Quantum Anomalous Hall Insulators-Based as Majorana platforms



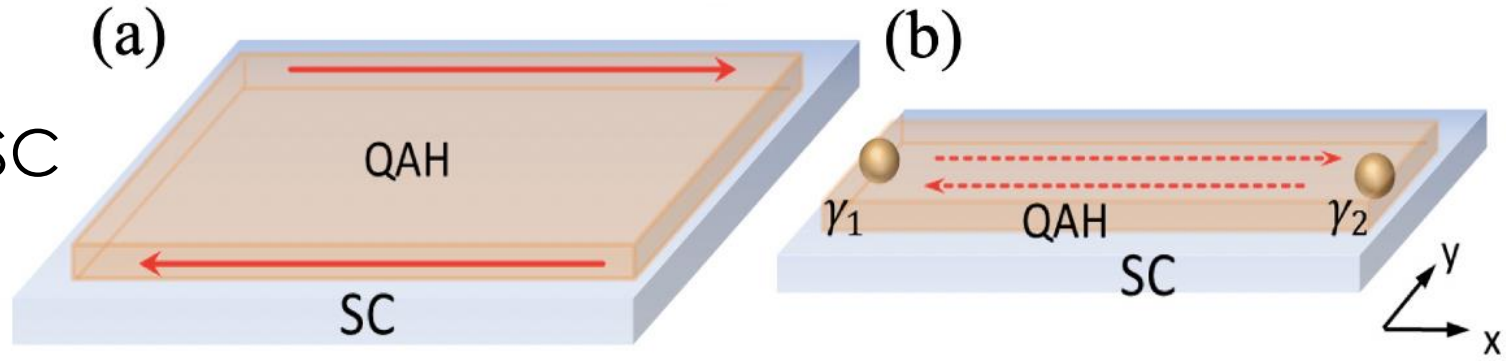
Vic K. T. Law

The Hong Kong University of Science and Technology

Quantum Anomalous Hall Insulators are promising Majorana platforms!

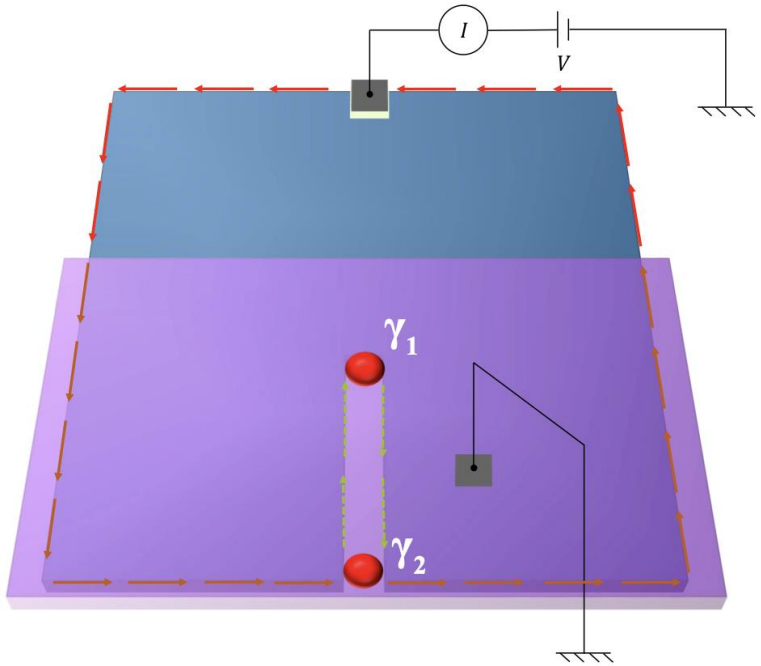
1. Quasi-1D QAHI/SC

PRB 2018, Cell Reports Physical Science 2023



2. QAHI with a cut

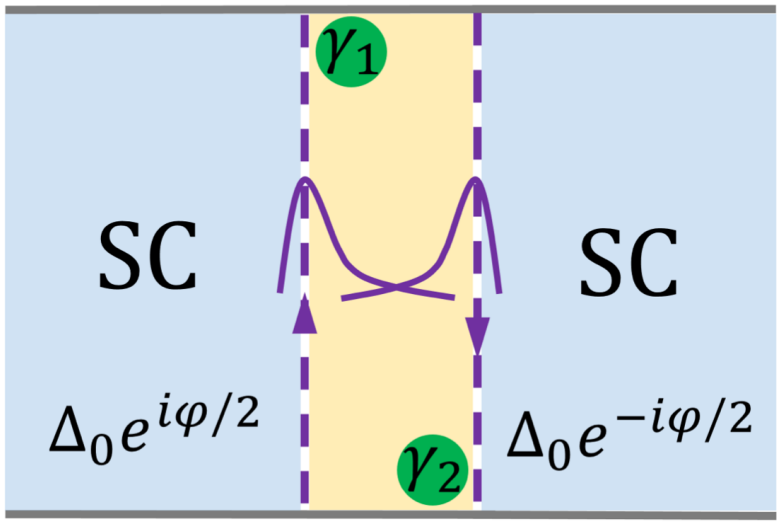
US Patent 2020, arXiv:2012.15523



Quantum Anomalous Hall Insulators are the most promising Majorana platforms!

3. SC/QAHI/SC junction

PRL 133, 056601 (2024)



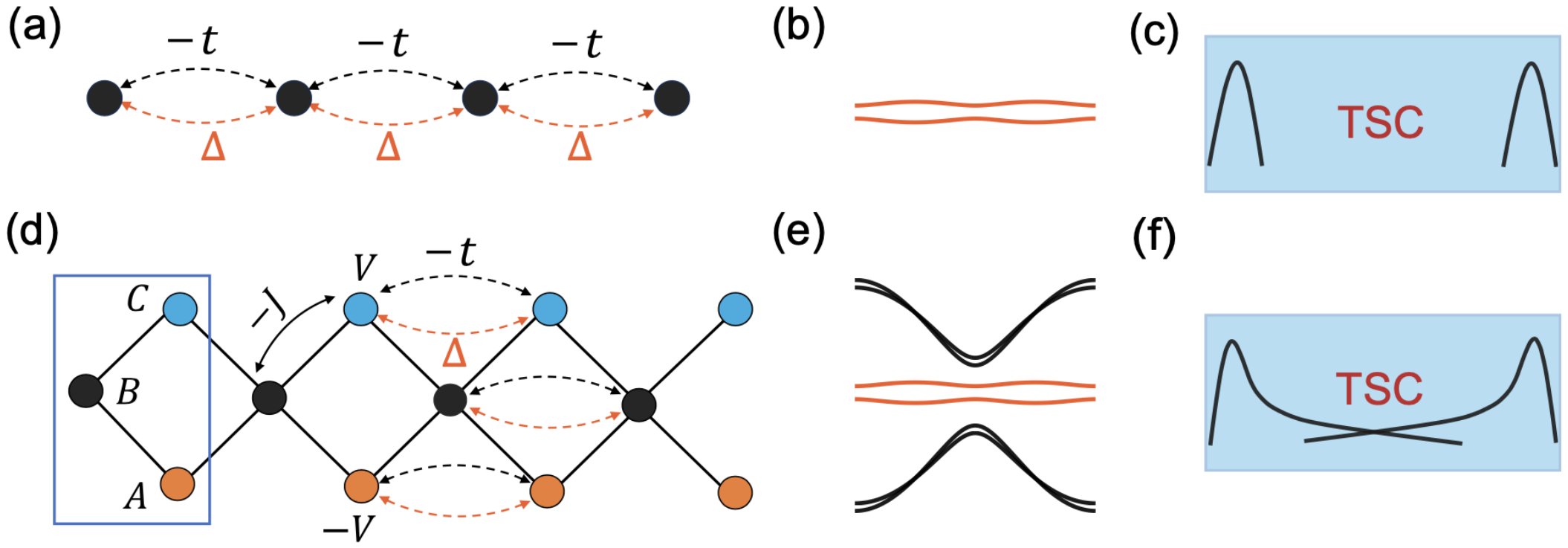
4. Gate-defined Josephson junction with interaction-driven QAHI

Nature Comm. 2023; PRL 2023



5. Majorana Zero Modes in Lieb-Kitaev Model with quantum metric

When $t = \Delta$ the MF are localized at one site



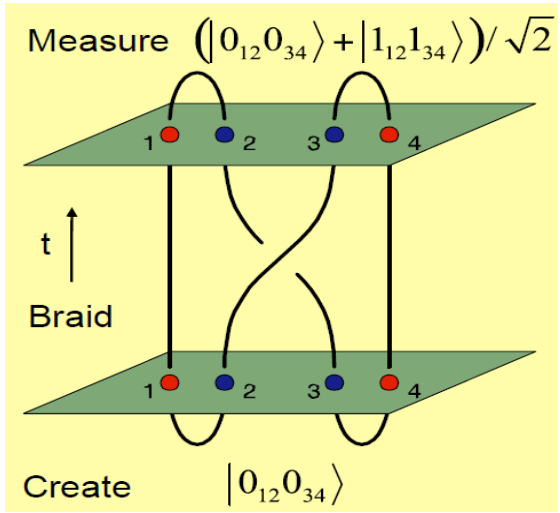
When $t = \Delta$ the MF can be highly delocalized in the Lieb-Kitaev model.

Kitaev model of 1D-superconductor with Majorana end states



Kitaev, Physics-Uspekhi (2001)

Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^\dagger$



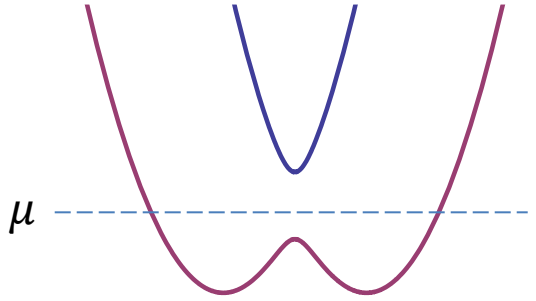
Real part of a Dirac fermion :

$$\begin{cases} \gamma_1 = \Psi + \Psi^\dagger & ; \quad \Psi = \gamma_1 + i\gamma_2 \\ \gamma_2 = -i(\Psi - \Psi^\dagger) & ; \quad \Psi^\dagger = \gamma_1 - i\gamma_2 \end{cases} \qquad \gamma_i^2 = 1 \qquad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

Non-abelian particles and braiding

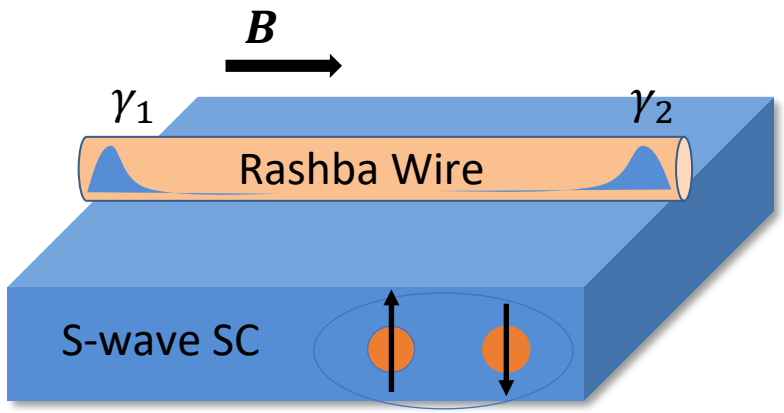
The key ingredients of realizing Majorana end states:

1. There is one band at the Fermi energy.
2. There is superconducting pairing.



Majorana in quantum wires with Rashba spin-orbit coupling (SOC)

S-wave + Rashba Wire + Zeeman field



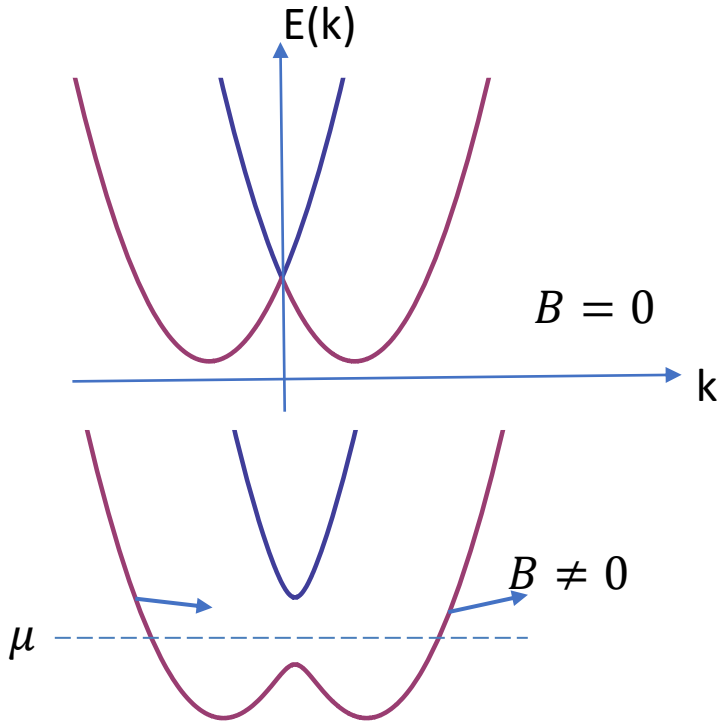
$$H_{1D}(k) = [(\frac{k^2}{2m} + \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k\sigma_y)]\tau_z - \Delta\sigma_y\tau_y$$

Rashba spin-orbit coupling

Majorana end states exist when

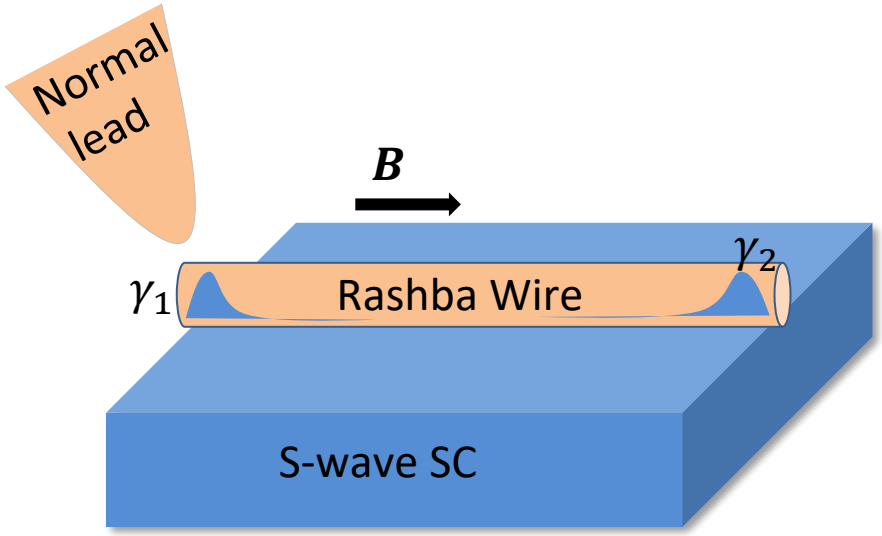
$$B > \sqrt{\mu^2 + \Delta^2}$$

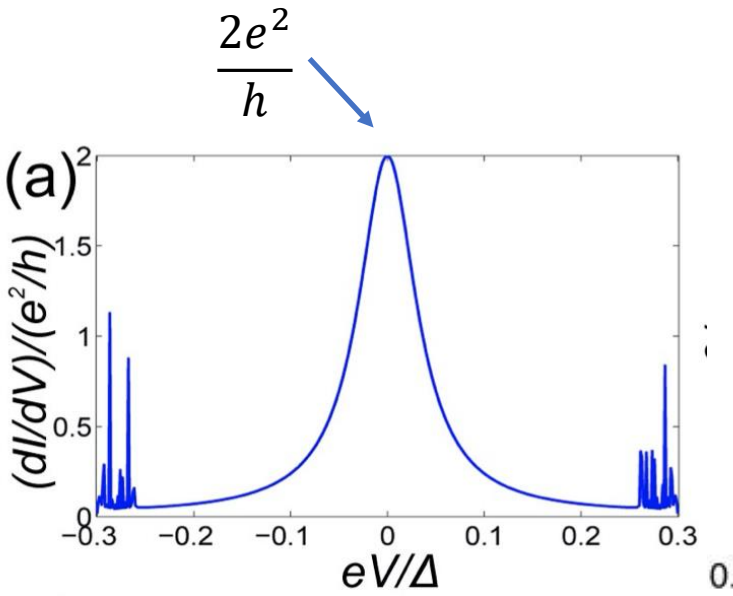
M. Sato, et al., PRL (2009); J. Alicea, PRB (2010);
Y. Oreg et al., PRL (2010); J. Sau et al., PRL (2010).

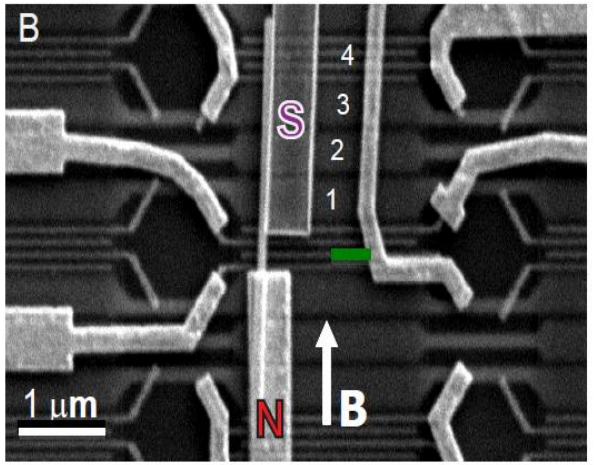


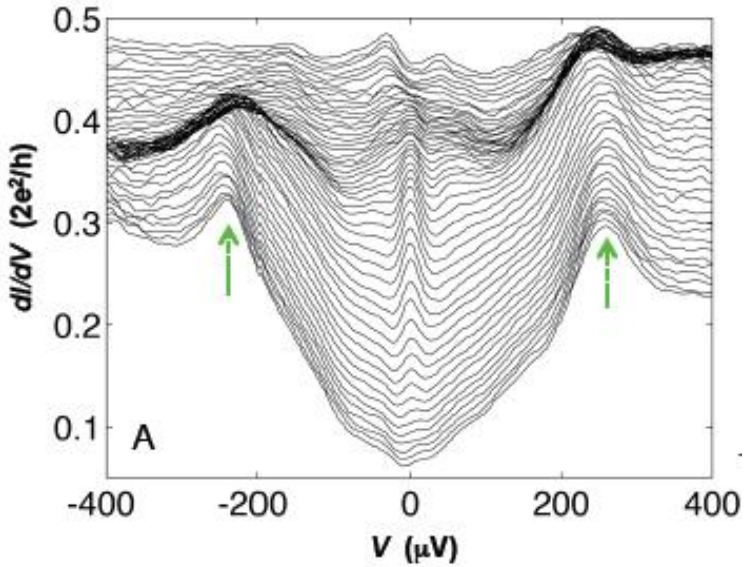
Disadvantages:
1.Need large Zeeman field to open a topological regime.
2. Large Zeeman field will reduce pairing amplitude

Charge signature – quantized zero conductance peak









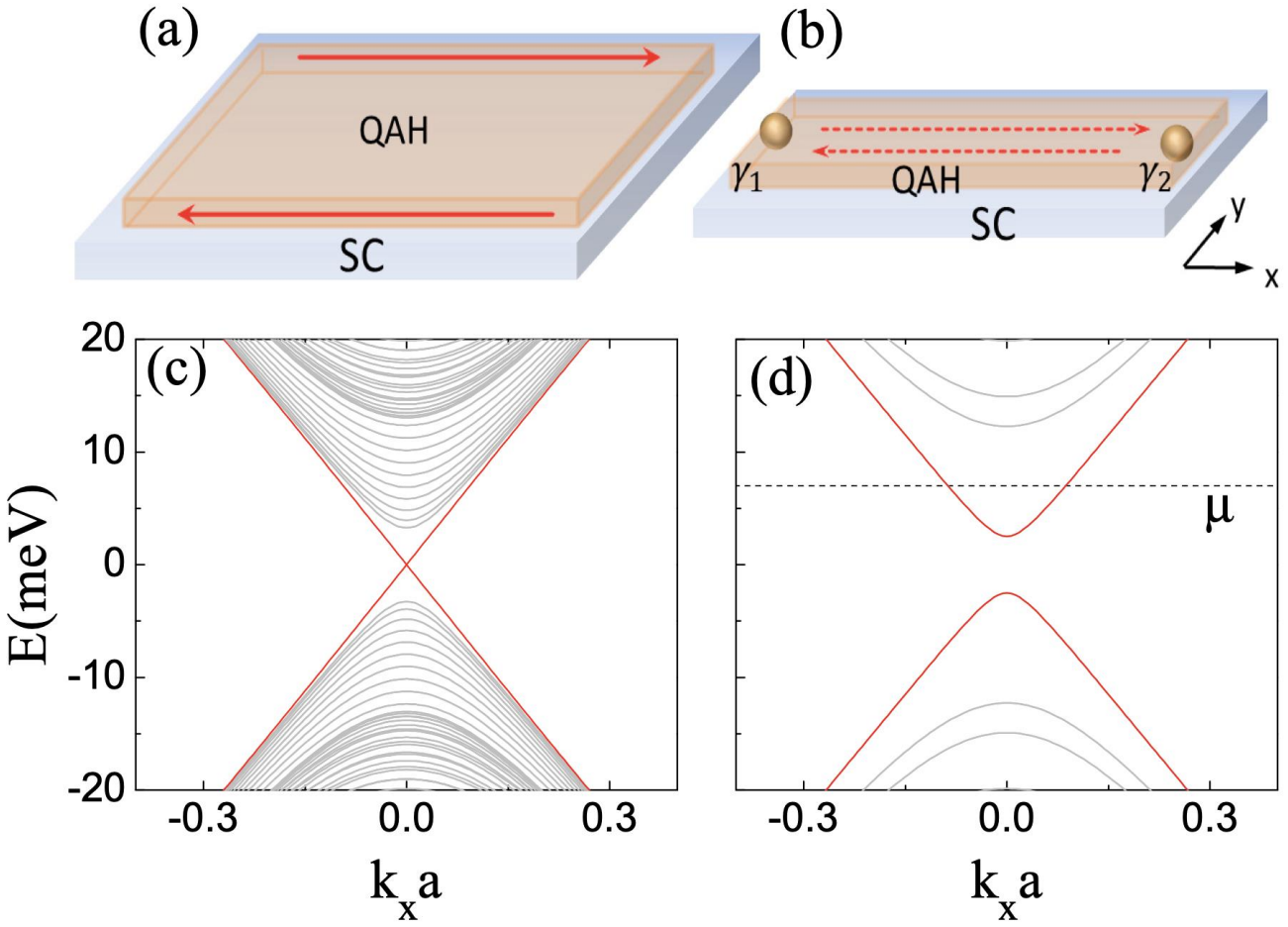
- Resonant Andreev reflection and quantized zero-bias conductance peak

$$\left.\frac{dI}{dV}\right|_{T=0,V=0} = \frac{2e^2}{h}$$

K. T. Law et al., PRL (2009)
Jie Liu et al., PRL (2012)

Kouwenhowen et al., Science (2012)

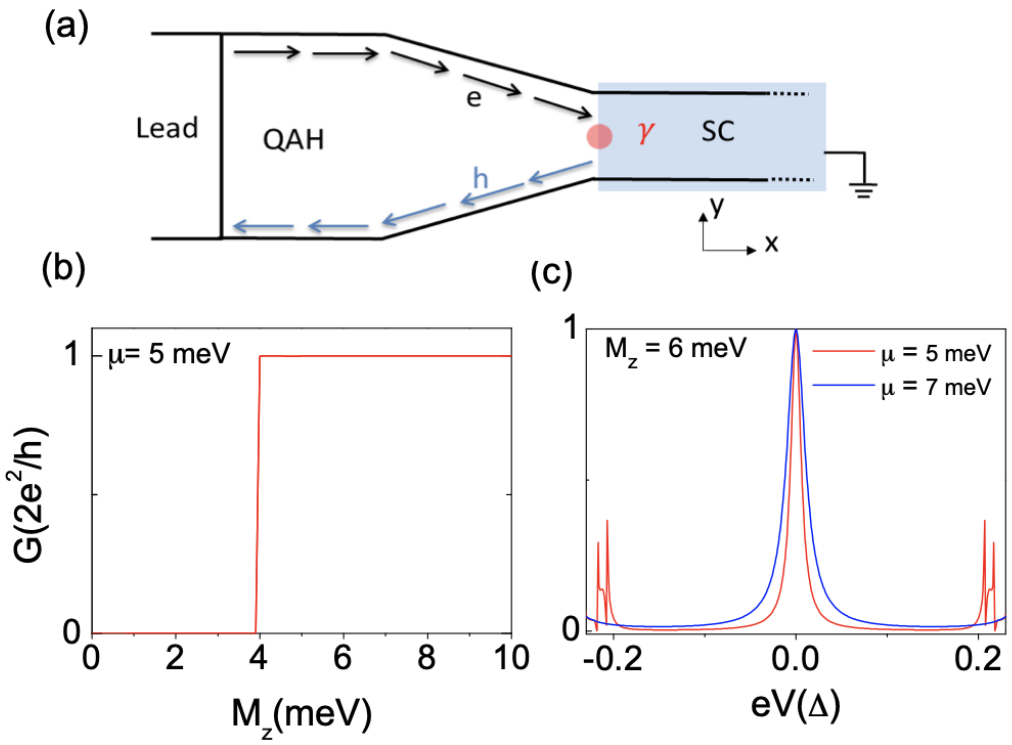
1. Majorana fermions in quasi-1D QAH platforms



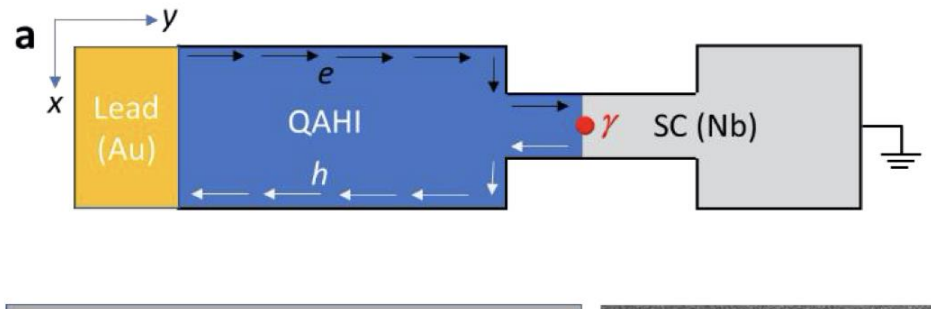
The separation of the edge states is comparable to the superconducting coherence length of the superconductor.

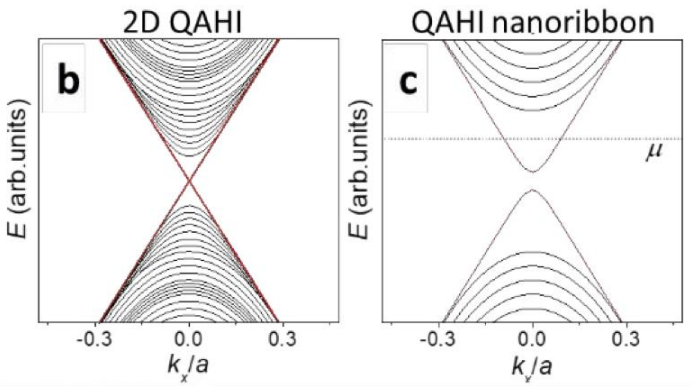
Yingming Xie et al. PRB 2018

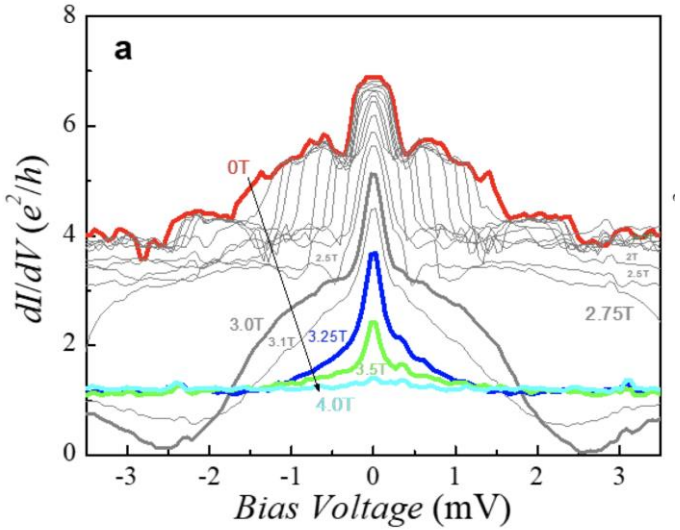
Detection

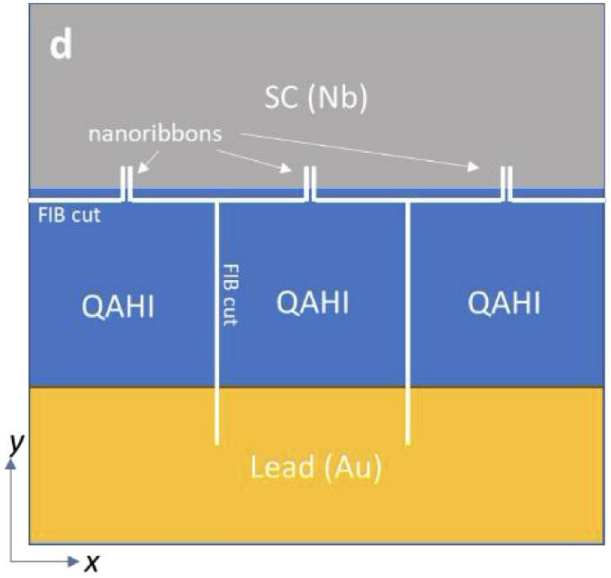


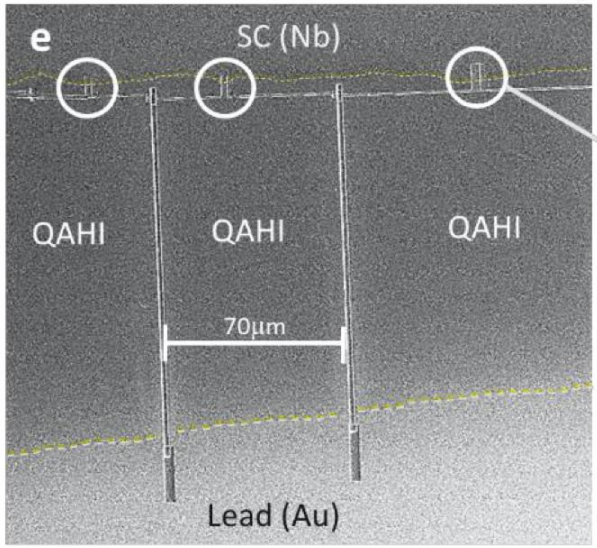
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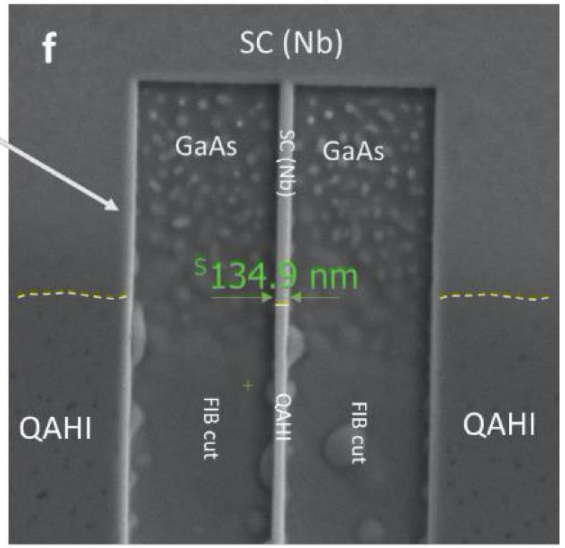


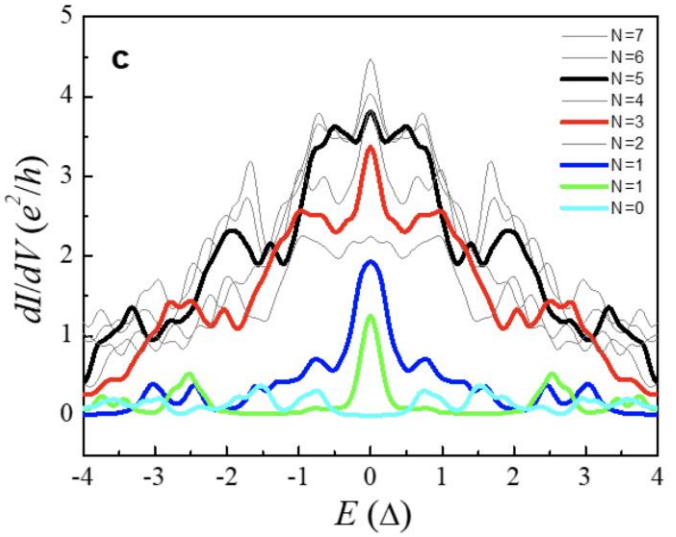






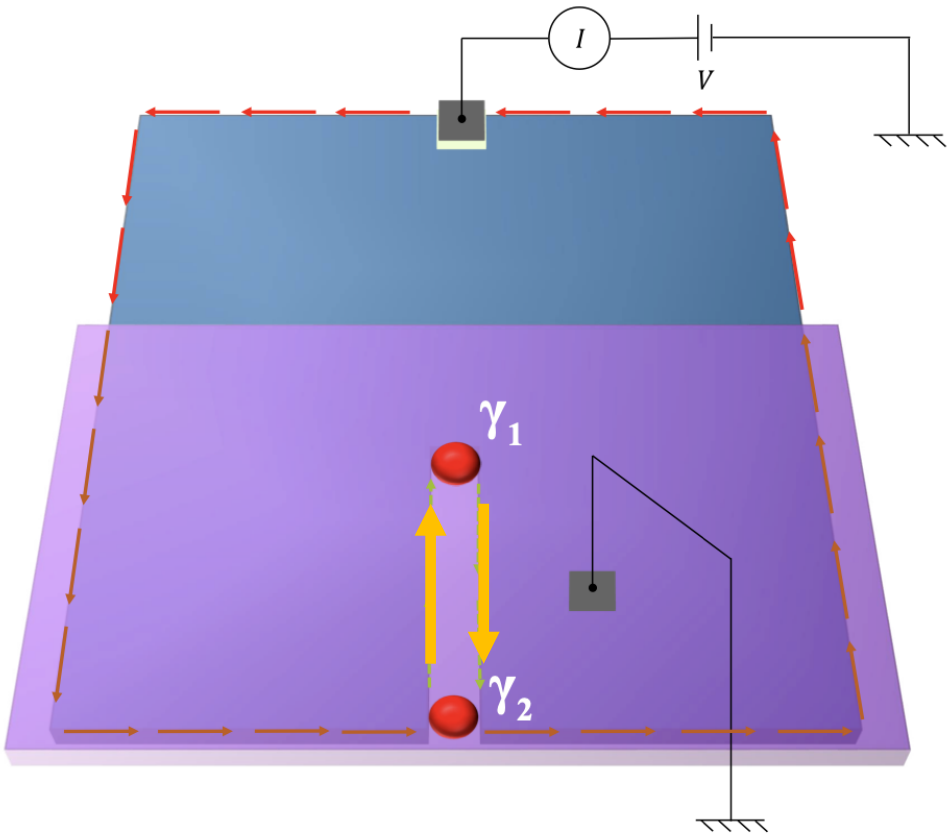






Cell reports physical science 2023 with Rolf Lort's group at HKUST

2. QAHI with a cut



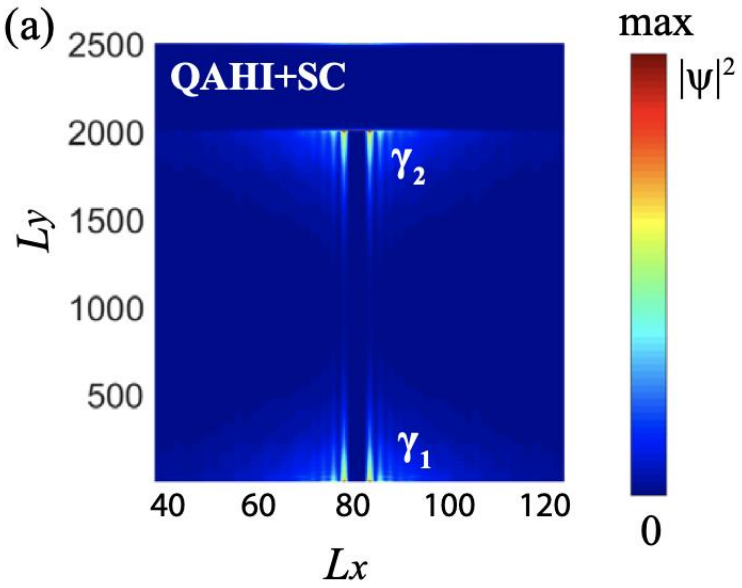
Superconductor

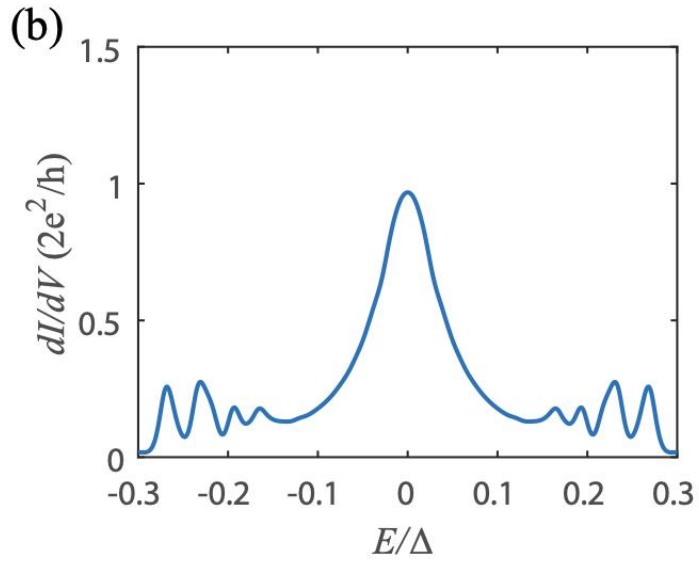
QAHI

Dielectric Barrier

Electrode

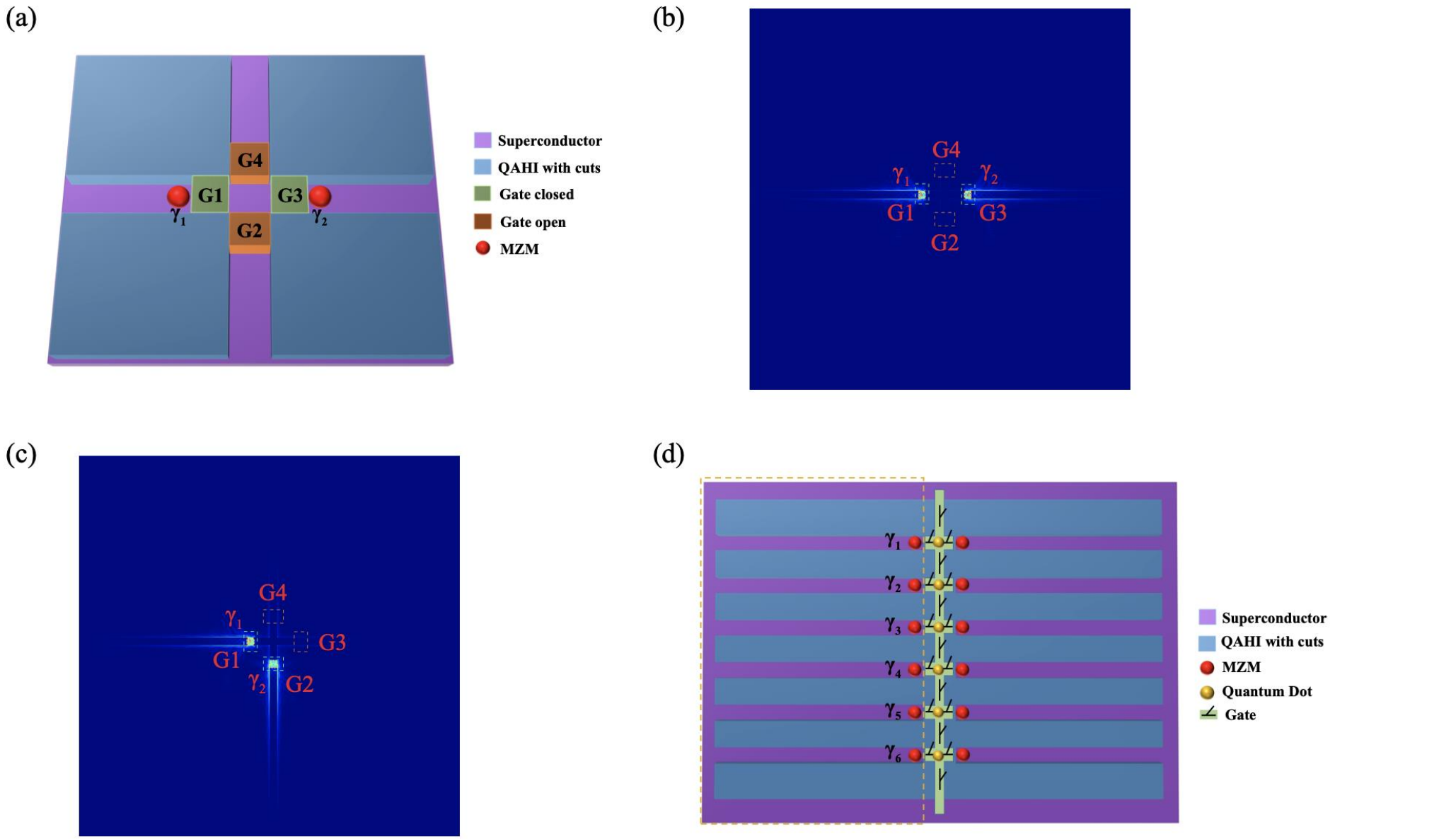
Majorana zero mode



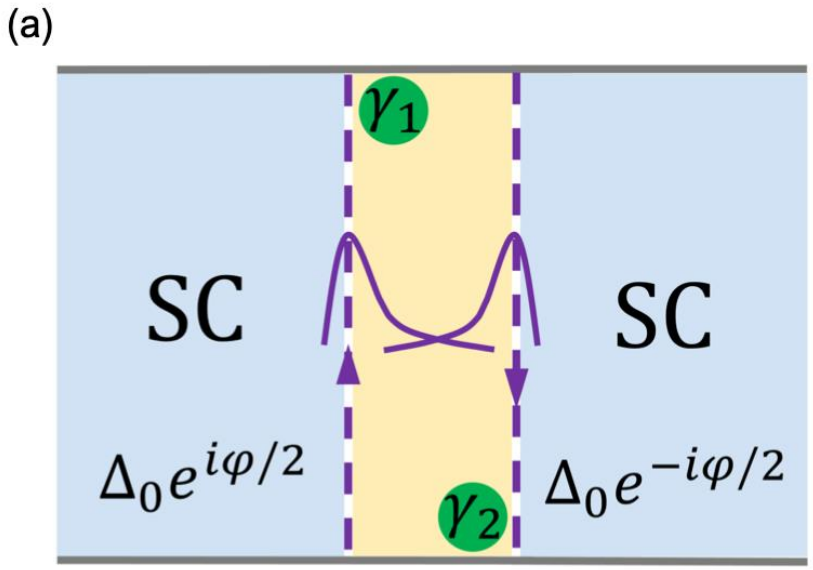


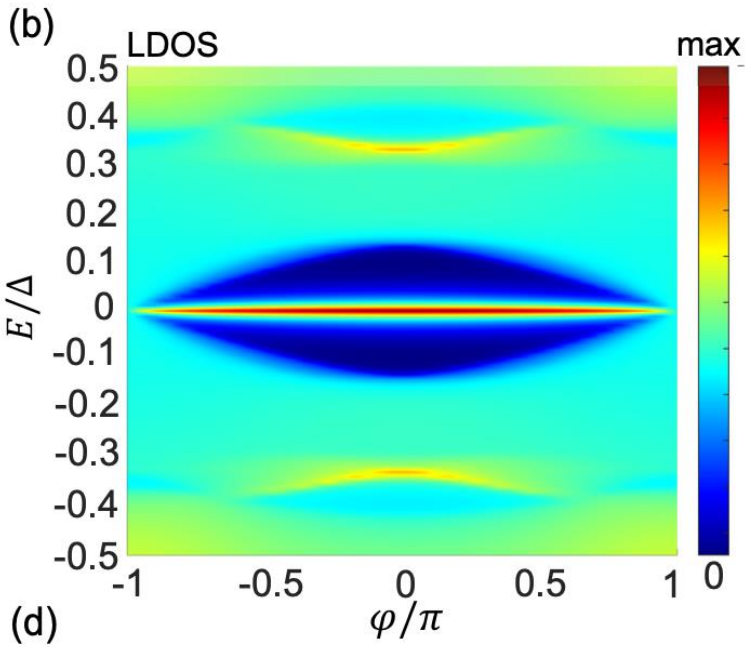
US Patent 2020, arXiv:2012.15523

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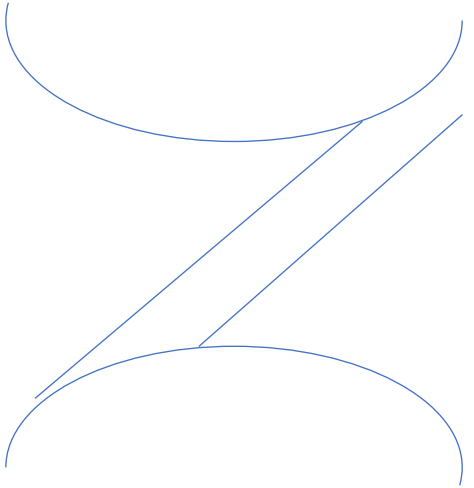


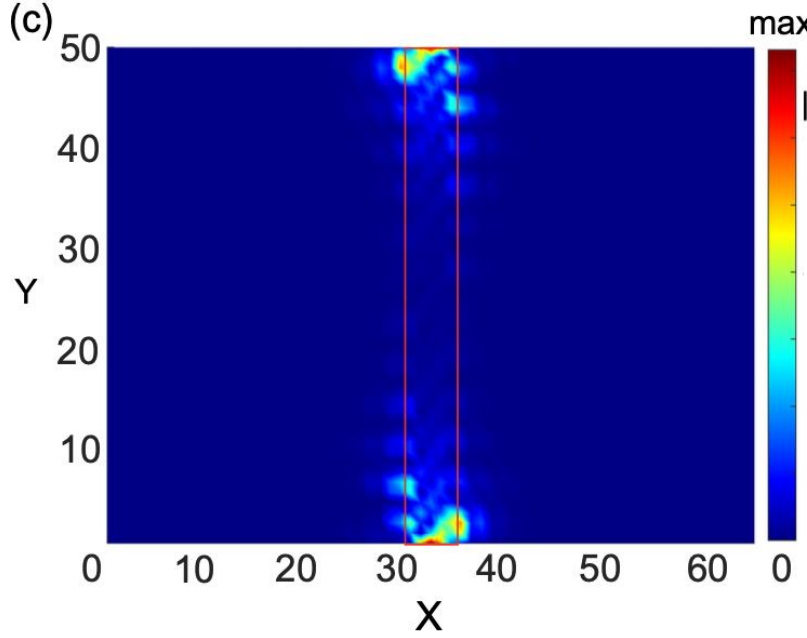
3. SC/QAHI/SC Josephson Junctions

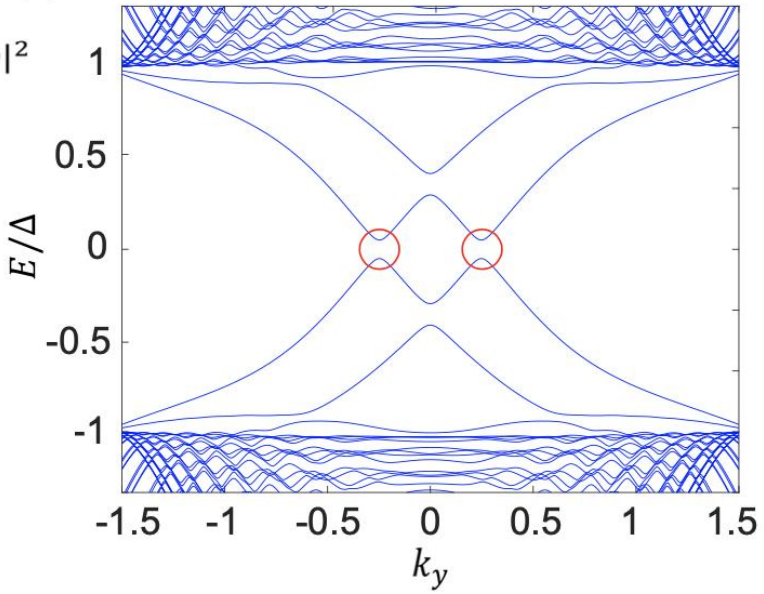


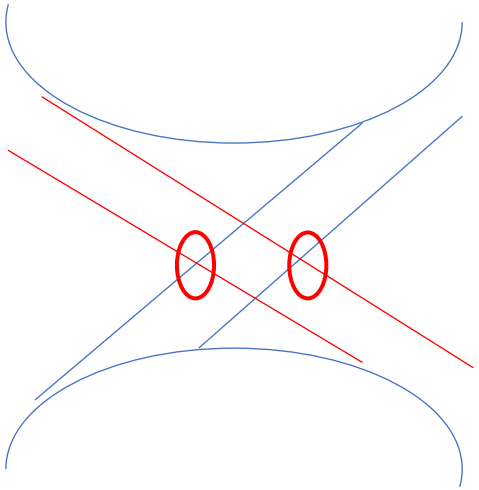


Chiral Andreev edge state









nature communications

Article

https://doi.org/10.1038/s41467-023-38005-7

Symmetry-broken Josephson junctions and superconducting diodes in magic-angle twisted bilayer graphene

Received: 16 January 2023

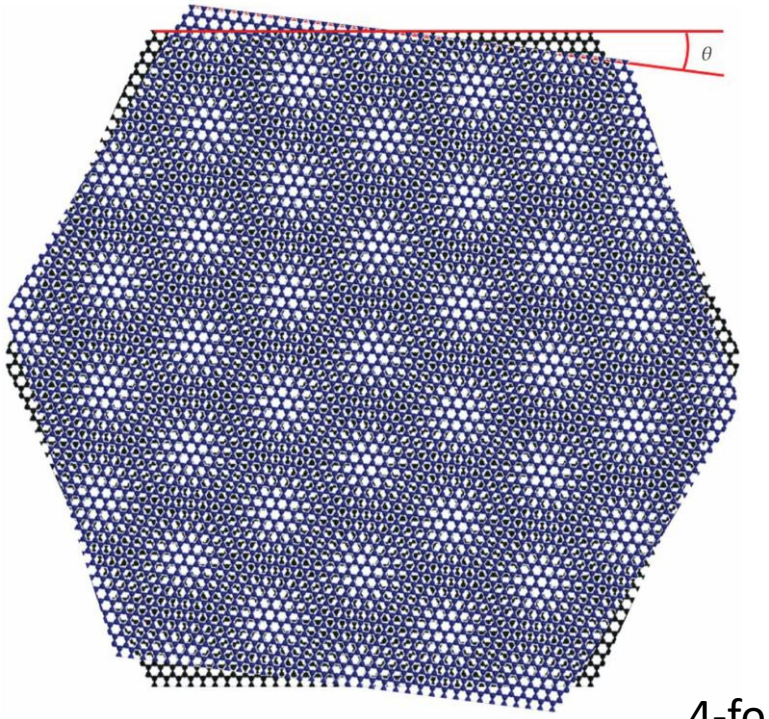
Accepted: 6 April 2023

Published online: 26 April 2023

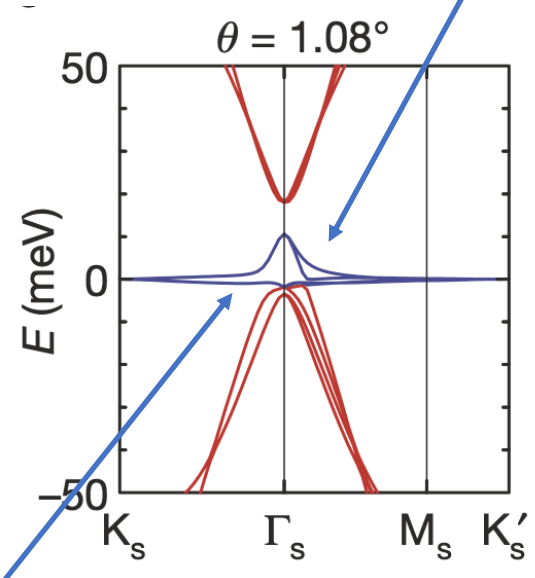
J. Díez-Mérida [1], A. Díez-Carlón [1], S. Y. Yang[1], Y.-M. Xie [2], X.-J. Gao[2], J. Senior [3], K. Watanabe [4], T. Taniguchi [4], X. Lu [1], A. P. Higginbotham [3], K. T. Law [2] & Dmitri K. Efetov [1]



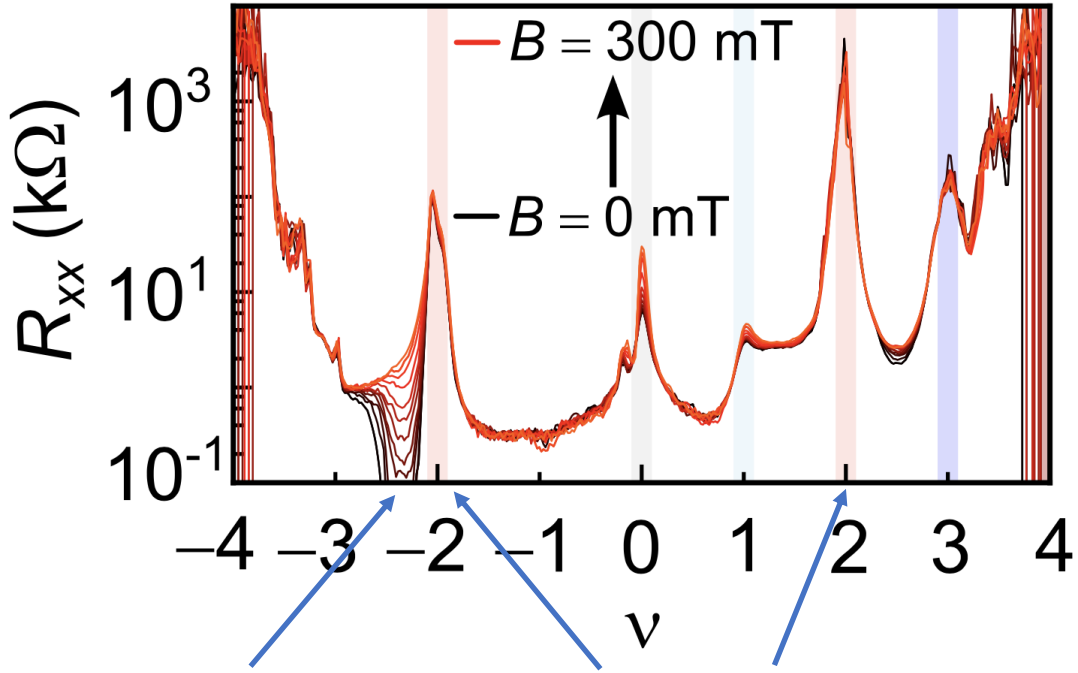
Correlated Insulator and Superconductivity in Twisted bilayer graphene



4-fold degenerate electron bands



4-fold degenerate electron bands



Superconducting state

Correlated insulating state

Cao, Yuan, et al.
Nature 556.7699 (2018). Nature 556.7699 (2018).
Adrian HC PO, Zou, Ashvin, SenthilPhys. Rev. X 8, 031089 (2018)

Nature Communications 14 : 2396 (2023)
KTL with Dima Efetov's group

nature communications

Article

https://doi.org/10.1038/s41467-023-38005-7

Symmetry-broken Josephson junctions and superconducting diodes in magic-angle twisted bilayer graphene

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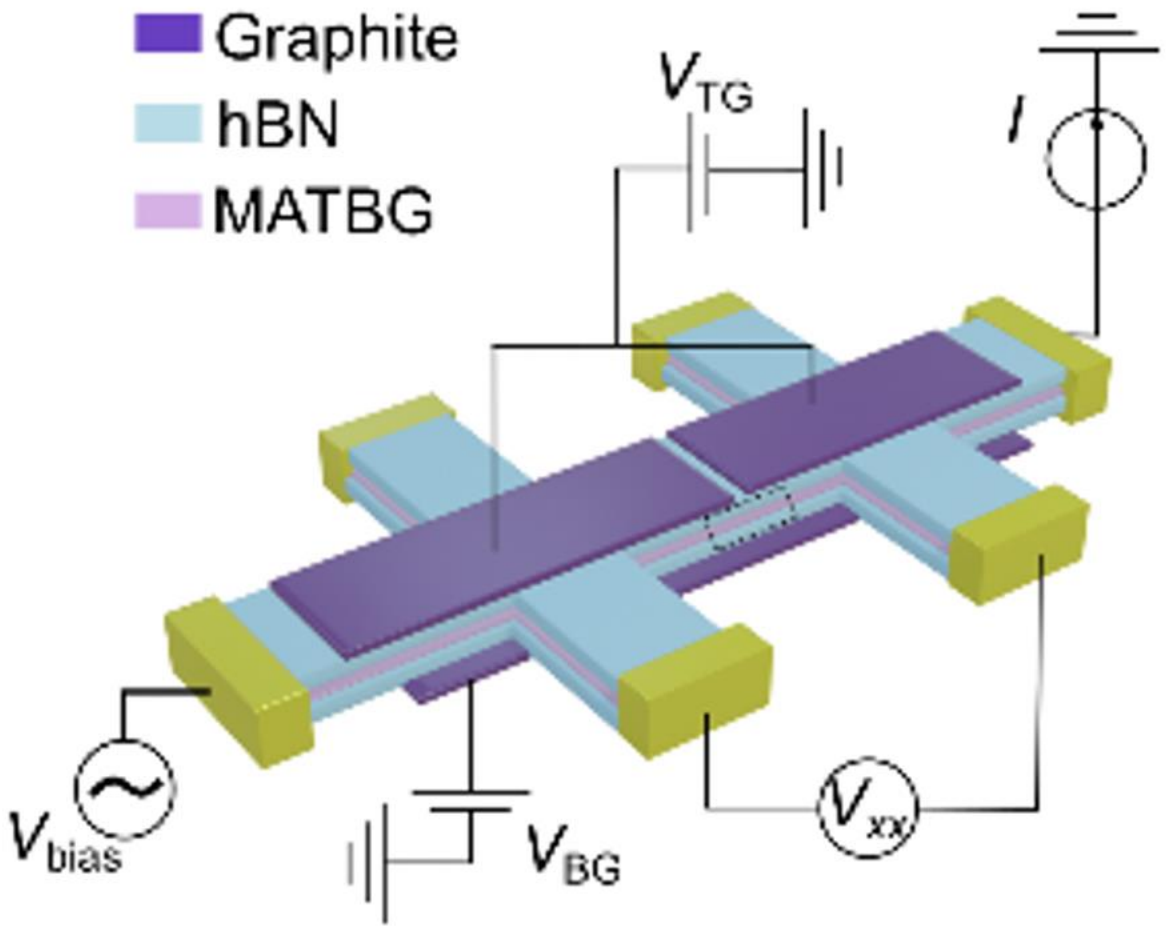
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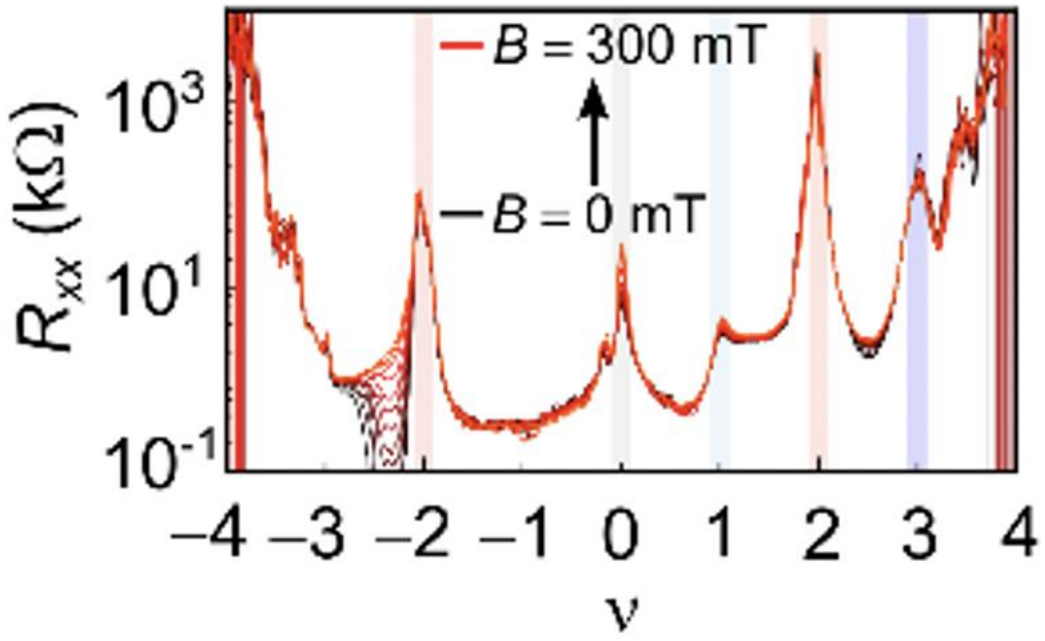
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J. Díez-Mérida [1], A. Díez-Carlón [1], S. Y. Yang[1], Y.-M. Xie [2], X.-J. Gao[2], J. Senior [3], K. Watanabe [4], T. Taniguchi [4], X. Lu [1], A. P. Higginbotham [3], K. T. Law [2] & Dmitri K. Efetov [1]



Superconductor/correlated insulator/superconductor junction







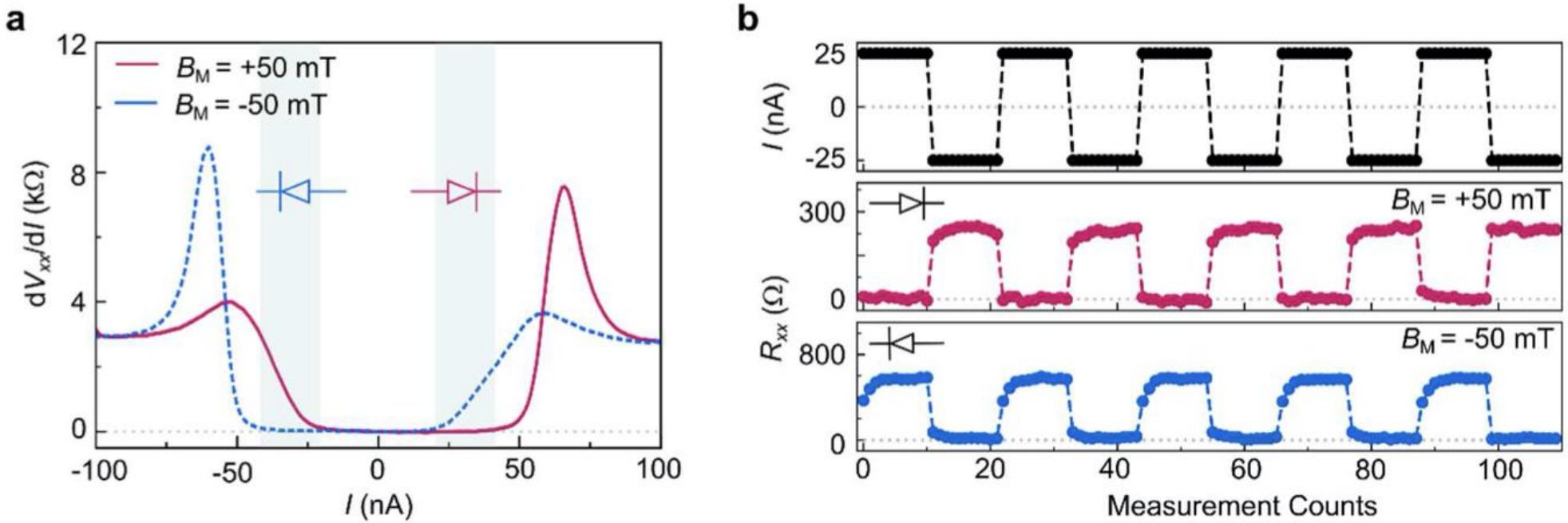
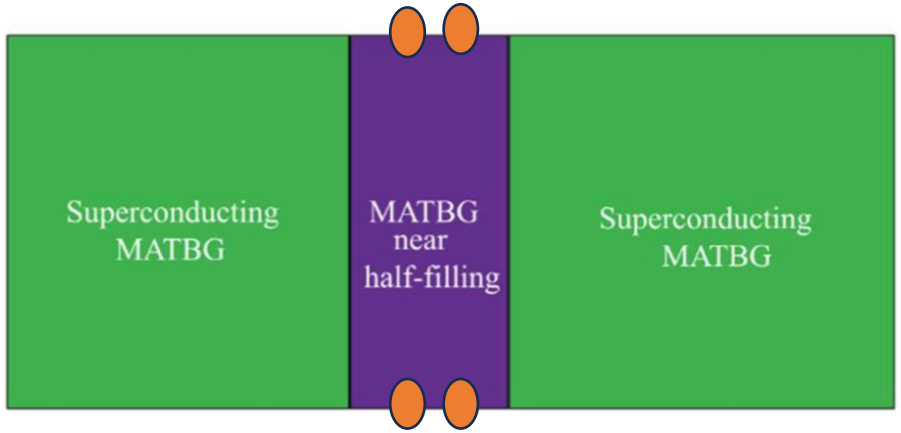
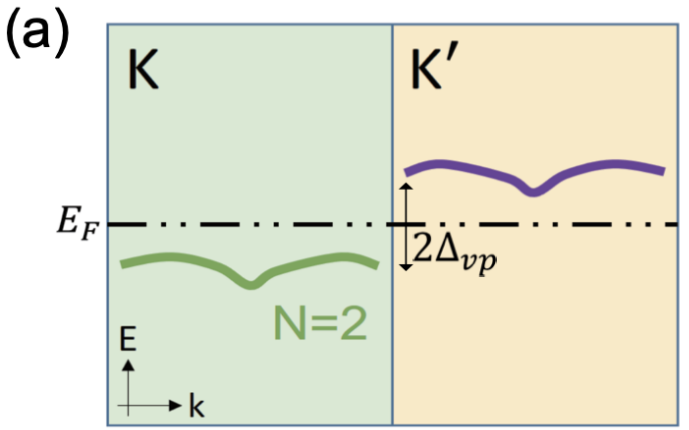


Fig. 4. Zero-field switchable superconducting Josephson diode. a, dV_{xx}/dI vs. I measured setting n_J = -1.56×10^{12} cm^{-2} at 35 mK. All curves are taken at B=0 mT after pre-magnetizing the sample at B_M = +50 mT (red) or B_M = -50 mT (blue). The shaded gray regions mark the values of current at which the diode behavior is observed. **b,** Switching between resistive and superconducting state by changing the direction of I as shown in the top panel. By applying opposite B_M, the diode behavior is inverted (red and blue curves).

Josephson Diode Effect Induced by Valley Polarization in Twisted Bilayer Graphene

Jin-Xin Hu,* Zi-Ting Sun,* Ying-Ming Xie,† and K. T. Law‡
Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

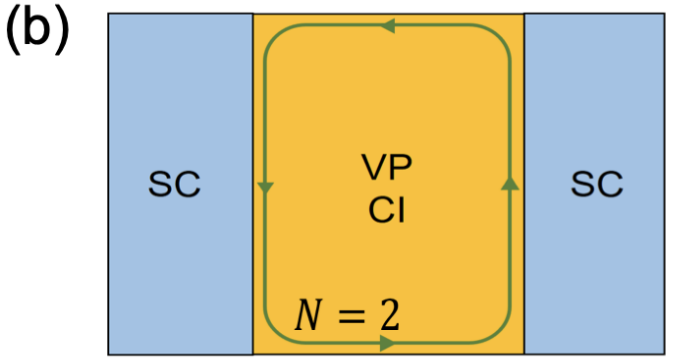




$$H_\tau^{2D} = \lambda_0(k_x^2 + k_y^2) + \tau\lambda_1 k_x(k_x^2 - 3k_y^2) + \tau\Delta_{vp} - \mu$$

$$I_s(\varphi) = -\frac{4e}{\hbar\beta}\frac{d}{d\varphi}\sum_{n=0}^{\infty}\ln\det[1 - S_A(i\omega_n,\varphi)S_N(i\omega_n,\varphi)]$$

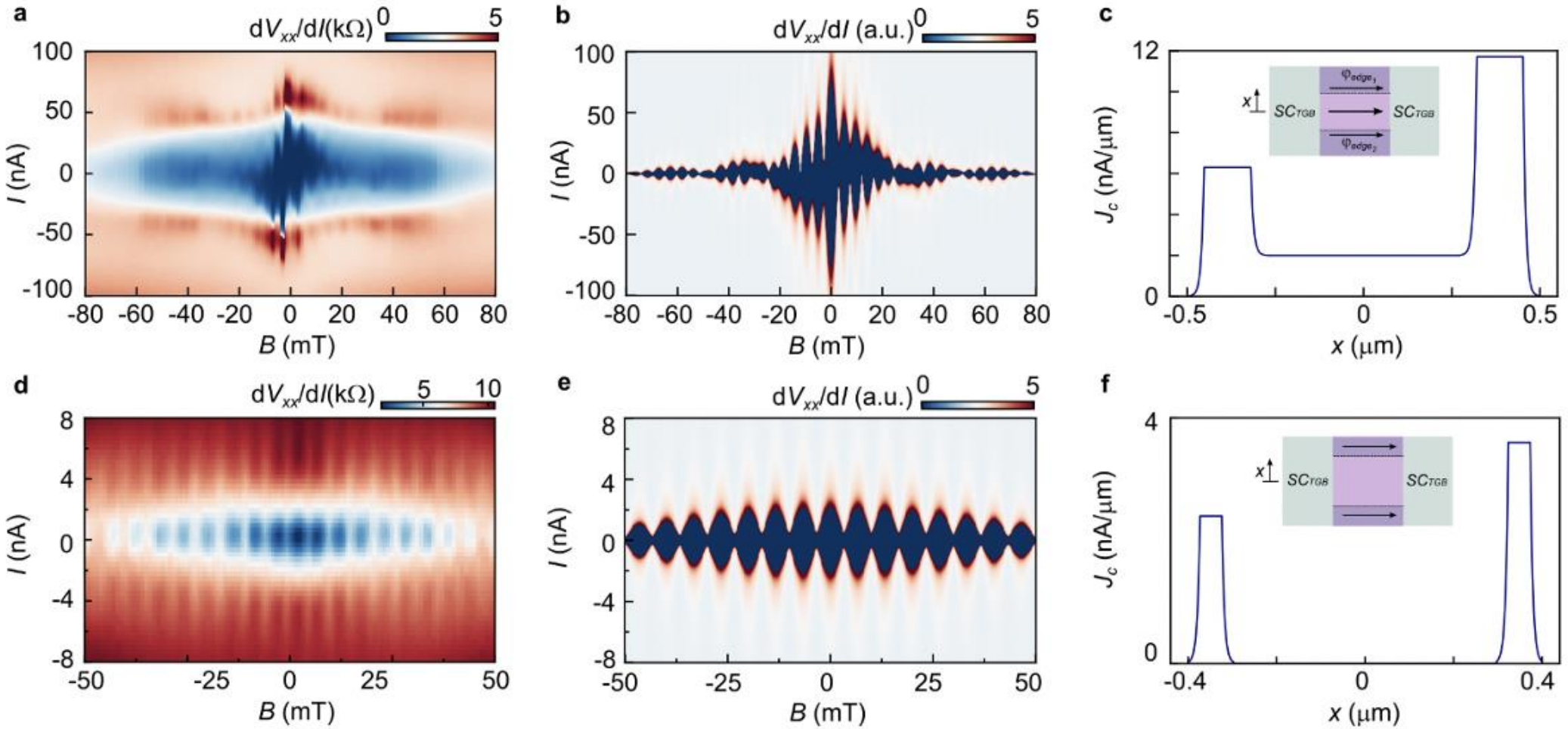
$$J_s(\varphi) = j_1\sin(\tilde{\varphi} + \delta) + j_2\sin(2\tilde{\varphi})$$



Fraunhofer pattern of two devices

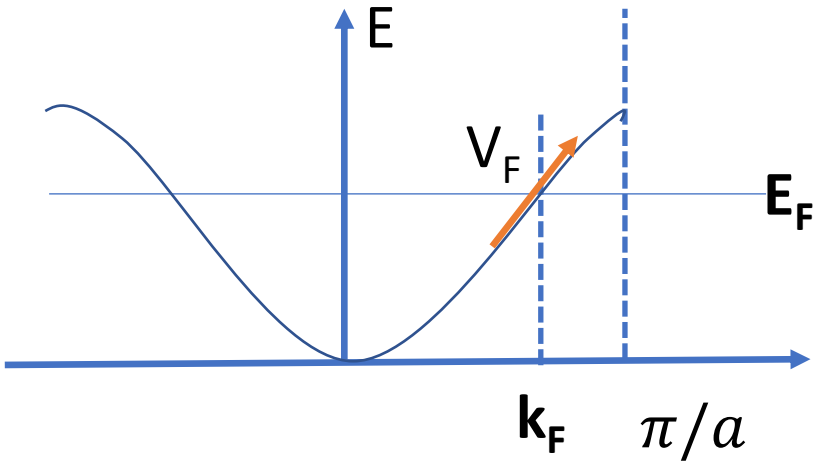
First device

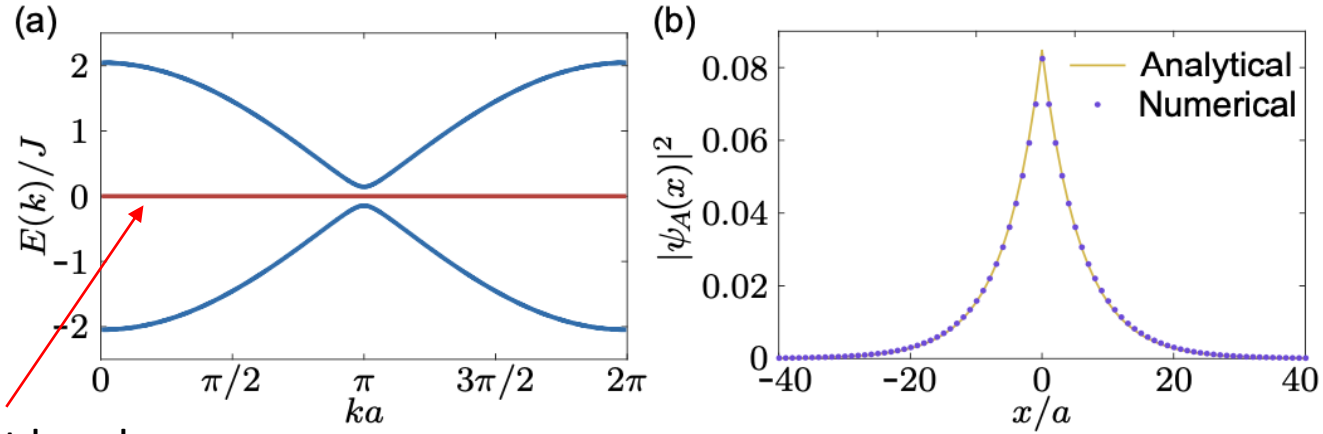
Second device



5. Majorana modes in flat band topological superconductors

Quantum Metric Lengths





Flat band

1. For a dispersive band, there is a length scale Fermi wavelength $\lambda_F=2\pi/k_F$.
It controls, for example, the Friedel oscillation period.

2. V_F is associated with many length scales such as the BCS coherence length:

$$\xi_{BCS} = \hbar v_F/\Delta$$

3. For flat bands, k_F is not well-defined and V_F is zero.
What is the fundamental length scale for superconducting coherence length, Andreev bound states? Answer: **The quantum metric lengths**.

Outline

1. Flat band superconductivity with quantum metric – coherence length

Shuai Chen, KTL PRL 132, 026002 (2024);

2. Majorana Zero Modes in Lieb-Kitaev Model with Tuneable Quantum Metric

Xingyao Guo, Xinglei Ma, Xuzhe Ying and KTL, arXiv: 2406.05789.

1. Flat band superconductivity with quantum metric – coherence length

Article | Published: 15 February 2023

Evidence for Dirac flat band superconductivity enabled by quantum geometry

Haidong Tian, Xueshi Gao, Yuxin Zhang, Shi Che, Tianyi Xu, Patrick Cheung, Kenji Watanabe, Takashi Taniguchi, Mohit Randeria, Fan Zhang, Chun Ning Lau ✉ & Marc W. Bockrath ✉

Nature **614**, 440–444 (2023) | Cite this article

PHYSICAL REVIEW LETTERS **132,** 026002 (2024)

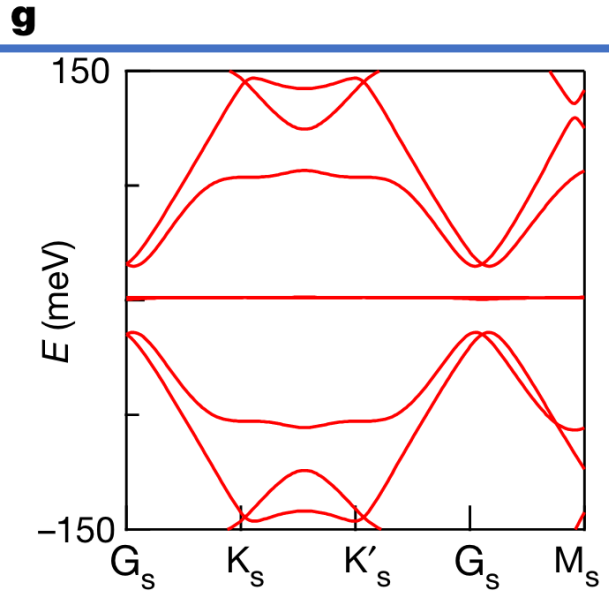
Editors' Suggestion

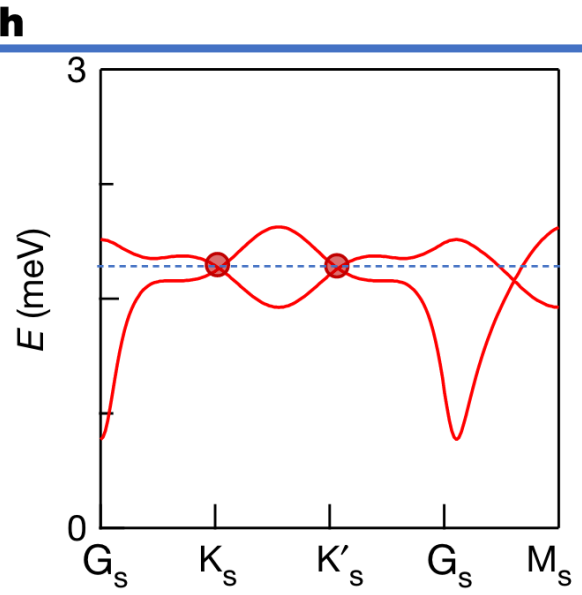
Ginzburg-Landau Theory of Flat-Band Superconductors with Quantum Metric

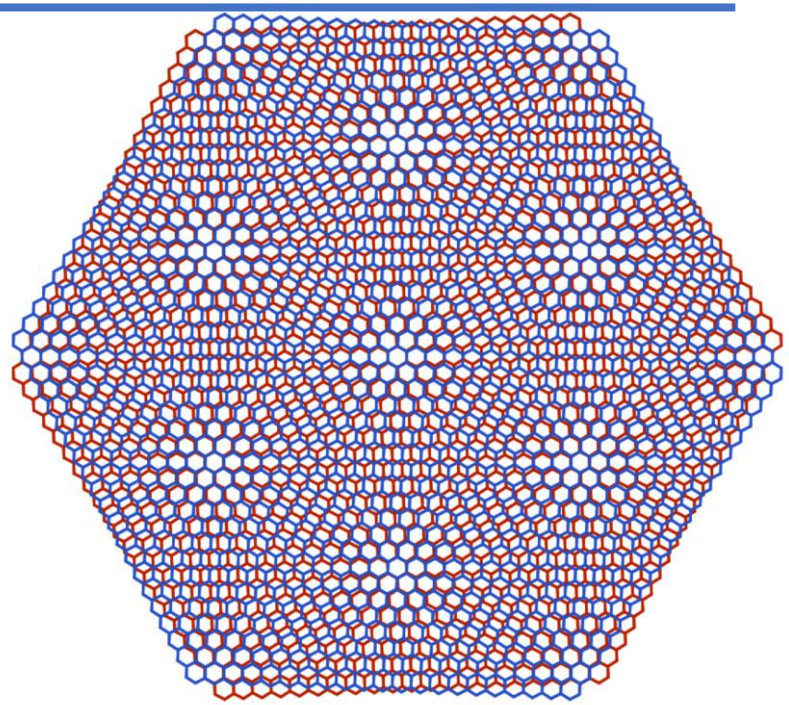
Shuai A. Chen* and K. T. Law†

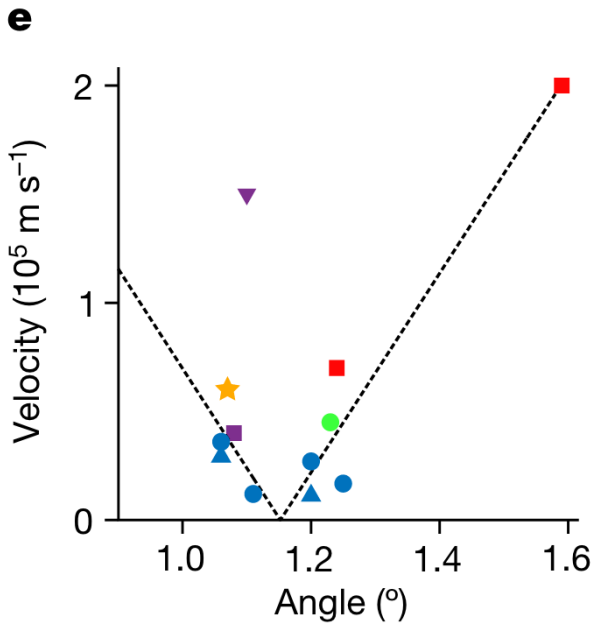
Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

Flat bands in twisted bilayer graphene









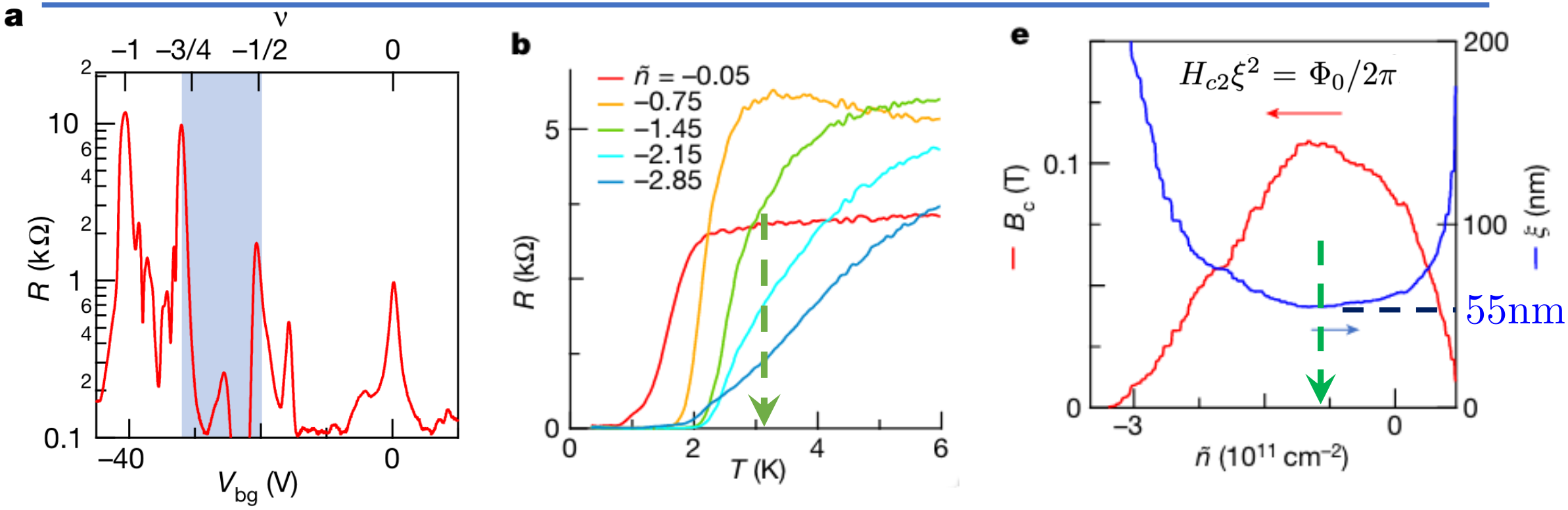
Flat/Narrow band

$\theta = 1.08^\circ$

$v_F \sim 1000 m/s$

Y. Cao et al *Nature* **556**, 80 (2018),
Marc W. Bockrath et al. *Nature* **614**, 440–444 (2023)

Superconducting coherence length



$v_F \sim 1000m/s$ $\quad T_c = 2.2\text{K}$ $\quad \xi_{BCS} = \hbar v_F/\Delta$

Experiments	Conventional SC
	$\Delta/T_c \sim 1.7$
$\xi \sim 55\text{nm}$	$\xi \sim v_F/T_c \sim 2.6\text{nm}$

Marc W. Bockrath et al. *Nature* **614**, 440–444 (2023)

Quantum distance

Parameter-dependent Hamiltonian $$H(\boldsymbol{\kappa})|\Psi(\boldsymbol{\kappa})\rangle = E(\boldsymbol{\kappa})|\Psi(\boldsymbol{\kappa})\rangle$$

- Bures Distance $$D_{12}^2 = 1 - \left|\langle \Psi_0(\boldsymbol{\kappa}_1) \mid \Psi_0(\boldsymbol{\kappa}_2)\rangle\right|^2$$

  - Vanishing when two states physically coincide.
  - D=1, when two states are orthogonal.

$$D^2_{\boldsymbol{\kappa},\boldsymbol{\kappa}+d\boldsymbol{\kappa}} = \sum_{\alpha,\beta=1}^{d} g_{\alpha\beta}(\boldsymbol{\kappa}) d\kappa_\alpha d\kappa_\beta$$

Quantum metric tensor
(components have units of length square!)

$$g_{\alpha\beta}(\kappa) = \mathrm{Re}[\langle \partial_\alpha \Psi_0(\kappa)|(1 - |\Psi_0\rangle\langle\Psi_0|)|\partial_\beta \Psi_0\rangle]$$

Framework of Ginzburg-Landau theory

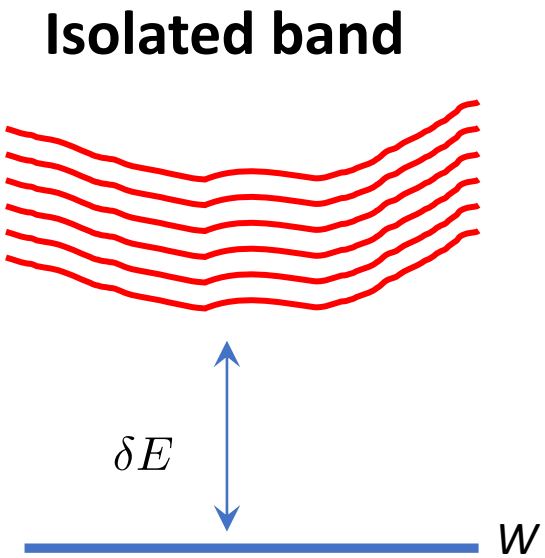
We focus on the flat/narrow band around Fermi energy

$$H = H_0 + H_{\text{int}}$$

$a_\xi(\mathbf{r})$: Fermion at orbital basis

$$H_0 = -\int d\mathbf{r}\mu(a_+^\dagger(\mathbf{r})a_+(\mathbf{r}) + a_-^\dagger(\mathbf{r})a_-(\mathbf{r}))$$

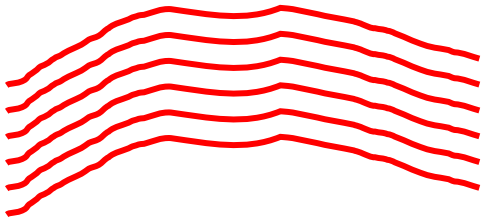
$$H_{\text{int}} = -U\int d\mathbf{r}a_+^\dagger(\mathbf{r})a_-^\dagger(\mathbf{r})a_-(\mathbf{r})a_+(\mathbf{r})$$



Bosonic field through the Hubbard-Stratonovich transformation: $\Delta(\mathbf{r}) = a_-(\mathbf{r})a_+(\mathbf{r})$,

$$\mathcal{L}[a,\bar{a},\Delta,\bar{\Delta}] = (\partial_\tau - \mu)(\bar{a}_+(\mathbf{r})a_+(\mathbf{r}) + \bar{a}_-(\mathbf{r})a_-(\mathbf{r})) - U\left[\Delta(\mathbf{r})\bar{a}_+(\mathbf{r})\bar{a}_-(\mathbf{r}) + h.c.\right]$$

$$\mathcal{Z}[\Delta,\bar{\Delta}] = \int \mathcal{D}[c,\bar{c}]e^{-\int_0^\beta d\tau \int d\mathbf{r}\mathcal{L}[a,\bar{a},\Delta,\bar{\Delta}]}$$



Ginzburg-Landau theory for flat band superconductor with quantum metric

$$\mathcal{L}[a,\bar{a},\Delta,\bar{\Delta}] =(\partial_\tau-\mu)(\bar{a}_+(\mathbf{r})a_+(\mathbf{r})+\bar{a}_-(\mathbf{r})a_-(\mathbf{r}))-U\left[\Delta(\mathbf{r})\bar{a}_+(\mathbf{r})\bar{a}_-(\mathbf{r})+h.c.\right]$$

$$a_\xi(\mathbf{r})\to\frac{1}{\sqrt{N}}\sum_{\mathbf{q}}\sum_{\alpha}e^{i\mathbf{q}\cdot\mathbf{r}}u^*_{\mathbf{q}\xi}(\alpha)c_{\mathbf{q}\xi}$$

$$\mathcal{L}[c,\bar{c},\Delta,\bar{\Delta}] =(\partial_\tau-\mu)(\bar{c}_{\mathbf{q},+}c_{\mathbf{q},+}+\bar{c}_{\mathbf{q},-}c_{\mathbf{q},-})-\sum_{\mathbf{k}}U[\Gamma(\mathbf{q},\mathbf{k})\Delta(\mathbf{k})\bar{c}_{\mathbf{q}+\frac{\mathbf{k}}{2},+}\bar{c}_{-\mathbf{q}+\frac{\mathbf{k}}{2},-}+h.c.]$$

$$\mathcal{Z}[\Delta,\bar{\Delta}]=\int\mathcal{D}[c,\bar{c}]e^{-\int_0^\beta d\tau\int d\mathbf{r}\mathcal{L}[a,\bar{a},\Delta,\bar{\Delta}]}$$

$$F[\Delta,\bar{\Delta}]=\sum_{\mathbf{k}}U|\Delta(\mathbf{k})|^2-T\ln\mathcal{Z}[\Delta,\bar{\Delta}]$$

Form factor

$$\sum_\alpha u_{-\mathbf{q}+\mathbf{k}/2,+}(\alpha)u_{\mathbf{q}+\mathbf{k}/2,-}(\alpha)$$

$$|\Gamma(\mathbf{q},\mathbf{k})|^2=\gamma_0(\mathbf{q})-\sum_{ab}g_{ab}(\mathbf{q})k_ak_b,$$

Quantum metric! →

$$g_{ab}(\mathbf{q})\equiv\mathrm{Re}\langle\partial_{\mathbf{q}_a}u_{\mathbf{q}}|(1-|u_{\mathbf{q}}\rangle\langle u_{\mathbf{q}}|)|\partial_{\mathbf{q}_b}u_{\mathbf{q}}\rangle,$$

Superconducting coherence length and quantum metric

$$\mathcal{L}[\delta\Delta] = \frac{1}{2m^*}|\nabla\delta\Delta|^2 + a(T)|\delta\Delta|^2 + \mathcal{O}(|\delta\Delta|^4)$$

$$\frac{1}{2m^*} = \frac{\beta U^2 \bar{\nu}}{4\mathcal{A}_{\rm uc}}\sqrt{\det(\bar{g}_{ab})},$$

$$a(T) = \frac{4U - U^2\beta\bar{\nu}\bar{\gamma}_0}{4\mathcal{A}_{\rm uc}}.$$

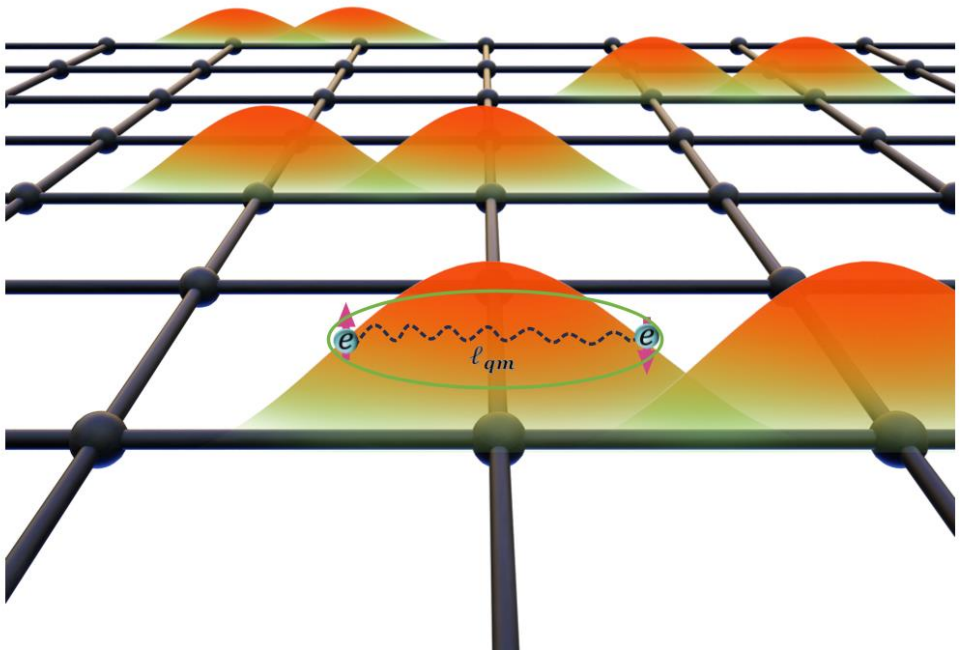
Minimal coupling to include magnetic field: $-i\nabla \to -i\nabla + 2e\mathbf{A}$

$$H_{c2} = \frac{\Phi_0}{2\pi\sqrt{\det(\bar{g}_{ab})}}\frac{|T - T_{\rm MF}|}{T_{\rm MF}}, \quad H_{c2}\xi^2 = \Phi_0/2\pi, \quad \xi = \sqrt{\frac{T_{\rm MF}}{|T - T_{\rm MF}|}}[\det(\bar{g}_{ab})]^{\frac{1}{4}}.$$

The superconducting coherence length is proportional to the quantum metric length!

Shuai Chen and KTL, Phys. Rev. Lett. 132, 026002 (2024)

Physical significance of the quantum metric



$$|\mathbf{r}_i\alpha\rangle = \frac{\mathcal{A}_{\rm uc}}{(2\pi)^d}\int_{\rm BZ} d^d\mathbf{k}\, e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_i)}\sum_{n\in\mathcal{V}}(\mathcal{U}_\mathbf{k})_{n,\alpha}|u_{n\mathbf{k}}\rangle$$

$$|u_{n\mathbf{k}}\rangle = \sum_{\mathbf{r}_i}\sum_{\alpha} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_i)}(\mathcal{U}^\dagger_\mathbf{k})_{\alpha,n}|\mathbf{r}_i\alpha\rangle.$$

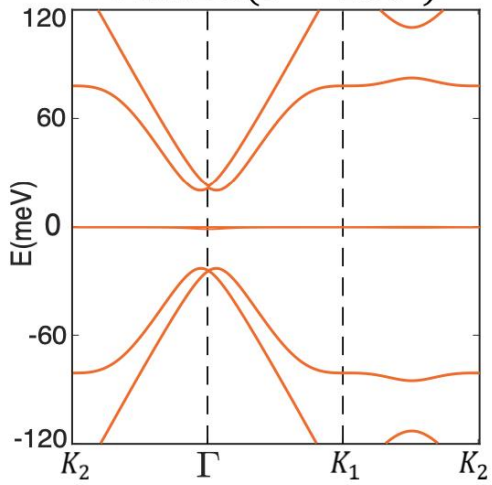
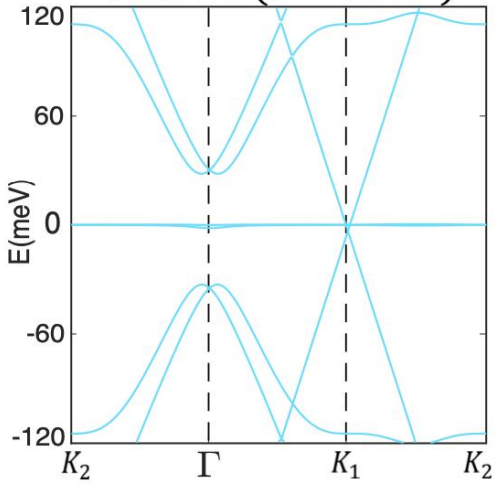
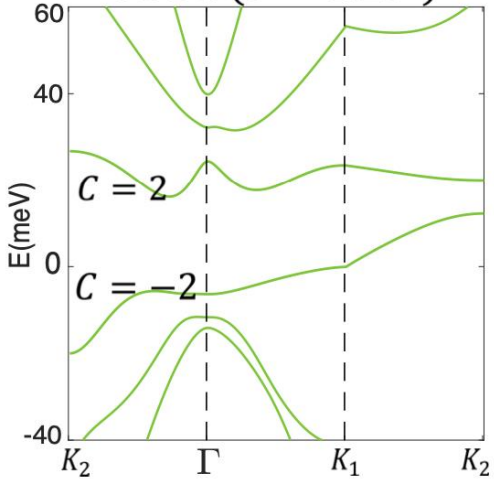
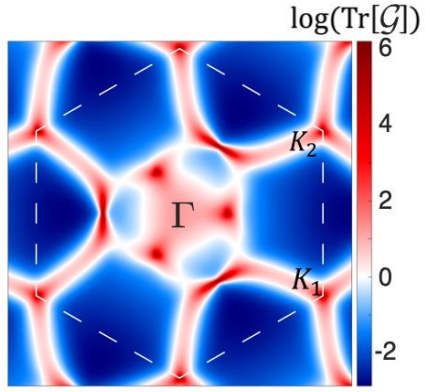
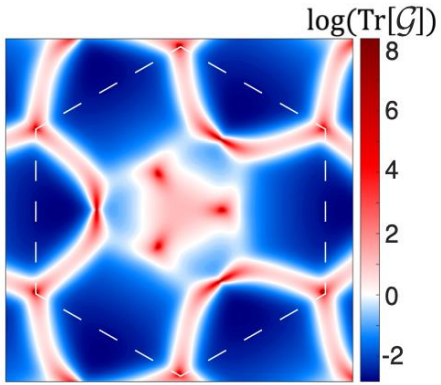
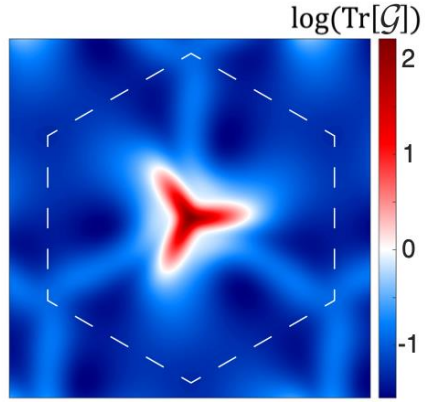
$$F = \sum_\alpha \left[\langle\mathbf{0}\alpha|\, r^2\,|\mathbf{0}\alpha\rangle - |\langle\mathbf{0}\alpha|\,\mathbf{r}\,|\mathbf{0}\alpha\rangle|^2\right]$$

The quantum metric determines the size (the quadratic spread) of an optimally localized Wannier function of a band

Nicola Marzari, Rev. Mod. Phys. 84, 1419 (20 12)

For a completely flat band, the Fermi wavelength $1/k_F$ is not well defined. BCS superconducting coherence length $\xi_{BCS} = \hbar v_F/\Delta$ goes to zero. The quantum metric length is the natural electronic length scale for flat band materials.

Superconducting graphene family

	MATBG($\theta = 1.08°$)	MATTG ($\theta = 1.53°$)	TDBG ($\theta = 1.26°$)
Moiré bands			
Quantum metric			
ℓ_{qm}	1 L_M (~13 nm)	1.2 L_M(~ 11 nm)	0.5 L_M(~ 6 nm)

5. Majorana Zero Modes in Kitaev Model

1D Kitaev model with spinless Fermions

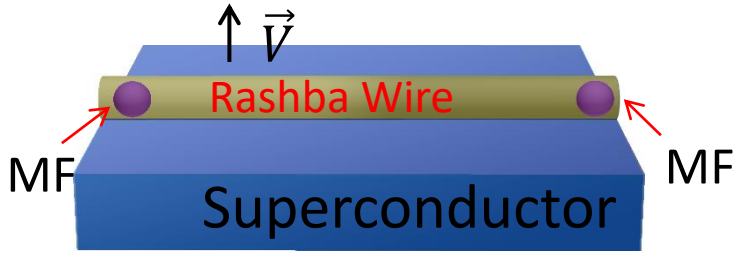
When $t = \Delta$ the MF are localized at one site

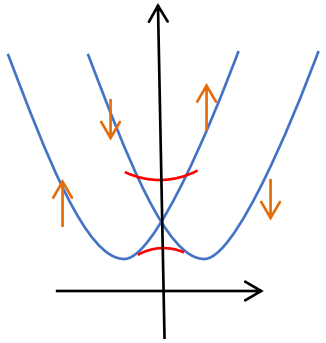
Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^\dagger$



Kitaev 2000

Realistic realization of Kitaev's model using Rashba wires



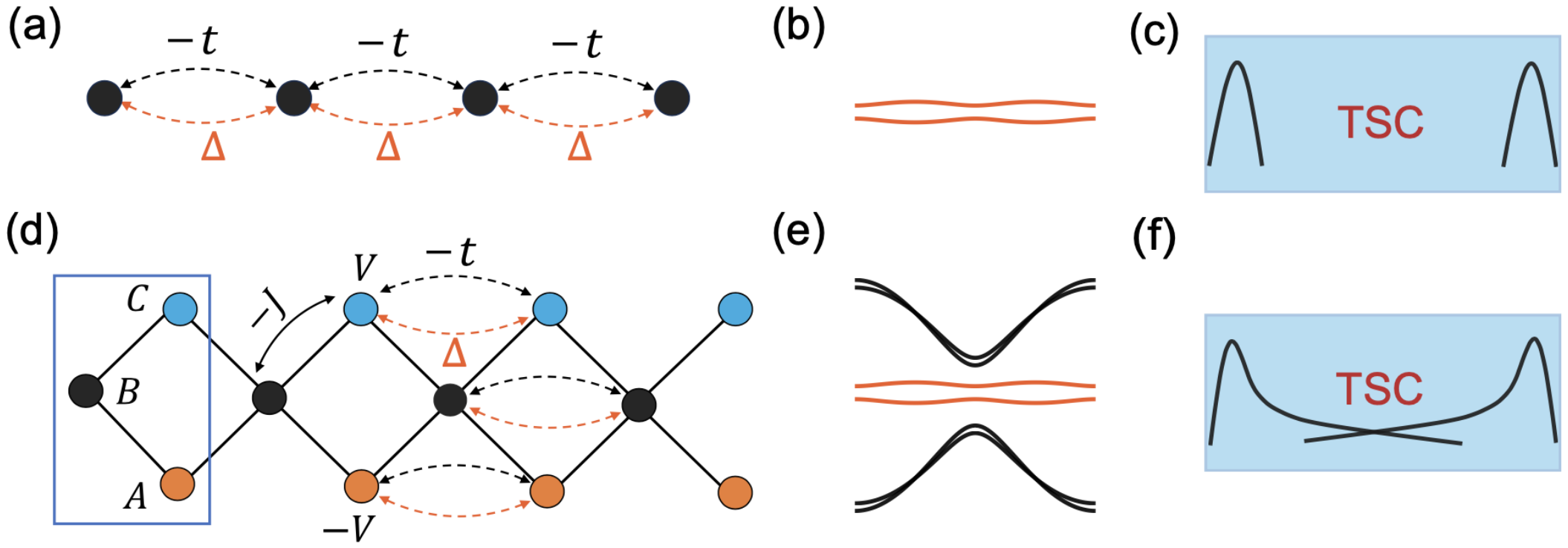


Rashba + Zeeman

$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k\sigma_y)]\tau_z - \Delta\sigma_y\tau_y$$

2. Majorana Zero Modes in Lieb-Kitaev Model

When $t = \Delta$ the MF are localized at one site



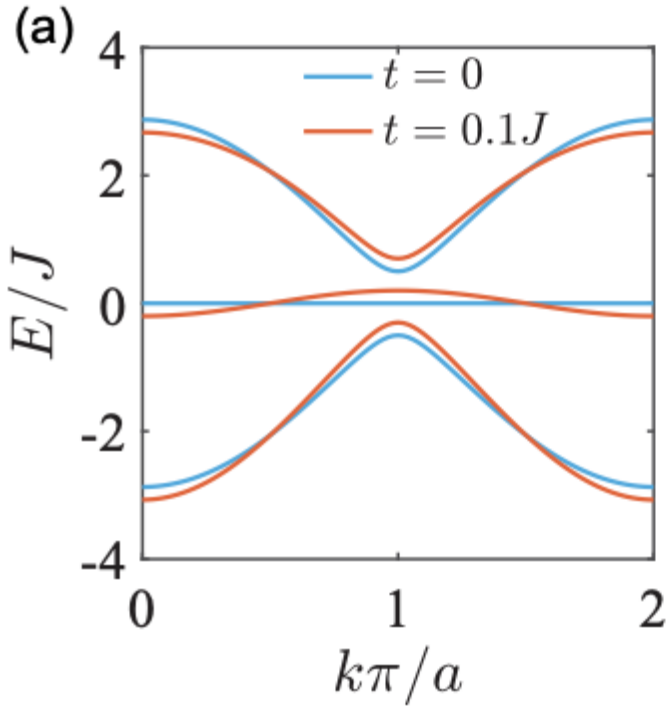
When $t = \Delta$ the MF can be highly delocalized in the Lieb-Kitaev model.

$$h(k) = (-2t\cos ka - \mu)\mathbb{I}_3 + \begin{pmatrix} -V & a_k & 0 \\ a_k^* & 0 & a_k^* \\ 0 & a_k & V \end{pmatrix}$$

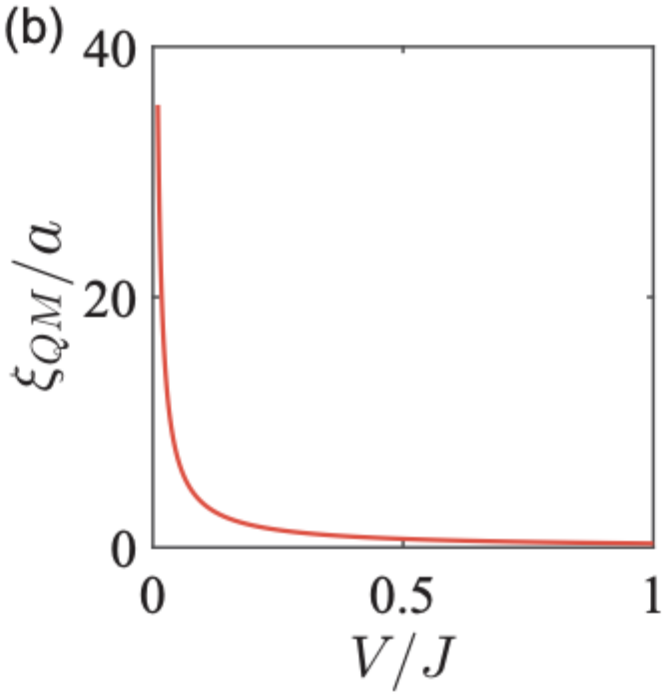
$$H_{BdG} = \sum_k \hat{\Psi}^\dagger(k) \begin{pmatrix} h(k) & -i2\Delta \sin ka\mathbb{I}_3 \\ i2\Delta \sin ka\mathbb{I}_3 & -h^*(-k) \end{pmatrix} \hat{\Psi}(k)$$

Topological superconductor with quantum metric

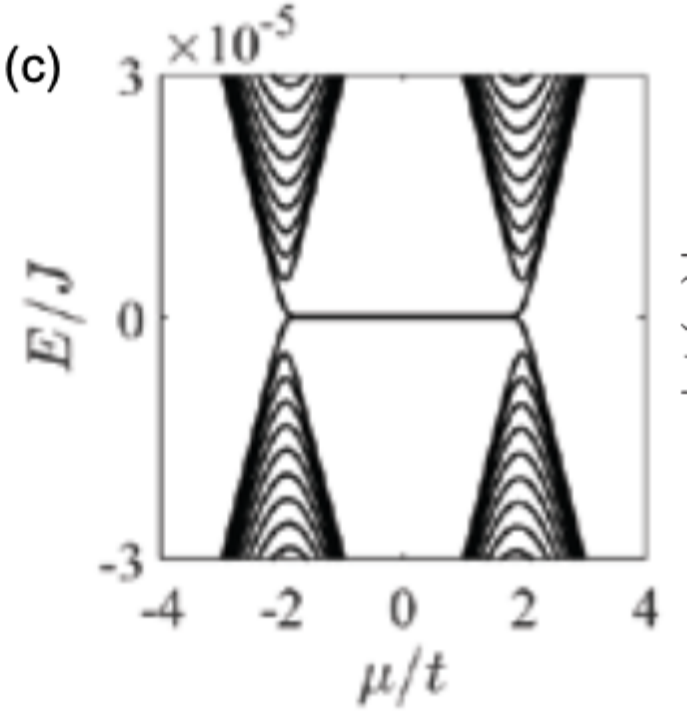
Energy dispersion of Lieb-Kitaev model



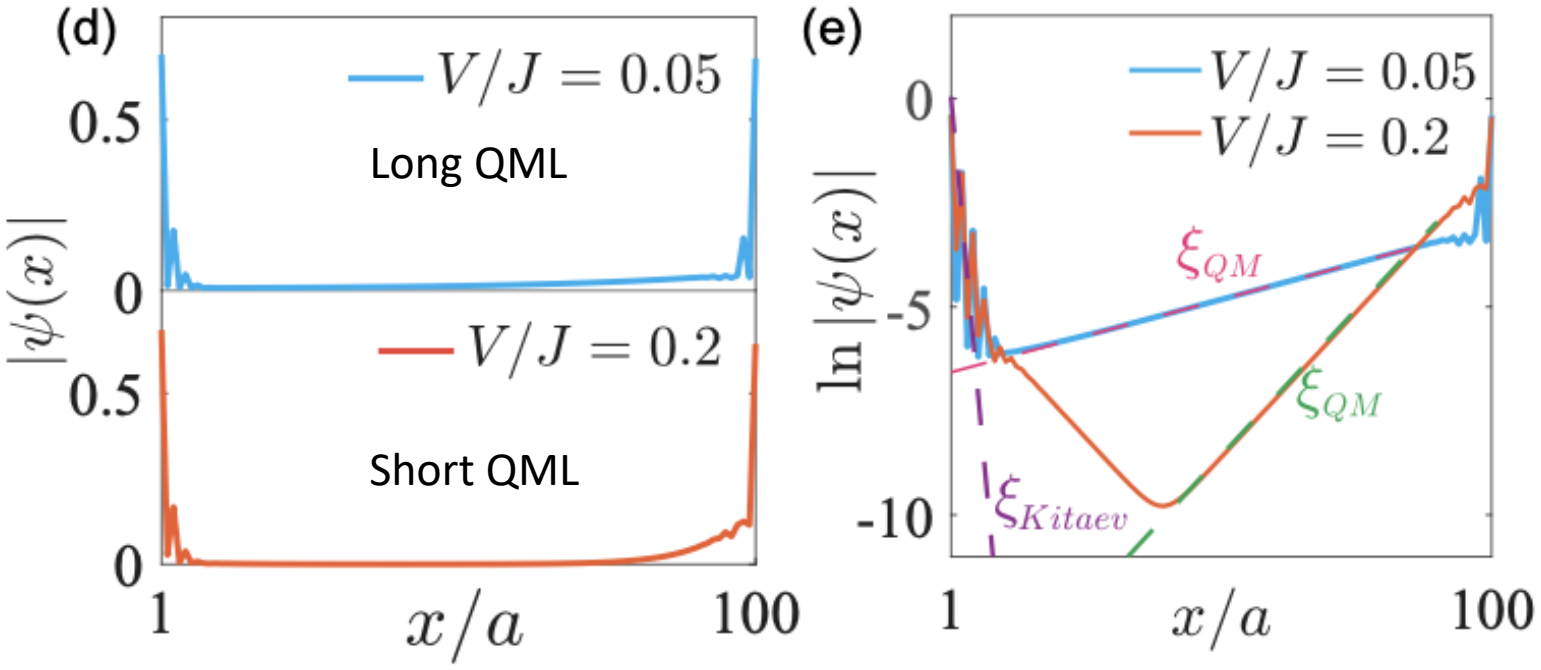
Quantum Metric length as a function of parameters



Energy spectrum



Majorana modes tuned by quantum metric

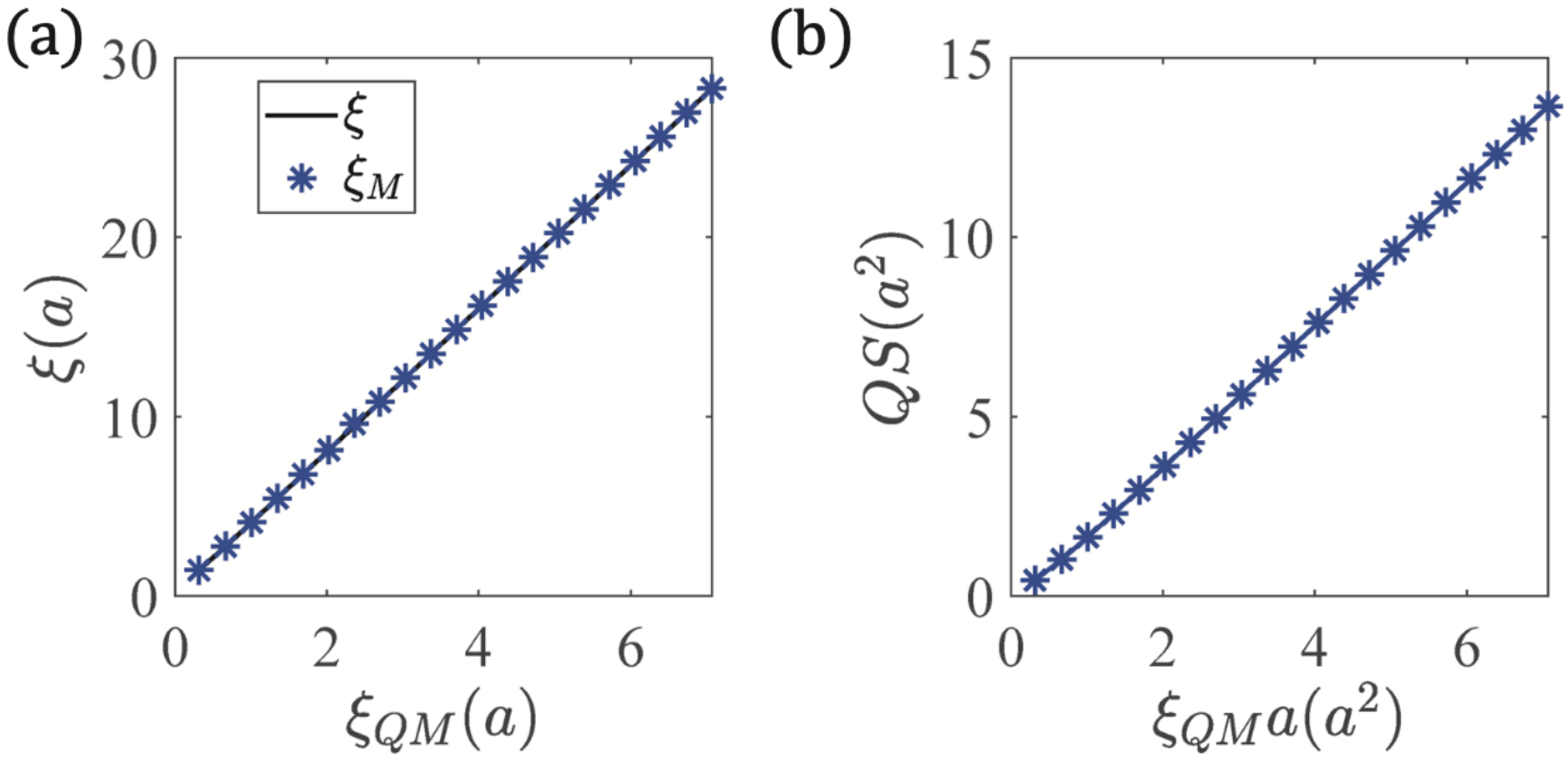


$$\boldsymbol{u}_0(k) = (a_k, V, -a_k)^T / \sqrt{4|J|^2(1+\cos ka) + V^2},$$

$$\mathcal{G}(k) = \langle \partial_k \boldsymbol{u}_0(k)| \left[\mathbb{I} - |\boldsymbol{u}_0(k)\rangle \langle \boldsymbol{u}_0(k)|\right] |\partial_k \boldsymbol{u}_0(k)\rangle$$

$$\xi_{QM} \equiv \int_{\mathrm{BZ}} \mathcal{G}(k) \frac{dk}{2\pi} \xrightarrow{\frac{V}{J}\to 0} \frac{\sqrt{2}}{4} \frac{J}{V} a.$$

“Size” of Majorana modes tuned by quantum metric

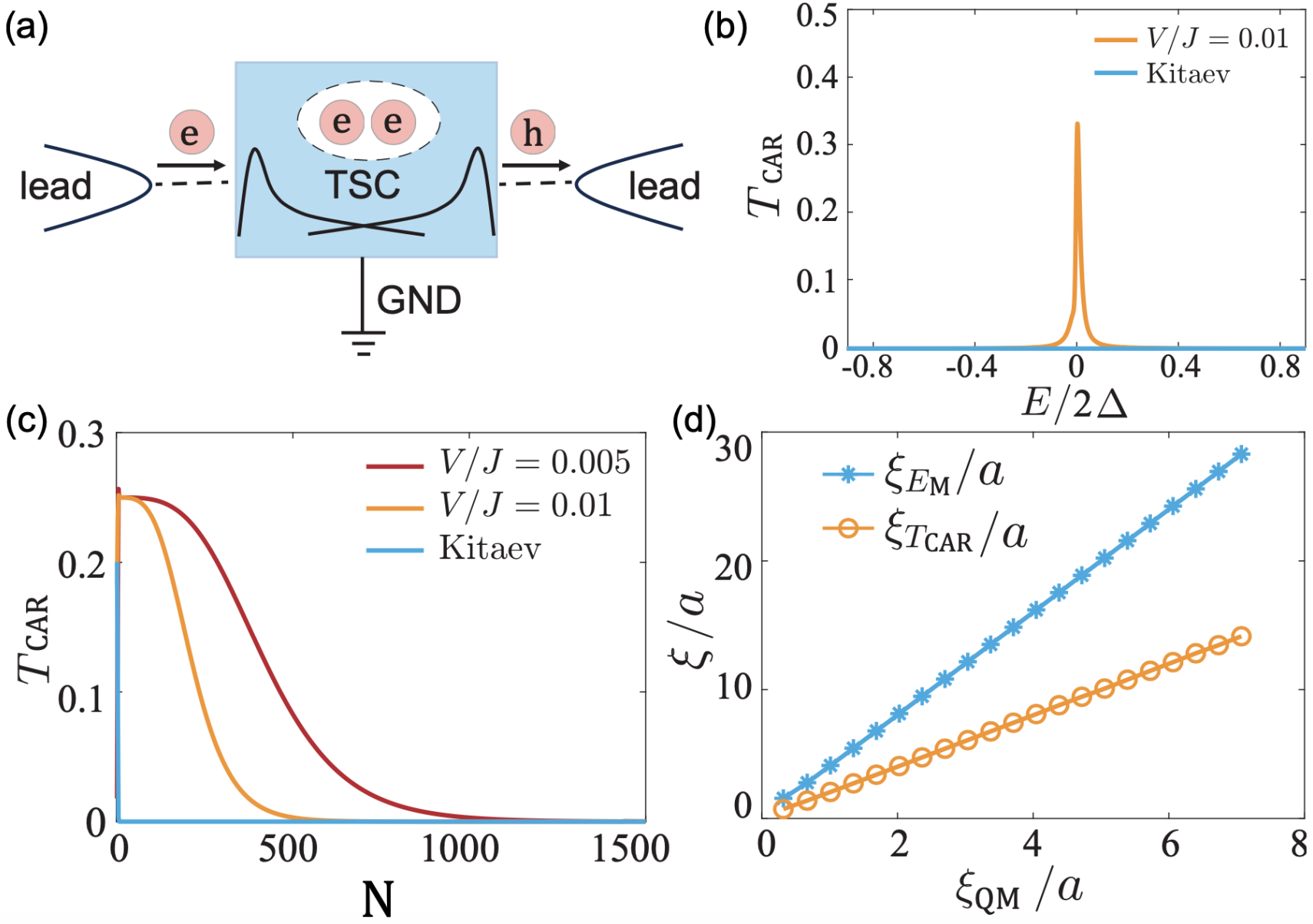


$$\boldsymbol{\psi}(x) = -M(x,a;E)U_{01}^{\dagger}\boldsymbol{\psi}(Na) - M(x,Na;E)U_{01}\boldsymbol{\psi}(a), \tag{6}$$

where

$$M(x_i,x_j;E) = \frac{a}{2\pi}\sum_{n=\pm}\int_{BZ}\frac{\boldsymbol{g}_n(k)\boldsymbol{g}_n^{\dagger}(k)}{E-\varepsilon_n(k)}e^{ik(x_i-x_j)}dk$$

Long range maximal crossed Andreev reflections



Thank you!

Josephson Diode Effect in Twisted bilayer graphene

nature communications

Article

https://doi.org/10.1038/s41467-023-38005-7

Symmetry-broken Josephson junctions and superconducting diodes in magic-angle twisted bilayer graphene

Received: 16 January 2023

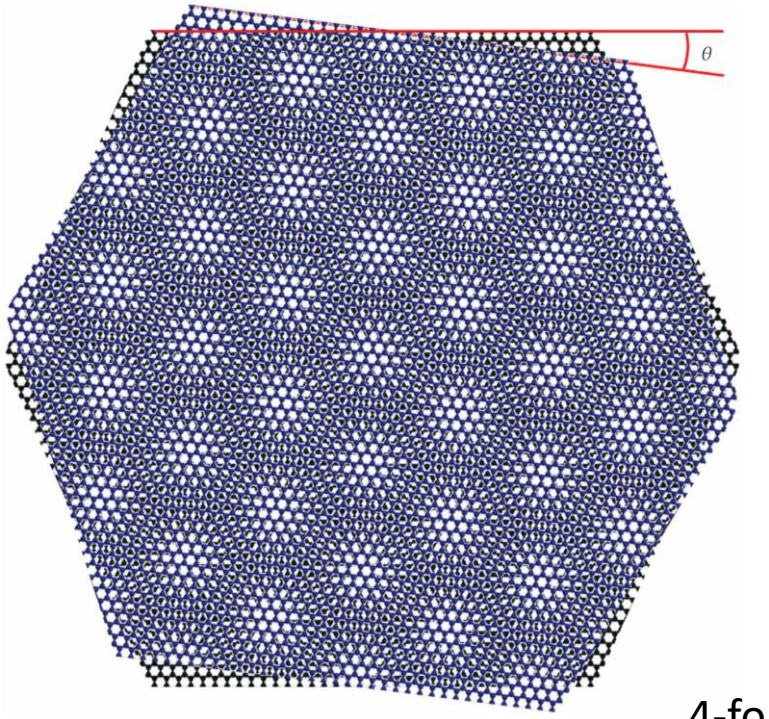
Accepted: 6 April 2023

Published online: 26 April 2023

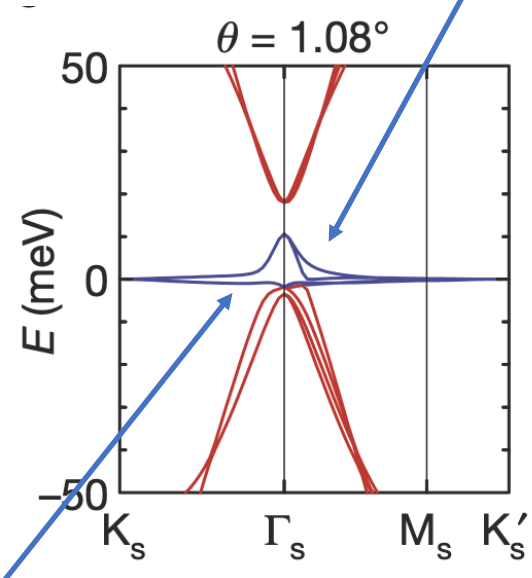
J. Díez-Mérida [1], A. Díez-Carlón [1], S. Y. Yang[1], Y.-M. Xie [2], X.-J. Gao[2], J. Senior [3], K. Watanabe [4], T. Taniguchi [4], X. Lu [1], A. P. Higginbotham [3], K. T. Law [2] & Dmitri K. Efetov [1]



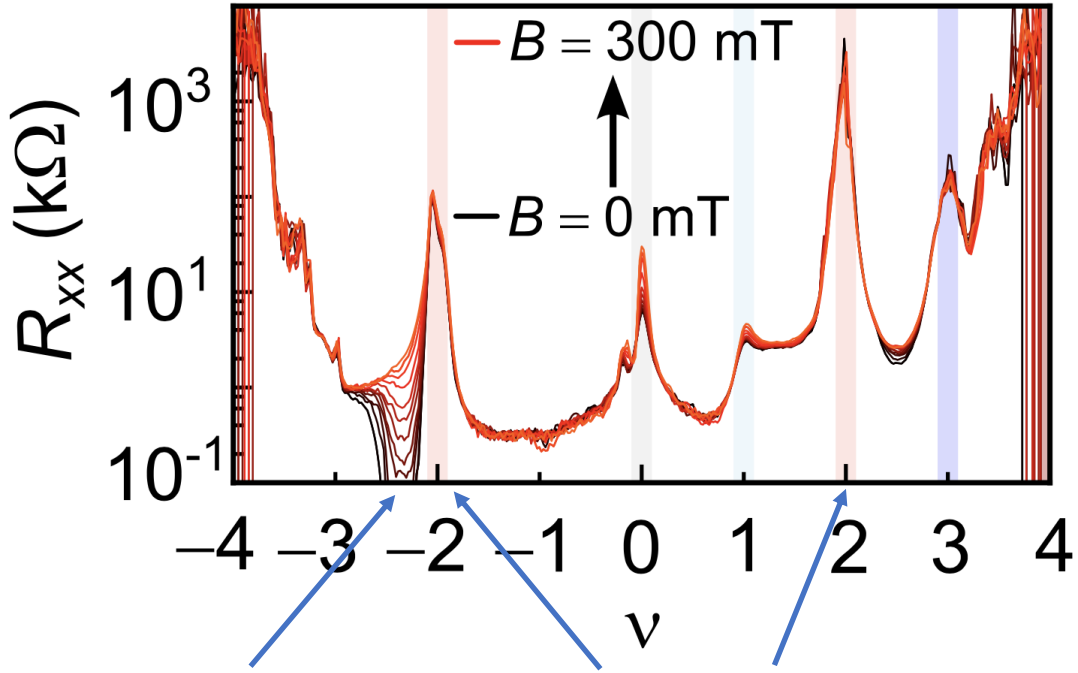
Correlated Insulator and Superconductivity in Twisted bilayer graphene



4-fold degenerate electron bands



4-fold degenerate hole bands



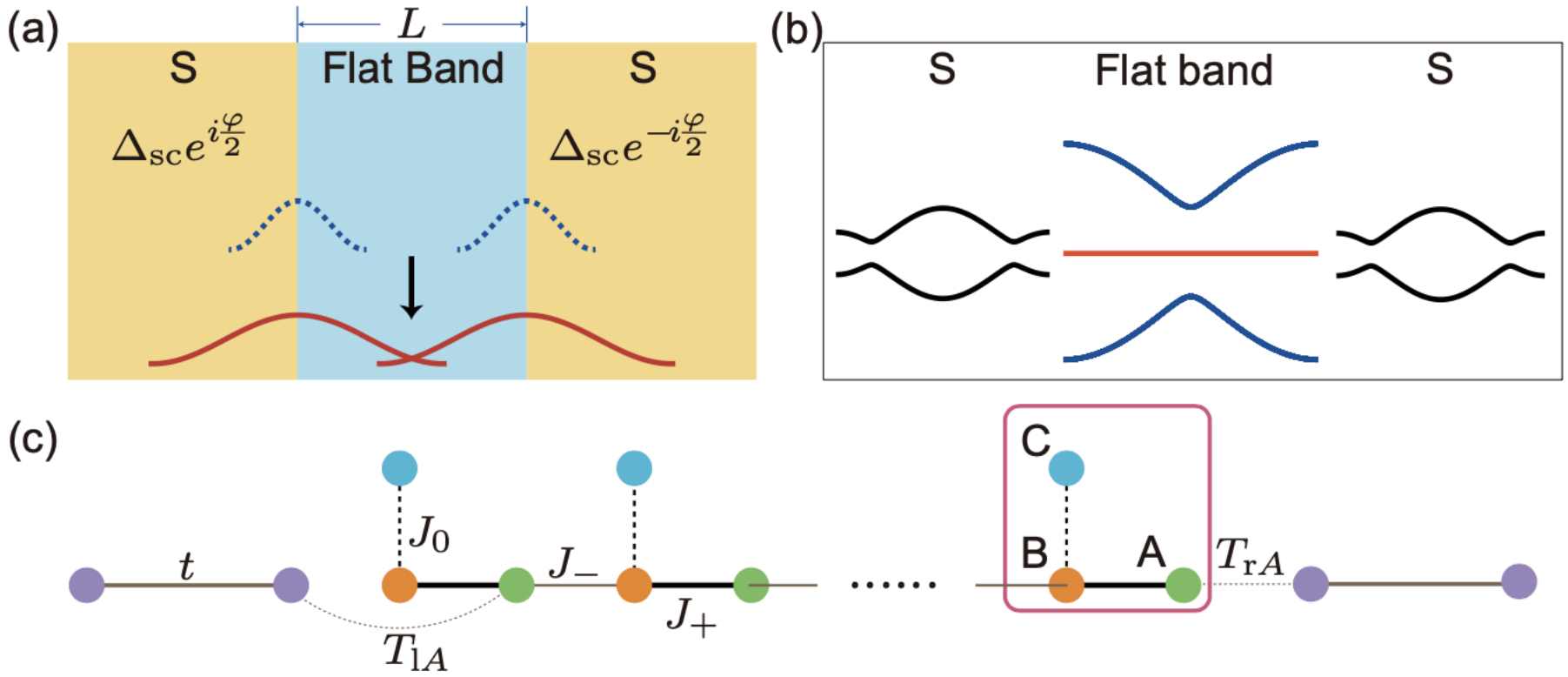
Superconducting state

Correlated insulating state

Cao, Yuan, et al.
Nature 556.7699 (2018). Nature 556.7699 (2018).
Bistritzer and MacDonald PNAS, 2011

Nature Communications 14 : 2396 (2023)
KTL with Dima Efetov's group

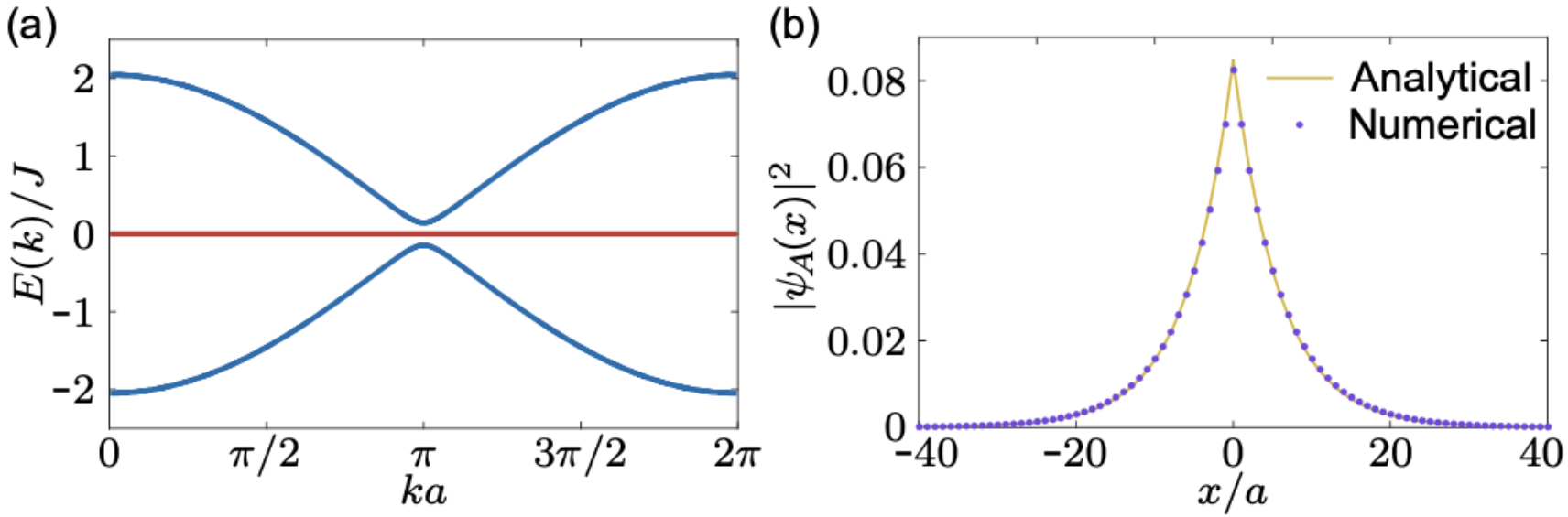
3. Flat band Josephson junction

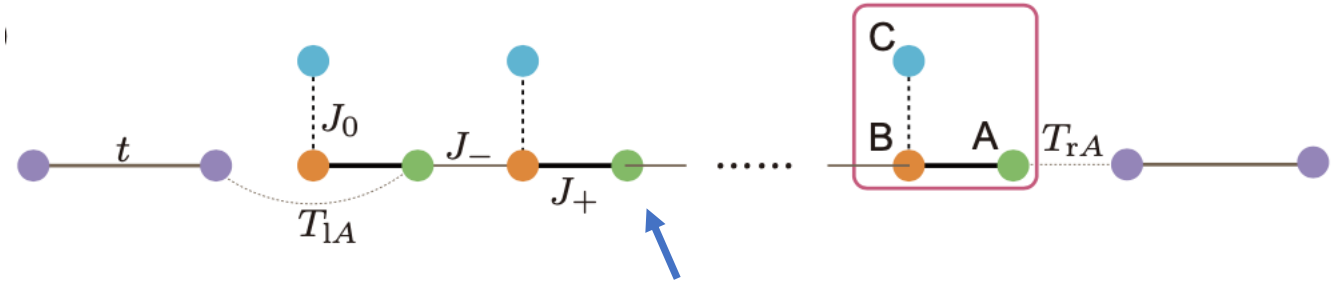


$$H_{\text{Lieb}} = \sum_{i\sigma} (J_+ a^\dagger_{i\text{A}\sigma} a_{i\text{B}\sigma} + J_0 a^\dagger_{i\text{C}\sigma} a_{i\text{B}\sigma} + J_- a^\dagger_{i-1\text{A}\sigma} a_{i\text{B}\sigma} + h.c.) - \sum_{\alpha i\sigma} \mu_N a^\dagger_{i\alpha\sigma} a_{i\alpha\sigma}.$$

$$J_\pm = J(1 \pm \delta),\ J_0 = \delta J$$

Bound state size in flat bands





Create a local potential

Bound state size in flat bands

$$h(k) = 2J \begin{pmatrix} 0 & a_k & 0 \\ a_k^* & 0 & b_k \\ 0 & b_k^* & 0 \end{pmatrix}, |u_0(k)\rangle = \begin{pmatrix} \frac{b_k}{\sqrt{|a_k|^2+|b_k|^2}} \\ 0 \\ -\frac{a_k^*}{\sqrt{|a_k|^2+|b_k|^2}} \end{pmatrix}$$

Quantum metric

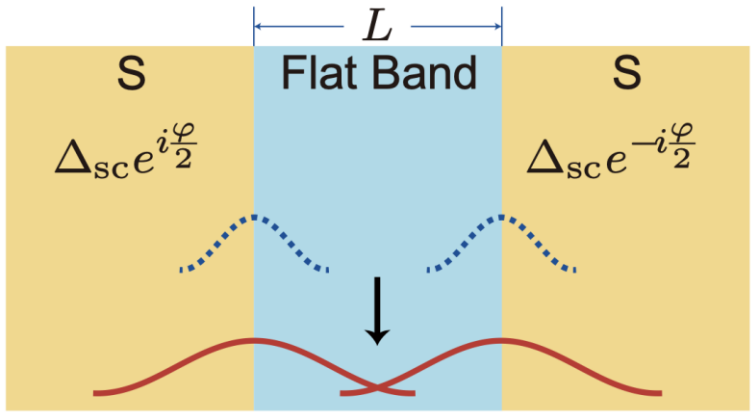
$$g^0(k) = \mathrm{Re}\langle \partial_k u_0(k)|(1 - |u_0(k)\rangle\langle u_0(k)|)|\partial_k u_0(k)\rangle.$$

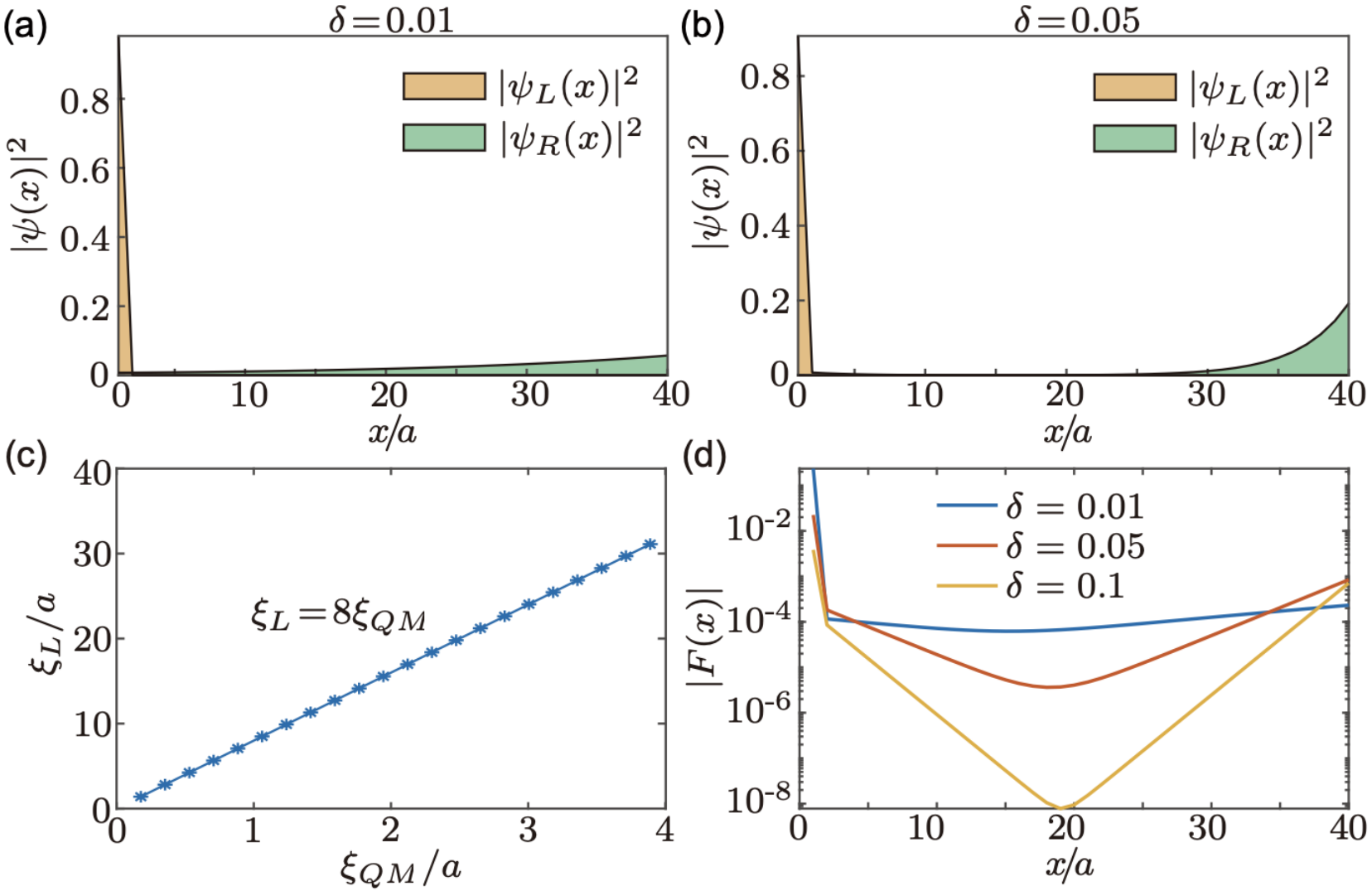
Quantum metric length

$$\xi_{QM} = \int_{-\pi/a}^{\pi/a} g^0(k) \frac{dk}{2\pi}.$$

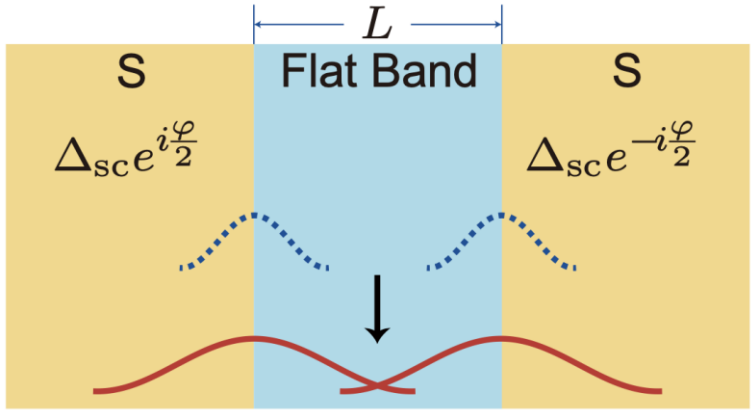
$$\psi_A(x) \approx \sum_k e^{ikx} |u_A(k)|^2 \propto \exp\left(-|x|/8\xi_{QM}\right)$$

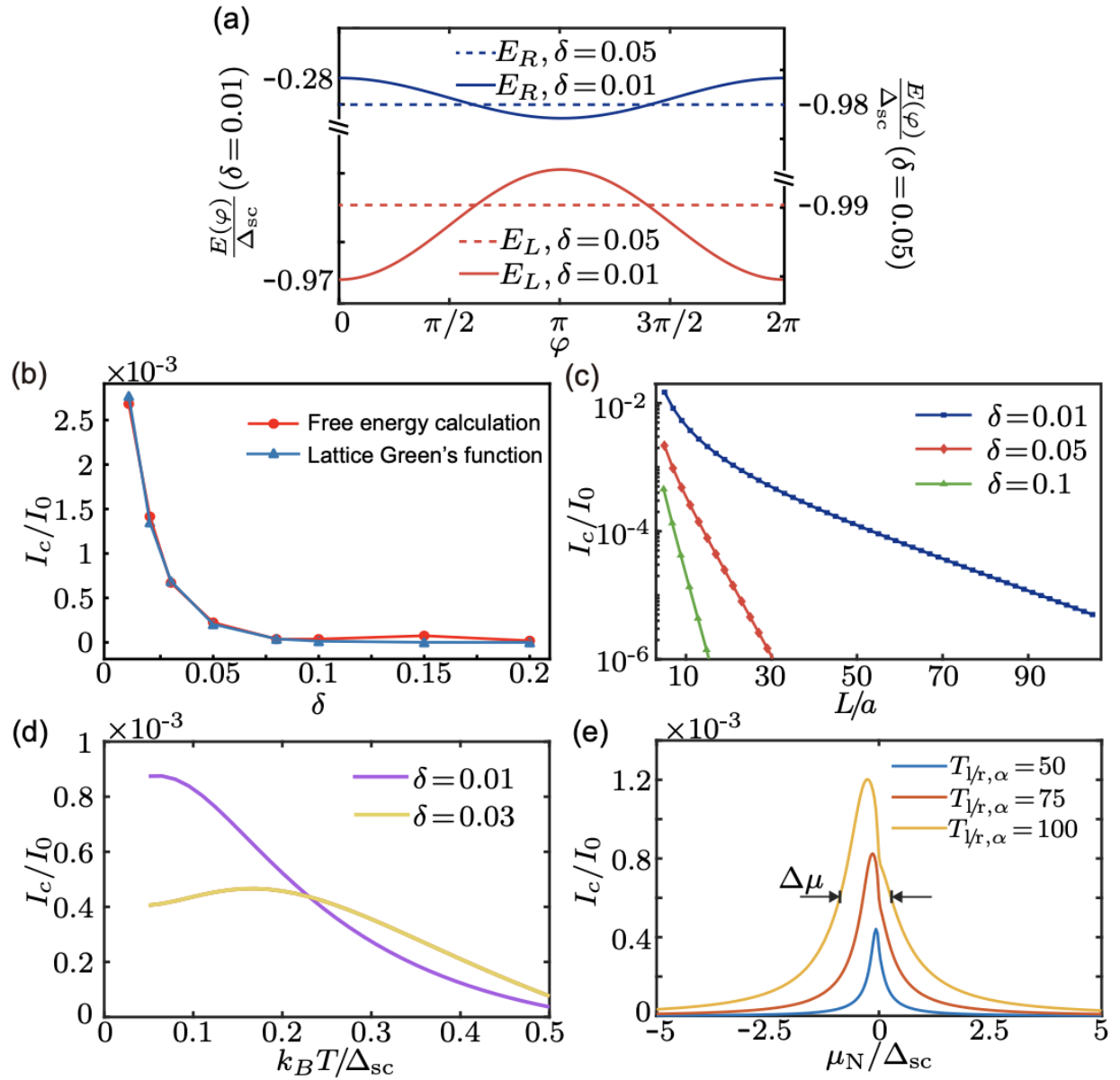
S/FB Interface states





Josephson current controlled by quantum metric





Quantum metric in the presence of band dispersion

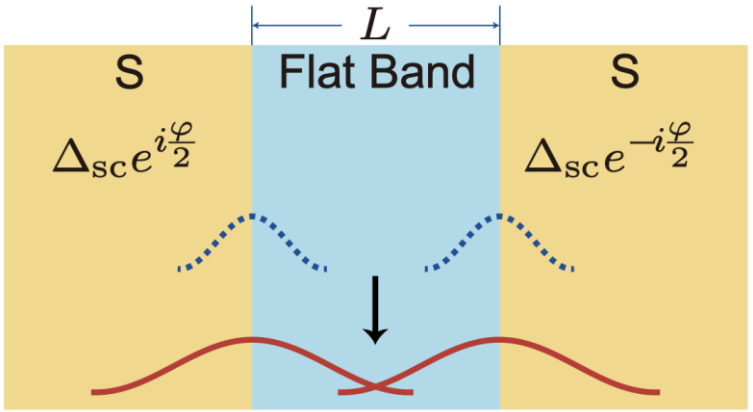
Critical current as a function of length L of weak link

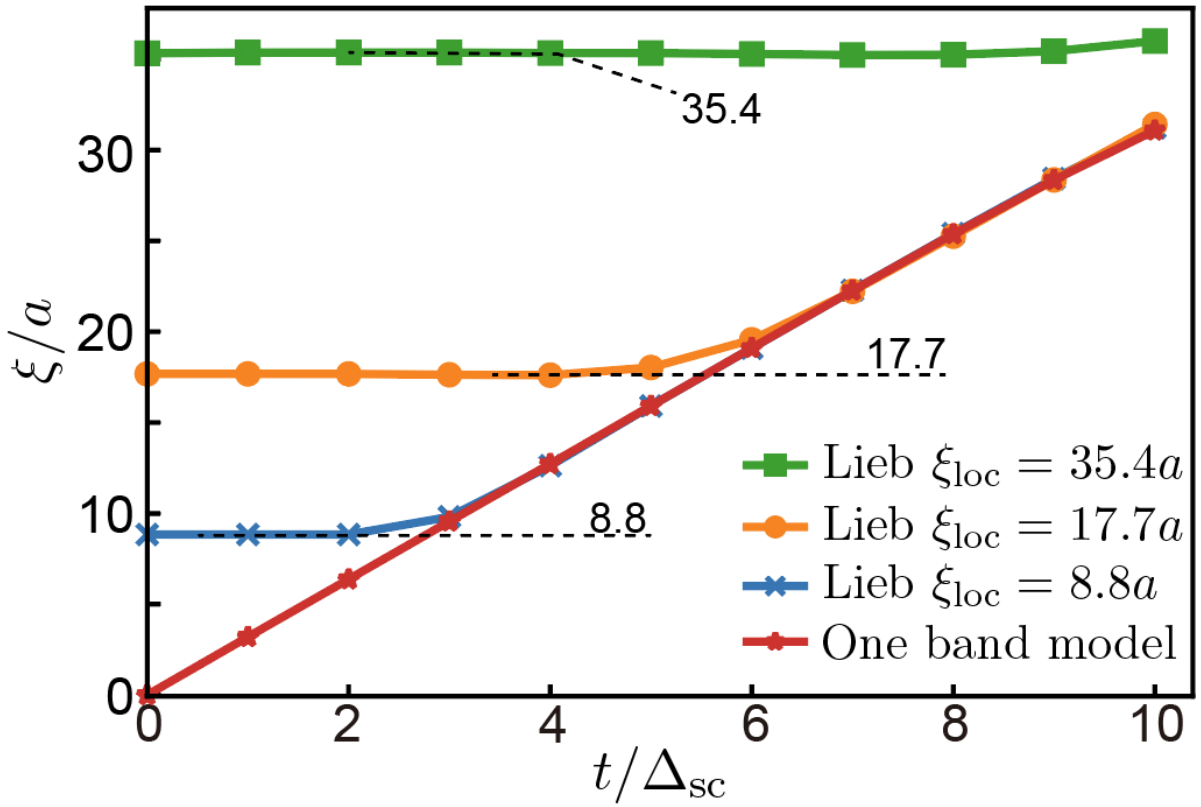
$$I_c \propto e^{-L/\xi}$$

For conventional metallic weak links:

$$\xi \approx V_F/2\pi T$$

For nearly flat bands, ξ is dominated by quantum metric length.





4. Majorana Zero Modes in Kitaev Model

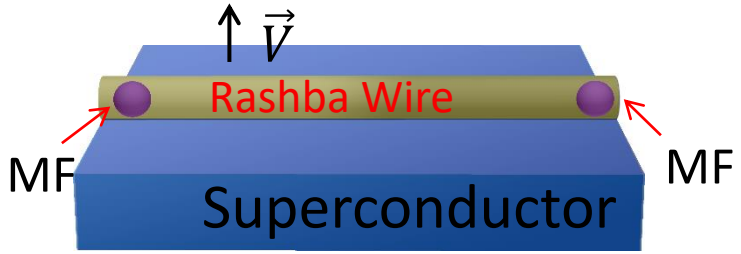
1D Kitaev model with spinless Fermions

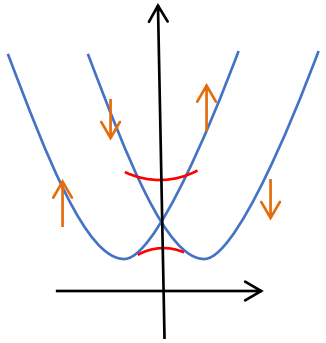
Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^\dagger$



Kitaev 2000

Realistic realization of Kitaev's model using Rashba wires

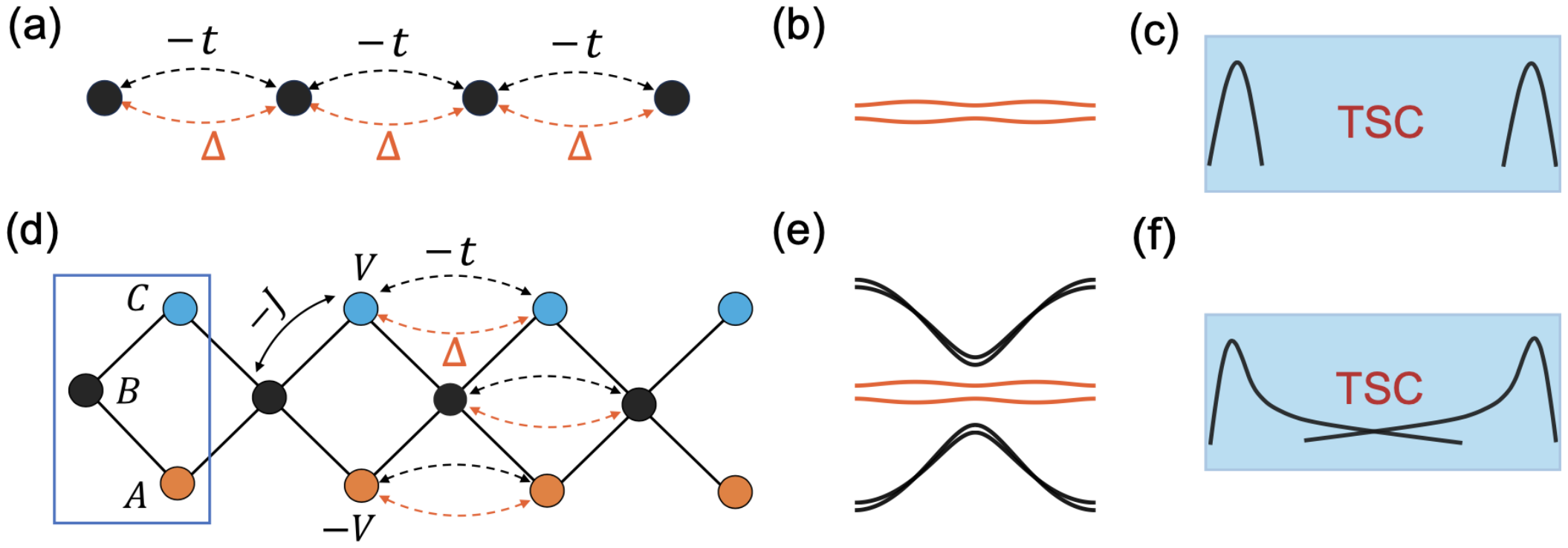




Rashba + Zeeman

$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k\sigma_y)]\tau_z - \Delta\sigma_y\tau_y$$

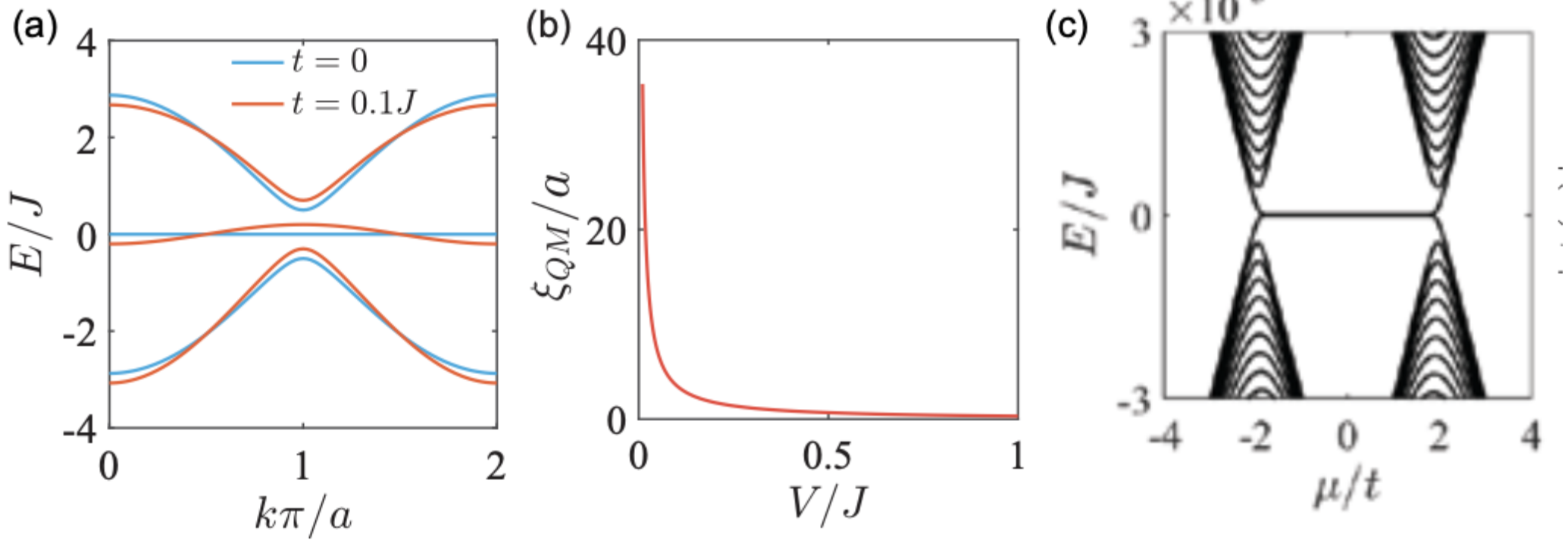
4. Majorana Zero Modes in Lieb-Kitaev Model



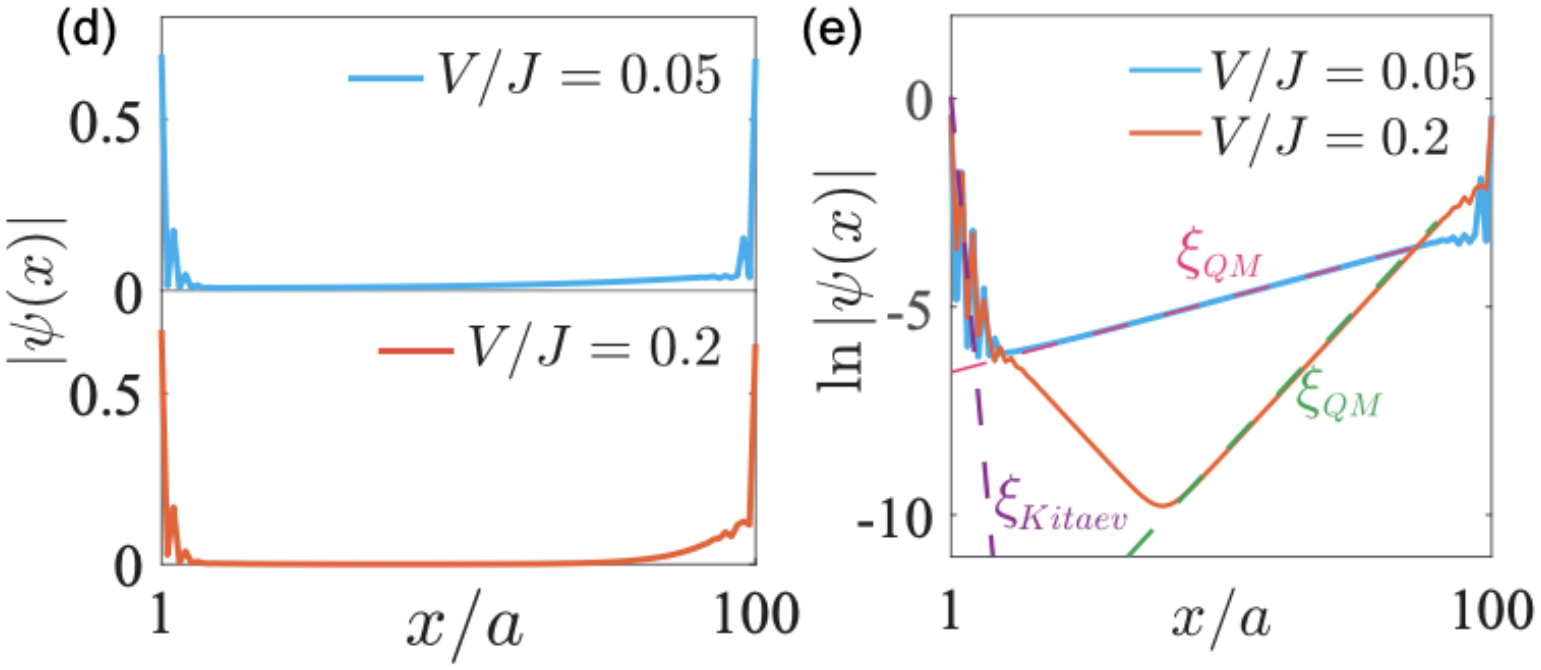
$$h(k) = (-2t\cos ka - \mu)\mathbb{I}_3 + \begin{pmatrix} -V & a_k & 0 \\ a_k^* & 0 & a_k^* \\ 0 & a_k & V \end{pmatrix}$$

$$H_{BdG} = \sum_k \hat{\Psi}^\dagger(k) \begin{pmatrix} h(k) & -i2\Delta \sin ka\mathbb{I}_3 \\ i2\Delta \sin ka\mathbb{I}_3 & -h^*(-k) \end{pmatrix} \hat{\Psi}(k)$$

Topological superconductor with quantum metric



Majorana modes tuned by quantum metric

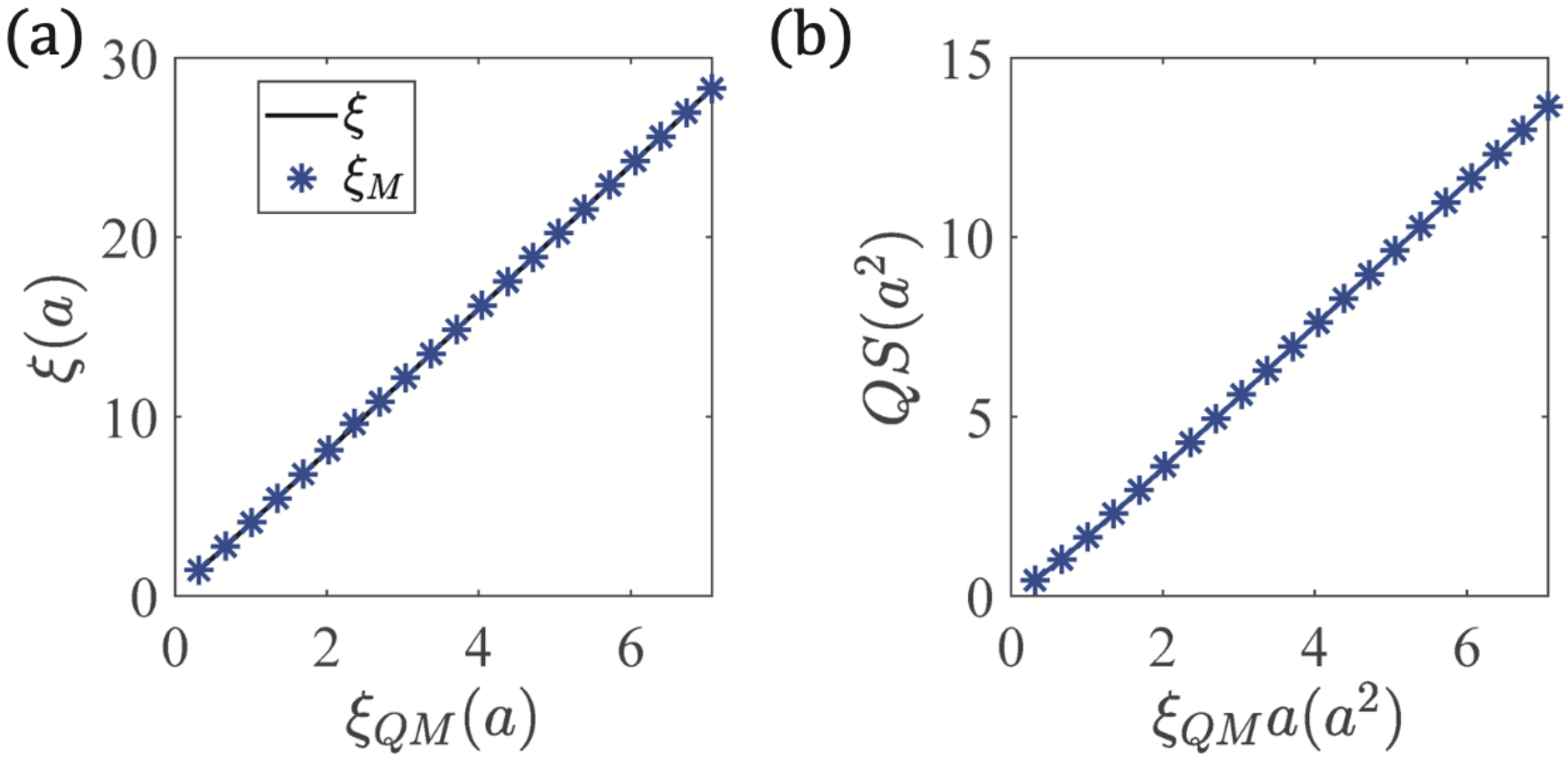


$$\boldsymbol{u}_0(k) = (a_k, V, -a_k)^T / \sqrt{4|J|^2(1+\cos ka) + V^2},$$

$$\mathcal{G}(k) = \langle \partial_k \boldsymbol{u}_0(k)| \left[\mathbb{I} - |\boldsymbol{u}_0(k)\rangle \langle \boldsymbol{u}_0(k)|\right] |\partial_k \boldsymbol{u}_0(k)\rangle$$

$$\xi_{QM} \equiv \int_{\mathrm{BZ}} \mathcal{G}(k) \frac{dk}{2\pi} \xrightarrow{\frac{V}{J}\to 0} \frac{\sqrt{2}}{4} \frac{J}{V} a.$$

“Size” of Majorana modes tuned by quantum metric

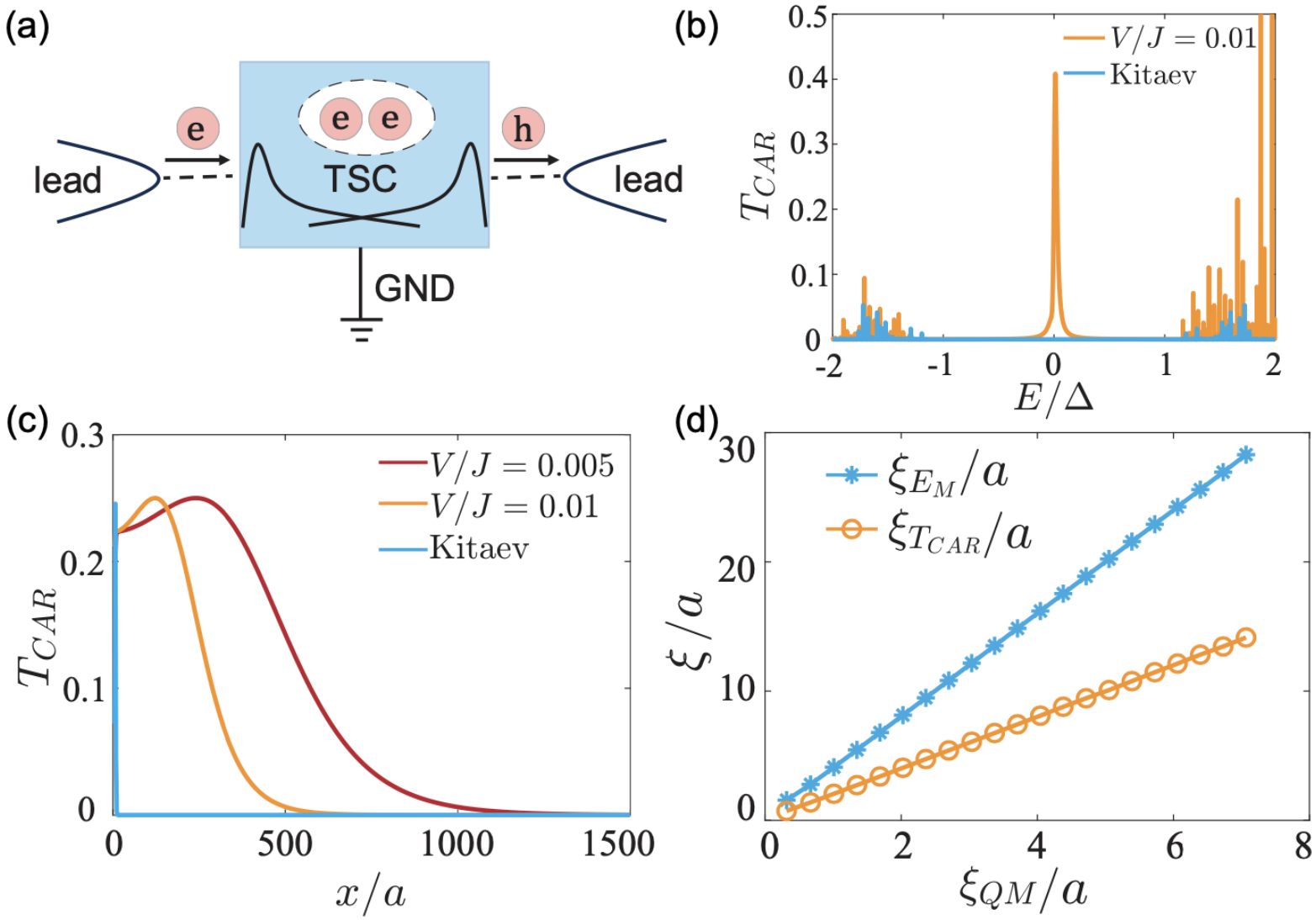


$$\boldsymbol{\psi}(x) = -M(x,a;E)U_{01}^{\dagger}\boldsymbol{\psi}(Na) - M(x,Na;E)U_{01}\boldsymbol{\psi}(a), \tag{6}$$

where

$$M(x_i,x_j;E) = \frac{a}{2\pi}\sum_{n=\pm}\int_{BZ}\frac{\boldsymbol{g}_n(k)\boldsymbol{g}_n^{\dagger}(k)}{E-\varepsilon_n(k)}e^{ik(x_i-x_j)}dk$$

Long range maximal crossed Andreev reflections



Thank you!