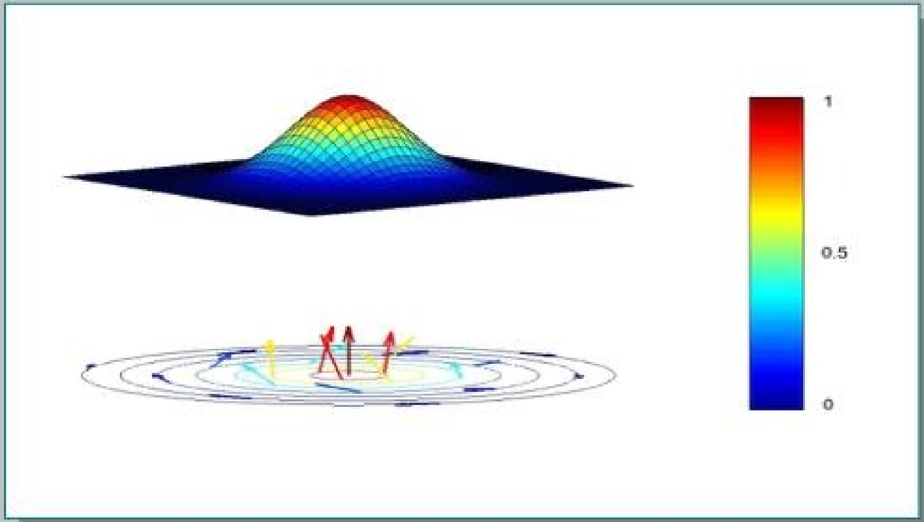
SO(5) Theory of High Tc Superconductivity

Shou-cheng Zhang



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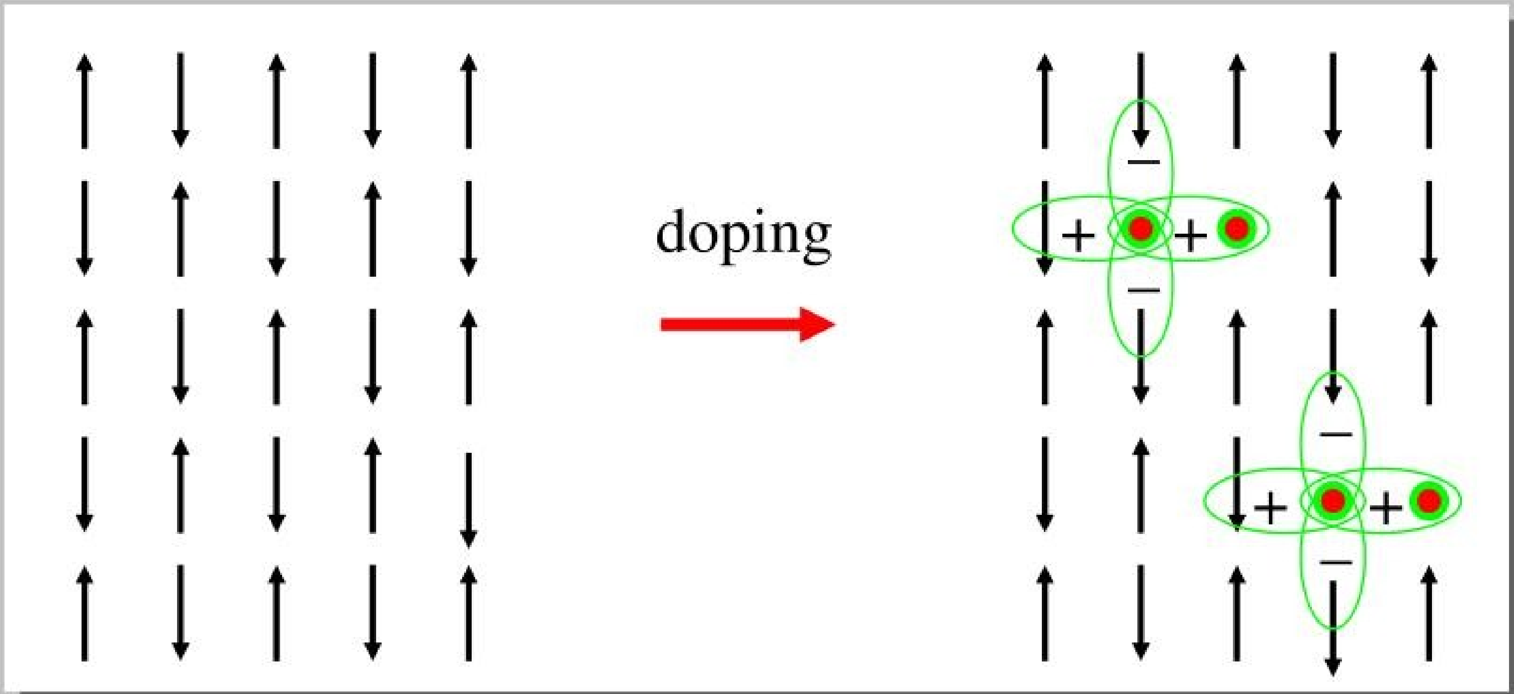
Outline

- **Introduction to high Tc superconductivity and SO(5) theory**
  - The central question: AF & dSC
- **T-J model and the pSO(5) model**
  - LG theory not sufficient.
- **Comparison with numerical results**
  - AF/SC coexistence state, multiplets, phase diagram
- **Experimental consequences**
  - AF vortex core, phase diagram…
- **Conclusions**

Microscopic models of high Tc

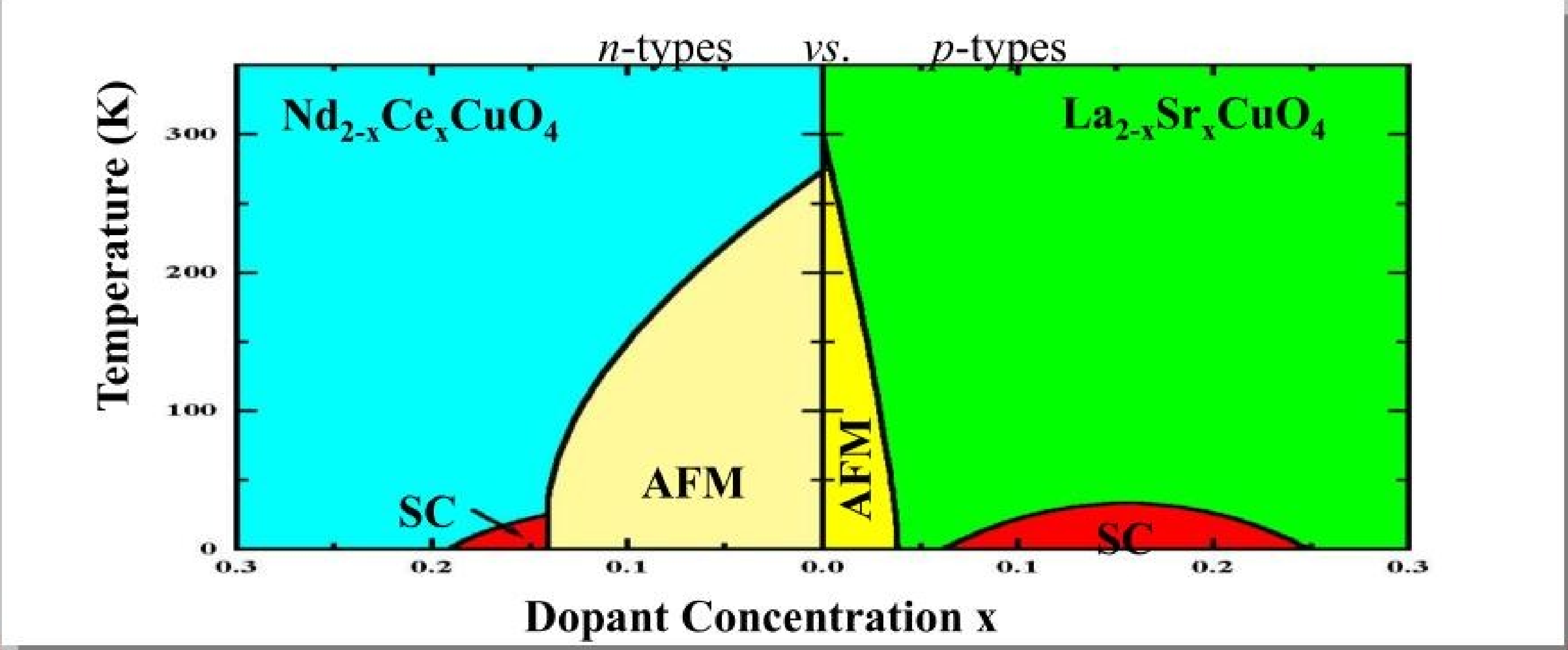
- **t-J model of spins and holes**

$$H = -t \sum_{<i,j>} c_\sigma^+(i) c_\sigma(j) + J \sum_{<i,j>} S^\alpha(i) S^\alpha(j)$$

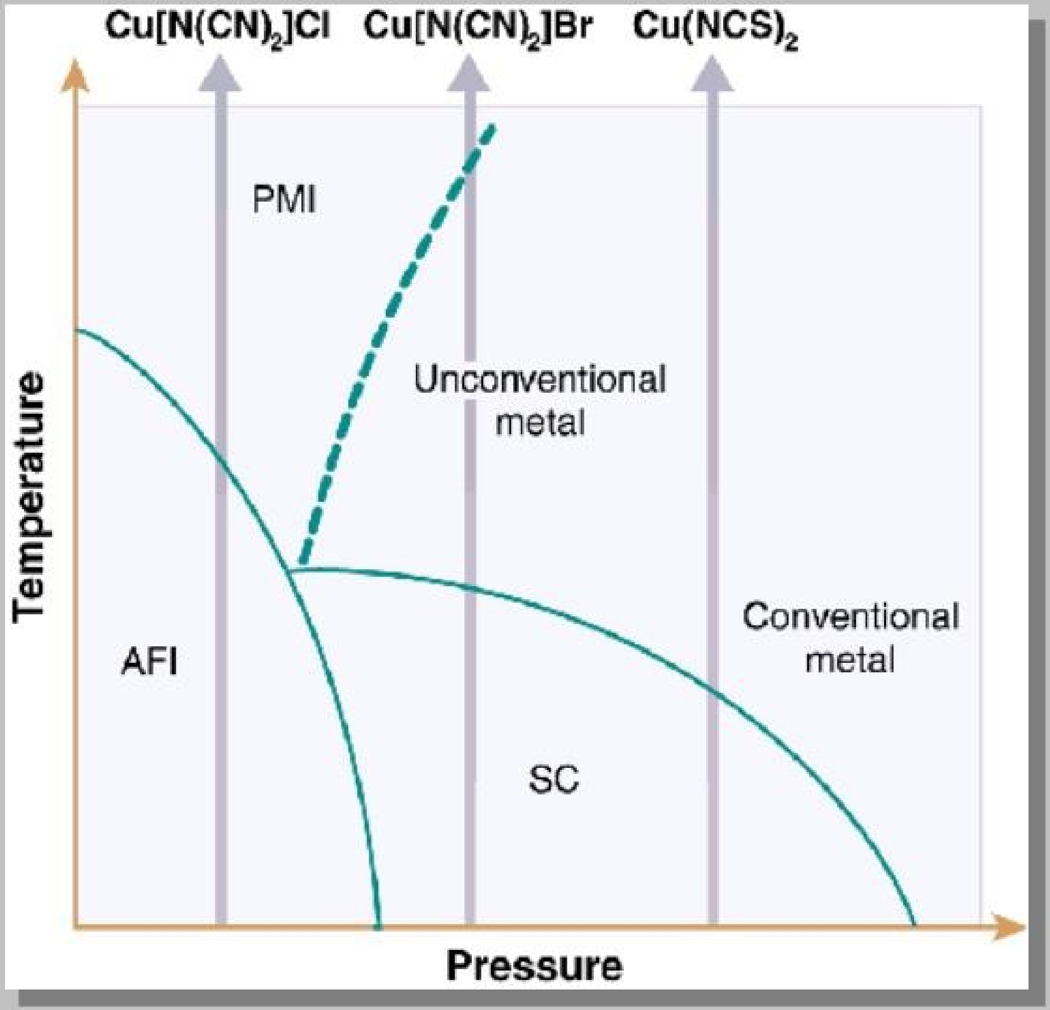


Fundamental questions

- What is the relationship between AF and SC?
  - How do we understand the phase diagram?
  - Does AF lead to SC pairing?



Phase diagram of the κ-bedt salt



SO(5) order parameters

- **AF order parameter**

$$N_i = (N_x, N_y, N_z) = \sum_k c^+_{Q+k} \sigma_i c_k, \qquad Q = (\pi, \pi)$$

- **SC order parameter**

$$\Delta_i = (\mathrm{Re}\,\Delta, \mathrm{Im}\,\Delta), \Delta = \sum_k g(k) c_{k\uparrow} c_{-k\downarrow}, g(k) = \cos k_x - \cos k_y$$

- **SO(5) superspin order parameter**

$$n_a = (\mathrm{Re}\,\Delta, N_x, N_y, N_z, \mathrm{Im}\,\Delta)$$

SO(5) algebra

- **The π operators:**

$$\pi_i = \sum_k g(k) c_{Q+k} \sigma_i \sigma_y c_{-k}$$

- **The SO(5) algebra:**

$$[L_{ab}, L_{cd}] = i\delta_{ac} L_{bd} + perm.$$

$$L_{ab} = \begin{pmatrix} 0 & & & & \\ \mathrm{Re}\,\pi_x & 0 & & & \\ \mathrm{Re}\,\pi_y & -S_z & 0 & & \\ \mathrm{Re}\,\pi_z & S_y & -S_x & 0 & \\ Q & \mathrm{Im}\,\pi_x & \mathrm{Im}\,\pi_y & \mathrm{Im}\,\pi_z & 0 \end{pmatrix}$$

SO(5) effective field theory

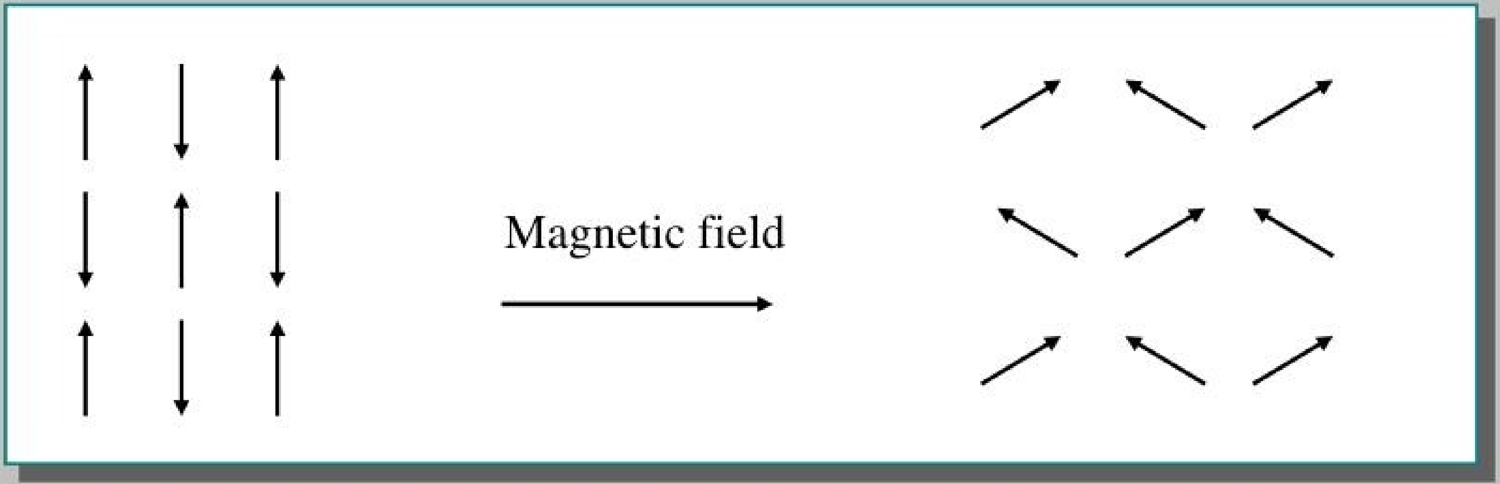
- **Quantum rotor model:**

$$H = \frac{1}{2\chi}\sum_i {L_{ab}}^2(i) + \frac{\rho}{2}\sum_{<i,j>} n_a(i) n_a(j) - g\sum_i ({n_2}^2 + {n_3}^2 + {n_4}^2) - \mu \sum_i Q(i)$$

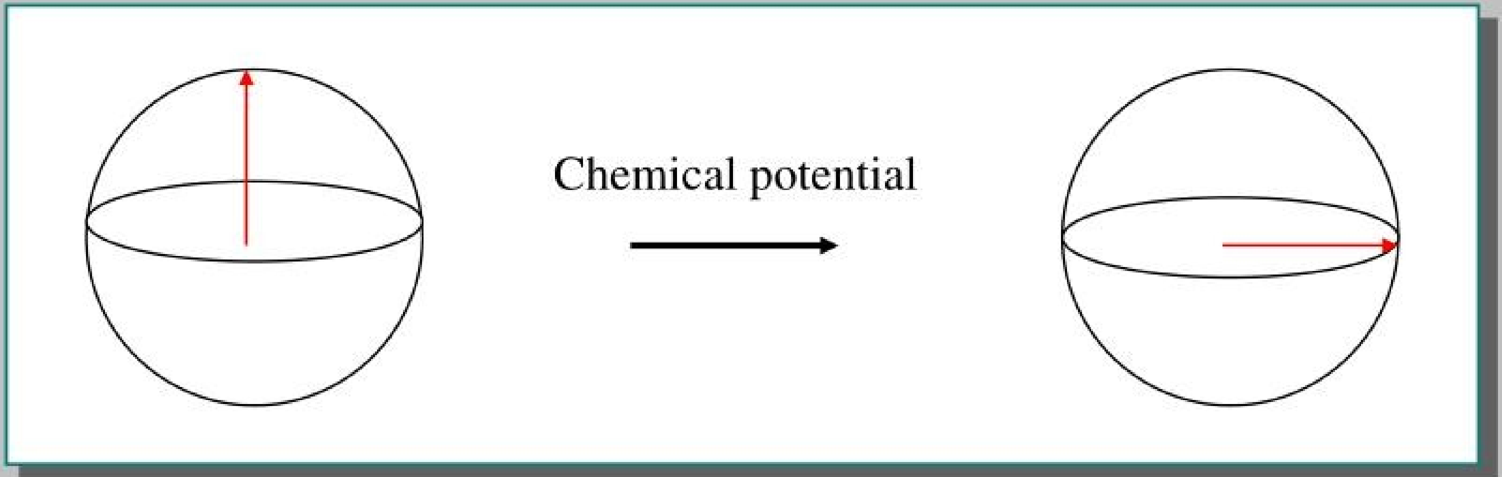
- **g term describe the anisotropy in SO(5) space, the chemical potential μ term describe the effect of doping. These two terms compete with each other.**

The superspin flop transition

- **Easy axis AF to easy plane AF transition**

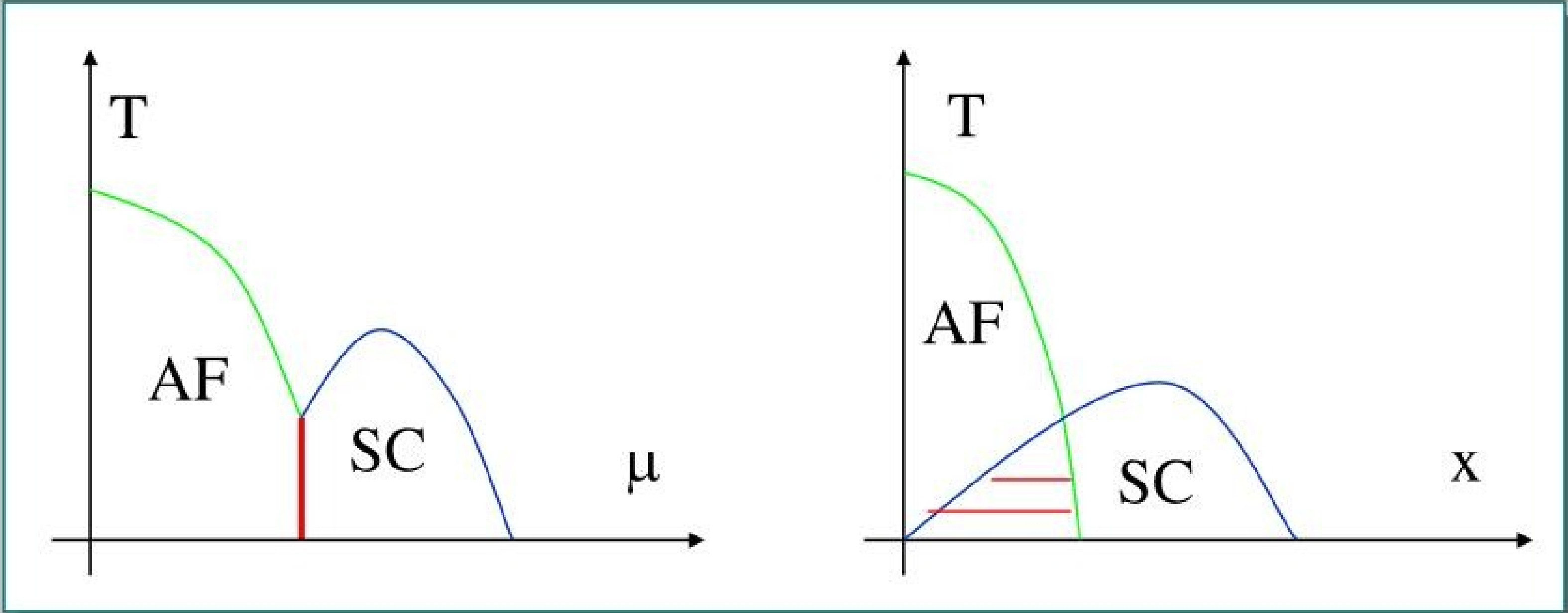


- **AF to SC transition**



Phase diagram of the SO(5) theory

- **SO(5) phase diagram predicts**
  - SO(5) bicritical point
  - Coexistence of AF and SC as a function of x
  - Pseudogap=preformed SO(5) superspin

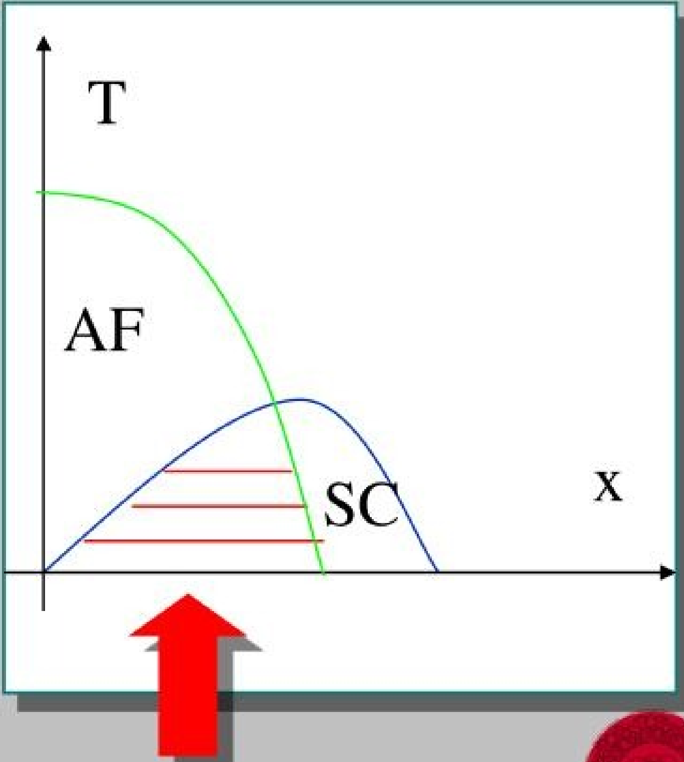


Phase separation vs uniform mix state

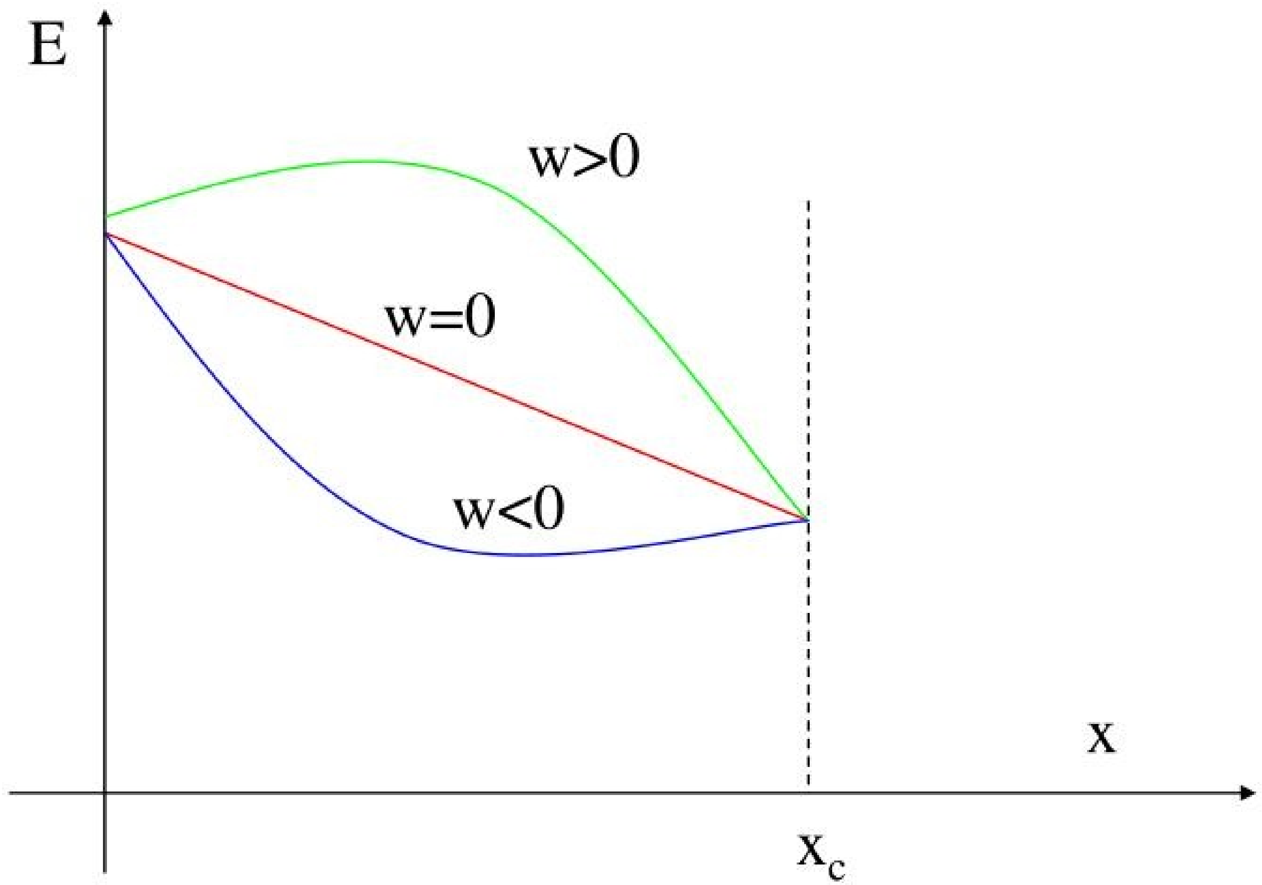
- **General form of the free energy:**

$$F \propto a\Delta^2 + bN^2 + u(\Delta^2 + N^2)^2 + w\Delta^2 N^2$$

- For w>0, (type 1) phase separation or stripes. =>LSCO
- For w<0, (type 2) uniform mix phase. =>YBCO
- w=0, (type 1.5) => SO(5)
- Since LSCO and YBCO are not very different, w must be close to zero. =>SO(5) symmetric point!



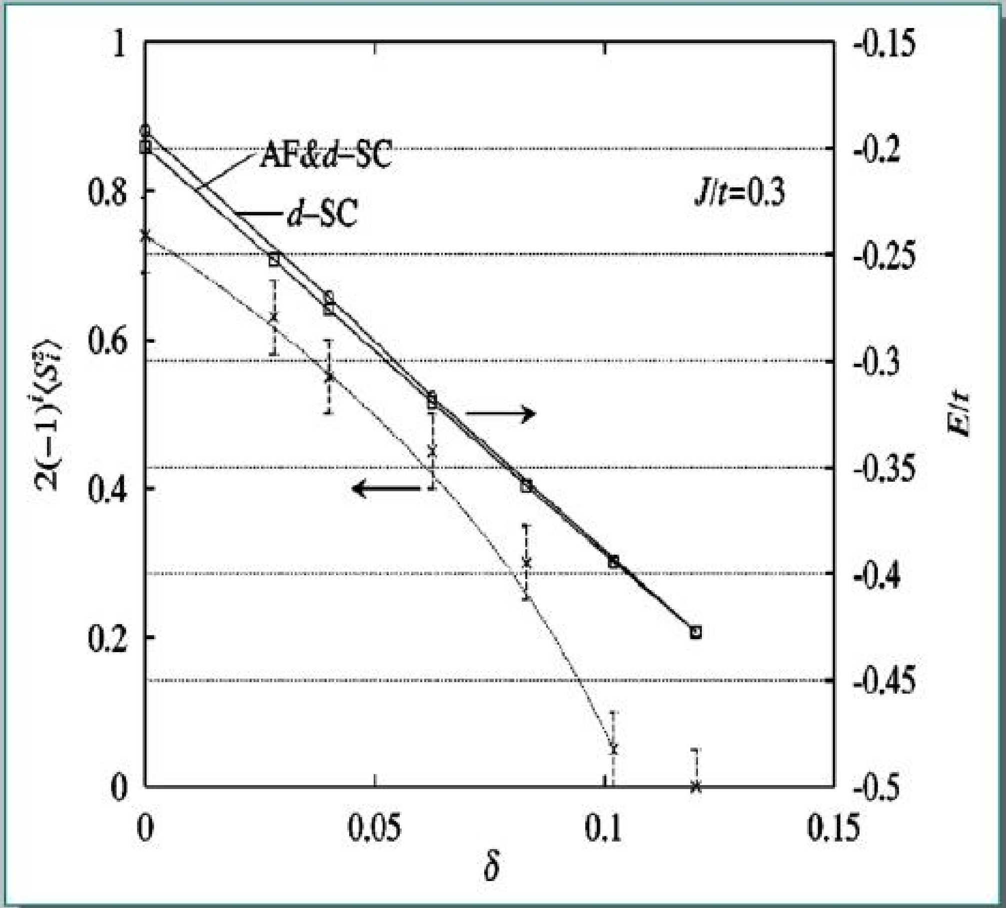
Doping dependence of ground state energy

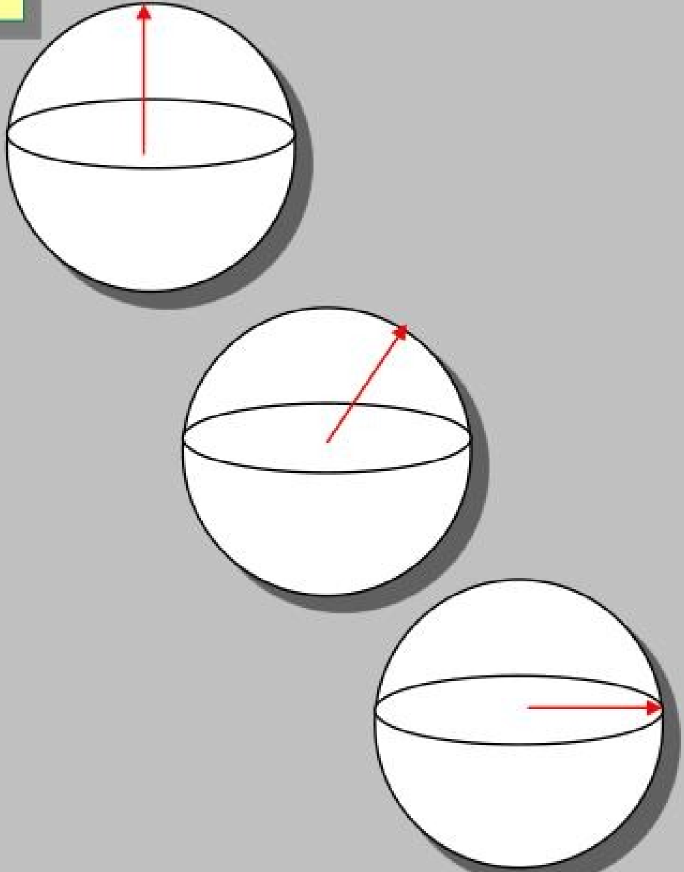


Microscopic evidence of SO(5)

Himeda and Ogata 1999

$$|\Psi\rangle = P_d P_N \,|\, \Delta_{dSC}, \Delta_{AF}, \mu\rangle$$

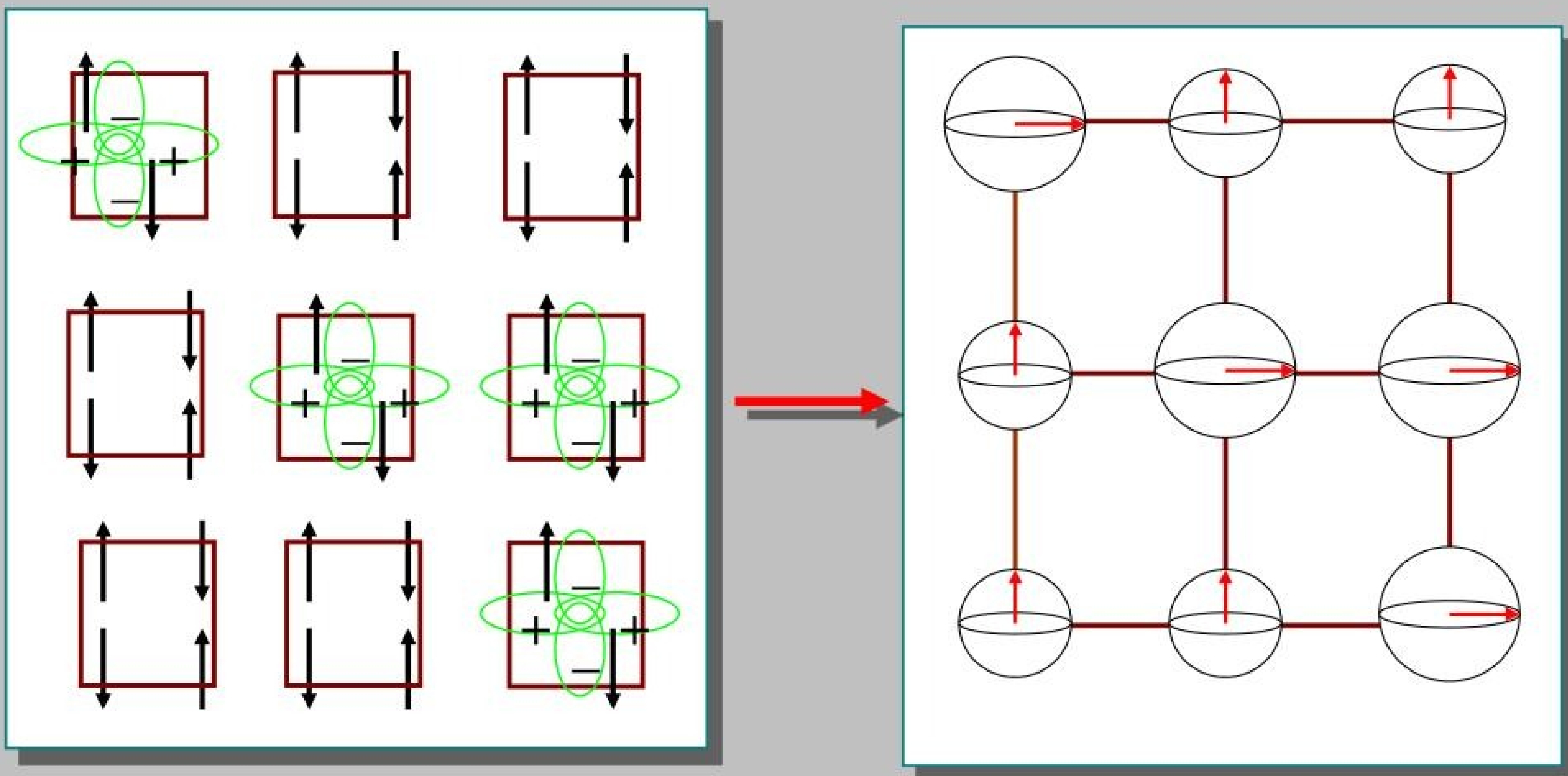




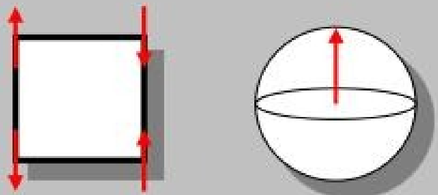
From the t-J model to the SO(5) model

Zhang et al, Altman and Auerbach

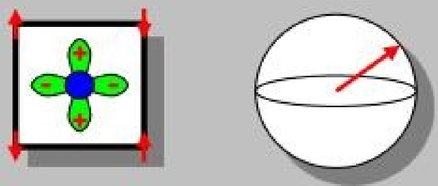
- **one step real space RG**



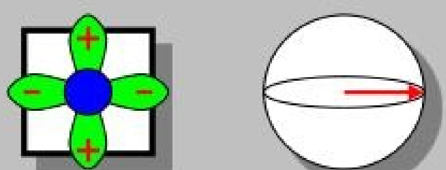
States on a plaquette



$$(Cos\theta + Sin\theta t_{\alpha}^{+}) \,|\, \Omega\rangle$$

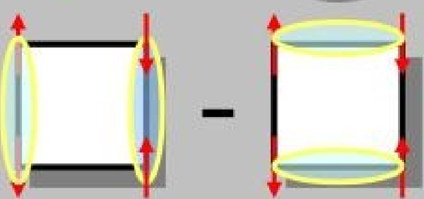


$$(Cos\theta + Sin\theta(Cos\alpha t_{\alpha}^{+} + Sin\alpha t_{h}^{+})) \,|\, \Omega\rangle$$

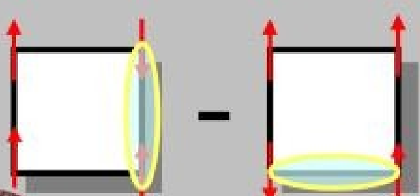


$$(Cos\theta + Sin\theta t_{h}^{+}) \,|\, \Omega\rangle$$

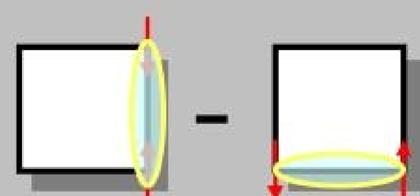
Represents hole density



$$|\, \Omega\rangle$$



$$t_{\alpha}^{+} \,|\, \Omega\rangle$$



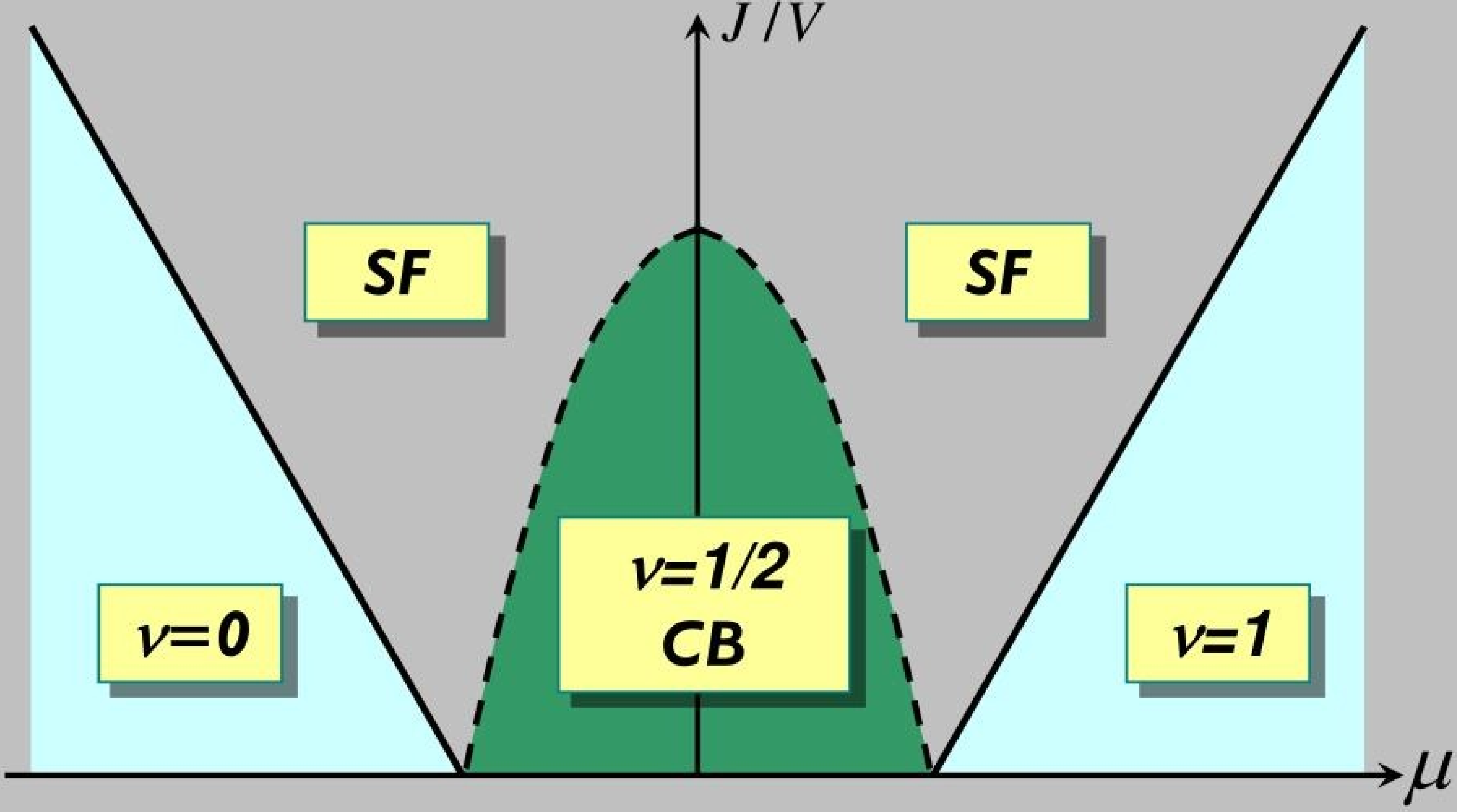
$$t_{h}^{+} \,|\, \Omega\rangle$$

Projected SO(5) model

$$H = \Delta_s \sum_x t_\alpha^+ t_\alpha + (\Delta_c - \mu) \sum_x t_h^+ t_h + J_s \sum_{xy} n_\alpha(x) n_\alpha(y)$$
$$+ J_c \sum_{xy} t_h^+(x) t_h(y) + c.c + V_1 \sum_{xy} \rho(x)\rho(y) + V_2 \sum_{xy'} \rho(x)\rho(y')$$

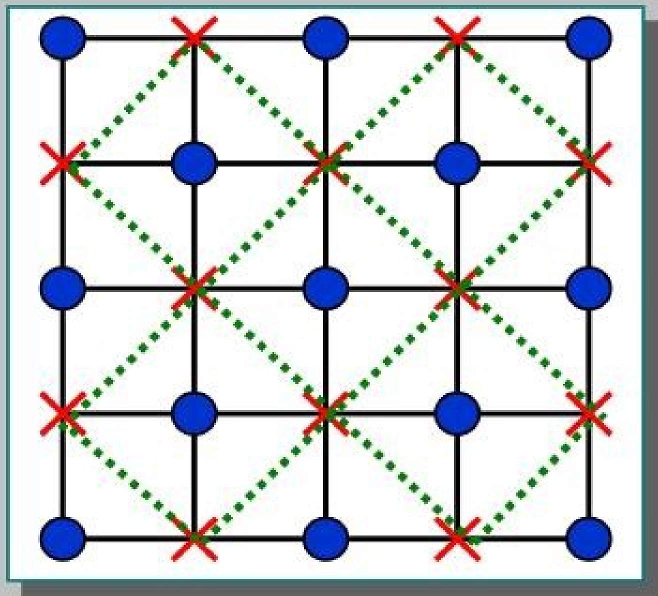
- **Each site on the SO(5) model represents a 2x2 square in the real lattice.**
- **Competition: Magnon and hole pair kinetic energies J_s and J_c favor uniform phases. Coulomb interactions V_1 and V_2 favor checkerboard charge ordering.**
- **If we ignore the magnetic degree of freedom, this reduces to a hard-core boson model, with well-understood phase diagram.**

Phase diagram of the pSO(5) model: Charge sector

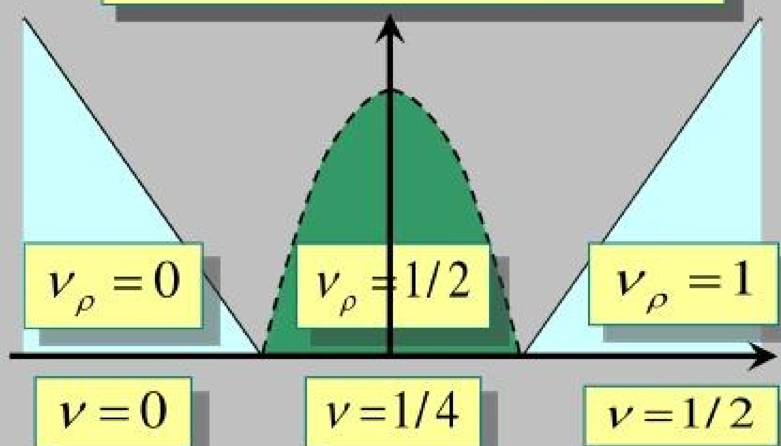


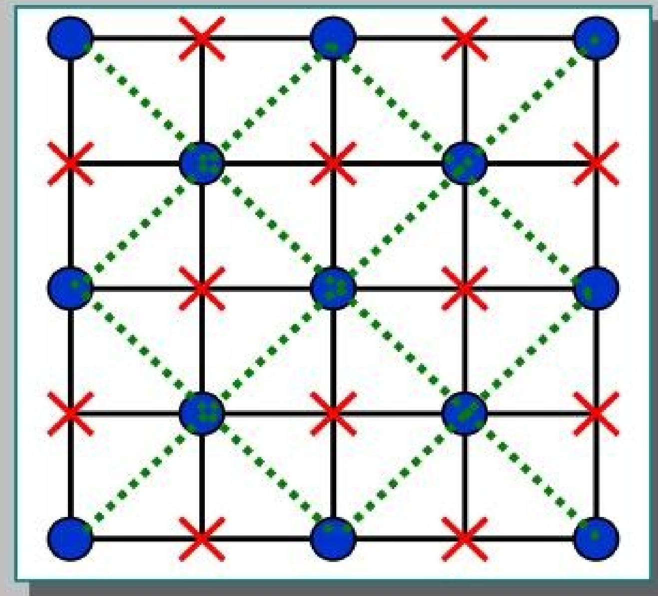
SF = Superfluid   CB = Checkerboard

Superlattice and Quarter Filling

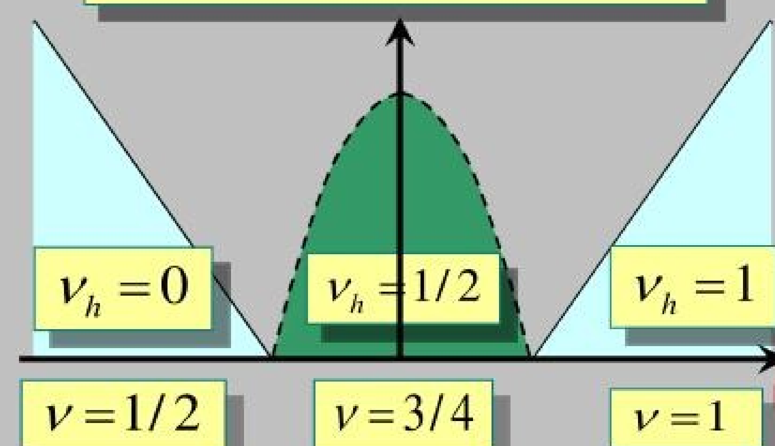


Particle superlattice interpolates between n = 0 & n = 1/2

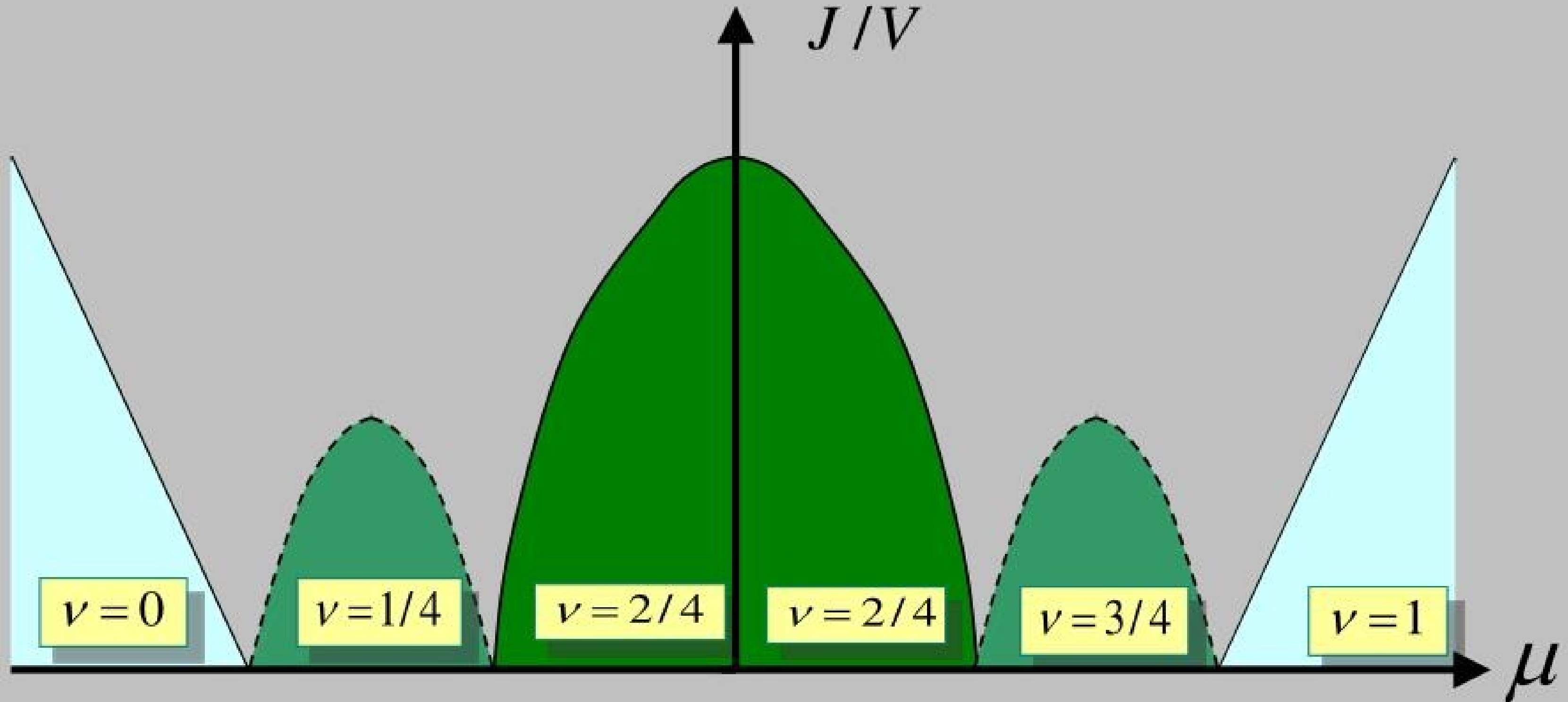




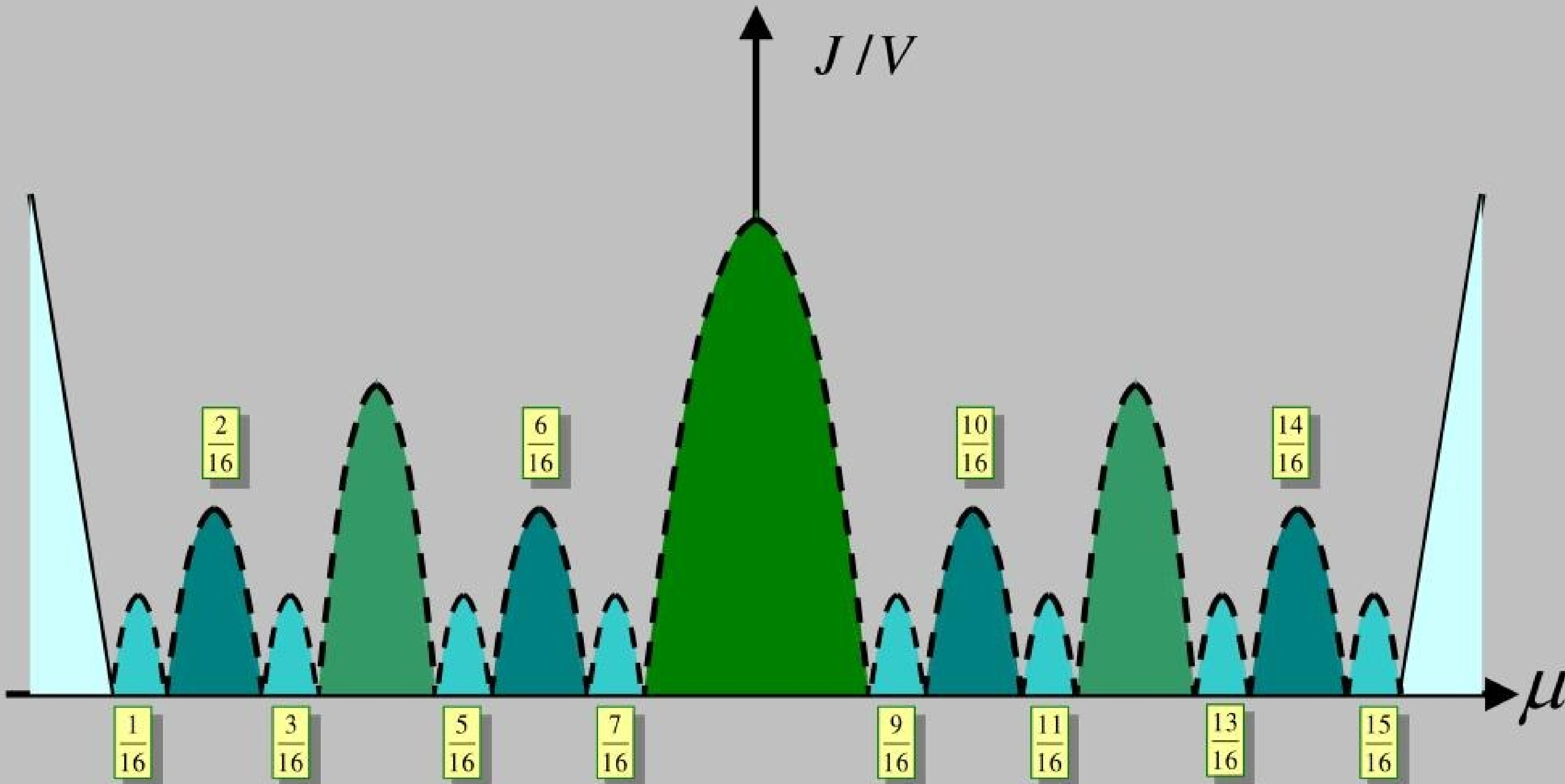
Hole superlattice interpolates between n = 1/2 & n = 1



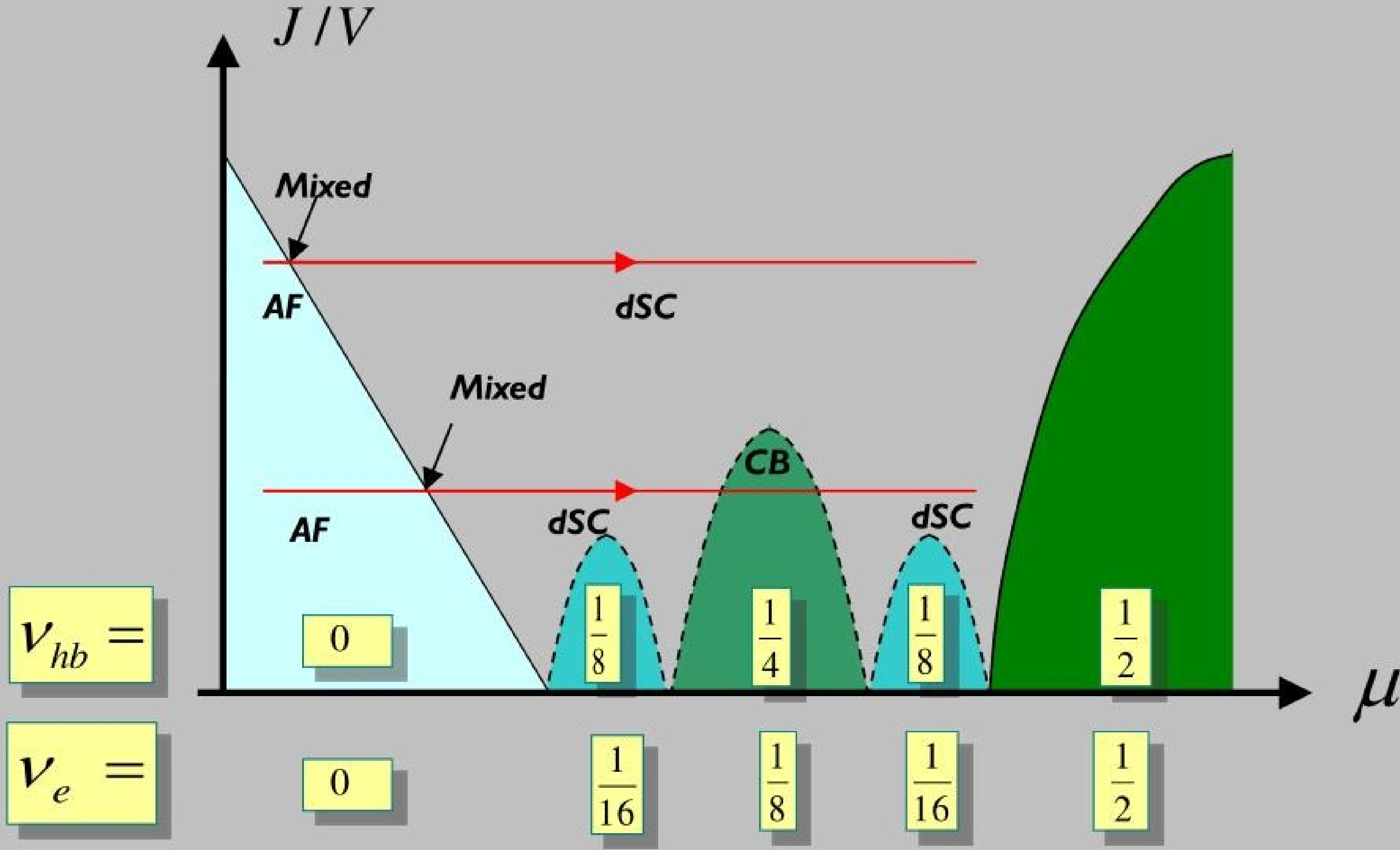
Combine n=1/2 CB State

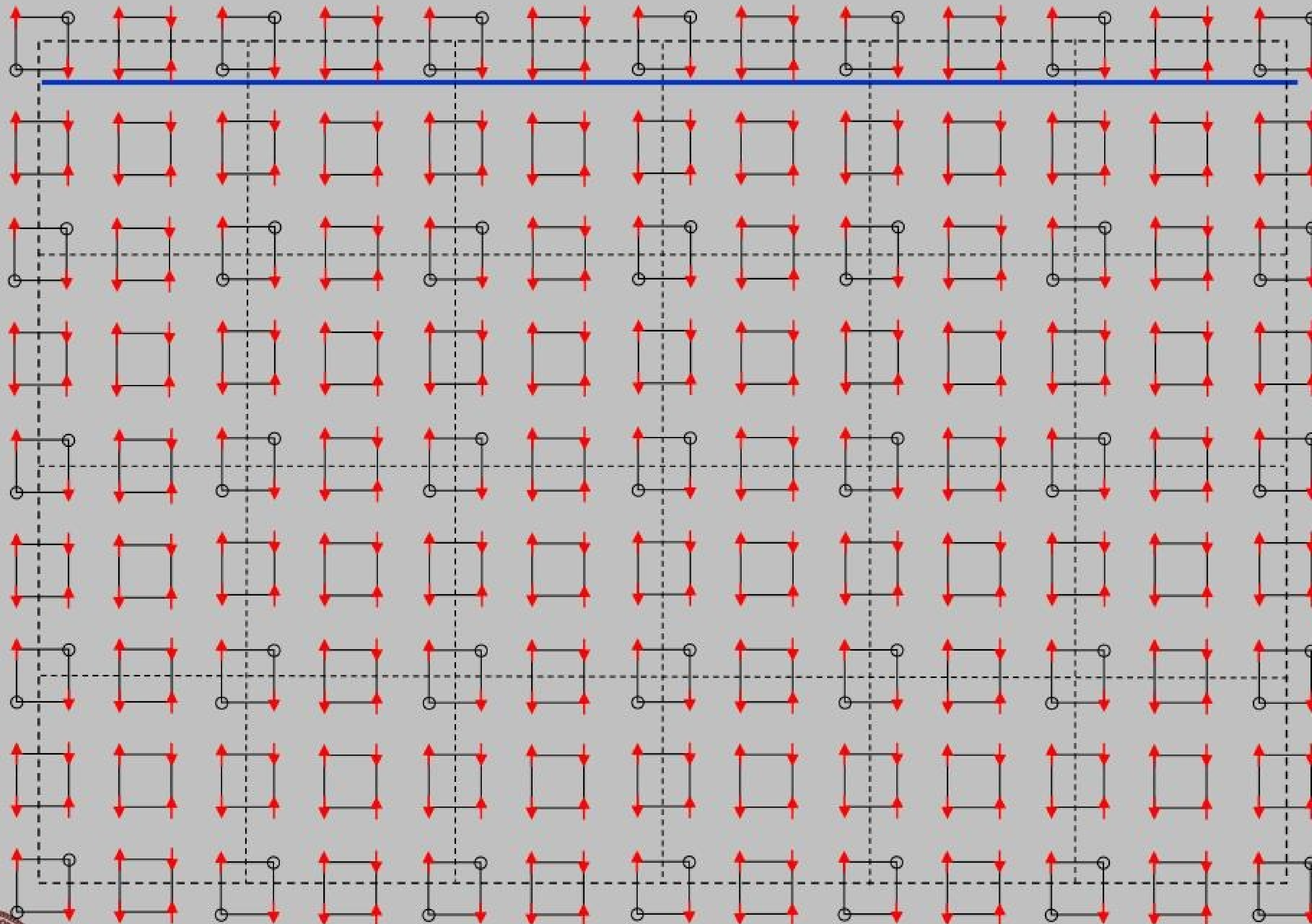


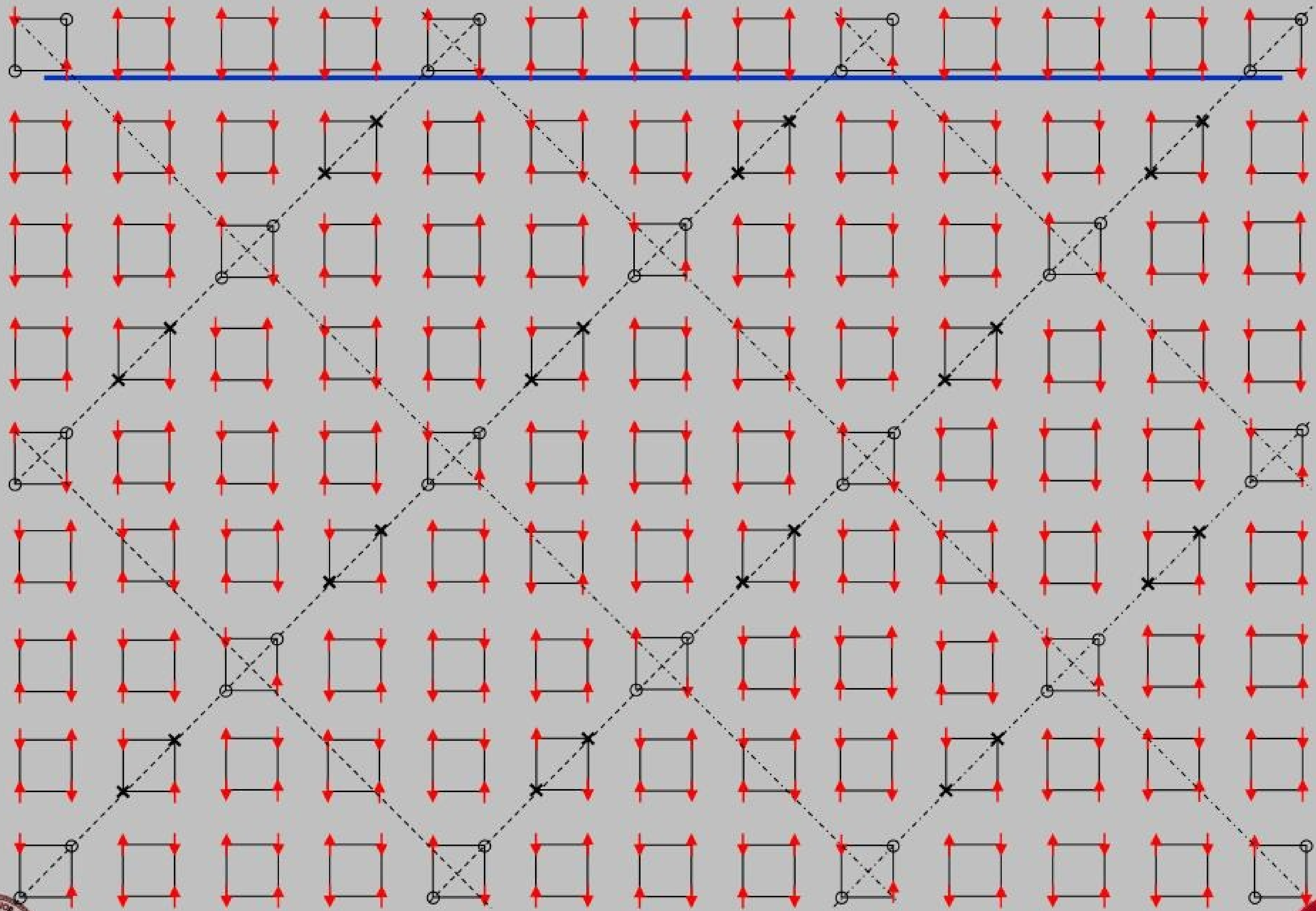
Global Phase diagram



Different Types of Behavior of High Tc

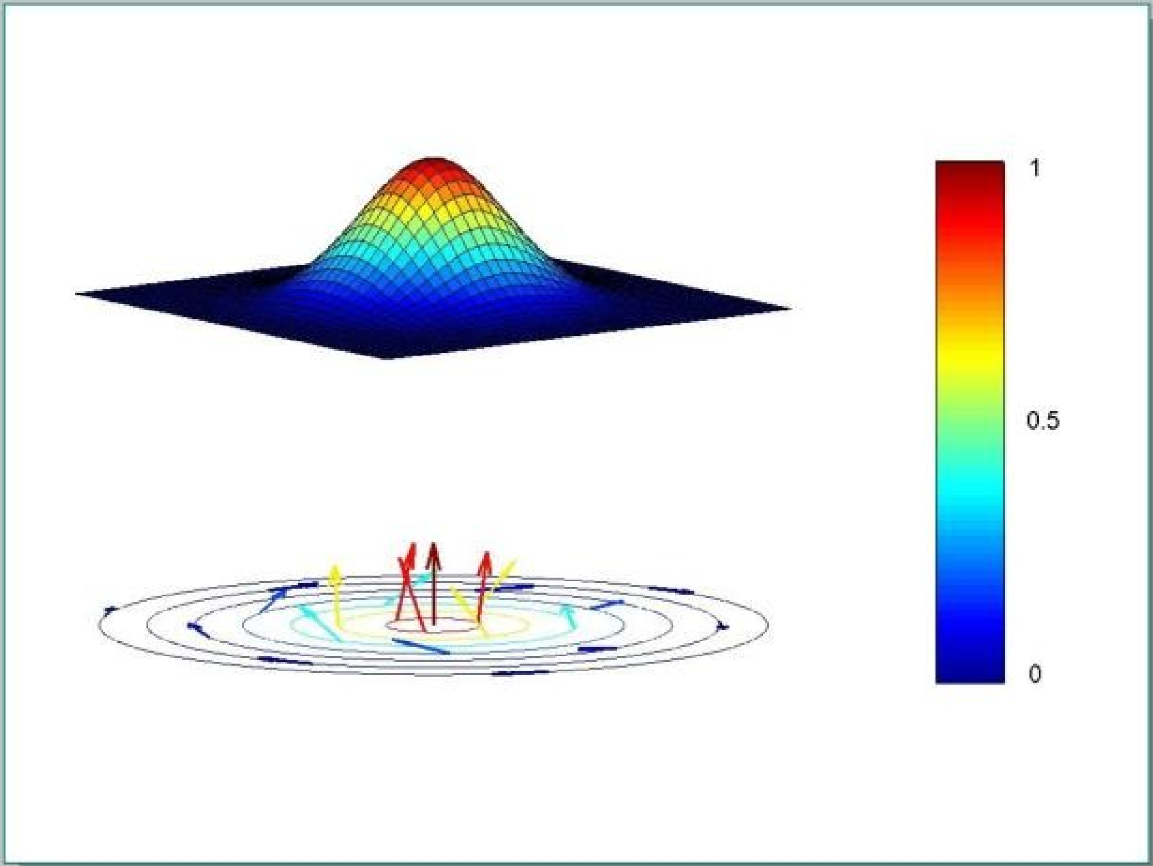






SO(5) prediction of the AF vortex state

- **Rotation of the superspin as the center of the vortex core is approached**
  - Field induced AF moment is proportional to the applied B field.
  - We can tune a new nob, the magnetic field, to study Mott insulator to SC transition.
  - Theoretical prediction first confirmed by the numerical calculations on the t-J model.

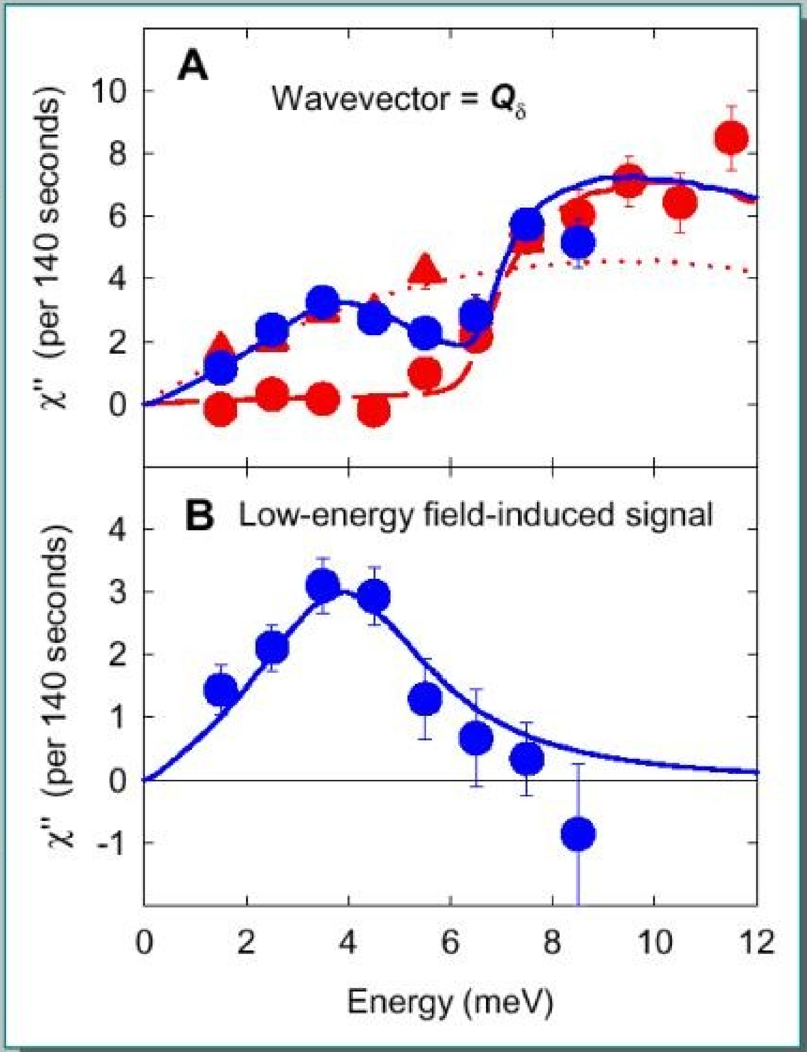


Experimental evidence of the AF vortex state

- **Neutron scattering in LSCO**
  - Field induced moment is proportional to the field
- **μSR in underdoped YBCO**
  - Staggered magnetic field of 18 Gauss from the vortex core centers
- **NMR in optimally doped YBCO and TlBCO, under high magnetic field**
  - Increases in $1/T_1$ rate *inside* the vortex core
- **STM measurement of the four unit cell checkerboard pattern around the vortex core**

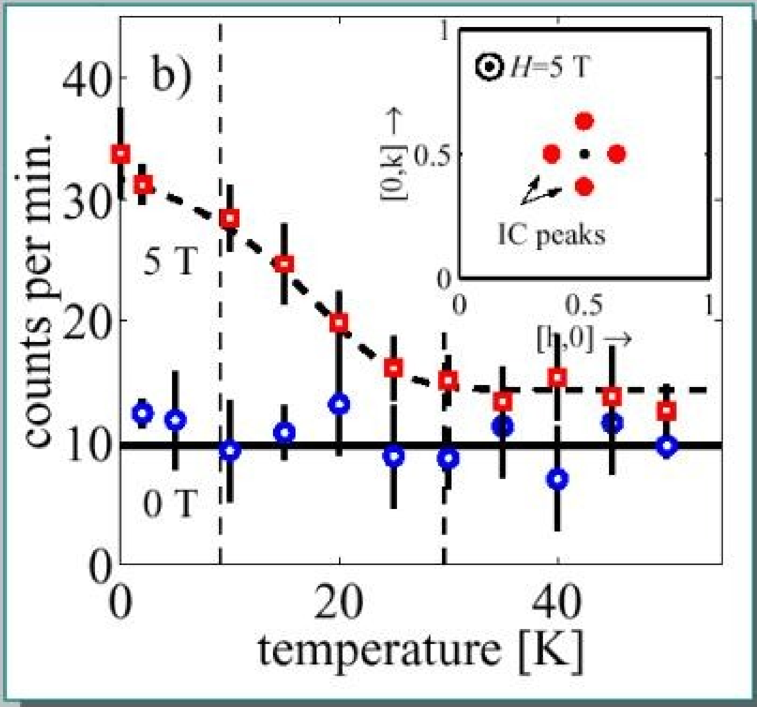
Exp. observation of the AF vortex core

- **Recent experiments by Lake, Aeppli et al observed slow AF fluctuations in the vortex core, in optimally doped LaSrCuO.**
- **Static AF moments in underdoped LaSrCuO.**

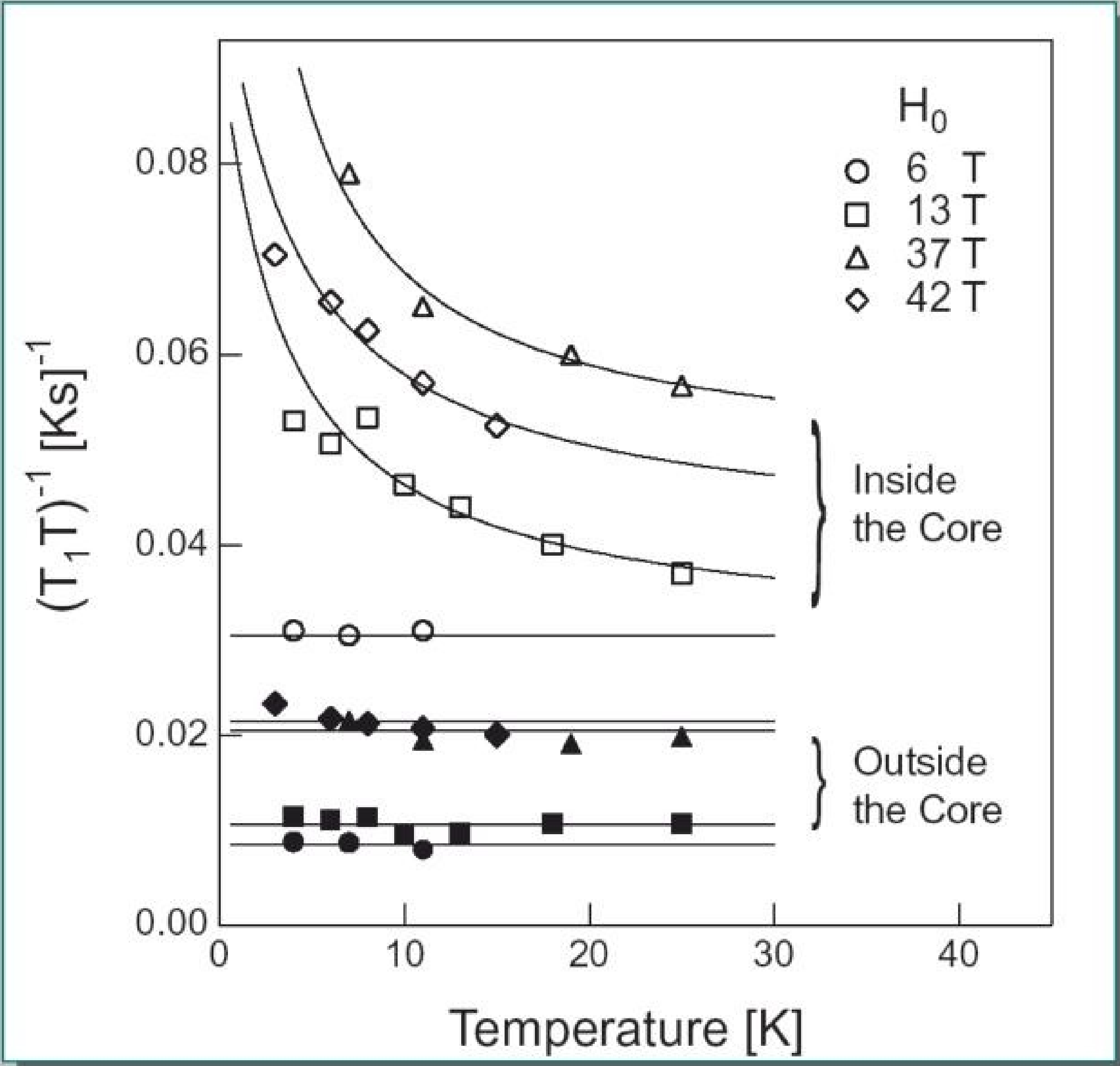


Neutron scattering on AF vortex core

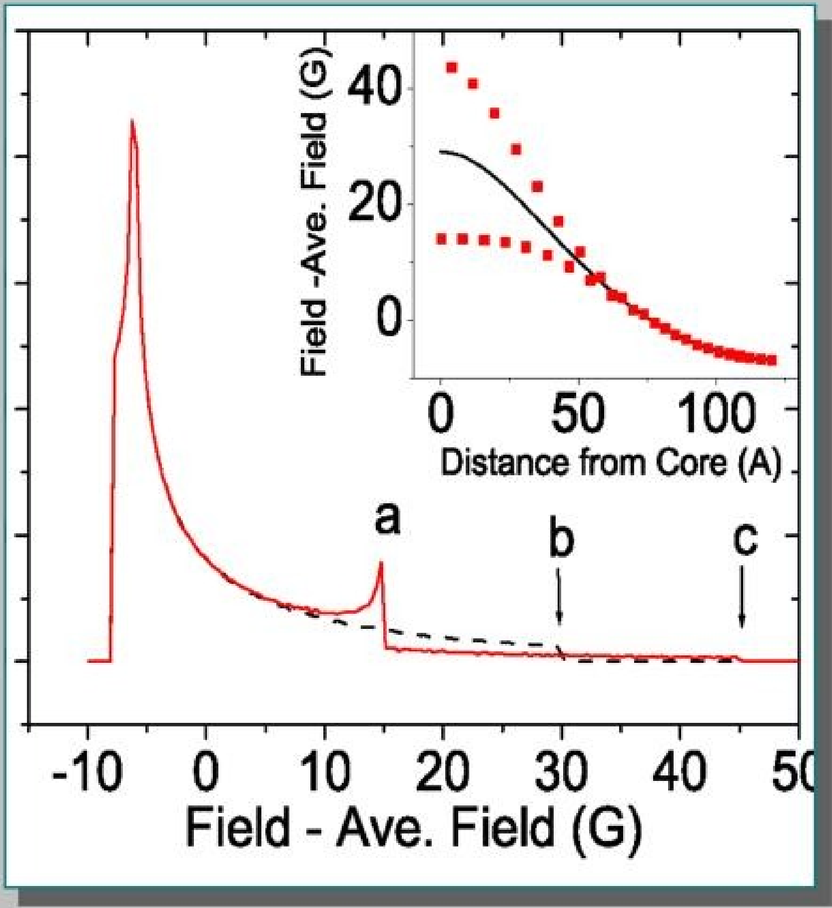
- **Significant increase of the static AF moment in the vortex state is observed in the underdoped LaSrCuO with x=0.10.**

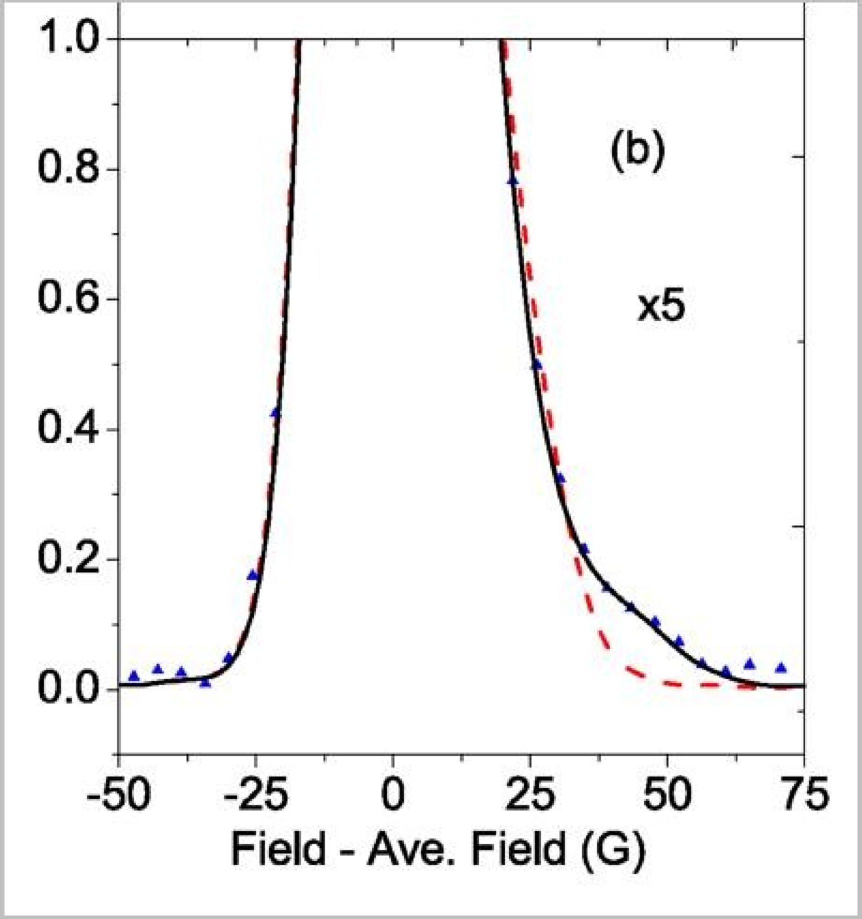


O NMR on optimally doped YBCO

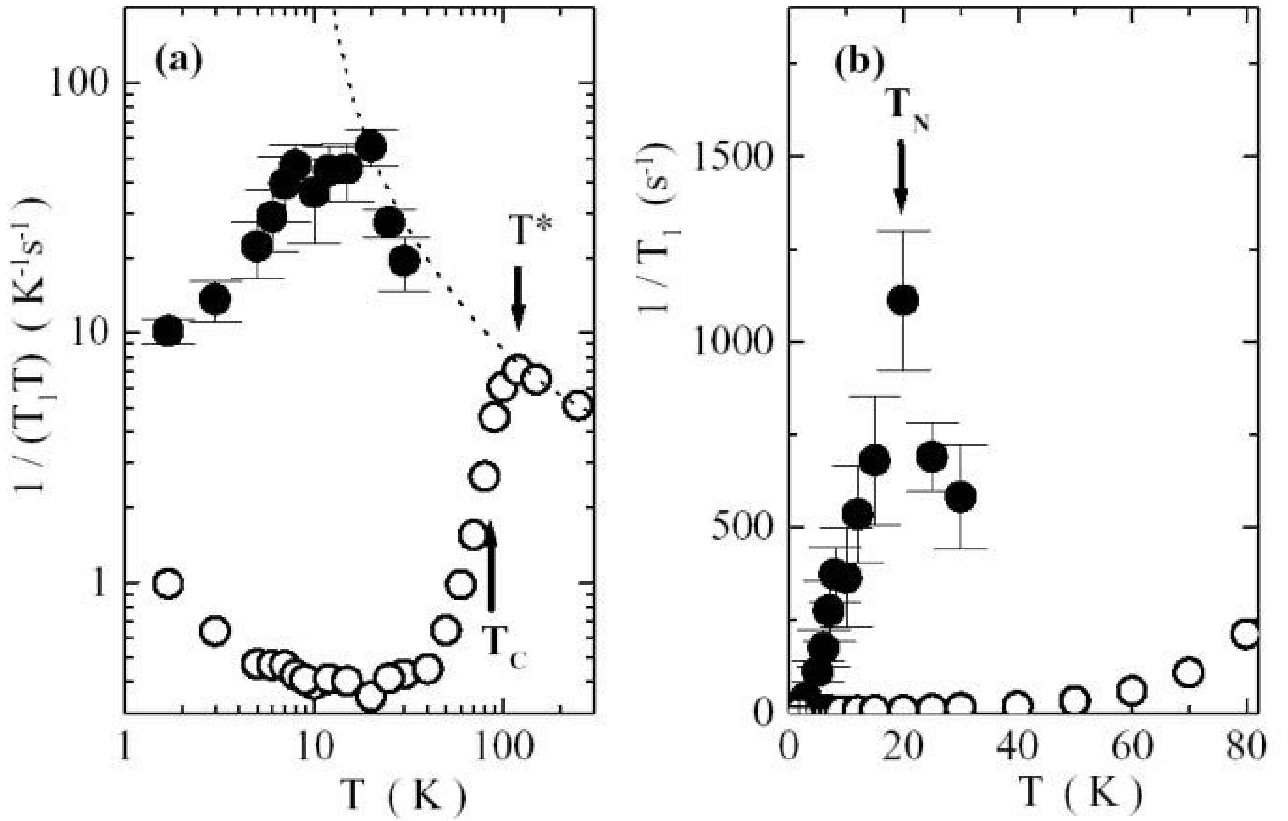


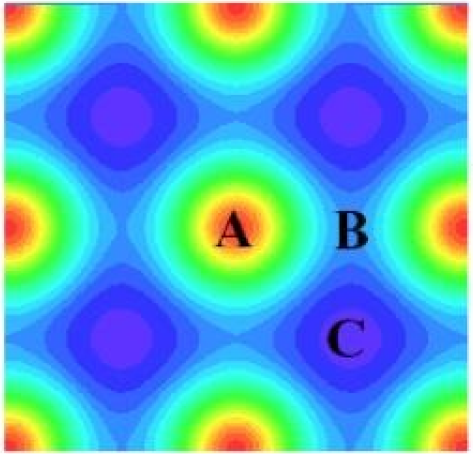
μsR on underdoped YBCO

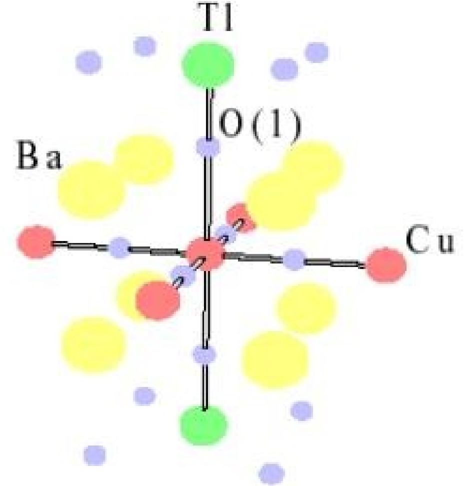




Tl NMR on optimally doped TlBaCuO

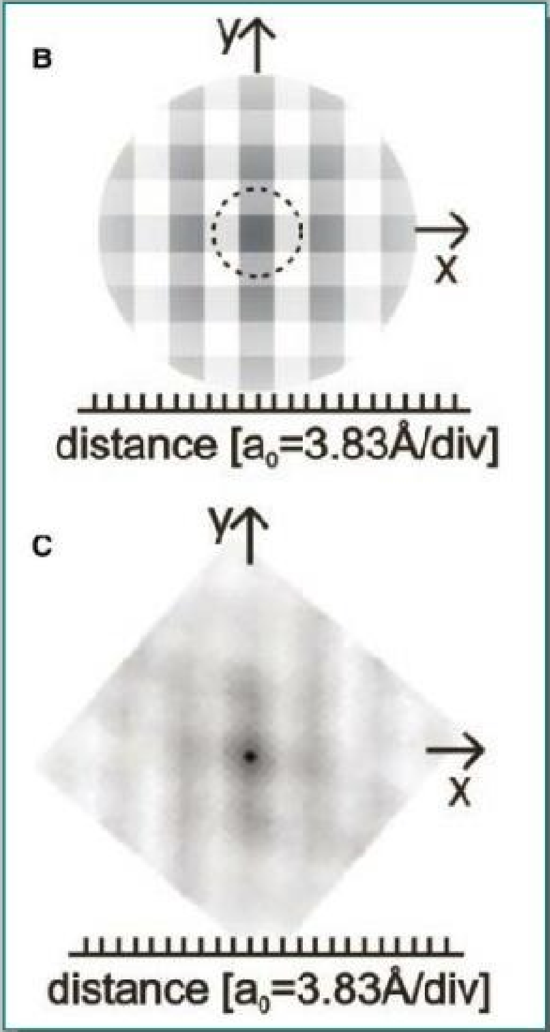






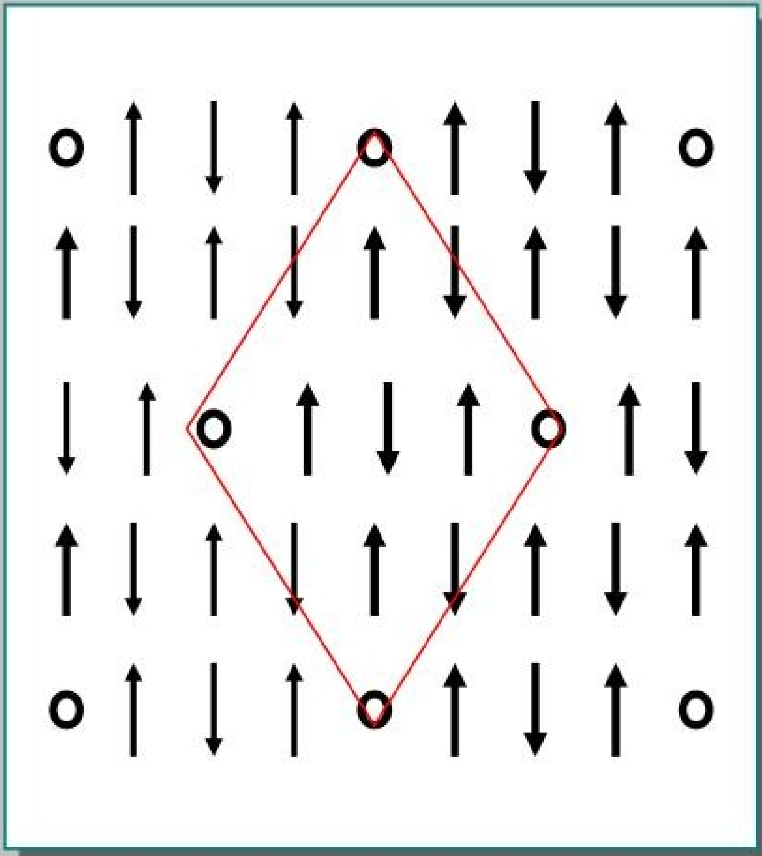
The checkerboard pattern

- 4ax4a charge unit cell
- Charge modulation is exponentially localized near the vortex core, with a decay length of 35A.
- x and y directions are roughly symmetric.



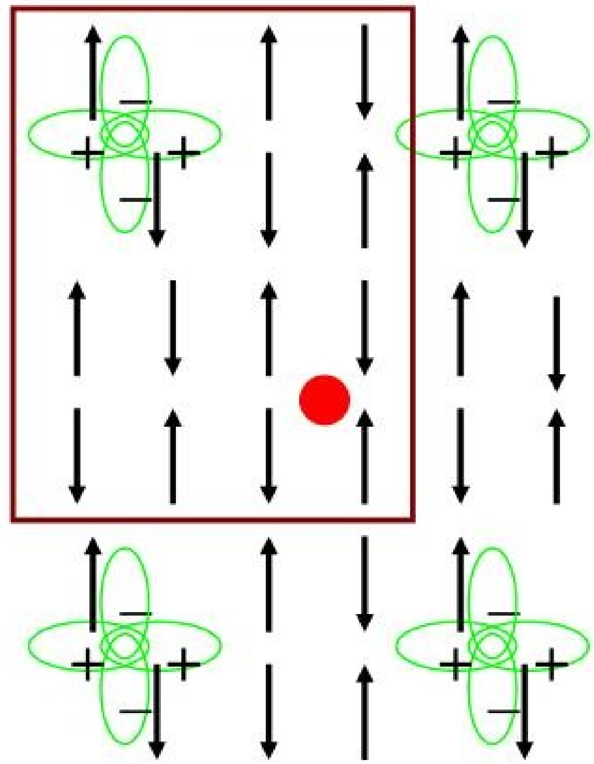
Wigner crystal of holes in AF background?

- **At or near x=1/8, holes would form a $\sqrt{8}a \times \sqrt{8}a$ superlattice, inconsistent with the 4ax4a pattern observed in the experiment.**
- **In the $2k_F$ fermi surface nesting explanation, the modulation vector depends on energy and doping.**

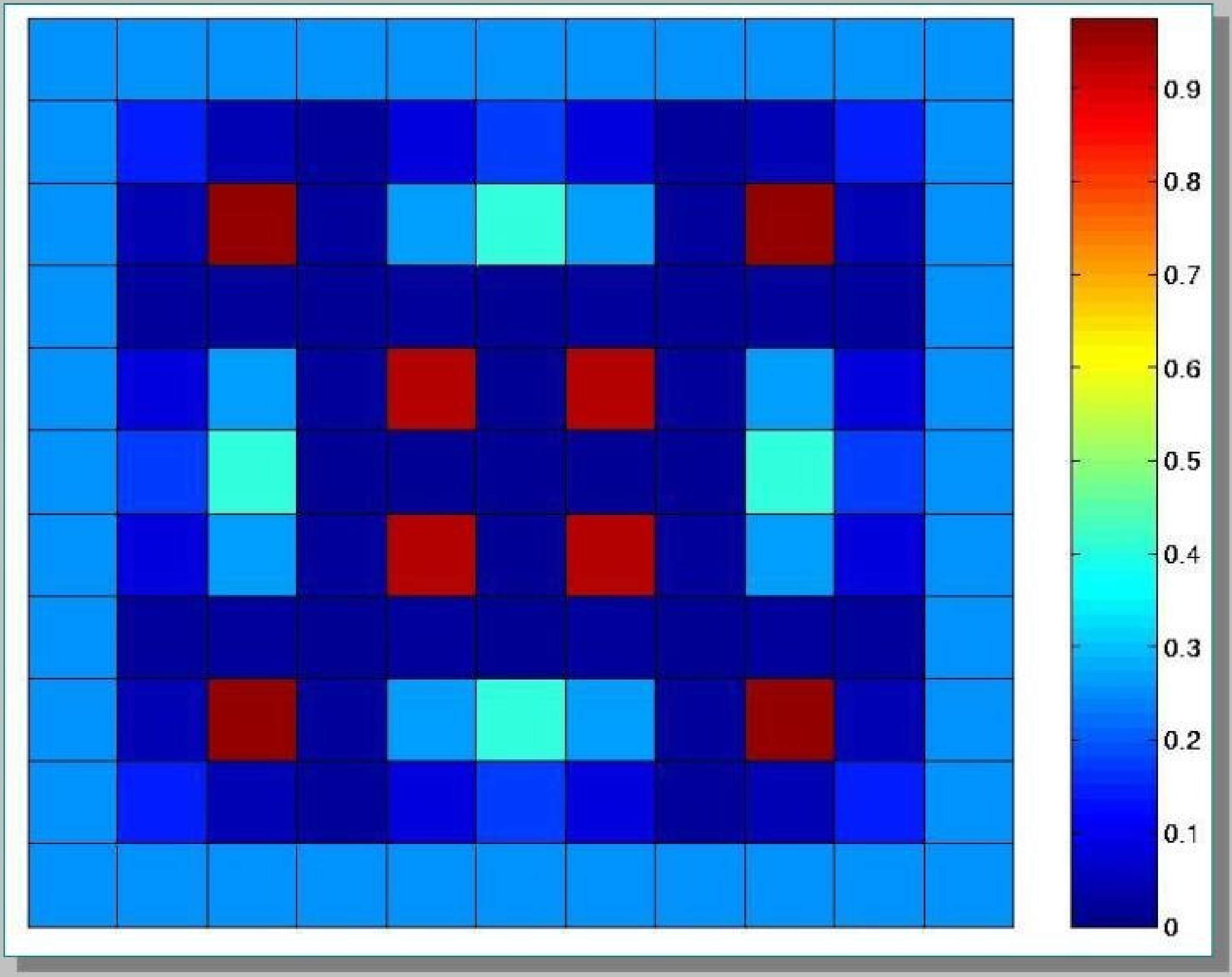


Cooper (or pair) crystal state at x=1/8

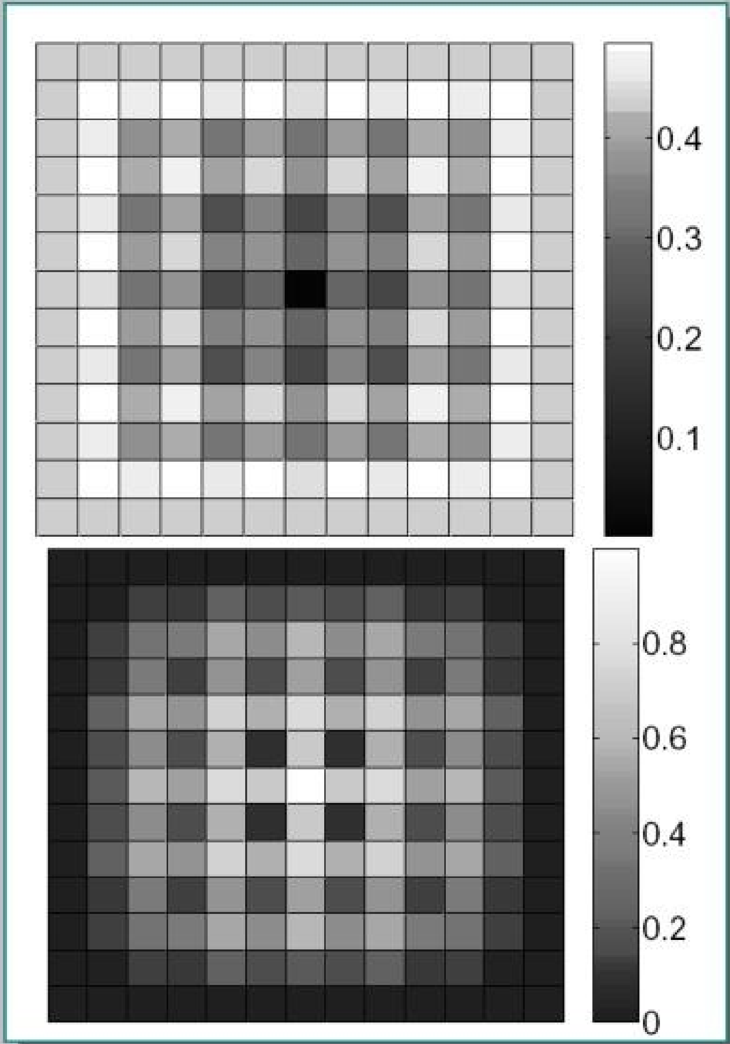
- **Inside the vortex core, SC is destroyed, but the Cooper pairs are simply localized!**
- **Alternating d-wave hole pairs in an antiferromagnetic background, forming 4ax4a charge unit cell.**
- **Spin order can be incommensurate stripe or commensurate checkerboard order, depending on details**

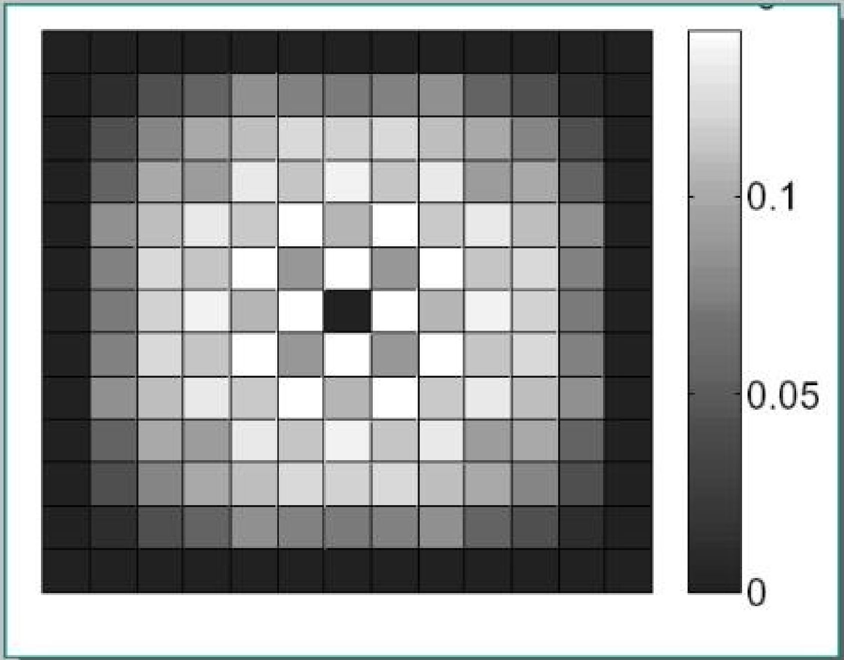


Charge distribution around a vortex



Order parameter distribution around a vortex





$$\langle L_{ab}\rangle\langle n_c\rangle+\langle L_{bc}\rangle\langle n_a\rangle+\langle L_{ca}\rangle\langle n_b\rangle=0$$

Quasi-particle interference vs 2nd order parameter

- **Friedel oscillation is a precursor of the CDW or SDW formation.**
- **Case for order parameter competition can only be established when both ordered states can be reached.**
- **Go above Hc2!**
  - A new insulating state with AF order, and a crystal of Cooper pairs.
  - Charge and heat insulator, $\kappa = a\,T + b\,T^3$, $a = 0$

Conclusions:

- **A new symmetry principle unifying DLRO (AF) and ODLRO (SC)**
- **AF and SC both corporate and compete.**
  - Corporation: condensation energy
  - Competition: AF vortex core
- **Precise relationship between microscopic t-J model and the pSO(5) model.**
  - Global phase diagram
- **Experimental predictions**
  - SO(5) bicritical point and AF/SC coexistence
  - AF vortex state
  - T and B dependence of the π resonance
  - Quantitative relations on the condensation energy