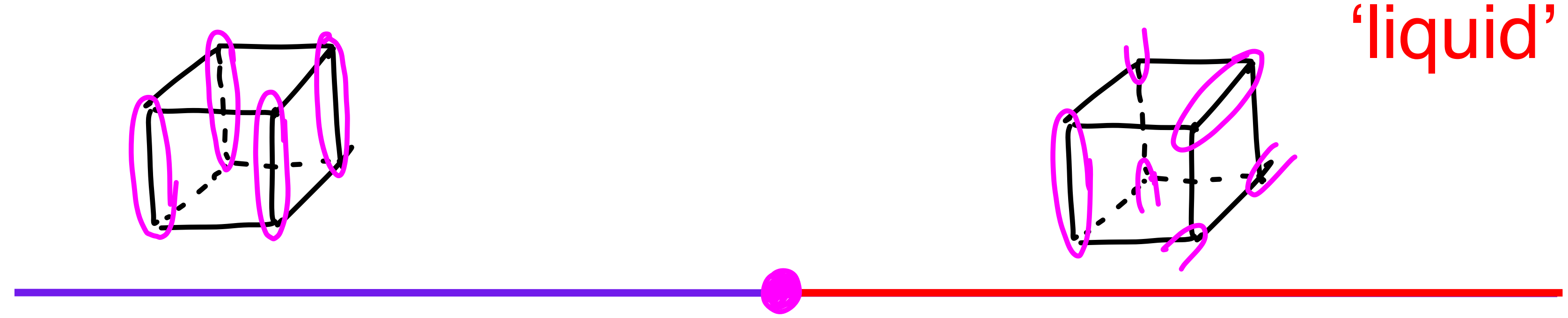


Numerical phase diagram:

Alet et al 06
Charrier and Alet
....

columnar order

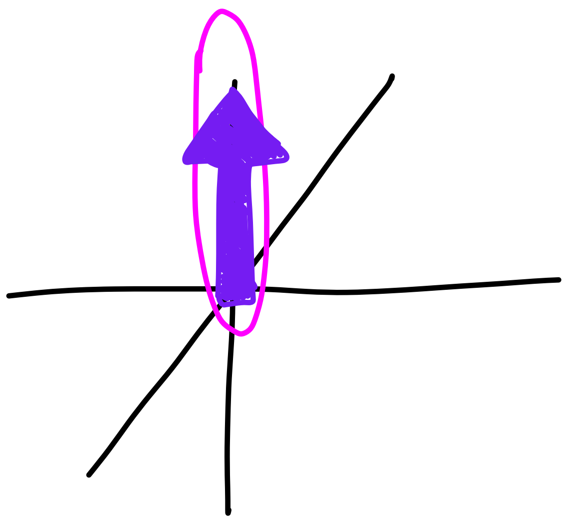
power-law correlated 'liquid'

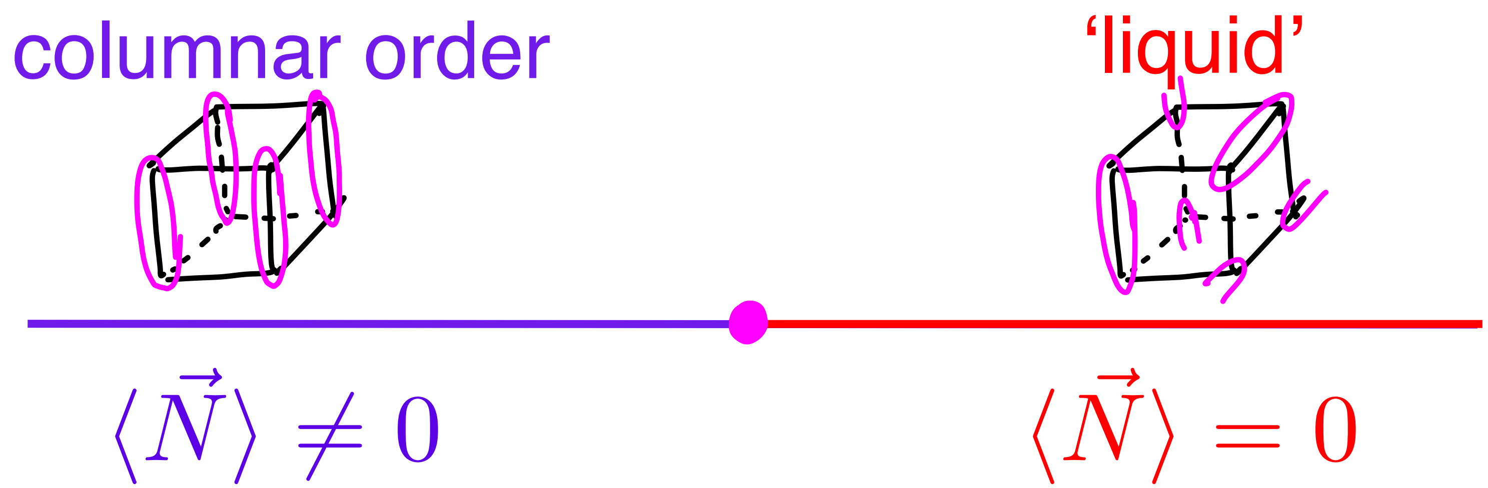


temperature →

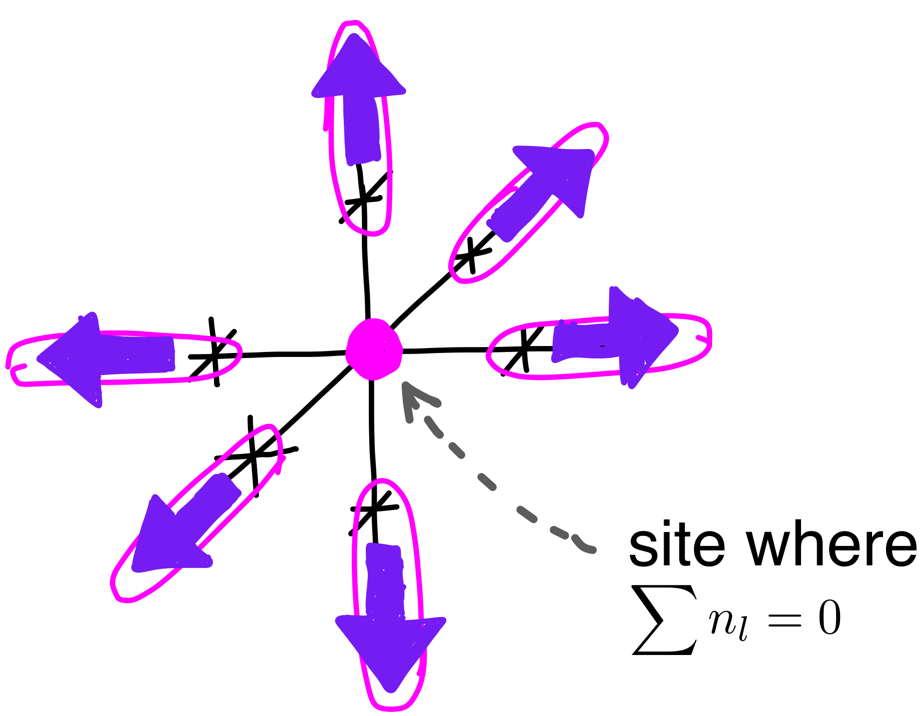
Dimer model = an anisotropic O(3) model

O(3) vector $\vec{N}$ → columnar order parameter





Vacancy = hedgehog. Full-packing → no hedgehogs!



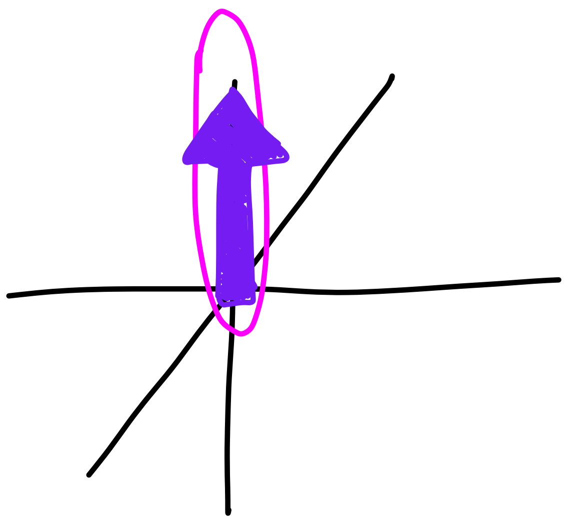
The dimer model is an (eccentric) regularization of the Hedgehog-free O(3) model!

plus **cubic anisotropy** $\sim \sum_{i=1}^{3} N_i^4$

(Only important in ordered phase)

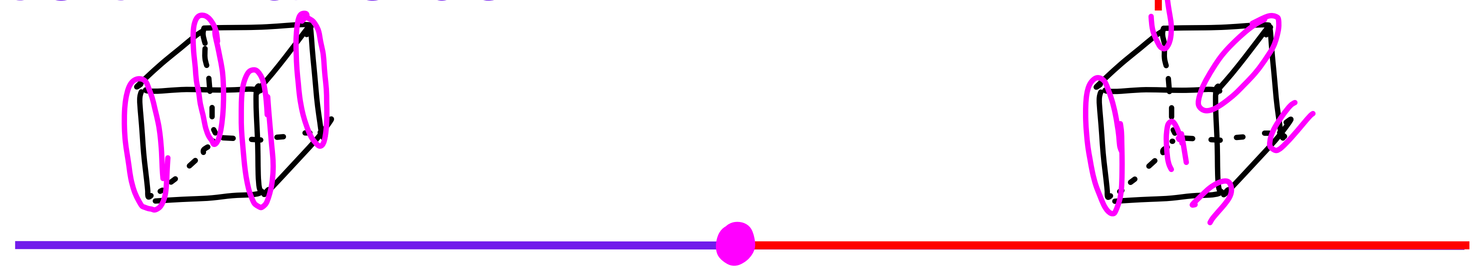
Dimer model = an anisotropic O(3) model

O(3) vector $\vec{N}$ → columnar order parameter



columnar order

'liquid'

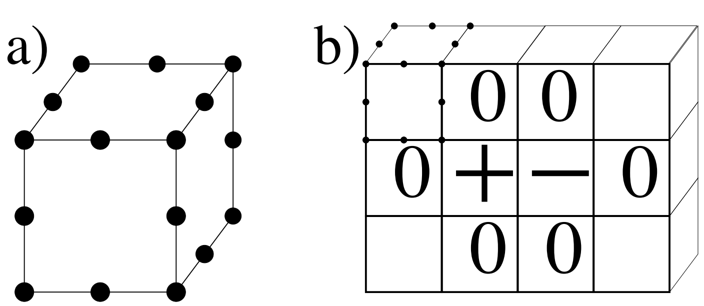


$\langle \vec{N} \rangle \neq 0$

$\langle \vec{N} \rangle = 0$

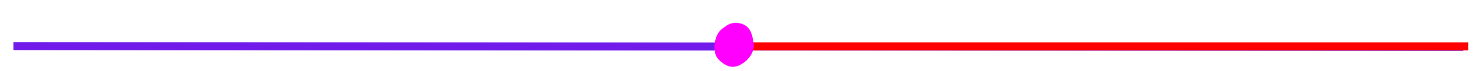
Equivalent phase diagram found numerically in a more conventional O(3) model without hedgehogs

Motrunich & Vishwanath 04



O(3) ferromagnet

hedgehog-free paramagnet



$\langle \vec{N} \rangle \neq 0$

$\langle \vec{N} \rangle = 0$

Two order parameters

from $\prod_{\mathbf{r}} \delta_{\sum n_l, 1}$

$$Z = \sum_{\{n\}} \int \mathcal{D}\theta \; e^{-\beta E[n] + i \sum_{\mathbf{r}} \theta_{\mathbf{r}} (\nabla . J^{\text{top}}[n])_{\mathbf{r}}}$$

U(1) symm: $\theta \to \theta + \text{const.}$ **U(1) current:** $J^{\text{top}}_{\mu} \sim \partial_{\mu} \theta$

$e^{\pm i \theta_{\mathbf{R}}}$ inserts a hedgehog (modifies delta function)

Imagine coarse-graining, retaining **both** order parameters:

$\vec{N} = (N_1, N_2, N_3)$	$\varphi \sim e^{i\theta}$
"O(3)" vector	hedgehog operator

Effective field theory?

$\vec{N} = (N_1, N_2, N_3)$ "O(3)" vector

$\vec{\varphi} \sim (\cos\theta, \sin\theta)$ hedgehog operator

Landau-Ginsburg theory? $\mathcal{L} \stackrel{?}{=} (\nabla \vec{N})^2 + (\nabla \vec{\phi})^2 + m_N^2 \vec{N}^2 + m_\phi^2 \phi^2 + (\vec{N}^2)^2 + \ldots$

No — fails to capture topological intertwining of two order parameters: $\varphi_x + i\varphi_y$ inserts topo. defect in $\vec{N}$

Instead, either:

- Gauge theory (fractionalise *N*)

  Powell Chalker 08, Charrier Alet Pujol 08, Chen Gukelberger Trebst Alet Balents 09
  (cf also Hedgehog-free O(3): Motrunich Vishwanath 04, DCP: Senthil Vishwanath Balents Sachdev Fisher 04)

- Sigma model with topological term

Effective field theory?

Need: well-defined continuum version of the term

$$“ \int \mathcal{D}\theta e^{i \int d^3x\, \theta\, (\nabla . J^{\text{top}})} ”$$

[recall $J_\mu^{\text{top}} = \frac{1}{8\pi} \epsilon_{\mu\nu\lambda} \vec{N} \cdot (\partial_\nu \vec{N} \times \partial_\lambda \vec{N})$]

One way to do this is to embed two order parameters in

$$\vec{n} = (N_x, N_y, N_z, \varphi_x, \varphi_y) \qquad \vec{n}^2 = 1$$

(Note: n can be well-defined everywhere: $|N|^2=0$ at hedgehogs)

NLσM for five real ‘order parameters’

Claim: desired imaginary term is Wess-Zumino-Witten term

$$\int \mathcal{D}\vec{n} \exp\left[-\frac{1}{2g} \int d^3x (\nabla \vec{n})^2 + \ldots + \frac{2\pi i}{\text{area}(S^4)} \int_0^1 du \int d^3x\, \epsilon^{abcde} n^a \partial_x n^b \partial_y n^c \partial_z n^d \partial_u n^e \right]$$

Let’s consider a simpler example: 1D (1+0D!)

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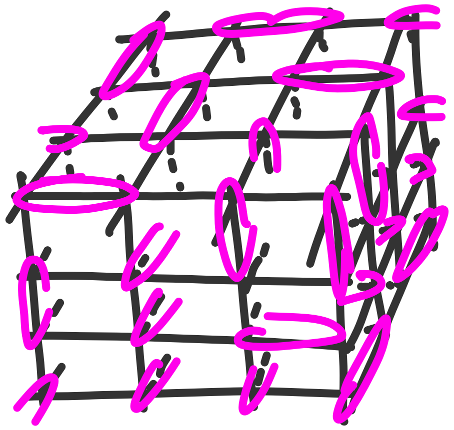
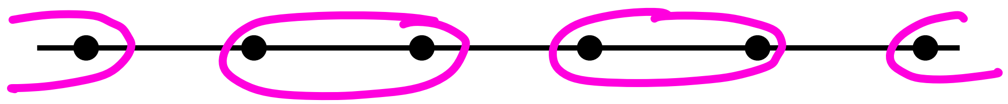


U(1) symmetry from topological constraint; dimer model

Toy example for appearance of WZW term: 1D dimers



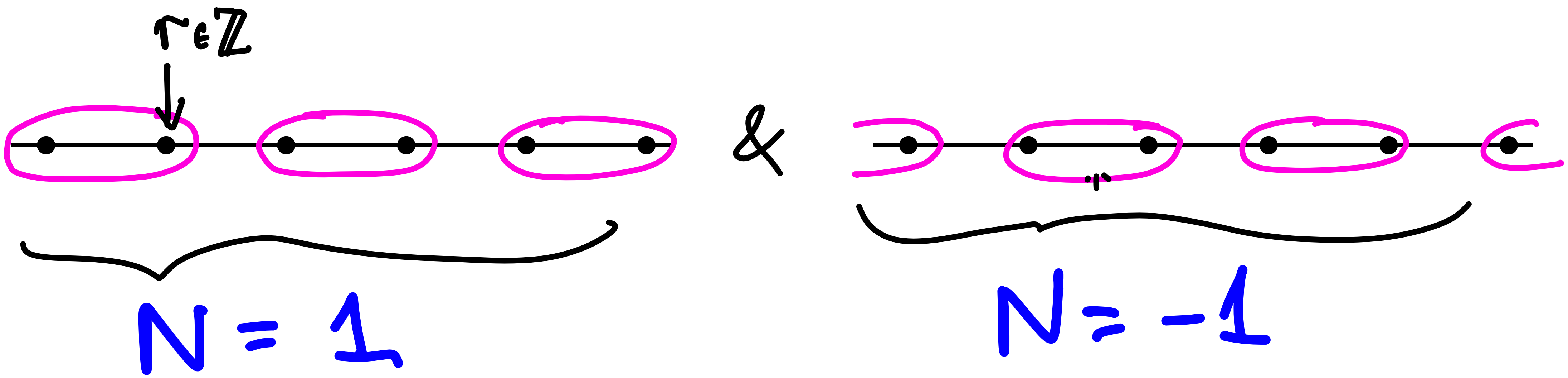
Five-component NLσM for the O(3) model without hedgehogs

Emergent symmetry

1D classical dimer model

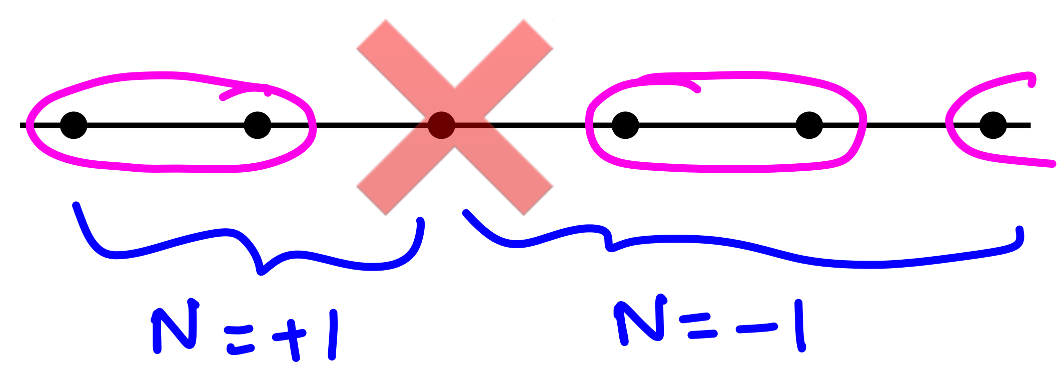
The simplest lattice model you will meet in this school!
Full-packing allows only 2 configs (PBCs, *L* even):

$$Z = 2$$



Have defined Ising order parameter $N_{r+1/2}$ on links (staggered defn)

= "Domain-wall free Ising model":

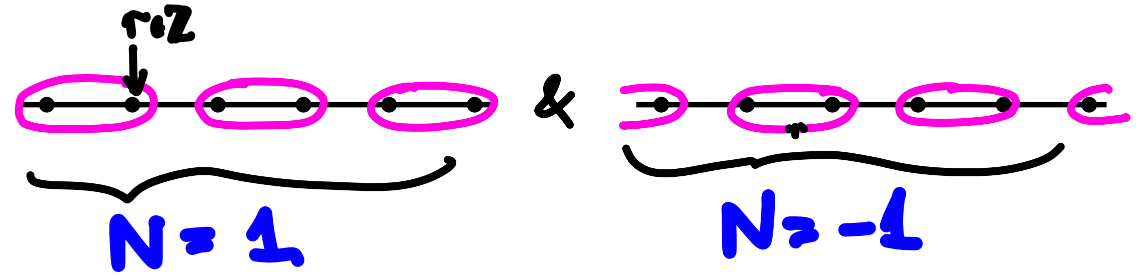


As before, introduce **θ** to impose the no-defect constraint:

$$Z = \int \mathcal{D}N\, \mathcal{D}\theta\, e^{i\sum_r \theta_r\left[N_{r+1/2} - N_{r-1/2}\right]/2}$$

$$\int \prod_r \frac{d\theta_r}{2\pi} \sum_{\{N\}}$$

1D dimers



Handwaving continuum limit: write $N = \cos\chi$, $\quad \vec{n} = \begin{pmatrix} \cos\chi \\ \cos\theta \sin\chi \\ \sin\theta\sin\chi \end{pmatrix}$

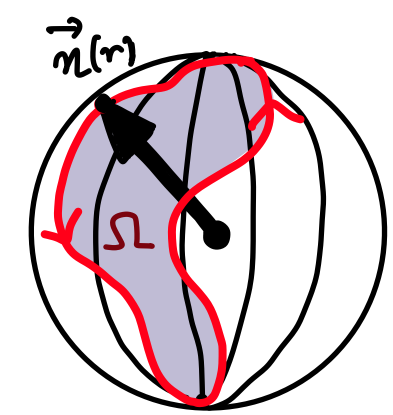
$$Z = \int \mathcal{D}N\, \mathcal{D}\theta\, e^{\frac{i}{2}\sum_r \theta_r\,(N_{r+1/2} - N_{r-1/2})}$$

$$= \int \mathcal{D}N\, \mathcal{D}\theta\, e^{-\frac{i}{2}\sum_r (\theta_{r+1} - \theta_{r-1})\,(1 + N_{r+1/2})}$$

$$\to \int \mathcal{D}\vec{n}\; e^{-\frac{i}{2}\int dr\,(\partial_r\theta)(1 + \cos\chi)}$$

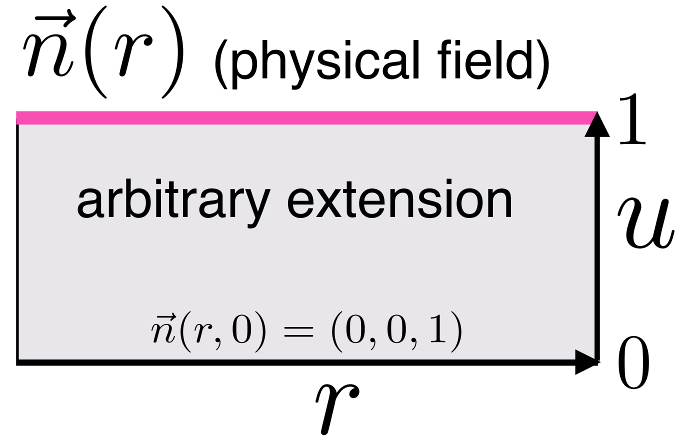
This is the Wess-Zumino term that occurs in path integral for spin-1/2!

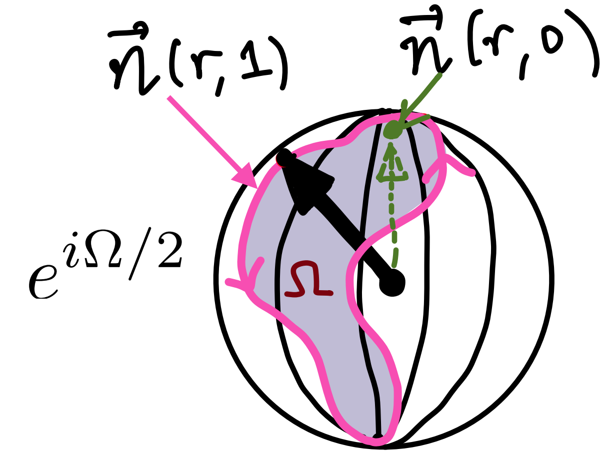
$e^{i\Omega/2}$



It can be written in an SO(3)-invariant form in terms of $\vec{n}(u, r)$, i.e. field extended into a fictitious second dimension *u*

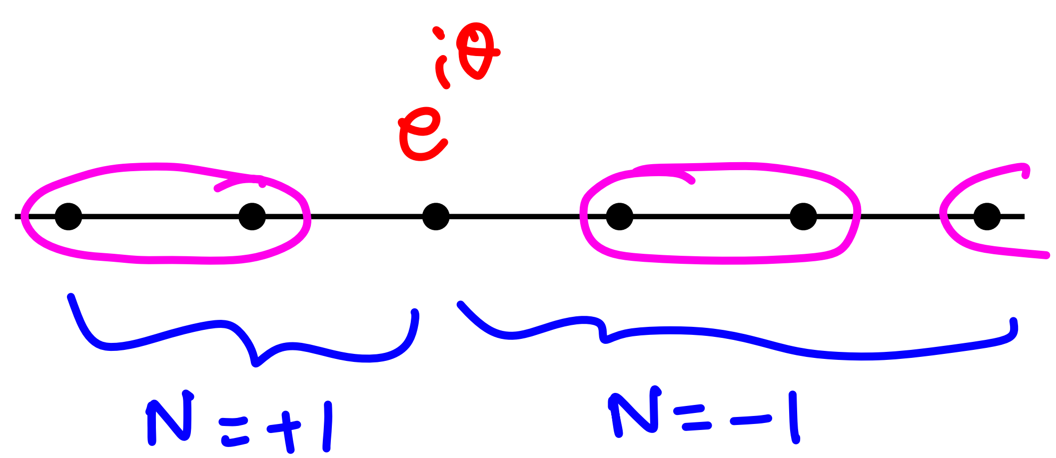
WZ term: $\vec{n}(r,u)$



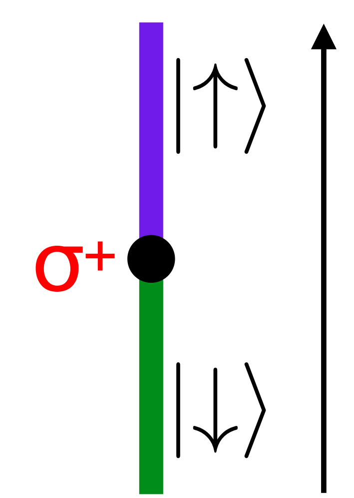


$$Z=\int \mathcal{D}\vec{n}\,\exp\left[-\frac{2\pi i}{\mathrm{area}(S^2)}\int_0^1 du\int dr\,\vec{n}\cdot(\partial_r\vec{n}\times\partial_u\vec{n})\right]$$

1D WZ term yields “delta function”: topo defects in $N\sim n_1$ only allowed at insertions of $e^{i\theta}\sim n_2+in_3$



Aside: The above action, with r→t, describes a spin-1/2. There we are familiar with the ‘topological intertwining’ of $N\sim\sigma^z$ and $e^{i\theta}\sim\sigma^+$:



Heuristic picture for WZW term: killing defects

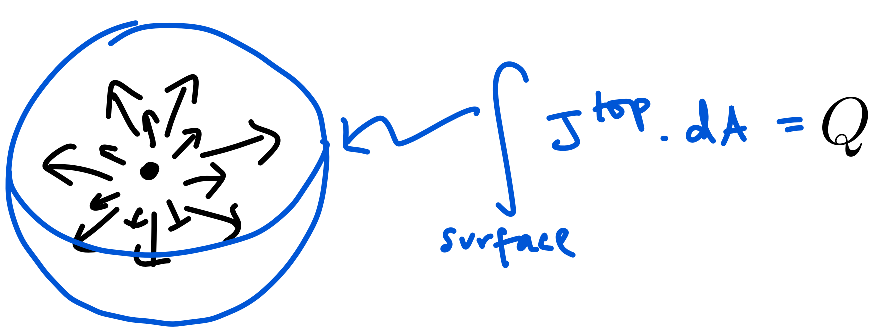
1D WZ term yields "delta function": topo defects in $N \sim n_1$ only allowed at insertions of $e^{i\theta} \sim n_2 + in_3$

This generalizes to WZW model in 3D.

$$\vec{n} = \Big(\sin\chi\,(\cos\theta, \sin\theta), \cos\chi\,\vec{N}\Big)$$

Heuristically: integral over **θ** imposes topological constraint forbidding point defects in *N*.

Exercise: fix config on a sphere inside 3D system to $\vec{n} = (0, 0, \vec{N}_Q)$, where N_Q has topological number Q. (If Q≠0, then *N* has hedgehog(s) inside the sphere.)



$$\int \mathcal{D}\theta \exp \frac{2\pi i}{\text{area}(S^4)} \int_0^1 du \int d^3x\, \epsilon^{abcde} n^a \partial_x n^b \partial_y n^c \partial_z n^d \partial_u n^e$$

Show that integrating over θ inside sphere gives zero if Q≠0.

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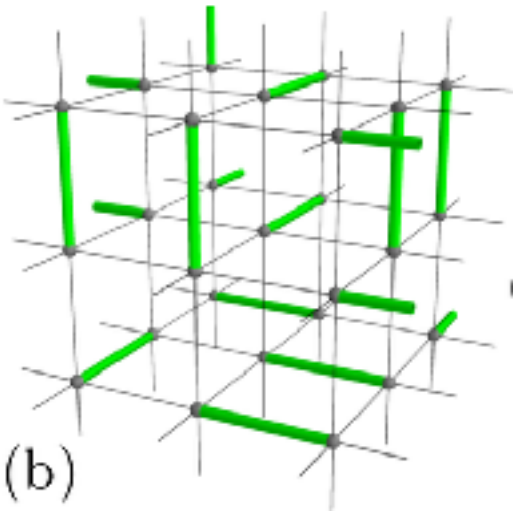
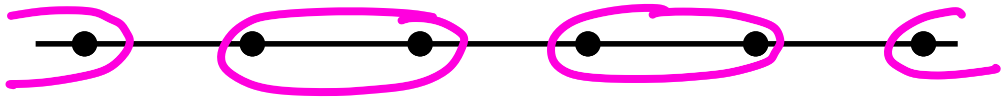
O(3) nonlinear sigma model with and without hedgehogs

U(1) symmetry from topological constraint; dimer model



(b)

Toy example: 1D dimer model

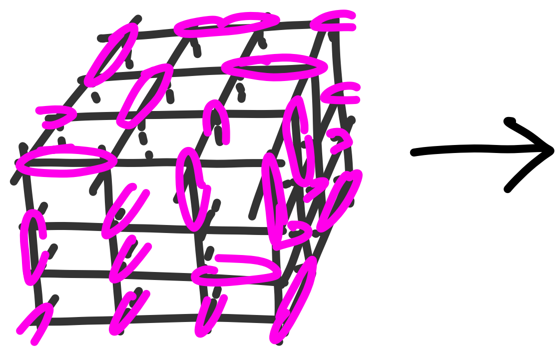


Five-component NLσM for the O(3) model without hedgehogs

Emergent symmetry

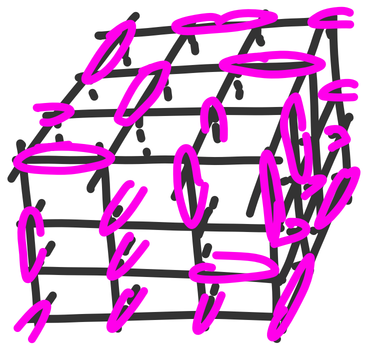
Hedgehog-free O(3) model → 5 cpt. NLσM

or



$$\int \mathcal{D}\vec{n}\, \exp\left[-\frac{1}{2g}\int d^3x(\nabla\vec{n})^2 - S_{\text{WZW}} + \dots\right]$$

Hedgehog-free O(3) model → 5 cpt. NLσM

 or  $\longrightarrow \int \mathcal{D}\vec{n}\, \exp\left[-\frac{1}{2g}\int d^3x (\nabla\vec{n})^2 + \text{anisotropies} + S_{\text{WZW}} \right]$

Anisotropies! Constrained by microscopic symmetries.
These are a **subgroup of SO(5)** rotations of $(n_1, \ldots, n_5)$

SO(3) rotations of $(n_1, n_2, n_3) = \vec{N}$ [cubic subgp O_h in dimer case]

U(1) rotations of $(n_4, n_5) = \vec{\varphi}$

SO(3) or O(3)? Are improper rotations of **N** allowed? (E.g. N → - N)

Yes: so O(3). But they exchange hedgehogs ↔ antihedgehogs, so must be combined with **φ→φ*** (improper rotation of **φ**)

SO(3) x O(2)

E.g. unit x-translation in dimer model: $N_1 \to -N_1$, **φ→φ***

Emergent SO(5)?

NLσM explains two phases, and suggests interesting possibility: emergent symmetry.

Let's ask what happens if NLσM has SO(5)-invariant fixed point $\mathcal{L}^*_{SO(5)}$ with only 1 relevant perturbation allowed by microscopic symm.

(probably not this simple in reality)

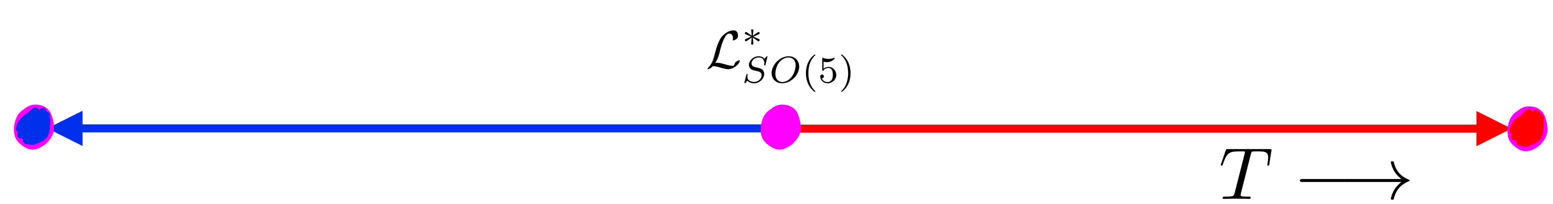
Classify perturbations in SO(5) reps:

$$n_a, \quad X^{(2)}_{ab} = n_a n_b - \frac{1}{5} n^2 \delta_{ab} \qquad X^{(4)}_{abcd} = n_a n_b n_c n_d - (\ldots), \quad \ldots, \quad \text{singlets}, \quad \ldots$$

——— relevant ——— ——— irrelevant! ———

Microscopic SO(3)xO(2) [or O_h x O(2)] allows only 1 relevant perturbation:

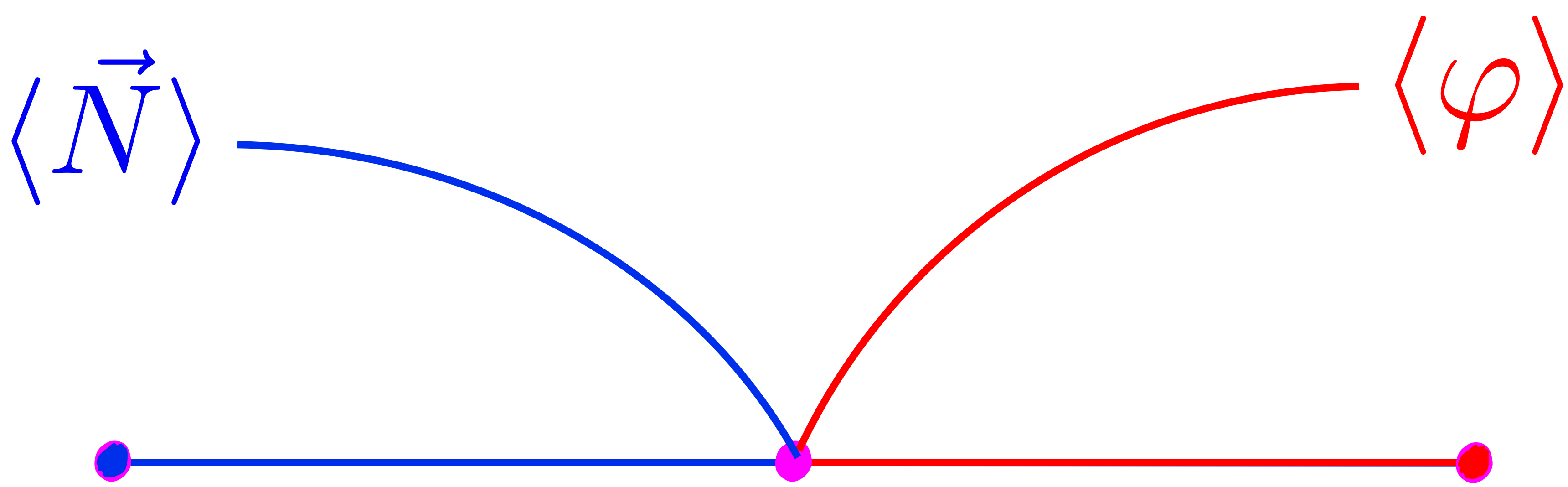
$$\mathcal{L}^*_{SO(5)} + (T - T_c)\left(2\vec{N}^2 - 3\vec{\varphi}^2\right) + \text{irrel}$$



Two phases → the two different orders

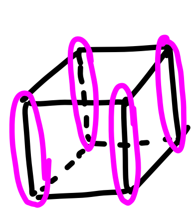
$$\mathcal{L} = \mathcal{L}^{*}_{SO(5)} + (T - T_c)\big(2\vec{N}^2 - 3\vec{\varphi}^2\big)$$

Anisotropy drives phase transition between ordered phases:



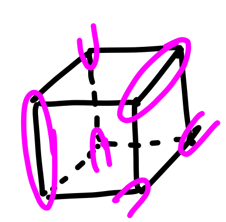
Ordered phase for N

φ massive, integrate out

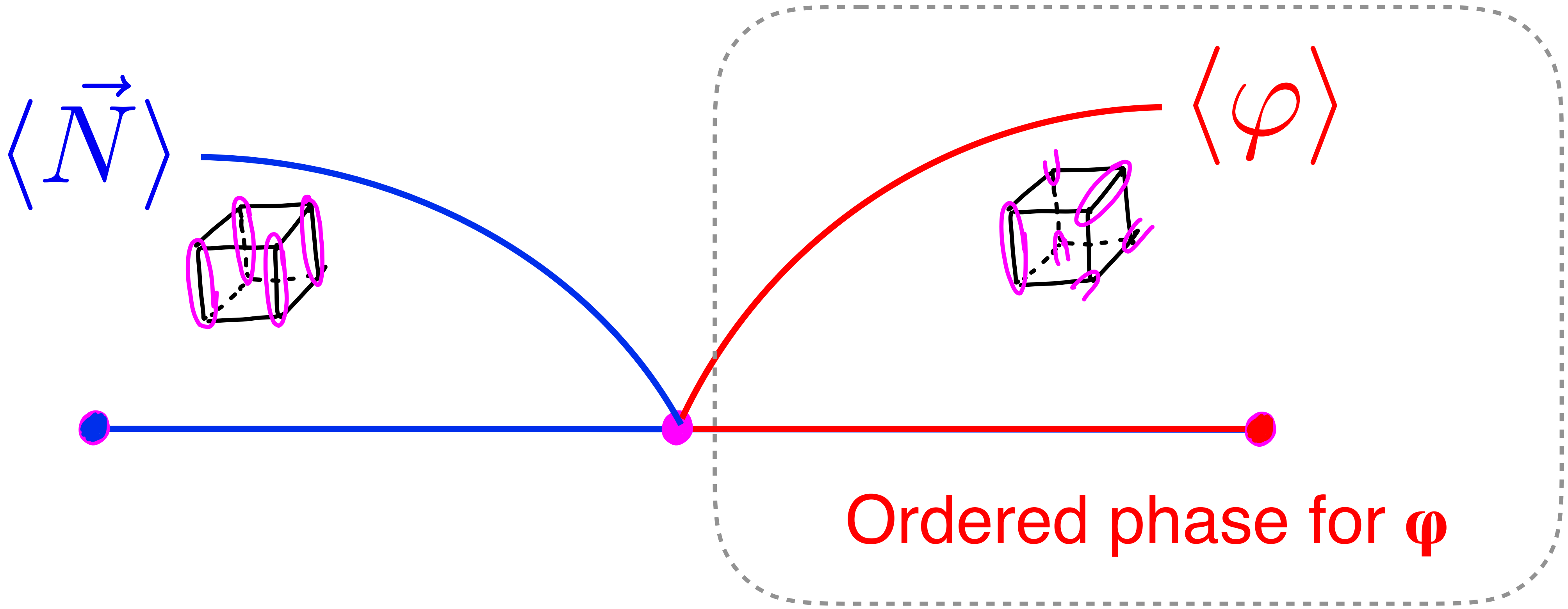


Ordered phase for **φ**

N massive, integrate out



Paramagnetic (dimer liquid) phase

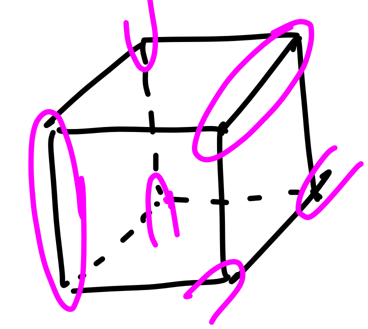


Meaning of LRO in $\boldsymbol{\varphi}$? Hedgehogs are deconfined:

$$\langle \varphi(\mathbf{r})\varphi^*(\mathbf{r}')\rangle = e^{-\beta \Delta F_{\text{hedgehogs}}} \sim \begin{cases} e^{-O(1)} & N\text{-paramagnet (dimer liquid)} \\ e^{-|r-r'|/\xi} & N\text{-ordered phase} \end{cases}$$

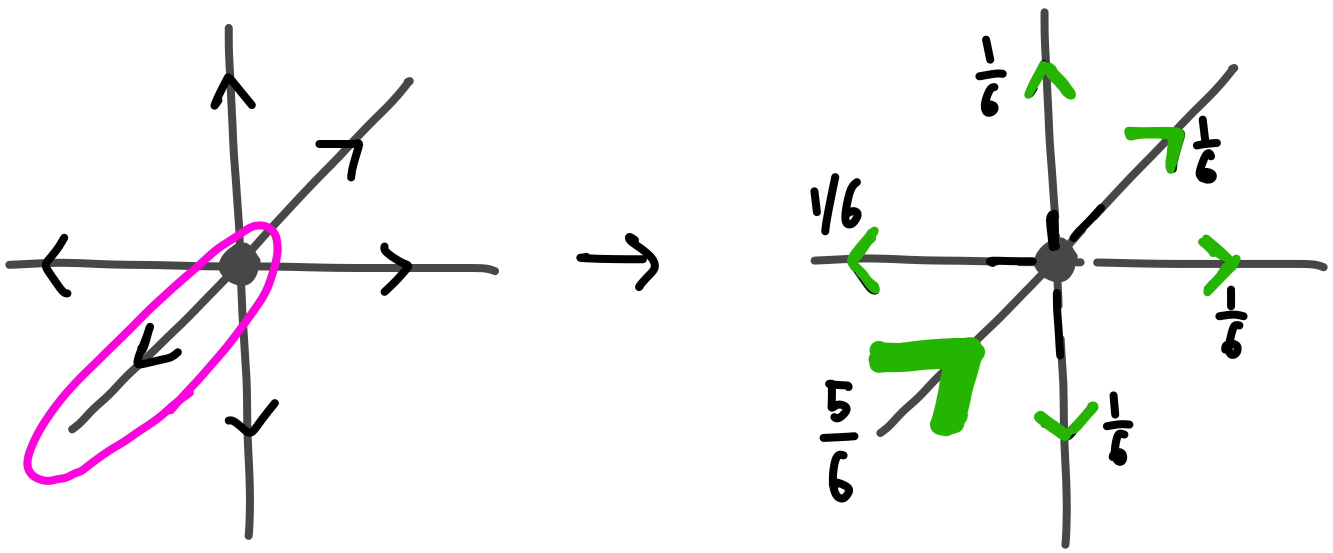
Also power-law correlations due to goldstone mode of $\boldsymbol{\varphi} \sim e^{i\theta}$:

Aside: liquid phase in dimer language



Known that full-packing implies divergence-free flux

cf C. Laumann's lecture on spin ice



J_μ

(use bipartite structure)

This implies power-law correlations in liquid phase (cf. spin ice)

Usually we write $J_\mu \propto \varepsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$ and $\mathcal{L} \propto (\varepsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$

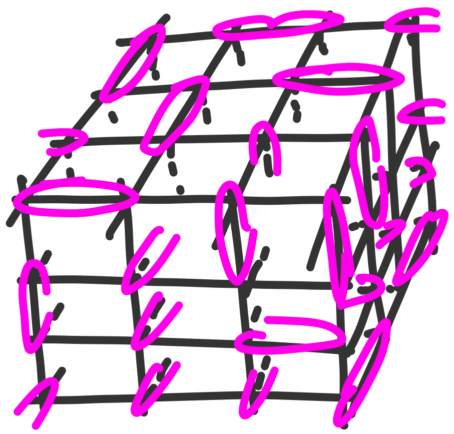
$$\rightarrow \langle J_\mu(r) J_\nu(0)\rangle \sim \frac{3r_\mu r_\nu - \delta_{\mu\nu} r^2}{r^5}$$

This is an equivalent (dual) description: $\varepsilon_{\mu\nu\lambda}\partial_\nu A_\lambda \sim \partial_\mu \theta$

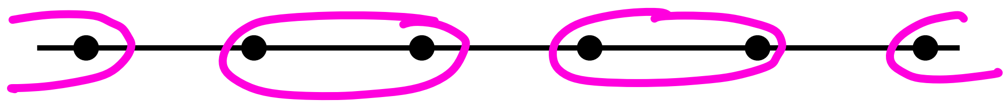
Plan: Part I

O(3) nonlinear sigma model with and without hedgehogs

U(1) symmetry from topological constraint; dimer model



Toy example: 1D dimer model



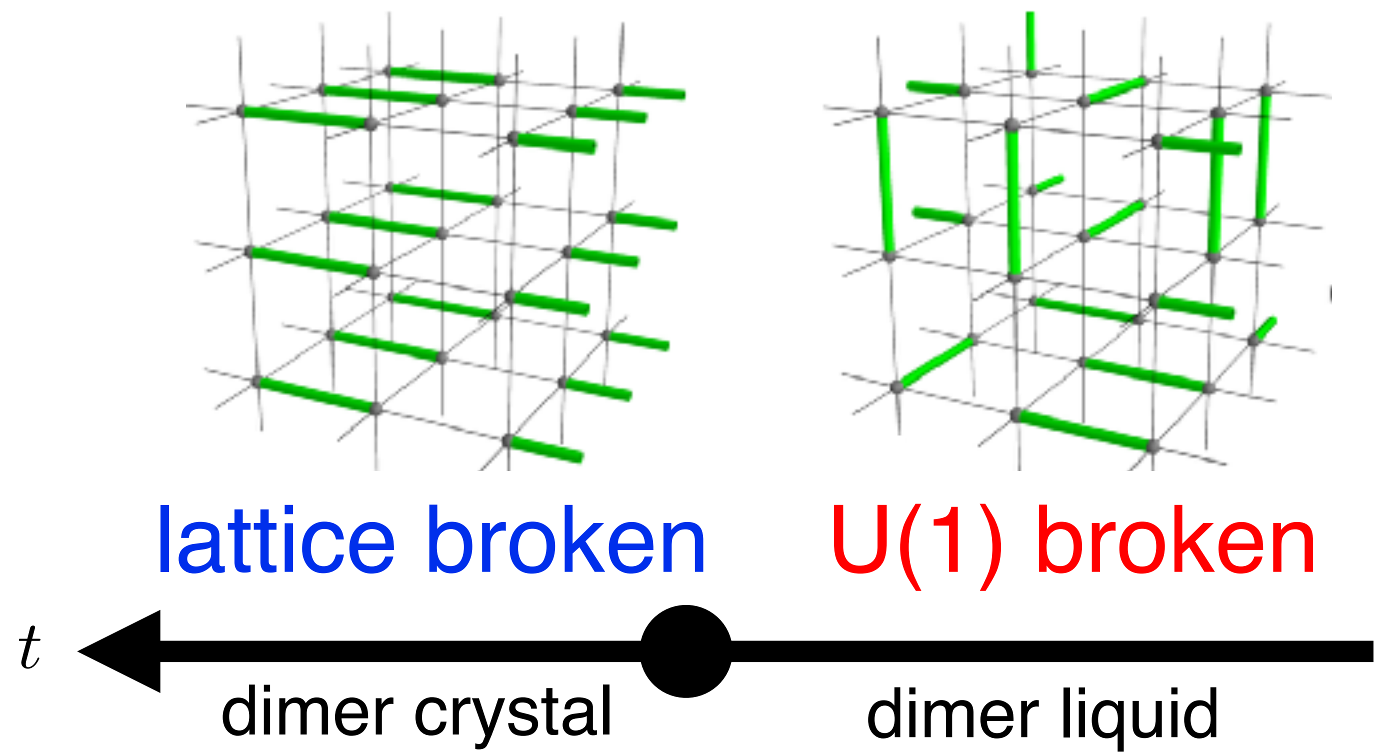
Five-component NLσM for the O(3) model without hedgehogs

Emergent symmetry at T_c

Emergent SO(5)?

What happens at the critical point? Examine using numerics on dimer model   Sreejith, Powell, Nahum 18

$$Z = \sum_{\substack{\text{fully-packed}\\ \text{dimer configs}}} e^{-E/T}$$



Charrier & Alet 10; Powell, Chalker 08, Charrier Alet Pujol 08, Chen Gukelberger Trebst Alet Balents 09

Vary interaction t on squares
Also, fixed interaction on cubes

Simulated efficiently using loop updates.
There is an apparent critical point (may be extremely weakly 1st order).

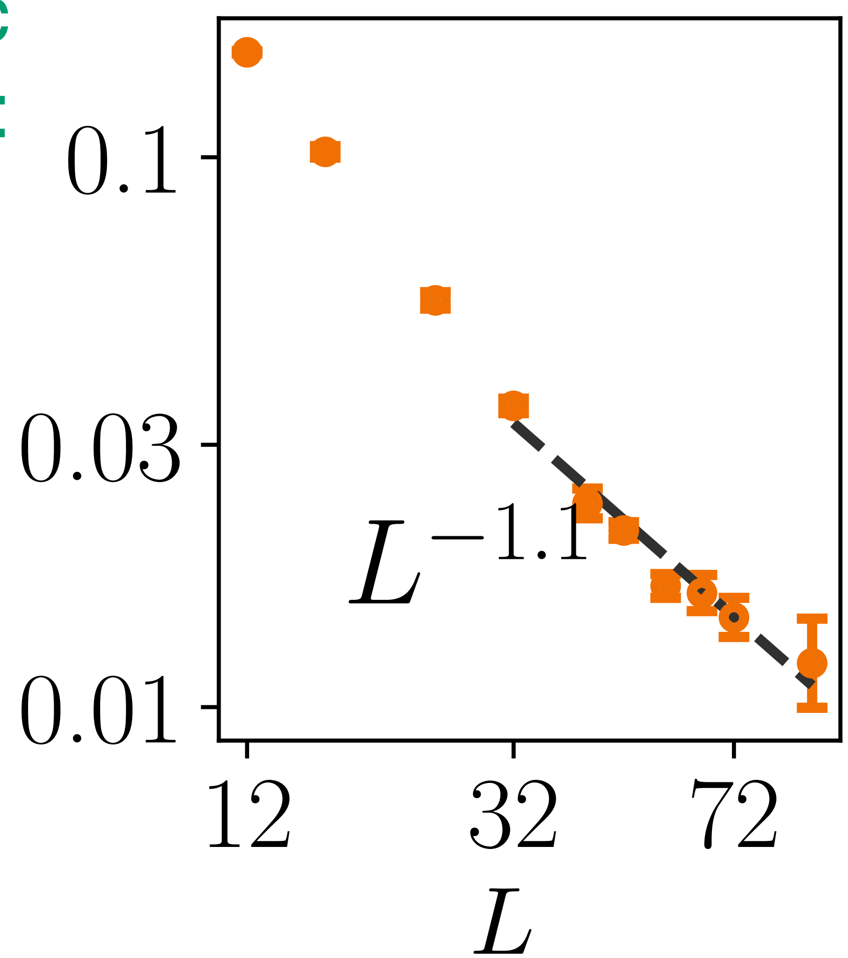
Dimers: valid “regularization” of hedgehog-free O(3)?

I.e. is cubic anisotropy irrelevant at critical point?

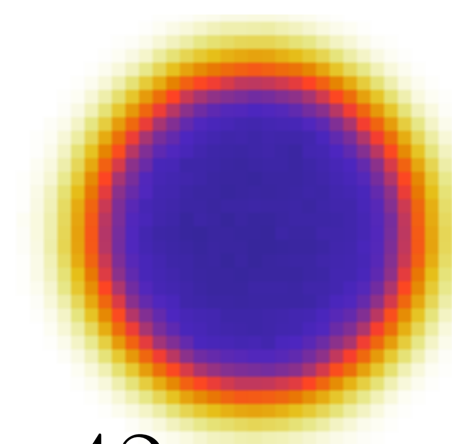
Empirically: yes, cubic anisotropy small & decreasing with L over numerically accessible range

Measure of cubic anisotropy:

$$1 - 6\frac{\langle N_x^2 N_y^2 \rangle}{\langle N_x^4 + N_y^4 \rangle}$$



Slice thru N_z histogram:



$L = 42$

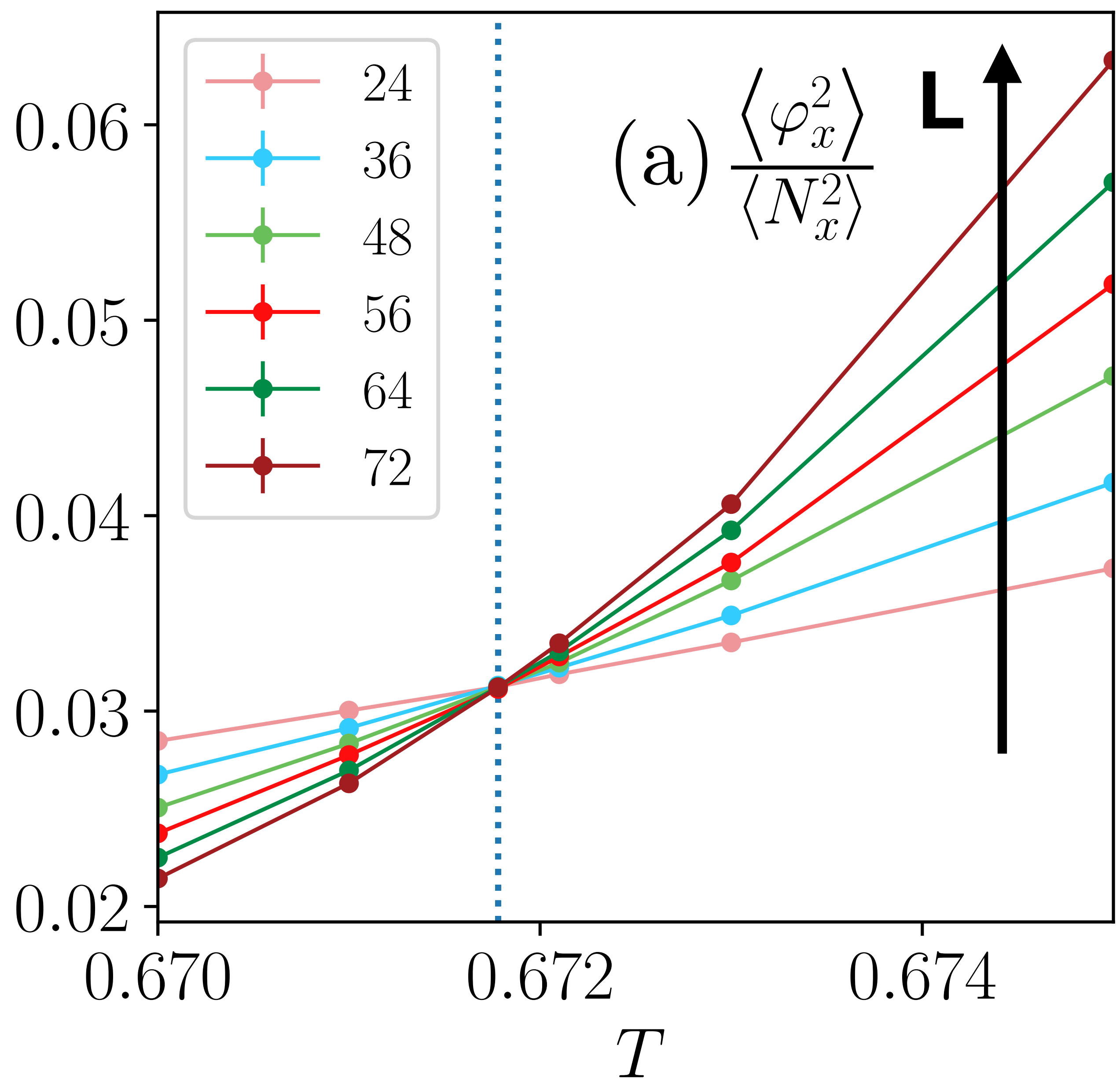
Sreejith, Powell, Nahum 2019

Emergent SO(5) at Tc? Test 1 $(\varphi_x, \varphi_y, N_x, N_y, N_z)$

Check emergent U(1) symmetry for (N_x, φ_x)

First test: $\langle {\varphi_x}^2 \rangle / \langle {N_x}^2 \rangle$ should be independent of size L at T_c

(Compare $\langle {\varphi_x}^2 \rangle / \langle {N_x}^2 \rangle \sim L^{2(\Delta_N - \Delta_\varphi)}$ at non-symmetric CFT)

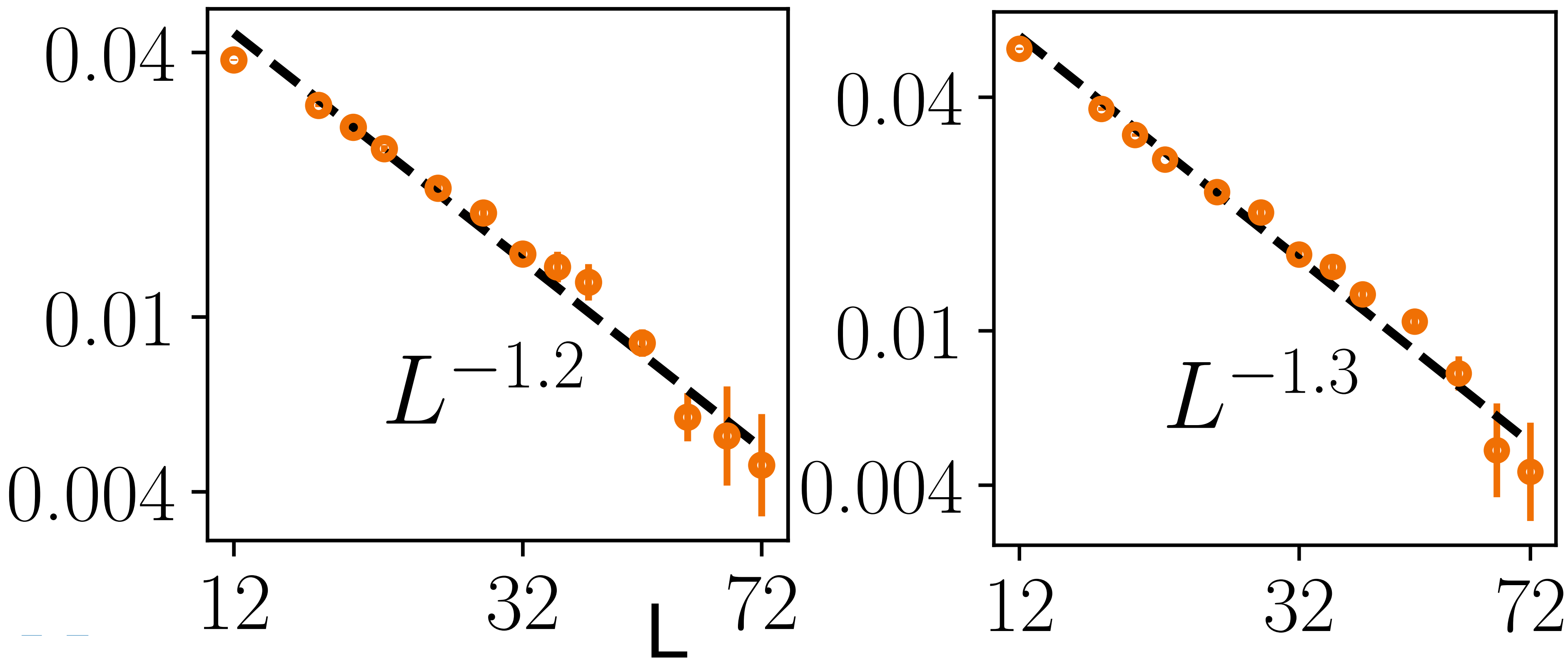


Emergent SO(5) at Tc? Test 2 $(\varphi_x, \varphi_y, N_x, N_y, N_z)$

SO(5) constrains moments to vanish:

$$\frac{\langle \tilde{N}_x{}^4 \rangle}{\langle \tilde{\varphi}_x{}^4 \rangle} - 1 = 0 \qquad \frac{\langle \tilde{N}_x{}^4 \rangle}{3\langle \tilde{N}_x{}^2 \tilde{\varphi}_x{}^2 \rangle} - 1 = 0$$

(fields normalized to have unit variance)



Agreement with SO(5) improves with L over entire range.

Whether exact or approx, looks like exact IR symmetry over accessible range of scales!

Remarks

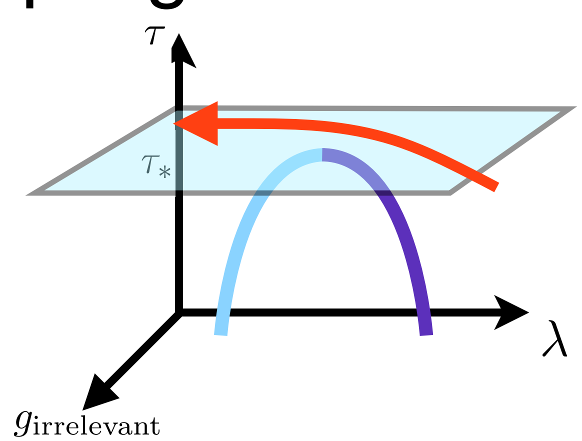
Very precise emergent SO(5) → 5-component sigma model is natural effective field theory for hedgehog-free transition

Similar evidence for SO(5) in 2+1D Neel-VBS (Part 2)

However: these transitions may not truly be continuous (although the relevant lengthscales are extremely large). In that case SO(5) is approximate.

The apparent continuous transition may be due to a 'nearby' SO(5) invariant fixed point that is inaccessible:
e.g. d slightly different from 3, or at slightly complex coupling.

AN, Chalker, Serna, Ortuno, Somoza 15
Wang, AN, Metlitski, Xu, Senthil PRX 17

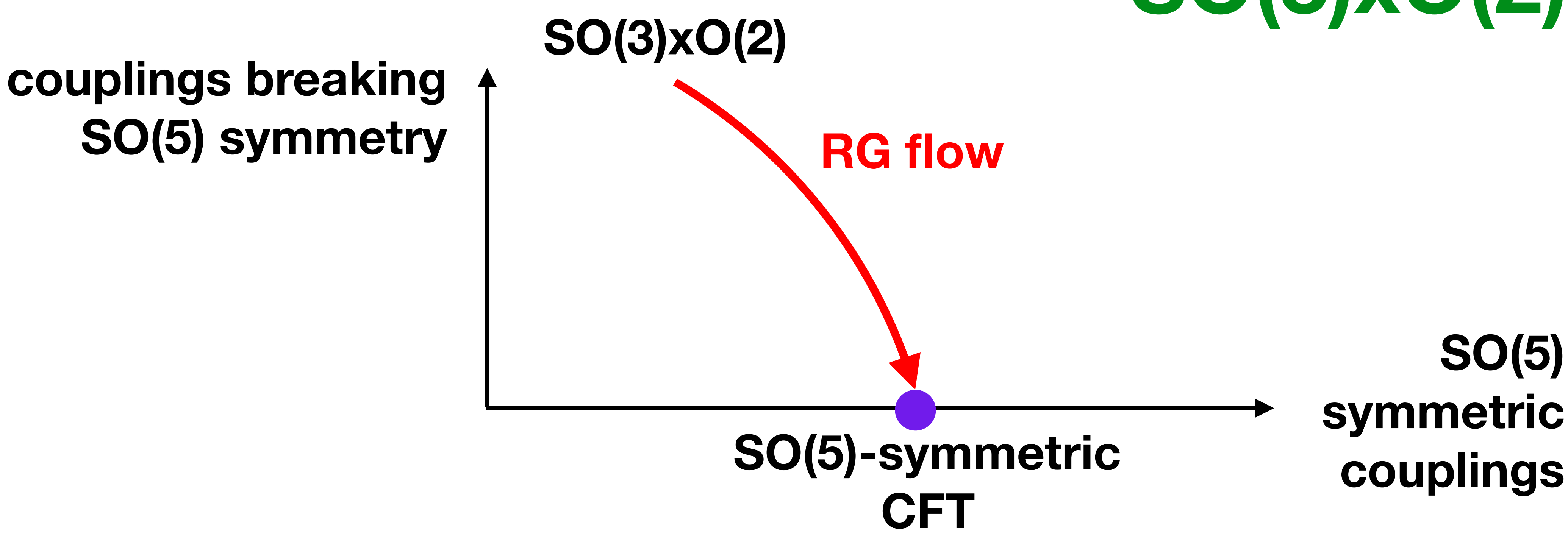


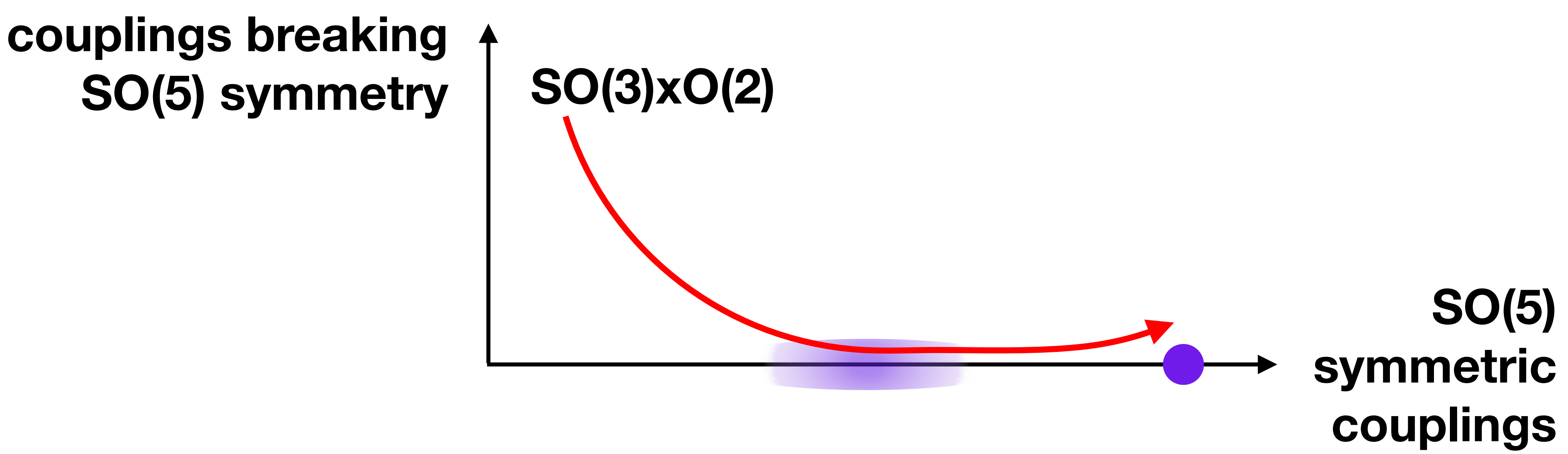
Still under debate

Shao, Guo, Sandvik '16

Symmetry enhancement under RG flow

SO(3)xO(2) ⊂ SO(5)





See Serna, AN ’18 and Wang, AN, Metlitski, Xu, Senthil ’17 for more info

Summary (1)

Adding the topological constraint to the O(3) model led to a new conserved flux, & corresponding U(1) symmetry and U(1) order parameter.

Dimer model gives concrete lattice regularization (with cubic anisotropy).

Aside: the 2+epsilon expansion for the O(3) model is really about the hedgehog-free case, so does not describe the same fixed point as the 4-epsilon expansion for O(3)!

AN, Chalker, Serna, Ortuno, Somoza '15
Cardy Hamber 80

Summary (2)

A useful effective field theory for O(3) without hedgehogs:
SO(5) sigma model, with anisotropies: SO(5)→O(2)xSO(3)

Heuristically, "extra" components ~ integral rep of delta function, killing topological defects.

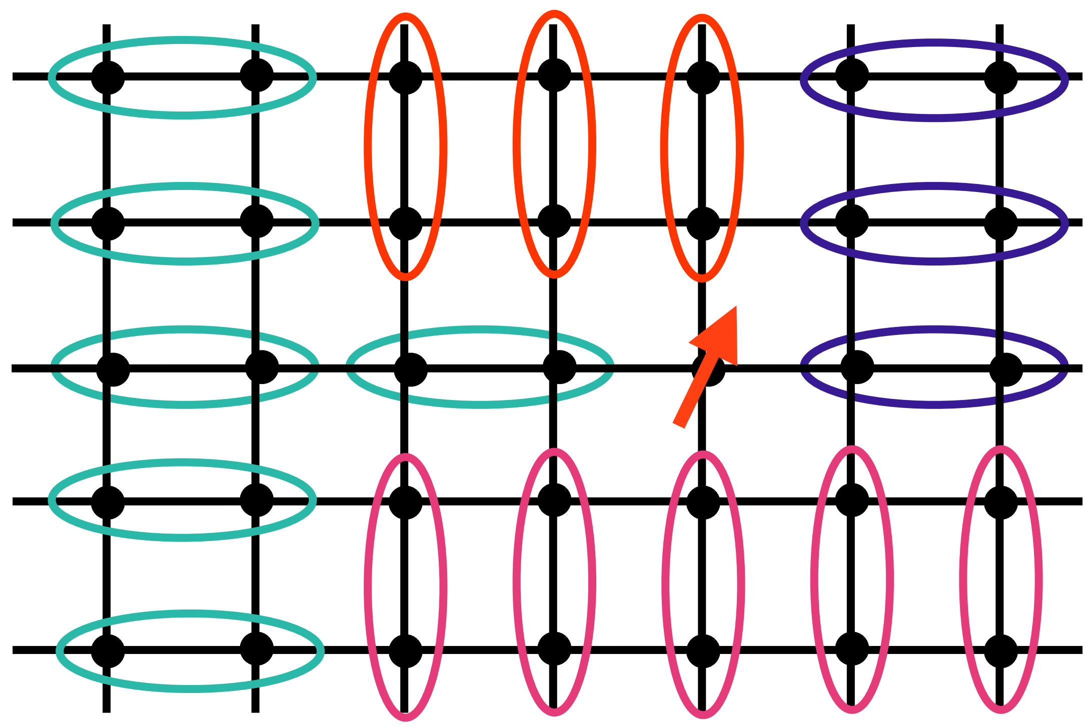
SO(5) breaking terms seem to be (effectively) irrelevant at the phase transition: emergent symmetry unifying two very different operators

However this emergent symmetry may be only approximate (conformal bootstrap, issues with scaling etc.)

This in itself is interesting: how to get "quasiuniversal" behaviour at a (weak) first order transition?

Intro to deconfined criticality & related ideas

Part 2: The Neel-VBS transition



Adam Nahum (Oxford)

Les Houches, 8 Sept 2019

Plan: Lecture 2

The Neel to Valence-Bond-Solid (VBS) transition

Lightning summary: “particle-vortex duality” for 2+1D XY model

Effective NCCP1 field theory via vortices

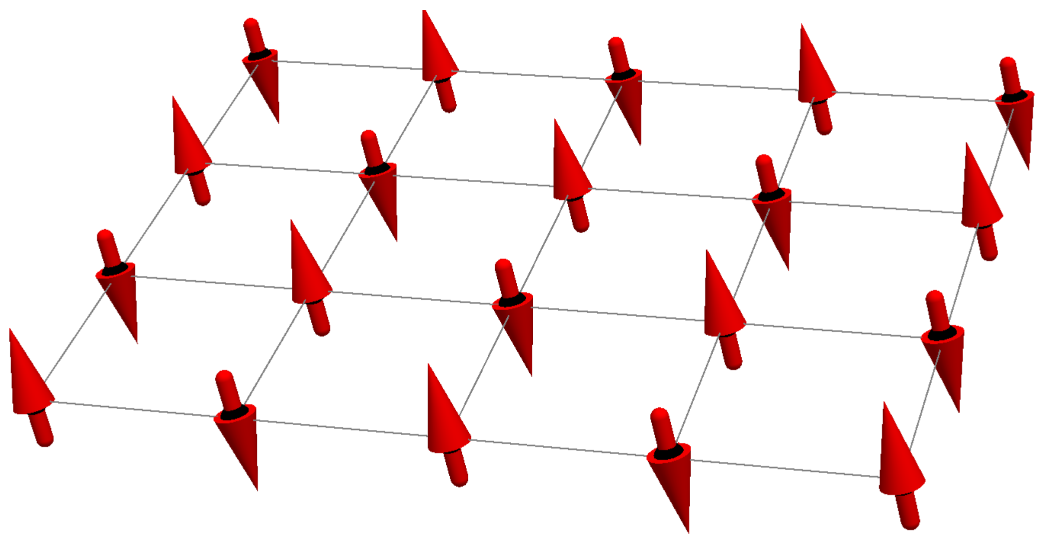
Deconfined criticality RG flows

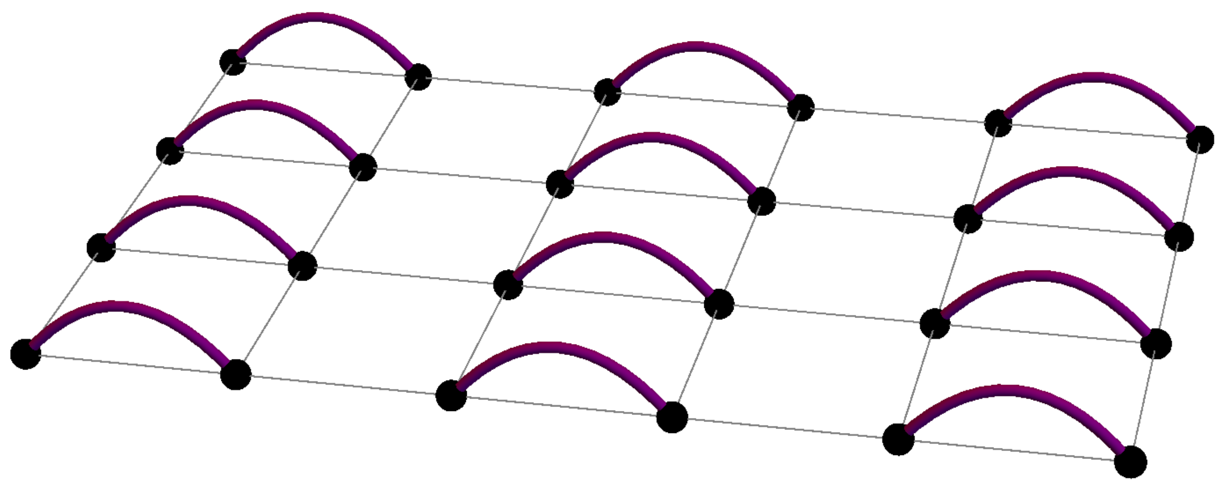
Relation to WZW model

Relation to hedgehog-free O(3)

Spin-1/2s on square lattice: $H = J\sum \vec{S}_i.\vec{S}_j + \ldots$

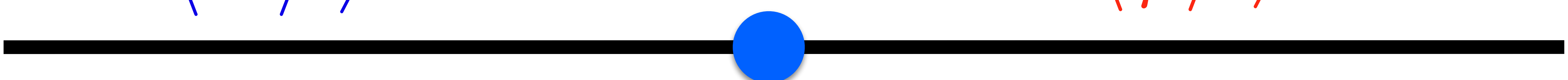
$\vec{N} = (-1)^{x+y}\vec{S}$





$\langle \vec{N} \rangle \neq 0$

$\langle \vec{\varphi} \rangle \neq 0$



Neel order

DCP

VBS order

$\vec{N} = (N_x, N_y, N_z)$

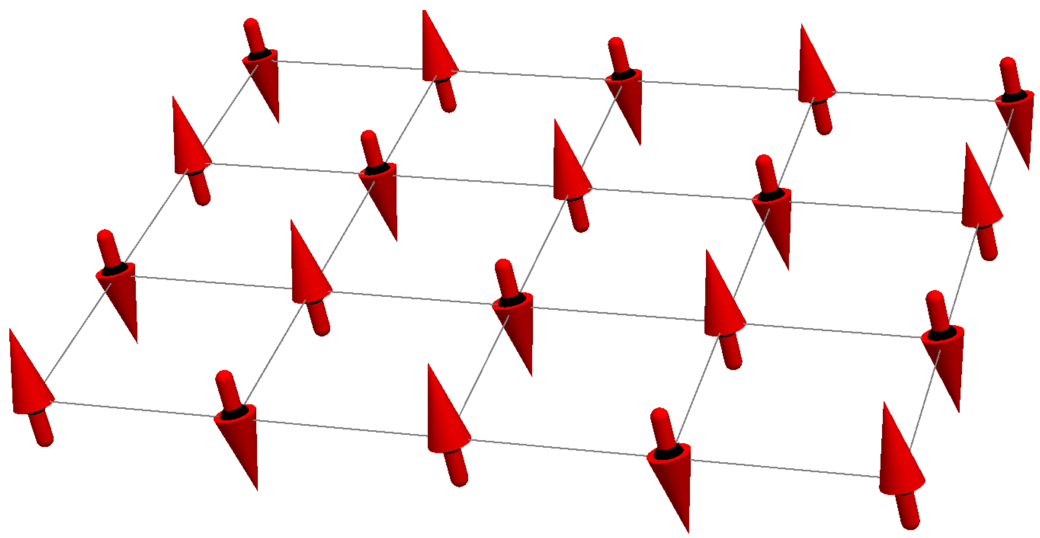
$\vec{\varphi} = (\varphi_x, \varphi_y)$

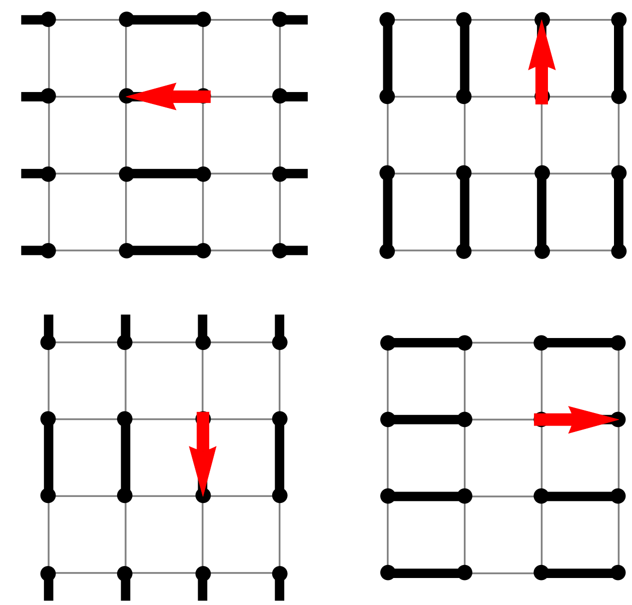
Senthil, Vishwanath, Balents, Sachdev, Fisher ’04

Drive transition with e.g. **sign-free** 4-spin interaction (J-Q model) Sandvik ’07

Spin-1/2s on square lattice: $H = J \sum \vec{S}_i . \vec{S}_j + \ldots$

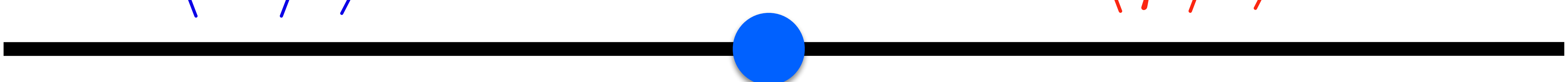
$\vec{N} = (-1)^{x+y} \vec{S}$





"XY-like" order parameter (φ_x, φ_y)

$\langle \vec{N} \rangle \neq 0$ $\langle \vec{\varphi} \rangle \neq 0$



Neel order DCP VBS order

$\vec{N} = (N_x, N_y, N_z)$ $\vec{\varphi} = (\varphi_x, \varphi_y)$

Senthil, Vishwanath, Balents, Sachdev, Fisher '04

Drive transition with e.g. **sign-free** 4-spin interaction (J-Q model) Sandvik '07

Plan: Part 2

The Neel-VBS transition

Lightning summary: “particle-vortex duality” for 2+1D XY model

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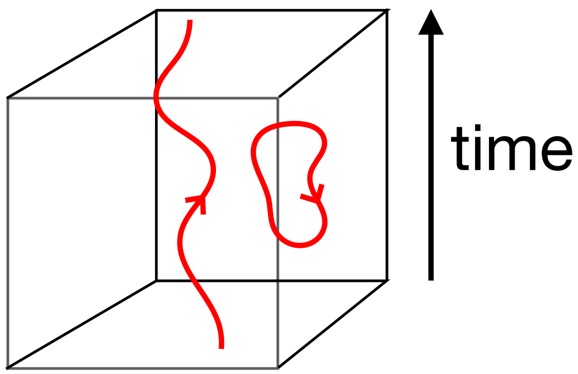
Relation to hedgehog-free O(3)

Duality between XY model and abelian Higgs model (2+1D)

Wilson-Fisher transition for XY 'spin' $\phi = \phi_x + i\phi_y$



vortex in $\boldsymbol{\phi}$ = particle of $\mathbf{w}$ field

cf D. Son's lecture

$$\mathcal{L}_{\mathrm{XY}} = |\nabla\phi|^2 + m^2|\phi|^2 + |\phi|^4$$

$$\mathcal{L}_{\mathrm{aH}} = |(\nabla - ia)w|^2 - m^2|w|^2 + |w|^4 + (\nabla \times a)^2$$

Phase diagram

$m^2 \longrightarrow$

XY order	XY disorder
photon: "$\langle w \rangle = 0$"	Higgs: "$\langle w \rangle \neq 0$"

Conserved U(1) current ($\partial_\mu J_\mu = 0$ except at charged operator insertions)

$$J_\mu \propto i(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) \qquad J_\mu \propto (\nabla \times a)_\mu$$

U(1) order parameter (charged operator)

ϕ (XY field)

$\mathcal{M}_a$ Inserts Dirac monopole (source of quantised flux)

Duality between XY model and abelian Higgs model (2+1D)

Closer look at the XY **ordered** phase $m^2 < 0$

$$\mathcal{L}_{\rm XY} = |\nabla\phi|^2 + m^2|\phi|^2 + |\phi|^4 \longrightarrow \mathcal{L} \propto (\nabla\theta)^2$$

$\varphi \sim e^{i\theta}$
Goldstone mode

Dual language: **w** massive (no vortices!)

$$\mathcal{L}_{\rm aH} = |(\nabla - ia)w|^2 - m^2|w|^2 + |w|^4 + (\nabla\times a)^2 \longrightarrow \mathcal{L} \propto (\nabla\times a)^2$$

'noncompact': no monopoles

What if we add **explicit** symmetry breaking?

$$\mathcal{L} \propto (\nabla\theta)^2 + \lambda_p \cos(p\theta) \qquad U(1) \to \mathbb{Z}_p$$

RG **relevant**. Expand near minimum: mass for Goldstone mode.

Dual language: $\mathcal{L} \propto (\nabla\times a)^2 + \lambda_p\,(\mathcal{M}_a^p + \mathcal{M}_a^{*p})$ $\qquad e^{ip\theta} \sim \phi^p \sim \mathcal{M}_a^p$

Now a 'compact' gauge field with strength-p Dirac monopoles.
Duality implies monopoles are relevant and lead to a **massive** theory (in fact a confining theory).

Polyakov

Plan: Part 2

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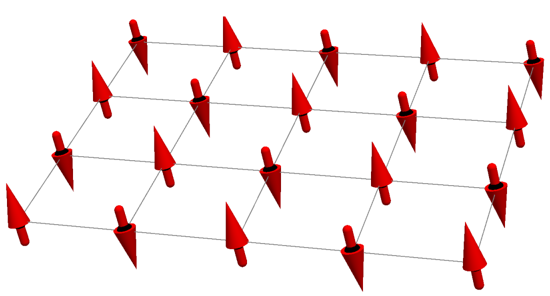
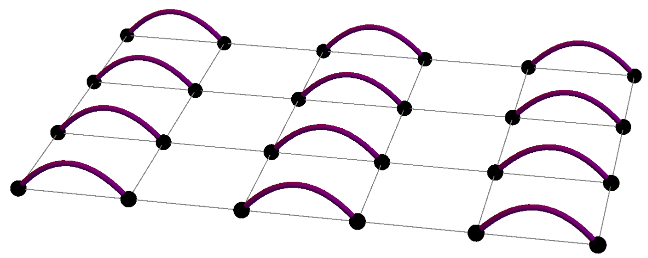




Relation to WZW model

Relation to hedgehog-free O(3)

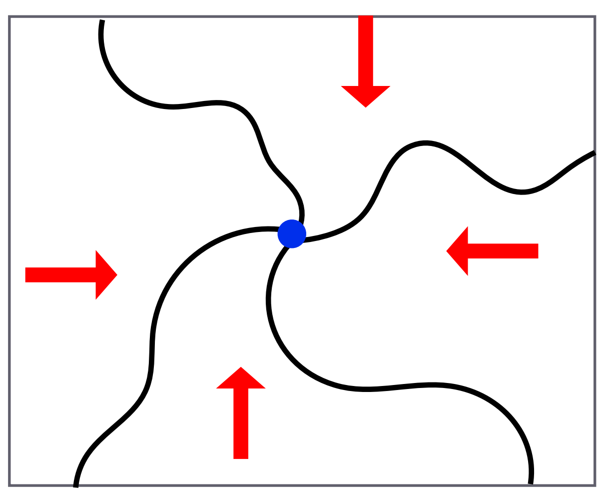
Neel-VBS: Effective field theory via vortices

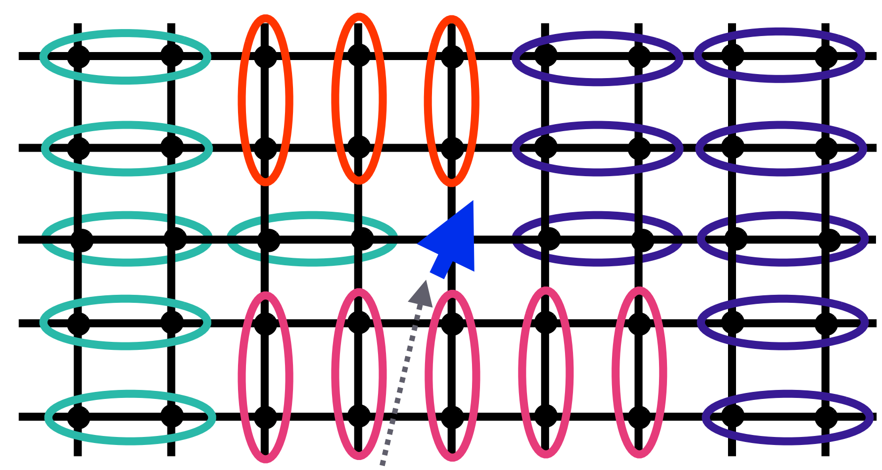
Levin & Senthil 04

First consider a standard Landau theory for an "XY-like" order parameter $\varphi=\varphi_x+i\varphi_y$ with 4-fold anisotropy

$$\mathcal{L}_{\rm LG} = |\nabla\phi|^2 + m^2|\phi|^2 + |\phi|^4 + \lambda_4(\phi^4 + \phi^{*4})$$

Not right! Fails to capture quantum #s of vortices in $\vec{\phi}$:

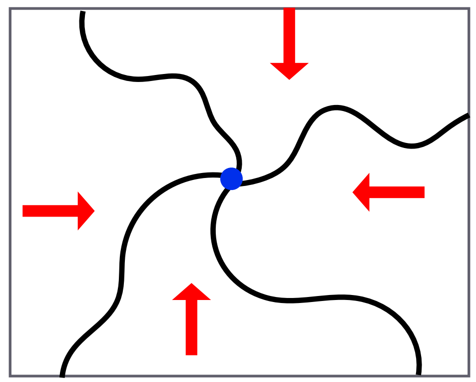


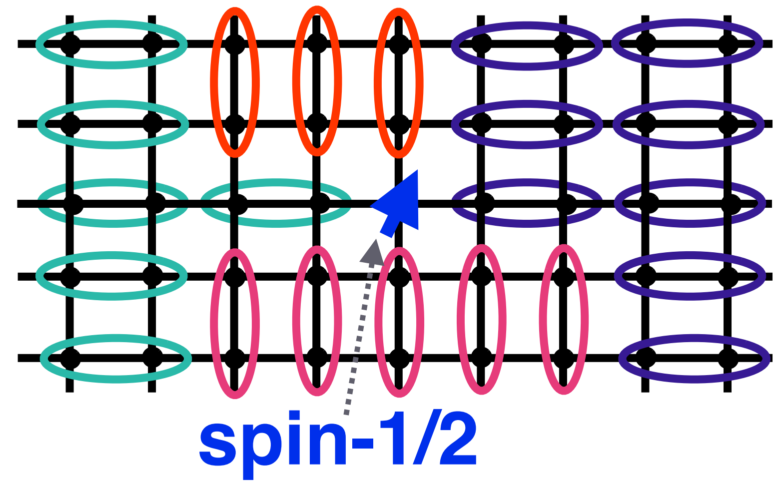


In the Landau theory there is no simple way to correct quantum #s of vortex. But in the dual theory this is easily done…

Neel-VBS: Effective field theory via vortices

Levin & Senthil 04





$$\mathcal{L}_{\mathrm{LG}} = |\nabla\phi|^2 + m^2|\phi|^2 + |\phi|^4 + \lambda_4(\phi^4 + \phi^{*4})$$

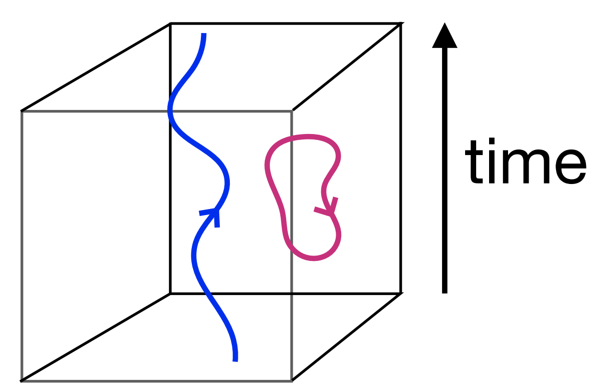
duality →

$$\mathcal{L}_{\mathrm{aH}} = |(\nabla - ia)w| - m^2|w|^2 + |w|^4 + (\nabla \times a)^2 + \lambda_4\left(\mathcal{M}_a^4 + \mathcal{M}_a^{*4}\right)$$

Now upgrade the "vortex field" to a SU(2)$_{\text{spin}}$ spinor!

$$\mathcal{L}_{\mathrm{NCCP}^1} = |(\nabla - ia)\mathbf{z}| - m^2|\mathbf{z}|^2 + |\mathbf{z}|^4 + (\nabla \times a)^2 + \lambda_4\left(\mathcal{M}_a^4 + \mathcal{M}_a^{*4}\right)$$

$$\mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$



Neel-VBS: Effective field theory

Senthil et al 04

$$\mathcal{L}_{\mathrm{NCCP}^1} = |(\nabla - ia)\mathbf{z}| - m^2|\mathbf{z}|^2 + |\mathbf{z}|^4 + (\nabla \times a)^2$$

Neel order parameter

$$\vec{N} = \mathbf{z}^\dagger \vec{\sigma} \mathbf{z}$$

VBS order parameter

$$\varphi_x + i\varphi_y = \mathcal{M}_a$$

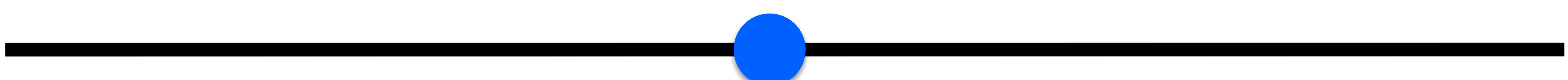
First pass at phase diagram
(building on XY duality and neglecting λ_4):

Higgs phase:

Condense **z**

Photon phase?

z massive



Neel order

$$\langle \vec{N} \rangle \neq 0$$

Critical point?

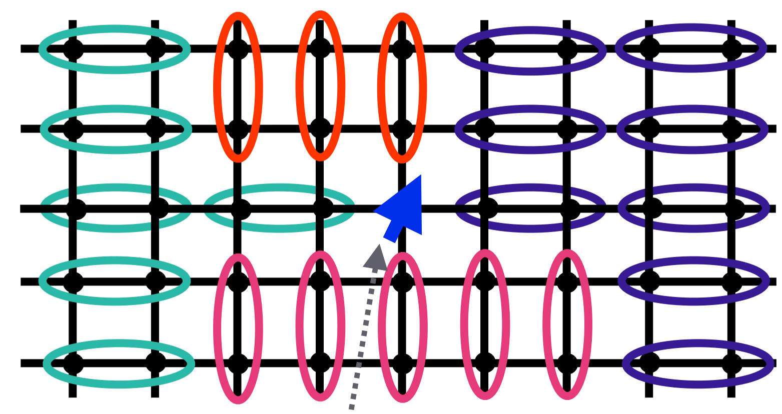
VBS order

$$\langle \vec{\varphi} \rangle \neq 0$$

Neel-VBS: SU(n→∞) gives a solvable limit

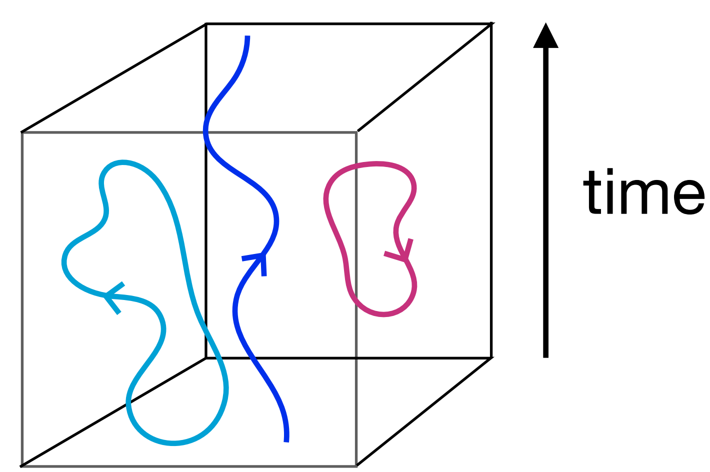
We may consider the same phase transition for SU(n) spins

Place SU(n) fundamental (antifundamental) on A (B) sublattice.



$$\mathcal{L}_{\mathrm{NCCP}^{n-1}} = |(\nabla - ia)\mathbf{z}| - m^2|\mathbf{z}|^2 + |\mathbf{z}|^4 + (\nabla \times a)^2 + \lambda_4 \left(\mathcal{M}_a^4 + \mathcal{M}_a^{*4}\right)$$

$$\mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \\ \ldots \\ z_n \end{pmatrix}$$



Large n: theory with $\lambda_4=0$ solvable by saddle point/diagrams.

Cts phase transition, where λ_4 is strongly RG irrelevant ($x_4 \propto n$).

Plan: Part 2

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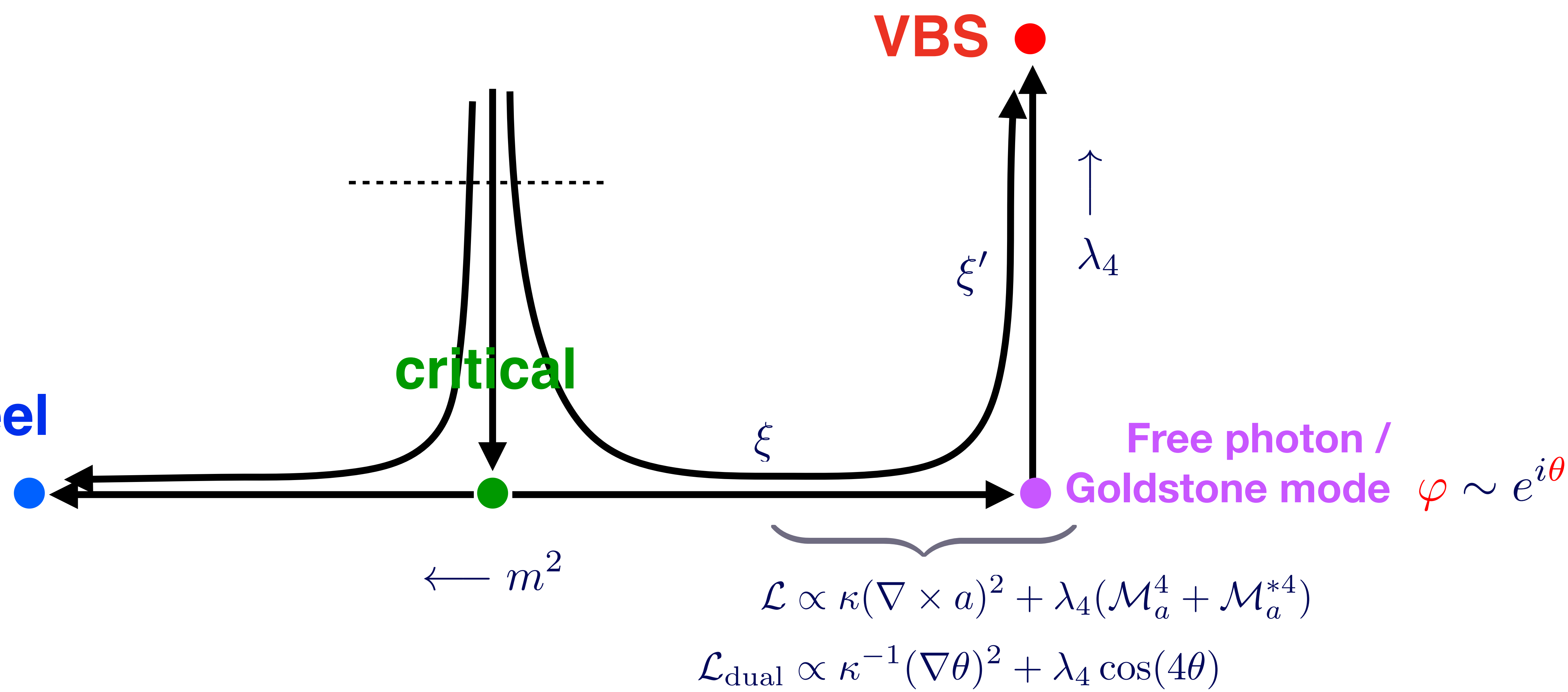
Deconfined criticality RG flows

Senthil et al 04

$$\mathcal{L}_{\mathrm{NCCP}^{n-1}} = |(\nabla - ia)\mathbf{z}| - m^2|\mathbf{z}|^2 + |\mathbf{z}|^4 + (\nabla \times a)^2 + \lambda_4 \left(\mathcal{M}_a^4 + \mathcal{M}_a^{*4}\right)$$

Critical point: emergent **noncompact** gauge field: a critical spin liquid.



Near-critical VBS phase: has **regime** $\xi < L < \xi'$ with massive **deconfined** spinons (**z**).

L> ξ': spinons confined (Polyakov mechanism).

Deconfined criticality RG flows

Senthil et al 04

Higgs phase:
Condense **z**

Confined phase
z massive

Neel order
$\langle \vec{N} \rangle \neq 0$

DCP

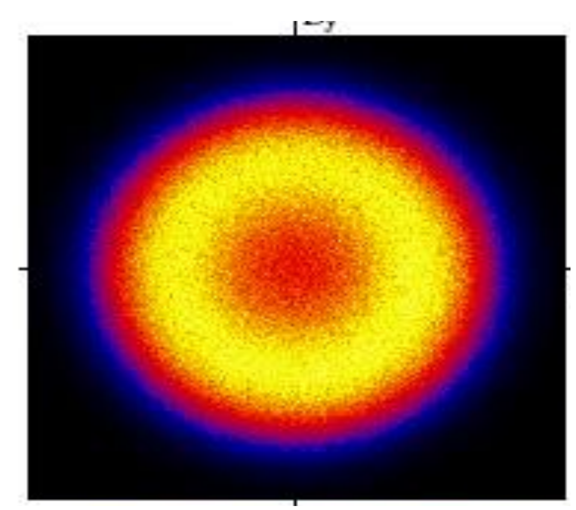
VBS order
$\langle \vec{\varphi} \rangle \neq 0$

* at **n=2,** DCP may in fact be very weakly 1st order transition with very large but finite correlation length, and 'quasiuniversal' behaviour

AN, Chalker, Serna, Ortuno, Somoza 15,
Wang, AN, Metlitski, Xu, Senthil 17

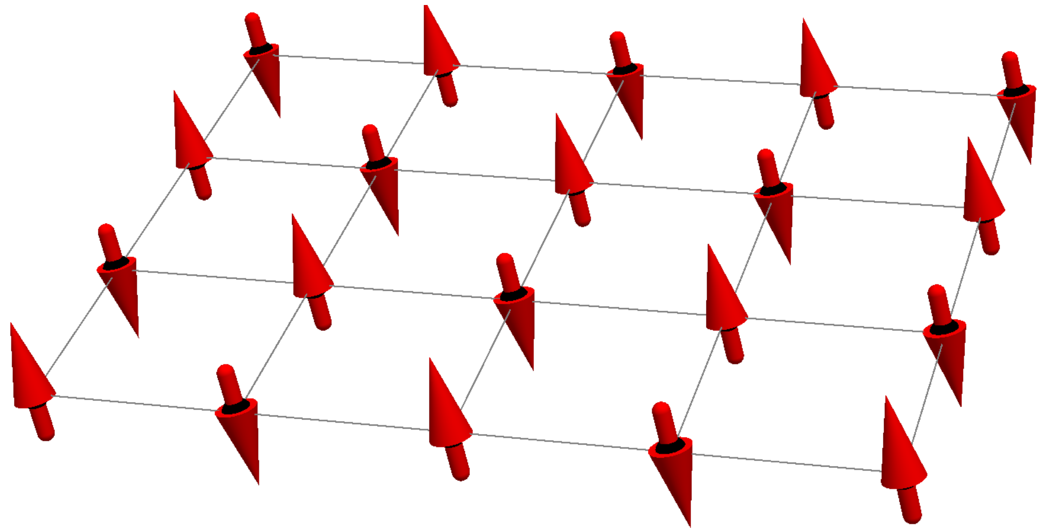
Alternative scenario see:
Shao, Guo, Sandvik '16

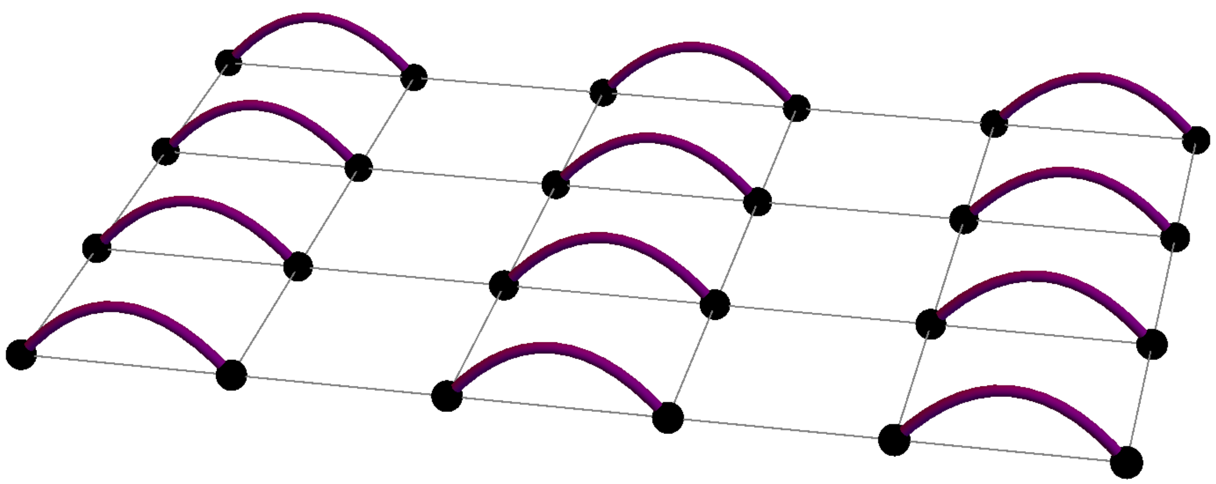
In particular, the deconfined regime is clearly seen as emergent U(1) symmetry in the distribution of (φ_x,φ_y):



Sandvik 07

Intermediate summary



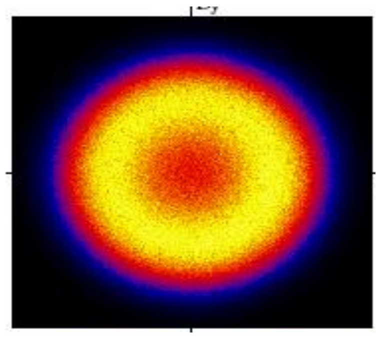


Higgs transitions in simple U(1) gauge theories can describe direct continuous transitions between distinct ordered phases.

This is not possible in Landau-Ginsburg (without fine tuning).

$$\mathcal{L}_{\mathrm{NCCP}^{n-1}} = |(\nabla - ia)\mathbf{z}| - m^2|\mathbf{z}|^2 + |\mathbf{z}|^4 + (\nabla \times a)^2 + \lambda_4 \left(\mathcal{M}_a^4 + \mathcal{M}_a^{*4}\right)$$

Irrelevance of monopoles at critical point →
Emergent noncompactness of gauge field.
Equivalent to emergent U(1) symmetry for VBS.

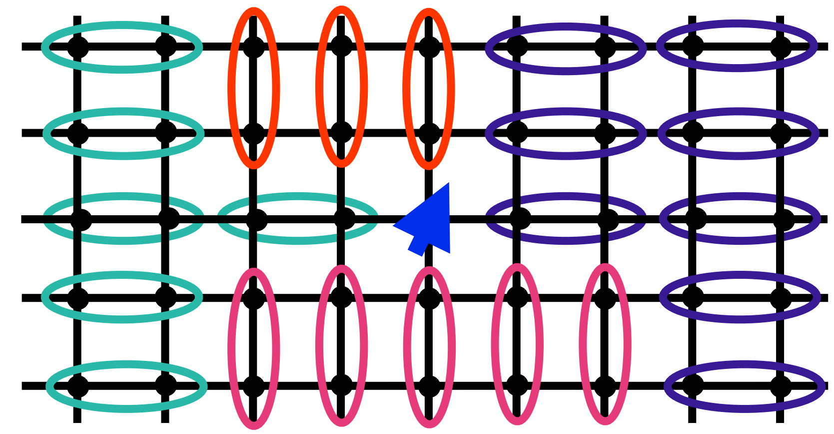


Aside: Vortices and LSM

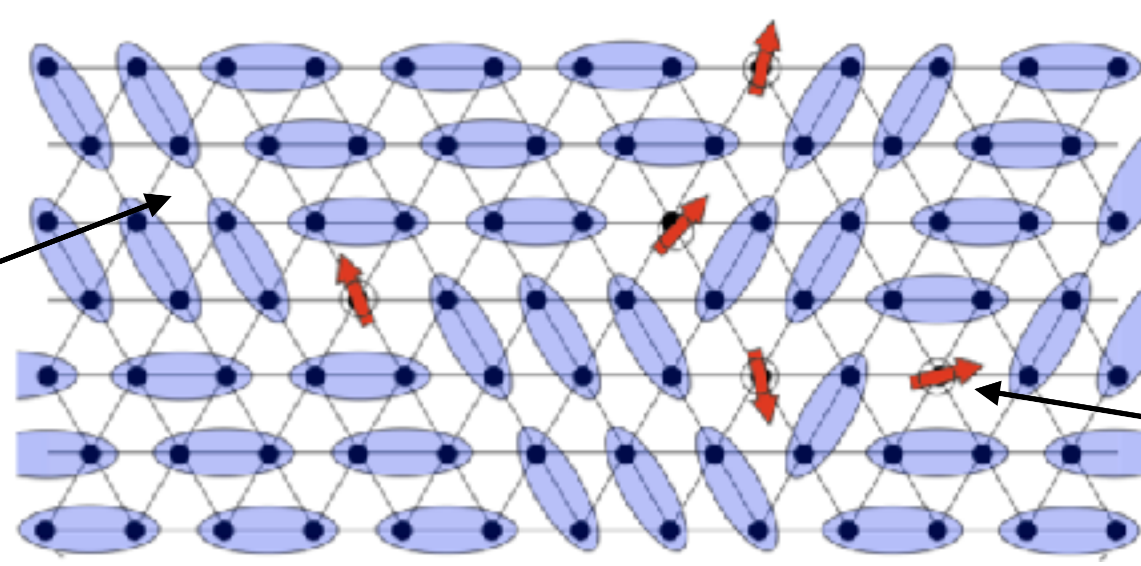
2+1D Lieb-Schultz-Mattis thm: Spin-1/2 per unit cell
⇒ no trivial paramagnet that preserves all symm

Hastings 03
cf S. Parameswaran's lecture

Spin-1/2 VBS vortex 'enforces' LSM:
makes sure we can't get a trivial phase by disordering the VBS.



Spin-1/2 VBS vortex also prevents a trivial phase when we destroy VBS long range order by pinning with quenched bond randomness



Kimchi, AN, Senthil 18
Liu, Shao, Lin, Guo, Sandvik 18

Plan: Part 2

The Neel-VBS transition

Lightning summary: “particle-vortex duality” for 2+1D XY model

Effective NCCP1 field theory via vortices

Deconfined criticality RG flows

Relation to WZW model

Relation to hedgehog-free O(3)

An alternative effective field theory for Neel-VBS

From now on specialize to the original n=2 case.

$$\mathcal{L}_{\mathrm{NCCP}^1} = |(\nabla - ia)\mathbf{z}| - m^2|\mathbf{z}|^2 + |\mathbf{z}|^4 + (\nabla \times a)^2$$

(Numerics support emergent U(1) near the transition, so I have neglected monopoles.)

There is an alternative effective field theory for this transition, which does not use 'partons'.

It is the **5-component sigma model** we met in Part 1.

$$\int \mathcal{D}\vec{n} \, \exp\left[-\frac{1}{2g}\int d^3x (\nabla\vec{n})^2 + \mathrm{anisotropies} + S_{\mathrm{WZW}}\right]$$

$$\vec{n} = (N_x, N_y, N_z, \varphi_x, \varphi_y)$$

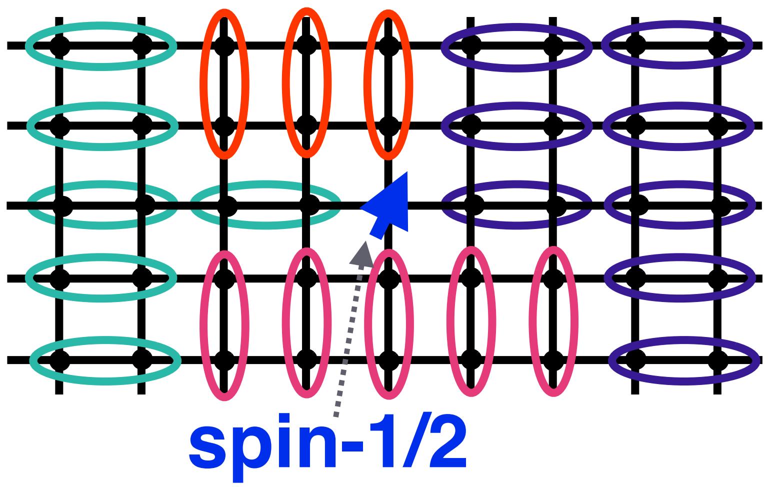
Sigma model from vortex considerations

One derivation of sigma model: introduce fermionic partons and integrate out. Tanaka Hu 05, Senthil Fisher 06, Abanov Weigmann

Instead, let's use similar logic to above.
Start with effective theory for all order params w / o topo term:

$$\int \mathcal{D}\vec{n} \exp\left[-\frac{1}{2g}\int d^3x(\nabla\vec{n})^2 + \text{anisotropies}\right] \qquad \vec{n} = (N_x, N_y, N_z, \varphi_x, \varphi_y)$$

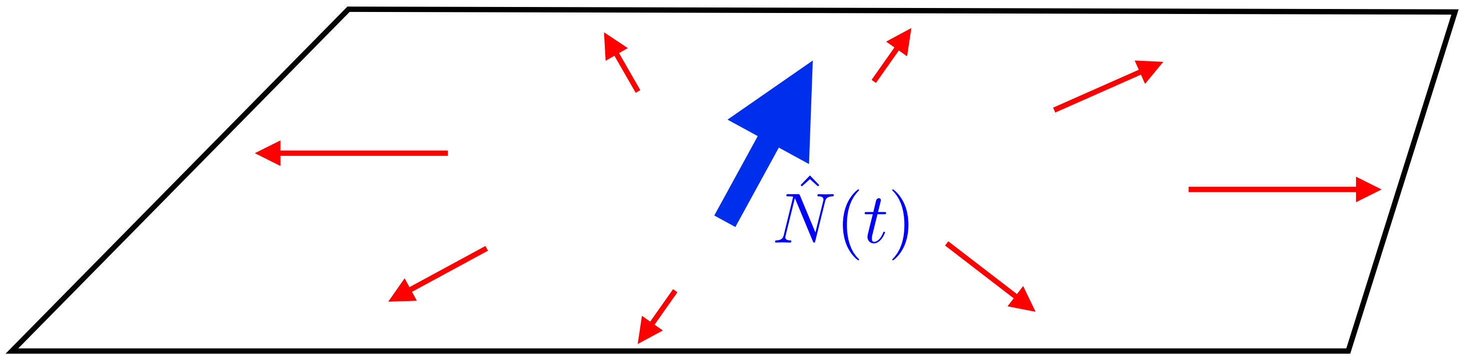
Problem: in this theory **φ** vortex is featureless.
Should carry spin-1/2!



Claim (exercise): **the WZW term solves this problem**

Heuristic picture for WZW term 2: vortices

$$\int \mathcal{D}\vec{n}\, \exp\left[-\frac{1}{2g}\int d^3x(\nabla\vec{n})^2 + \frac{2\pi i}{\mathrm{area}(S^4)}\int_0^1 du \int d^3x\, \epsilon^{abcde} n^a \partial_x n^b \partial_y n^c \partial_z n^d \partial_u n^e\right]$$



Consider a **static vortex configuration** of **φ =** (n1, n2).
Must have nonzero *N(t)* in core, since n² = 1:

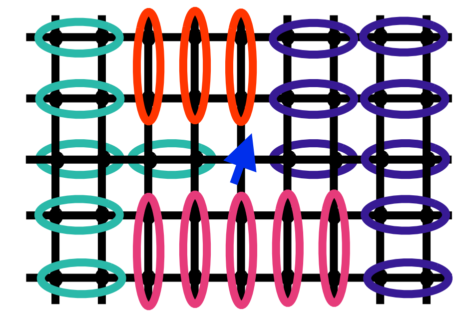
radial coord in plane — polar coord in plane — time

$$\vec{n} = \Big(\sin\chi(r)(\cos\psi, \sin\psi), \cos\chi(r)\hat{N}(t)\Big)$$

$\sin\chi=1$ at infinity,
$\cos\chi=1$ at origin

Exercise: show that the path integral for unit vector $\hat{N}(t)$ reduces to the 0+1D path integral for a spin 1/2.

Sigma model from vortex considerations



$$\int \mathcal{D}\vec{n} \exp\left[-\frac{1}{2g}\int d^3x (\nabla \vec{n})^2 + \text{anisotropies} - S_{\text{WZW}}\right]$$

$$\vec{n} = (N_x, N_y, N_z, \varphi_x, \varphi_y)$$

Topo term corrects spin of the **φ** vortex.

Anisotropies play a similar role to case of the dimer model.
(The higher order terms are different because of different microscopic symmetries.)

$$\mathcal{L}^*_{SO(5)} + (K - K_c)\left(2\vec{N}^2 - 3\vec{\varphi}^2\right) + \ldots$$

Same effective theory as hedgehog-free O(3) [near critical point]

Senthil et al 04

Again very accurate emergent SO(5) at the critical point.

The emergent U(1) [=deconfinement] is a subgroup of SO(5).

AN, Chalker, Serna, Somoza, Ortuno 15
Suwa, Sen, Sandvik 16

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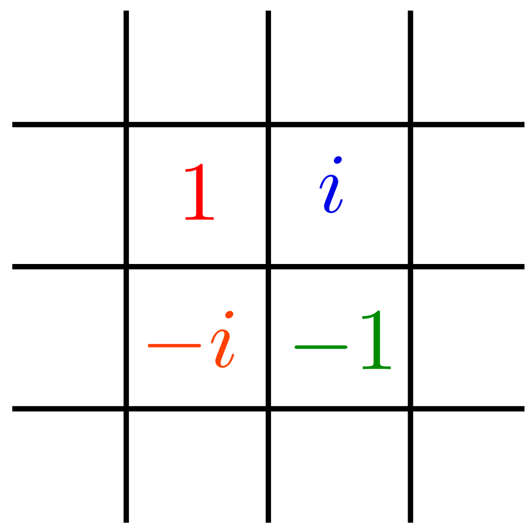
Relation to hedgehog-free O(3)

Previous heuristic picture for WZW term: kills hedgehogs in N.
So are hedgehogs absent here too? Yes (at critical point).

Microscopic calculation (Haldane 88) shows that hedgehog (in **spacetime**) gives imaginary contribution to action.
Phase depends on **spatial** location on square lattice:

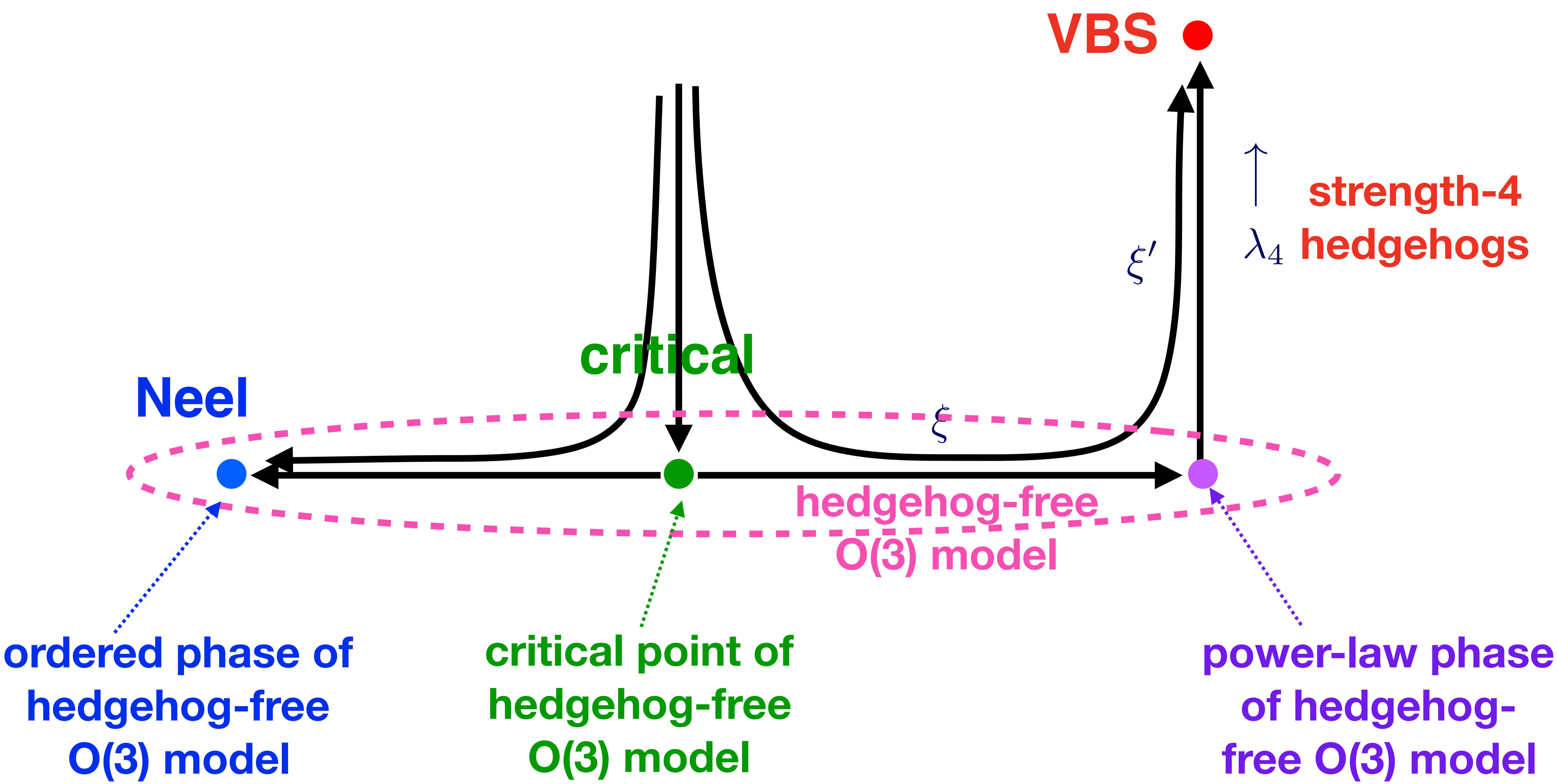
$$e^{-S}$$



Isolated hedgehogs suppressed by phase cancellation!

In fact hedgehog = monopole in the gauge theory:
λ_4 is a fugacity for strength-4 hedgehogs.

Relation to hedgehog-free O(3)



Summary

Deconfined critical points: playground for many mechanisms in critical phenomena, with simple lattice realisations

topological terms

emergent gauge fields

emergent symmetries

topological defects

non-Wilson-Fisher fixed points

anomalies

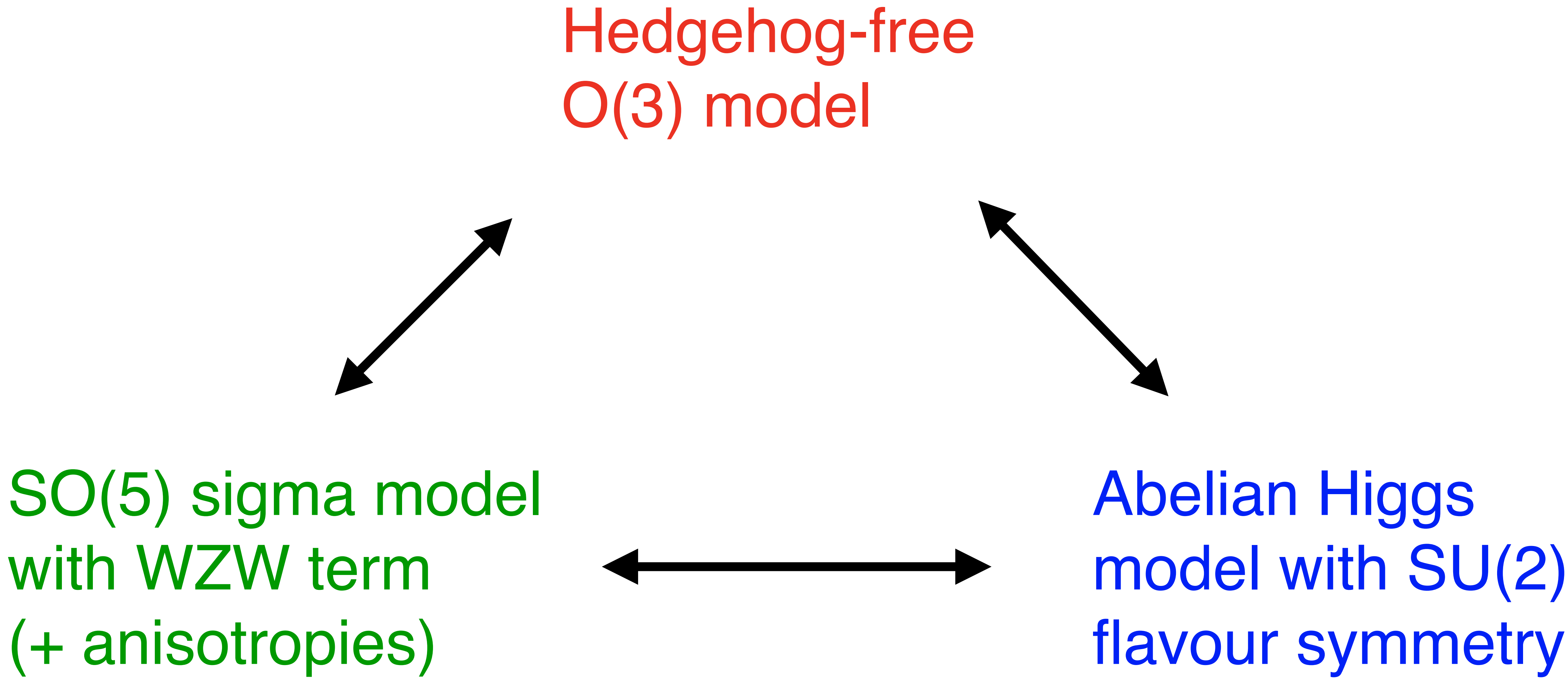
field theory dualities

quasiuniversality

no time today - see Wang, AN, Metlitski, Xu, Senthil 17

Many insights for more complex systems (e.g. other order parameters, other symmetries, with fermions, etc.)

Summary


Hedgehog-free
O(3) model
SO(5) sigma model
with WZW term
(+ anisotropies)
Abelian Higgs
model with SU(2)
flavour symmetry

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Some extensions Deconfined criticality with 4 components

Deconfined criticality with 4 cpts

We had 5 components going ‘soft’ at the transition:

$$\vec{n} = (N_x, N_y, N_z, \varphi_x, \varphi_y)$$

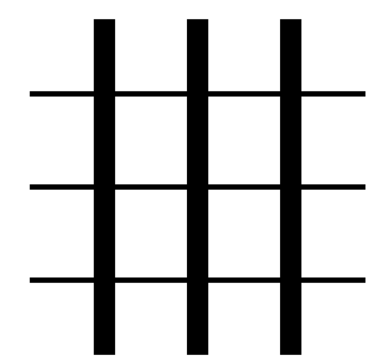
If we reduce symmetry, we can gap out a component.

Easy-plane: reduce spin symmetry SO(3)→O(2) $\delta H \sim N_z^2$

$$\vec{n} = (N_x, N_y, \varphi_x, \varphi_y)$$

Qin, He, You, Lu, Sen, Sandvik, Xu, Meng 17
Motrunich Vishwanath 04
…

Favour one cpt of VBS, e.g. rectangular lattice:



$$\vec{n} = (N_x, N_y, N_z, \varphi_y)$$

Sato, Hohenadler Assaad 17, Metlitski Thorngren 18,
Zhao Weinberg Sandvik 18, Serna, AN 18,
Wang Kivelson Lee 15, Komargodski et al 18, …

Many other models with the symmetry of one of these

Deconfined criticality with 4 cpts

The 4-cpt case has similar descriptions to the 5-cpt case:

Abelian Higgs model

4-cpt sigma model with theta term

Interesting transition with emergent (although probably approximate) O(4)

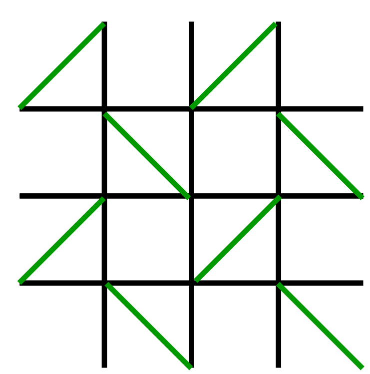
Wang, AN, Metlitski, Xu, Senthil 17
Qin, He, You, Lu, Sen, Sandvik, Xu, Meng 17
….

Unusual (weak) 1st order transition with emergent symmetry!

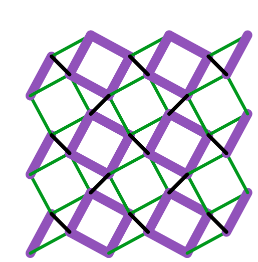
Zhao Weinberg Sandvik 18
Serna, AN 18

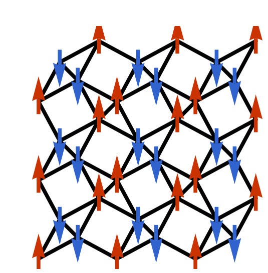
Exciting possible application: Shastry Sutherland lattice

$$\vec{n} = (N_x, N_y, N_z, \varphi_x + \varphi_y)$$



Zayed et al 17
Zhao Weinberg Sandvik 18
Lee, You, Sachdev, Vishwanath '19
Guo, Sun, Zhao, Wang, Hong et al, '19





Experimental implications of DCP? $\mathbf{SrCu_2(BO_3)_2}$