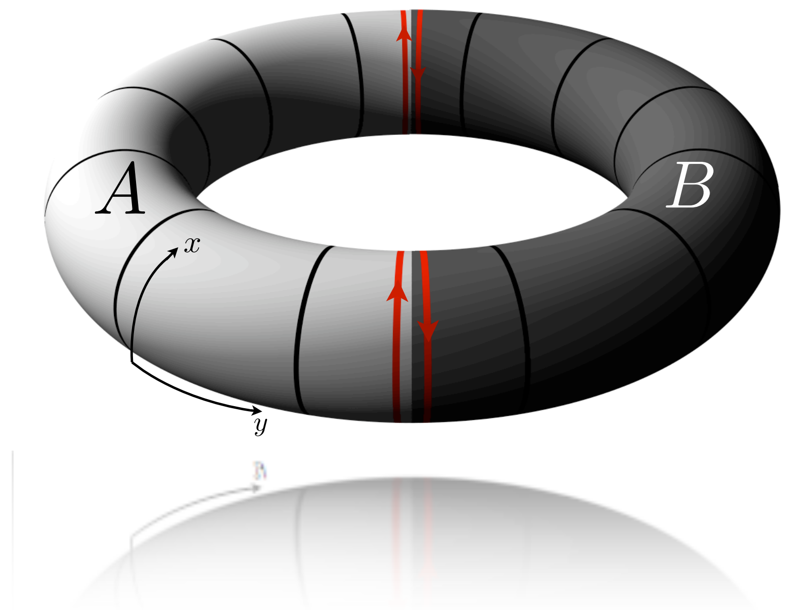


Entanglement Spectra of Fractional Quantum Hall States

Andreas Läuchli
"New states of quantum matter"
MPI für Physik komplexer Systeme - Dresden

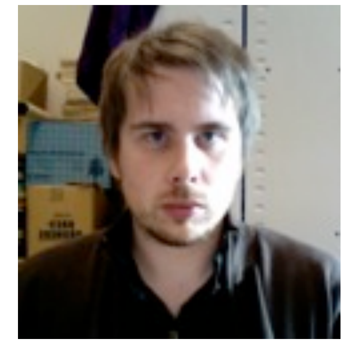
http://www.pks.mpg.de/~aml

ICTS Workshop on Condensed Matter Physics, Mahabaleshwar, 12/9/2009

Collaborators

- J. Suorsa, Oslo

UNIVERSITY OF OSLO



- Emil J. Bergholtz, Dresden

mpipks

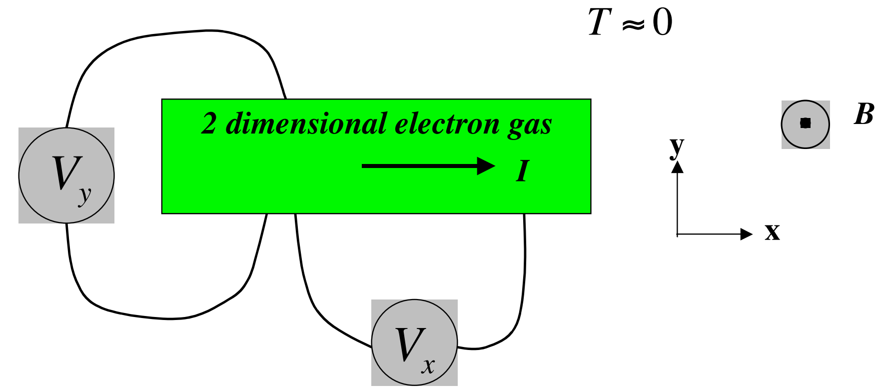
- Masudul Haque, Dresden

mpipks

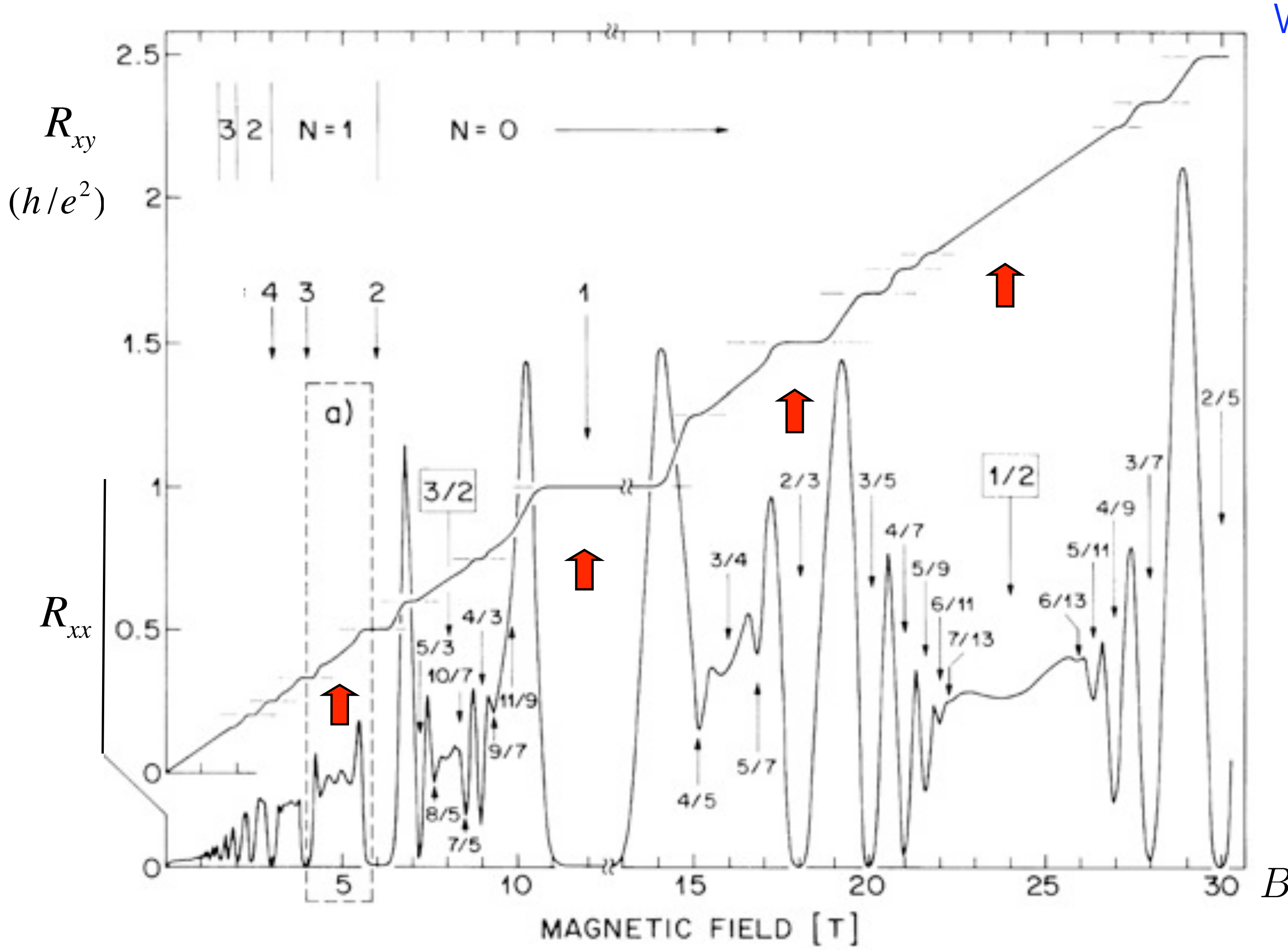
Outline

- Introduction: Fractional Quantum Effect
- Topological Entanglement Entropy
- Entanglement Spectra

Quantum Hall Effect



Willet et al.,1987



Quantum Hall states

$$R_{xy} = \frac{1}{s}\frac{h}{e^2} \qquad R_{xx} = 0$$

h Planck's constant
e electron charge

$s = 1, 2, 3, \ldots$ Integer QHE

von Klitzing, Dorda, Pepper 1980

$s = \frac{1}{3}, \frac{2}{5}, \frac{3}{7} \cdots$ Fractional QHE

Tsui, Störmer, Gossard 1982

$s = \frac{5}{2}, \ldots$ Non-abelian Fractional QHE?

Willet et al 1987

(incompressible quantum liquids)

Metallic states

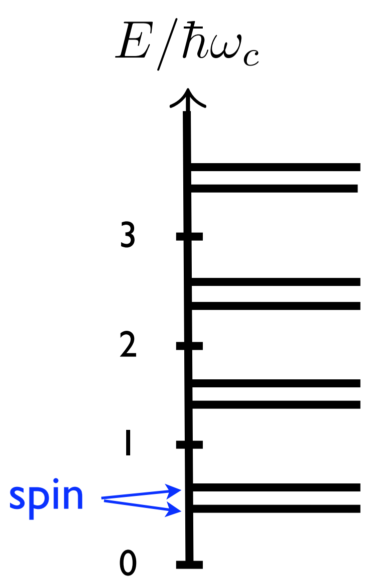
$$s = \frac{1}{2}, \frac{1}{4}, \ldots$$

(compressible quantum liquids)

....and much more

Stripes (high LL's)
Wigner Crystals (low filling)
.....

One electron in a magnetic field (2D)



- Kinetic energy quantized $E_n = (n + \frac{1}{2})\hbar\omega_c$ $\quad n = 0, 1, 2, 3...$

  Landau levels $\quad \omega_c = \frac{eB}{mc}$

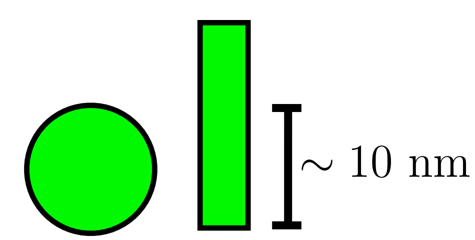
- Each Landau level (LL) is highly degenerate.

  # of states $\propto B$

  One state per flux quantum $\varphi_0 = hc/e$

- Quantized area of electron state

$$A = 2\pi\ell^2 \; ; \quad \ell \equiv \sqrt{\frac{\hbar c}{eB}} = \frac{257\text{Å}}{\sqrt{\frac{B}{1 \text{ tesla}}}}$$



(within a Landau level)

Define $\quad \nu$ = number of filled LL *(filling factor)*

The crucial parameter!

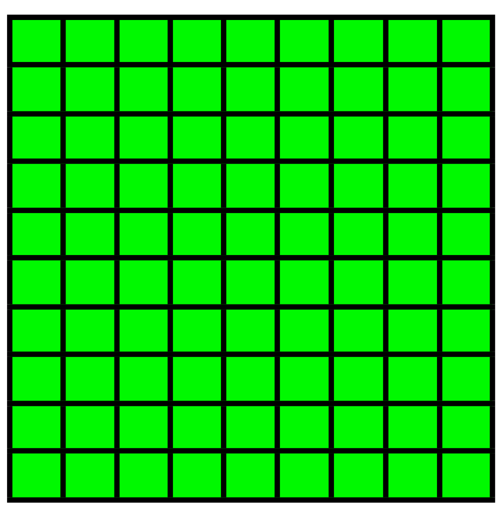
Integer versus fractional filling

Fixed area per state *(one state per flux quantum* $\varphi_0 = hc/e$*)*

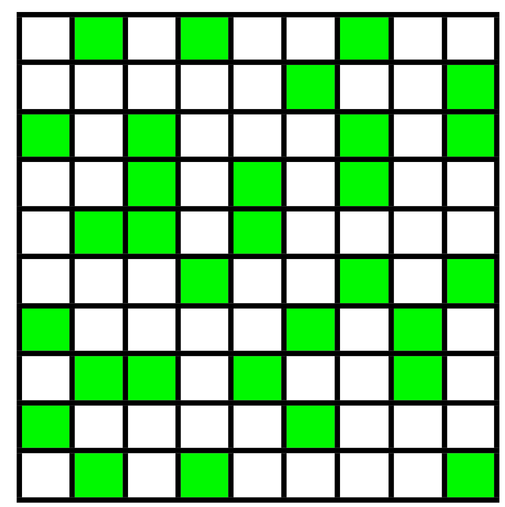
□ empty one electron state

■ filled one electron state

IQHE $\nu = 1$



FQHE $\nu = 1/3$



I ~ 10 nm

(Real samples much bigger)

unique state

many, many degenerate states

!

?

incompressible liquid

mpipks

Standard theory - brief review

$\nu = 1/(2m+1)$ **incompressible quantum liquid with** $e^{\star} = e/(2m+1)$ **particles**

Many-body wave functions *(Laughlin '83)*

But other fractions $\nu = p/q$ less clear. *(All $\nu = p/q$, q odd, experimentally similar.)*

Hierarchy: The fractionally charged qp's condense and form an incompressible liquid just as electrons condensed at 1/(2m+1). Iterating this gives all $\nu = p/q$, q odd.

(Haldane '83, Halperin '83)

and/or

Composite fermions: Electrons form new particles, composite fermions, by absorbing magnetic flux. FQHE is IQHE of these composite fermions. Gives $\nu = p/(2mp+1)$ directly. *(Jain '89)*

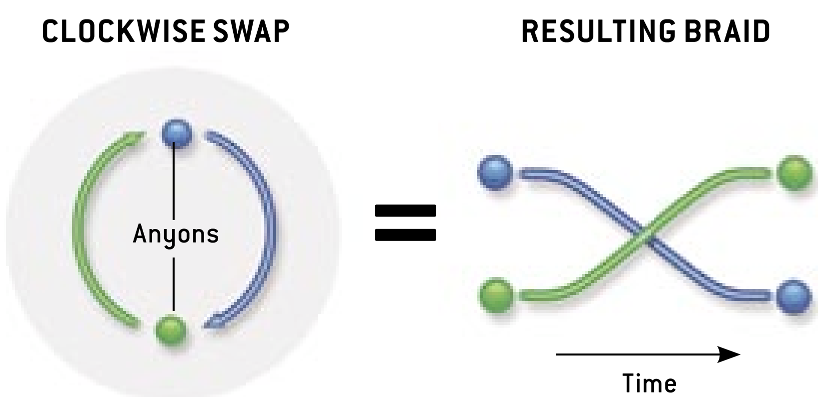
Gapless states Half filled Landau level; free composite fermions *(Halperin, Lee and Read '93)*

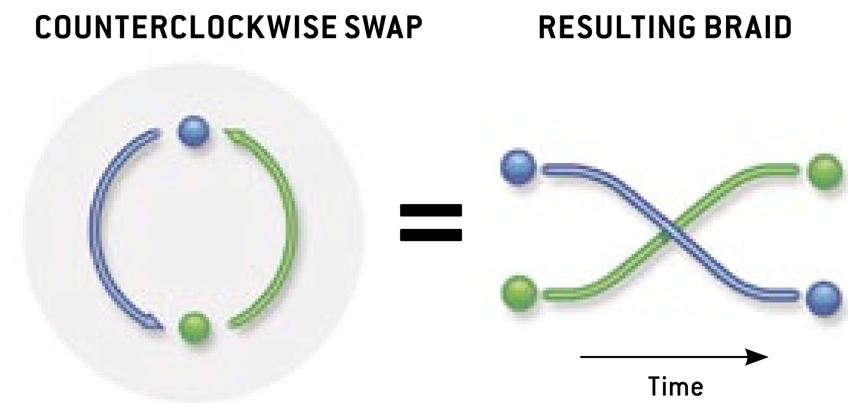
Non-abelions Appear (?) in higher Landau levels (and/or in rotating condensates) Motivated by conformal field theory (CFT-FQHE correspondence) *(Moore and Read '91)*

...and it goes on.....

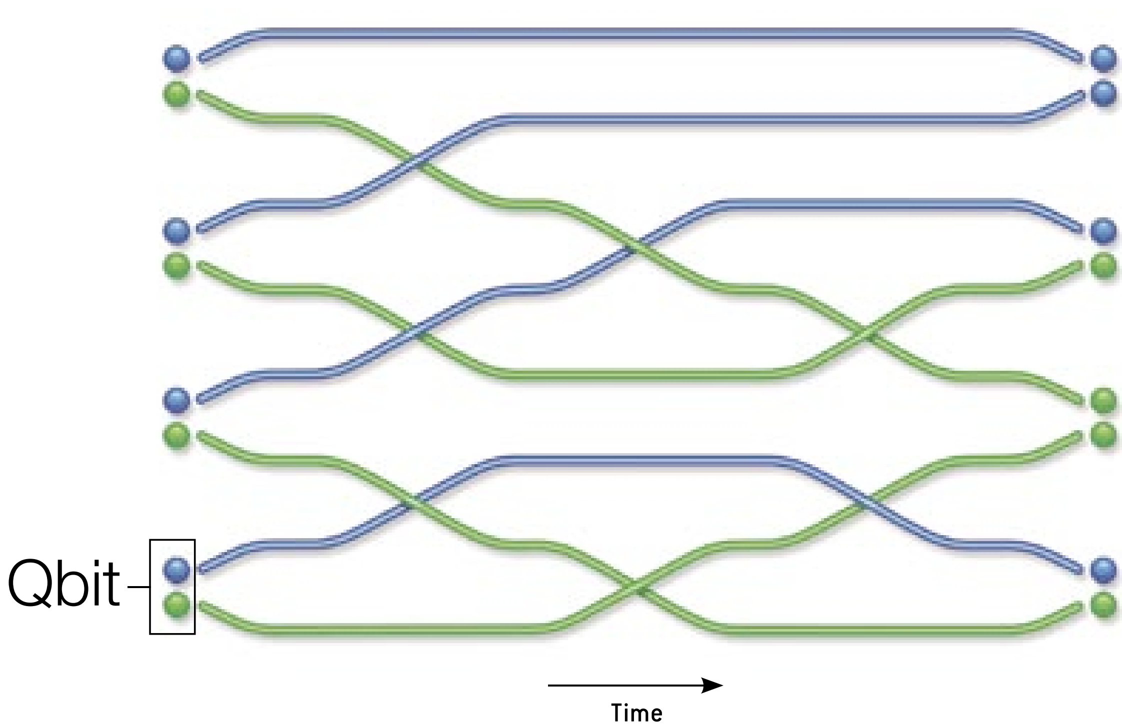
Topological Quantum Computing

- Nonabelian Anyons: Quantum particles which know which way they are braided





- Braiding many nonabelian anyons encodes computation in a quantum computer



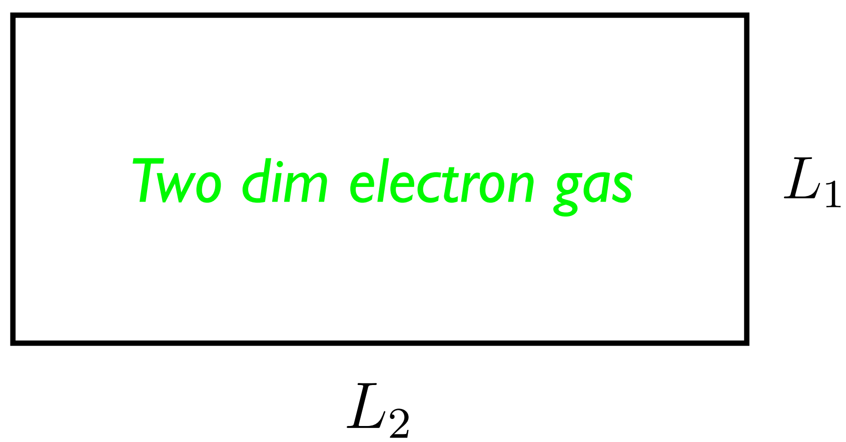
Topology of braids insensitive to “small” amount of noise

Feasible with ν=5/2 or 12/5 ?

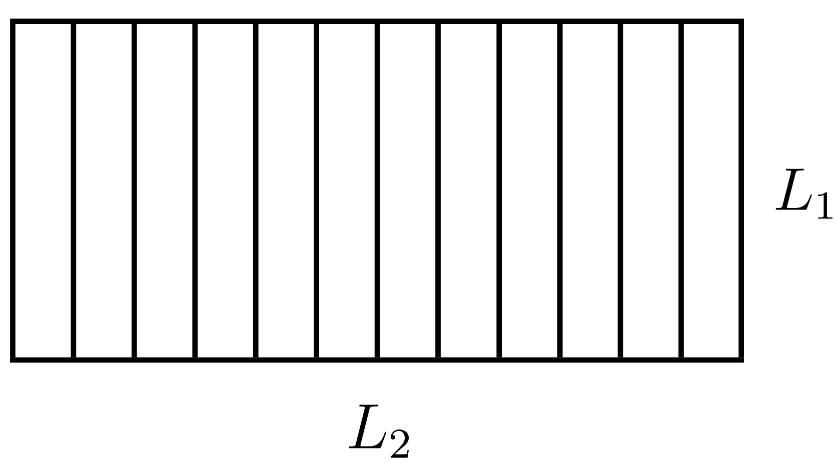
Nayak et al., RMP ’08
Source: Scientific American

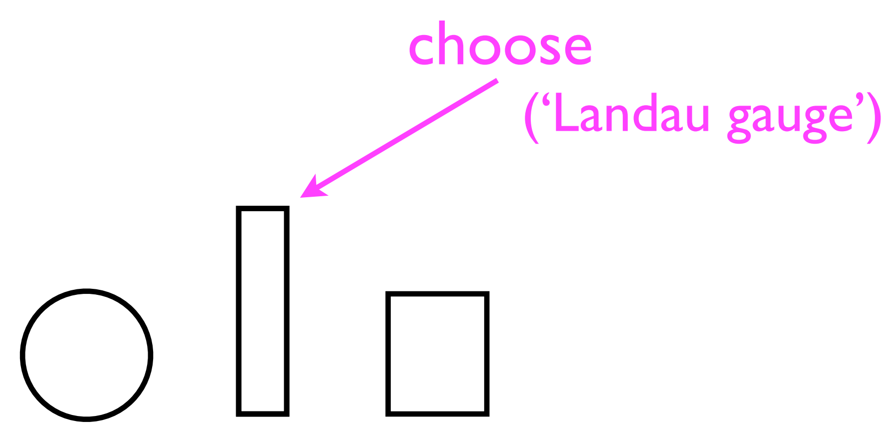
A “one-dimensional” microscopic approach

Consider sample with lengths $L_1,\ L_2$



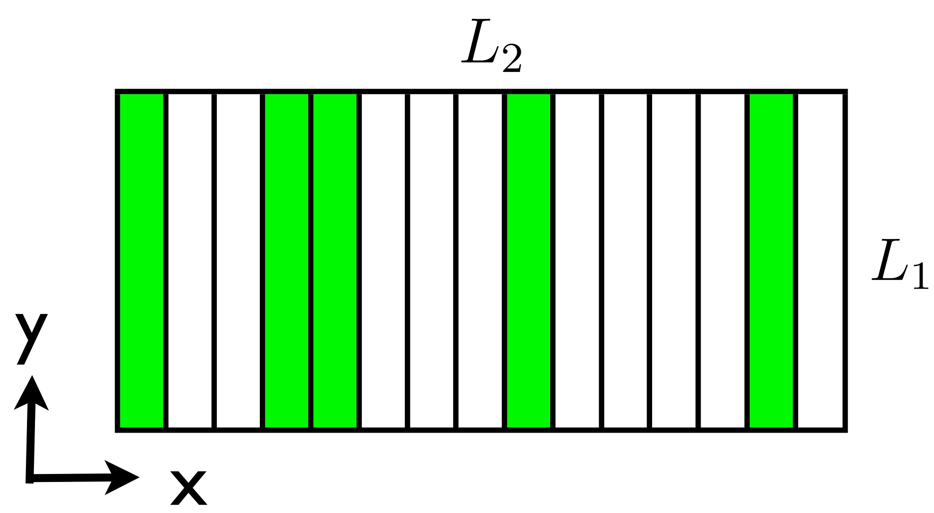
Each electron state has fixed area





(any shape is OK)

A "one-dimensional" microscopic approach



1 0 0 1 1 0 0 0 1 0 0 0 0 1 0

Each box is either empty 0 or filled 1

$$\psi_k \sim e^{ik\frac{2\pi}{L_1}y} e^{-(x-k\frac{2\pi}{L_1})^2/2}$$

(Landau gauge)

A possible state at $\nu = 1/3$

Hamiltonian (ee-interaction) *(eg Coulomb $V(r)=e^2/r$)* No kinetic energy!

$1..0.....0..1 \leftrightarrow 0..1.....1..0$ $V_{k,m}$ *(all ee-terms that preserve position of CM)*

k+m k-m

including

$1......1$ $V_{k,0}$ *(electrostatic repulsion)*

k

Exact mapping of a single Landau level!

Exact solution at $L_1 \to 0$ (Bergholtz et al., '05-'09, Seidel et al)

Hopping $1..0.....0..1 \leftrightarrow 0..1.....1..0$ makes ground state complicated.

But, when $L_1 \to 0$

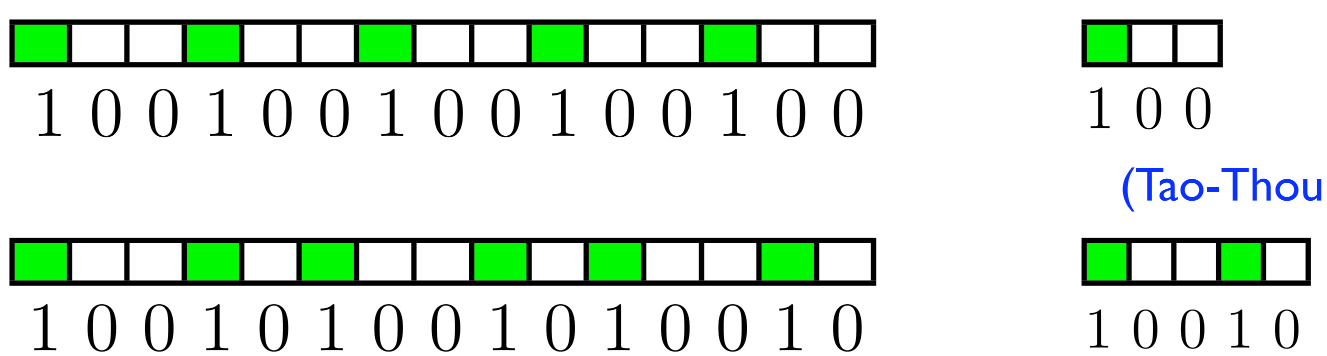
hopping vanishes and only electrostatic repulsion remains: $1......1$

This is a simple classical electrostatics problem!

States with electrons in fixed positions are the energy eigenstates - groundstate obtained by separating the electrons as much as possible:

Unit cell

$\nu = 1/3$



$\nu = 2/5$

At $\nu = p/q$ ground state is TT-state with p electrons in unit cell of length q.

↑ *'gapped crystal'*

For several fractions this TT state is adiabatically connected to the bulk FQH state !

Exact Diagonalization: Main Idea

- Solve the Schrödinger equation of a quantum many body system numerically

$$\mathcal{H}|\psi\rangle = E|\psi\rangle$$

- Sparse matrix, but for quantum many body systems the vector space dimension grows exponentially!

- But you can get a tremendous amount of physical information out of a finite system and the reward is a powerful:

Quantum Mechanics Toolbox

Exact Diagonalization: Present Day Limits

- Fractional quantum hall effect
  different filling fractions ν, up to 16-20 electrons
  up to 300 million basis states, up to several billion in the near future

- Spin S=1/2 models:
  40 spins square lattice, 39 sites triangular, 42 sites star lattice at S^z=0
  64 spins or more in elevated magnetization sectors
  up to 1.5 billion(=10^9) basis states with symmetries, up to 4.5 billion without

- t-J models:
  32 sites checkerboard with 2 holes
  32 sites square lattice with 4 holes
  up to 2.8 billion basis states

- Hubbard models
  21 sites triangular lattice at half filling, 20 sites quantum dot structure
  22-25 sites in ultracold atoms setting
  up to 160 billion basis states

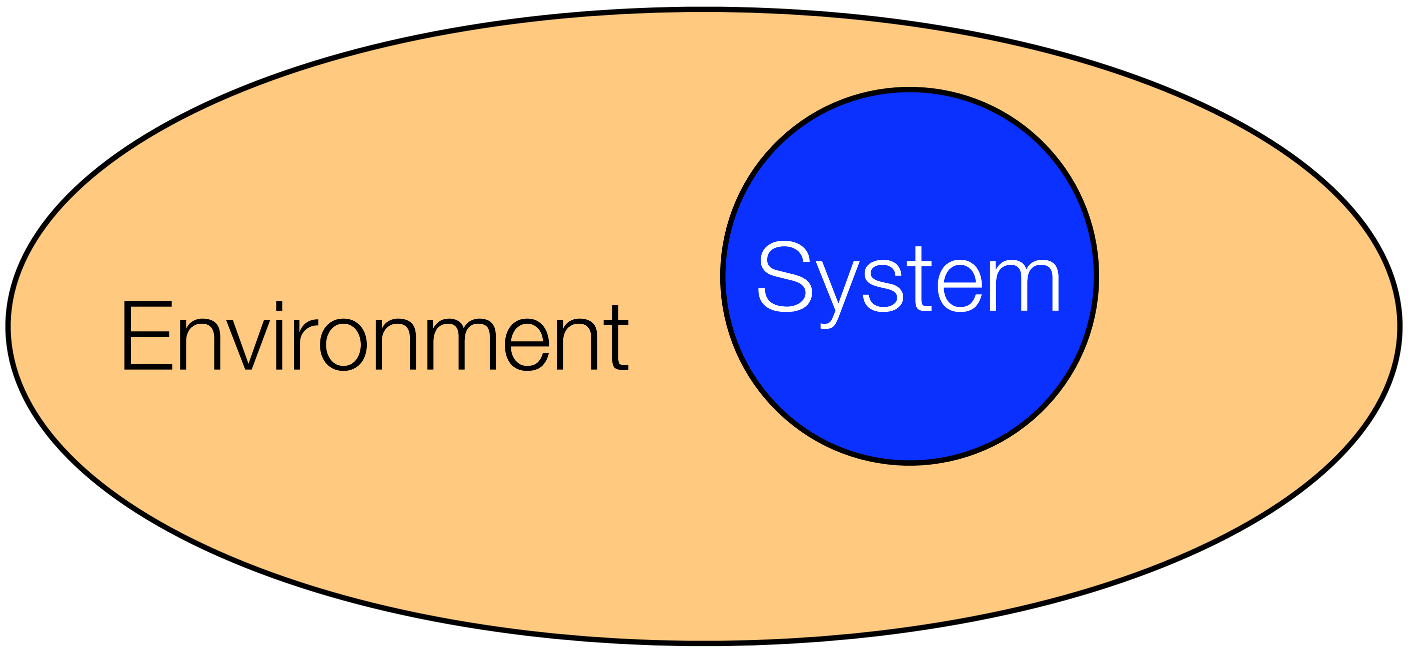
low-lying eigenvalues, not full diagonalization

Outline

- Introduction: Fractional Quantum Effect
- Topological Entanglement Entropy
- Entanglement Spectra

(Topological) Entanglement Entropy

- Let us look at reduced density matrices, and their entanglement entropies



$$\rho = \mathrm{Tr}_E |\psi\rangle\langle\psi|$$

$$S(\rho) = \mathrm{Tr}[-\rho \log \rho]$$

For topologically ordered phases:

Perimeter/Area Law

$$S(\rho) = \alpha L - \gamma + \cdots$$

Topological entanglement entropy

$$\gamma = \log \mathcal{D}$$

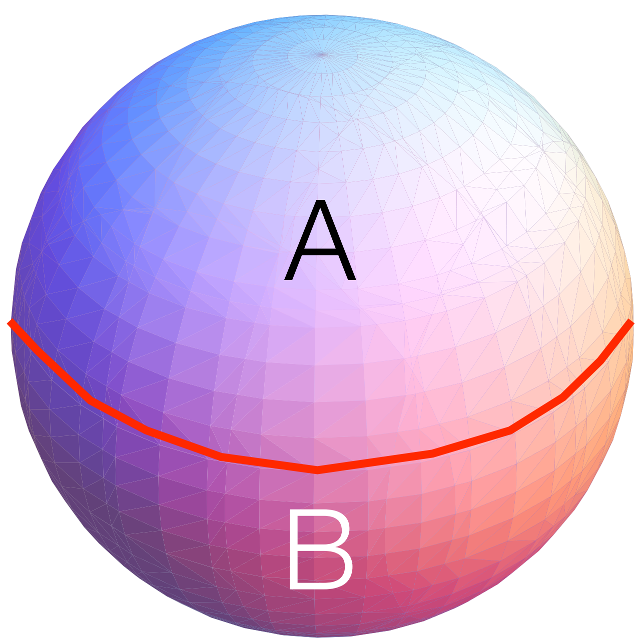
$\mathcal{D}$ Total quantum dimension

Kitaev & Preskill PRL '06
Levin & Wen PRL '06

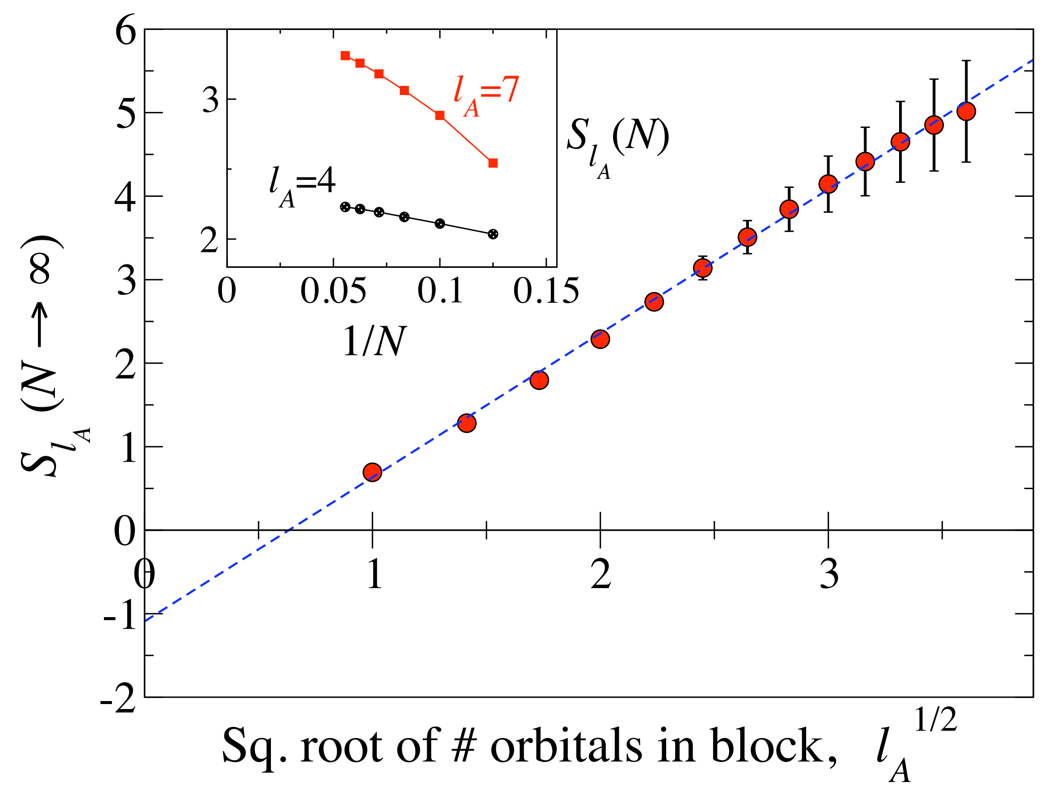
Topological entanglement entropy for FQH states on the sphere

- FQH model states are known to have topological order

- Can one extract γ based on the entanglement entropy ?



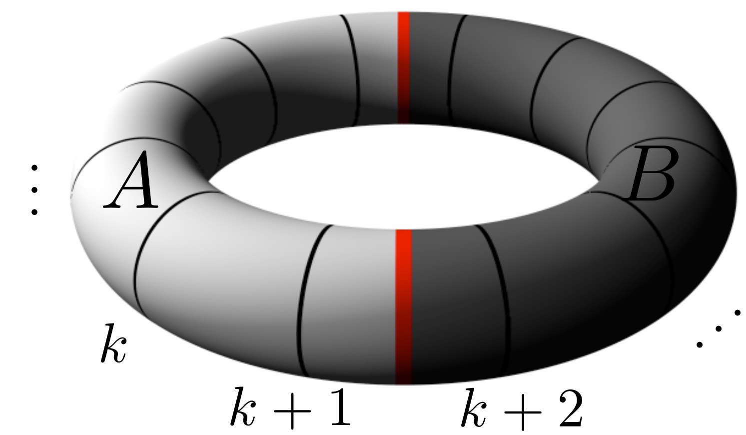
Haque, Zozulya & Schoutens, PRL ’07

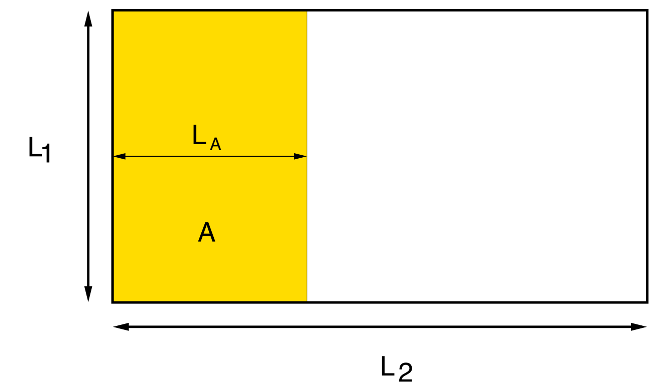


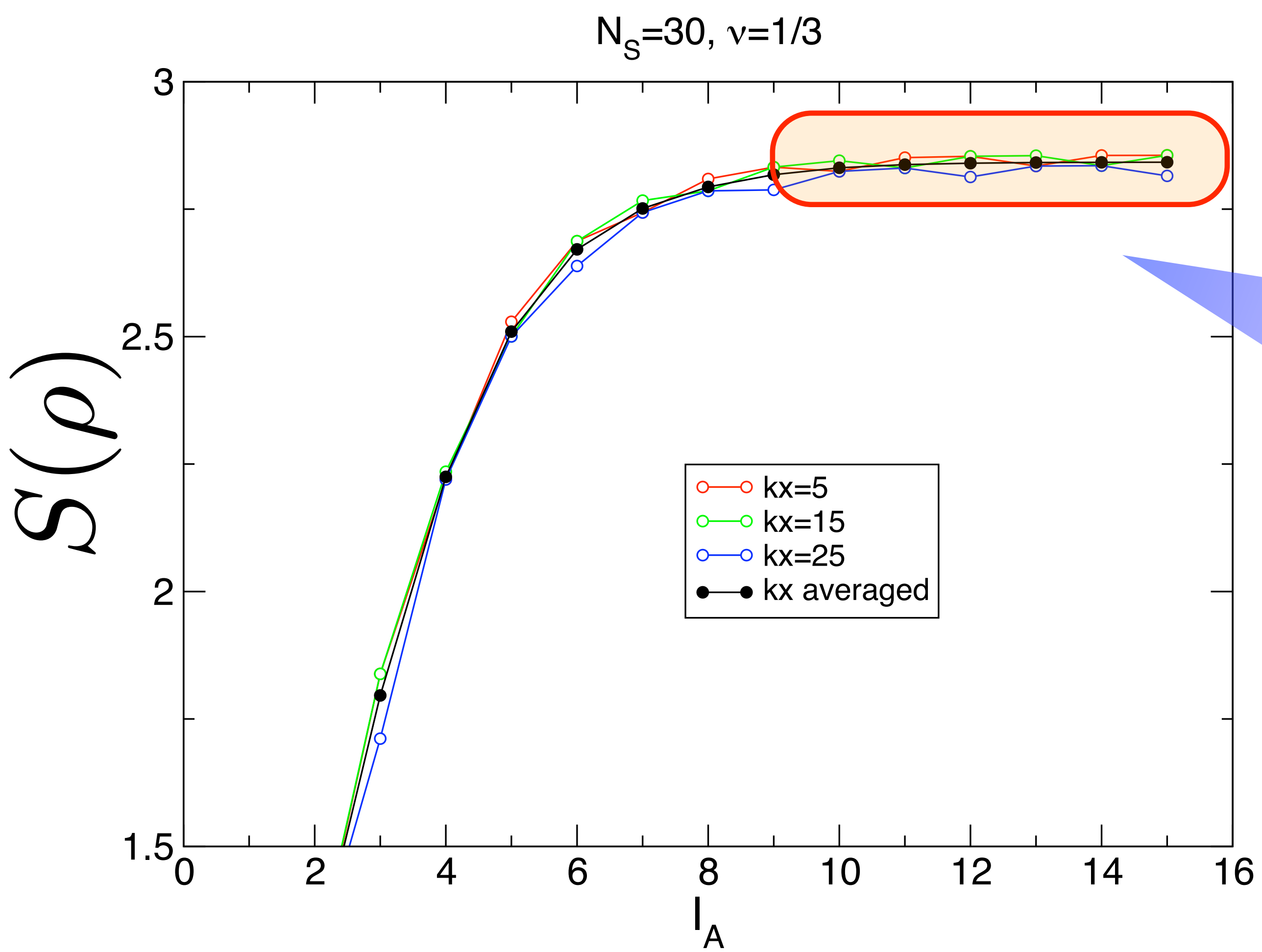
- Feasible, but tricky on the sphere.
  Complications due to varying length as a function of latitude

How to do this on the torus

- The torus can be tuned continuously by varying L_1 and L_2 ($L_1 L_2 = 2\pi N_s$).
- Determine $S(L_1)$ without extrapolation, then use $S(\rho) = \alpha L - \gamma + \cdots$ relation





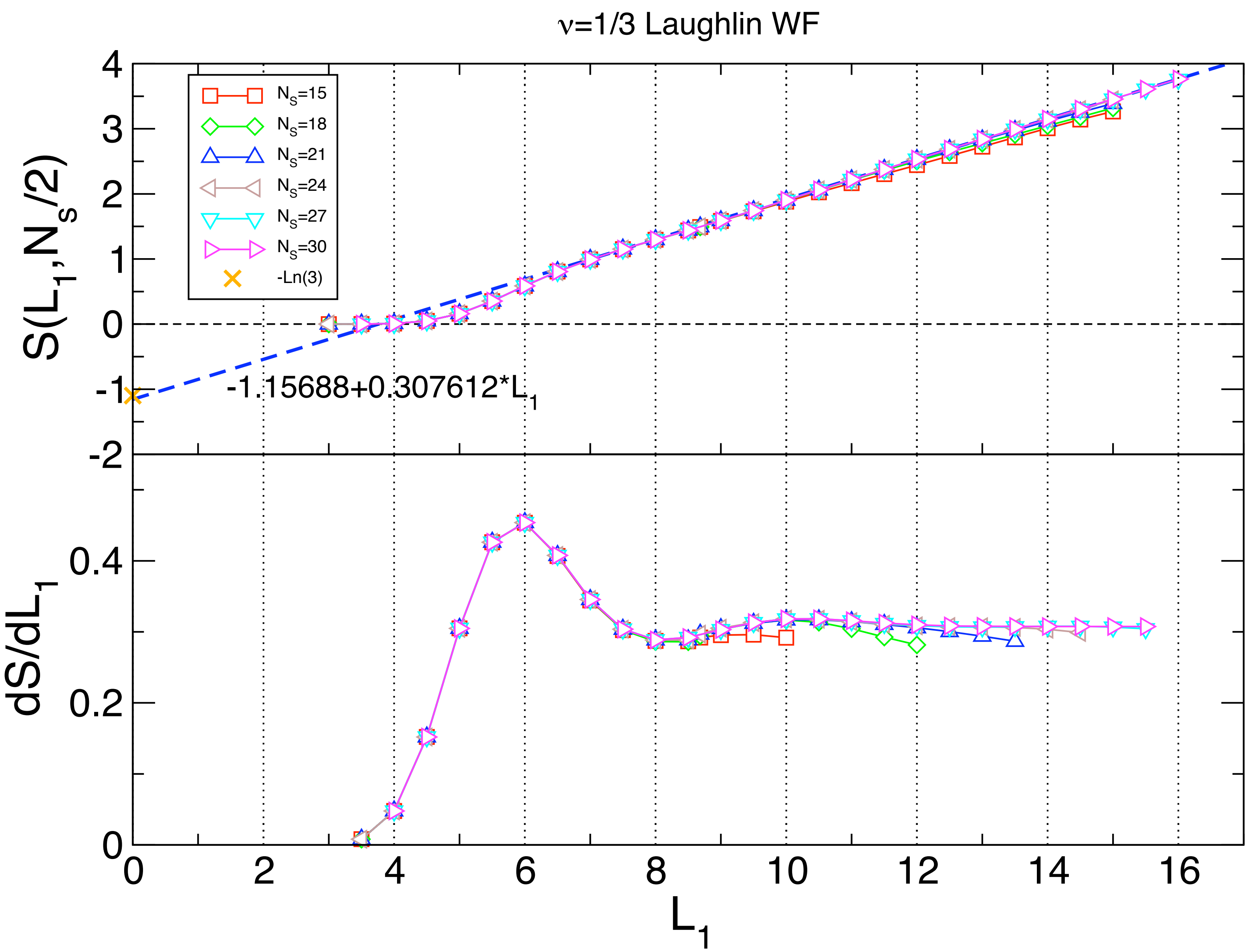


In the thin torus limit (here L_1=13) one can see the area law at work

AML, Bergholtz & Haque, unpublished

How to do this on the torus

- Determine $S(L_1)$ by looking at blocks which are long enough ($L_2 >> 1$)



- Control over the subleading L_1 effects !
  Better accuracy than on the sphere (a few %)

AML, Bergholtz & Haque, unpublished

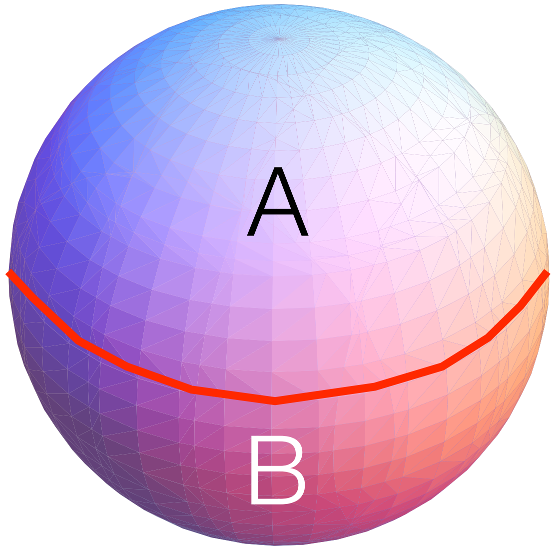
Outline

- Introduction: Fractional Quantum Effect
- Topological Entanglement Entropy
- Entanglement Spectra

Entanglement Spectra (Li & Haldane PRL '08)

- The entanglement entropy is a single number !

- Is there more one can extract from the reduced density matrix ?



- One can always write

$$\rho =: \exp[-H_{\mathrm{Entanglement}}]$$

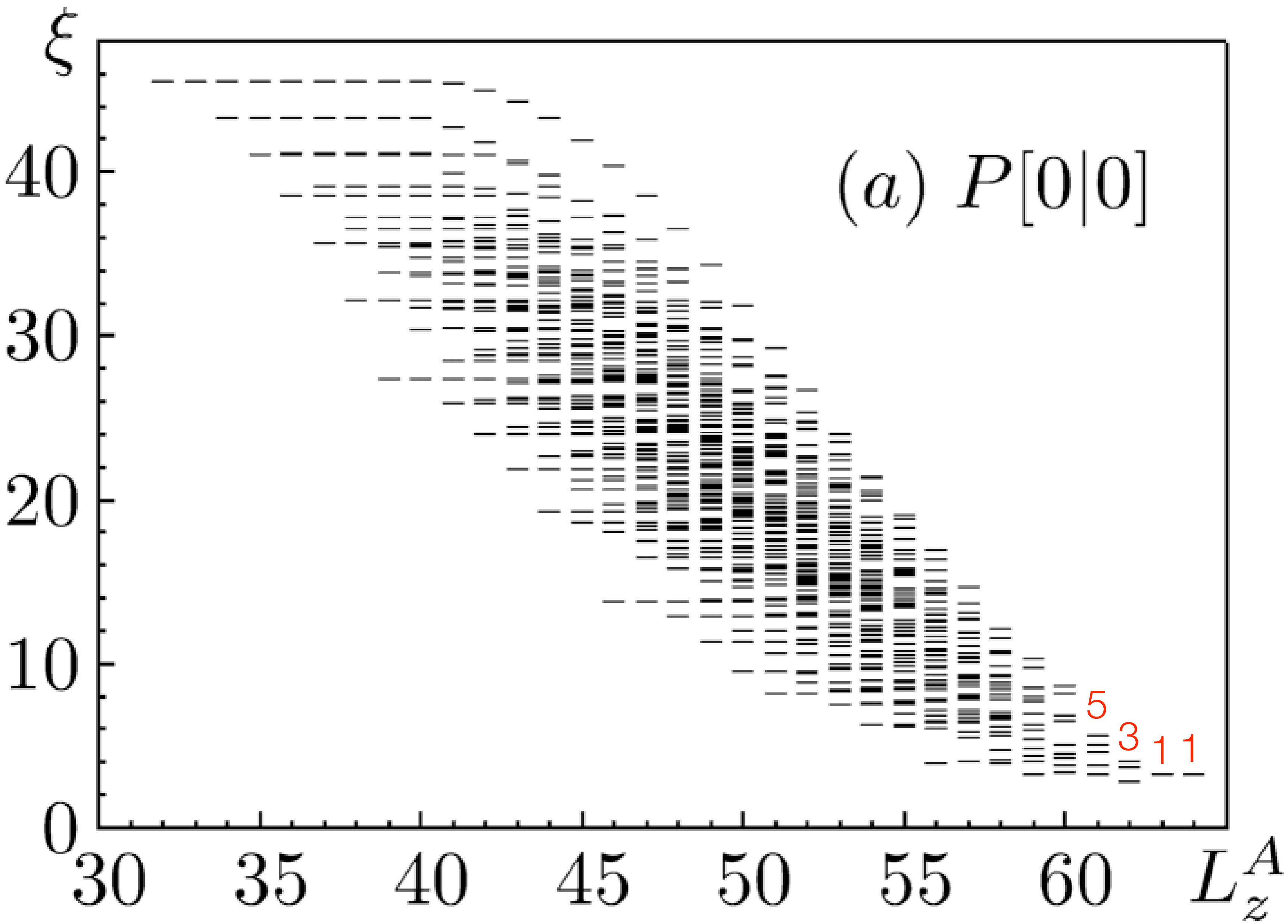
$$\mathcal{S} = \sum_i \xi_i \exp(-\xi_i),$$

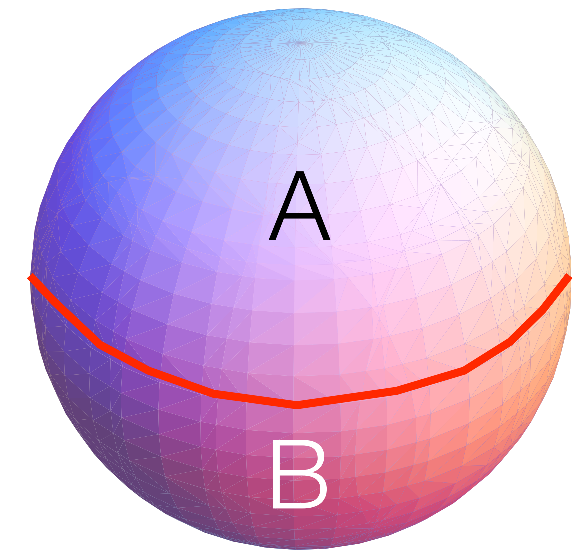
- Assuming that the entanglement Hamiltonian and the physical Hamiltonian are "similar", then one expects to see some features related to the open boundary block structure in the spectrum of the reduced density matrix

- FQH states have interesting edge physics, visible in entanglement spectrum ?

Moore-Read state on the sphere (Li & Haldane, PRL '08)

- Entanglement spectrum has dispersive structure



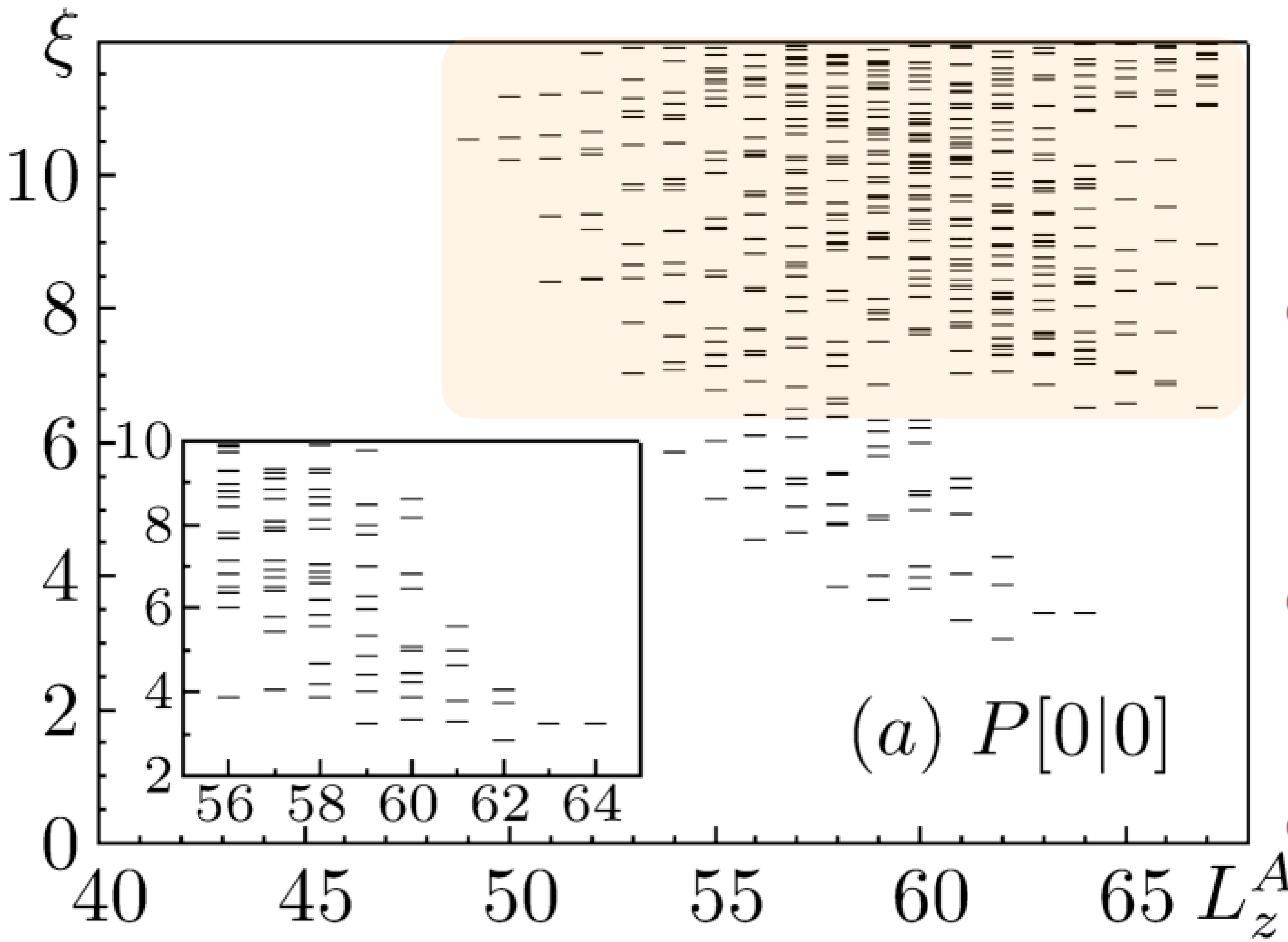


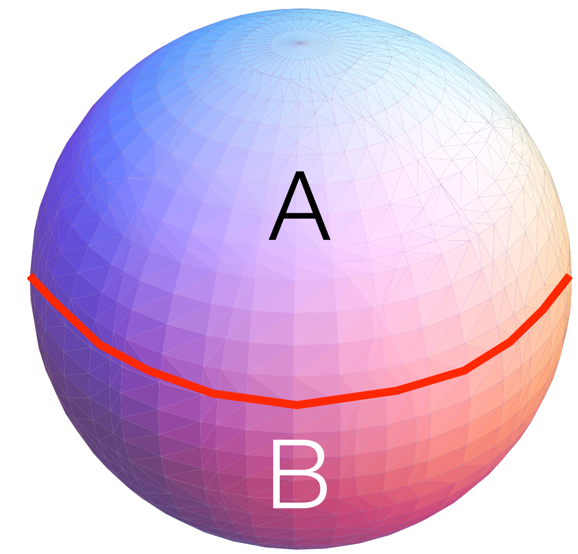
- Degeneracy at large momenta follows CFT counting rule (edge theory of the Pfaffian is U(1)+Majorana)

Wen, PRL '93

Moore-Read state on the sphere (Li & Haldane, PRL '08)

- Now for "realistic" Coulomb Hamiltonian at ν=5/2

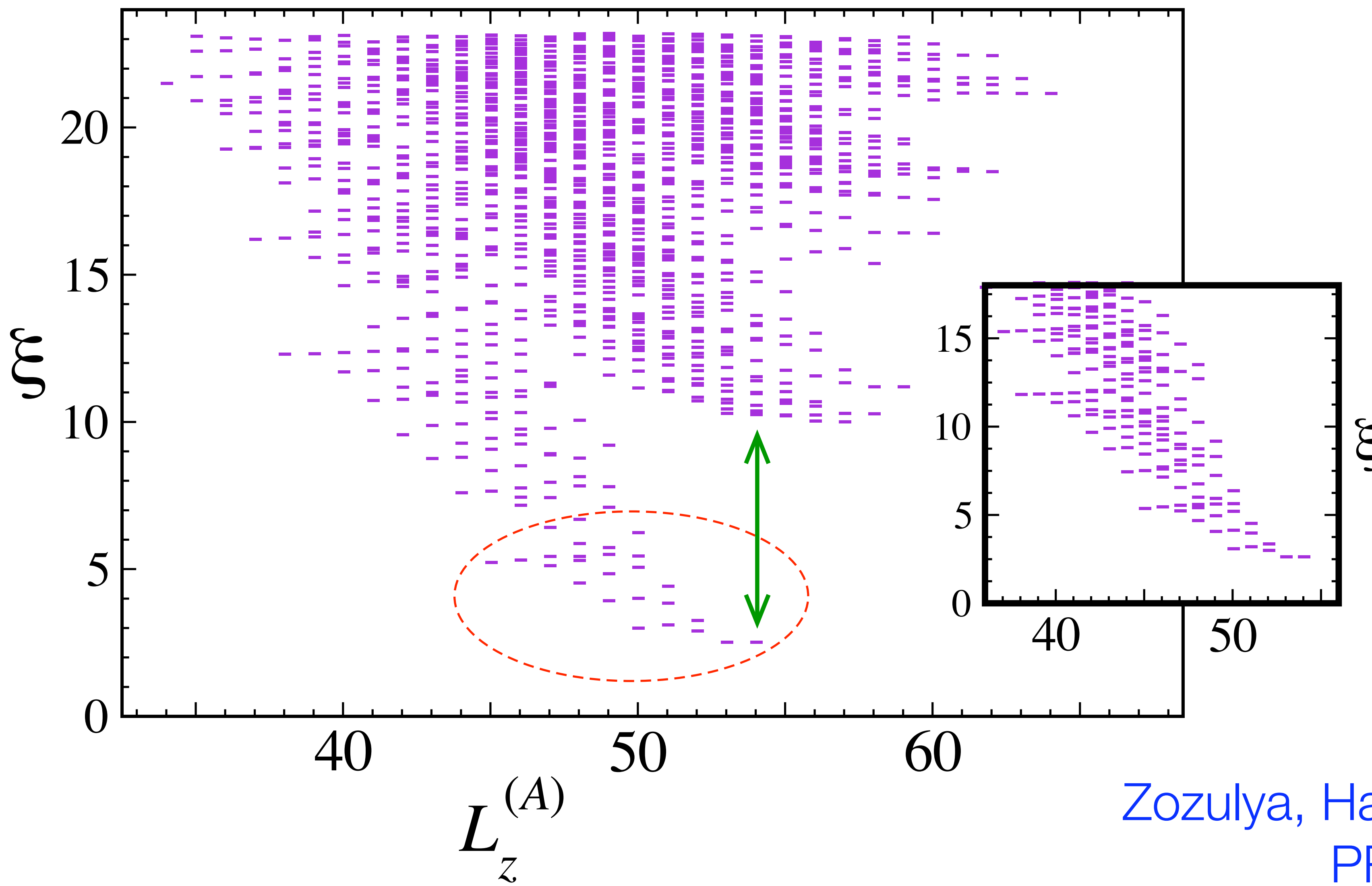




- Lower part of entanglement spectrum similar to model state
- Pollution by generic levels above "entanglement gap"
- Energetics not understood

Entanglement Spectrum at ν=1/3 (Coulomb)

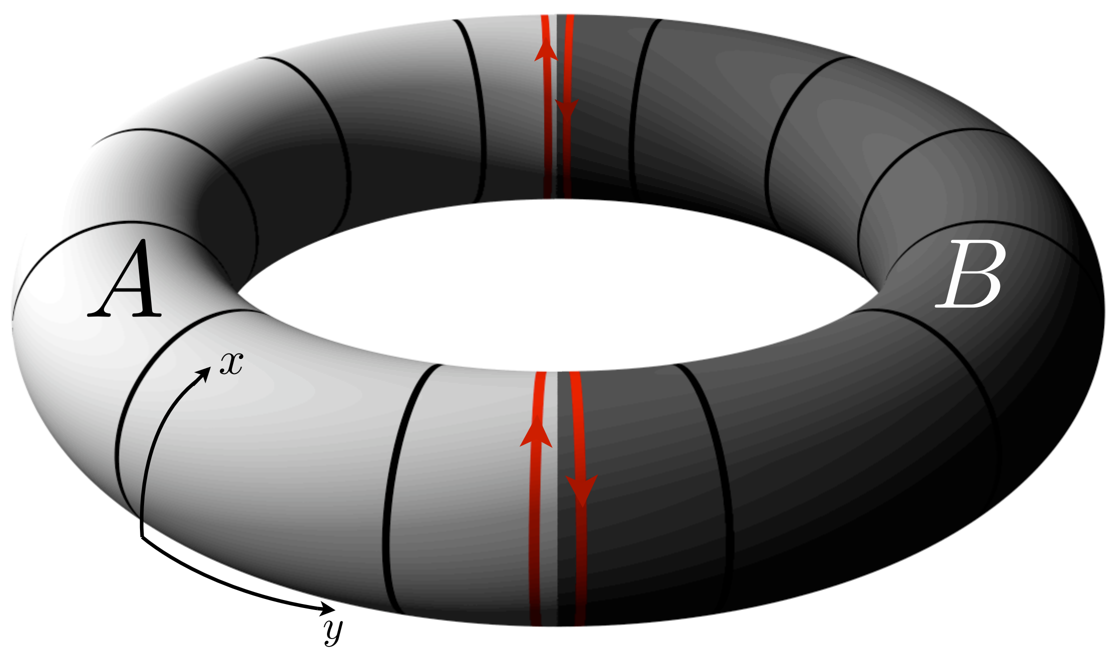
- Chiral low energy mode with an entanglement gap to generic levels
- Satisfies degeneracy count for a chiral U(1) theory (1-1-2-3-5-7-11-....)



Zozulya, Haque & Regnault
PRB ’09

And now for something different, the torus

- The natural partition of Landau level orbitals leads to blocks having two edges

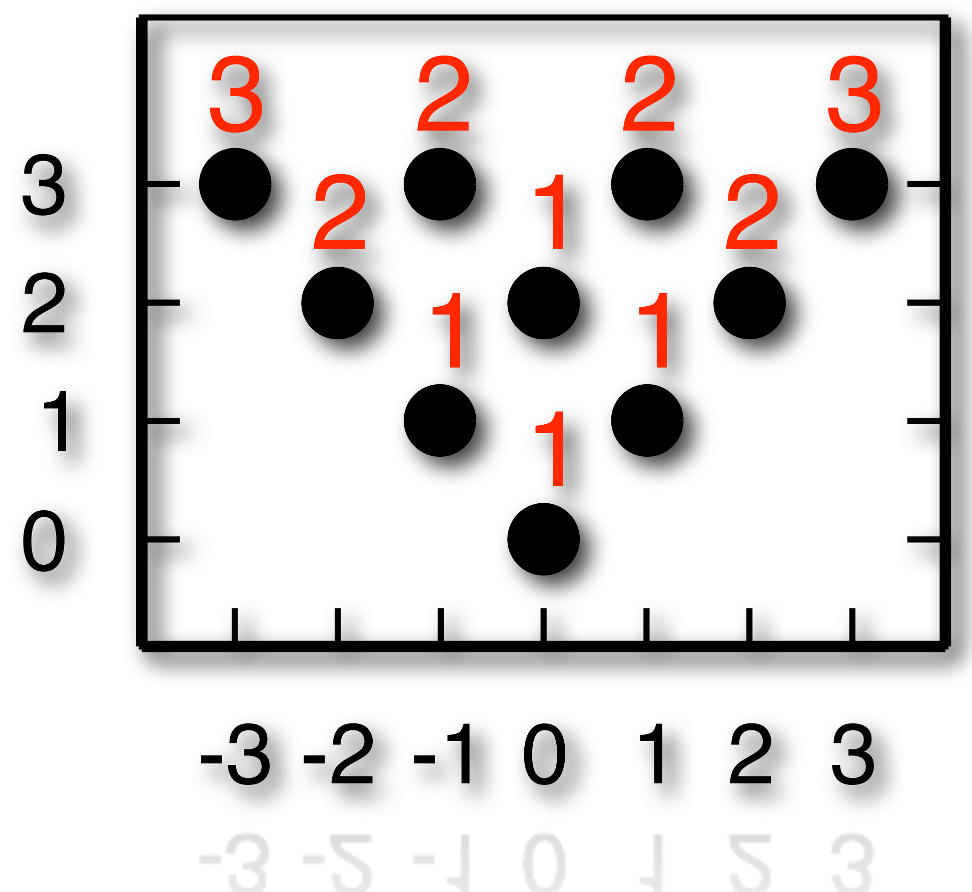


- How do the two chiral edges combine in the entanglement spectrum ?

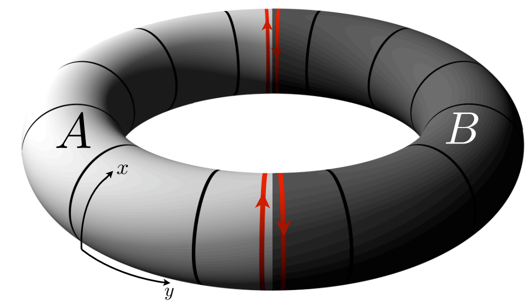
- Can we exploit the tunability of the aspect ratio to understand the entanglement spectrum quantitatively ?

Combining two chiral U(1) edges

- What do we expect to see when there are two linearly dispersing chiral U(1) modes?

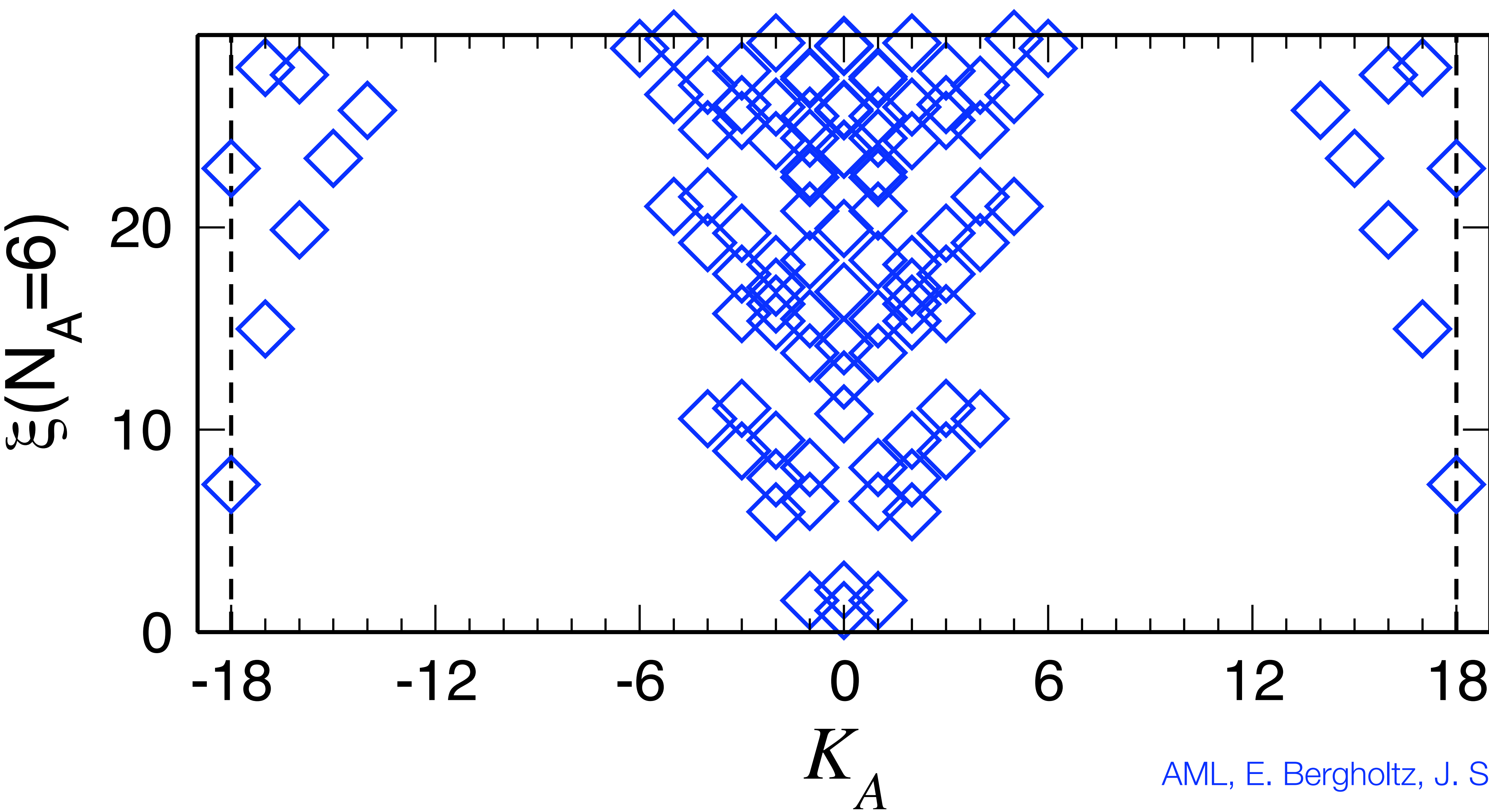


- This well known in the excitation spectrum of e.g. Luttinger liquids in spin chains



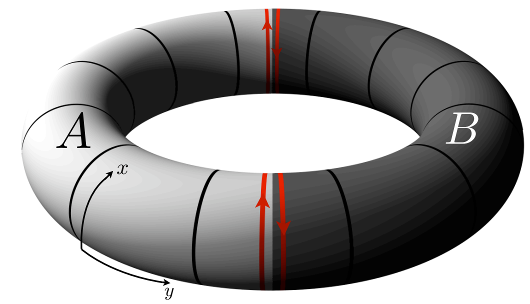
Torus entanglement spectrum

- $\nu=1/3$ Lauglin state, $N_s=36$, $L_1=10$, $L_A=N_s/2=18$, $N_A=6$

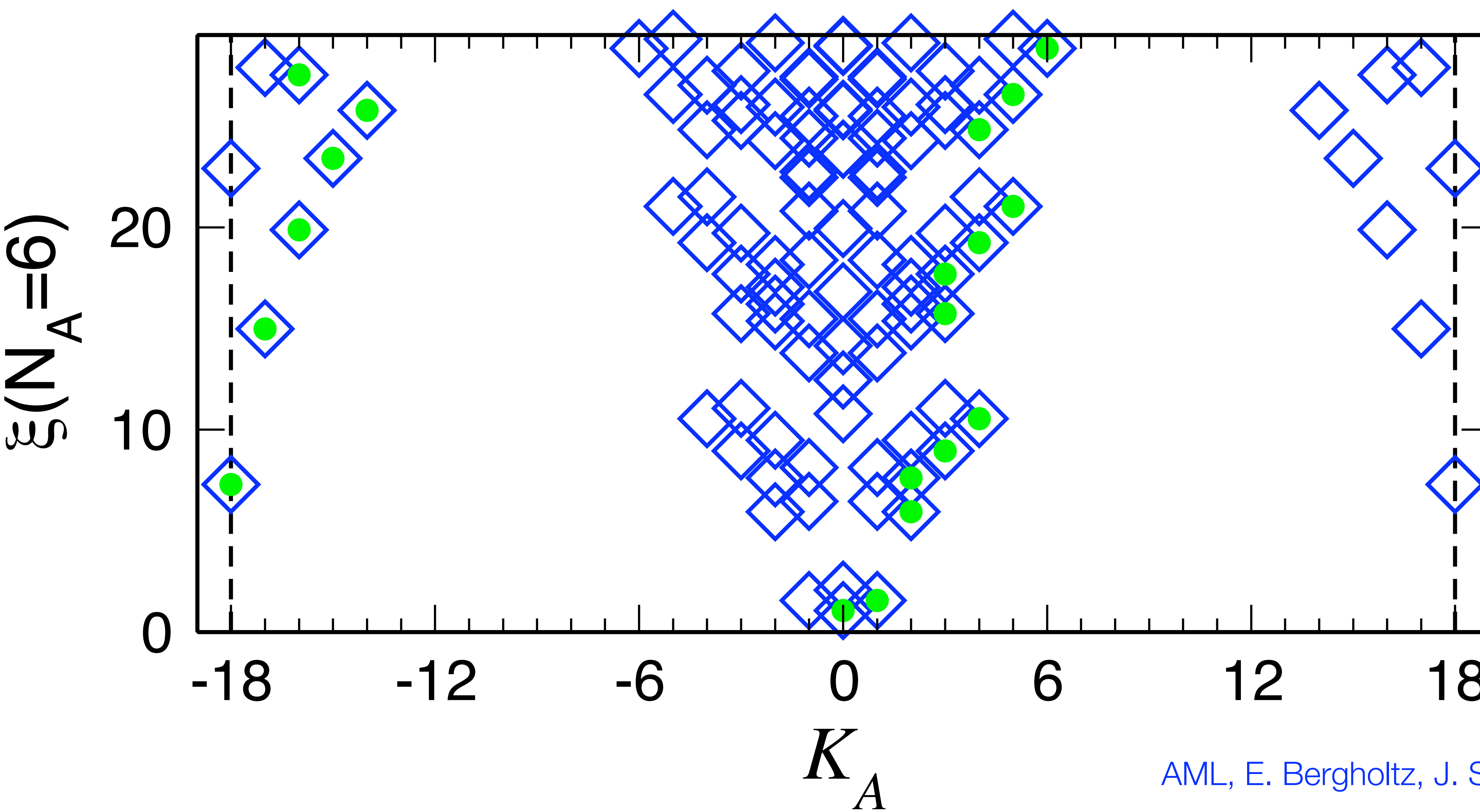


AML, E. Bergholtz, J. Suorsa & M. Haque
arXiv:0911.5477

Assigning edge levels

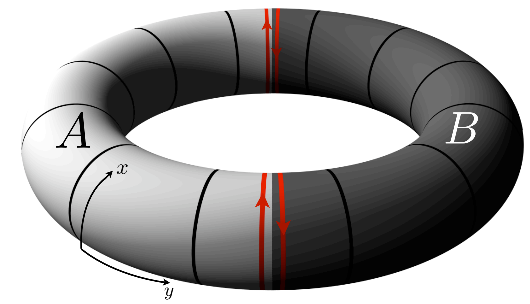


- Key step: find algorithmically all edge levels by relying on a two independent edge hypothesis

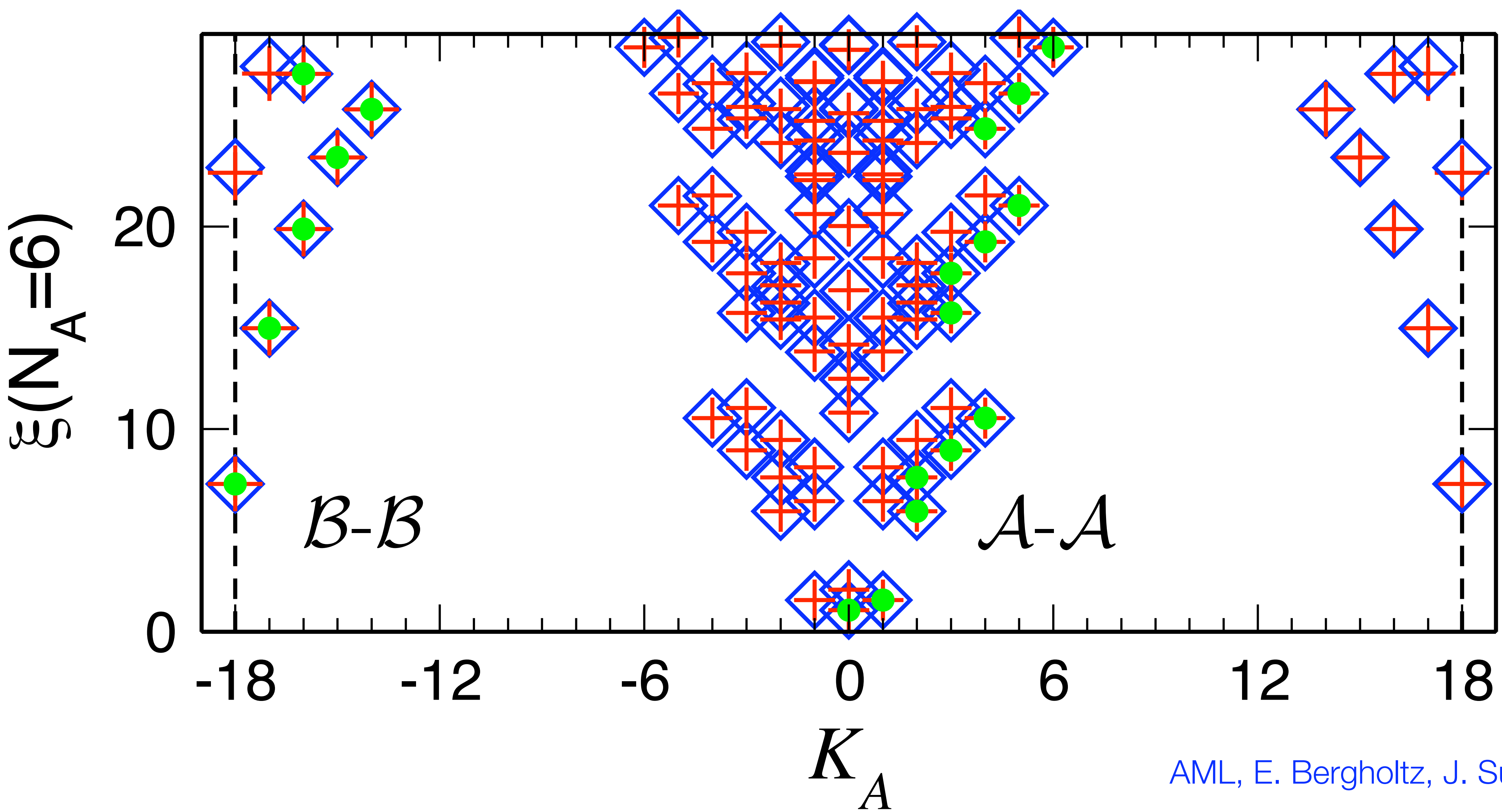


AML, E. Bergholtz, J. Suorsa & M. Haque
arXiv:0911.5477

The two edge hypothesis at work



- Excellent match between the actual entanglement spectrum (tilted squares) and the two edge prediction (crosses)



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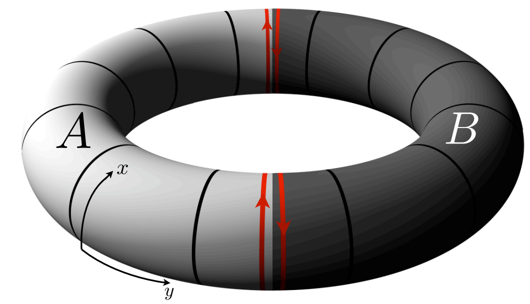
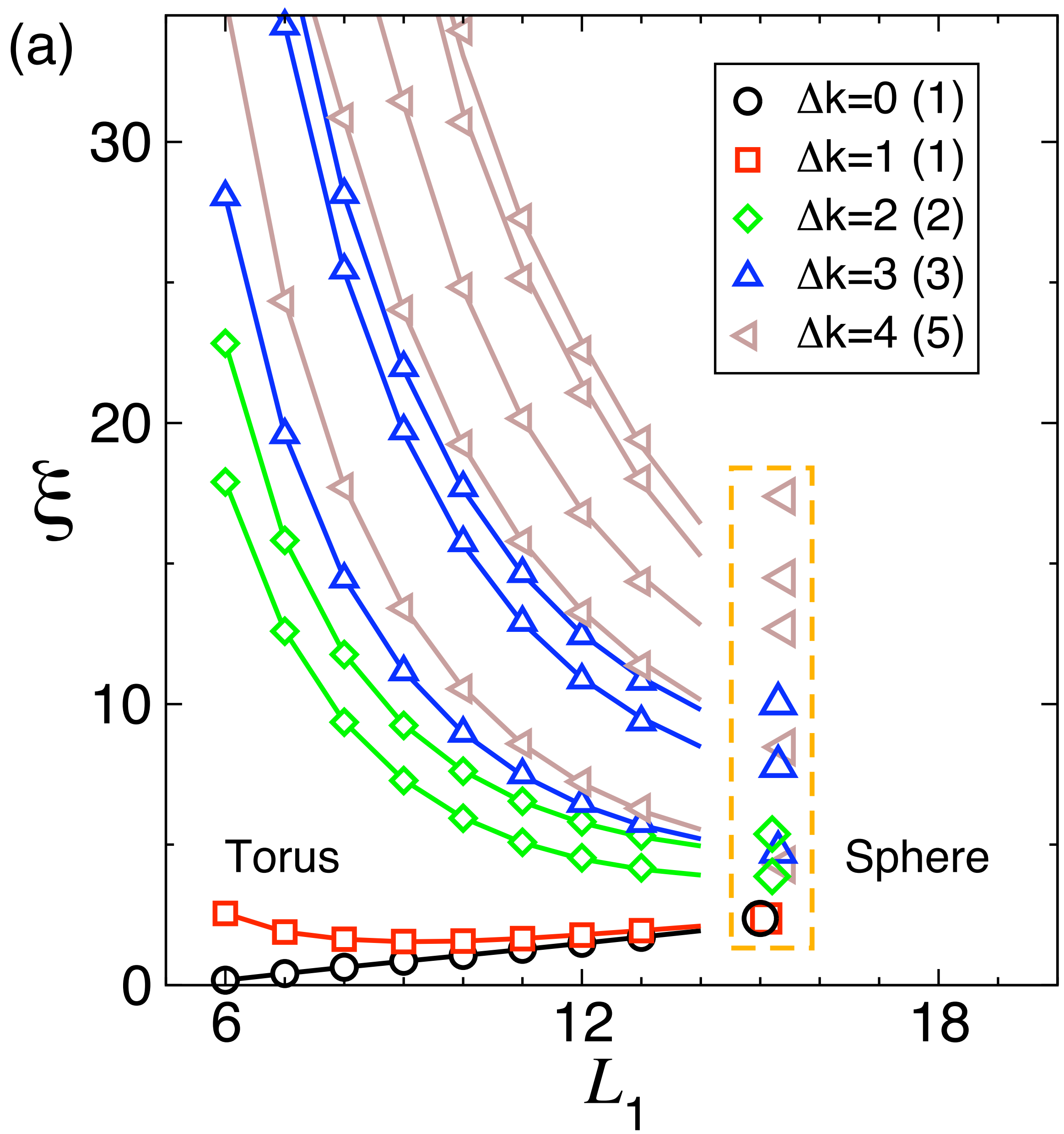
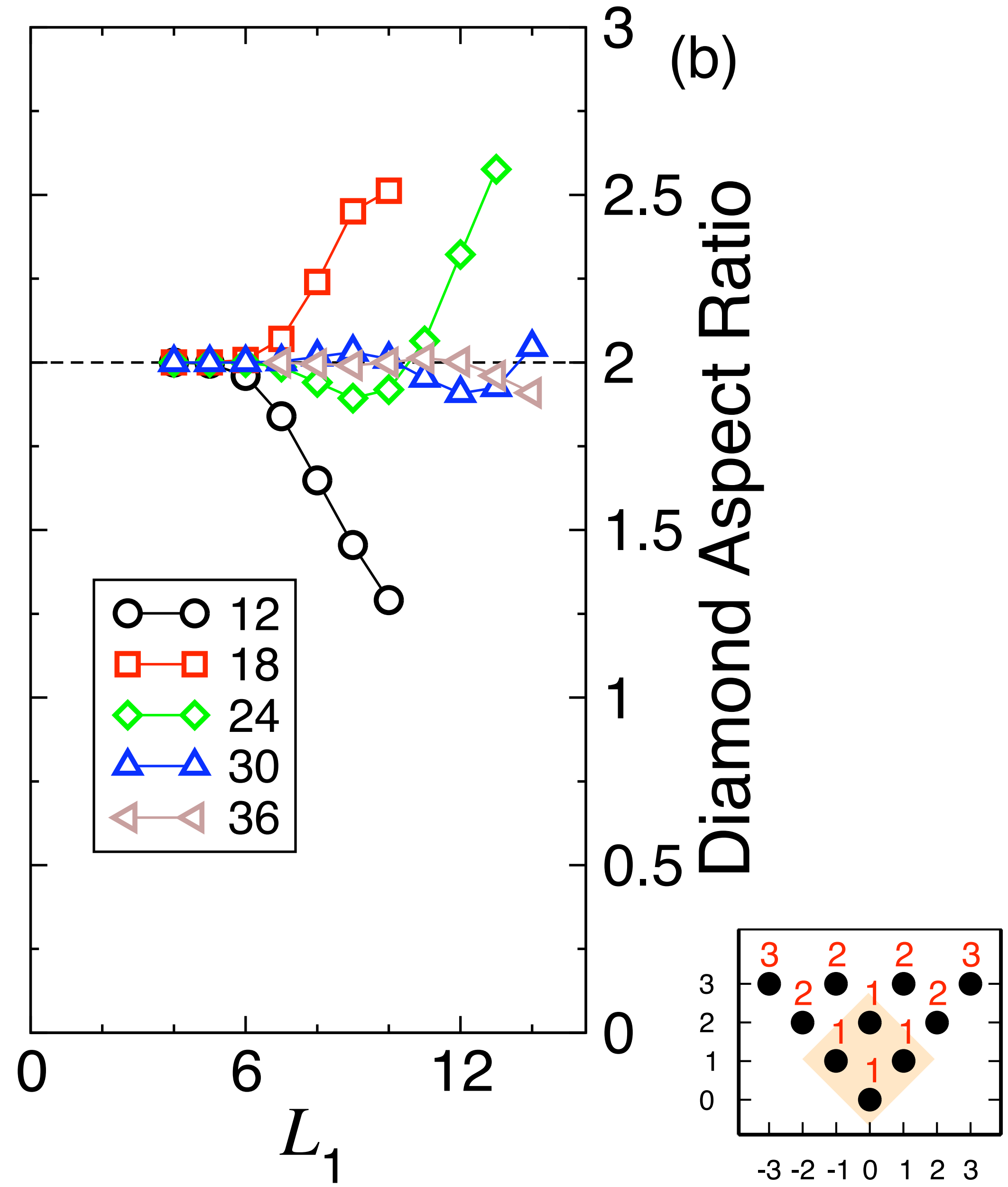
L1 dependence of chiral edge levels



- Chiral edge theory has the correct U(1) count [1-1-2-3-5-....] (not enforced) !





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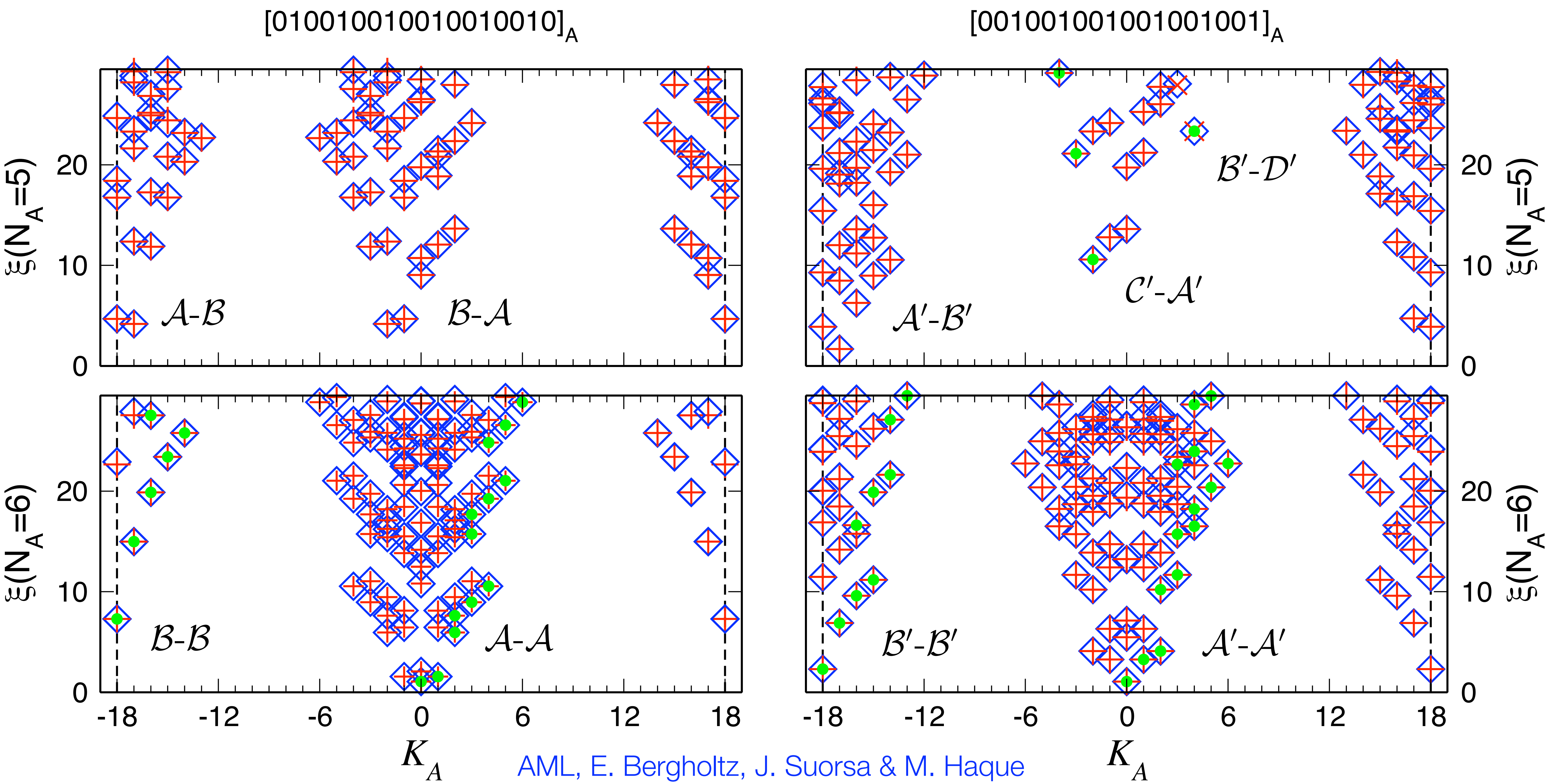
arXiv:0911.5477

- Adiabatic evolution: perform perturbation theory for small L_1

The predictive power of the two edge hypothesis

- Based on some simple microscopic picture, one can predict the occurrence and the type of energetics of many towers, even with different N_A



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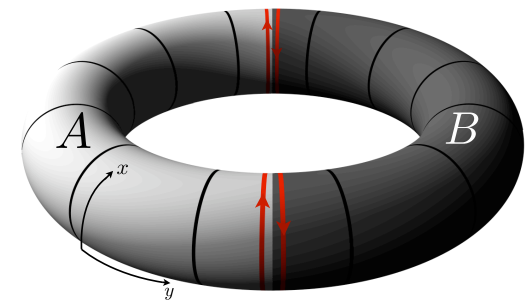
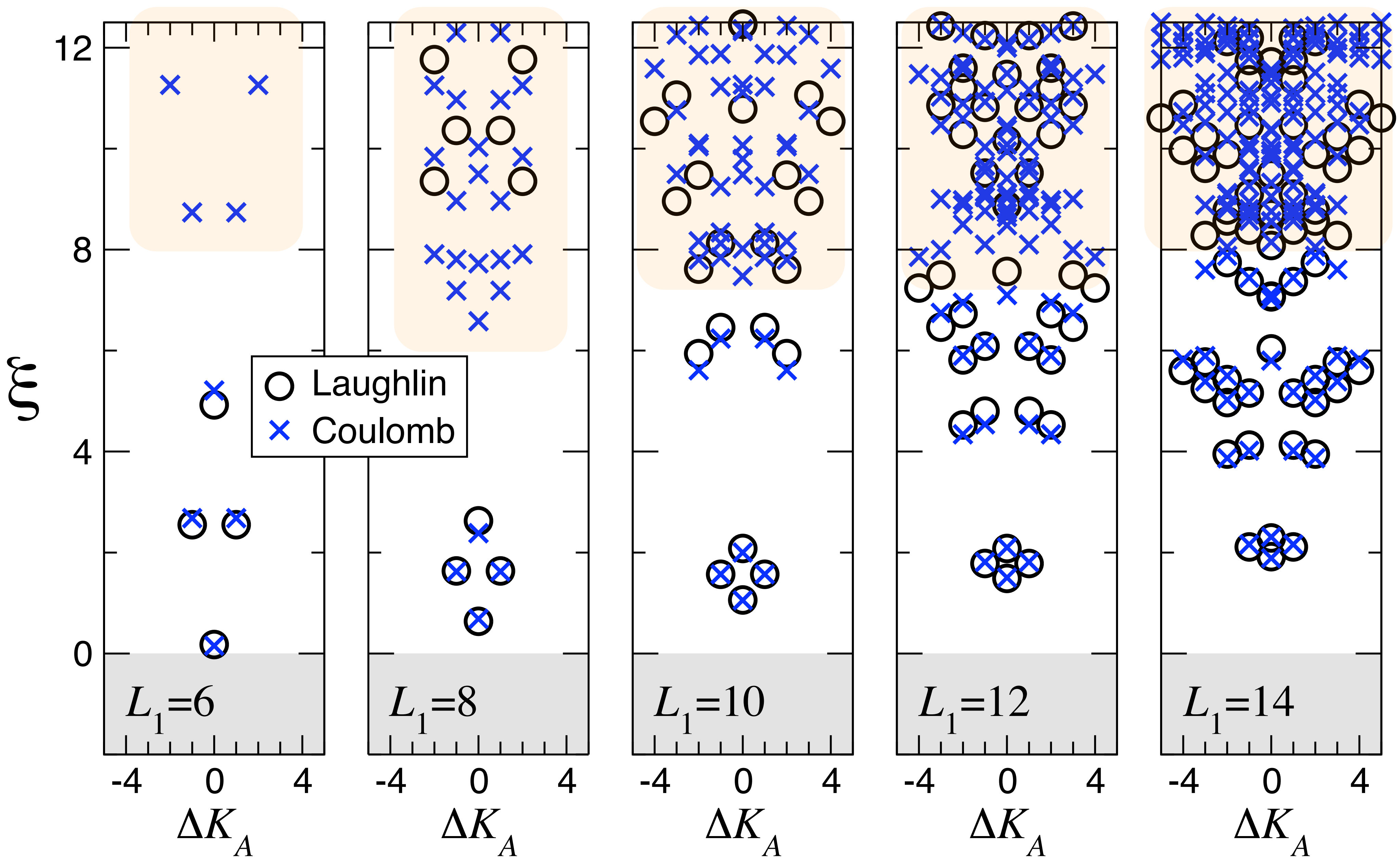
Coulomb vs Laughlin states at ν=1/3



- More and more U(1) structure emerging in the Coulomb state with increasing L_1



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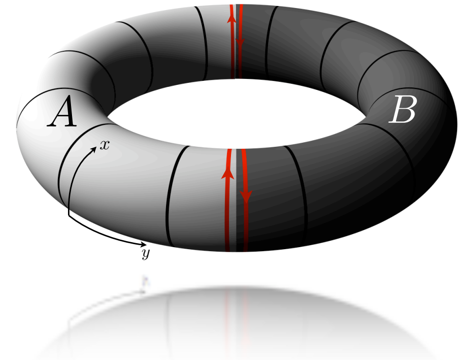
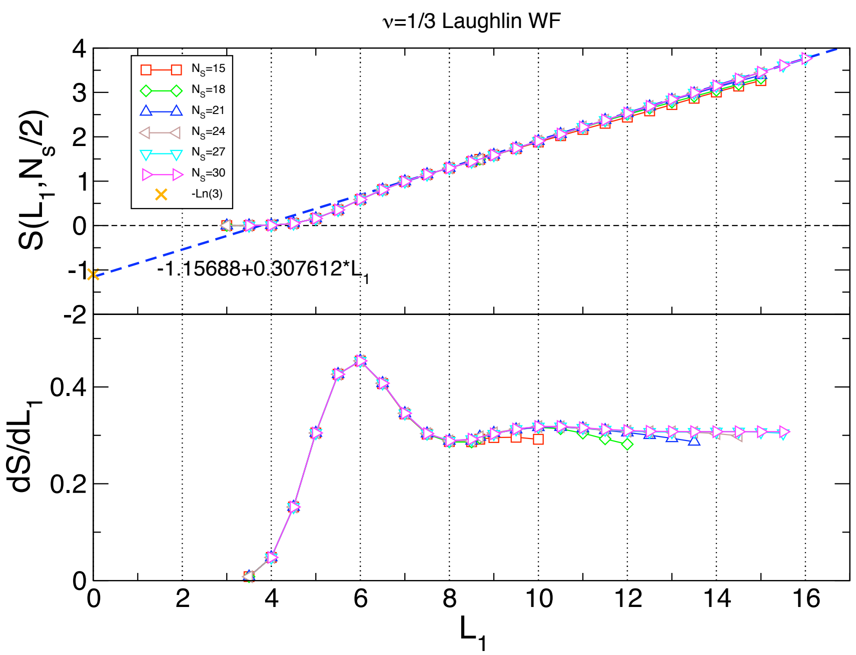
arXiv:0911.5477



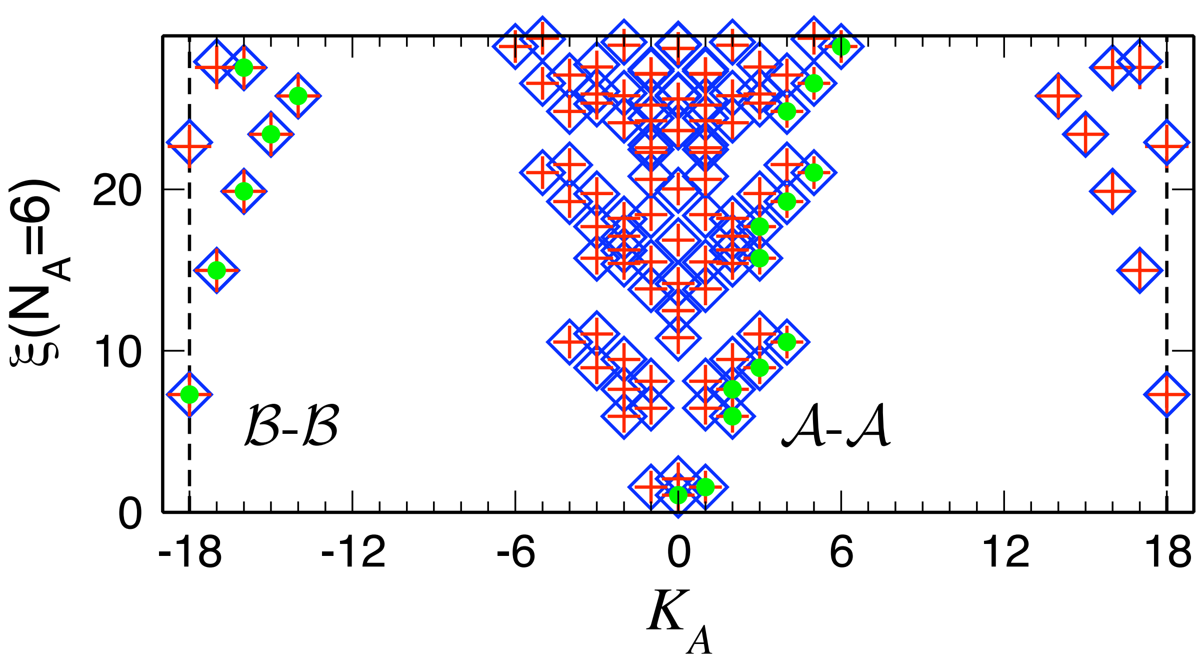
Conclusions

Topological Entanglement Entropy



- Exploiting the advantage of the torus to continuously change the circumference allows to get a significantly better estimates for the topological entanglement entropy.

Entanglement Spectrum



- Fascinating combination of two spatially separated edges to form conformal towers with correct Virasoro count. Applying now to more complicated fractions, such as $\nu=5/2$. Interesting also for lattice models.

AML, E. Bergholtz, J. Suorsa & M. Haque
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Thank you !