Global inconsistency and 't Hooft anomaly in bifundamental gauge theories at finite theta angles

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Abstract: we study possible vacuum structures of $SU(n) \times SU(n)$ gauge theory with bifundamental fermions by using the global inconsistency and 't Hooft anomaly.

't Hooft anomaly and anomaly matching

't Hooft anomaly is an obstruction to gauging the global symmetry. ['t Hooft 1980]

Consider a theory with the global symmetry G. Let $\mathcal{Z}[A]$ be the partition function under background G-gauge field A. G has an 't Hooft anomaly if

$$\mathcal{Z}[A + \mathrm{d}\theta] = \mathcal{Z}[A] \exp(\mathrm{i}\mathcal{A}[A, \theta])$$

under the G-gauge transformation $A \mapsto A + \mathrm{d}\theta$.

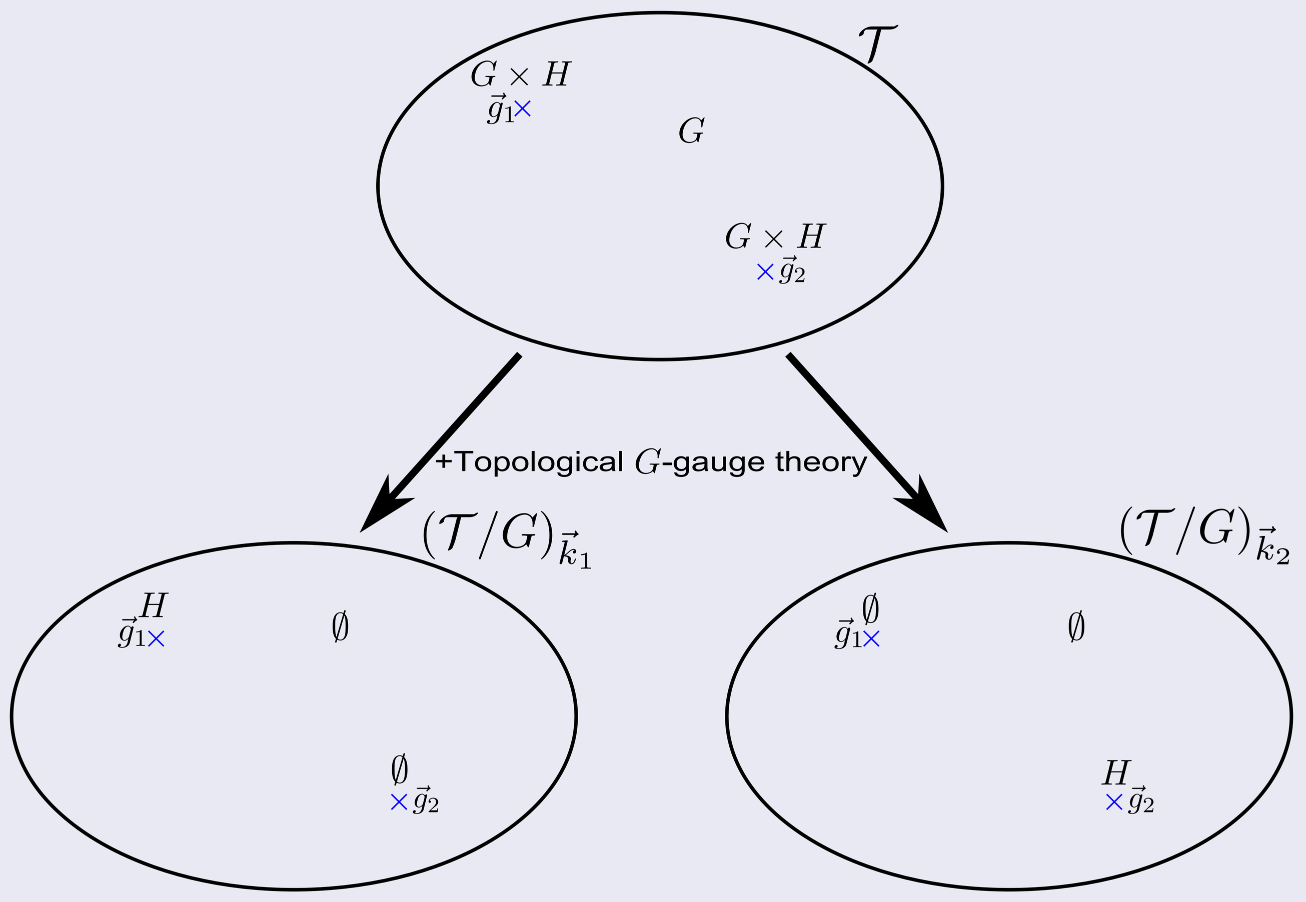
't Hooft anomaly matching rules out the trivial IR theory:

- G is unbroken and the theory contains massless excitations.
- G is unbroken, vacuum is gapped, and the theory is described by TQFT.
- G is broken $\Rightarrow$ degenerate vacua, NG modes.

Global inconsistency

Consider a QFT $\mathcal{T}$ with a set of **continuous parameters** $\vec{g}$.

- G - the symmetry at generic $\vec{g}$.
- $\hat{G}$ - the symmetry enhanced by H at **high symmetry points** $\vec{g}_1$, $\vec{g}_2$.



Gauge G-symmetry at high symmetry points by coupling to topological G-gauge theory with discrete parameters $\vec{k}$.

- Assume $\hat{G}$ has no 't Hooft anomaly $\Rightarrow \exists \vec{k}_i$ s.t.
$$\mathcal{Z}_{\vec{g}_i, \vec{k}_i}[h \cdot A] = \mathcal{Z}_{\vec{g}_i, \vec{k}_i}[A]$$
- Choose $\vec{k}_1$ at both $\vec{g}_1$ and $\vec{g}_2 \Rightarrow \hat{G}$ has a **global inconsistency** if
$$\mathcal{Z}_{\vec{g}_2, \vec{k}_1}[h \cdot A] = \mathcal{Z}_{\vec{g}_2, \vec{k}_1}[A] \exp(\mathrm{i}\mathcal{A}_{\vec{g}_2, \vec{k}_1}[h, A]).$$
- There exists no $\vec{k}$ which is compatible with the H-invariance at $\vec{g}_1$ and $\vec{g}_2$.

IR theory compatible with the global inconsistency:

- The vacuum either at $\vec{g}_1$ or $\vec{g}_2$ is nontrivial. [Gaiotto, Kapustin, Komargodski, Seiberg 2017]
- $\vec{g}_1$ and $\vec{g}_2$ are separated by a phase transition.

$SU(n) \times SU(n)$ bifundamental gauge theory

The Euclidean classical action of $SU(n)_1 \times SU(n)_2$ bifundamental gauge theory:

$$S = -\frac{1}{2g_1^2} \int \mathrm{Tr}[G_1 \wedge * G_1] - \frac{1}{2g_2^2} \int \mathrm{Tr}[G_2 \wedge * G_2] + \int \mathrm{Tr}[\bar{\Psi}(\not{D} + m)\Psi] + \frac{\mathrm{i}\theta_1}{8\pi^2} \int \mathrm{Tr}[G_1 \wedge G_1] + \frac{\mathrm{i}\theta_2}{8\pi^2} \int \mathrm{Tr}[G_2 \wedge G_2],$$

with

$$\bar{\Psi}(\not{D} + m)\Psi = \bar{\Psi}\gamma^\mu(\partial_\mu + a_{1,\mu}\Psi - \Psi a_{2,\mu}).$$

- Ψ - Dirac field belonging to fund. rep. of $SU(n)_1$ and anti-fund. rep. of $SU(n)_2$.
- a_i - $SU(n)_i$ gauge field. $G_i = \mathrm{d}a_i + a_i \wedge a_i$.
- $SU(n)_1 \times SU(n)_2$-gauge transformation (u_1, u_2):
$$\Psi \mapsto u_1 \Psi u_2^\dagger, \quad a_i \mapsto u_i a_i u_i^\dagger + u_i \mathrm{d}u_i^\dagger.$$

Symmetry

- $(\mathbb{Z}_n)_{\mathrm{center}}$ 1-form symmetry
$$(\mathbb{Z}_n)_{\mathrm{center}}: W_1(C) \mapsto \omega_n W_1(C), \quad W_2(C) \mapsto \omega_n W_2(C),$$
with $\omega_n = \exp(2\pi \mathrm{i}/n)$ and Wilson lines
$$W_1(C) = \mathrm{Tr}\left[\mathcal{P}\exp\left(\oint_C a_1\right)\right], \quad W_2(C) = \mathrm{Tr}\left[\mathcal{P}\exp\left(\oint_C a_2\right)\right].$$
- CP symmetry exists at $(0,0), (0,\pi), (\pi,0), (\pi,\pi)$: high symmetry points.

Gauging $\mathbb{Z}_n$ center symmetry

Prepare $\mathbb{Z}_n$ topological gauge theory [Kapustin, Seiberg 2014]

$$S_{\mathrm{TFT}} = \frac{\mathrm{i}}{2\pi} \int F \wedge (\mathrm{d}A + nB) + \frac{\mathrm{i}nk}{4\pi} \int B \wedge B.$$

- A: $U(1)$ 1-form gauge field, B: $U(1)$ 2-form gauge field, F 2-form auxiliary field.
- $U(1)$ 1-form gauge transformation:
$$A \mapsto A - n\lambda, \quad B \mapsto B + \mathrm{d}\lambda, \quad F \mapsto F - k\lambda.$$
Gauge invariance $\Rightarrow k \in \mathbb{Z}_n$: a discrete parameter.
- Eom for $F \Rightarrow \mathrm{d}A = -nB$.

Couple the bifundamental gauge theory to the topological gauge theory

- Extend the gauge group:
$$SU(n)_1 \times SU(n)_2 \to \frac{SU(n)_1 \times SU(n)_2 \times U(1)}{\mathbb{Z}_n}$$
- Identify $U(1)$ gauge field with A in the topological theory.
- Replace $SU(n)$ gauge fields a_i by U(n) gauge fields $\mathcal{A}_i$:
$$\mathcal{A}_1 = a_1 + \frac{1}{n}A, \quad \mathcal{A}_2 = a_2 + \frac{1}{n}A, \quad A = \mathrm{Tr}[\mathcal{A}_1] = \mathrm{Tr}[\mathcal{A}_2].$$
- $U(1)$ gauge field A does not couple to the bifundamental fermion.
- Replace G_i by $\mathcal{G}_i + B$ with $U(n)$ field strengths $\mathcal{G}_i$.

Resultant θ term:

$$S_\theta = \sum_{i=1,2} \frac{\mathrm{i}\theta_i}{8\pi^2} \int \mathrm{Tr}[(\mathcal{G}_i + B) \wedge (\mathcal{G}_i + B)] = \sum_{i=1,2} \frac{\mathrm{i}\theta_i}{8\pi^2} \int (\mathrm{Tr}[\mathcal{G}_i \wedge \mathcal{G}_i] - nB \wedge B).$$

't Hooft anomaly and global inconsistency for $\mathbb{Z}_n \times CP$ symmetry

Now, we have the partition function $\mathcal{Z}_{(\theta_1,\theta_2),k}[A, B]$ with the $\mathbb{Z}_n$ gauge symmetry.

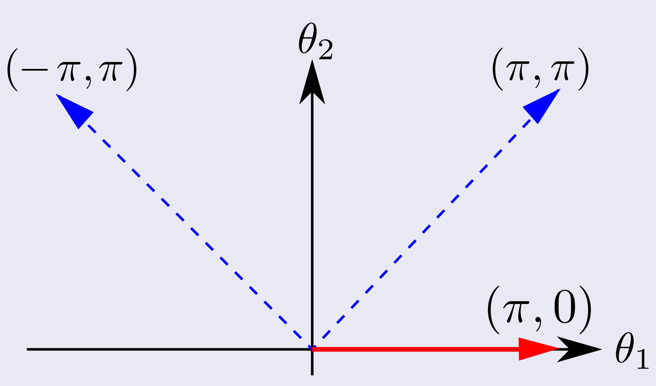
CP transformation [c.f. Gaiotto, Kapustin, Komargodski, Seiberg, 2017]

- θ term at the high symmetry points
$$S_\theta \mapsto S_\theta - \sum_{i=1,2} \frac{2\mathrm{i}\theta_i}{8\pi^2} \int \mathrm{Tr}[\mathcal{G}_i \wedge \mathcal{G}_i] + \sum_{i=1,2} \frac{2\mathrm{i}\theta_i n}{8\pi^2} \int B \wedge B.$$
The second term is in $2\pi\mathrm{i}\mathbb{Z}$.
- The topological term
$$S_{\mathrm{TFT}} = k\frac{\mathrm{i}n}{4\pi} \int B \wedge B \mapsto -k\frac{\mathrm{i}n}{4\pi} \int B \wedge B.$$
- The partition function transforms as
$$\mathcal{Z}_{(\theta_1,\theta_2),k}[\mathsf{T} \cdot (A, B)] \mapsto \mathcal{Z}_{(\theta_1,\theta_2),k}[A, B] \exp\left(\left(\frac{\sum_i \theta_i}{\pi} - 2k\right) \frac{\mathrm{i}n}{4\pi} \int B \wedge B\right).$$

Global inconsistency and 't Hooft anomaly

- $(\theta_1, \theta_2) = (0,0), (\pi, -\pi)$,
$$k = 0 \ (\mathrm{mod}\ n).$$
- $(\theta_1, \theta_2) = (0,\pi), (\pi, 0)$,
$$1 - 2k = 0 \ (\mathrm{mod}\ n) \Rightarrow \begin{cases} \text{'t Hooft anomaly for even } n, \\ k = (n+1)/2 \text{ for odd } n. \end{cases}$$
- $(\theta_1, \theta_2) = (\pi, \pi)$,
$$2 - 2k = 0 \ (\mathrm{mod}\ n) \Rightarrow k = 1.$$

- For even n, global inconsistences exists between $(0,0)$ and (π,π), and between $(\pi,-\pi)$ and (π,π).
- For odd n, global inconsistences exists between $(0,0)$ and (π,π), between $(0,0)$ and $(\pi,0)$, ...



Phase diagram

A phase diagram compatible with the constraints by the global inconsistencies and 't Hooft anomalies.

- $(\theta_1, \theta_2) = (\pi, \pi)$ and $(\pi, -\pi)$ must be identified.
- T is broken at $(\pi, 0)$ and $(0, \pi)$.
- T is unbroken at (π,π) but separated from $(0,0)$ by a phase transition $\to$ the global inconsistency does not lead to nontrivial vacuum at (π,π).
- $(0,0)$ and (π,π) are smoothly connected.
- More exotic phase diagram is possible..