Novel Orbital Phases of Cold Atoms– Unconventional BEC, Cooper pairing, and frustration

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Fermions: W. C. Lee, C. Wu, S. Das Sarma, arXiv:0905.1146.
C. Wu, PRL 101, 168807 (2008).
C. Wu, PRL 100, 200406 (2008).
C. Wu, and S. Das Sarma, PRB 77, 235107 (2008).
S. Z. Zhang , H. H. Hung, and C. Wu, arXiv:0805.3031.
C. Wu, D. Bergman, L. Balents, and S. Das Sarma, PRL 99, 67004(2007).

Bosons: C. Wu, Mod. Phys. Lett. 23, 1(2009) (brief review);
V. M. Stojanovic, C. Wu, W. V. Liu and S. Das Sarma, PRL 101, 125301(2008).
C. Wu, W. V. Liu, J. Moore and S. Das Sarma, PRL 97, 190406 (2006).
W. V. Liu and C. Wu, PRA 74, 13607 (2006).

July 29, 2009, KITP

Collaborators:

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Outline

- **Introduction:**

  A brief review of cold atom physics. What is orbital physics?

  Orbital physics with cold atoms: a new promising research direction. Pioneering experiments.

- Bosons: Unconventional BEC beyond the "no-node" theorem; meta-stable states of bosons.

- Fermions in the honeycomb lattice: $p_{x,y}$-orbital "graphene".

1. Strong correlation from flat bands: Wigner crystal and ferromagnetism.

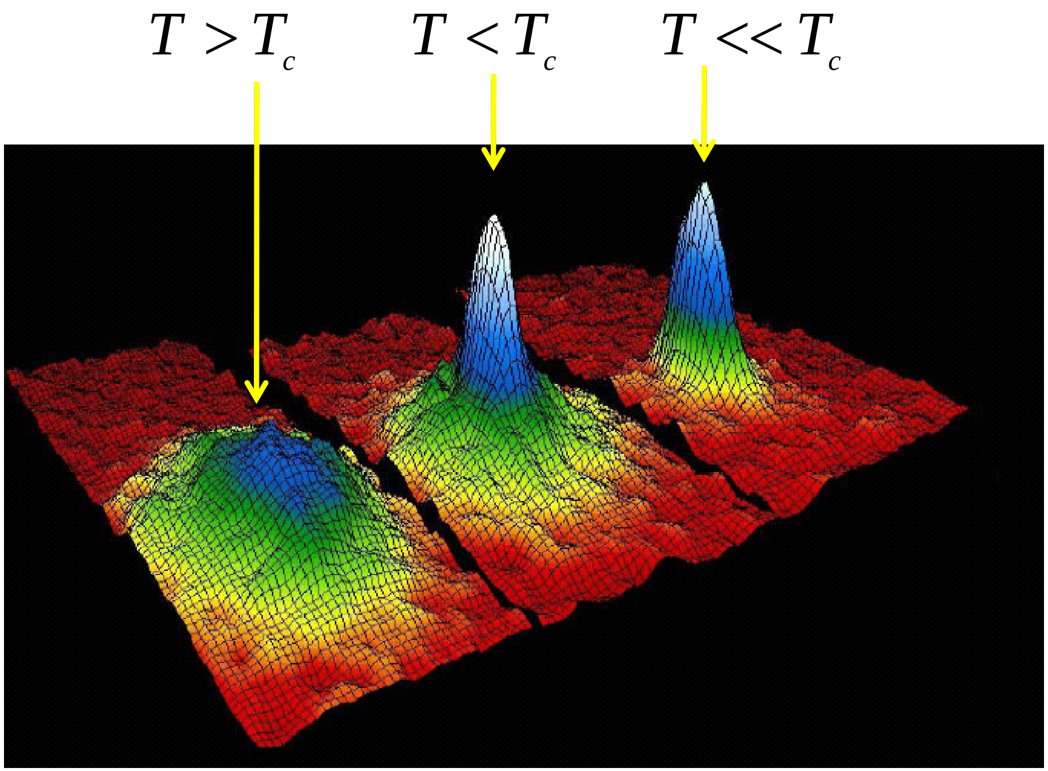
2. A new mechanism for unconventional pairing – the f-wave.

3. p-orbital Mott insulator: orbital exchange; a new type of frustrated "magnet"-like mode.

4. Band insulator (topological): quantum anomalous Hall effect by orbital angular momentum polarization.

Bose-Einstein condensation

- Bosons in magnetic traps: dilute and weakly interacting systems.

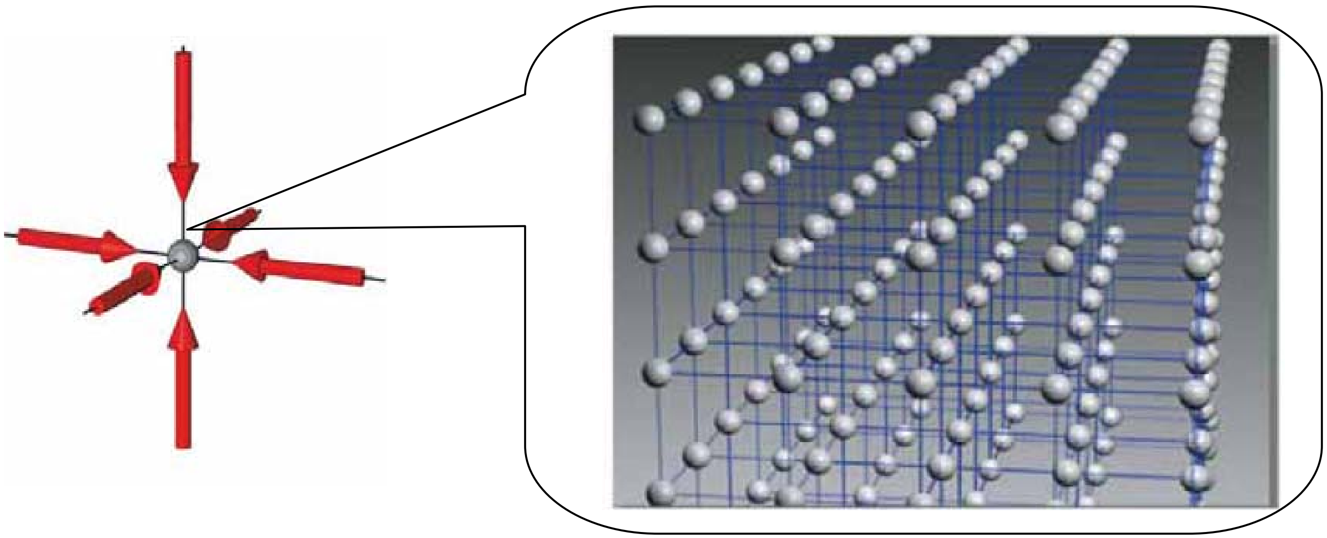


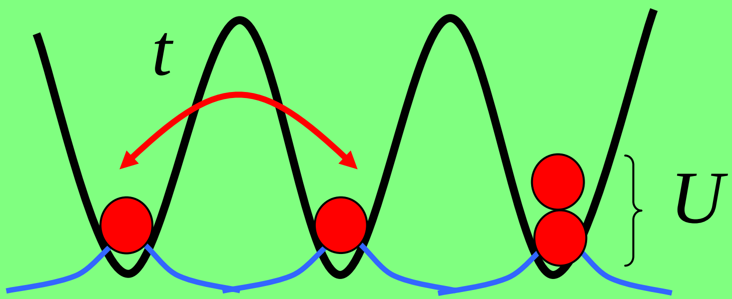
M. H. Anderson et al., Science 269, 198 (1995)

$$T_{BEC} \sim 1 \mu K \quad n \sim 10^{14}\,\mathrm{cm}^{-3}$$

New era of cold atom physics: optical lattices

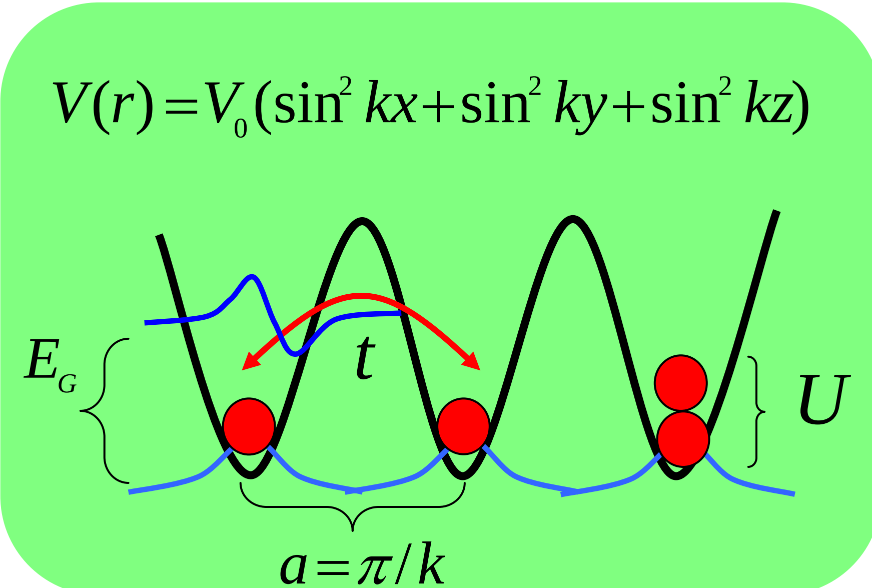
- Strongly correlated systems.
- Interaction effects are tunable by varying laser intensity.





t : inter-site tunneling
U: on-site interaction

Typical length and energy scales



T. L. Ho and Q. Zhou, Phys. Rev. Lett. **99**, 120404 (2007)

^{87}Rb $a_{scattering} = 5.5nm$

$$E_R = \frac{\hbar k^2}{2M} \approx 150nK \qquad a = 425nm$$

$$\frac{t}{E_R} \approx 1.43\left(\frac{V_0}{E_R}\right)^{0.98} e^{-2\sqrt{\frac{V_0}{E_R}}}$$

$$\frac{U}{E_R} \approx 3\left(\frac{a_s}{a}\right)\left(\frac{V_0}{E_R}\right)^{0.88}$$

V_o/E_R	3	5	10	15	20	30
E_G/E_R	0.58	1.91	4.42	6.23	7.63	9.79
E_G (nK)	90	294	678	956	1171	1503
U (nK)	15.5	24.2	44.6	63.7	82.0	117.2
t (nK)	17.9	10.4	3.01	1.03	0.39	0.073
t^2/U (nK)	20.66	4.45	0.20	0.0166	0.00019	0.00005

Research focuses of cold atom physics

- Great success of cold atom physics in the past decade:

BEC; superfluid-Mott insulator transition;

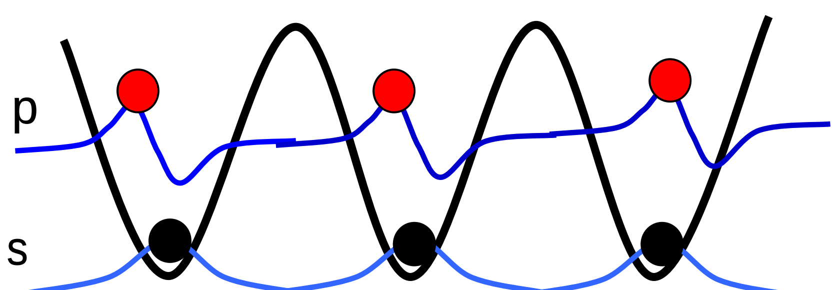
fermion superfluidity and BEC-BCS crossover; ... ...

- **Orbital** Physics: new physics of bosons and fermions in high-orbital bands in optical lattices.

Here orbital refers to the different energy levels (e.g. s, p) of each optical site. Please don't be confused by the electron orbitals inside the atom. Atoms in optical lattices replace the role of electrons in solid state lattices.

Good timing: pioneering experiments on orbital-bosons.

Square lattice (Mainz); double well lattice (NIST).

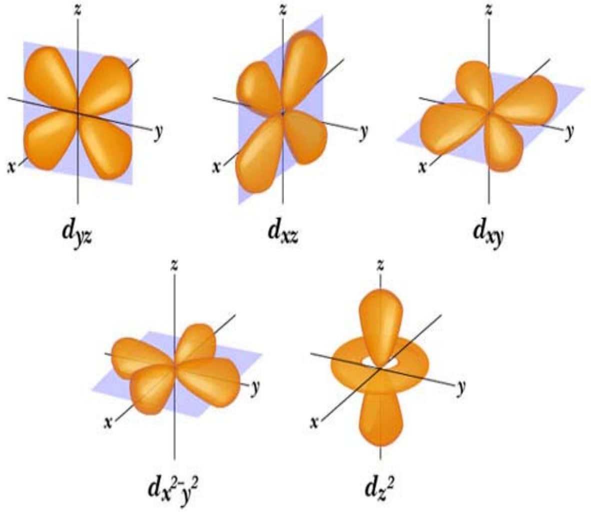


J. J. Sebby-Strabley, et al., PRA 73, 33605 (2006); T. Mueller et al., Phys. Rev. Lett. 99, 200405 (2007); C. W. Lai et al., Nature 450, 529 (2007).

Orbital physics

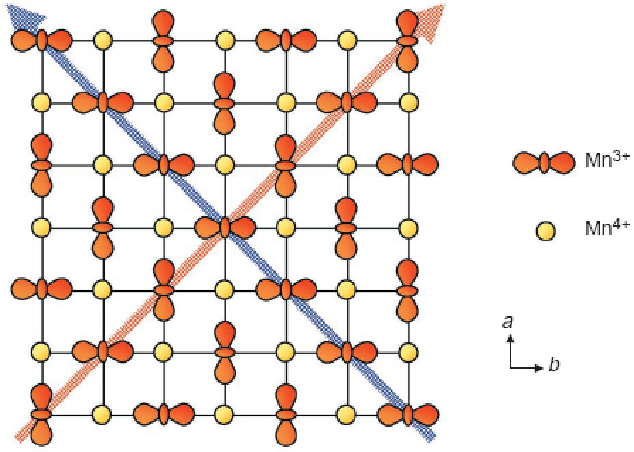
- Orbital: a degree of freedom independent of charge and spin.

  Tokura, et al., science 288, 462, (2000).

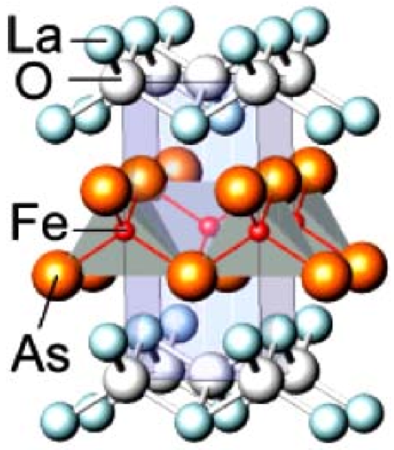


- Orbital degeneracy and spatial anisotropy.

- *cf.* transition metal oxides (*d*-orbital bands with electrons).



$La_{1-x}Sr_{1+x}MnO_4$



LaOFeAs

Advantage of cold atom orbital systems

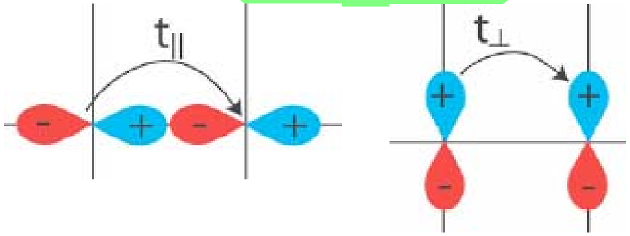
• Solid state orbital systems: Electron orbitals in lattices of positive ions.

Jahn-Teller distortion quenches orbital degree of freedom;

only fermions;

correlation effects in *p*-orbitals are weak. *p*-orbital Mott-insulators?

$$t_{//} >> t_{\perp}$$



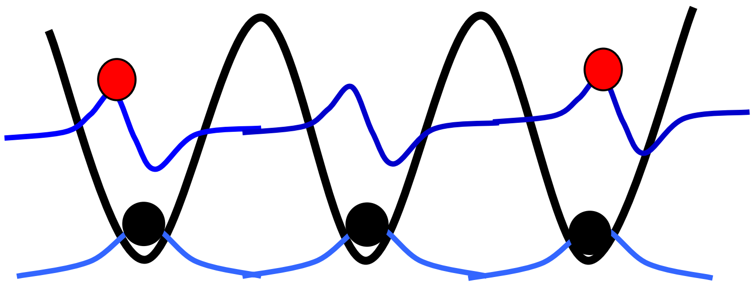
σ-bond π-bond

• Cold atom orbital systems: atoms in external optical lattices.

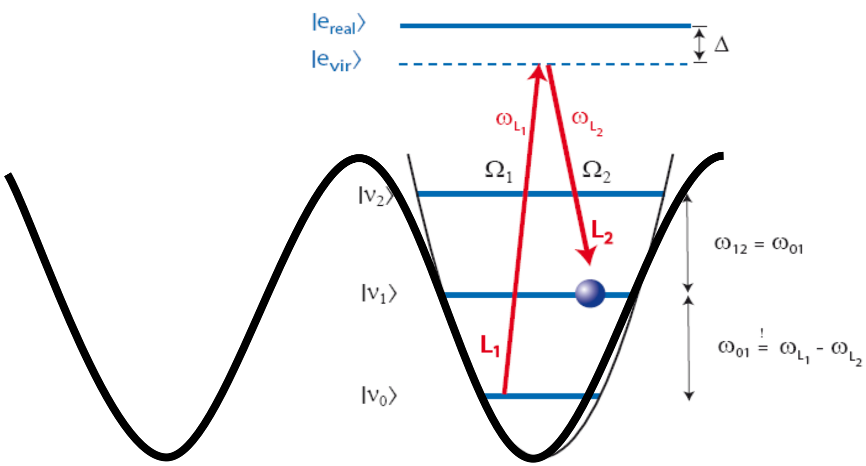
rigid lattice free of distortion;

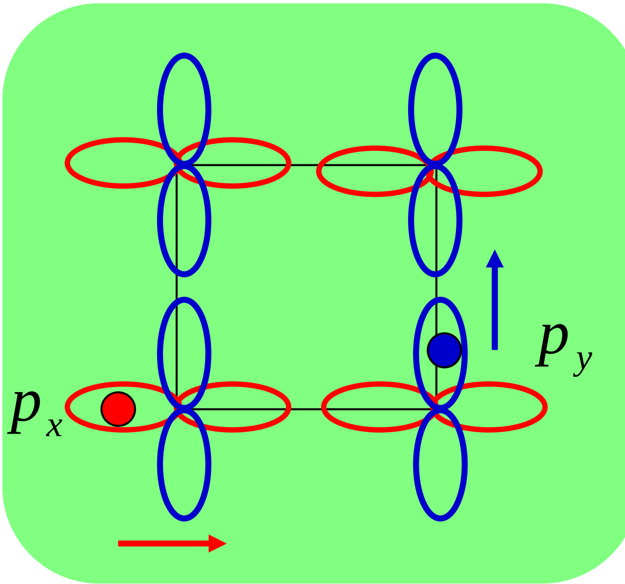
New materials: orbital bosons (meta-stable excited states with long life time);

New materials: strongly correlated *p*-orbital systems; strongest anisotropy + strong correlation.



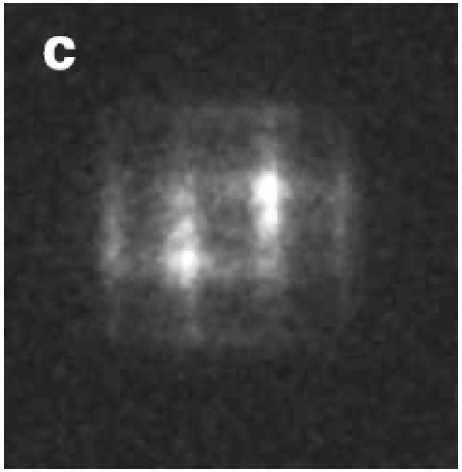
Orbital bosons: pumping bosons by Raman transition





- Long life-time: phase coherence.

- Quasi-1d feature in the square lattice.



T. Mueller, I. Bloch et al., Phys. Rev. Lett. 99, 200405 (2007).

Outline

• Introduction: orbital physics, a new promising direction for cold atom physics.

• Orbital bosons: they do not obey the “no-node” theorem of the ground state wavefunctions of bosons. Instead, they exhibit unconventional BEC with complex-valued wavefunctions, which breaks time reversal symmetry spontaneously.

C. Wu, Mod. Phys. Lett. 23, 1(2009) (brief review);
V. M. Stojanovic, C. Wu, W. V. Liu and S. Das Sarma, PRL 125301(2008);
C. Wu, W. V. Liu, J. Moore and S. Das Sarma, PRL 97, 190406 (2006);
W. V. Liu and C. Wu, PRA 74, 13607 (2006).

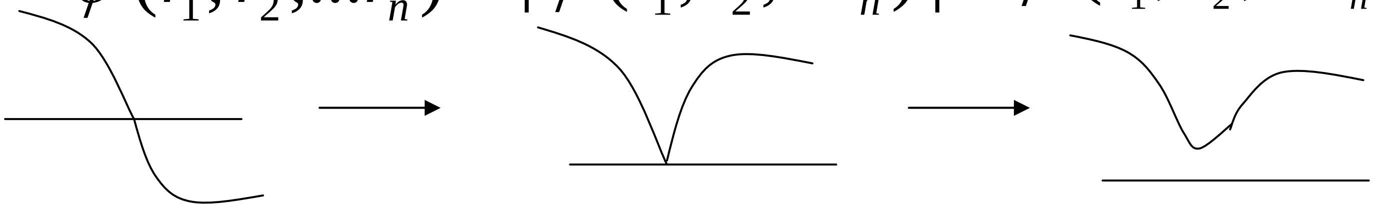
Other group’s related work: V. W. Scarola *et. al,* PRL, 2005; A. Isacsson *et. al.,* PRA 2005; A. B. Kuklov, PRL 97, 2006; C. Xu *et al*., cond-mat/0611620 .

• Fermions in hexagonal lattice: $p_{x,y}$-orbital counterpart of graphene.

Paradigm for bosons: Feynman's "no-node" theorem

- The many-body ground state wavefunctions (WF) of boson in the coordinate-representation are positive-definite in the absence of rotation.

$\psi(r_1, r_2, \ldots r_n)$ $\quad |\psi(r_1, r_2, \ldots r_n)|$ $\quad \psi(r_1, r_2, \ldots r_n) > 0$



$$\langle G | H | G \rangle = \int dr_1 \ldots dr_n \; \frac{\hbar^2}{2m} \sum_{i=1}^{n} |\nabla_i \psi(r_1, \ldots r_n)|^2 + |\psi(r_1, .. r_n)|^2 \sum_{i=1}^{n} U_{ex}(r_i)$$
$$+ |\psi(r_1, .. r_n)|^2 \sum_{i<j} V_{\text{int}}(r_i - r_j)$$

- Feynman's statement applies to all of superfluid, Mott-insulating, super-solid, density-wave ground states, etc.

- Quantum Monte-Carlo has no sign problem. In principle, we can study the ground state wavefunctions of bosons exactly.

How to go beyond the "no-node" theorem?

- Strong constraint from the no-node theorem!

Complex-valued WF reduced to a positive-definite distribution.
→ **Time-reversal symmetry cannot be broken!**

- Goal: **complex**-valued many-body wavefunctions whose physics should be richer. (e.g. spontaneous time-reversal symmetry breaking.)

Solution 1: **Excited (meta-stable**): bosons in high orbital bands ---- **orbital physics of bosons**.

Solution 2: Ground states of bosons with **spin-orbit (SO) coupling**. Its linear dependence on momentum invalidates Feynman's proof. (e.g. excitons, cold atoms with artificial spin-orbit coupling).

Half-quantum vortex with skymion spin-texture!

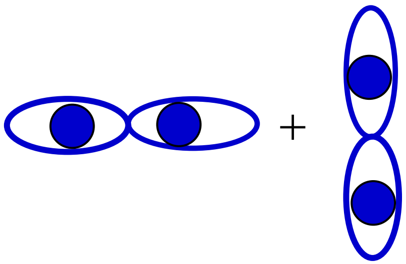
c.f. C. Wu and I. Mondragon-shem, arxiv:0809.3532; C. Wu, Mod. Phys. Lett. 23, 1 (2009).

Solution 1: Ferro-orbital interaction for spinless p-orbital bosons

- A single site problem: two orbitals p_x and p_y with two spinless bosons.

$$V(r_1-r_2)=g\delta(r_1-r_2) \qquad U=g\int dr\,|\phi_x(r)|^4=g\int dr\,|\phi_y(r)|^4$$

$$x^2+y^2 \quad L_z=0: \frac{1}{\sqrt{2}}(p_x^+p_x^+ + p_y^+p_y^+)|0\rangle$$



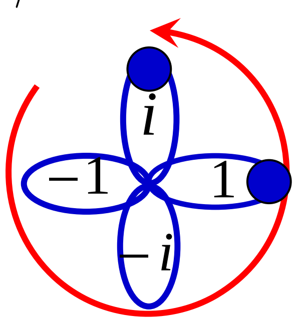
polar

$$E_0=\tfrac{4}{3}U$$

$$(x\pm iy)^2 \quad L_z=\pm 2:\ \{\frac{1}{\sqrt{2}}(p_x^+ \pm i p_y^+)\}^2\,|0\rangle$$

$$=\{\frac{1}{2}(p_x^+p_x^+ - p_y^+p_y^+)\pm\frac{i}{\sqrt{2}}p_x^+p_y^+\}|0\rangle$$

axial



$$E_{\pm 2}=\tfrac{2}{3}U$$

Axial states are spatially more extended!

$$E_{\pm 2}<E_0$$

Orbital Hund's rule for bosons

$$H_{\text{int}} = \frac{U}{2}\sum_r \{n_r^2 - \frac{1}{3}(L_r^z)^2\}$$

$$n = p_x^+ p_x + p_y^+ p_y, L_z = -i(p_x^+ p_y - p_y^+ p_x)$$

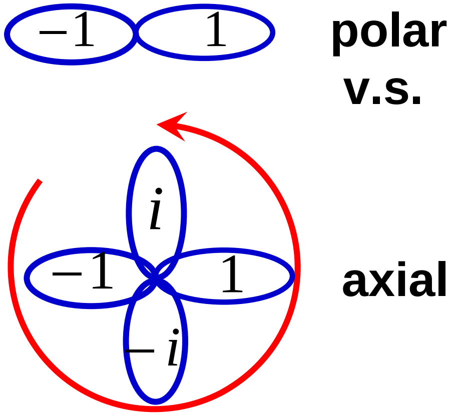
- If more than two bosons occupy in the same site, they prefer to aggregate into the same single-particle state.

- The same axial state (e.g. p+ip) instead of the polar state (e.g. p_x) to minimize repulsion and maximize **orbital angular momentum** simultaneously.

- *cf*. Second Hund's rule for electrons.

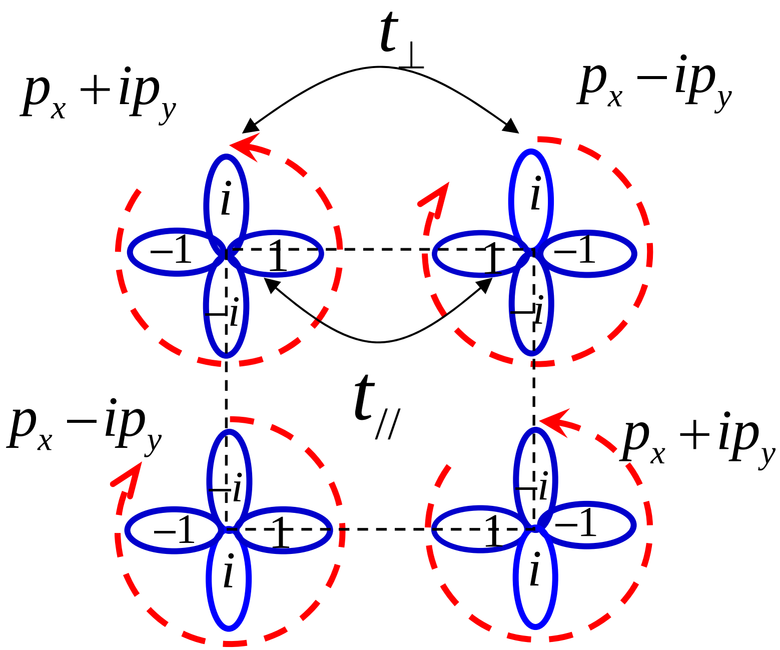
- c.f. p+ip superconductors.

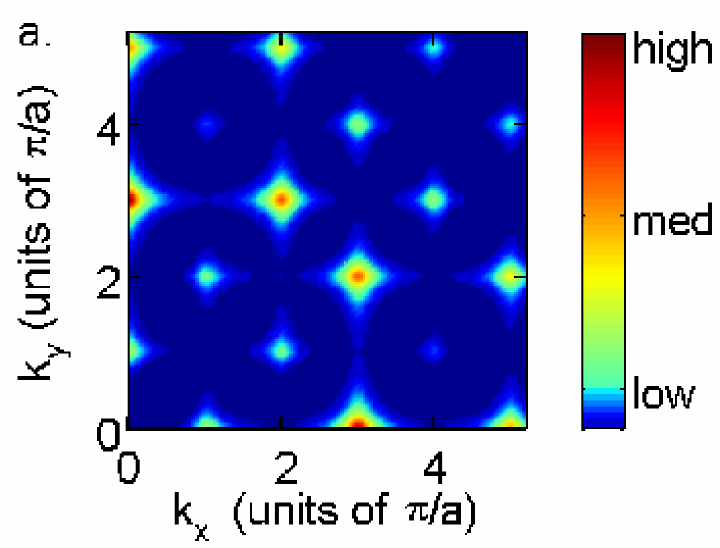
- New material: we are filling the shell structure of each site with bosons. Orbital physics with bosons!



"Complex" superfluidity with time-reversal symmetry breaking

- Inter-site tunneling orders the onsite orbital angular momentum (OAM) moments. Staggered ordering of OAM in the square lattice.





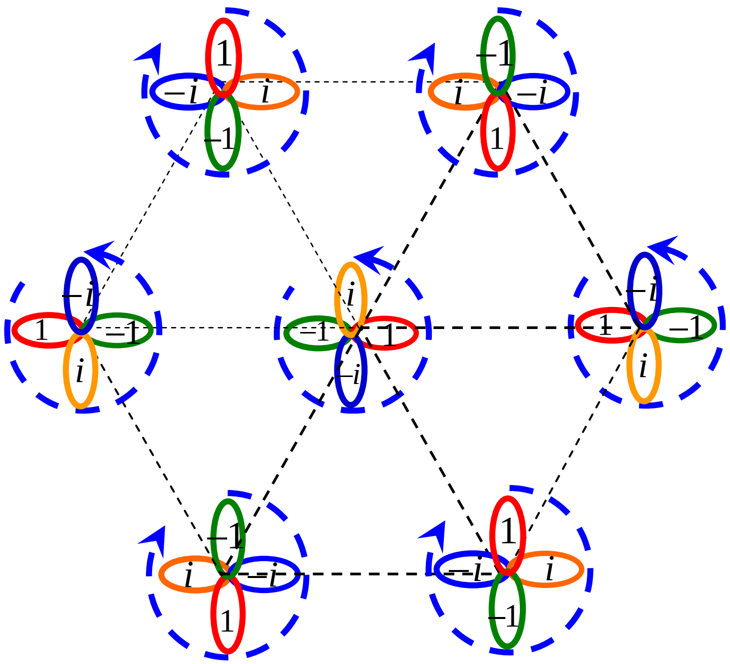
- Time of flight (zero temperature): Bragg peaks located at fractional values of reciprocal lattice vectors.

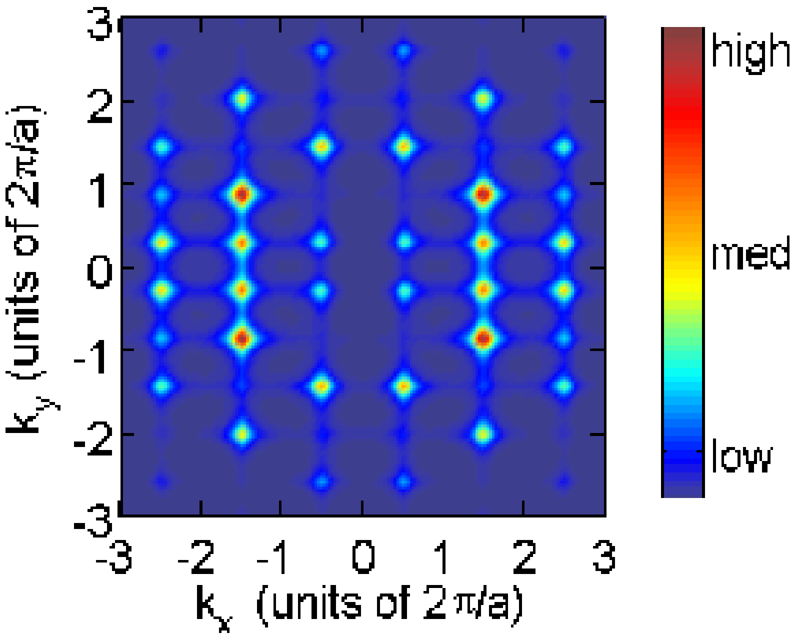
$$((m+\tfrac{1}{2})\frac{\pi}{a},0) \quad (0,(n+\tfrac{1}{2})\frac{\pi}{a})$$

W. V. Liu and C. Wu, PRA 74, 13607 (2006).

"Complex" superfluidity with time-reversal symmetry breaking

• Stripe ordering of orbital angular momentum moment in the triangular lattice.

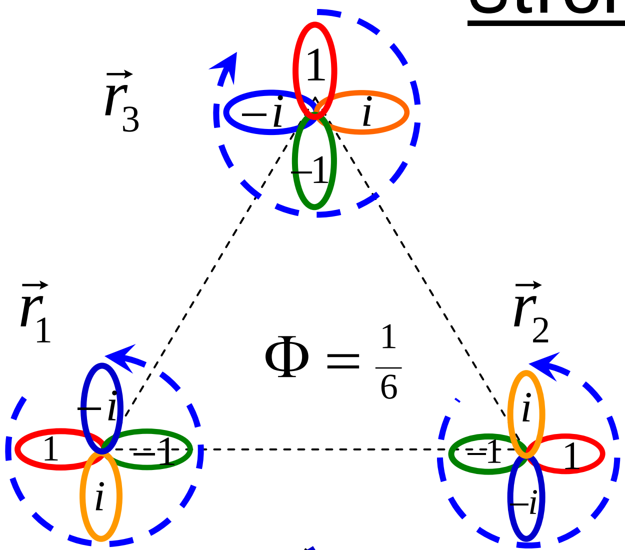


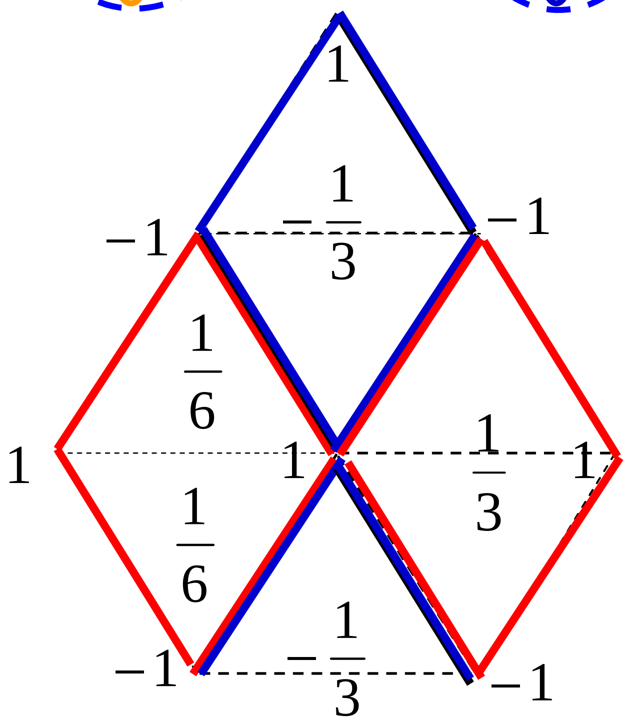


• Each site behaves like a vortex with long range interaction in the superfluid state. Stripe ordering to minimize the global vorticity.

C. Wu, W. V. Liu, J. Moore and S. Das Sarma, Phy. Rev. Lett. 97, 190406 (2006).

Strong coupling analysis





- Ising variable for vortex vorticity:

$$\sigma = \pm 1$$

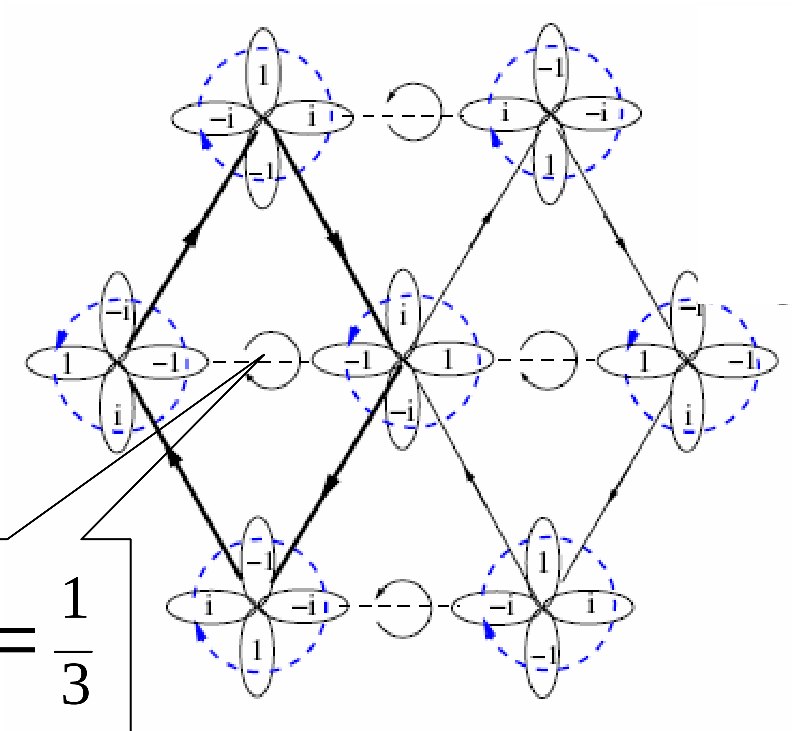
- The minimum of the effective flux per plaquette is $\pm 1/6$.

$$\Phi_i = \frac{1}{2\pi} \sum_{\langle r,r' \rangle} A_{r,r'} = \frac{1}{6}(\sigma_{r1} + \sigma_{r2} + \sigma_{r3})$$

- The stripe pattern minimizes the ground state vorticity.

- cf. The same analysis also applies to p+ip Josephson junction array.

Staggered plaquette orbital moment



$$j = n_0 t \sin \Delta\theta$$

$$\Delta\theta = \frac{\pi}{6}$$

$$\frac{\Phi}{2\pi} = \frac{1}{3}$$

cf. Current loop states in high Tc cuprate:
C. M. Varma, Phys. Rev. B **55**, 14554 (1997).
S. Chakravarty, R. B. Laughlin, Dirk K. Morr, and Chetan Nayak, Phys. Rev. B **63**, 094503 (2001)

Solution 2: Unconventional BEC with spin-orbit (SO) coupling

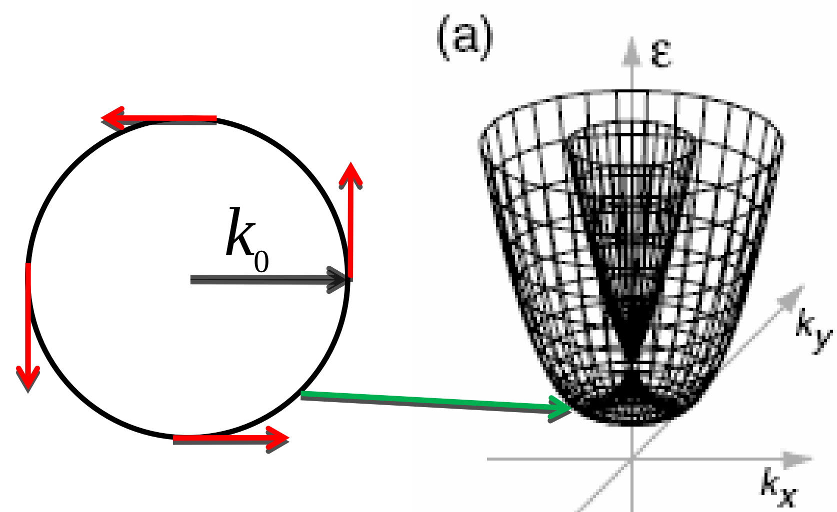
- Atoms are too heavy to exist SO coupling in the center of mass motion. Nevertheless, an effective SO coupling can be generated by the atom and light interaction.

Stenescu et al PRL 99, 110403 (2007); Ruseckas et al PRL 95, 10404 (2005).

- Rashba SO coupling for bosons. Spin $|\uparrow\rangle$, $|\downarrow\rangle$ refer to two internal states. The single particle ground states are degenerate along a ring.

$$H_k = \frac{\hbar^2 k^2}{2M} + \hbar\lambda(k_x\sigma_y - k_y\sigma_x)$$

helicity instead of spin is the good quantum number



$$|\psi(\vec{k})\rangle_+ = \frac{1}{\sqrt{2}}\left(e^{i\frac{\varphi_k}{2}}|\uparrow\rangle + ie^{-i\frac{\varphi_k}{2}}|\downarrow\rangle\right)$$

- Where do bosons condense into?

Use a harmonic trap to lift the degeneracy

- The trap length scale and SO length scale.

$$H=\int d^2\vec{r}\,\psi_\alpha^+\{-\frac{\hbar^2\nabla^2}{2M}-\mu+V_{tr}(\vec{r})\}\psi_\alpha+\hbar\lambda\,\psi_\alpha^+(-i\nabla_y\sigma_x+i\nabla_x\sigma_y)\psi_\beta+\frac{g}{2}\psi_\alpha^+\psi_\beta^+\psi_\beta\psi_\alpha$$

$$V_{trap}(r)=\tfrac{1}{2}M\omega^2r^2 \qquad l_{trap}=\sqrt{\hbar/(M\omega)} \qquad l_{SO}=1/k_0=\hbar/(M\lambda)$$

- The single particle ground state is a Kramer doublet.

$$\begin{matrix} & L_z & s_z \\ |\psi_{1/2}\rangle=\begin{pmatrix} f(r) \\ g(r)e^{i\phi}\end{pmatrix} & \begin{matrix}0\\1\end{matrix} & \begin{matrix}\frac{1}{2}\\ \frac{-1}{2}\end{matrix}\end{matrix} \qquad |\psi_{-1/2}\rangle=\begin{pmatrix} -g(r)e^{i\phi} \\ f(r)\end{pmatrix} \qquad j_z=L_z+s_z=\pm\tfrac{1}{2}$$

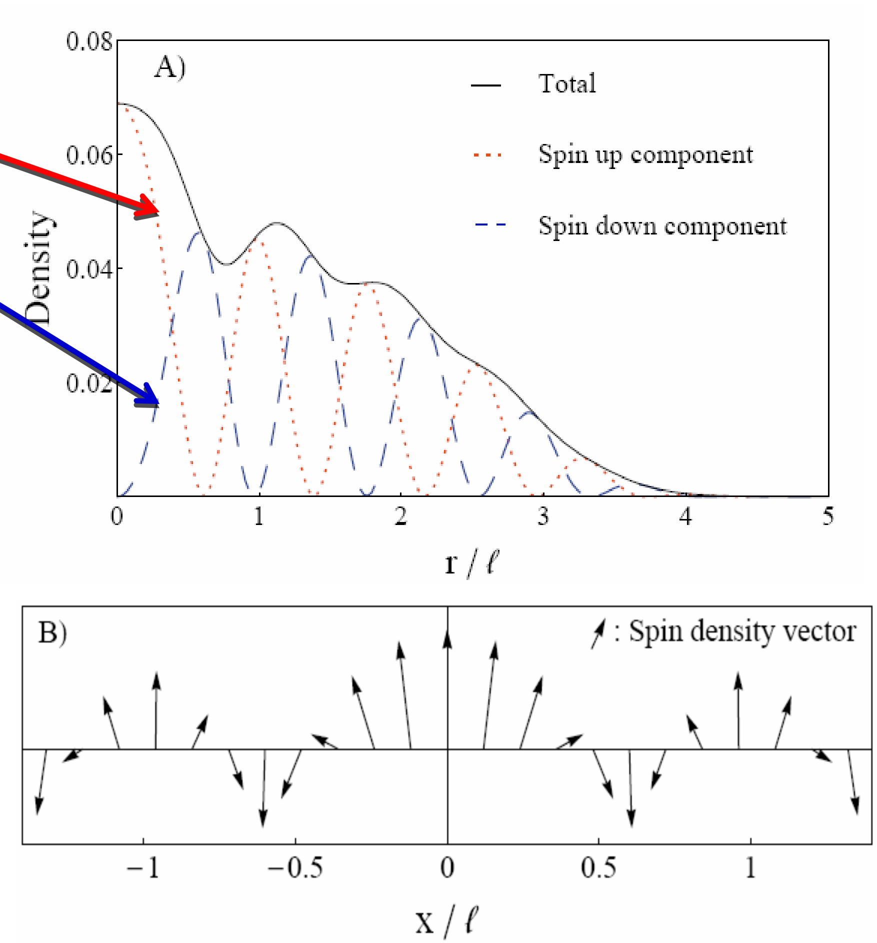
$$l_{SO}<<l_{trap} \qquad V_{trap}\approx\tfrac{1}{2}\hbar\omega\left(\frac{l_{so}}{l_{trap}}\right)^2(i\partial_{\varphi_k}-A_k)^2 \longrightarrow |\psi_{1/2}\rangle\approx\int d\varphi_k e^{i\frac{\varphi_k}{2}}|\psi_k\rangle_+$$

Half-quantum vortex as the skyrmion spin texture

• With many bosons, they condense into one of the doublet and spontaneously break TR symmetry.

$$|\psi_{1/2}\rangle = \begin{pmatrix} f(r) \\ g(r)e^{i\phi} \end{pmatrix}, \qquad |f(r)|^2 \qquad |g(r)|^2$$

a half quantum vortex



• f(r) and g(r) exhibit an oscillation roughly at periodicity l_{SO} , but with a relative phase shift of π/2.

$$S_z = \tfrac{1}{2}(|f(r)|^2 - |g(r)|^2)$$

$$S_x = \tfrac{1}{2} f(r)g(r)\cos\phi \quad S_y = \tfrac{1}{2} f(r)g(r)\sin\phi$$

P-orbital fermions: $p_{x,y}$-orbital counterpart of graphene

- Band flatness and strong correlation effect.

  (e.g. Wigner crystal, and flat band ferromagnetism.)

- P-orbital Mott insulators: orbital exchange; from Kitaev to quantum 120 degree model.

- Novel pairing state (attractive interaction): f-wave Cooper pairing.

- **Topological band insulators: quantum anomalous Hall effect by orbital angular momentum polarization.**

C. Wu, PRL 101, 168807 (2008).

P-orbital fermions: $p_{x,y}$-orbital counterpart of graphene

- **Strong correlation effect due to band flatness, e.g. Wigner crystal and ferromagnetism.**

C. Wu, and S. Das Sarma, PRB 77, 235107(2008);
C. Wu et al, PRL 99, 67004(2007).
Shi-zhong zhang, Hsiang-hsuan Hung, and C. Wu, arXiv:0805.3031.

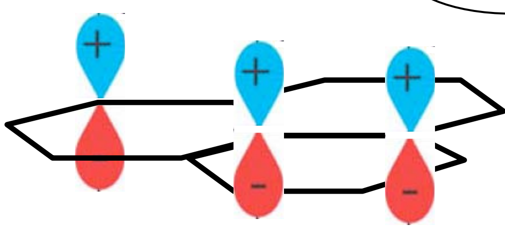
- A simple mechanism for unconventional pairing: the f-wave.

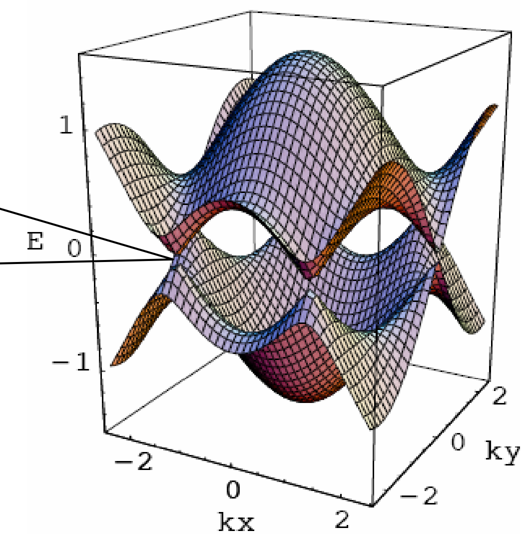
- p-orbital Mott insulators: frustrated orbital super-exchange; from Kitaev to quantum 120 degree model.

- Topological insulator: orbital analogy of quantum anomalous quantum Hall effect.

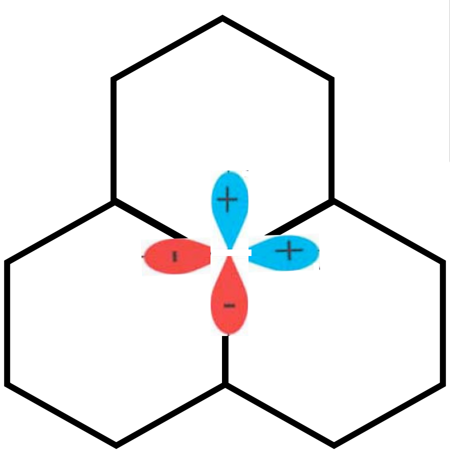
p-orbital fermions in honeycomb lattices

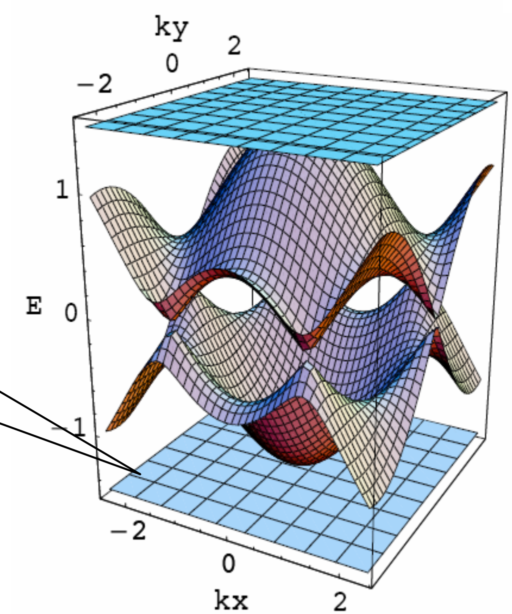
cf. graphene: a surge of research interest; p_z-orbital; Dirac cones.





p_x;p_y-orbitals: flat bands; interaction effects dominate.



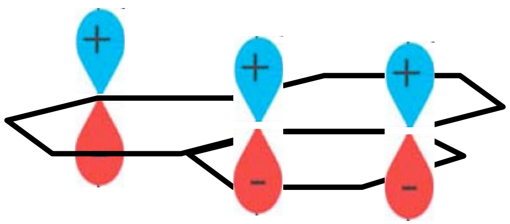


C. Wu, D. Bergman, L. Balents, and S. Das Sarma, PRL 99, 70401 (2007).

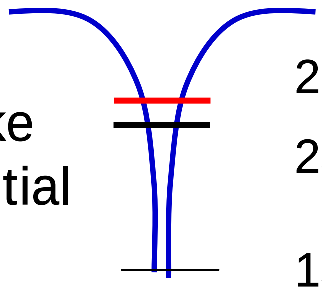
p_x, p_y orbital physics: why optical lattices?

- p_z-orbital band is not a good system for orbital physics.

isotropic within 2D; non-degenerate.

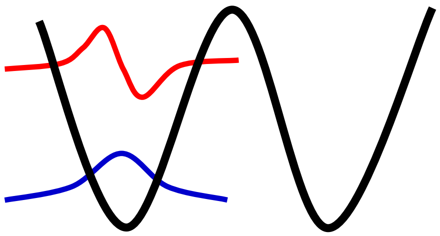


- Interesting orbital physics in the p_x, p_y-orbital bands.

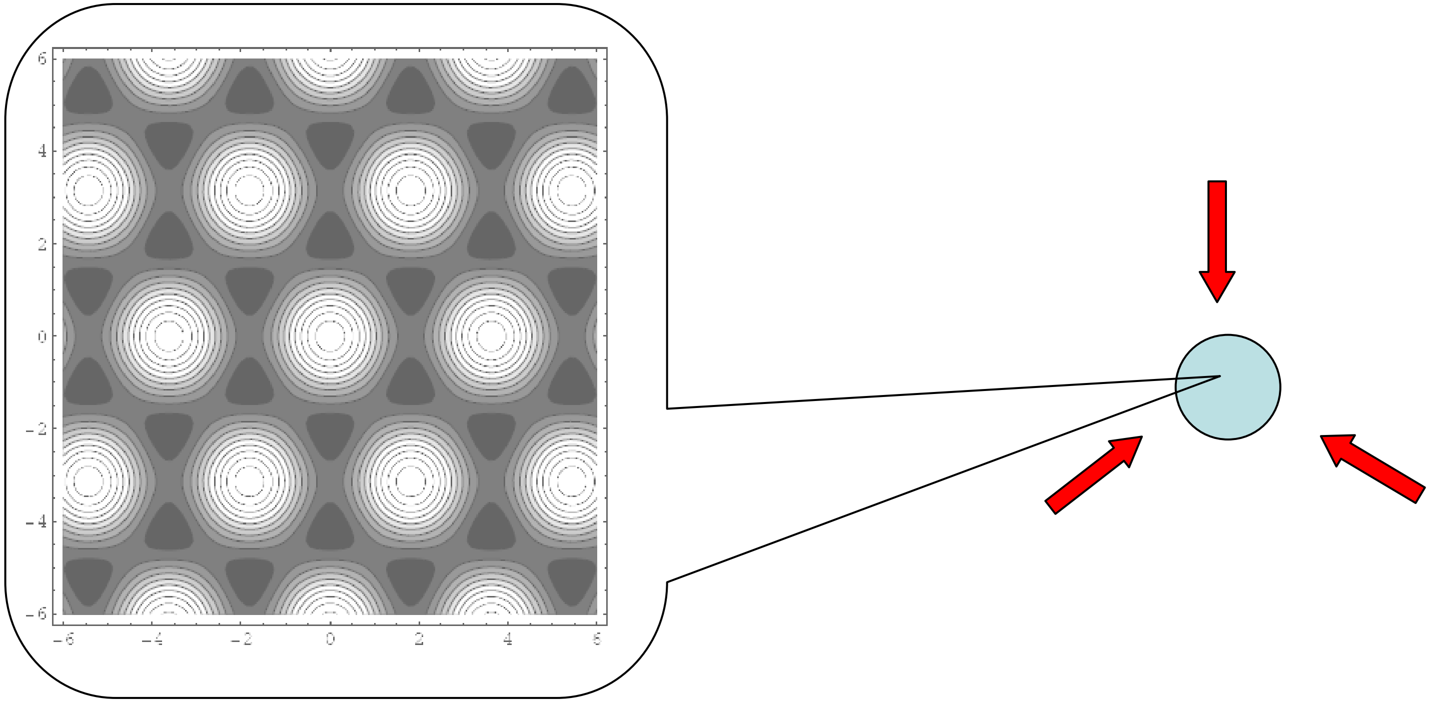


- However, in graphene, 2p_x and 2p_y are close to 2*s*, thus strong hybridization occurs.

- In optical lattices, p_x and p_y-orbital bands are well separated from *s*.



Honeycomb optical lattice with phase stability



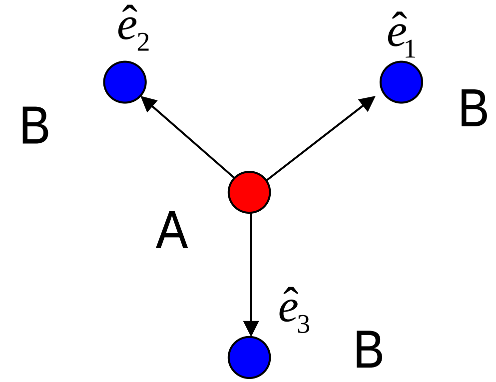
- Three coherent laser beams polarizing in the z-direction.

- Phase stability: phase drift of laser beams only causes an overall translation of the lattice but not the internal distortion.

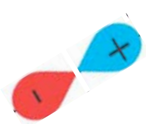
G. Grynberg et al., Phys. Rev. Lett. **70**, 2249 (1993).

Artificial graphene in optical lattices

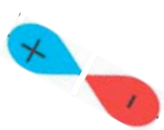
- Band Hamiltonian (σ-bonding) for spin-polarized fermions.



$$H_t = t_{//}\Big\{ \sum_{\vec{r}\in A} [p_1^+(\vec{r})p_1(\vec{r}+\hat{e}_1)+h.c.]$$
$$+[p_2^+(\vec{r})p_1(\vec{r}+\hat{e}_2)+h.c]$$
$$+[p_3^+(\vec{r})p_3(\vec{r}+\hat{e}_3)+h.c] \Big\}$$

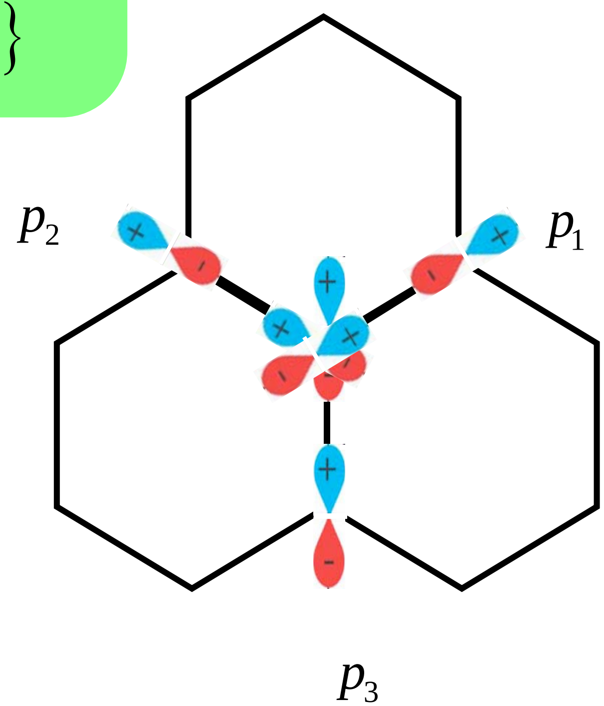


$$p_1 = \frac{\sqrt{3}}{2}p_x + \frac{1}{2}p_y$$

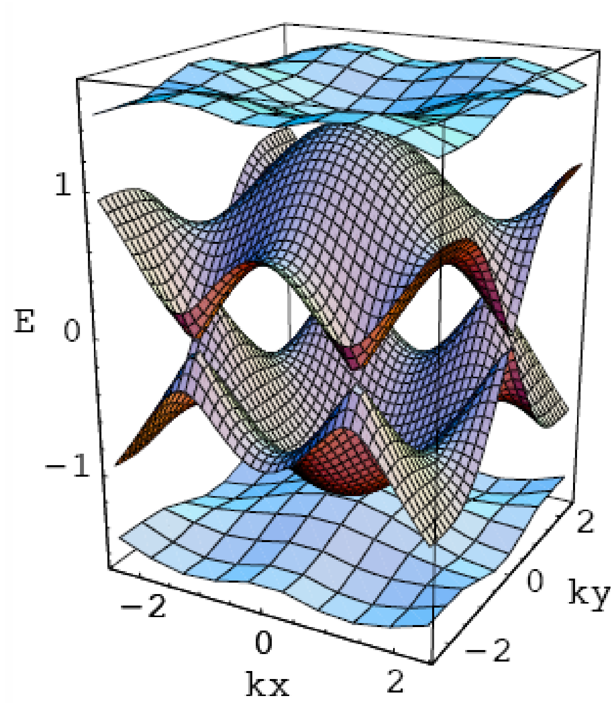


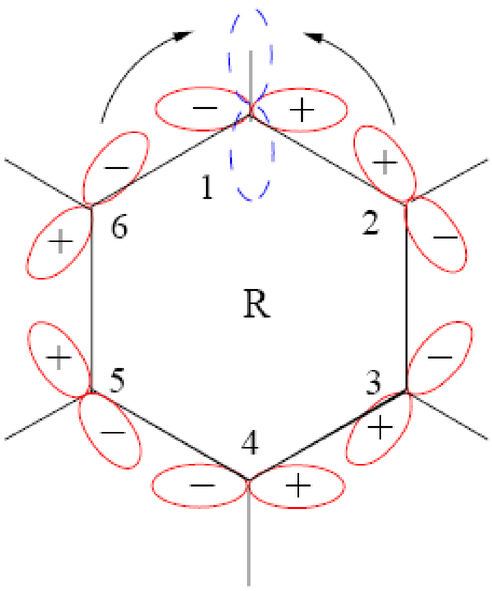
$$p_2 = -\frac{\sqrt{3}}{2}p_x + \frac{1}{2}p_y$$

$$p_3 = -p_y$$



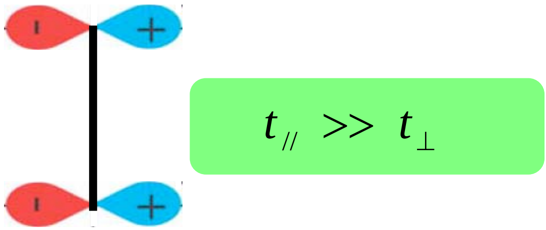
Flat bands in the entire Brillouin zone!





- Flat band + Dirac cone.
- localized eigenstates.

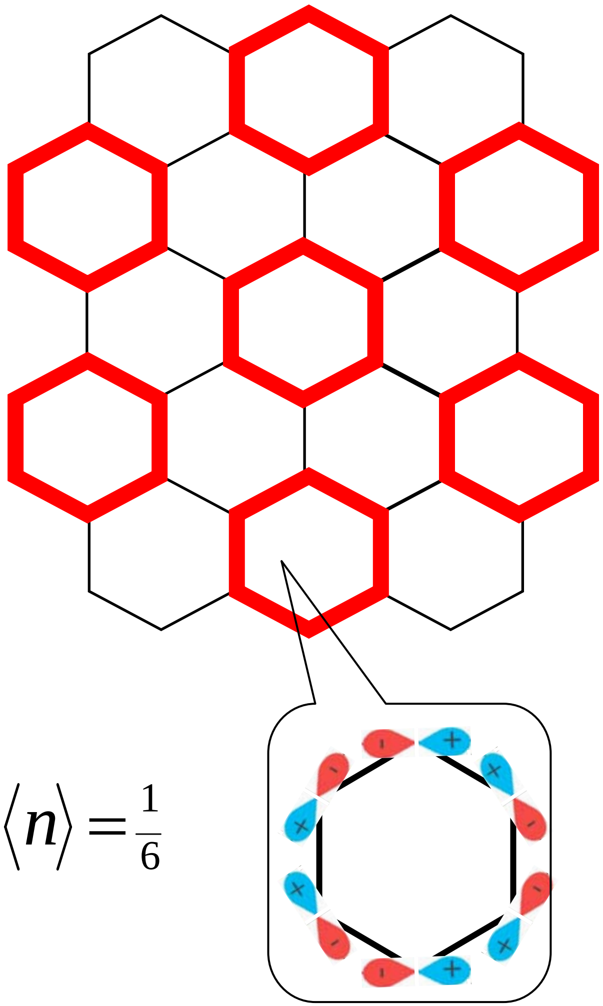
- If π-bonding is included, the flat bands acquire small width at the order of $t_{\perp}$. Realistic band structures show $t_{\perp} / t_{//} \rightarrow 1\%$



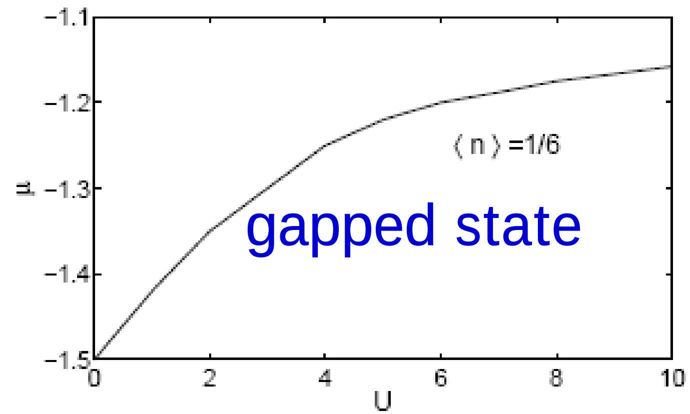
π-bond

Hubbard model for spinless fermions: Exact solution: Wigner crystallization

$$H_{\text{int}} = U \sum_{\vec{r} \in A,B} n_{p_x}(\vec{r}) n_{p_y}(\vec{r})$$



$$\langle n \rangle = \frac{1}{6}$$

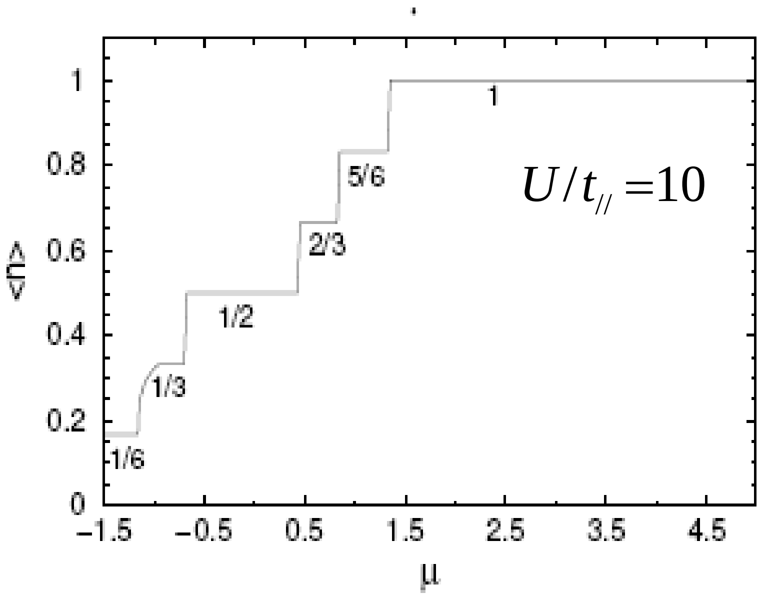


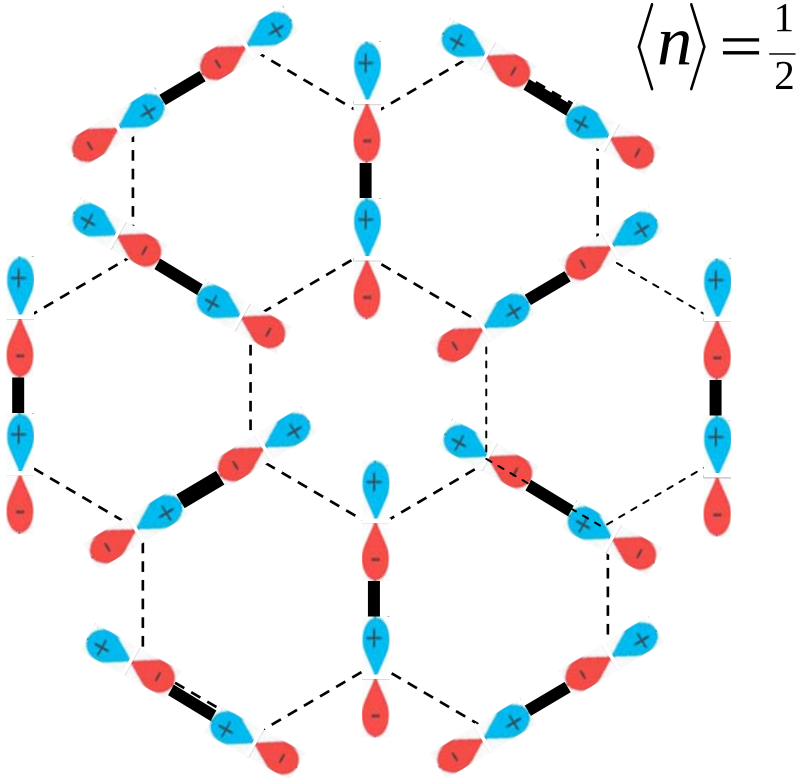
- Close-packed hexagons; avoiding repulsion.

- The crystalline ordered state is stable even with small $t_{\perp}$.

- Particle statistics is irrelevant. The result is also good for bosons.

Orbital ordering with strong repulsions





- Various orbital ordering insulating states at commensurate fillings.

- Dimerization at <n>=1/2! Each dimer is an entangled state of empty and occupied states.

Spinful Fermions: itinerant FM of cold atoms

• Ferromagnetism (FM) requires strong repulsive interactions, and thus has no well-defined weak coupling picture.

• It is difficult to achieve FM state in Hubbard type modes except with the flat band and Nagaoka limit.

A. Mielke and H. Tasaki, Comm. Math. Phys 158, 341 (1993).

• Very recently, evidence of FM with cold atoms has appeared through Feshbach resonance from the positive scattering length side.

G. B. Jo, et al., arXiv 0907.2888.

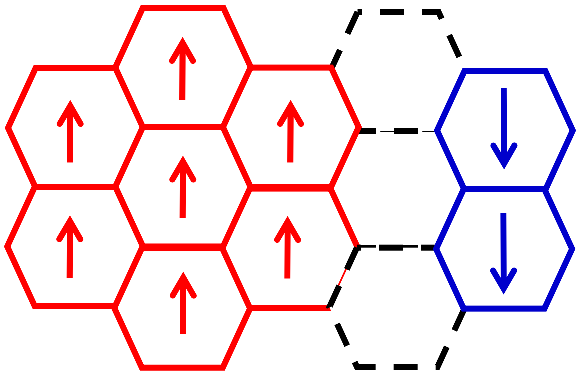
• Flat-band FM in the p-orbital honeycomb lattices.

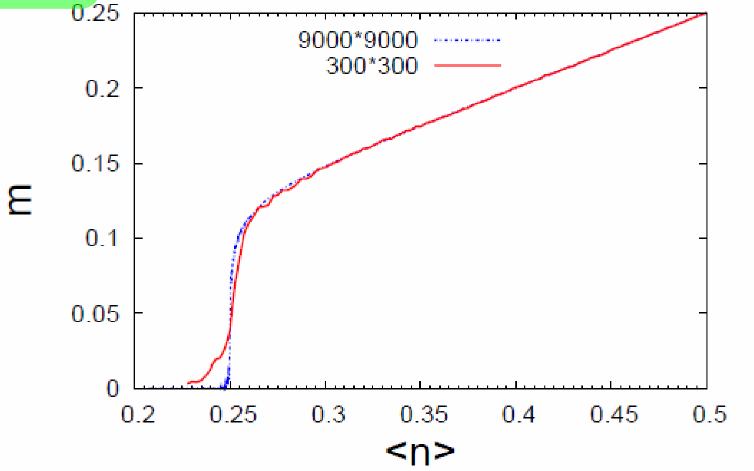
• Interaction amplified by the divergence of DOS. **FM with weak repulsive interactions in cold atom systems**. A route to spin-dependent transport with cold atoms.

Shizhong Zhang, Hsiang-hsuan Hung, and C. Wu, arXiv:0805.3031.

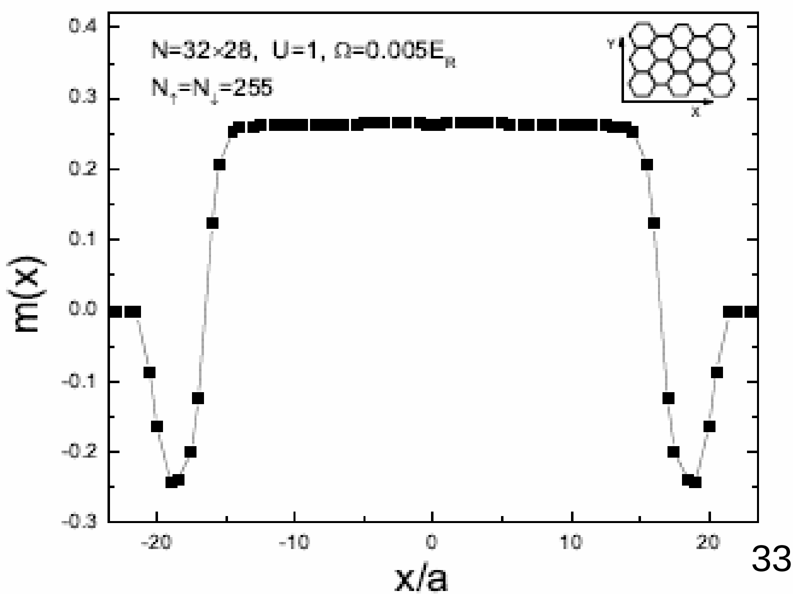
Flat-band itinerant FM in p-orbitals

- Percolation picture for flat band FM.





$$H_{\text{int}} = U\sum_{\vec{r}}(n_{p_{x,\uparrow}}(\vec{r})n_{p_x,\downarrow}(\vec{r}) + n_{p_y,\uparrow}(\vec{r})n_{p_y,\downarrow}(\vec{r}))$$
$$- J\sum_{r}(\vec{S}_{p_x}(\vec{r})\cdot\vec{S}_{p_y}(\vec{r}) - \frac{1}{4}n_{p_x}(\vec{r})n_{p_y}(\vec{r}))$$
$$+ \Delta\sum_{r}(p^{+}_{x,\uparrow}(\vec{r})p^{+}_{x,\downarrow}(\vec{r})p_{y,\downarrow}(\vec{r})p_{y,\downarrow}(\vec{r}) + h.c.)$$



- Self-consistent calculation for the FM phase separation with a soft harmonic trap.

P-orbital fermions: $p_{x,y}$-orbital counterpart of graphene

- Strong correlation effect from band flatness.

  (e.g. Wigner crystal, and flat band ferromagnetism.)

- **A simple mechanism: f-wave Cooper pairing.**

  W. C. Lee, C. Wu, S. Das Sarma, arXiv:0905.1146.

- P-orbital Mott insulators: orbital exchange; from Kitaev to quantum 120 degree model.

- Topological insulator: orbital analogy of quantum anomalous quantum Hall effect.

Simple mechanism for unconventional Cooper pairing

• Most of unconventional pairing states arise from strong correlation effects. Predictions and analysis are difficult.

p-wave: superfluid ^{3}He-A and B; Sr_2RuO_4;

d-wave: high T_c cuprates; some heavy fermion compounds(?),

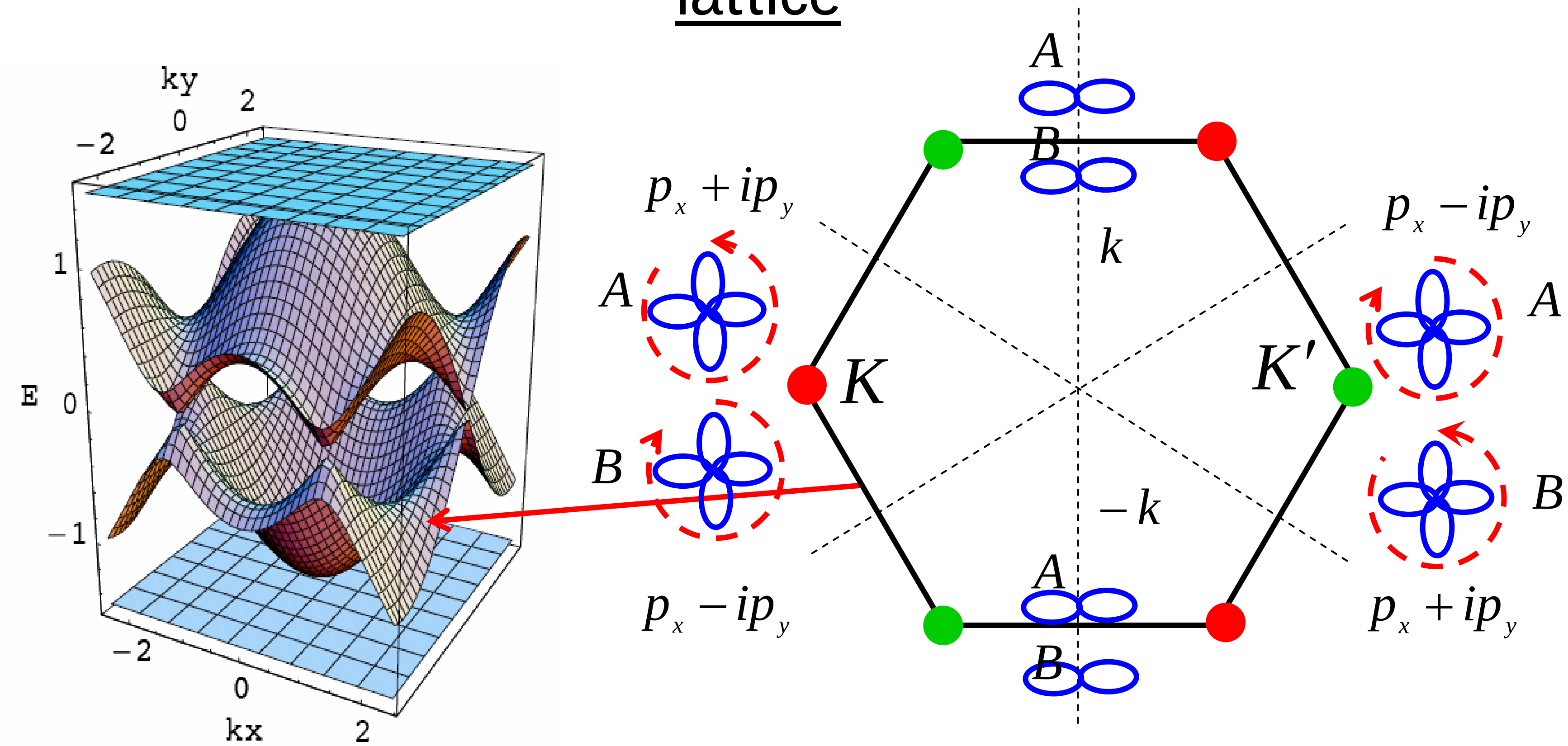
Extended s-wave: Iron-based superconductors (?);

Possible f-wave: UPt_3(?)

• Can we arrive at **unconventional** pairing in a simpler way, say, from conventional interactions but driven by **nontrivial band structures**?

• No strong correlation effect. Analysis is controllable.

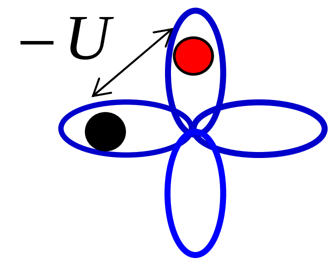
Nontrivial orbital hybridization: p-orbital hexagonal lattice



- Along the three middle lines of Brillouin zone, eigen-orbitals are real.
- At K and K', eigen-orbitals are complex and orthognoal.

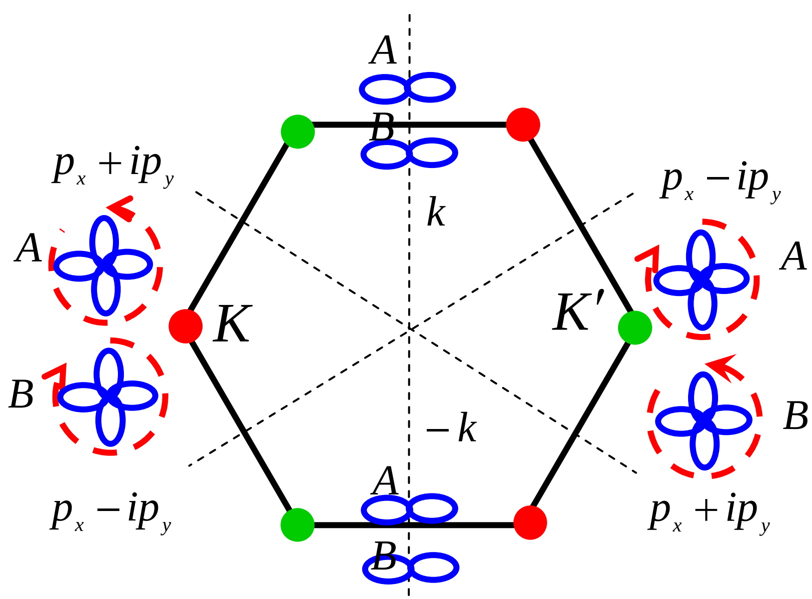
Onsite attraction for SPINLESS *p*-orbital fermions

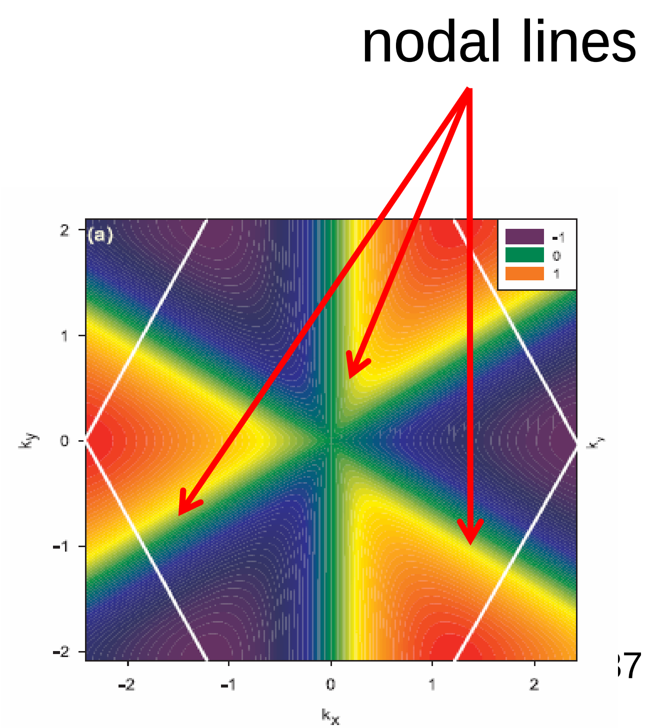
$$H_{\text{int}} = -U \sum_{\vec{r}} n_{p_x}(\vec{r}) n_{p_y}(\vec{r})$$
$$= -U \sum_{\vec{r}} n_{p_x+ip_y}(\vec{r}) n_{p_x-ip_y}(\vec{r})$$



- F-wave structure intra-band pairing. Pairing strength vanishes along three middle lines, and goes strongest at K and K'.

$$F(k) \propto \sin\tfrac{\sqrt{3}}{2}k_x(\cos\tfrac{\sqrt{3}}{2}k_x - \cos\tfrac{3}{2}k_y)$$

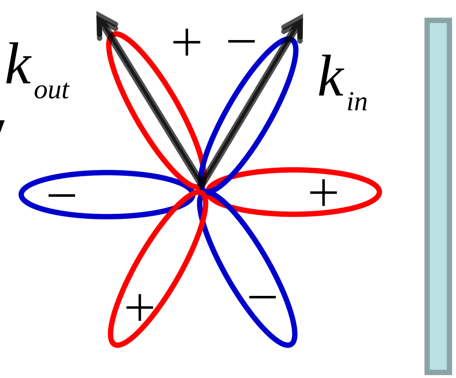


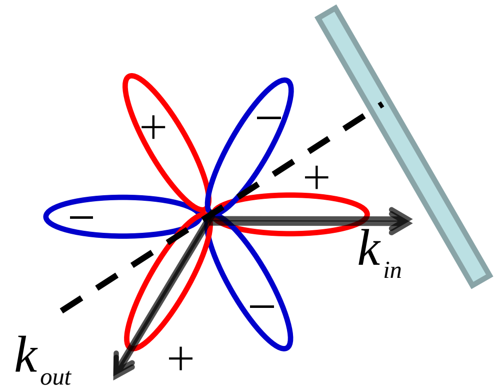


Phase sensitive detection: zero energy Andreev bound states

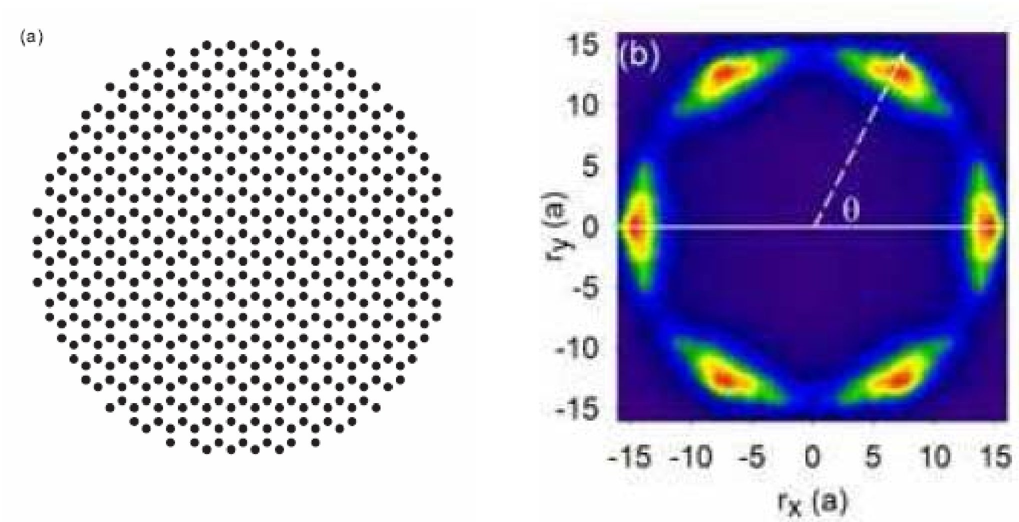
- If the boundary is perpendicular to the anti-nodal (nodal) direction, the zero energy Andreev bound states appear (vanish).

With Andreev Bound States





No Andreev Bound States



P-orbital fermions: $p_{x,y}$-orbital counterpart of graphene

- Strong correlation effect from band flatness.

  (e.g. Wigner crystal, and flat band ferromagnetism.)

- A simple mechanism: f-wave Cooper pairing.

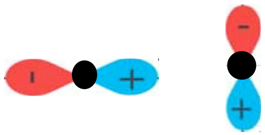
- **P-orbital Mott insulators: orbital exchange; from Kitaev to quantum 120 degree model; a new type of frustrated magnet-like mode.**

C. Wu, PRL 100, 200406 (2008).

- Topological insulator: orbital analogy of quantum anomalous quantum Hall effect.

Mott-insulators with orbital degrees of freedom: orbital exchange of **spinless** fermion

- Pseudo-spin representation.

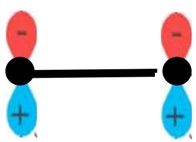


$$\tau_1 = \tfrac{1}{2}(p_x^+ p_x - p_y^+ p_y) \quad \tau_2 = \tfrac{1}{2}(p_x^+ p_y + p_y^+ p_x) \quad \tau_3 = \tfrac{i}{2}(p_x^+ p_y - p_y^+ p_x)$$

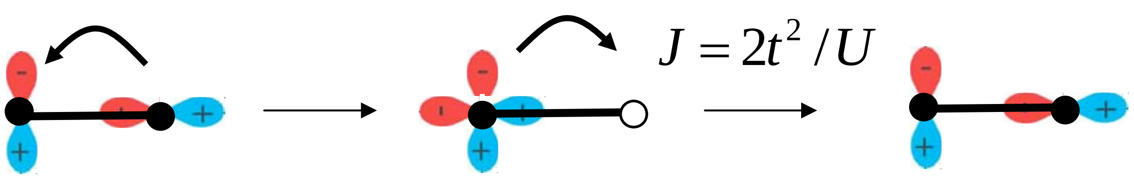
- No orbital-flip process. Antiferro-orbital Ising exchange.

$J = 0$

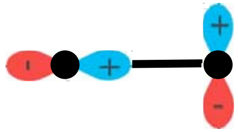
$$H_{ex} = J\tau_1(r)\tau_1(r+\hat{x})$$



$J = 0$



$J = 2t^2/U$

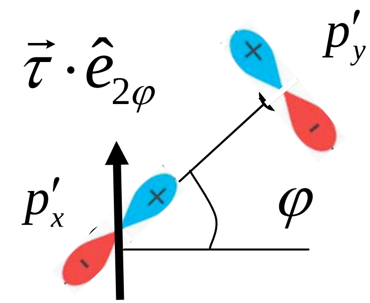


$J = 2t^2/U$

Hexagon lattice: quantum 120° model

• Non-triviality: Ising quantization axis depends on bond orientation. For a bond along the general direction $\hat{e}_\varphi$,

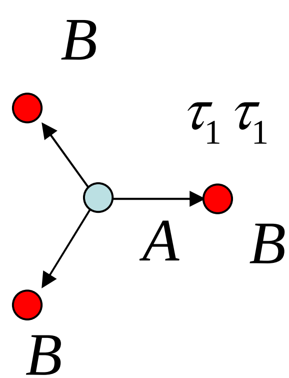
p'_x, p'_y : eigen-states of $\vec{\tau}\cdot\hat{e}_{2\varphi} = \cos 2\varphi\,\tau_x + \sin 2\varphi\,\tau_y$



$$H_{ex} = J(\vec{\tau}(r)\cdot\hat{e}_{2\varphi})(\vec{\tau}(r+\hat{e}_\varphi)\cdot\hat{e}_{2\varphi})$$

• After a suitable transformation, the Ising quantization axes can be chosen just as the three bond orientations.

$$(-\tfrac{1}{2}\tau_1+\tfrac{\sqrt{3}}{2}\tau_2)(-\tfrac{1}{2}\tau_1+\tfrac{\sqrt{3}}{2}\tau_2)$$



$$(-\tfrac{1}{2}\tau_1-\tfrac{\sqrt{3}}{2}\tau_2)(-\tfrac{1}{2}\tau_1-\tfrac{\sqrt{3}}{2}\tau_2)$$

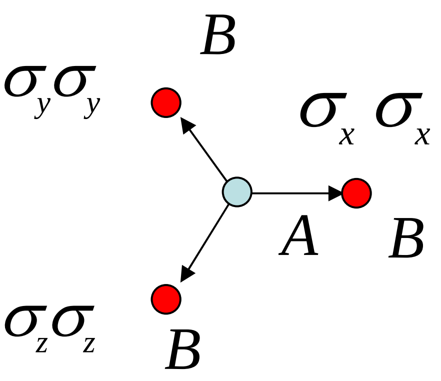
$$H_{ex} = -\sum_{r,r'} J(\vec{\tau}(r_i)\cdot\hat{e}_{ij})\,(\vec{\tau}(r'_j)\cdot\hat{e}_{ij})$$

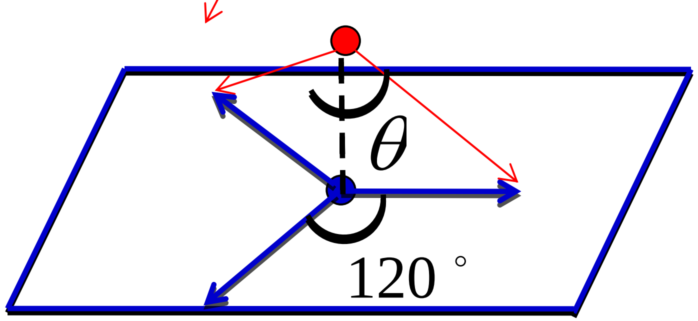
C. Wu et al, arxiv0701711v1; C. Wu, PRL 100, 200406 (2008). E. Zhao, and W. V. Liu, Phys. Rev. Lett. 100, 160403 (2008)

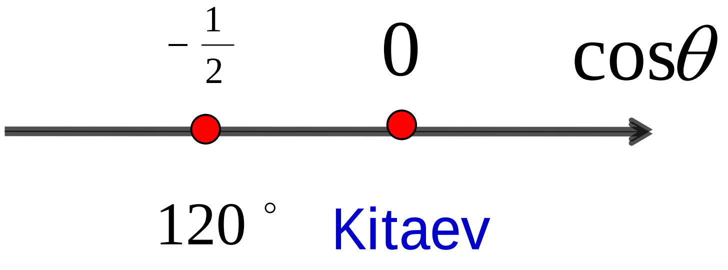
From the Kitaev model to 120 degree model

- cf. Kitaev model: Ising quantization axes form an orthogonal triad.

$$H_{kitaev} = -J\sum_{r\in A}(\sigma_x(r)\sigma_x(r+e_1)+\sigma_y(r)\sigma_y(r+e_2)+\sigma_z(r)\sigma_z(r+e_3))$$



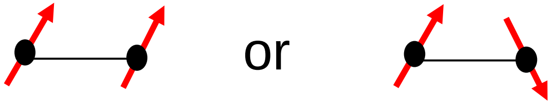




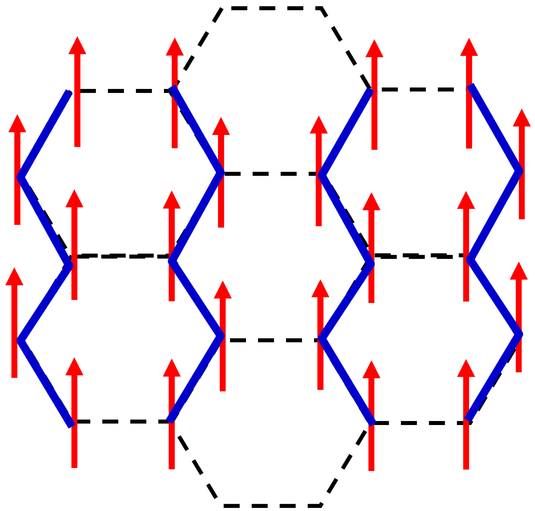
Large S picture: heavy-degeneracy (frustration) of classic ground states

- Ground state constraint: the two τ-vectors have the same projection along the bond orientation.

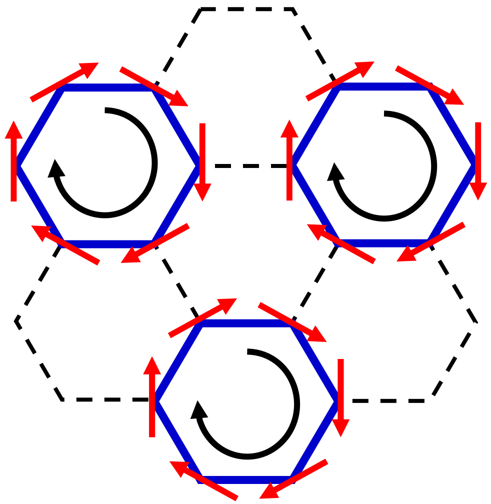
$$H_{ex} = \sum_{r,r'} J\{[(\vec{\tau}(r) - \vec{\tau}(r')] \cdot \hat{e}_{rr'}\}^2 + J \sum_{r} \tau_z^2(r)$$



- Ferro-orbital configurations.

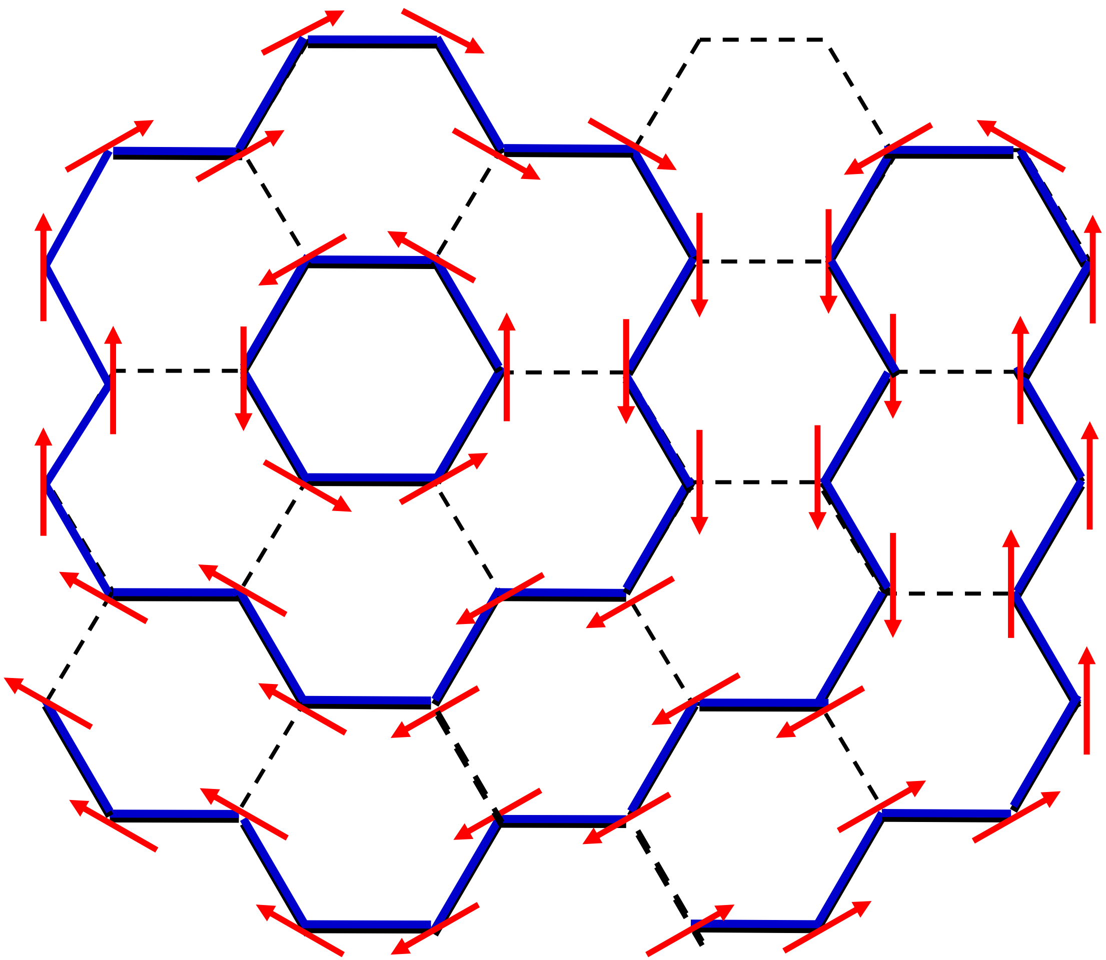


- Oriented loop config: τ-vectors along the tangential directions.



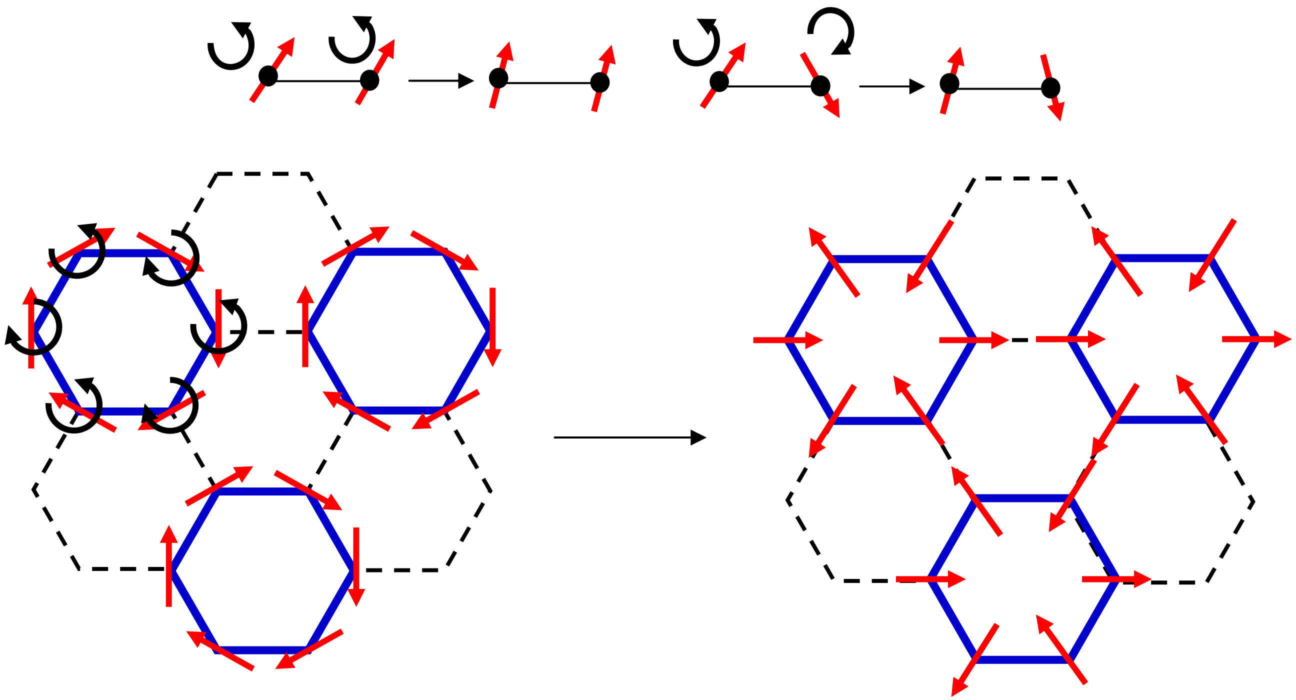
Heavy-degeneracy of classic ground states

- General loop configurations



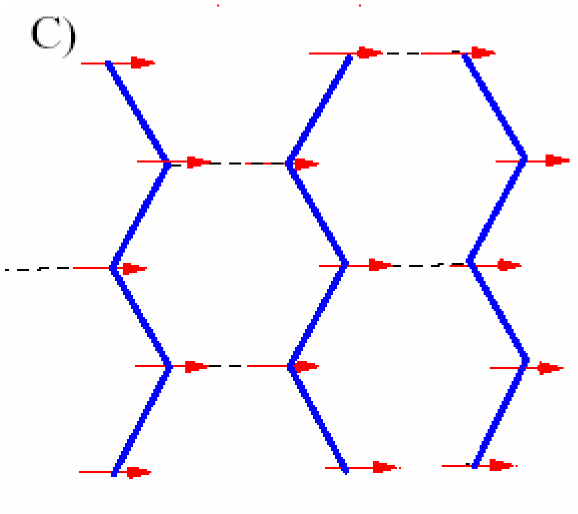
Global rotation degree of freedom

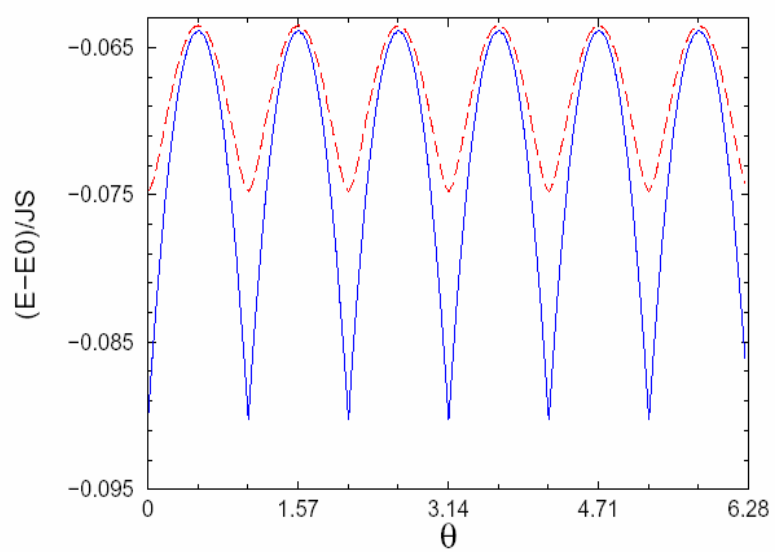
• Each loop config remains in the ground state manifold by a suitable arrangement of clockwise/anticlockwise rotation patterns.

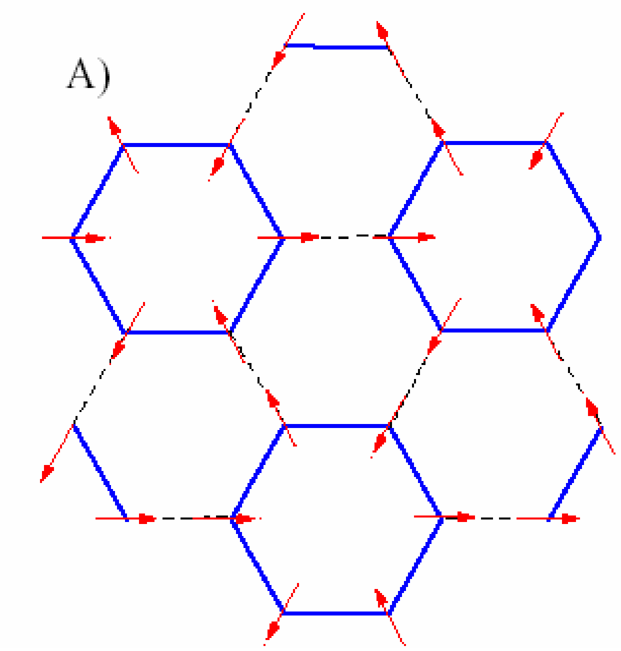


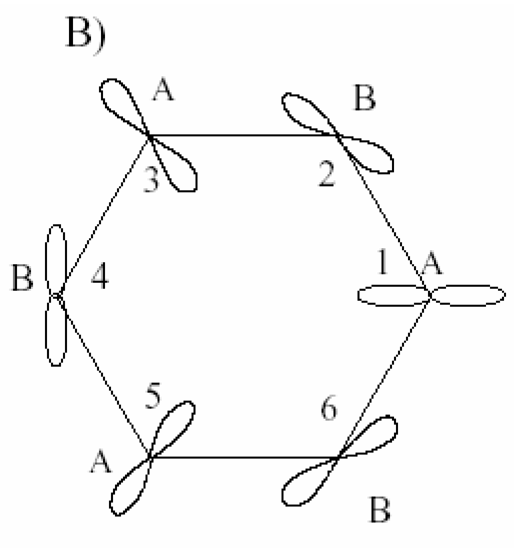
• Starting from an oriented loop config with fixed loop locations but an arbitrary chirality distribution, we arrive at the same unoriented loop config by performing rotations with angles of $\pm 30^\circ, \pm 90^\circ, \pm 150^\circ$.

“Order from disorder”: 1/S orbital-wave correction





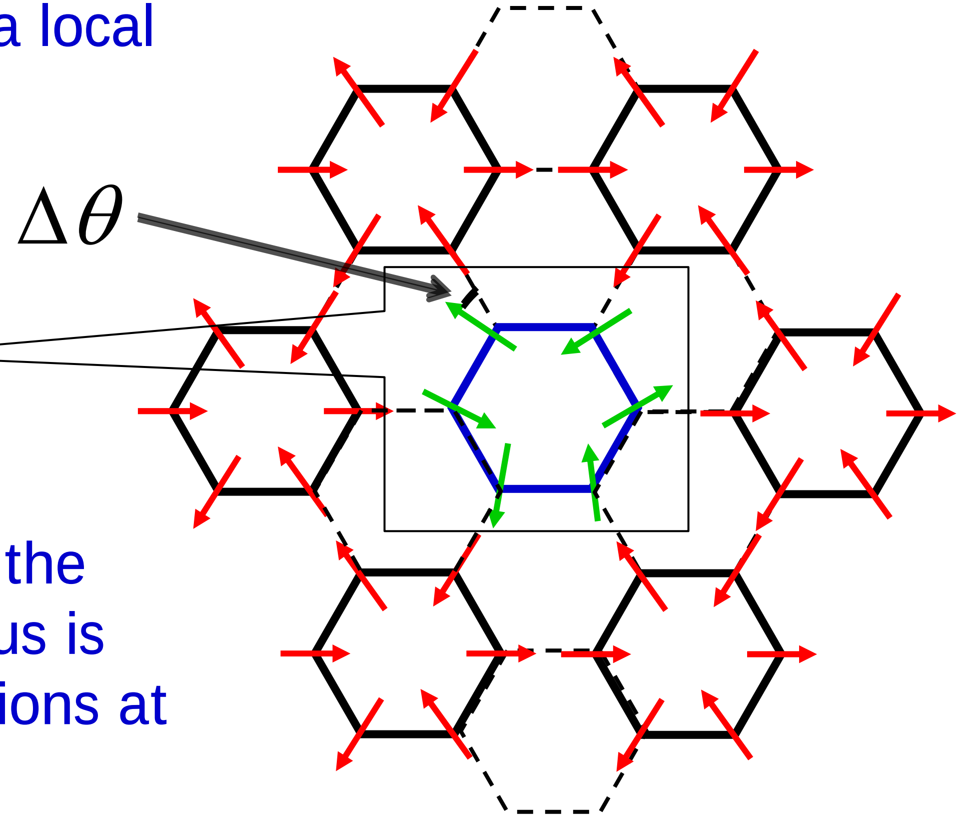




Zero energy flat band orbital fluctuations

- Each un-oriented loop has a local zero energy model up to the quadratic level.

$$\Delta E = 6JS^2(\Delta\theta)^4$$



- The above config. contains the maximal number of loops, thus is selected by quantum fluctuations at the 1/S level.

- Project under investigation: the quantum limit (s=1/2)? A promising route to arrive at the **orbital liquid state**?

P-orbital fermions: $p_{x,y}$-orbital counterpart of graphene

- Strong correlation effect from band flatness.

  (e.g. Wigner crystal, and flat band ferromagnetism.)

- A simple mechanism: f-wave Cooper pairing.

- *P*-orbital Mott insulators: orbital exchange; a new type of frustrated magnet-like mode.

- **Topological insulator: orbital analogy of quantum anomalous quantum Hall effect.**

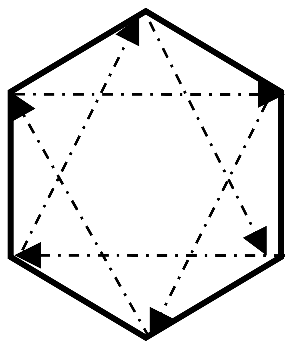
C. Wu, Phys. Rev. Lett. 101. 186807(2008).

Topological insulators

- **Connection between orbital physics with topological insulators --- quantum anomalous Hall effect.**

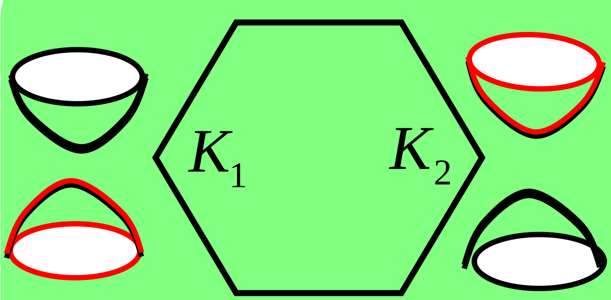
- Quantum Hall effect: band structure with non-vanishing Chern number. Landau level is not essential.

- Haldane's quantum anomalous Hall model: honeycomb lattice with complex-valued next-nearest neighbor hopping.



$$H_{NN} = -t \sum_{\vec{r} \in A} \{c^+(\vec{r}_A) c(\vec{r}_B) + h.c.\}$$

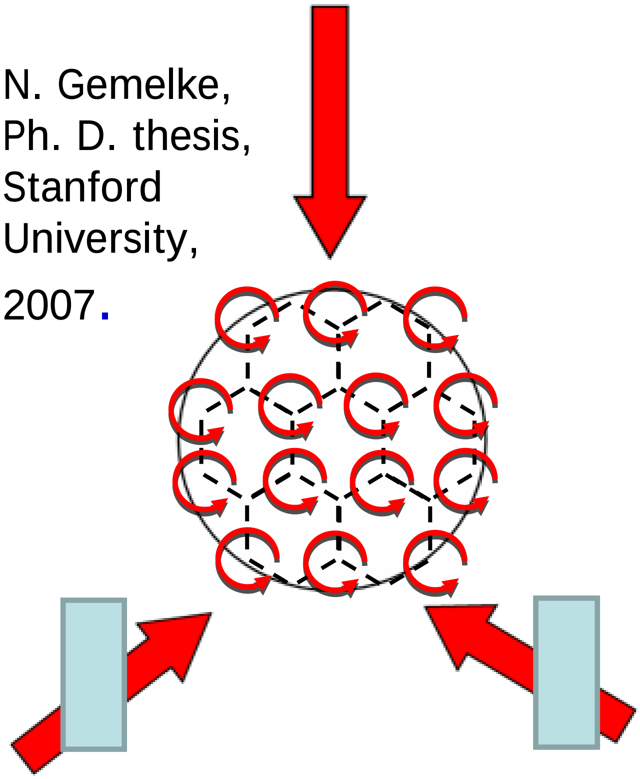
$$H_{NNN} = -\sum_{\vec{r}} t' \{e^{i\delta} c^+(\vec{r}_A) c(\vec{r}_A') + e^{i\delta} c^+(\vec{r}_B) c(\vec{r}_B') + h.c.\}$$

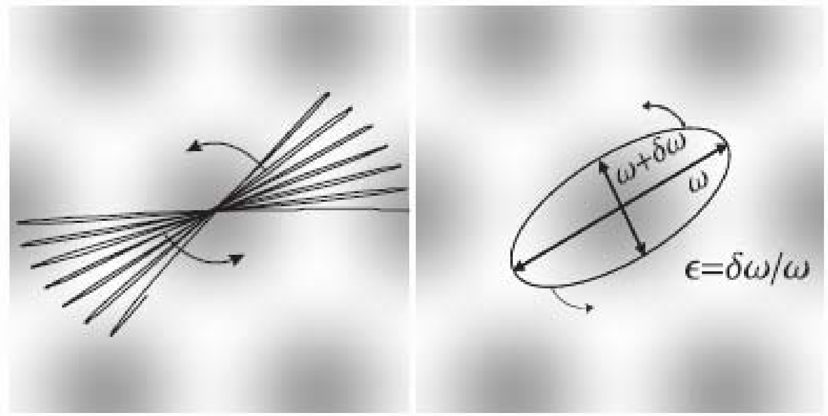


- Topological insulator if $\delta \neq 0, \pi$. Mass changes sign at $K_{1,2}$.

Rotate each site around its own center

N. Gemelke, Ph. D. thesis, Stanford University, 2007.





- Phase modulation on laser beams: a fast overall oscillation of the lattice. Atoms cannot follow and feel a slightly distorted averaged potential.

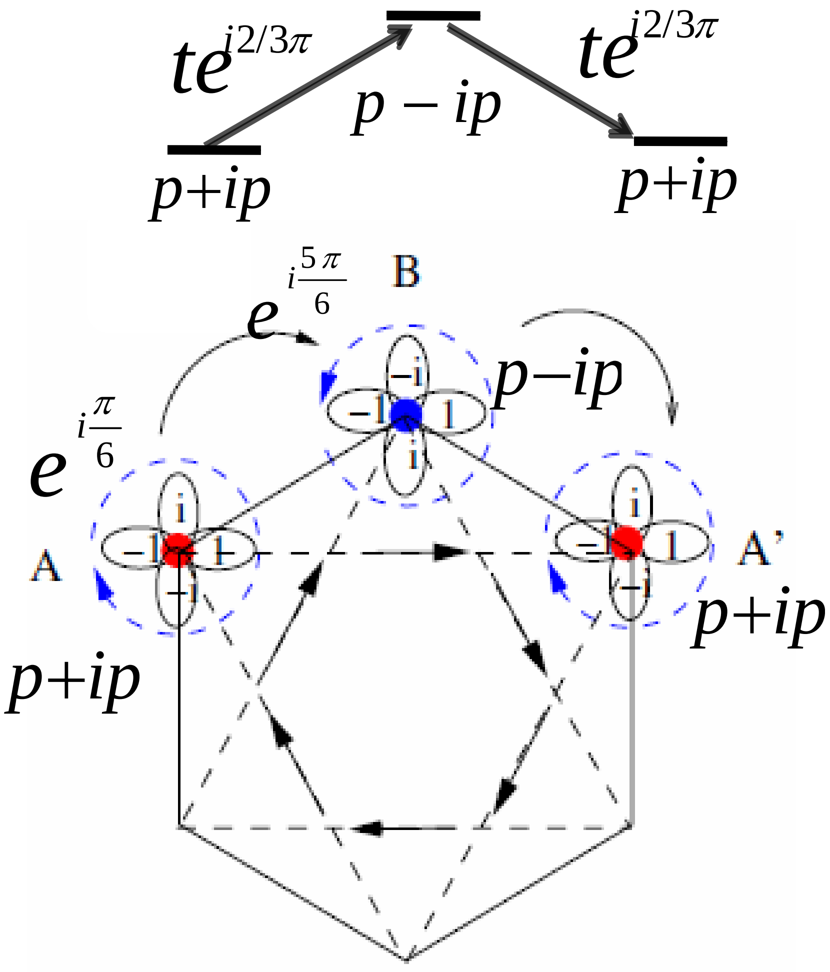
- Orbital Zeeman term (NOT spin!).

$$H_{zmn} = -\Omega \sum_{r \in A} L_z(\vec{r})$$
$$= i\Omega \sum_{\vec{r} \in A} \{ p_x^+(\vec{r}) p_y(\vec{r}) - p_y^+(\vec{r}) p_x(\vec{r}) \}$$

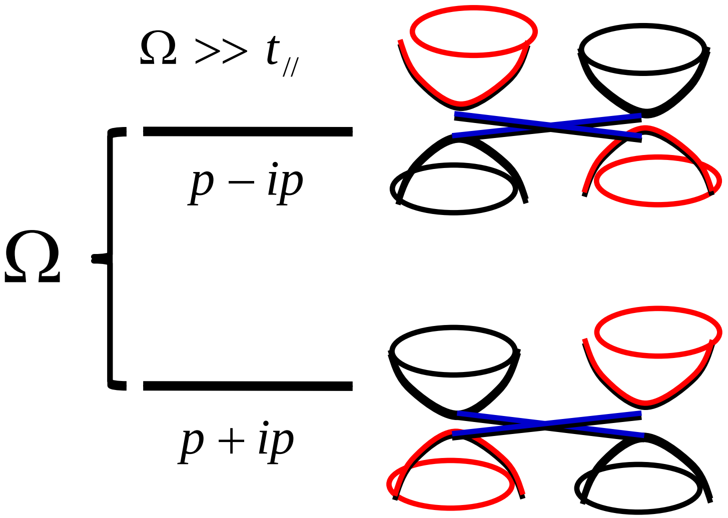
- The oscillation axis slowly precesses at the angular frequency of Ω .

Large rotation angular velocity $\Omega >> t_{//}$

- The second order perturbation generates the NNN complex hopping.

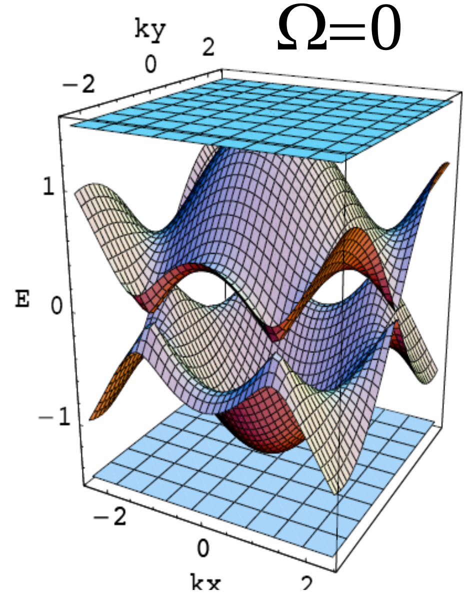


$$t' = -(te^{i2/3\pi})^2 / 2\Omega$$

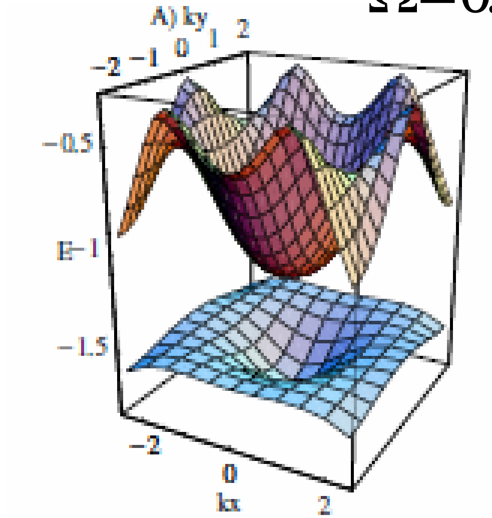


Small rotation angular velocity

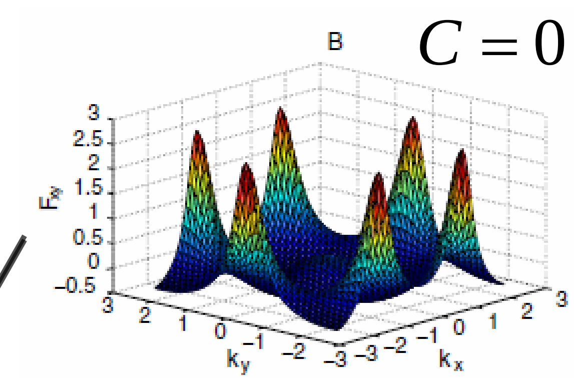
$\Omega=0$

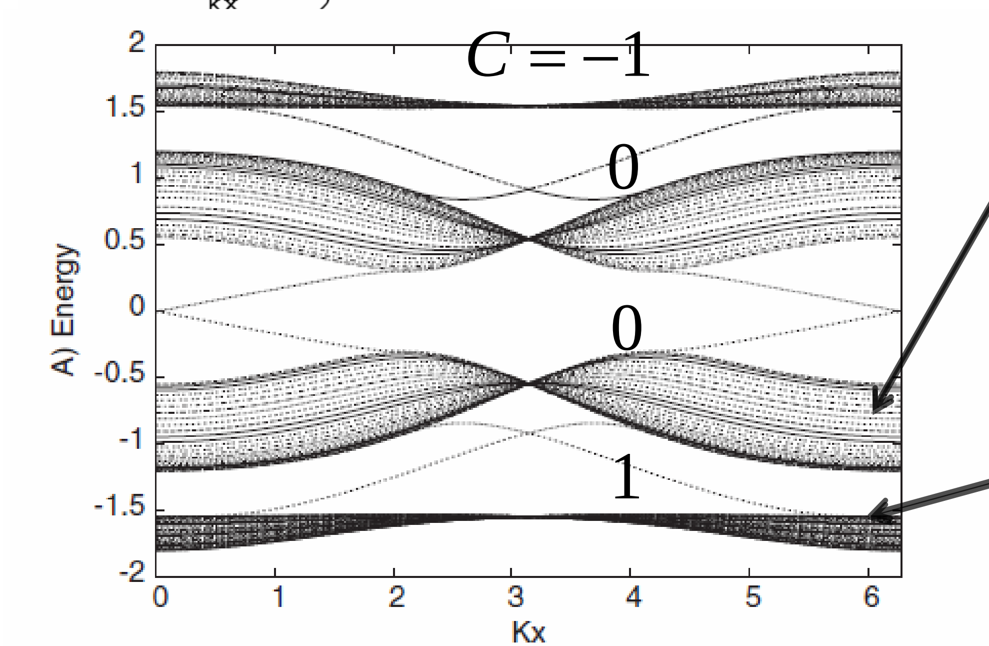


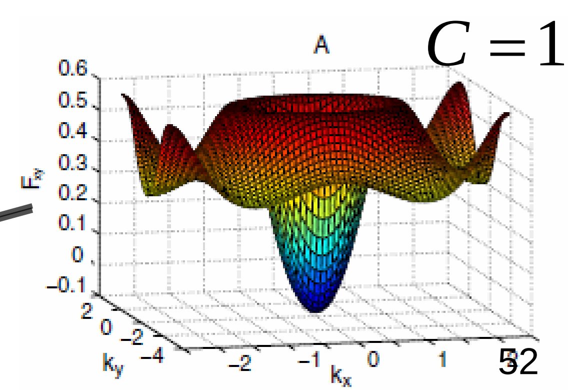
$\Omega=0.3t_{//}$



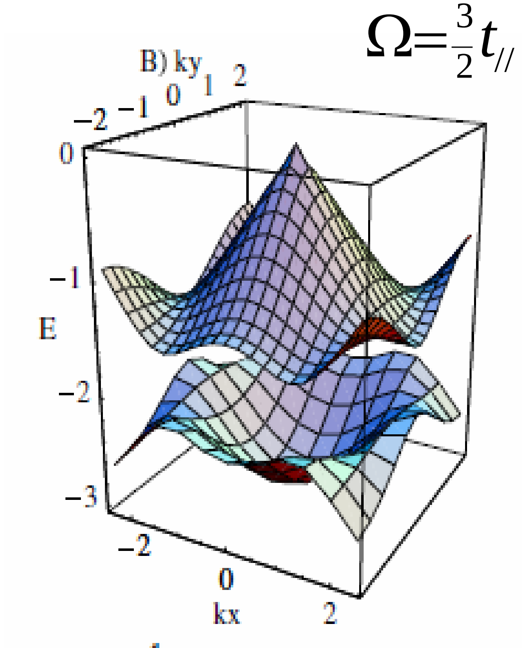
Berry curvature.

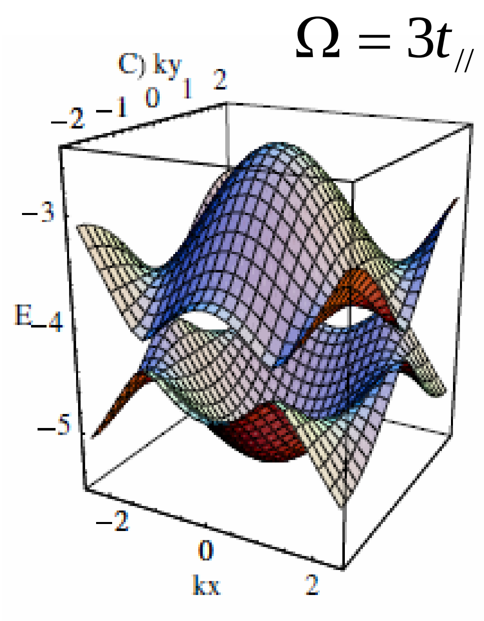




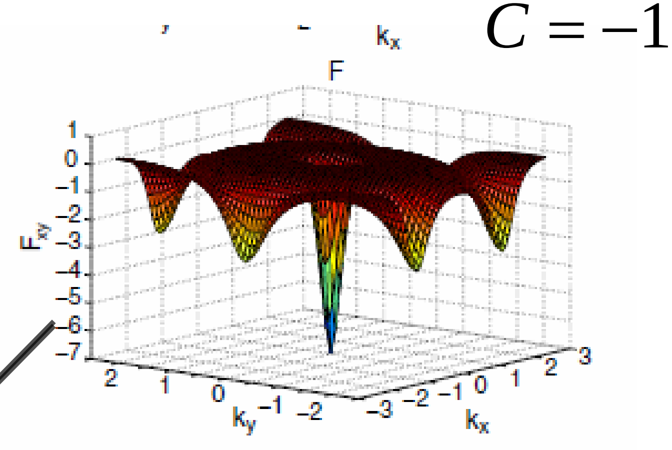


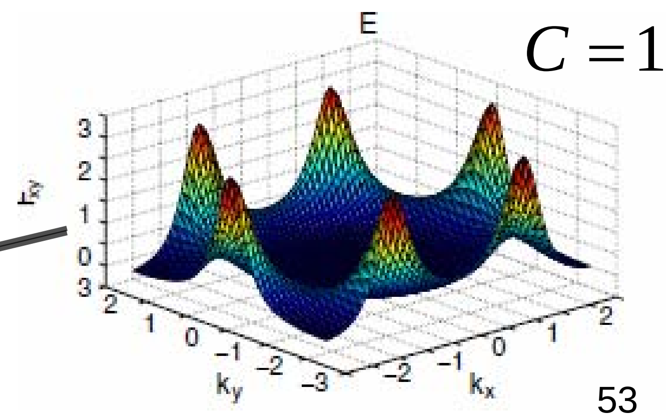
Large rotation angular velocity

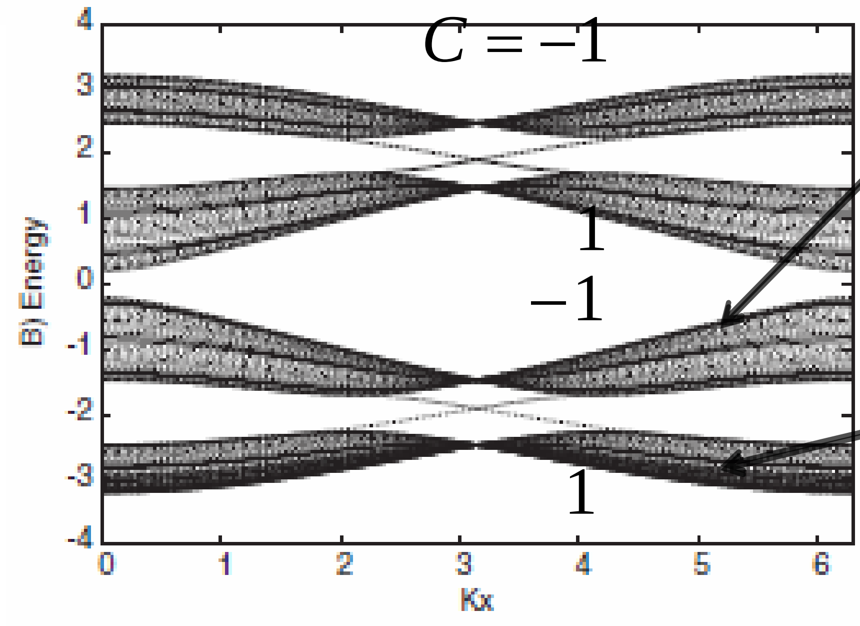




Berry curvature.



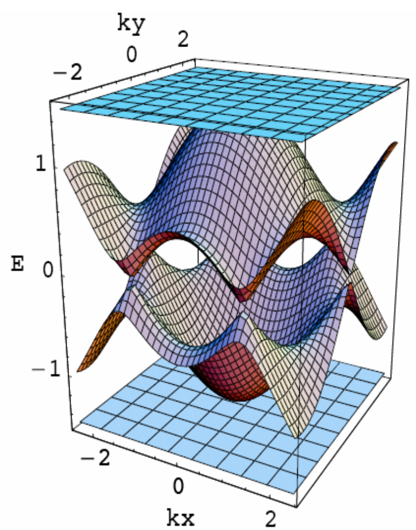


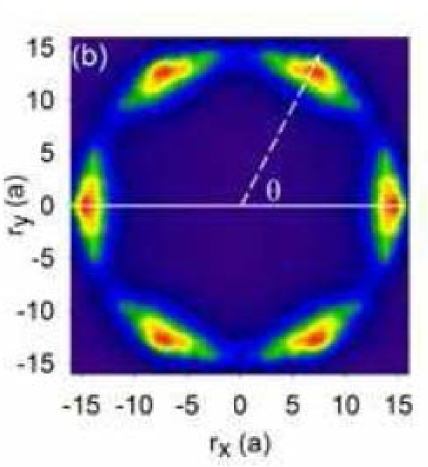


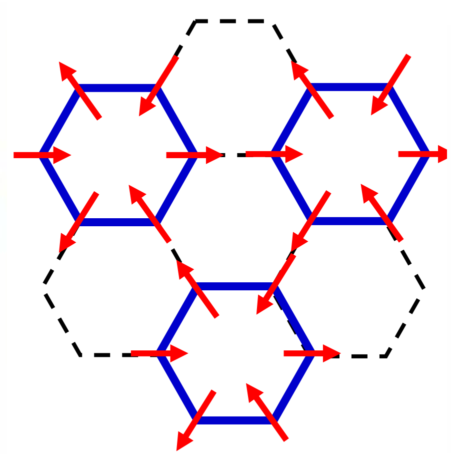
Summary: a new research direction of cold atoms in optical lattices

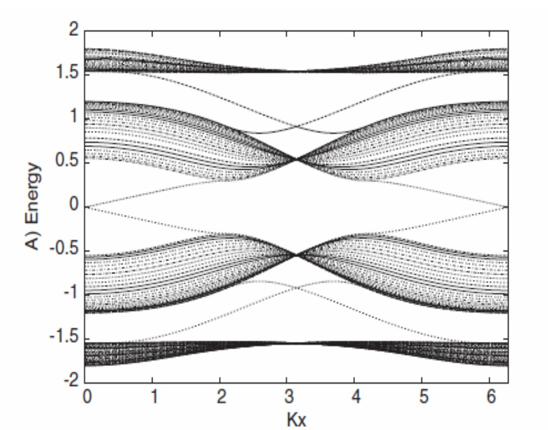
- Unconventional BEC beyond the “no-node” theorem.
- Strong correlation effect from band flatness: Wigner crystal and ferromagnetism;
- Simple mechanism for f-wave Cooper pairing;
- Orbital exchange: a new type of frustrated magnet-like model; a cousin of the Kitaev model;
- Topological band insulator: quantum anomalous Hall effect.

C. Wu, PRL 101, 168807 (2008).



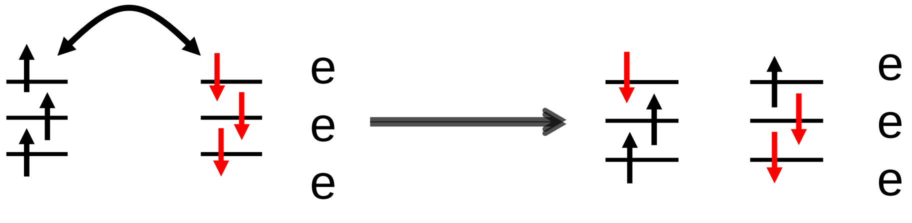






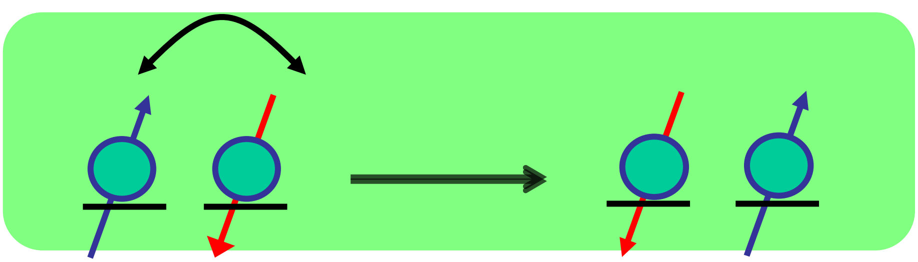
Large spin cold atoms: strong spin fluctuation, hidden symmetry, pairing and quartteting

- Solid state large spin: large S.



$$\Delta S_z = \pm 1 \rightarrow \vec{S} \cdot \vec{S}$$

- Cold atom large spin: hyperfine multiplet → large N.



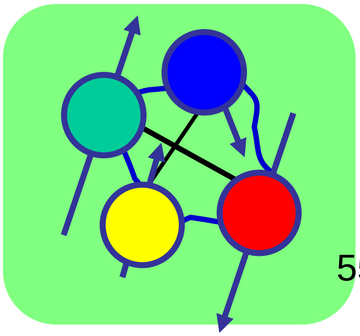
$$\vec{S} = \vec{S}_{nuclear} + \vec{S}_{electron}$$

$$\Delta S_z = \pm 1, \ldots \pm (2S+1) \rightarrow$$

$$\vec{S} \cdot \vec{S}, (\vec{S} \cdot \vec{S})^2, \ldots\ldots.$$

- Spin-3/2 Hubbard model: hidden Sp(4)/SO(5) symmetry without fine-tuning.

- Attractive interaction: multi-fermion instability → baryon like state.

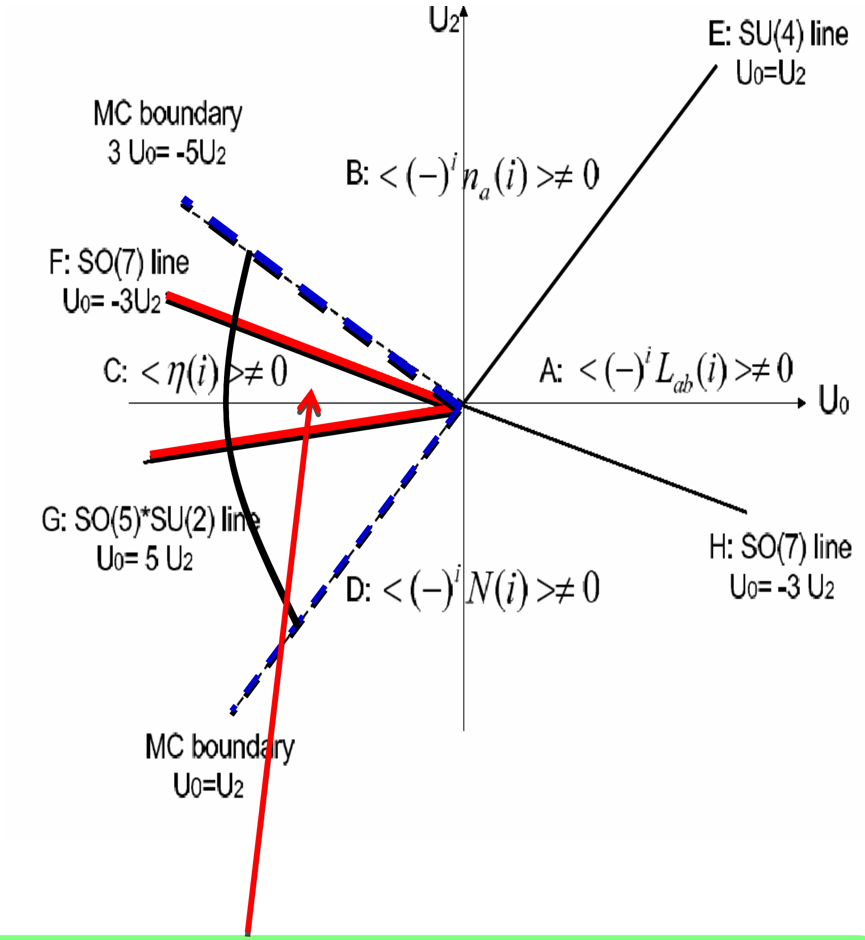


C. Wu, Mod. Phys. Lett. B 20, 1707 (2006) (brief review).

http://online.itp.ucsb.edu/online/coldatoms07/wu2/

Mean-field phase diagram at half-filling of the spin-3/2 Hubbard model

- A: 10-component AF spin-spin octupole phase.
- B: 5-component AF spin quadrupole phase.
- C: Singlet pairing phase.
- D: CDW phase.
- F: Exact SO(7) symmetry 5-AF spin quadrupole +singlet pairing
- G: Exact pseudo-spin SU(2). CDW + singlet pairing



- Quantum Monte Carlo regime: no sign problem even away from half-filling. Competition between AF, SC, CDW, and doping.