Lecture 3.2: Collective Excitations

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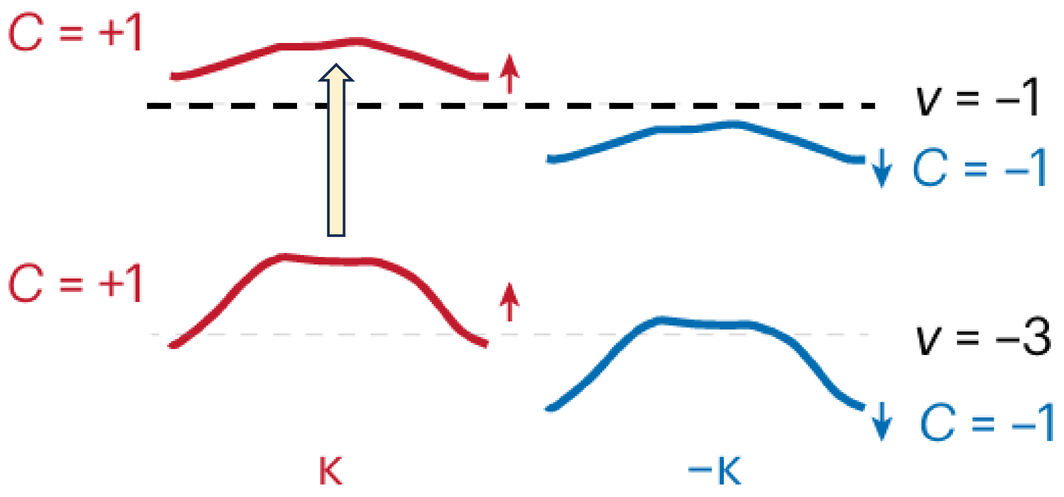
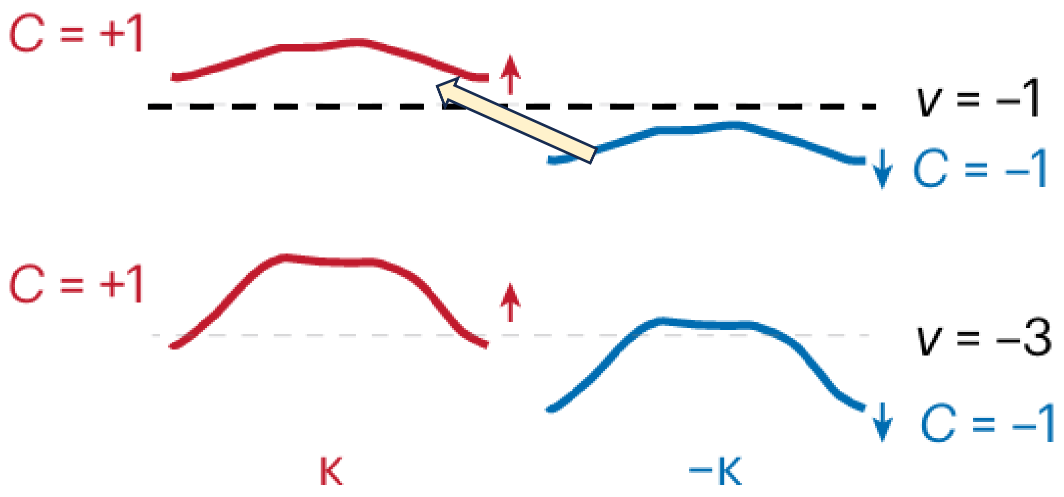
Collective Excitations in Integer QAHI





Exciton:

- Spin conserved collective excitation
- Optically active

Magnon:

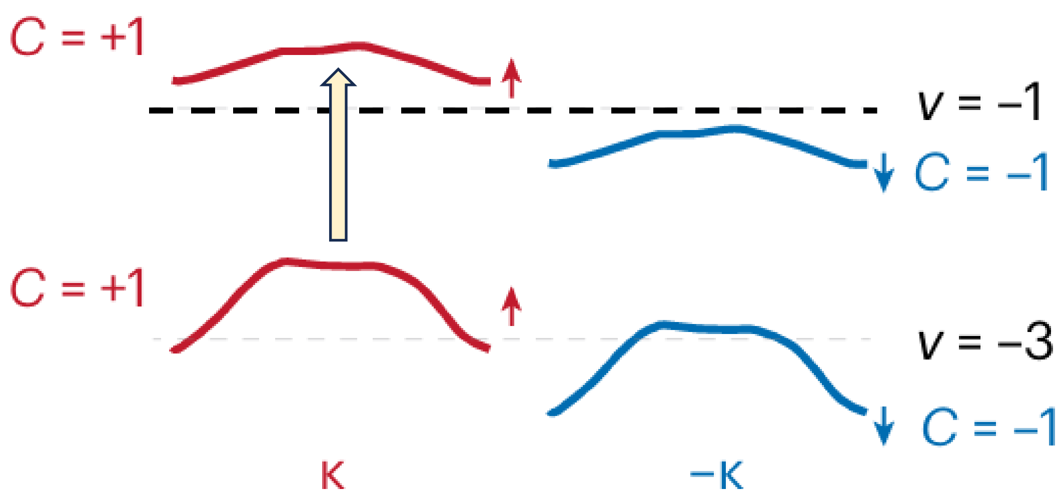
- Spin flip collective excitation

- Obtain the ground state using Hartree-Fock mean field theory.
- Calculate the collective excitations using Bethe-Salpeter equation.

Outline

- ✓ Excitonic Optical Response
- ✓ Topological Magnons

Bethe-Salpeter Equation



Exciton Wave Function

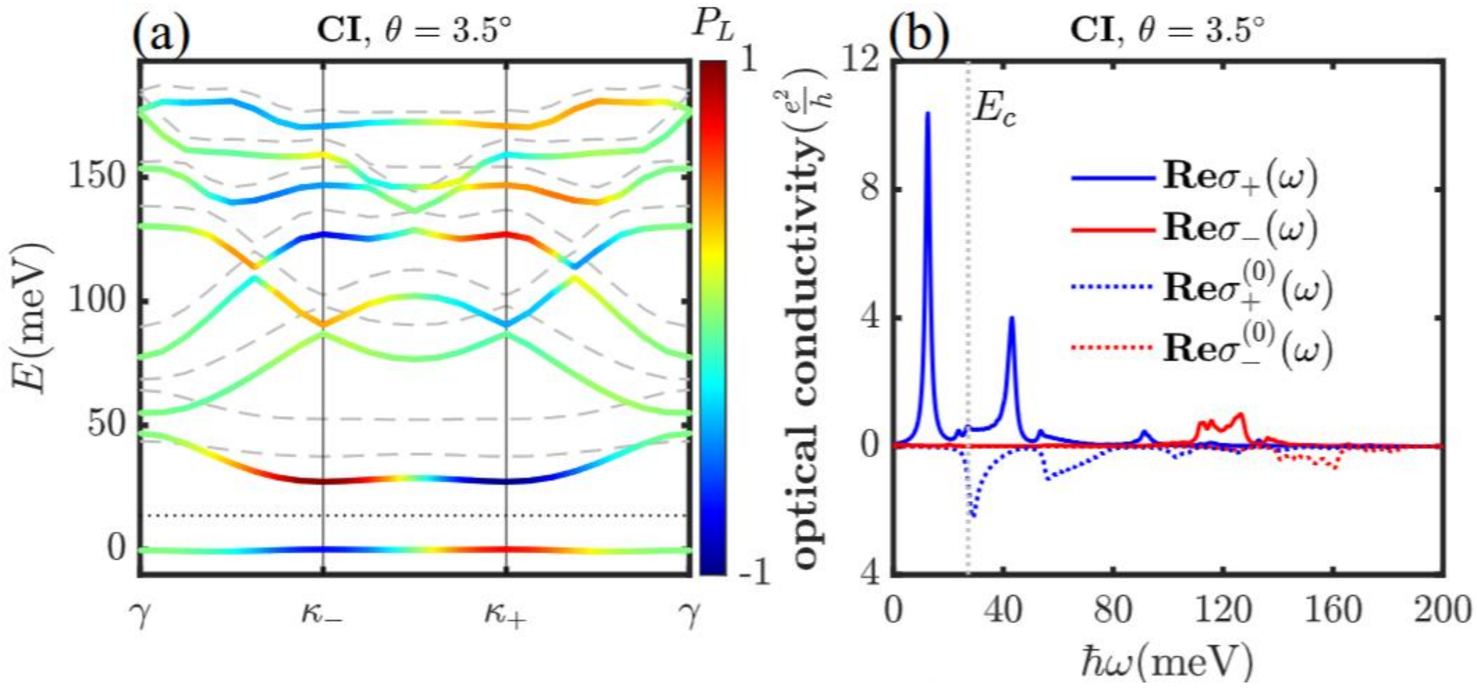
$$|\chi\rangle = \sum_{\boldsymbol{k},\lambda\geq 2} z_{\boldsymbol{k},\lambda}(\chi) f^{\dagger}_{\boldsymbol{k}\lambda+} f_{\boldsymbol{k}1+}|G\rangle$$

Bethe-Salpeter Equation

$$\mathcal{E}_{\chi} z_{\boldsymbol{k},\lambda}(\chi) = \sum_{\boldsymbol{p}\lambda'} \mathcal{H}^{\lambda\lambda'}_{\boldsymbol{kp}} z_{\boldsymbol{p},\lambda'}(\chi),$$

$$\mathcal{H}^{\lambda\lambda'}_{\boldsymbol{kp}} = (E^{\lambda+}_{\boldsymbol{k}} - E^{1+}_{\boldsymbol{k}})\delta_{\boldsymbol{kp}}\delta_{\lambda\lambda'} - (\tilde{V}^{1\lambda\lambda'1}_{\boldsymbol{pkpk}} - \tilde{V}^{\lambda 1\lambda' 1}_{\boldsymbol{kppk}})$$

Quantum Geometry Probed by Excitonic Optical Response in tMoTe2

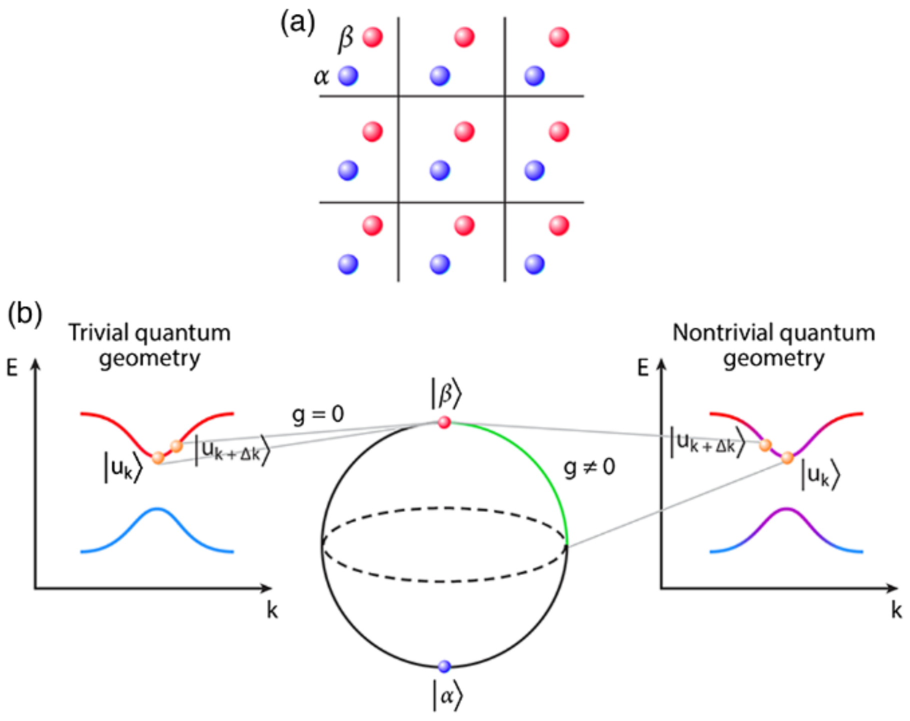


- A single chiral exciton dominates the optical sum rule. $p_\chi = \frac{w_{xx}^{(1)}(\chi)}{\sum_{\chi'} w_{xx}^{(1)}(\chi')} \approx 0.7$

- The absorption spectrum shows nearly perfect magnetic circular dichroism.
  ➔ nearly perfect quantum geometry of Chern insulator, $K \approx C$

W.-X. Qiu and F. Wu, PRB 111, L121104 (2025) [Editors' Suggestion]

Quantum Geometry



- ➢ **Euclidean geometry**: determines distances between two points on a plane and on a sphere

- ➢ **Quantum geometry:** captures the phase and amplitude distances between quantum states.

- Quantum geometric tensor

$$\underline{Q}_{ij}^{\mu\nu} = \langle \partial_\mu u_i | (1 - P) | \partial_\nu u_j \rangle \quad \text{with} \quad i, j = 1, \ldots, r,$$

$$P = \sum_{i=1}^{r} |u_i\rangle\langle u_i|$$

$$(\underline{Q}^{\mu\nu})^\dagger = \underline{Q}^{\nu\mu}$$

$$\underline{Q}^{\mu\nu} = \underline{G}^{\mu\nu} - (i/2)\underline{F}^{\mu\nu}$$

Quantum metric: amplitude distance
Berry curvature: phase distance

Päivi Törmä, PRL **131**, 240001 (2023)

Generalized Optical Sum Rules

- Kubo formula

$$\sigma_{\alpha\beta}(\omega) = i\frac{e^2}{\hbar}\frac{1}{\mathcal{A}}\sum_{\chi}\frac{1}{\mathcal{E}_\chi}\left[\frac{V^{\alpha}_{G\chi}V^{\beta}_{\chi G}}{\hbar\omega - \mathcal{E}_\chi + i\eta} + \frac{V^{\alpha}_{\chi G}V^{\beta}_{G\chi}}{\hbar\omega + \mathcal{E}_\chi + i\eta}\right],$$

$$V^{\alpha}_{\chi G} = \langle\chi|\hat{v}^{\alpha}|G\rangle \qquad \hat{\boldsymbol{v}} = i[\hat{\mathcal{H}}, \hat{\boldsymbol{r}}]$$

- Absorptive part of optical conductivity

$$\sigma^{L,H}_{\mu\nu}(\omega) = \frac{\sigma_{\mu\nu}(\omega) \pm \sigma_{\nu\mu}(\omega)}{2}$$

$$\sigma^{\text{abs}} \equiv \text{Re}\,\sigma^{L} + i\,\text{Im}\,\sigma^{H}$$

Re σ_L: absorption of linearly polarized light
Imσ_H: magnetic circular dichroism

- C_{3z} Symmetry

$$\sigma^{\text{abs}}_{\alpha\beta} = \delta_{\alpha\beta}\text{Re}\sigma_{\alpha\beta} + i(1-\delta_{\alpha\beta})\text{Im}\sigma_{\alpha\beta}$$

- Negative first moment

$$W^{(1)}_{\alpha\beta} = \int_0^{\infty} d\omega \frac{\sigma^{\text{abs}}_{\alpha\beta}(\omega)}{\omega} = \frac{e^2}{\hbar}\frac{\pi}{\mathcal{A}}\sum_{\chi} w^{(1)}_{\alpha\beta}(\chi)$$
$$= \frac{e^2}{\hbar}\frac{\pi}{\mathcal{A}}[\langle\hat{r}^{\alpha}\hat{r}^{\beta}\rangle_G - \langle\hat{r}^{\alpha}\rangle_G\langle\hat{r}^{\beta}\rangle_G],$$

$$w^{(1)}_{\alpha\beta}(\chi) = V^{\alpha}_{G\chi}V^{\beta}_{\chi G}/\mathcal{E}^2_\chi$$

- Position operator

$$\hat{\boldsymbol{r}} = \sum_{\boldsymbol{k}}\sum_{mn}\boldsymbol{r}_{mn}(\boldsymbol{k})f^{\dagger}_{\boldsymbol{k},m}f_{\boldsymbol{k},n},$$
$$\boldsymbol{r}_{mn}(\boldsymbol{k}) = \delta_{mn}i\partial_{\boldsymbol{k}} - \boldsymbol{A}_{mn}(\boldsymbol{k}),$$
$$\boldsymbol{A}_{mn}(\boldsymbol{k}) = -\langle u_{m\boldsymbol{k}}|i\partial_{\boldsymbol{k}}|u_{n\boldsymbol{k}}\rangle,$$

Generalized Optical Sum Rules

$$\int_0^\infty d\omega \frac{\mathrm{Re}[\sigma_{\alpha\alpha}(\omega)]}{\omega} = \frac{\pi}{2}\frac{e^2}{h}K, \quad \alpha = x \text{ or } y,$$

Quantum Weight: $K = \frac{1}{2\pi}\sum_\alpha \int d^2\boldsymbol{k}\mathrm{Tr}\, G^{\alpha\alpha}$

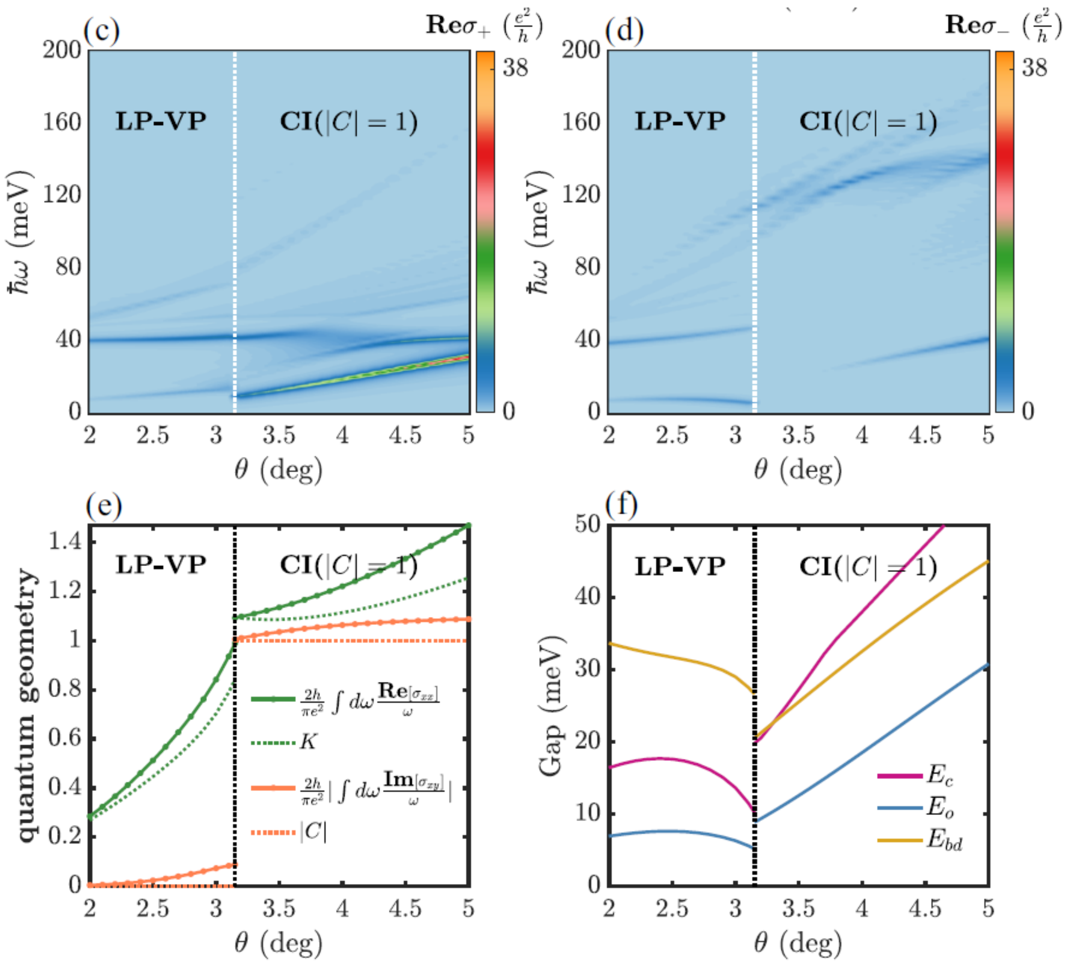
$$\int_0^\infty d\omega \frac{\mathrm{Im}[\sigma_{\alpha\beta}(\omega)]}{\omega} = \frac{\pi}{2}\frac{e^2}{h}\epsilon^{\alpha\beta}C, \quad \alpha \neq \beta,$$

Chern Number: $C = \frac{1}{2\pi}\int d^2\boldsymbol{k}\mathrm{Tr}\ \Omega$

Absorption of circularly polarized light:

$$\mathrm{Re}\sigma_\pm(\omega) = \mathrm{Re}\sigma_{xx}(\omega) \mp \mathrm{Im}\sigma_{xy}(\omega)$$

Quantum Geometry Probed by Excitonic Optical Response in tMoTe2



Sum Rules:

$$\int_0^\infty d\omega \frac{\mathrm{Re}[\sigma_{\alpha\alpha}(\omega)]}{\omega} = \frac{\pi}{2}\frac{e^2}{h}K, \quad \alpha = x \text{ or } y,$$

$$\int_0^\infty d\omega \frac{\mathrm{Im}[\sigma_{\alpha\beta}(\omega)]}{\omega} = \frac{\pi}{2}\frac{e^2}{h}\epsilon^{\alpha\beta}C, \quad \alpha \neq \beta,$$

$$\int_0^\infty d\omega \mathrm{Re}\sigma_{xx}(\omega) = \frac{\pi n_c e^2}{2m^*}$$

Bound on optical gap:

$$\int_0^\infty d\omega \frac{\mathrm{Re}\sigma_{xx}(\omega)}{\omega} \leq \frac{\hbar}{E_o}\int_0^\infty d\omega \mathrm{Re}\sigma_{xx}(\omega),$$

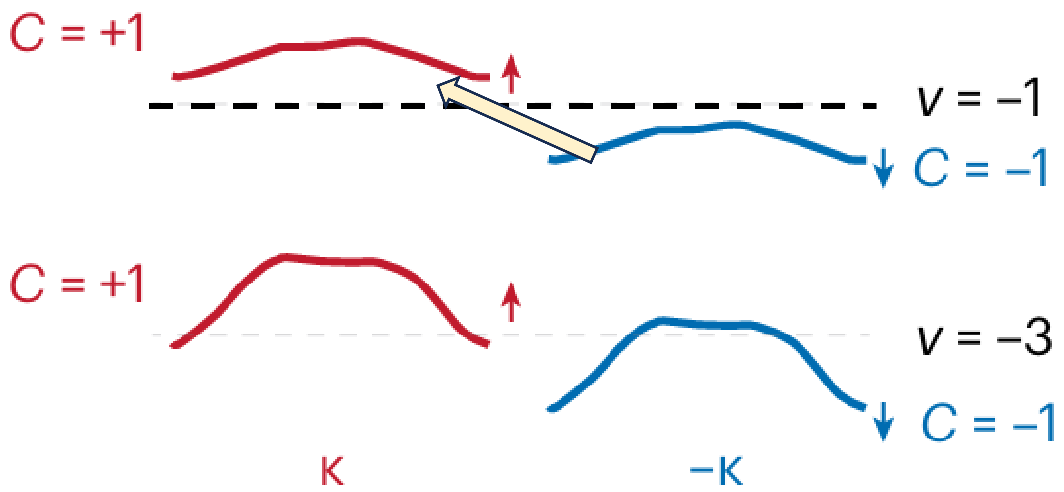
$$E_o \leq E_{bd} = \frac{2\pi\hbar^2 n_c}{m^* K}$$

W.-X. Qiu and F. Wu, PRB 111, L121104 (2025) [Editors' Suggestion]

Outline

- ✓ Excitonic Optical Response
- ✓ Topological Magnons

Bethe-Salpeter Equation



Exciton Wave Function

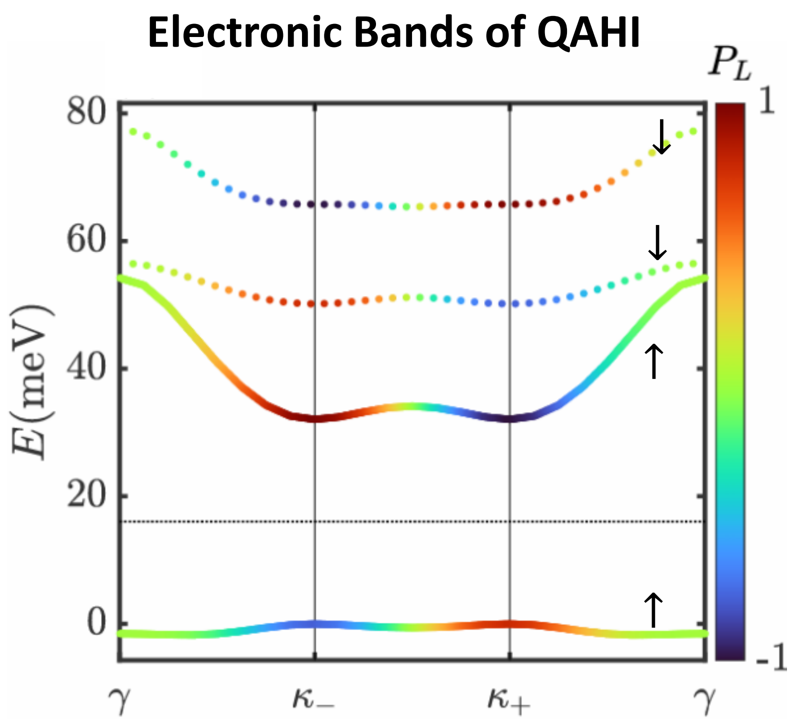
$$|\chi, \boldsymbol{q}\rangle = \sum_{\boldsymbol{k},\lambda} z^{\lambda}_{\boldsymbol{k}}(\chi, \boldsymbol{q}) f^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},\lambda,-} f_{\boldsymbol{k},1,+} |G\rangle$$

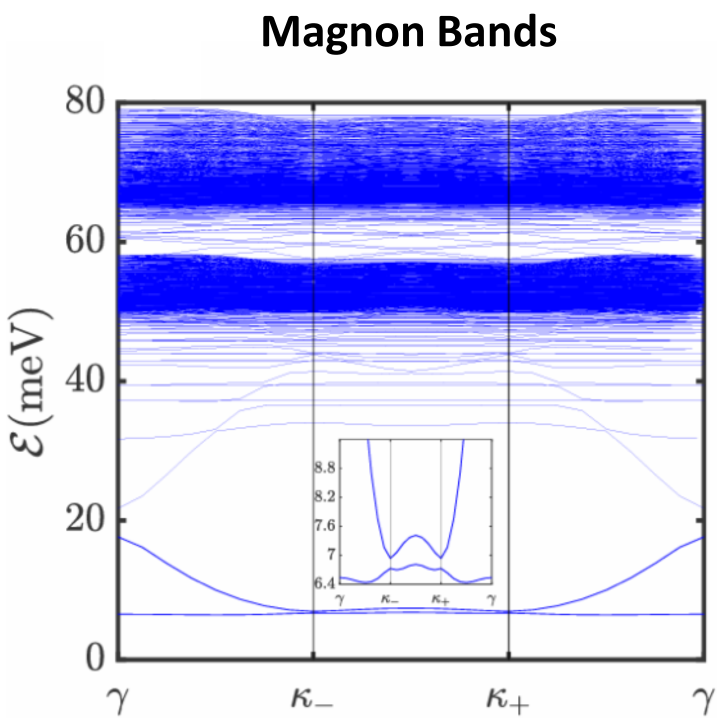
Bethe-Salpeter Equation

$$\mathcal{E}_{\chi,\boldsymbol{q}} z^{\lambda}_{\boldsymbol{k}}(\chi, \boldsymbol{q}) = \sum_{\boldsymbol{p}\lambda'} \mathcal{H}^{\lambda\lambda'}_{\boldsymbol{kp}}(\boldsymbol{q}) z^{\lambda'}_{\boldsymbol{p}}(\chi, \boldsymbol{q}),$$

$$\mathcal{H}^{\lambda\lambda'}_{\boldsymbol{kp}}(\boldsymbol{q}) = (E^{\lambda-}_{\boldsymbol{k}+\boldsymbol{q}} - E^{1+}_{\boldsymbol{k}})\delta_{\boldsymbol{kp}}\delta_{\lambda\lambda'} - \tilde{V}^{1\lambda\lambda'1}_{\boldsymbol{p},\boldsymbol{k}+\boldsymbol{q},\boldsymbol{p}+\boldsymbol{q},\boldsymbol{k}}(+,-)$$

Magnon Spectrum





- Magnon: Spin flip collective excitation
- Magnon Spectrum is gapped (~ 6 meV) ➔ Stability of QAHI

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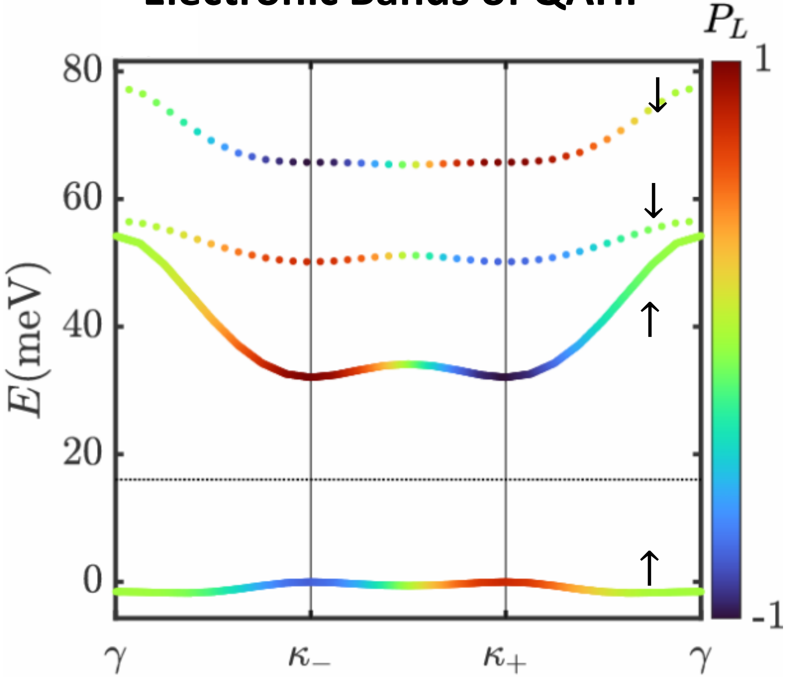
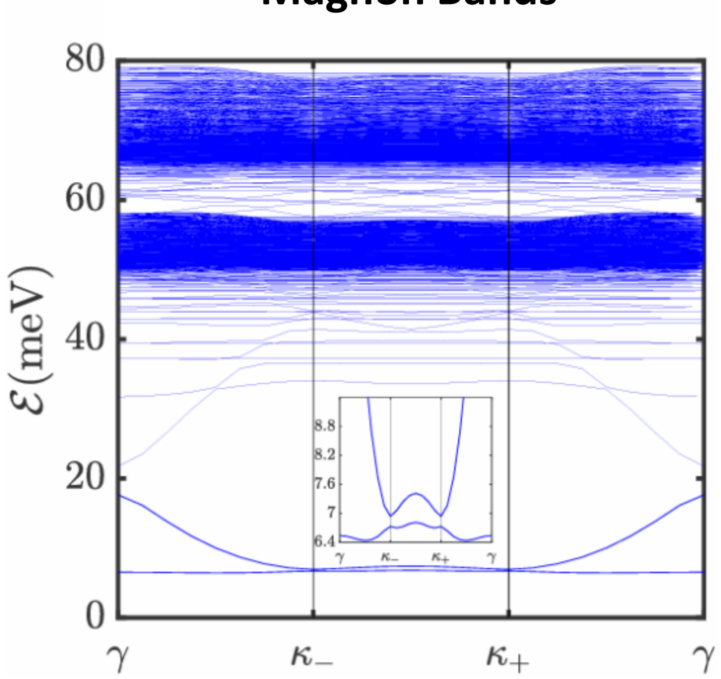
Magnon Spectrum





	Symmetry	QAHI Order Parameter	Symmetry Breaking?
Valley	U(1)	$\pi_z = n_{+K} - n_{-K}$	Valley U(1) symmetry NOT broken; $\hat{\mathcal{T}}$ broken

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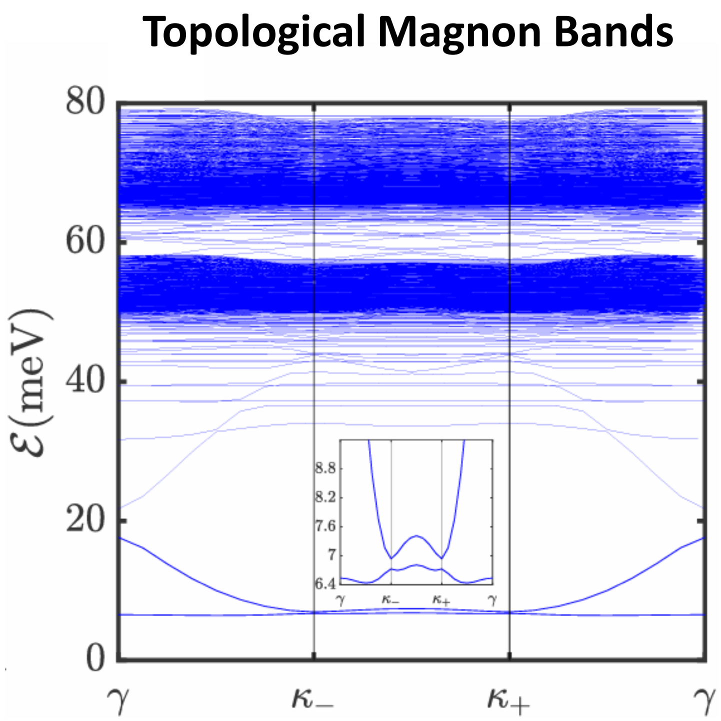
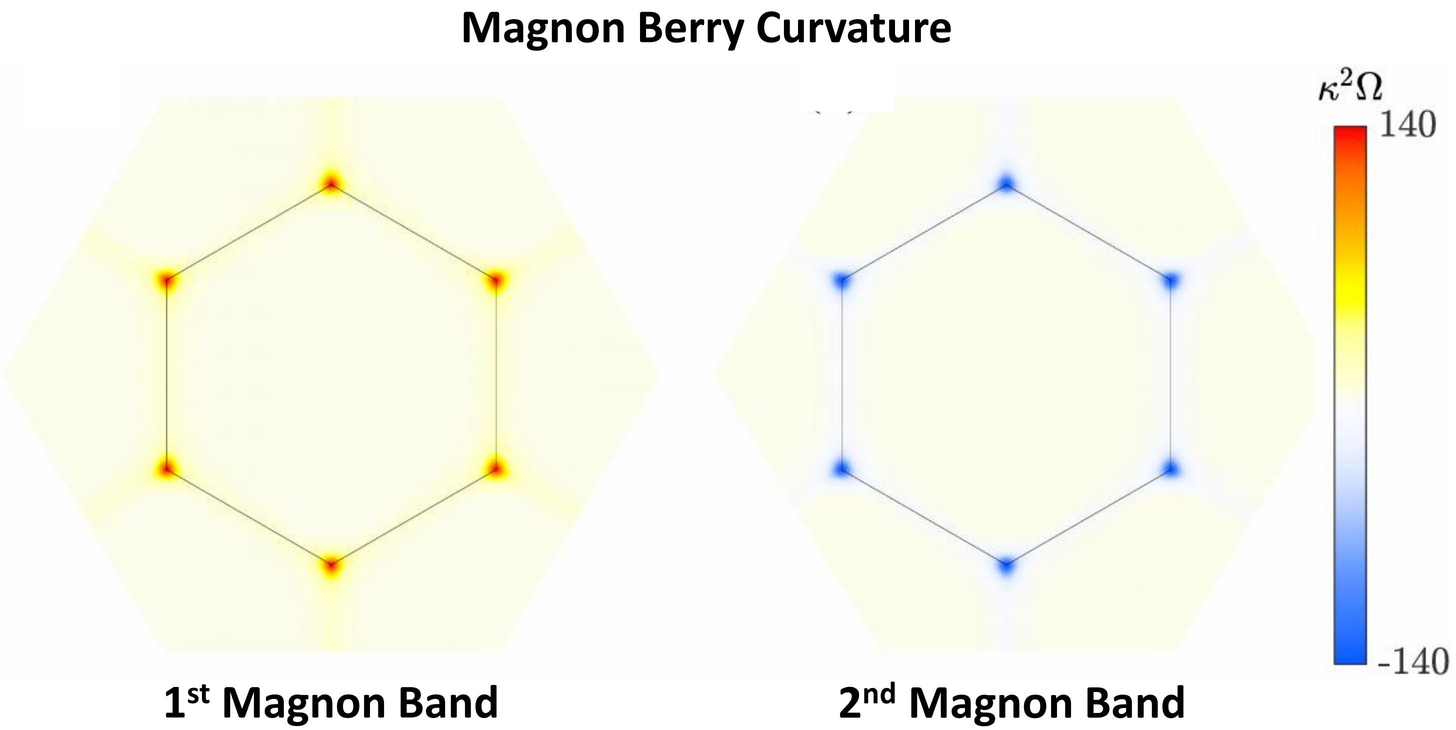
Magnon Topology





The first two magnon bands are topological with Chern numbers -1 *and* +1.

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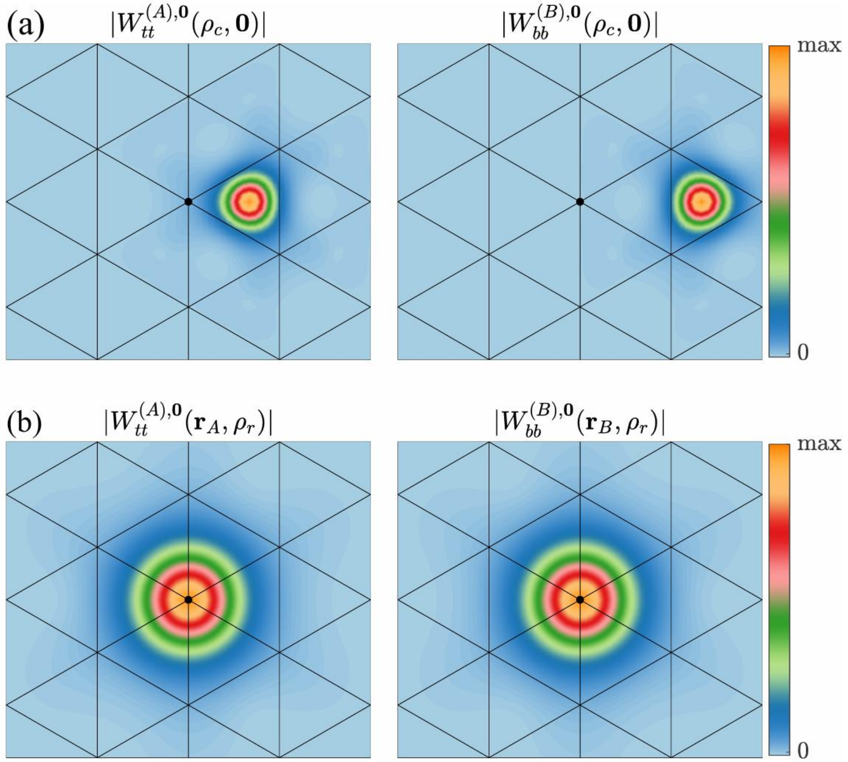
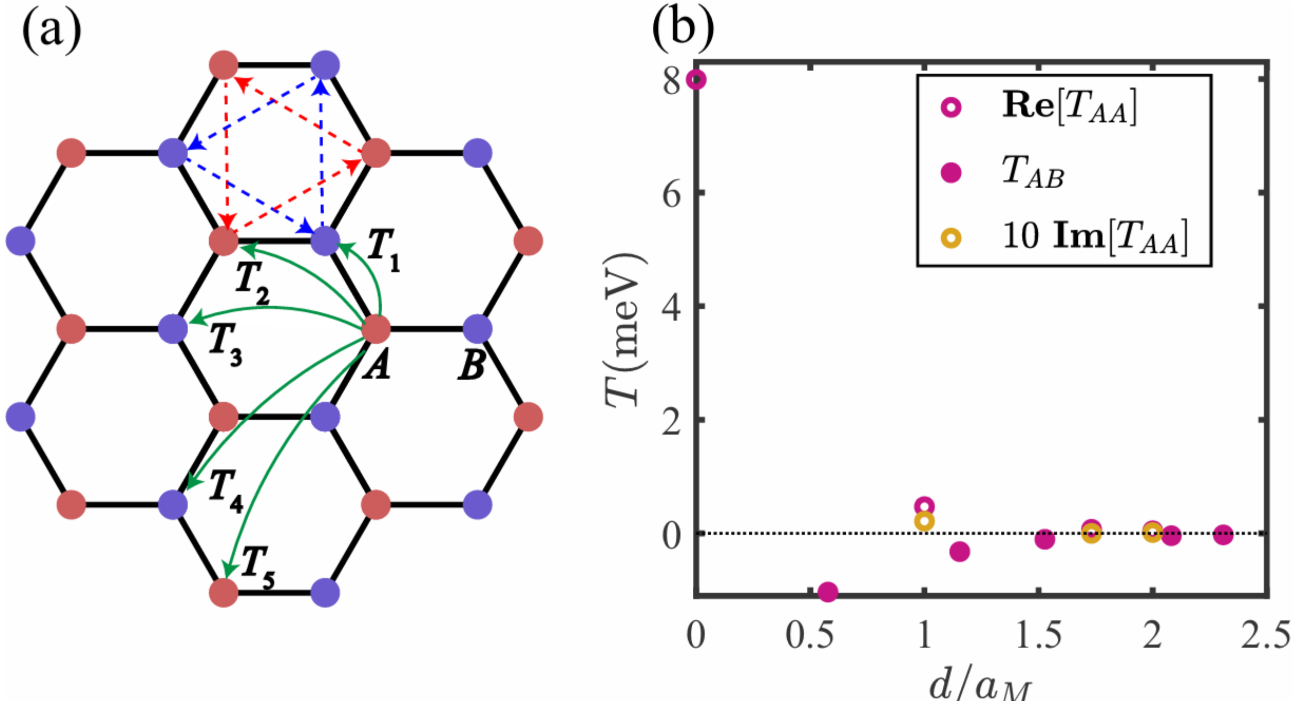
Magnon Haldane Model

Magnon Wannier States



Magnon Tight-Binding Model



$$H_{\mathrm{mag}} = \sum_{\alpha\beta} \sum_{\boldsymbol{R}\boldsymbol{R}'} T_{\alpha\beta}(\boldsymbol{R}' - \boldsymbol{R}) a^{\dagger}_{\boldsymbol{R}\alpha} a_{\boldsymbol{R}'\beta}$$

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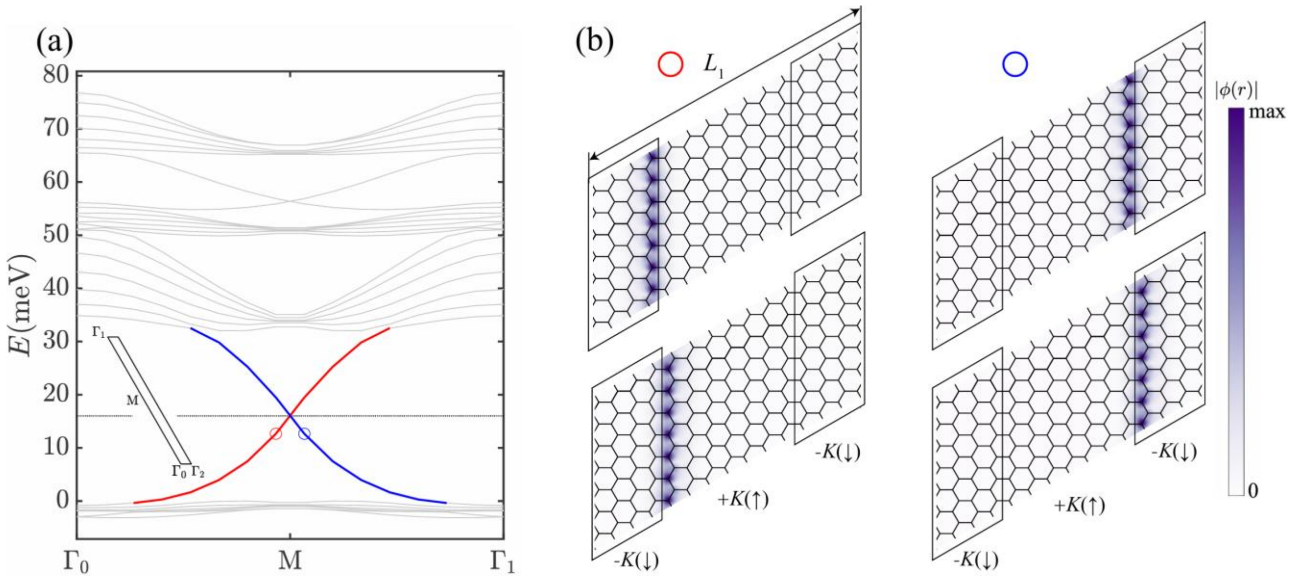
Domain Walls



Domain Wall

- Separates domains with opposite spin polarizations & Chern numbers
- Binds chiral edge states

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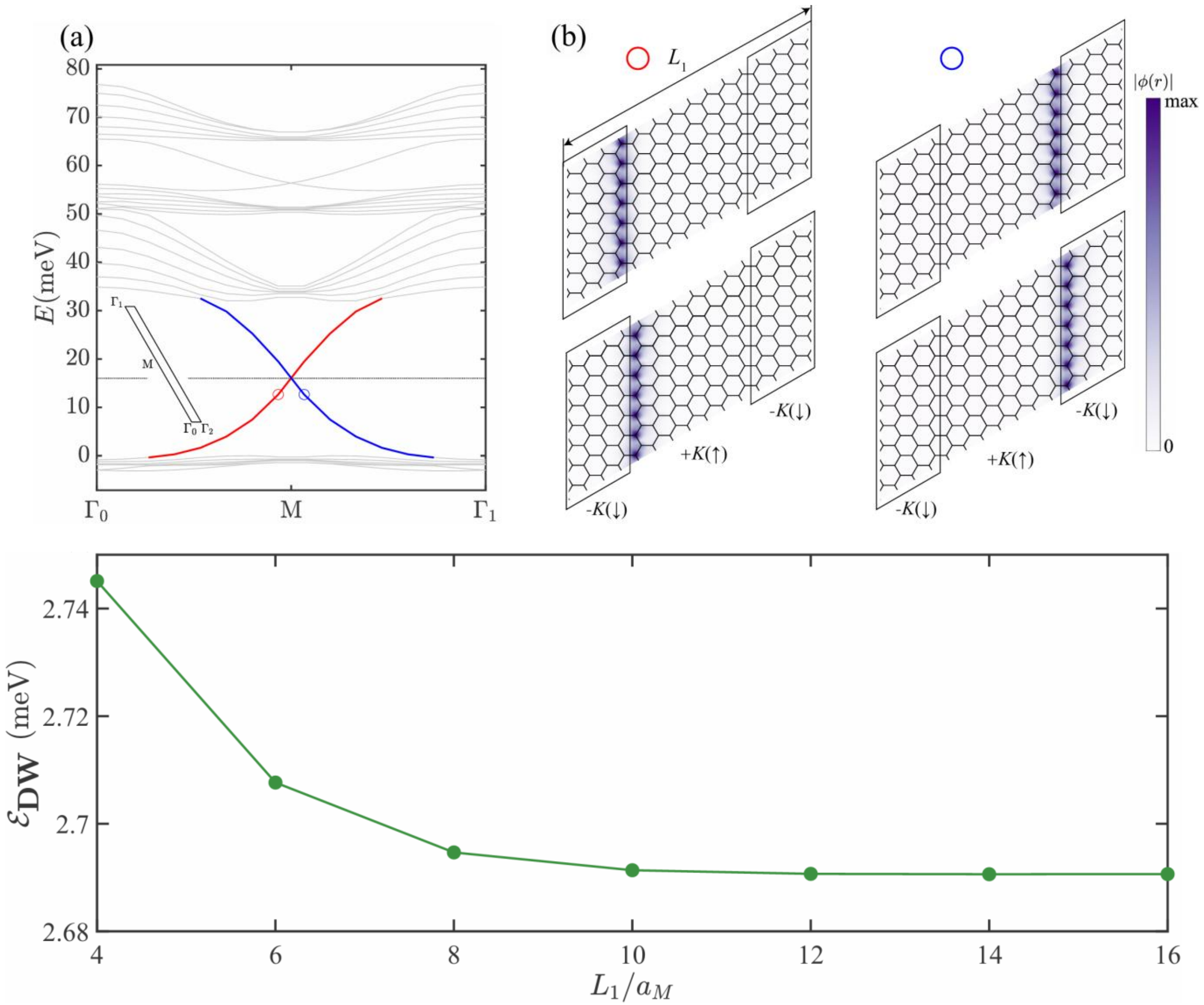
Domain Walls



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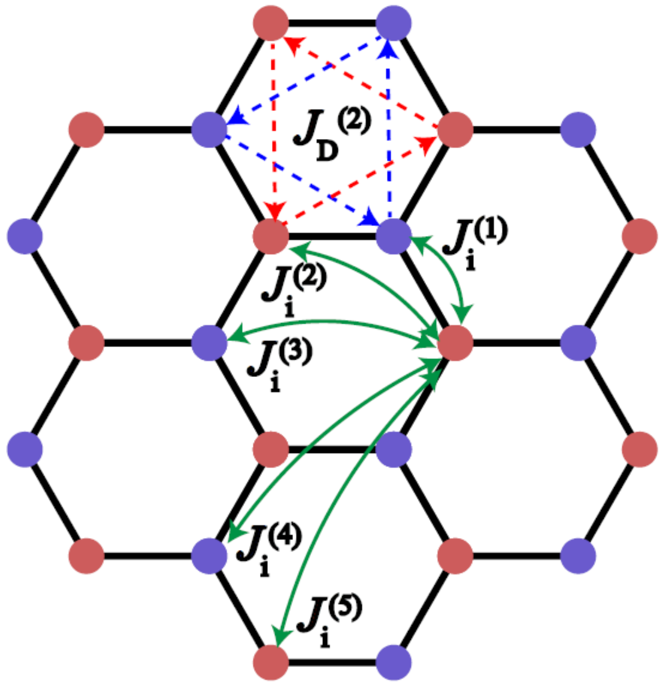
Domain Wall

- Separates domains with opposite spin polarizations & Chern numbers
- Binds chiral edge states
- Energy cost~2.7 meV per unit cell along the domain wall

Effective Spin Model



$$\mathcal{L}_S = \mathcal{B}_S - \mathcal{E}[\boldsymbol{m}],$$

$$\mathcal{B}_S = -\frac{\hbar}{2}\sum_{\boldsymbol{R}\alpha} n_\alpha \mathcal{A}[\boldsymbol{m}_{\boldsymbol{R}\alpha}] \cdot \partial_t \boldsymbol{m}_{\boldsymbol{R}\alpha},$$

$$\mathcal{E}[\boldsymbol{m}] = {\sum_{\boldsymbol{R}\boldsymbol{R}'\alpha\beta}}' \sum_i J_i^{\alpha\beta}(\boldsymbol{R}'-\boldsymbol{R}) m^i_{\boldsymbol{R}\alpha} m^i_{\boldsymbol{R}'\beta}$$

$$+ {\sum_{\boldsymbol{R}\boldsymbol{R}'\alpha}}' J_D^{\alpha\alpha}(\boldsymbol{R}'-\boldsymbol{R})(\boldsymbol{m}_{\boldsymbol{R}\alpha} \times \boldsymbol{m}_{\boldsymbol{R}'\alpha}) \cdot \hat{z} \longrightarrow \text{DMI}$$

- DMI accounts for the magnon topology.
- Coupling constants J are extracted from the magnon and domain wall energies.
- Estimation of Cuire temperature

$$T_c \approx -\frac{\sum_{\boldsymbol{R}\beta} J_z^{\alpha\beta}(\boldsymbol{R})}{3k_B} = \frac{T_{\alpha\alpha}(0)}{12k_B} \approx 7.7\ \text{K}.$$

Outline

- ✓ Excitonic Optical Response
- ✓ Topological Magnons