Generalized Lieb-Schultz-Mattis theorems from the SPT perspective

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Acknowledgements

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- Reference:

  CMJ, Z. Bi, and C. Xu, PRB 97, 054412 (2018)

  CMJ, A. Thomson, A. Rasmussen, Z. Bi, and C. Xu, ArXiv:1710.0668

  CMJ, A. Thomson, A. Rasmussen, and C. Xu, unpublished

Introduction

- Lieb-Schultz-Mattis (LSM) Theorem

A spin system with half-integer spin per unit cell with **translation** and **SO(3) spin rotation symmetry**

Possible the zero temperature phases are **gapless** or **symmetry breaking** or **topologically ordered** (or **fracton**)

[Lieb, Schultz and Mattis 1961; Oshikawa 2000; Hastings 2004]

Introduction

- Lieb-Schultz-Mattis (LSM) Theorem

A spin system with half-integer spin per unit cell with **translation** and **SO(3) spin rotation symmetry**

No-go theorem to a featureless ground state

A featureless ground state: a gapped, short-range entangled, fully symmetric state.

[Lieb, Schultz and Mattis 1961; Oshikawa 2000; Hastings 2004]

- Generalizing the LSM Theorem

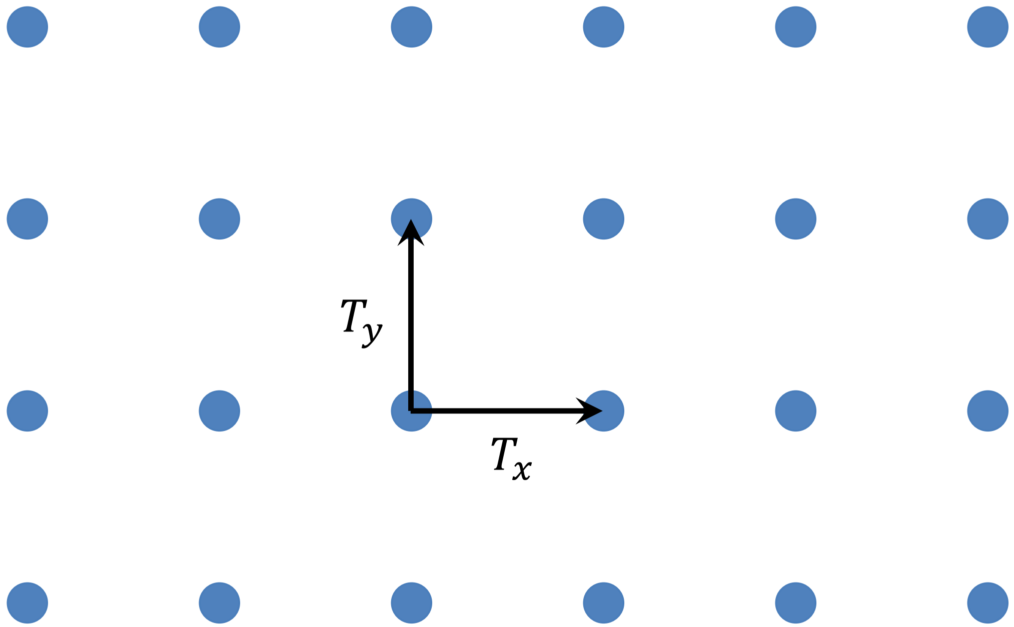
Space Group + **Internal symmetry** → **"no-go" to featureless ground state**

[Affleck 1988, Yamanaka et al 1997, Parameswaran et al 2013,…]

[Po, Watanabe, **CMJ** and Zaletel 2017]

Introduction

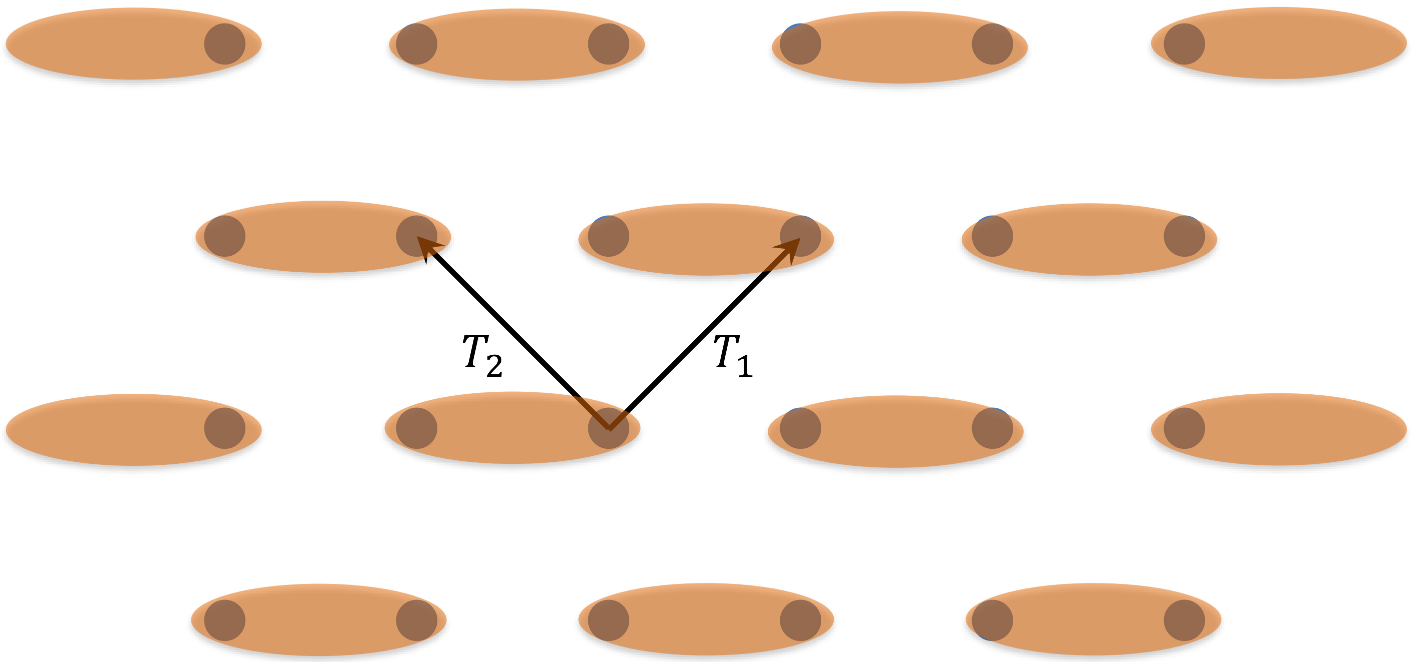
- A quick demonstration: a lattice of spin-1/2



Original LSM theorem applies

Introduction

- A quick demonstration: a lattice of spin-1/2 with doubled unit cell



Original LSM theorem does NOT applies

With site-centered C_4 rotation, no featureless ground state is allowed

Introduction

- Symmetry protected topological (SPT) states

(Gapped) short-range entangled states with **(internal) symmetries**.

Featureless states forbidden on the boundary of non-trivial SPT states.

- Similarity between LSM and SPT

LSM: **Space group** + **Internal symmetry** → **"no-go" to featureless ground state**

SPT: **Internal symmetry** → **"no-go" to featureless boundary state**

- Goal： To obtain generalized LSM using SPT knowledge

Strategy and Why

- Connection between LSM and SPT

  No-go to featureless states on the boundary of SPT is due to anomalies of global symmetry

  A generalized LSM should also be caused by anomalies of global symmetry in the continuum description of a spin system (e.g. near a critical point)

  Space group action → internal symmetry in the continuum description of spin systems

- Identifying generalized LSMs:

  In some cases, anomalies can be calculated directly

  e.g. [Metlitski,Thorngren 2017; Meng et al 2015; Cho et al 2017; CMJ et al unpublished]

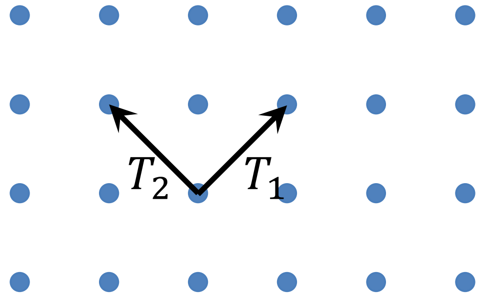
  Our stragy: Match the continuum description of a D-dimensional spin system with the boundary of a (D+1)-dimensional SPT

Example 1: spin-1/2 square lattice

- 2D square lattice with spin-1/2 per site and doubled unit cell

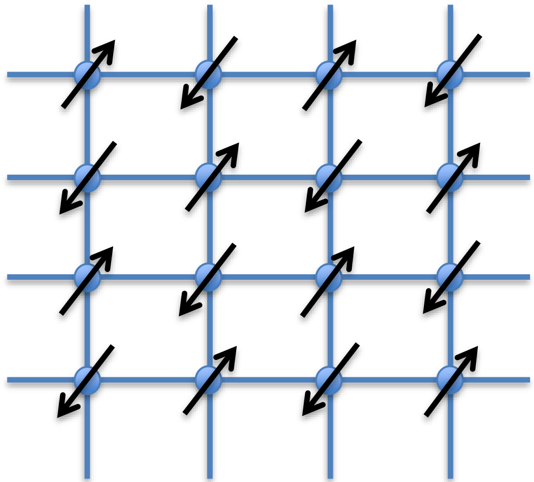
Space group: **site-centered C_4 rotation and $T_{1,2}$**

Internal symmetry: **spin SO(3) rotation**

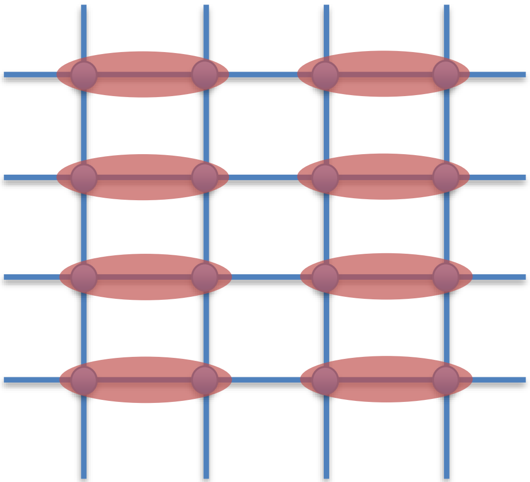


Consider the Neel-VBS critical point:

Neel:



VBS:



Example 1: spin-1/2 square lattice

- Neel-VBS critical point: Deconfined quantum critical point (dQCP)

Neel order parameter $\vec{n} = (n_1, n_2, n_3)$ transforms under SO(3)

VBS order parameter $\vec{v} = (v_1, v_2)$ transforms under C_4

[Senthil et al 2004]

Space group (Site-centered C_4) + **Internal symmetry (SO(3) spin)** ≈ **"Enhanced internal symmetry" ($Z_4 \times$ SO(3))**

2+1D O(5) non-linear sigma model with a level-1 WZW term with the O(5) vector $(n_1, n_2, n_3, v_1, v_2)$

[Wang et al 2017]

Example 1: spin-1/2 square lattice

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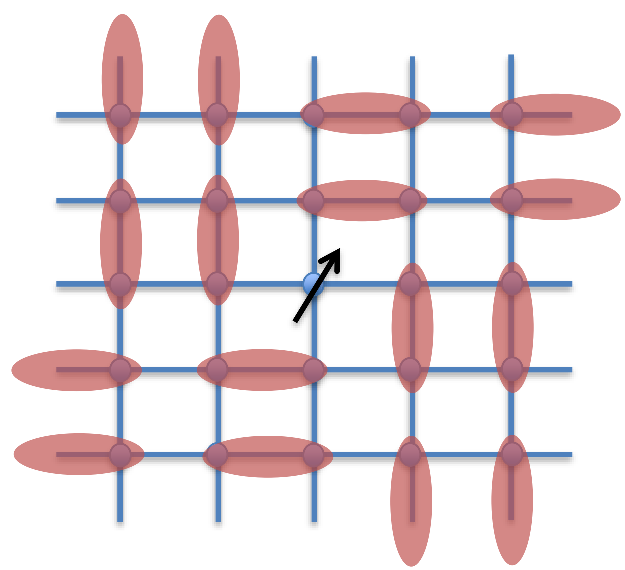
[Senthil et al 2004]

Space group (Site-centered C_4) + **Internal symmetry (SO(3) spin)** ≈ **"Enhanced internal symmetry" ($Z_4 \times$ SO(3))**

Essential physical consequence of the WZW term: C_4 vortex carries a spin-1/2

C_4 can be embedded in emergent U(1)



Example 1: spin-1/2 square lattice

- Identification as the boundary of 3+1D $Z_4 \times SO(3)$ SPT:

2+1D O(5) WZW Term on the boundary → 3+1D SPT with O(5) Θ-Term in the bulk

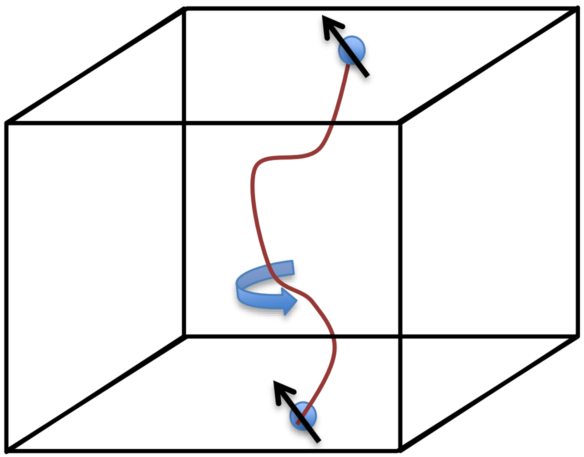
Physical Picture:

Start with $U(1) \times SO(3)$ symmetry

$U(1) \times SO(3)$ SPT as a liquid of fluctuating U(1) vortex lines

U(1) vortex line decorated by 1+1D SO(3) Haldane phase

End point of vortex line carries a spin-1/2



Example 1: spin-1/2 square lattice

- Identification as the boundary of 3+1D $Z_4 \times SO(3)$ SPT:

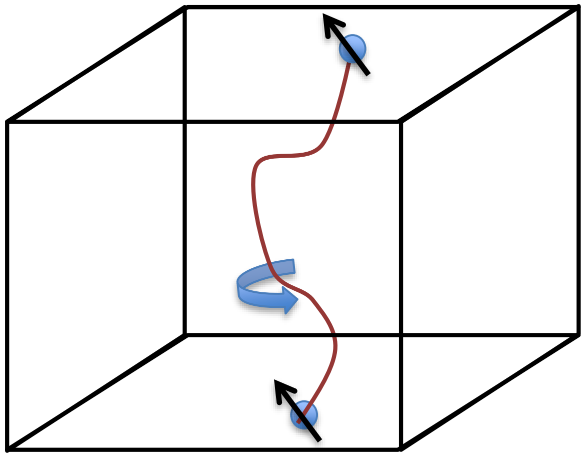
Reduce to a 3+1D $Z_4 \times SO(3)$ SPT :

U(1) vortex line → Z_4 vortex line; Vortictiy: $Z \to Z_4$

Decoration of vortex lines by SO(3) Haldane phase is stable

“no-go” to a featureless boundary state for 3+1D $Z_4 \times SO(3)$ SPT

Generalized LSM for the square lattice with spin-1/2 per site, site-centered C_4 rotation, doubled-unit-cell translation and SO(3) spin rotation symmetry



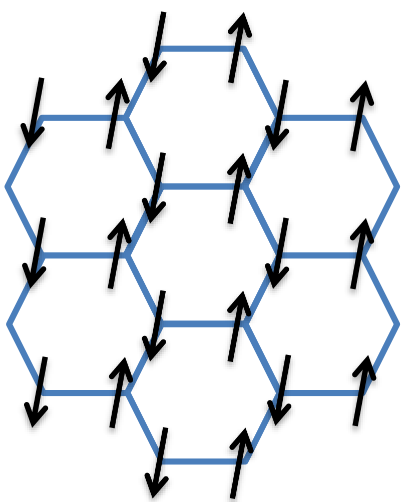
Example 2: spin-1/2 honeycomb lattice

- 2D honeycomb lattice with spin-1/2 per site

Consider **site-centered C_3 rotation + SO(3) spin rotation**

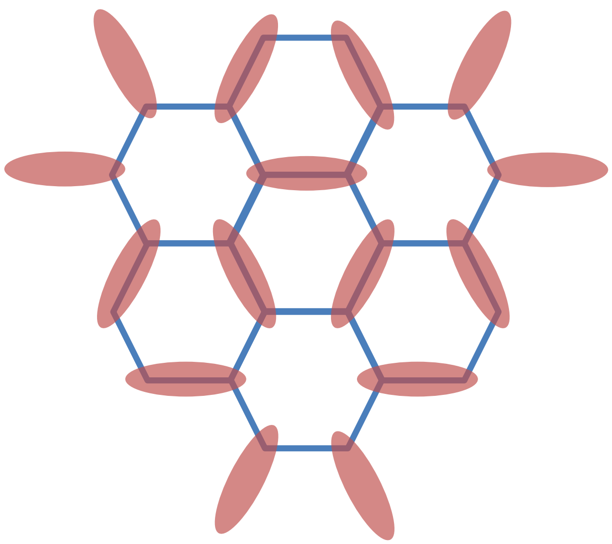
Consider the Neel-VBS critical point (dQCP):

Neel:



$\vec{n} = (n_1, n_2, n_3)$

VBS:



$\vec{v} = (v_1, v_2)$

Example 2: spin-1/2 honeycomb lattice

- Critical theory: 2+1D O(5) NLSM + level-1 WZW term

Space Group (site-centered C_3)	+	Internal symmetry (SO(3))	≈	"Enhanced Internal Symmetry" ($Z_3 \times SO(3)$)

C_3 embedded in U(1)

Identified as the boundary theory of 3+1D $U(1) \times SO(3)$ SPT with U(1) vortex line decorated by 1+1D SO(3) Haldane phase

SPT **trivialized** by reduced symmetry $U(1) \rightarrow Z_3$. **No LSM !**

Consistent with lattice homotopy [H. Po. et al 2017], PEPS construction [P. Kim et al 2015] and [Metlitski and Thorngren 2017]

Comment

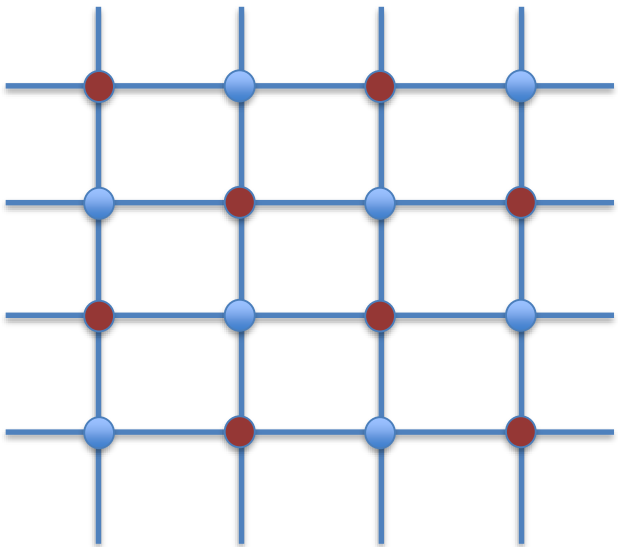
Such generalized LSMs obtained using SPT knowledge need to start with a continuum description of the spin model

Due to anomaly matching between UV and IR, we believe it captures more than just the vicinity of the continuum description (or the critical point)

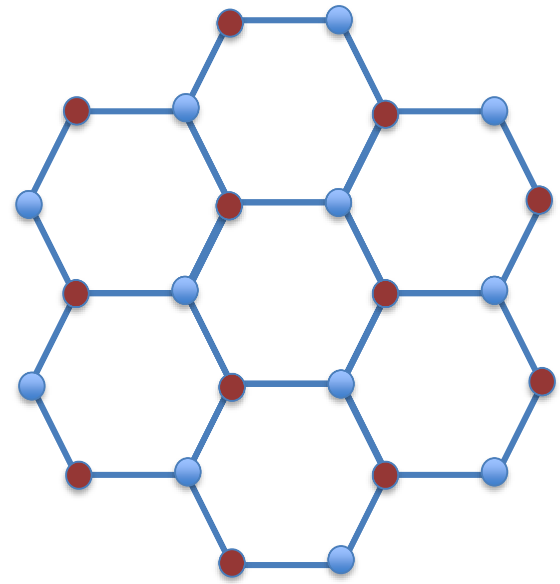
Argument can be even made at a “fine-tuned” critical point

2D lattices with SU(N) spins

- 2D square/honeycomb fund.-anti.-fund. lattice with **PSU(N) internal symmetry**



PSU(N) x site-centered C_4



PSU(N) x site-centered C_3

fundamental rep.of SU(N)

anti-fundamental rep.of SU(N)

2D lattices with SU(N) spins

- Neel-VBS critical point: nonlinear σ-model with WZW terms

PSU(N) Neel order parameter $P \in \frac{U(N)}{U(1)\times U(N-1)}$

VBS order parameter $\vec{v} = (v_1, v_2)$

Vortex of VBS carries a fundamental rep. of SU(N)

WZW term: write $V = \begin{pmatrix} \cos\theta\, P & \sin\theta\ (v_1+iv_2) \\ \sin\theta\ (v_1-iv_2) & -\cos\theta\, P \end{pmatrix} \in \frac{U(2N)}{U(N)\times U(N)}$

$$\frac{2\pi}{256\pi^2}\int du\, dx^2 d\tau\, \varepsilon_{\mu\nu\lambda\sigma}\, \mathrm{Tr}\left[V\partial_\mu V\partial_\nu V\partial_\lambda V\, \partial_\sigma V\right]$$

2D lattices with SU(N) spins

- Bulk SPT: VBS vortex line decorated by “PSU(N) Haldane chain”

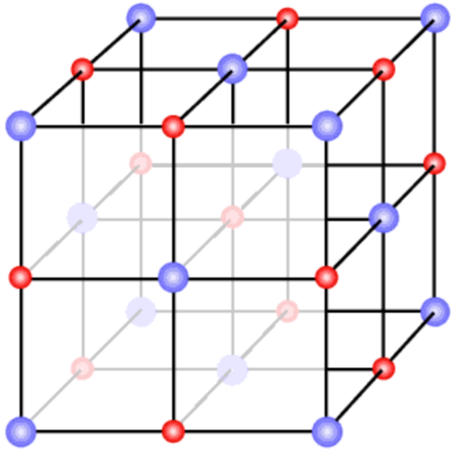
- **Generalized LSM exists for even N on the square lattice**

- **Generalized LSM exists for N a multiple of 3 on the honeycomb lattice**

- Featureless PEPS states can be constructed when generalized LSM doesn’t exist

3D cubic lattice with SU(N) spins

- 3D fund.-anti-fund. cubic lattice



Space group: point group of cubic lattice (F432)

Internal symmetry: PSU(N)

Neel-VBS critical point: PSU(N) × SO(3) critical point

Corresponding bulk SPT (in 4+1D):
SO(3) hedge-hog monopole line of VBS decorated by 1+1D PSU(N) Haldane phase

Bulk SPT is nontrivial for N even. Hence, **generalized LSM exist for even N**

Anomaly for N=2 calculated in [CMJ, Thomson, Rasmussen, Xu unpulished]

SO(N) LSM

From SPT analysis, generalizaed LSM exist on:

1D Spin(N) chain with a spinor rep. per site

1D SO(2N) chain with a vector rep. per site

2D SO(2N+1) square lattice with a spinor rep.per site

2D SO(2N) square lattice with a vector rep.per site

⋮

Connecting LSM with SPT on triangular lattice

- Triangular lattice with spin-1/2 per site

120° Neel to 12-site VBS critical point with emergent $SO(3)_s \times SO(3)_v$ symmetry

120° Neel order parameter manifold is SO(3)

VBS order parameter manifold is also SO(3)

VBS vortex (classified by $\pi_1(SO(3)_v) = Z_2$) carries a spin-1/2 of $SO(3)_s$

Corresponds to the boundary of 3+1D $SO(3)_s \times SO(3)_v$ SPT with $SO(3)_v$ vortex line decorated by $SO(3)_s$ "Haldane chain"

CMJ, A. Thomson, A. Rasmussen, Z. Bi, and C. Xu, ArXiv:1710.0668

Summary & future direction

Summary

- Connection between LSM in spin systems and boundary state of SPT in one higher dimension
- Obtain generalized LSM thereoms on square, honeycomb, cubic lattice for SU(N), SO(N) spins

Future directions

- Implication of the LSM theorem on topological orders (near the quantum critical point)
- LSM with fermions or with U(1) symmetry or both
- “no-go” to trivial SPT phase, e.g. [Lu 2017, Yang et al 2017]
- LSM for disordered systems, e.g. [Kimchi 17]