Introduction to Topological Superconductors



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Hong Kong University of Science & Technology

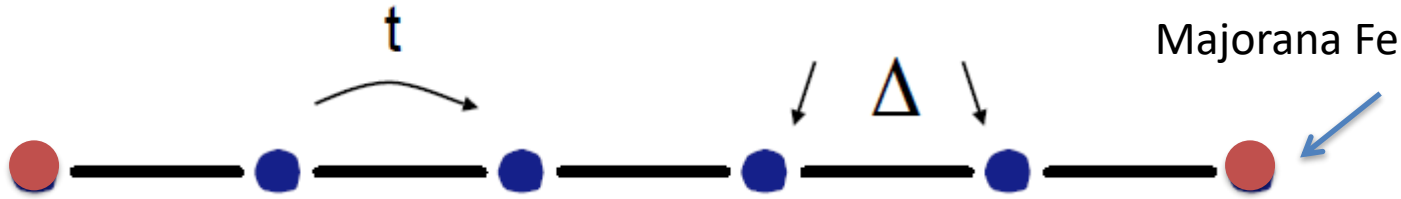
POSTECH, Feb 12, 2015

Collaborators:
Students: James He, Noah Yuan, Wenyu He, Chris Wong
(Former) Group members: TP Choy, Jie Liu, Xiong-Jun Liu
Senior Collaborators: Yukio Tanaka, FC Zhang, Tai Kai Ng, Patrick Lee

Outline Lecture 1 (Realization)

1. 1D Kitaev model

Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^{\dagger}$



Kitaev 2000

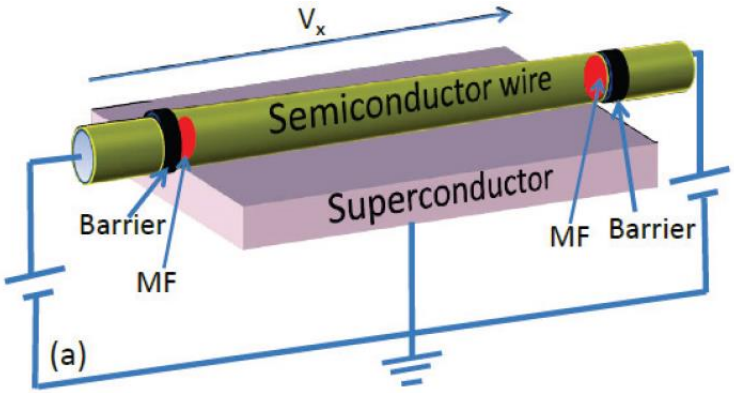
2. Realizing p-wave SC in condensed matter systems

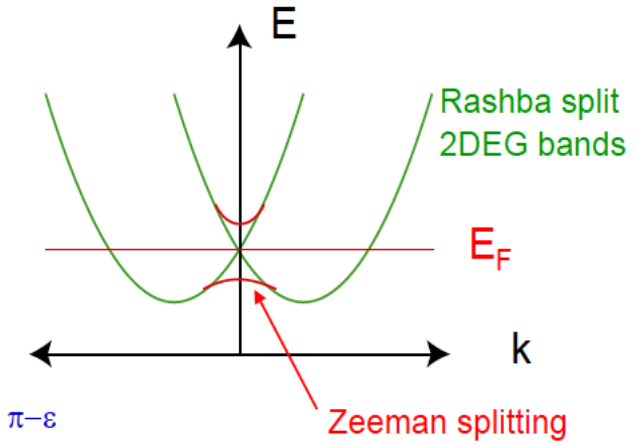
a. Topological insulator-superconductor heterostructure (2008)

b. Quantum wire with Rashba-spin orbit coupling (2009)

c. Superlattices on Rashba superconductors

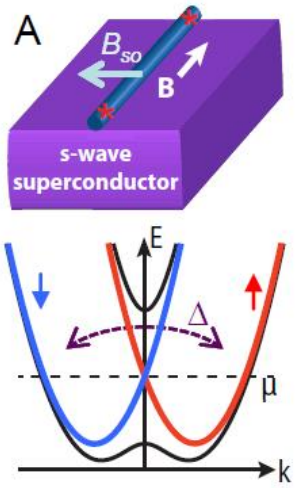
arXiv:1502.02548

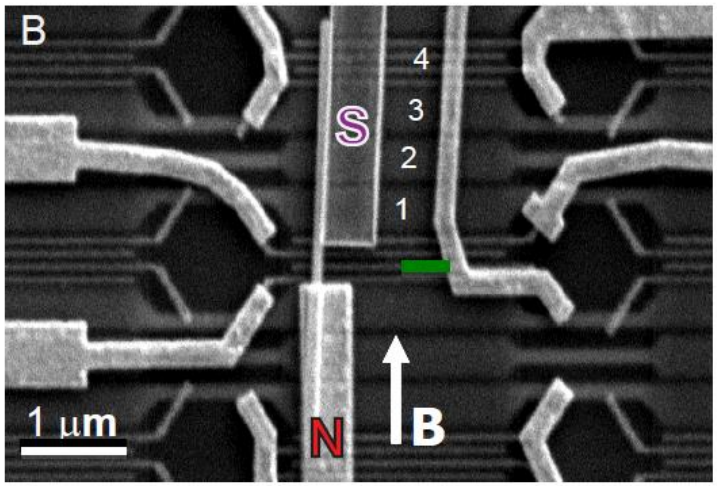


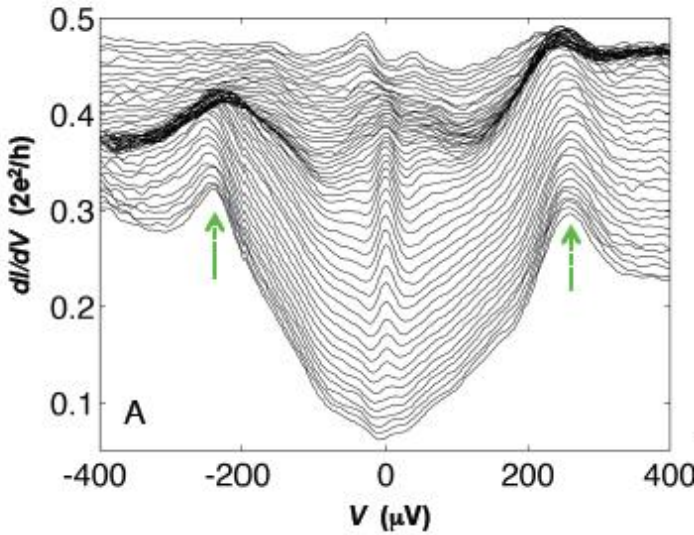


Outline Lecture 2 (Detection)

1. Majorana Fermion induced Resonant Andreev Reflections







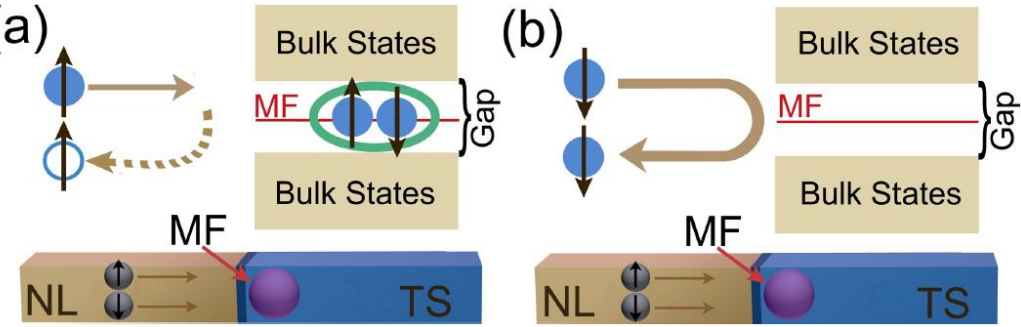
Experiments: Kowenhowen, HQ Xu, Heiblum (2012)

Jie Liu, A.C. Potter, KTL, P.A. Lee (PRL 2012)

KTL, TK Ng, PA Lee PRL (2009)

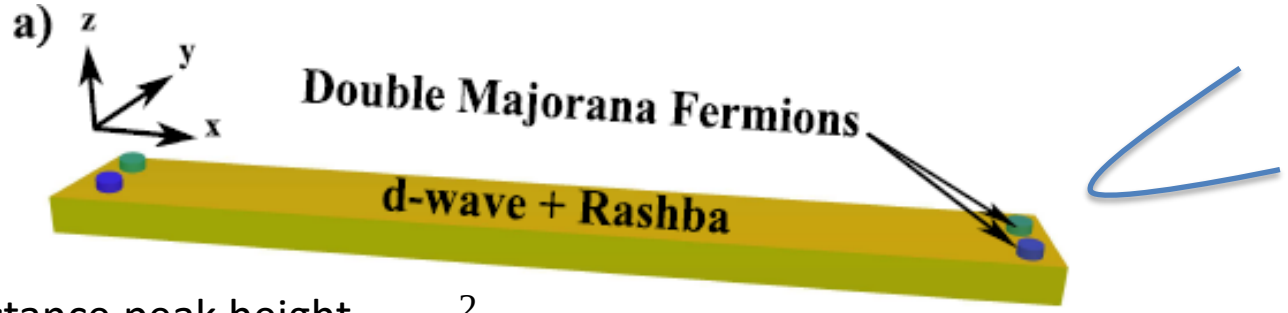
Other detection schemes:

1. Non-local transport (crossed Andreev reflections) (PRB 2013)
2. Majorana Fermion induced Selective Equal Spin Andreev reflections (PRL 2014)
3. Fractional Josephson effect (PRB 2011)



Outline Lecture 3 (Family)

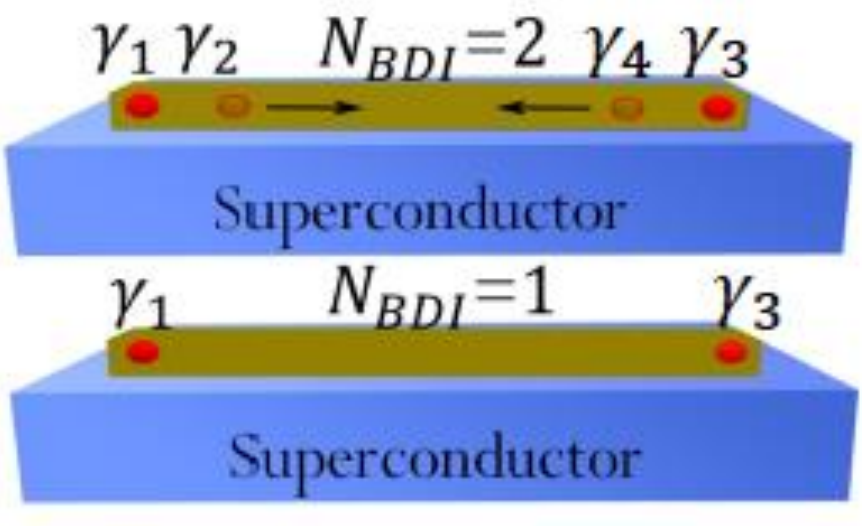
DIII class TS



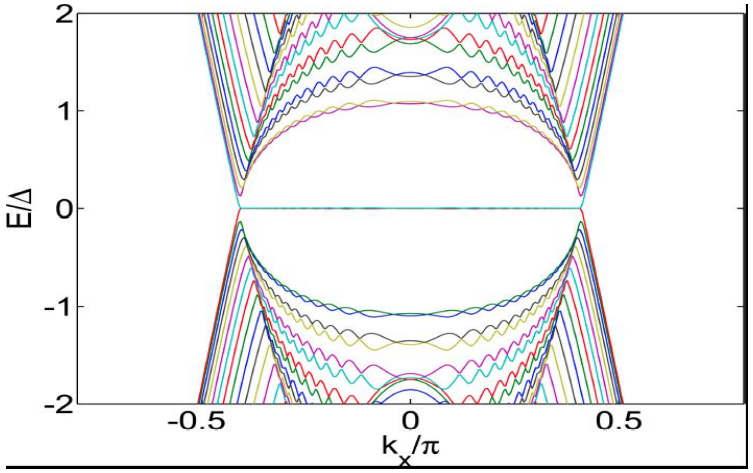
Zero Bias Conductance peak height $4\frac{e^2}{h}$

C. L.M. Wong, KTL 86, 184516 (2012)
Also Nakogai et al. (PRL 2013), Kane, Fu, Stern, Loss

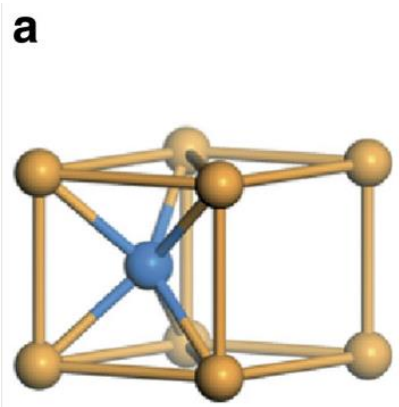
BDI Class TS



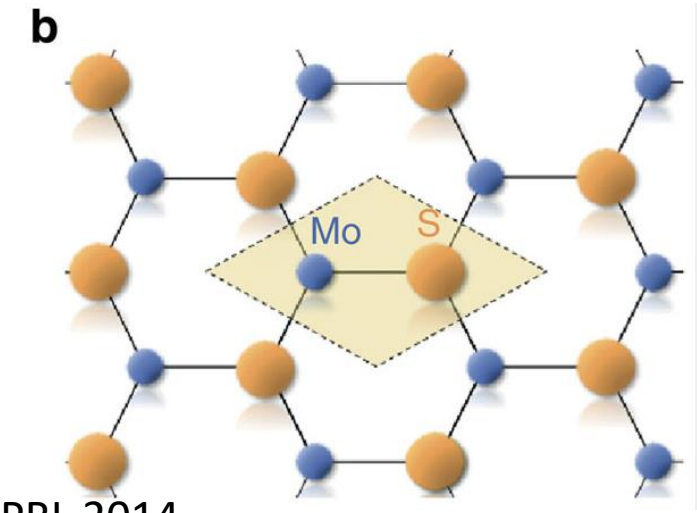
Nodal topological SC



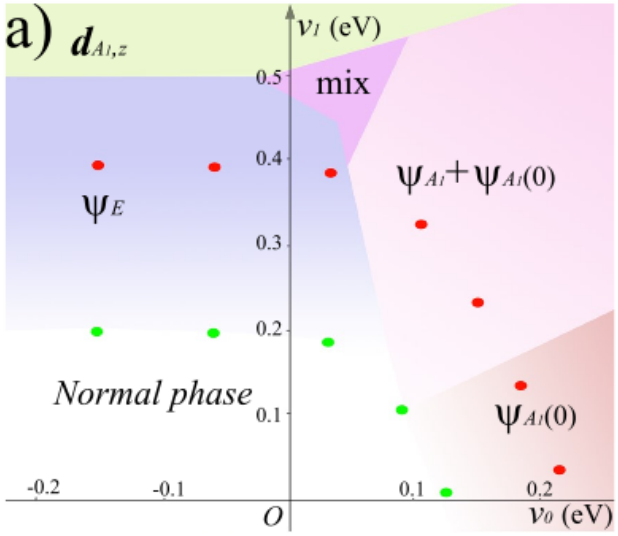
Outline Lecture 4 (Future)



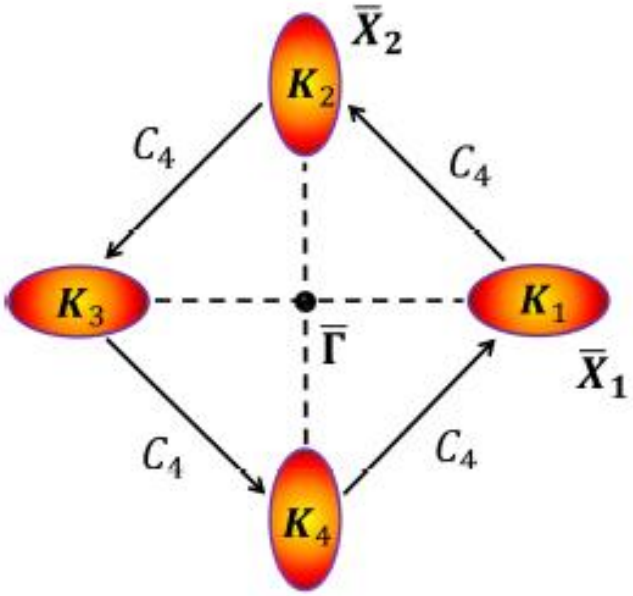
Noah, KF Mak, KTL PRL 2014

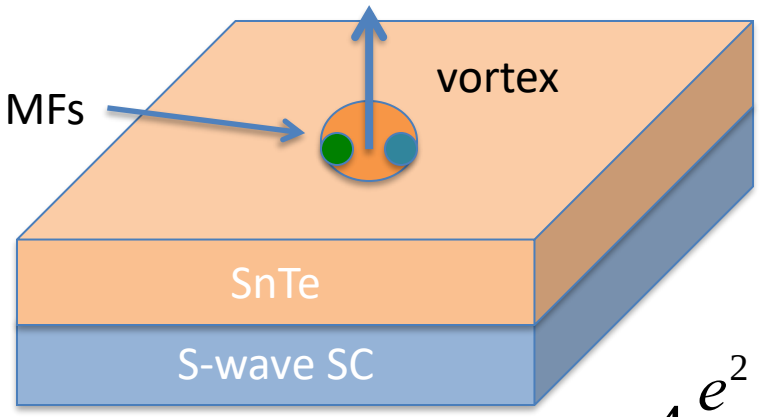


Postech-lectures



Vortex in a TCI hosts two symmetry protected Majoranas

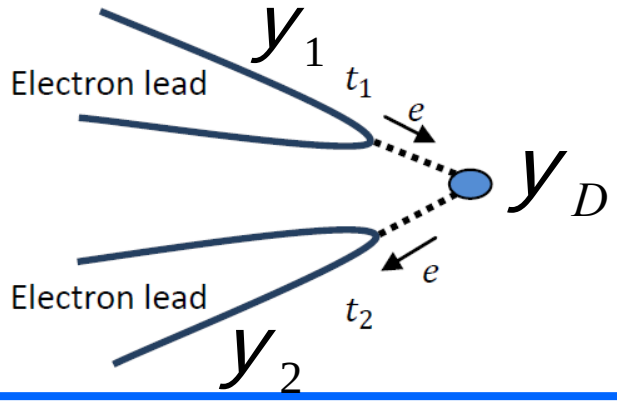




Zero Bias Conductance peak height $4\frac{e^2}{h}$

XJ Liu, JJ He, KTL PRB 2014

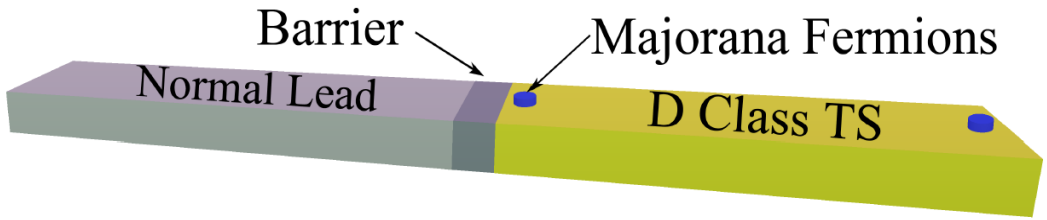
Majorana Fermion Induced Resonant Andreev Reflection

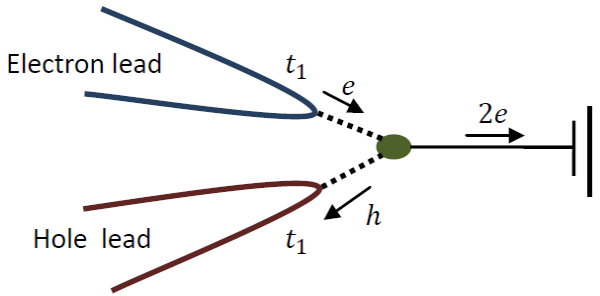


$$H_T = \gamma_D^+[t_1\gamma_1 + t_2\gamma_2] + h.c.$$

$$T(E) = \frac{(2t_1t_2)^2}{(E-E_0)^2 + (t_1^2+t_2^2)^2}$$

Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^\dagger$



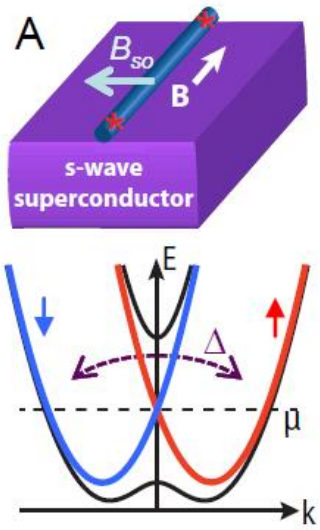


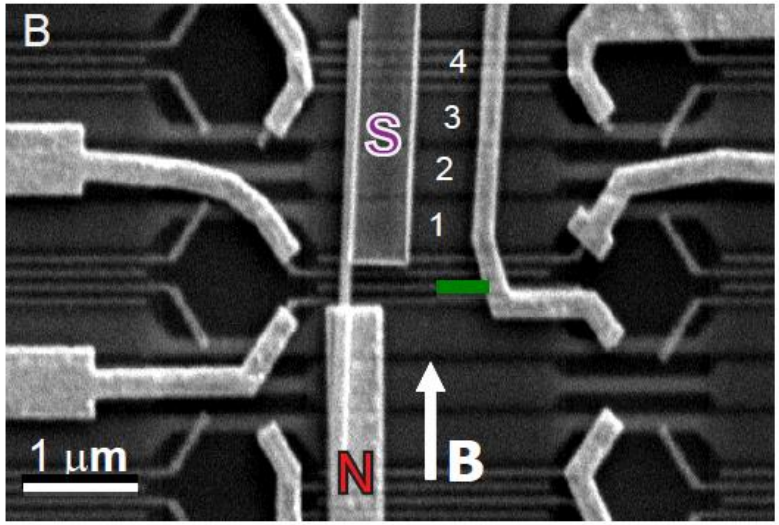
$$H_T = -it_1g[\gamma^+(0) + \gamma(0)]$$

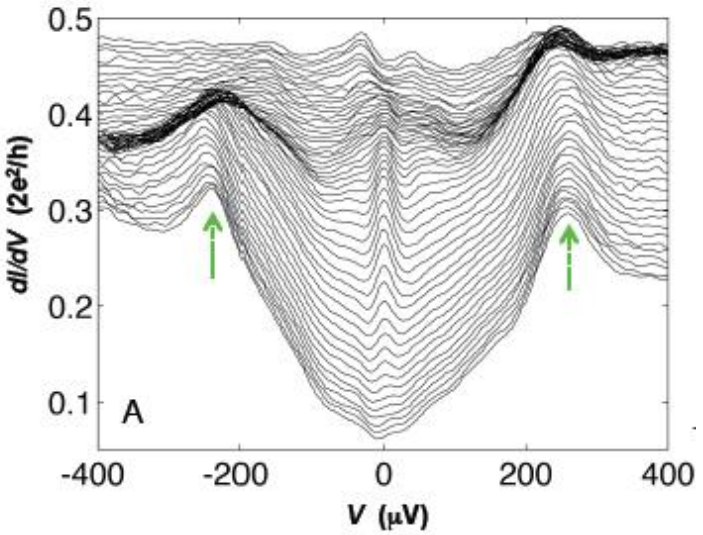
$$T(E) = \frac{(2t_1^2)^2}{(E-E_0)^2 + (2t_1^2)^2}$$

KTL, TK Ng, PA Lee PRL (2009)

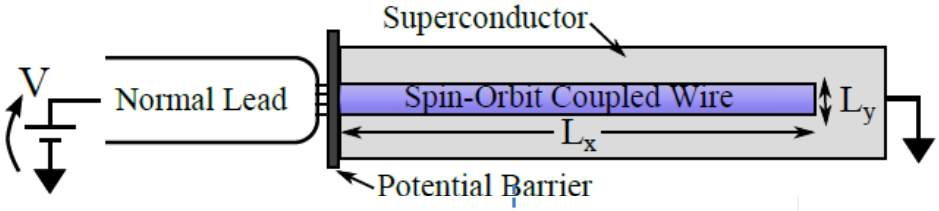
Zero Bias Peaks in Delft experiment

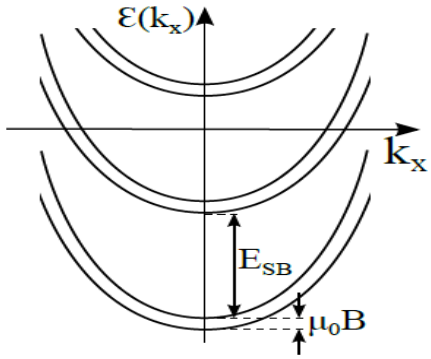


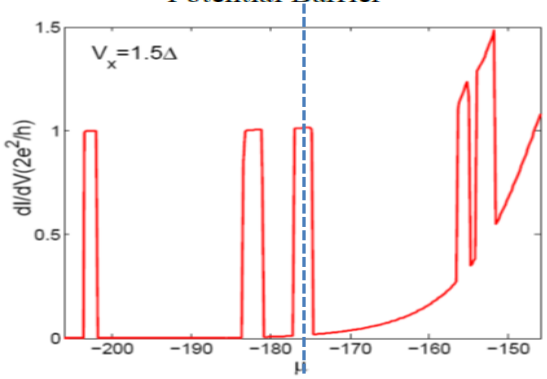


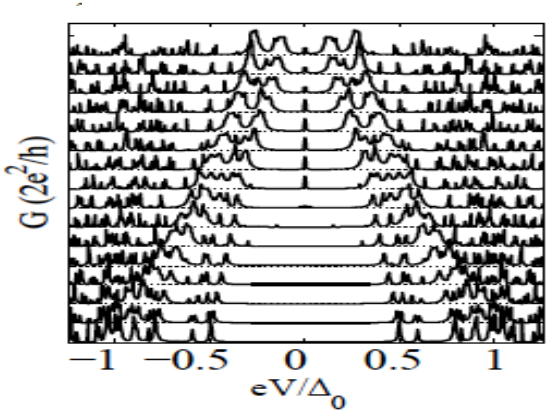


Experiments: Kowenhowen, HQ Xu, Heiblum (2012)



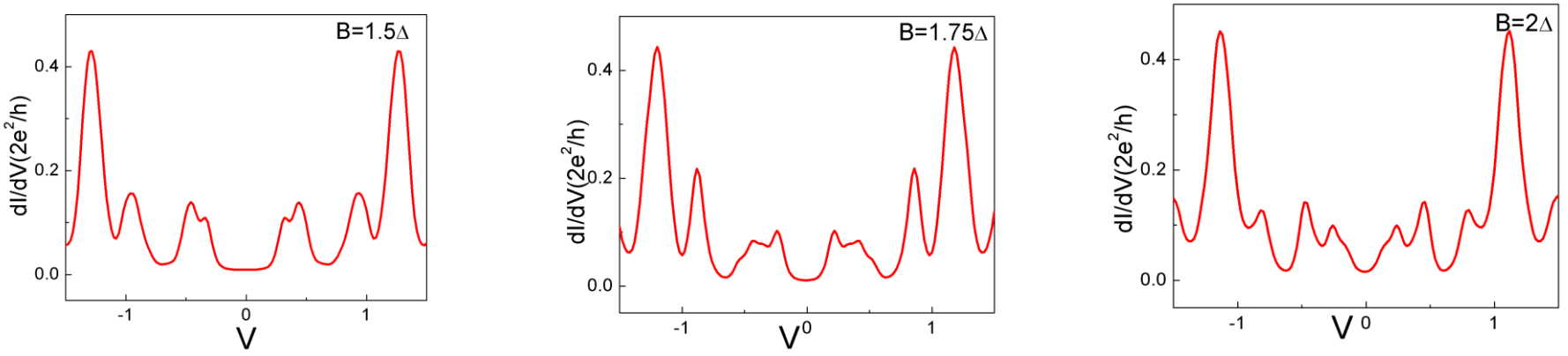


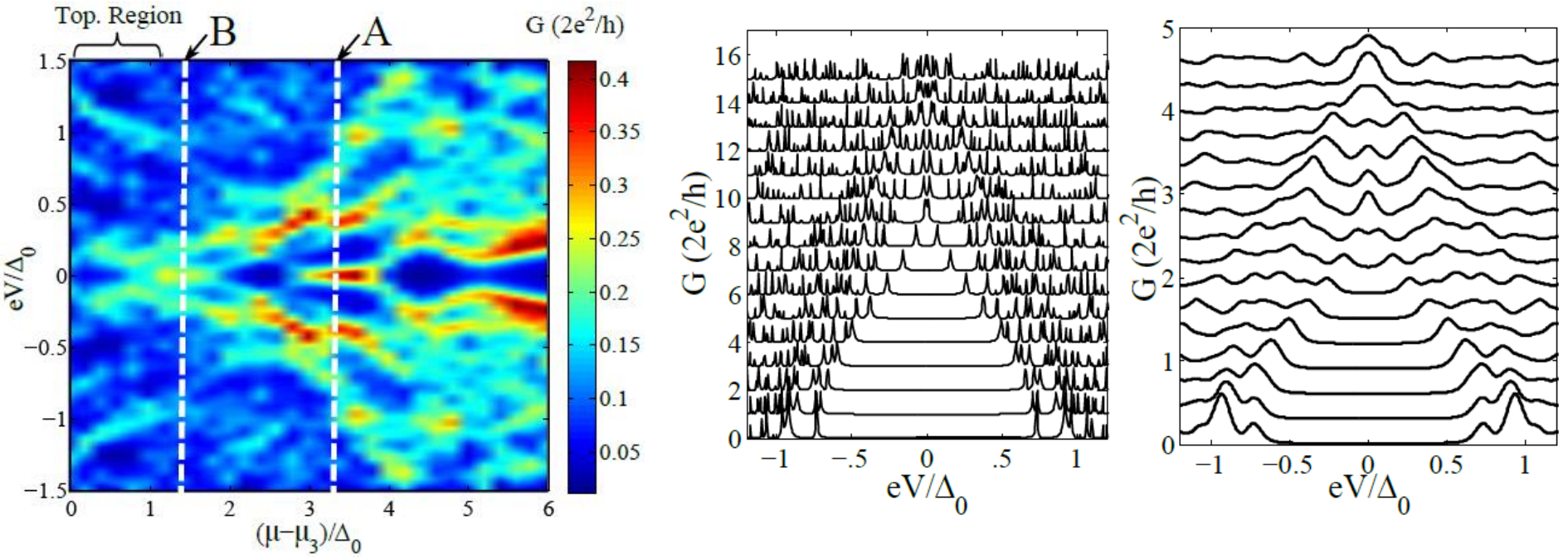




Disorder Induced ZBP in the absence of MFs

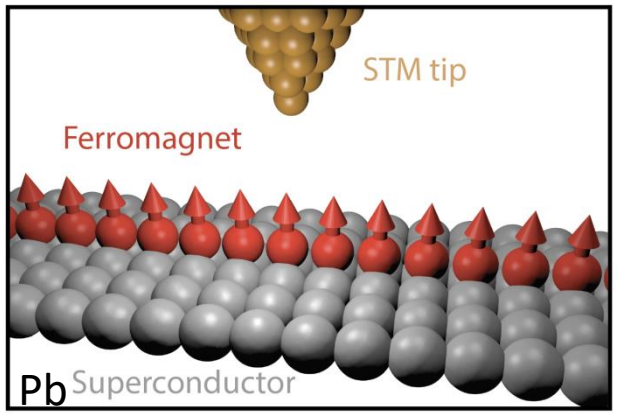
Presence of disorder with mean free path about 300nm

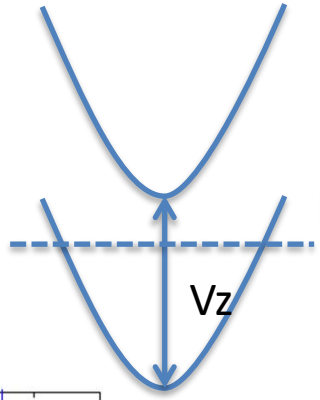


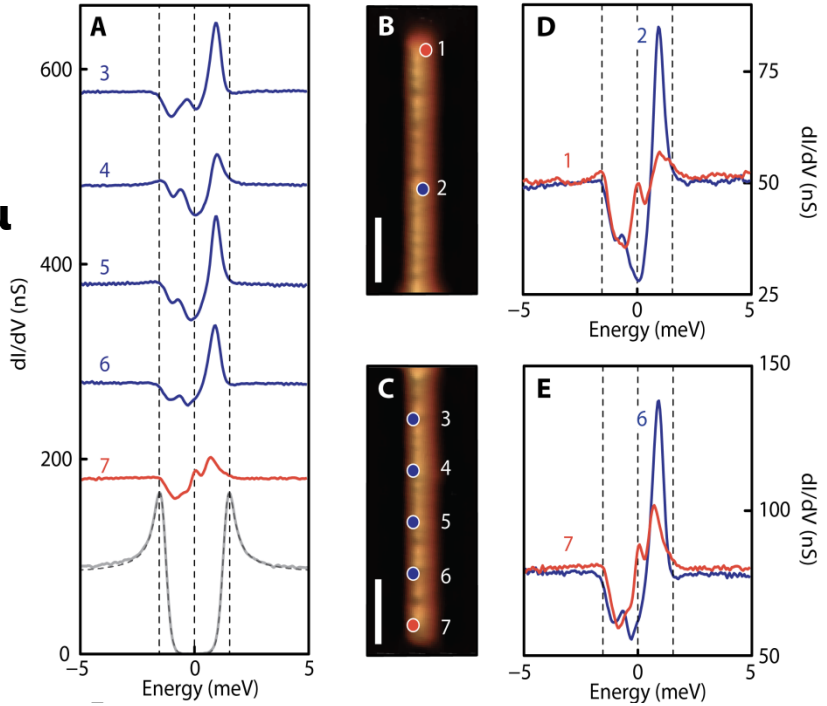


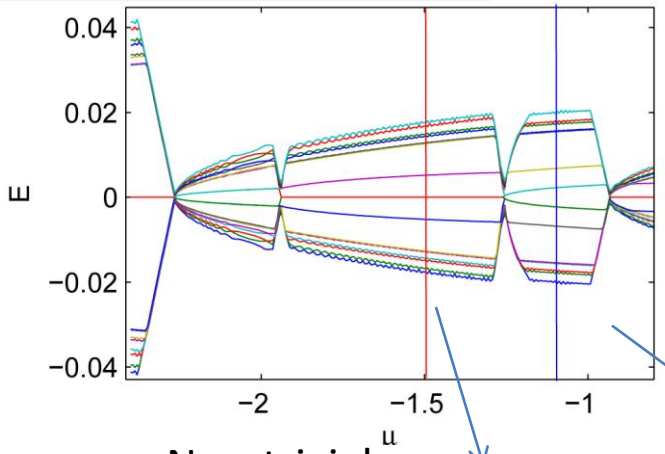
Jie Liu, A.C. Potter, KTL, P.A. Lee (PRL 2012)

Zero Bias Peaks in Princeton experiment



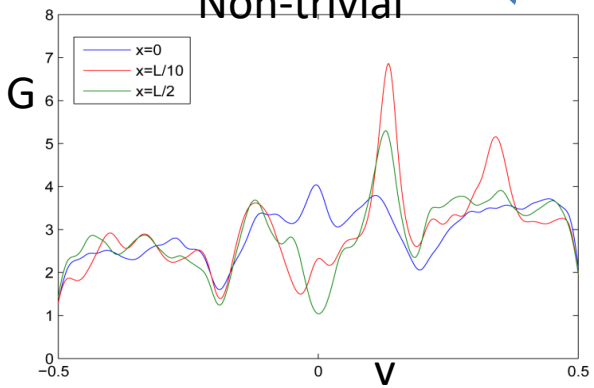




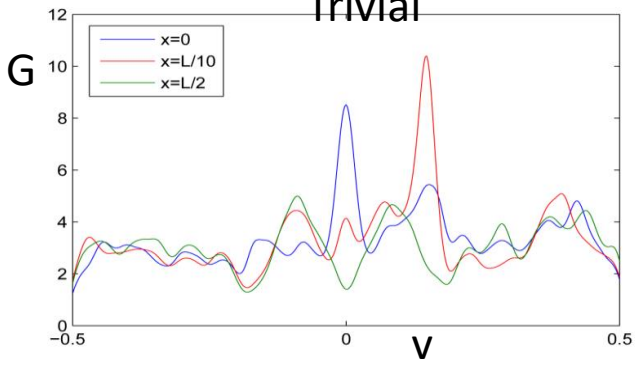


Science 346, 6209 (2014)

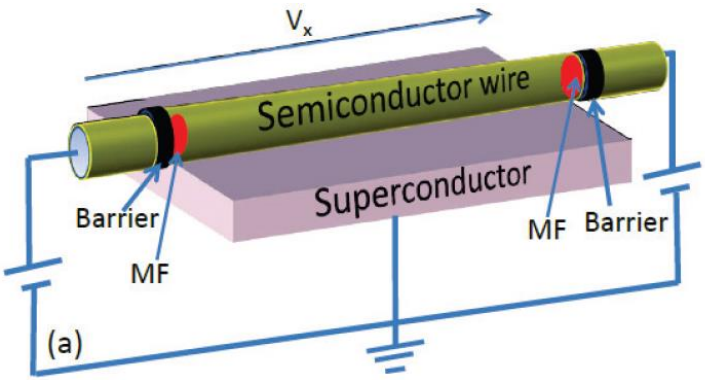
Non-trivial

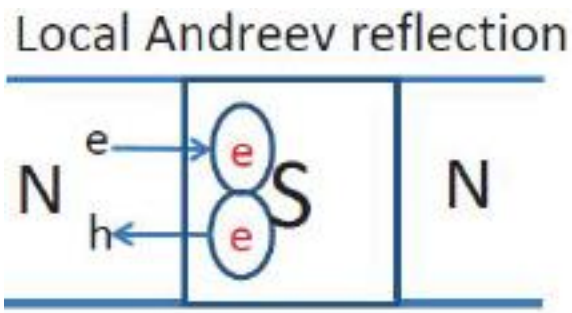


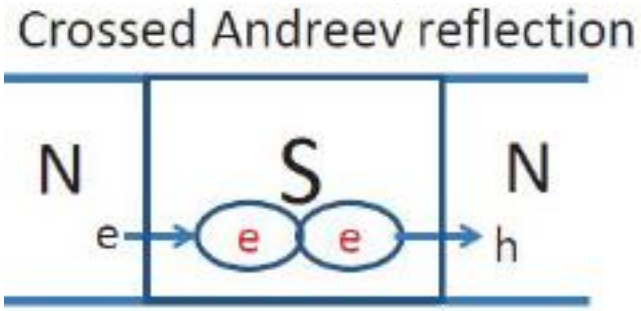
Trivial



MF induced Crossed Andreev Reflection







Crossed Andreev reflection dominate regime

1. Fano factor = C/I= e instead of 2e
2. Strong current-current correlations

Current Fluctuation

$$dI_i(t) = I_i(t) - \langle I_i \rangle$$

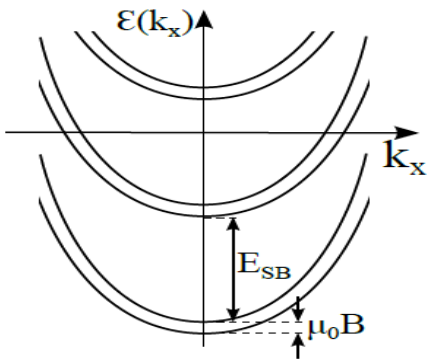
Shot Noise

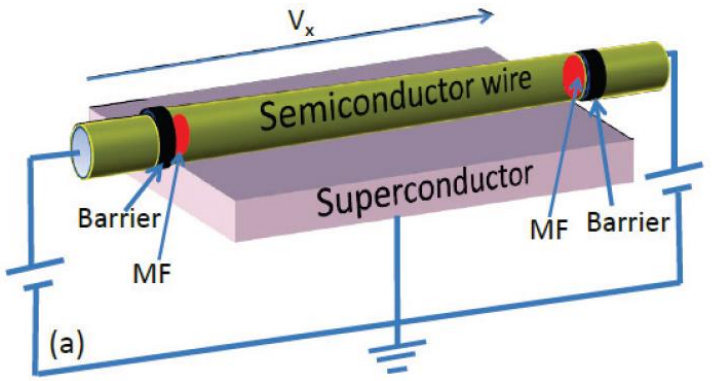
$$C_{ij} = \int_{-\infty}^{\infty} \langle dI_i(0) dI_j(t) \rangle dt$$

Fano Factor

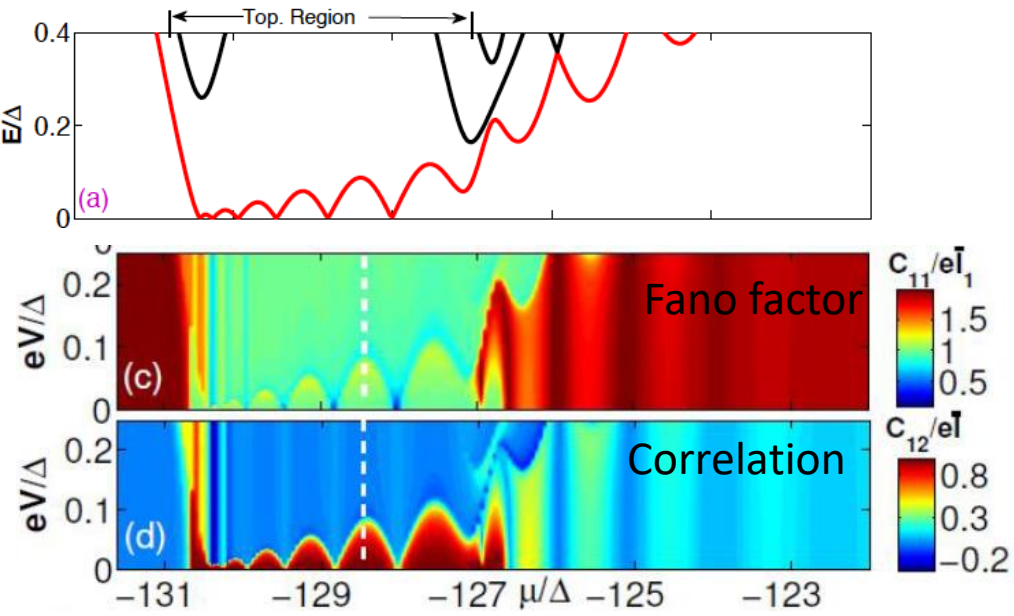
$$F_i = C_{ii} / I_i = e^*$$

MF induced Crossed Andreev Reflection

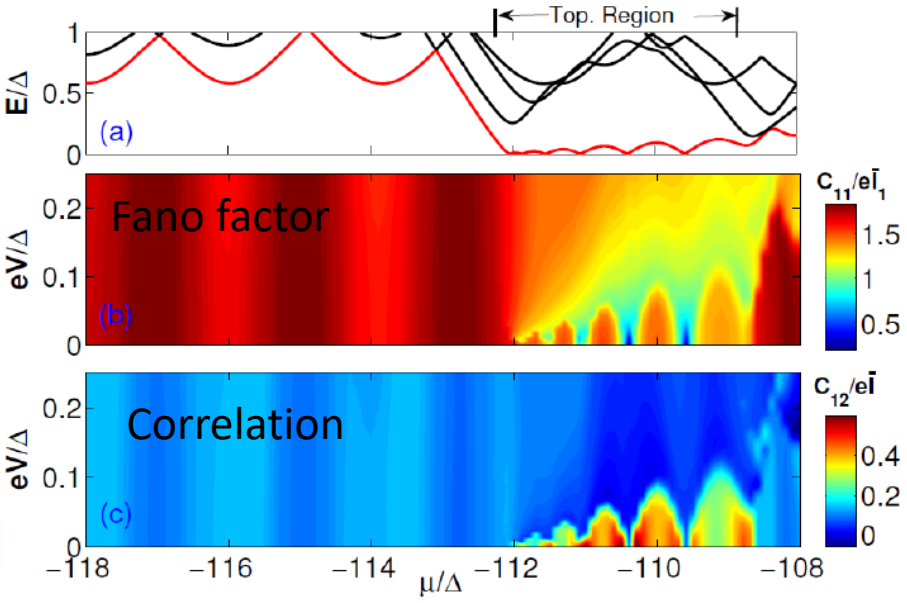




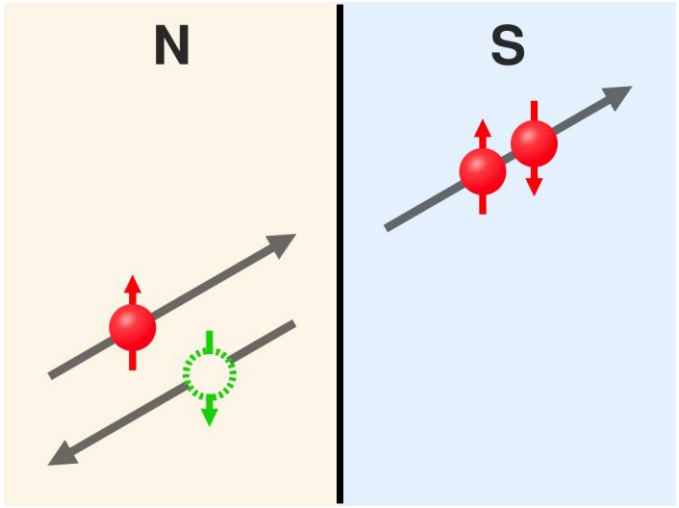
One transverse sub-band is occupied

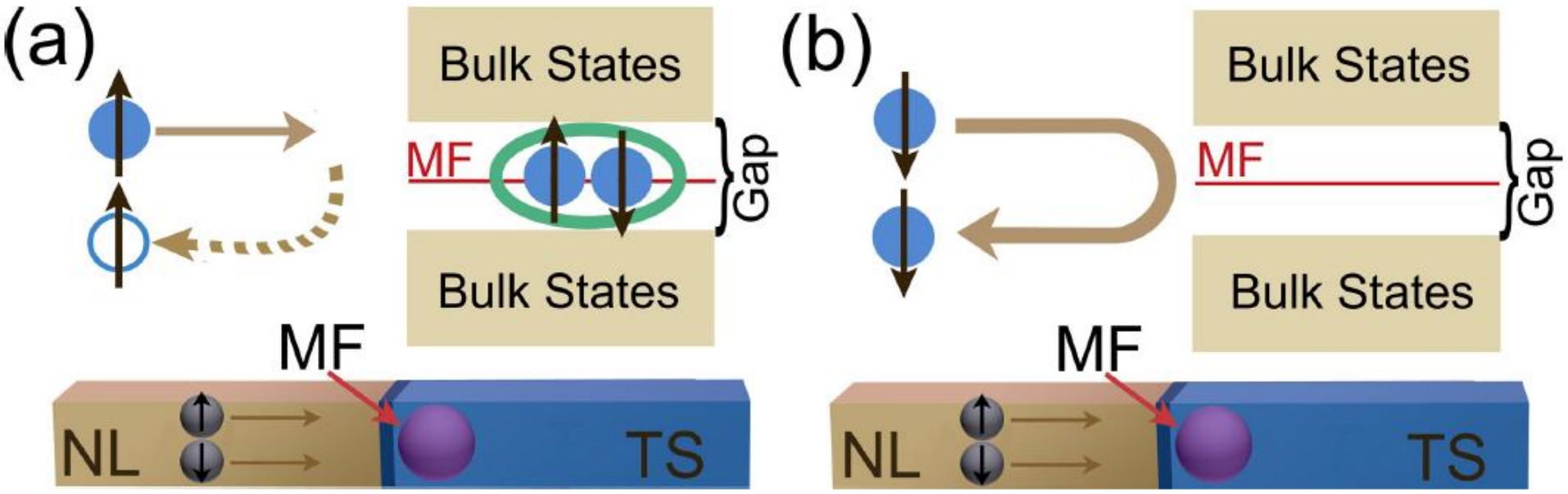


Three transverse sub-bands are occupied

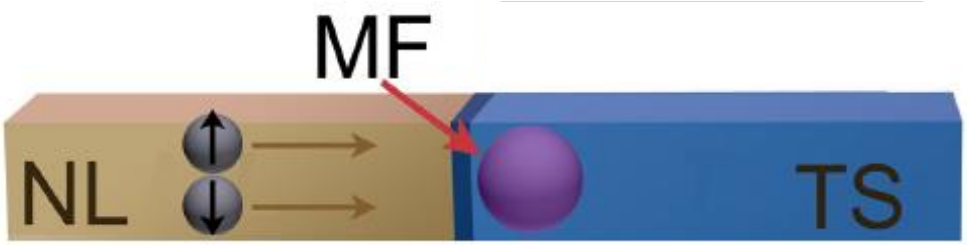


MF induced Selective Equal spin Andreev reflections





MF induced Selective Equal spin Andreev reflections



$$H_T = H_L + H_c,$$
$$H_L = -iv_F \sum_{\alpha\in\uparrow/\downarrow} \int_{-\infty}^{+\infty} \psi_\alpha^\dagger(x)\partial_x\psi_\alpha(x)dx,$$
$$H_c = \tilde{t}\gamma[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)]$$

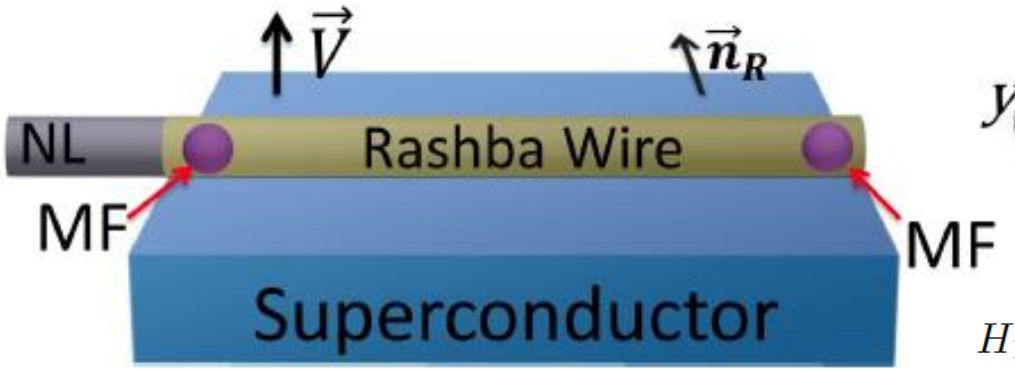
$$\Psi_1 = a\psi_\uparrow + b\psi_\downarrow \text{ and } \Psi_2 = -b^*\psi_\uparrow + a^*\psi_\downarrow$$
$$H_L = -iv_F \sum_{\alpha\in 1/2} \int_{-\infty}^{+\infty} \Psi_\alpha^\dagger(x)\partial_x\Psi_\alpha(x)dx,$$
$$H_c = \tilde{t}\gamma[\Psi_1(0) - \Psi_1^\dagger(0)].$$

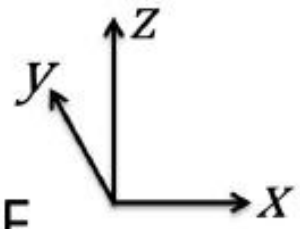
Scattering matrix

$$\begin{pmatrix} \Psi_{1E}(+) \\ \Psi_{1E}^\dagger(+) \end{pmatrix} = \frac{1}{\Gamma + iE} \begin{pmatrix} iE & \Gamma \\ \Gamma & iE \end{pmatrix} \begin{pmatrix} \Psi_{1E}(-) \\ \Psi_{1E}^\dagger(-) \end{pmatrix} \qquad \Gamma = 2\tilde{t}^2/v_F$$

Andreev reflection amplitude: $\Gamma/(\Gamma + iE)$

MF induced Selective Equal spin Andreev reflections





$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k\sigma_y)]\tau_z - \Delta\sigma_y\tau_y$$

Andreev reflection matrix:

$$r_{he}(V_z) = \begin{pmatrix} \frac{V_z - \sqrt{V_z^2-\Delta^2}}{2V_z} & -\frac{\Delta}{2V_z} \\ -\frac{\Delta}{2V_z} & \frac{\sqrt{V_z^2-\Delta^2}+V_z}{2V_z} \end{pmatrix}$$

Normal reflection matrix:

$$r_{ee}(V_z) = r_{he}(-V_z)e^{i\chi(k)}$$

$$r_{he}\vec{s}_0 = \vec{s}_0^* \text{ and } r_{ee}\vec{s}_0 = 0$$

$$\vec{s}_0 = (\cos\frac{\theta_0}{2}, e^{i\phi_0}\sin\frac{\theta_0}{2})^T$$

Electrons with spin polarization

$$\vec{\mathbf{n}}_0 = \langle\vec{s}_0|\vec{\sigma}|\vec{s}_0\rangle =$$

$$(\sin\theta_0\cos\phi_0, \sin\theta_0\sin\phi_0, \cos\theta_0)$$

are resonantly reflected as holes with the same spin.

On the contrary, we have

$$r_{ee}\vec{u}_0 = e^{i\chi}\vec{u}_0$$

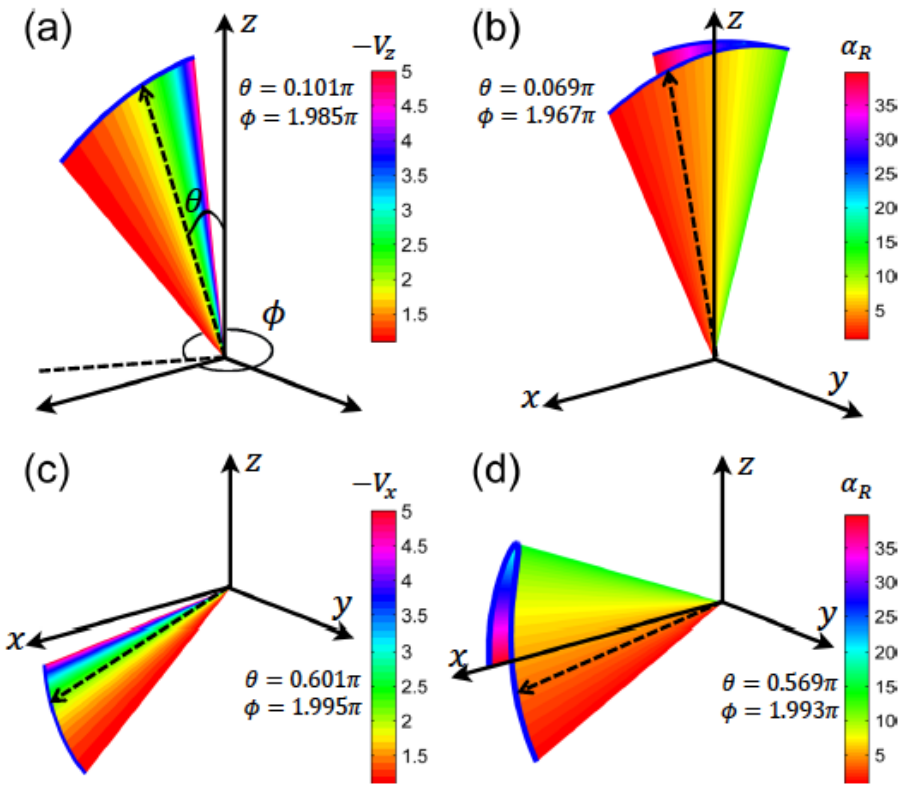
$$\vec{u}_0 = (-\sin\frac{\theta_0}{2}, e^{i\phi_0}\cos\frac{\theta_0}{2})$$

Electrons with spin polarization anti-parallel to $\vec{\mathbf{n}}_0$ are completely reflected as electrons.

MF induced Selective Equal spin Andreev reflections

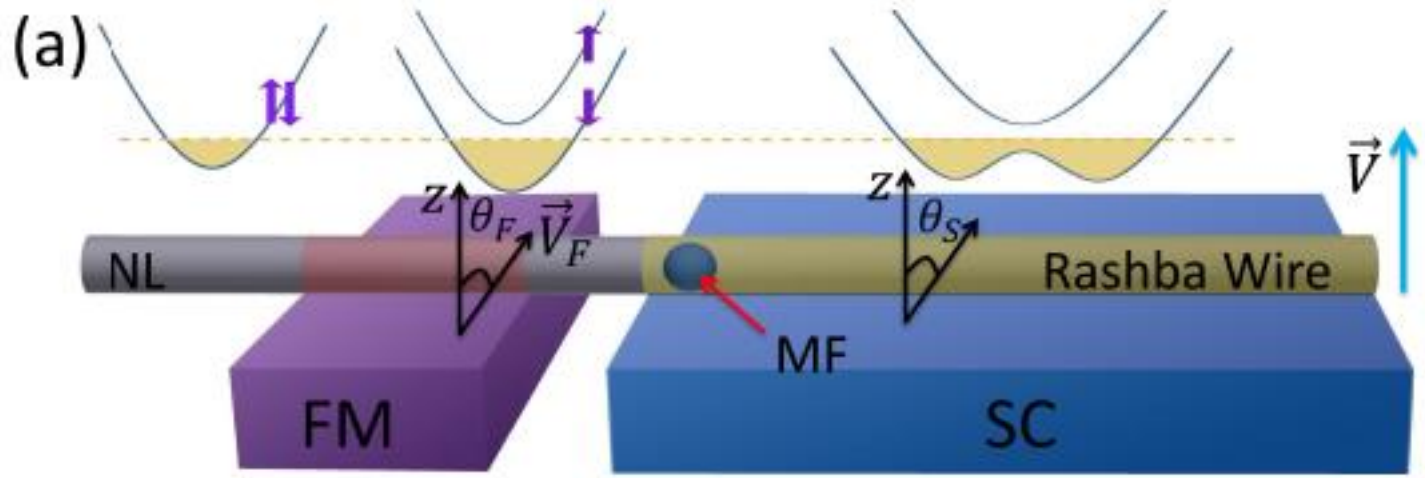
Spin polarization vector

$$\vec{\mathbf{n}}_0 = \langle \vec{s}_0 | \vec{\sigma} | \vec{s}_0 \rangle = (\sin\theta_0 \cos\phi_0, \sin\theta_0 \sin\phi_0, \cos\theta_0)$$



James He, TK Ng, PA Lee and KTL (PRL2014)

MF induced Equal spin Andreev reflections



Topological regime

