Majorana Induced Equal Spin Andreev Reflections



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7th SRC Winter Workshp
Feb 2015

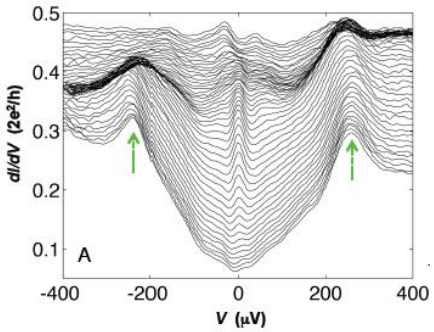
안녕하십니까

Collaborators:
James He, Noah Yuan, Chris Wong, TP Choy, Jie Liu, Xiong-Jun Liu
Senior Collaborators: Yukio Tanaka, FC Zhang, Tai Kai Ng, Patrick Lee

Outline

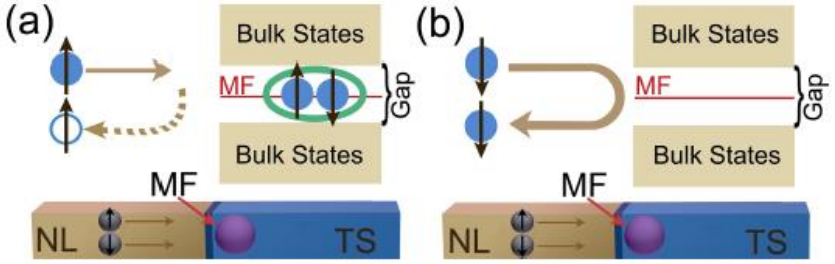
1. Introduction

a. Realizing Majorana fermions in topological superconductors
b. Detecting Majorana fermions
c. Origin of zero bias conductance peak



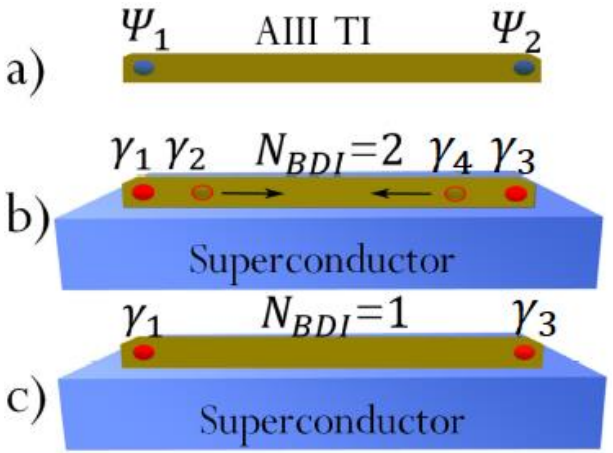
2. MF induced *Selective* Equal spin Andreev reflections

James He, TK Ng, PA Lee and KTL
Phys. Rev. Lett. 112, 037001 (2014)



3. Creating correlated spin polarized current using new types of BDI class TS

James HE, Jian-sheng Wu, Ting-pong Choy, Xiong-Jun Liu,
Yukio Tanaka and KTL
Nature Communications 5:3232 (2014)



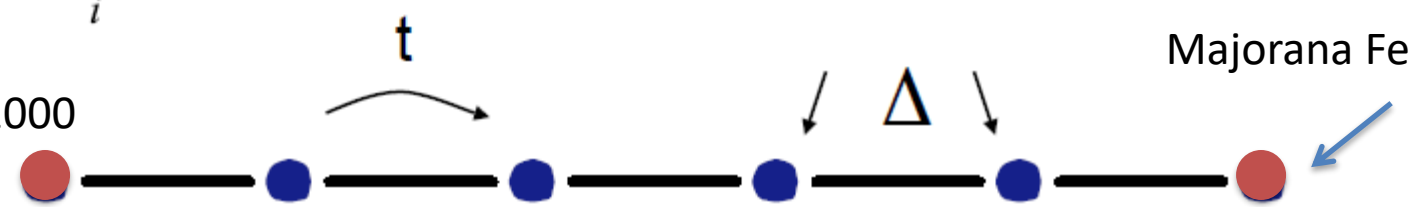
Realizing Topological Superconductors

Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^\dagger$

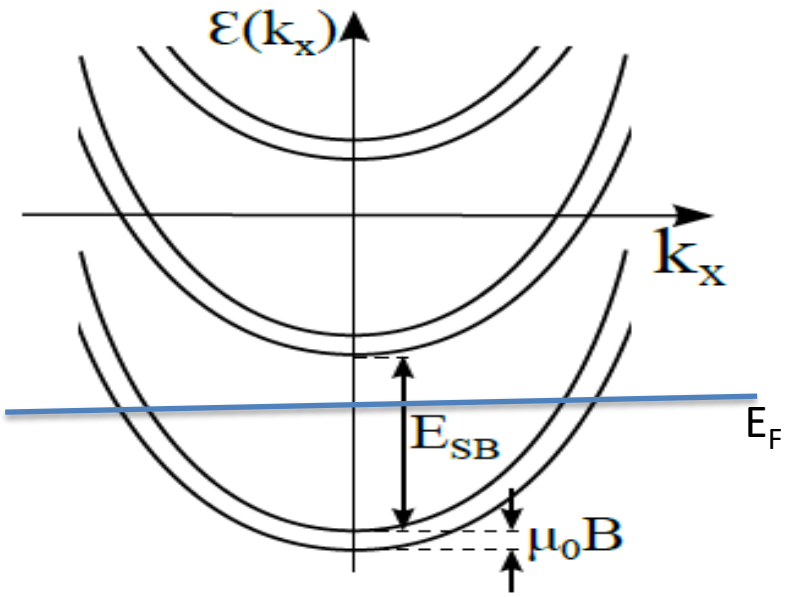
$$H - \mu N = \sum_i t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu c_i^\dagger c_i + \Delta(c_i c_{i+1} + c_{i+1}^\dagger c_i^\dagger)$$

Read and Green 2000

Kitaev 2000



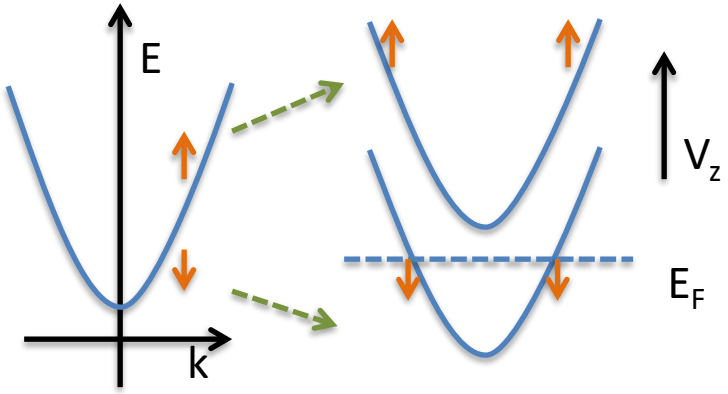
Ingredients: Spinless fermions + superconducting pairing



The key to realize a topological superconductor is:

1) to have the Fermi energy cuts through an odd number of subbands.
2) Introduce SC pairing at the Fermi energy.

Realizing effective spinless fermions by Zeeman field?



1. Zeeman field
2. Chemical potential in the Zeeman gap
3. Induce superconducting pairing

Metal wire + magnetic field + s-wave superconductor?

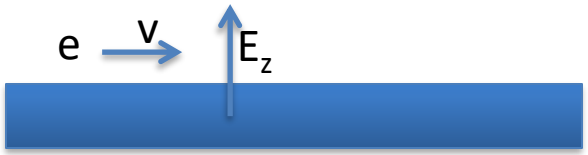


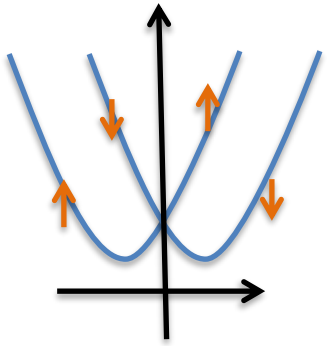
Unfortunately, s-wave superconductor cannot induce superconducting pairings on electrons with the same spin.

Spin-orbit coupling: Due to Electrons moving in an electric field experience an effective magnetic field

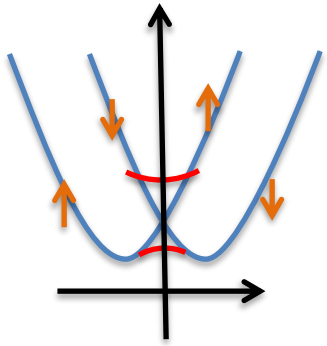
$SOC = \gamma\,(v \times E)\cdot \Leftrightarrow$

$= \alpha_R\, k\sigma_y$





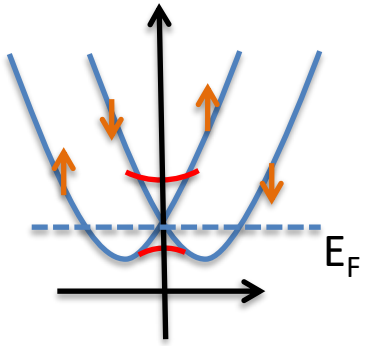
Rashba



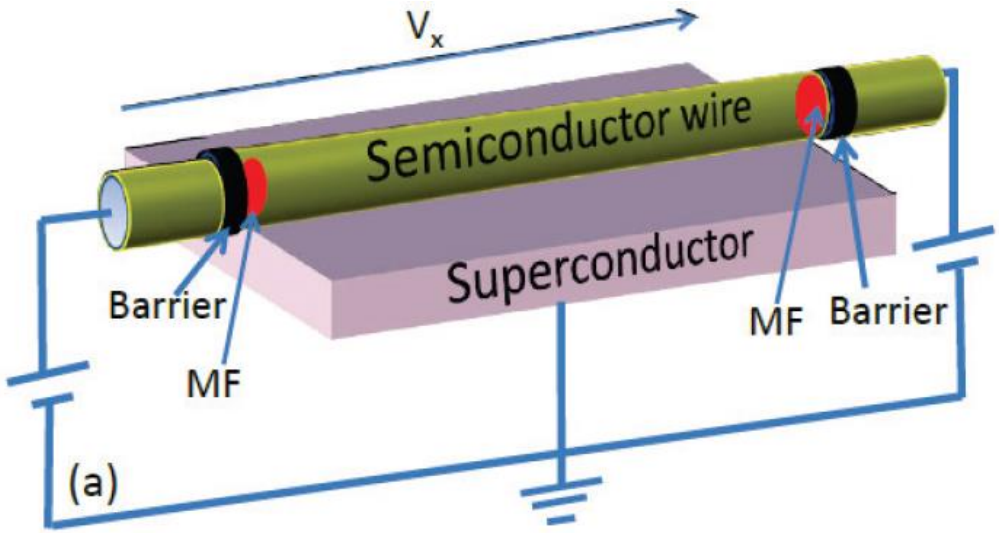
Rashba + Zeeman

Zeeman + Spin orbit coupling

$$H_{1D}(k) = [(\frac{k^2}{2m} + \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k\sigma_y)]\tau_z - \Delta\sigma_y\tau_y$$



Rashba + Zeeman

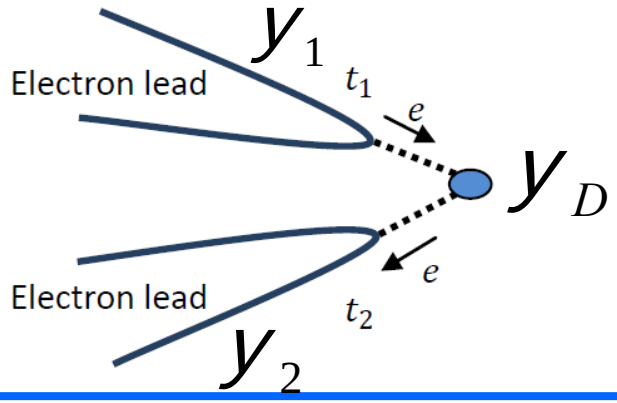


1. Zeeman gap makes single subband at E_F possible
2. Rashba SOC makes pairing possible

P-wave pairing gap ≈ (α_R k_F/|V|)Δ

M. Sato et al. PRB (2009)
Sau et al. PRL (2010)
Oreg et al. PRL (2010)
Alicea PRB (2010)

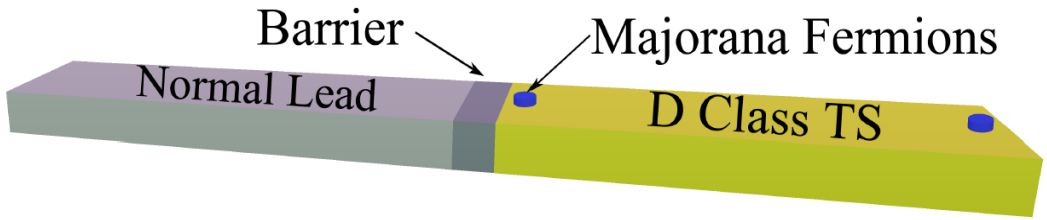
Majorana Fermion Induced Resonant Andreev Reflection

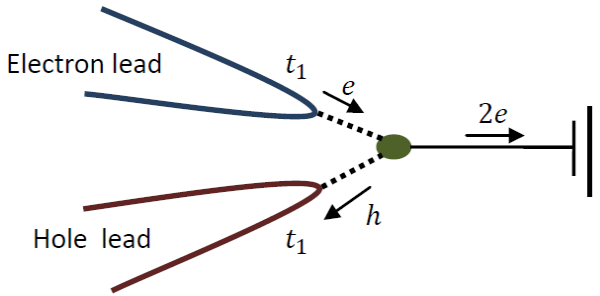


$$H_T = \gamma_D^+[t_1\gamma_1 + t_2\gamma_2] + h.c.$$

$$T(E) = \frac{(2t_1t_2)^2}{(E-E_0)^2 + (t_1^2+t_2^2)^2}$$

Majorana Fermion : Particle = Antiparticle $\gamma = \gamma^\dagger$



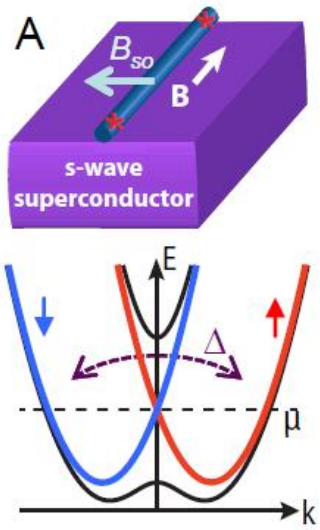


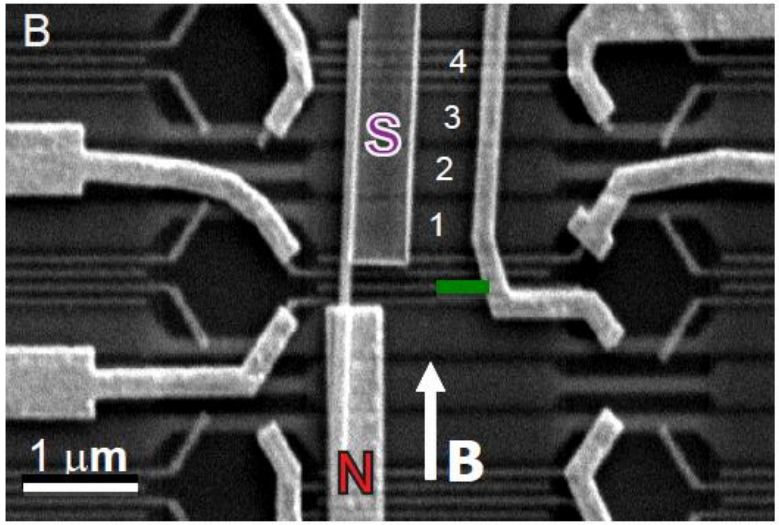
$$H_T = -it_1g[\gamma^+(0) + \gamma(0)]$$

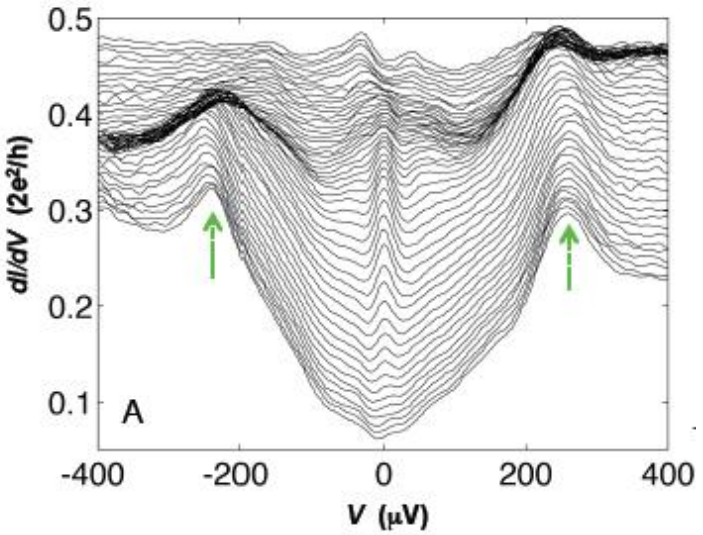
$$T(E) = \frac{(2t_1^2)^2}{(E-E_0)^2 + (2t_1^2)^2}$$

KTL, TK Ng, PA Lee PRL (2009)

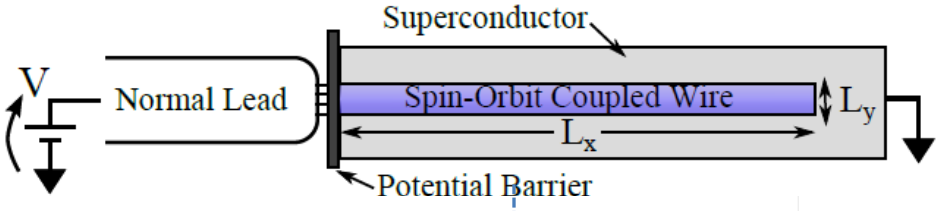
Zero Bias Peaks in Delft experiment

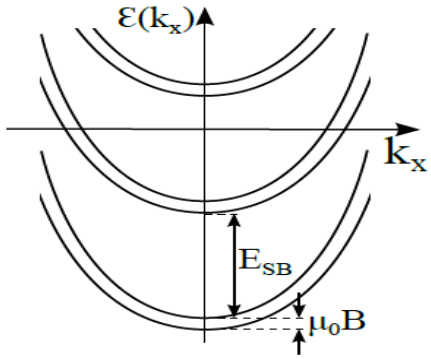


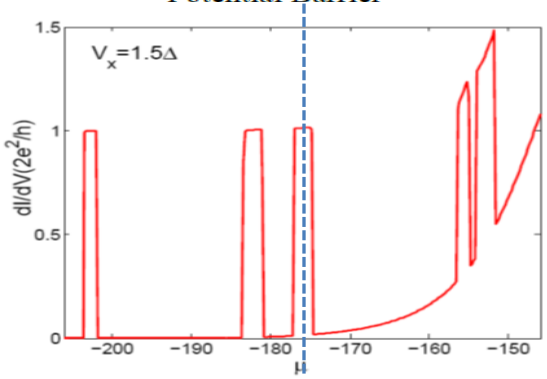


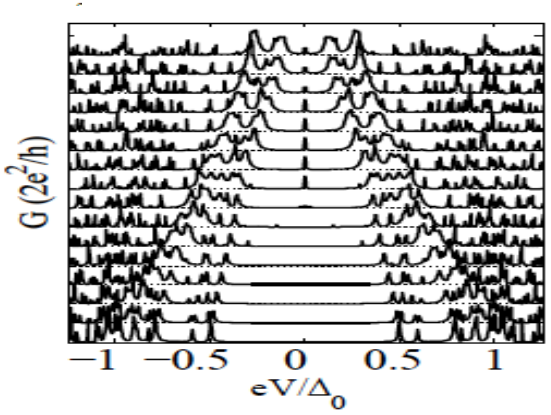


Experiments: Kowenhowen, HQ Xu, Heiblum (2012)



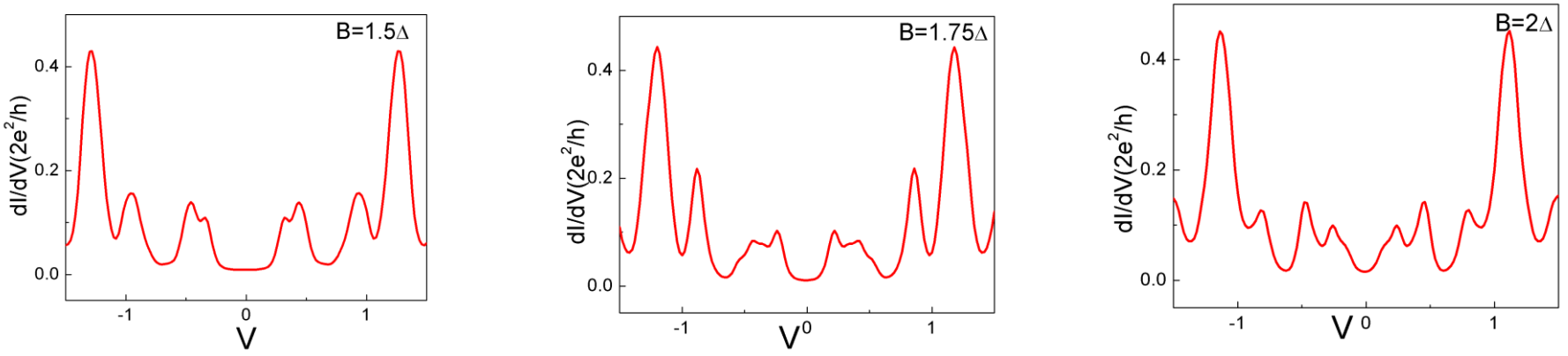


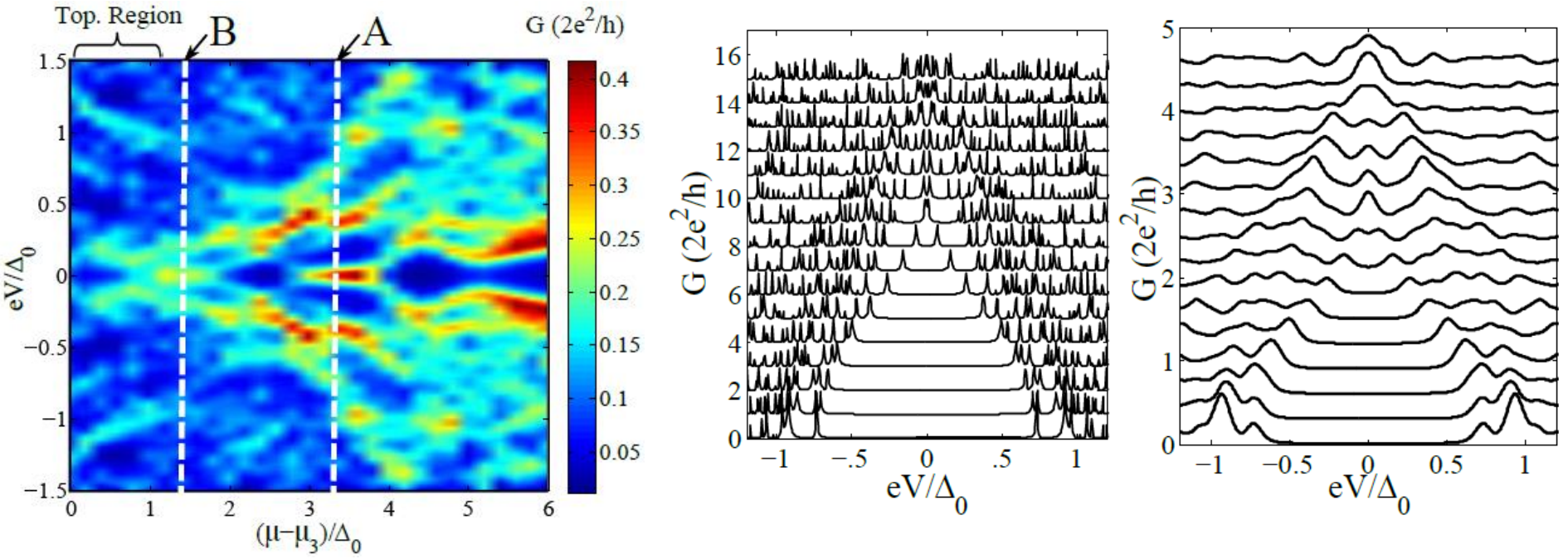




Disorder Induced ZBP in the absence of MFs

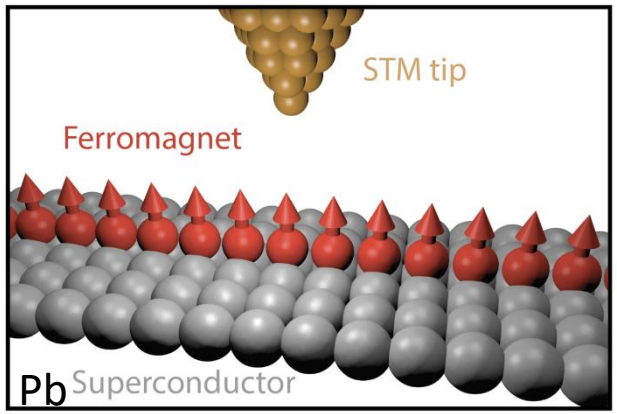
Presence of disorder with mean free path about 300nm

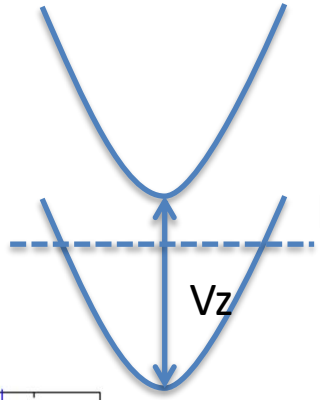


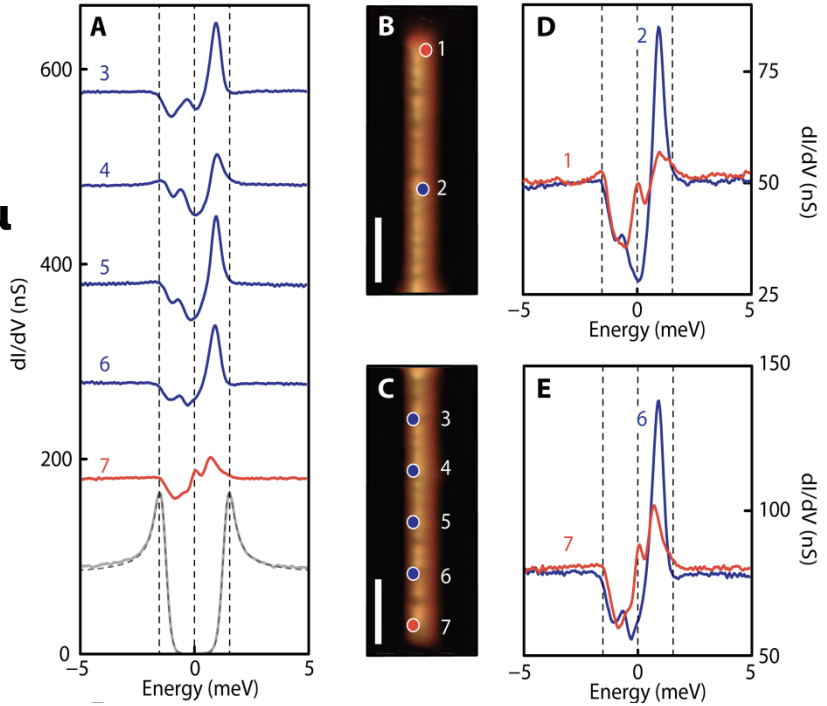


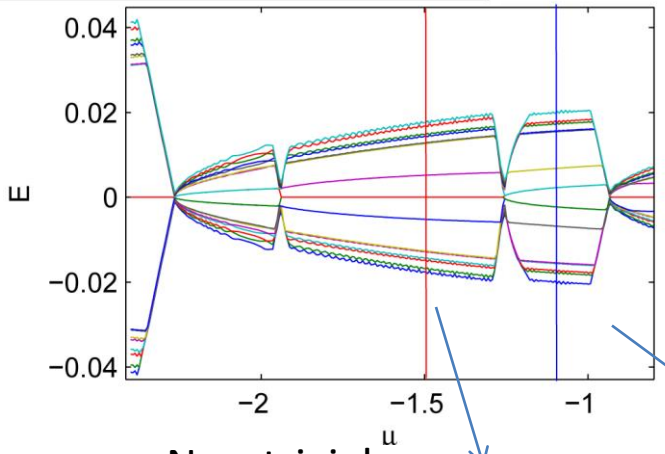
Jie Liu, A.C. Potter, KTL, P.A. Lee (PRL 2012)

Zero Bias Peaks in Princeton experiment



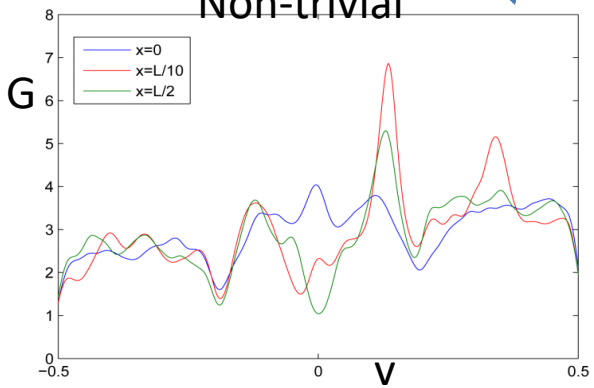




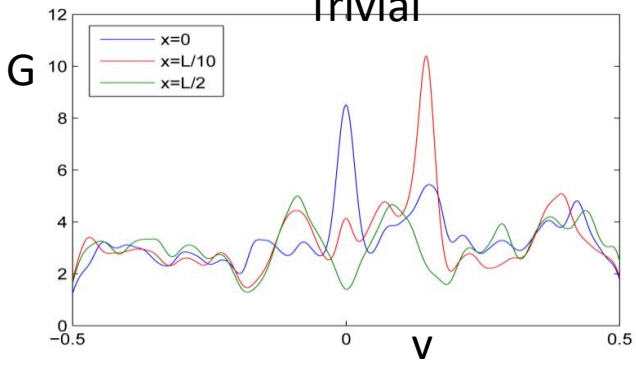


Science 346, 6209 (2014)

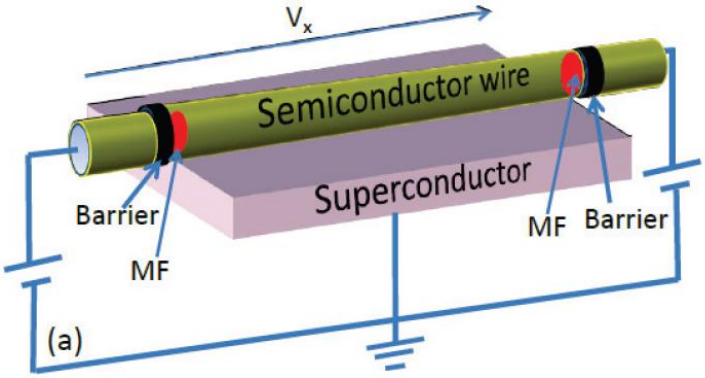
Non-trivial



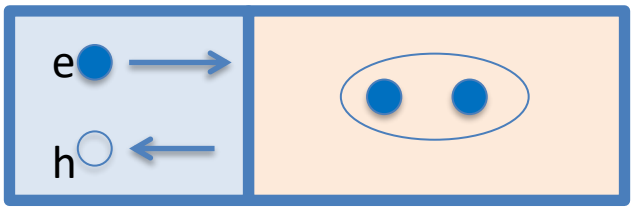
Trivial



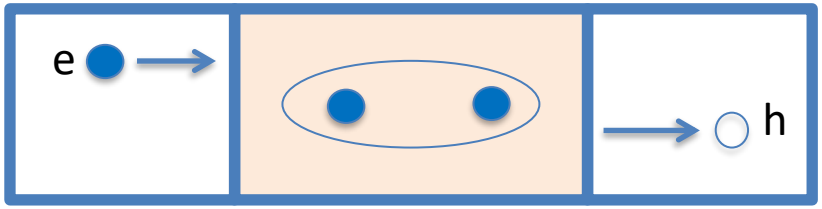
MF induced Crossed Andreev Reflection



Local Andreev Reflections



Crossed Andreev Reflections



Crossed Andreev reflection dominate regime

1. Fano factor = C/I= e instead of 2e
2. Strong current-current correlations

Current Fluctuation

$$dI_i(t) = I_i(t) - \langle I_i \rangle$$

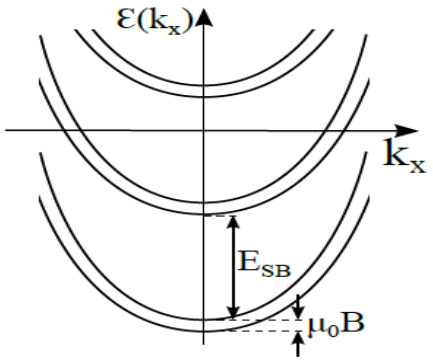
Shot Noise

$$C_{ij} = \int_{-\infty}^{\infty} \langle dI_i(0) dI_j(t) \rangle dt$$

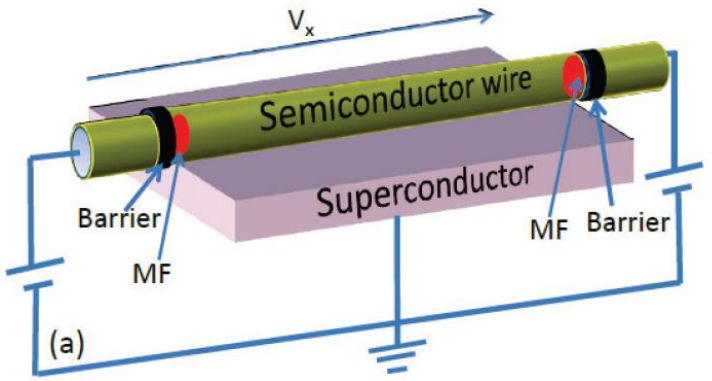
Fano Factor

$$F_i = C_{ii} / I_i = e^*$$

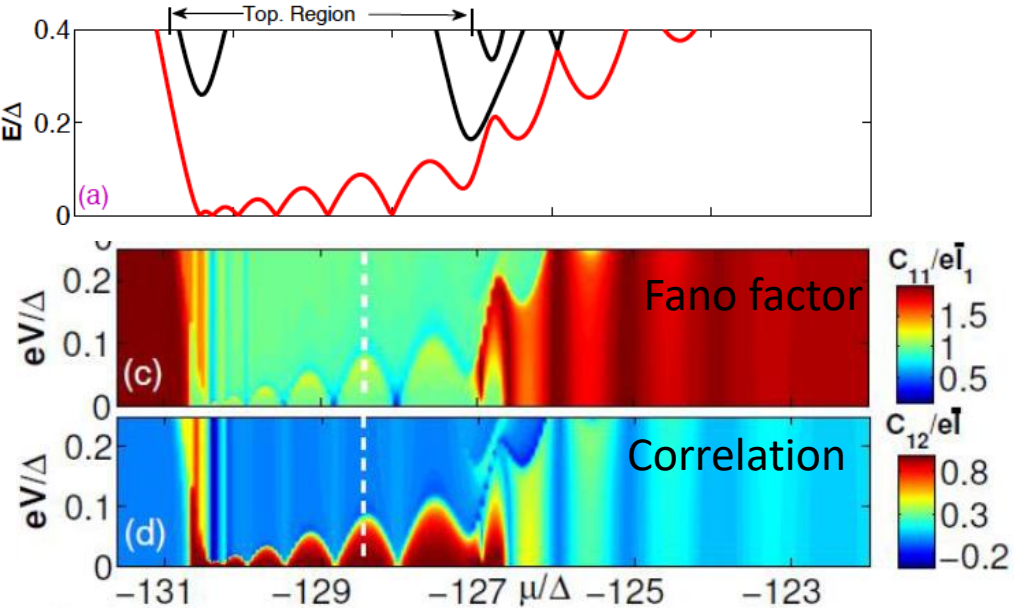
MF induced Crossed Andreev Reflection

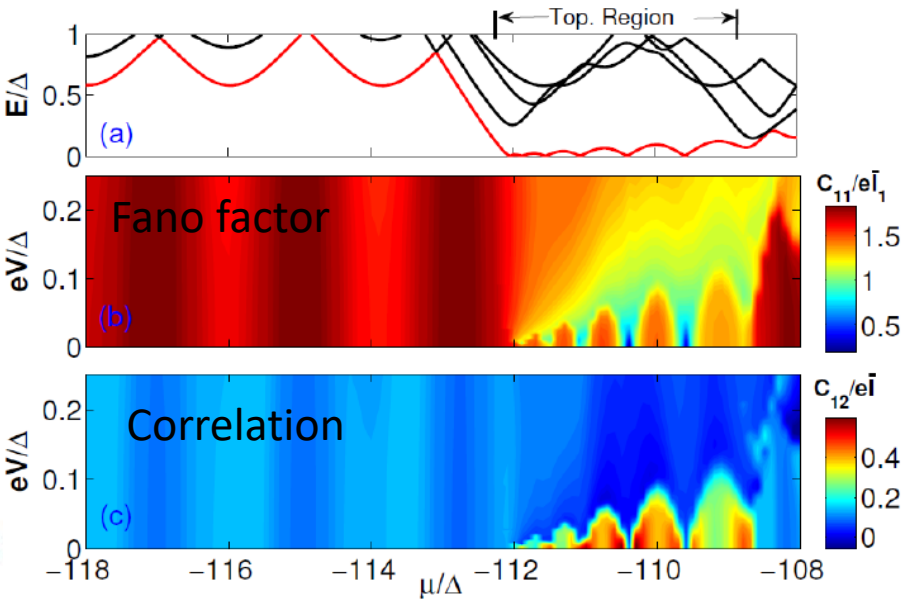


One transverse sub-band is occupied



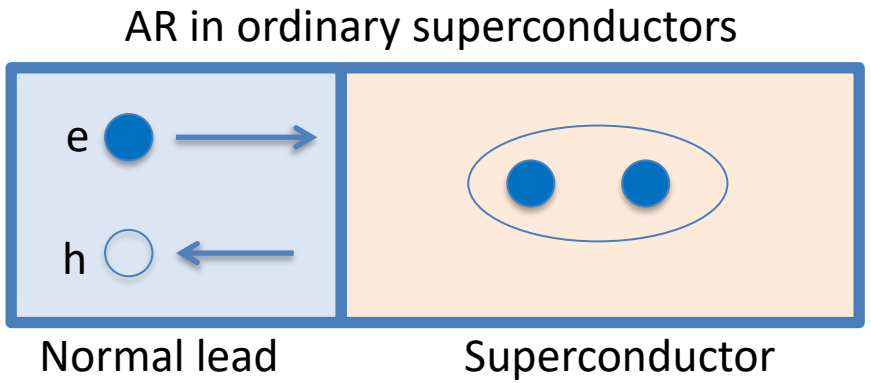
Three transverse sub-bands are occupied

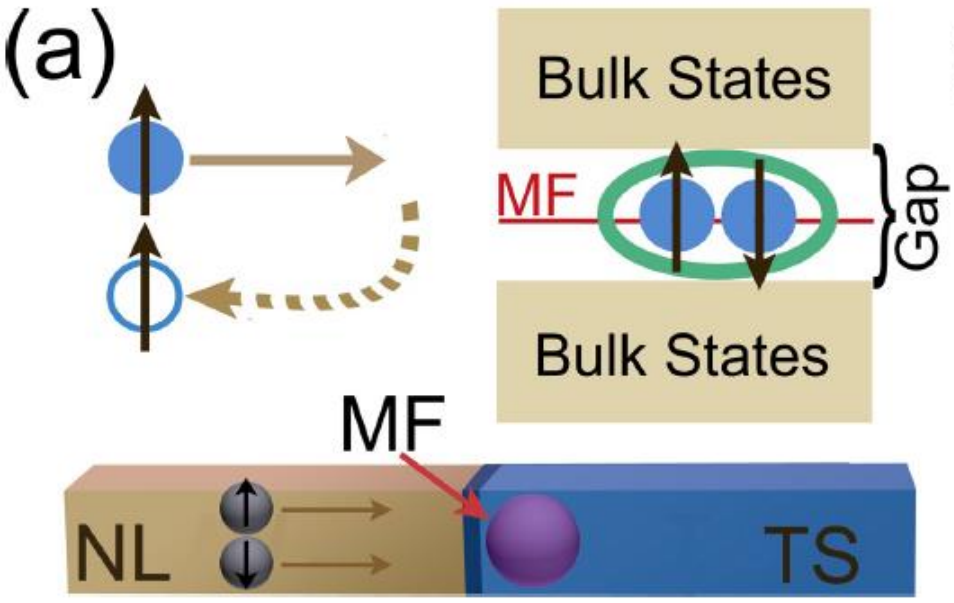


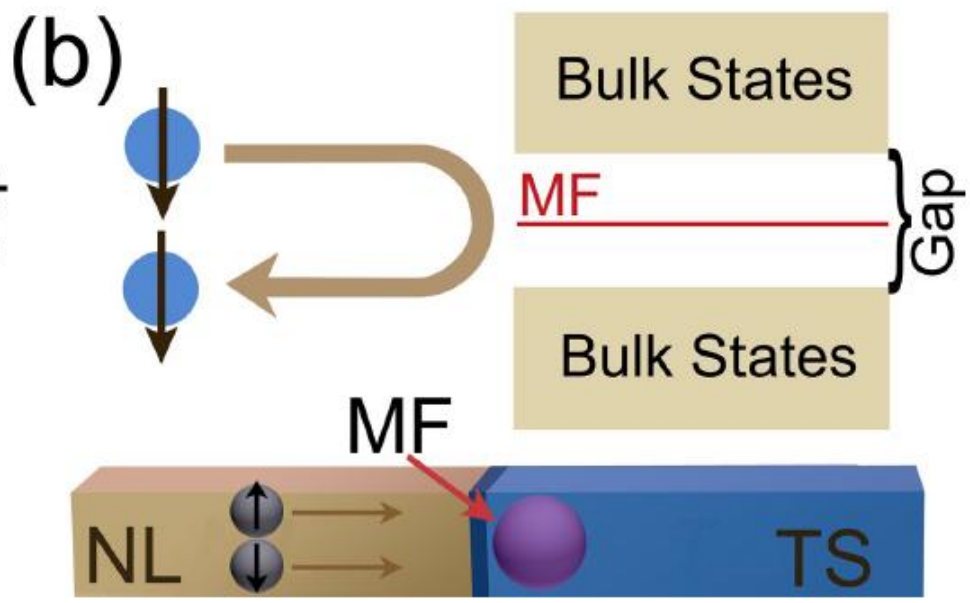


J Liu, FC Zhang, KTL PRB 88 , 064509 (2013)

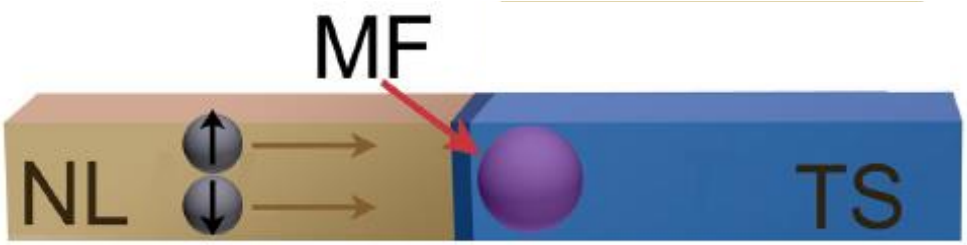
MF induced Selective Equal spin Andreev reflections







MF induced Selective Equal spin Andreev reflections



$$H_T = H_L + H_c,$$
$$H_L = -iv_F \sum_{\alpha\in\uparrow/\downarrow} \int_{-\infty}^{+\infty} \psi_\alpha^\dagger(x)\partial_x\psi_\alpha(x)dx,$$
$$H_c = \tilde{t}\gamma[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)]$$

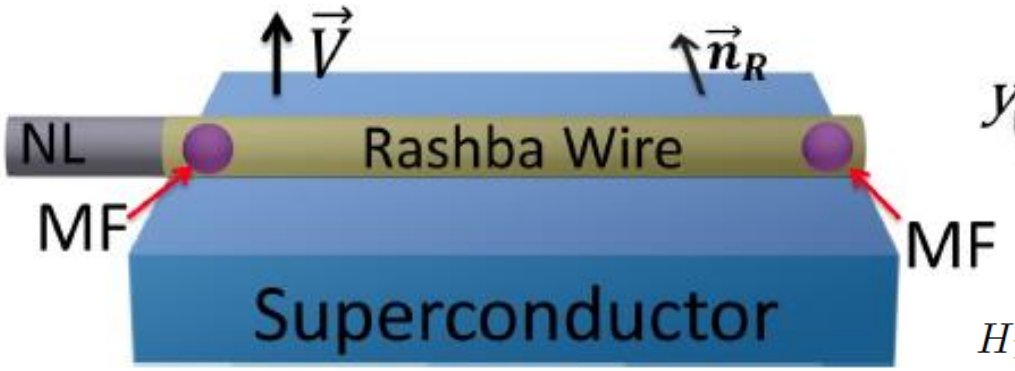
$$\Psi_1 = a\psi_\uparrow + b\psi_\downarrow \text{ and } \Psi_2 = -b^*\psi_\uparrow + a^*\psi_\downarrow$$
$$H_L = -iv_F \sum_{\alpha\in 1/2} \int_{-\infty}^{+\infty} \Psi_\alpha^\dagger(x)\partial_x\Psi_\alpha(x)dx,$$
$$H_c = \tilde{t}\gamma[\Psi_1(0) - \Psi_1^\dagger(0)].$$

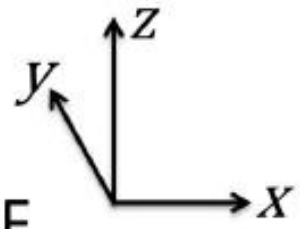
Scattering matrix

$$\begin{pmatrix} \Psi_{1E}(+) \\ \Psi_{1E}^\dagger(+) \end{pmatrix} = \frac{1}{\Gamma + iE} \begin{pmatrix} iE & \Gamma \\ \Gamma & iE \end{pmatrix} \begin{pmatrix} \Psi_{1E}(-) \\ \Psi_{1E}^\dagger(-) \end{pmatrix} \qquad \Gamma = 2\tilde{t}^2/v_F$$

Andreev reflection amplitude: $\Gamma/(\Gamma + iE)$

MF induced Selective Equal spin Andreev reflections





$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k\sigma_y)]\tau_z - \Delta\sigma_y\tau_y$$

Andreev reflection matrix:

$$r_{he}(V_z) = \begin{pmatrix} \frac{V_z - \sqrt{V_z^2-\Delta^2}}{2V_z} & -\frac{\Delta}{2V_z} \\ -\frac{\Delta}{2V_z} & \frac{\sqrt{V_z^2-\Delta^2}+V_z}{2V_z} \end{pmatrix}$$

Normal reflection matrix:

$$r_{ee}(V_z) = r_{he}(-V_z)e^{i\chi(k)}$$

$$r_{he}\vec{s}_0 = \vec{s}_0^* \text{ and } r_{ee}\vec{s}_0 = 0$$

$$\vec{s}_0 = (\cos\frac{\theta_0}{2}, e^{i\phi_0}\sin\frac{\theta_0}{2})^T$$

Electrons with spin polarization

$$\vec{\mathbf{n}}_0 = \langle\vec{s}_0|\vec{\sigma}|\vec{s}_0\rangle =$$

$$(\sin\theta_0\cos\phi_0, \sin\theta_0\sin\phi_0, \cos\theta_0)$$

are resonantly reflected as holes with the same spin.

On the contrary, we have

$$r_{ee}\vec{u}_0 = e^{i\chi}\vec{u}_0$$

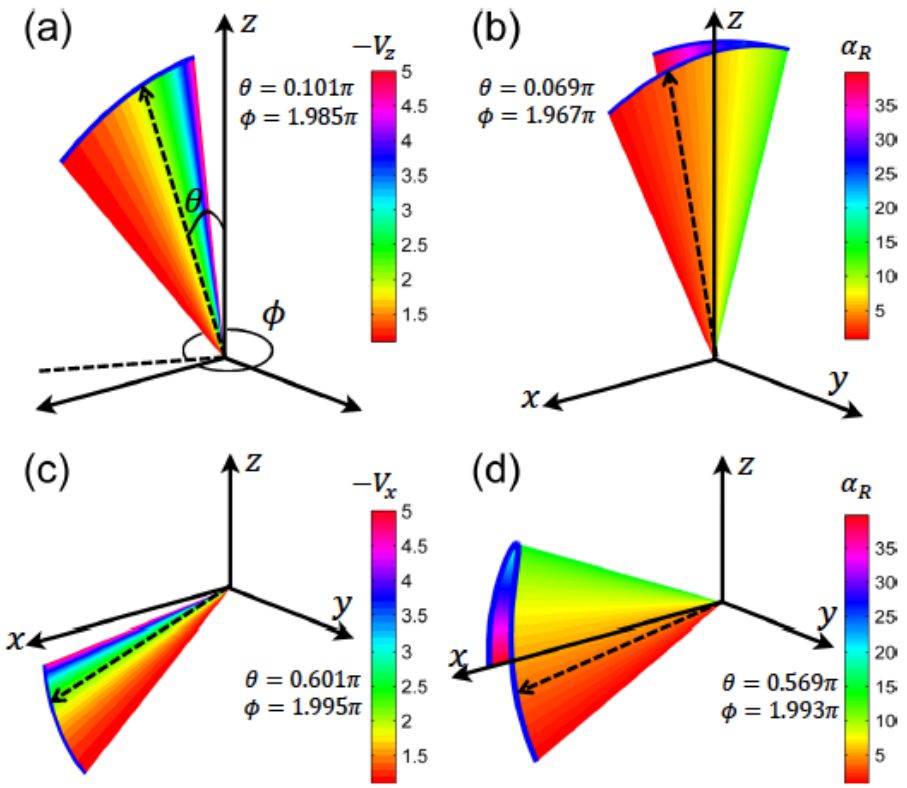
$$\vec{u}_0 = (-\sin\frac{\theta_0}{2}, e^{i\phi_0}\cos\frac{\theta_0}{2})$$

Electrons with spin polarization anti-parallel to $\vec{\mathbf{n}}_0$ are completely reflected as electrons.

MF induced Selective Equal spin Andreev reflections

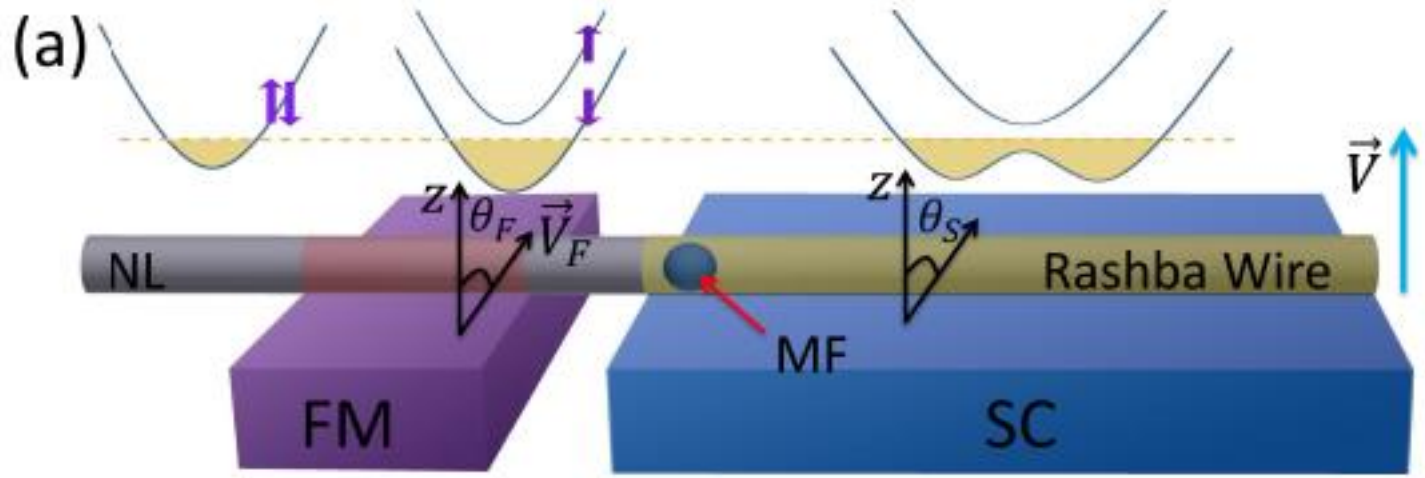
Spin polarization vector

$$\vec{\mathbf{n}}_0 = \langle \vec{s}_0 | \vec{\sigma} | \vec{s}_0 \rangle = (\sin\theta_0 \cos\phi_0, \sin\theta_0 \sin\phi_0, \cos\theta_0)$$

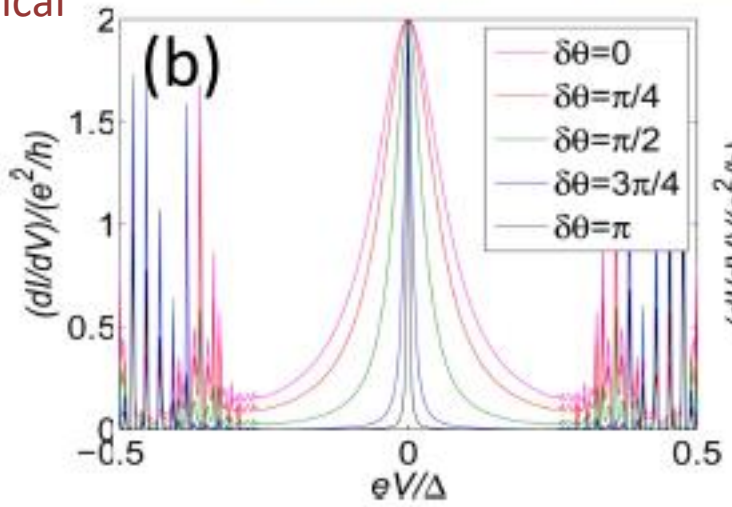


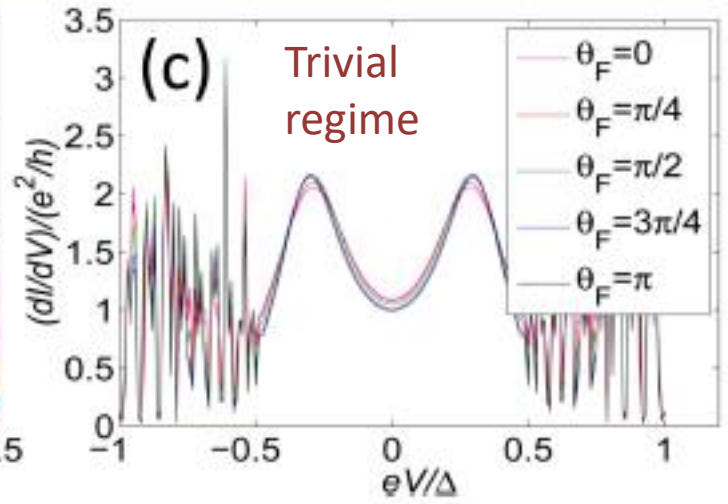
James He, TK Ng, PA Lee and KTL (PRL2014)

MF induced Equal spin Andreev reflections



Topological regime





James He, TK Ng, PA Lee and KTL (PRL2014)

Topological Classification

Symmetry				d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Quantum Hall

Kitaev Model

2D TI

p+ip

p+ip,p-ip

3D TI

Schnyder etc. PRB

Q

Time-reversal symmetry

X

Particle-hole symmetry

P

Chiral symmetry

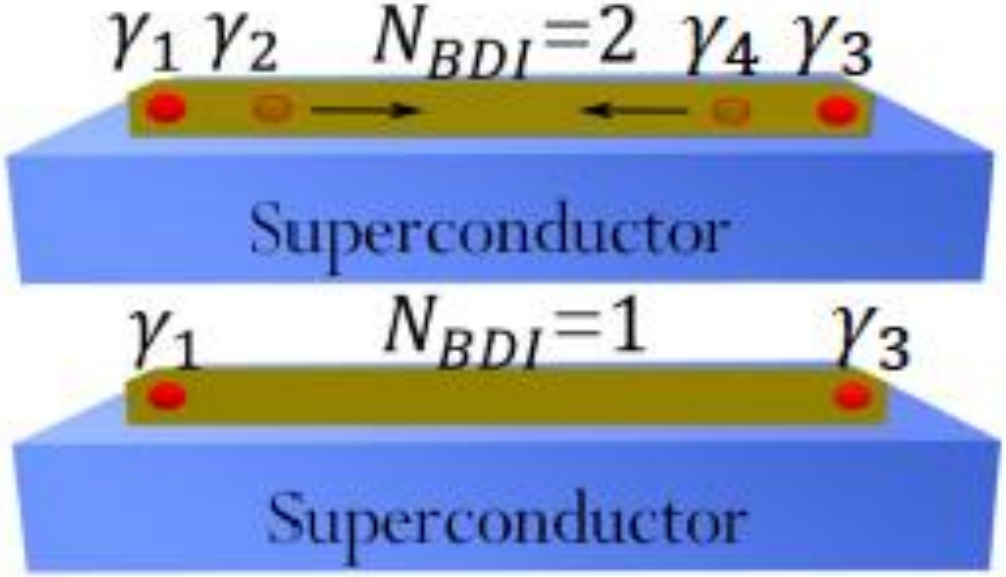
AIII class TI

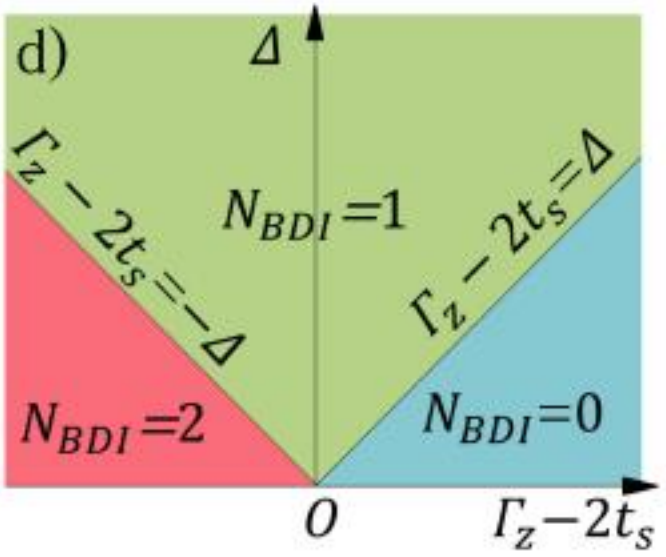
Fermionic
End states



$$H_{\mathrm{AIII}}(k) = (\Gamma_z - 2t_s \cos k)\sigma_z + 2t_{\mathrm{so}} \sin k\sigma_y \quad \sigma_x H(k)\sigma_x = -H(k)$$

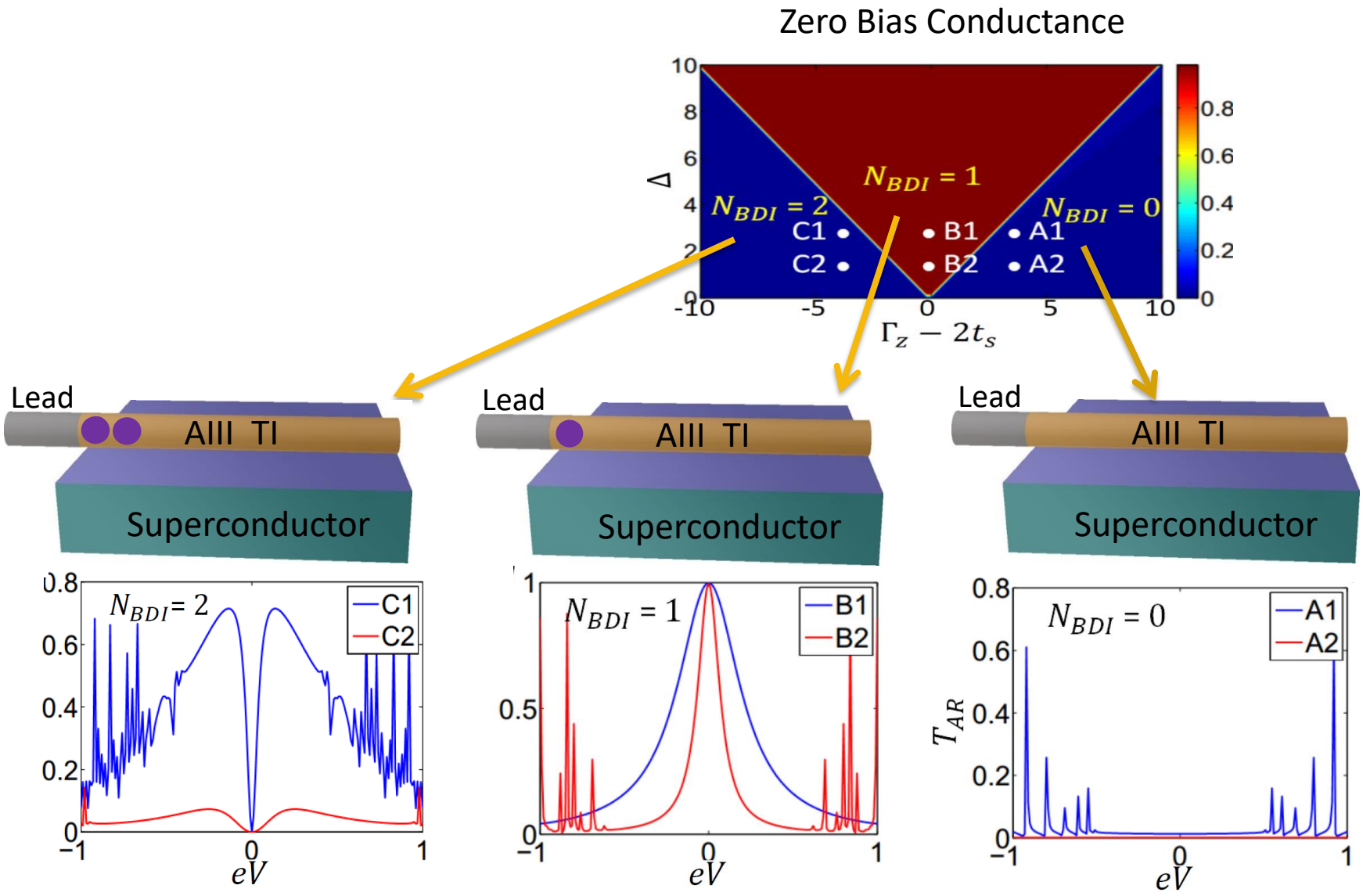
$$H_{\mathrm{BDI}}(k) = [(\Gamma_z - 2t_s \cos k)\,\sigma_z + 2t_{\mathrm{so}} \sin k\sigma_y]\,\tau_z + \Delta\sigma_y\tau_y$$



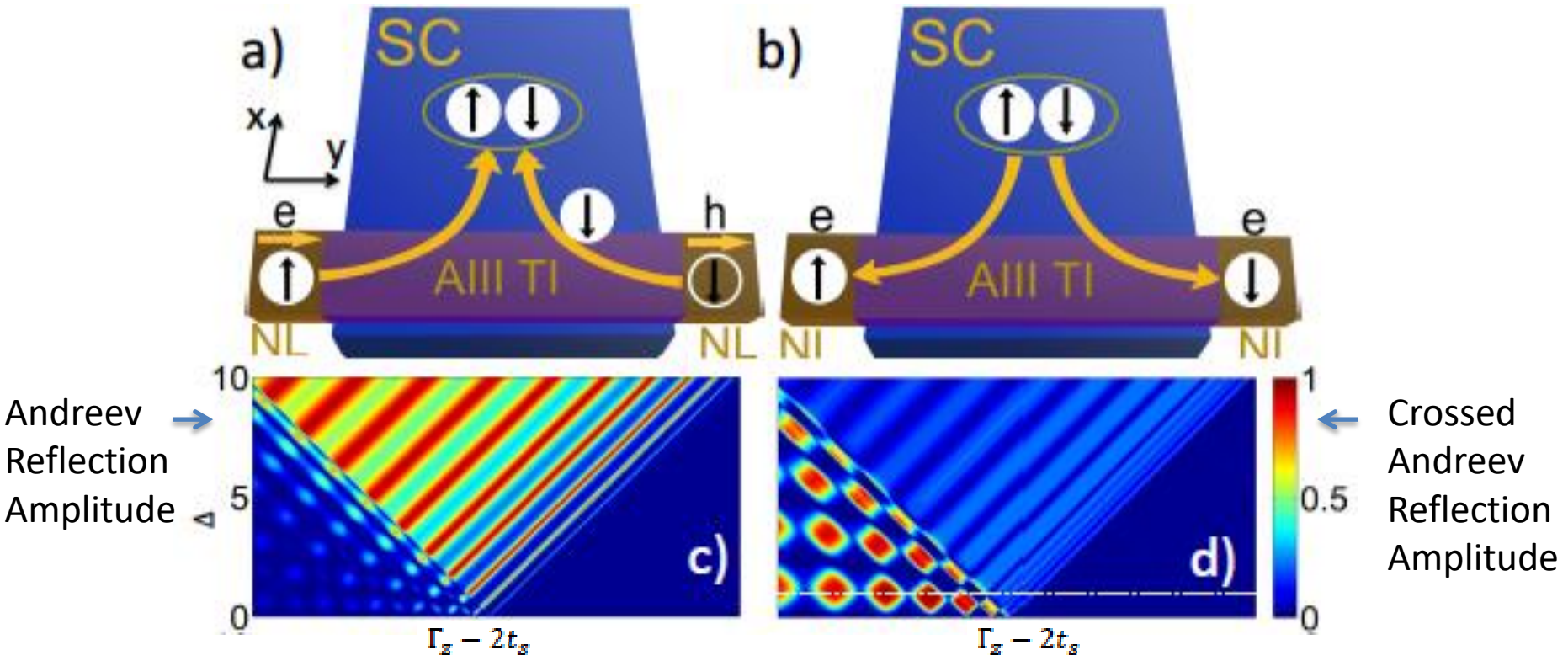


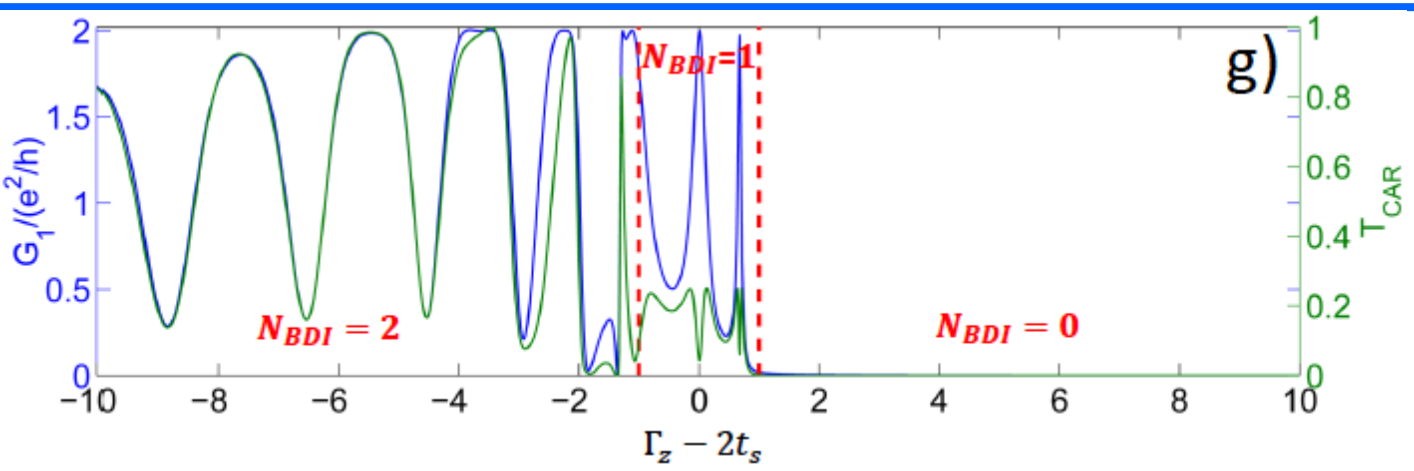
James HE, Jian-sheng Wu, Ting-pong Choy, Xiong-Jun Liu, Yukio Tanaka and KTL (Nature Comm 2014)

Suppression of local Andreev reflections

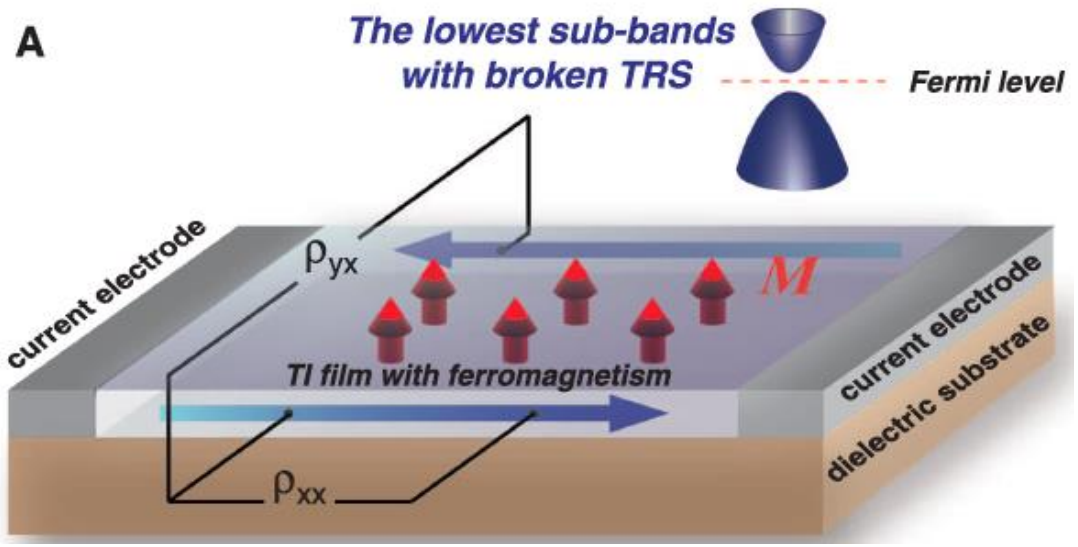


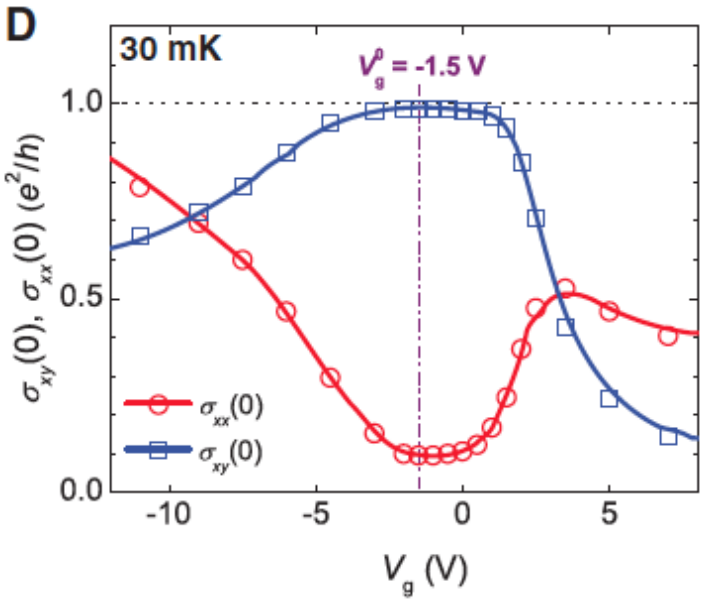
Resonant Crossed Andreev reflections



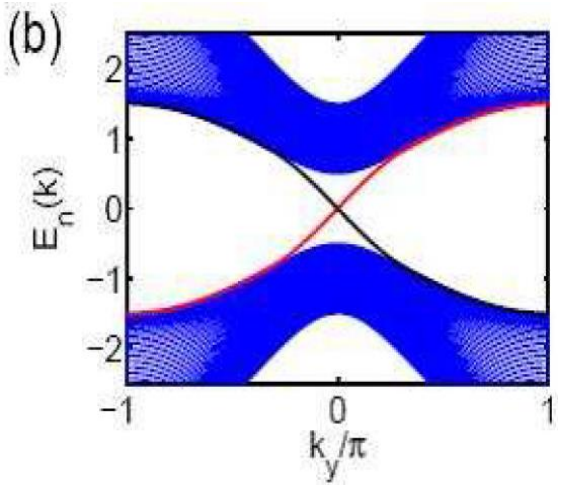


Quantum Anomalous Hall Insulators

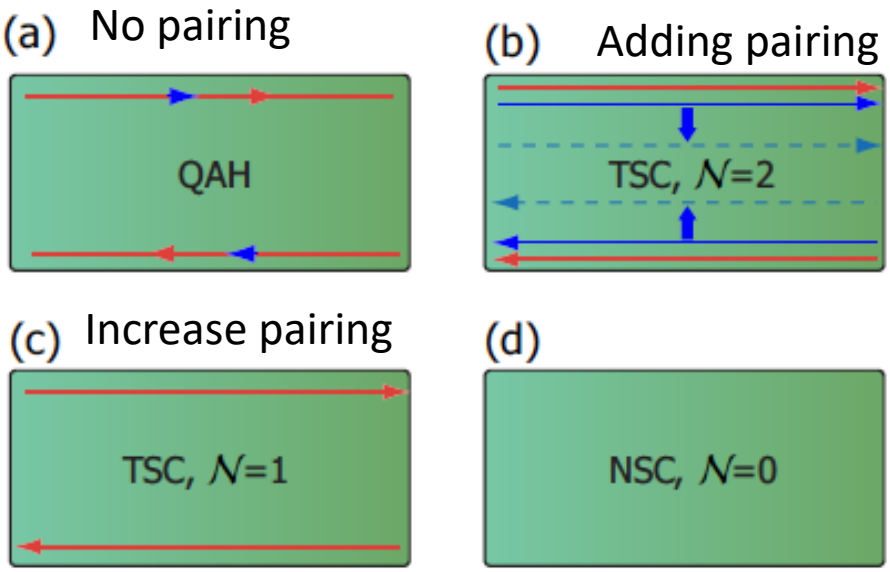




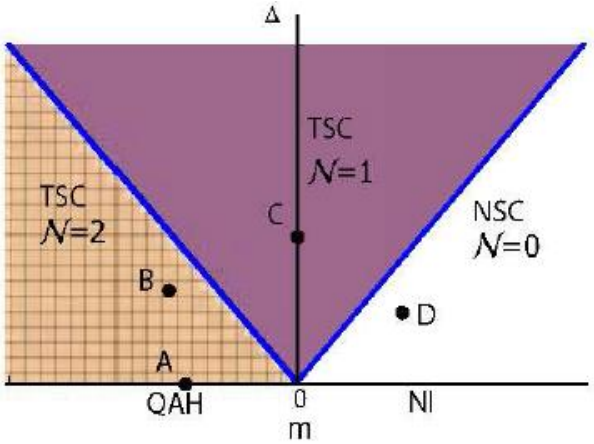
CZ Chang et al. Science 2013.



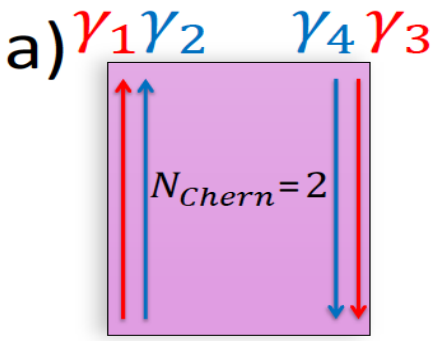
Qi et al. 2010 PRB

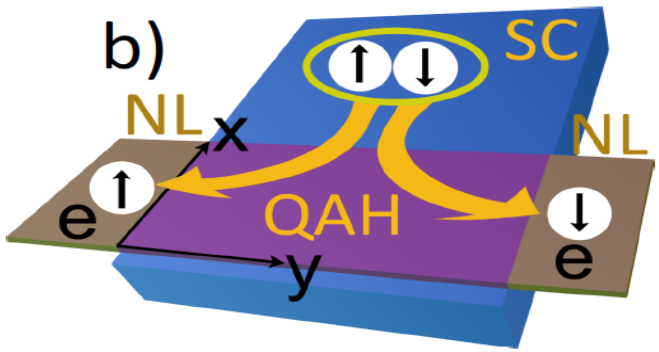


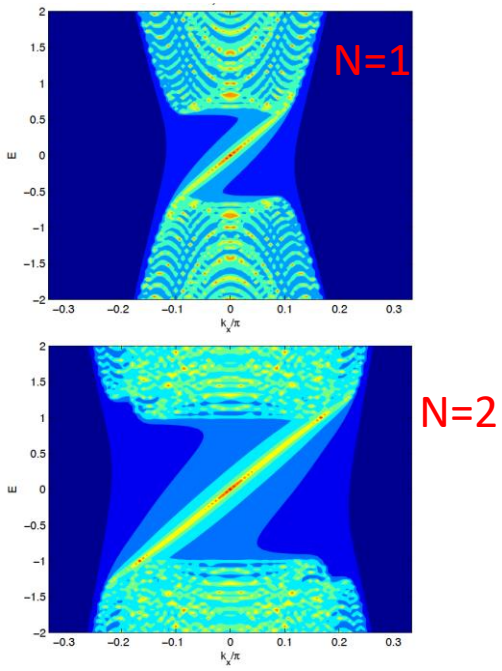
Quantum Anomalous Hall Insulator with SC

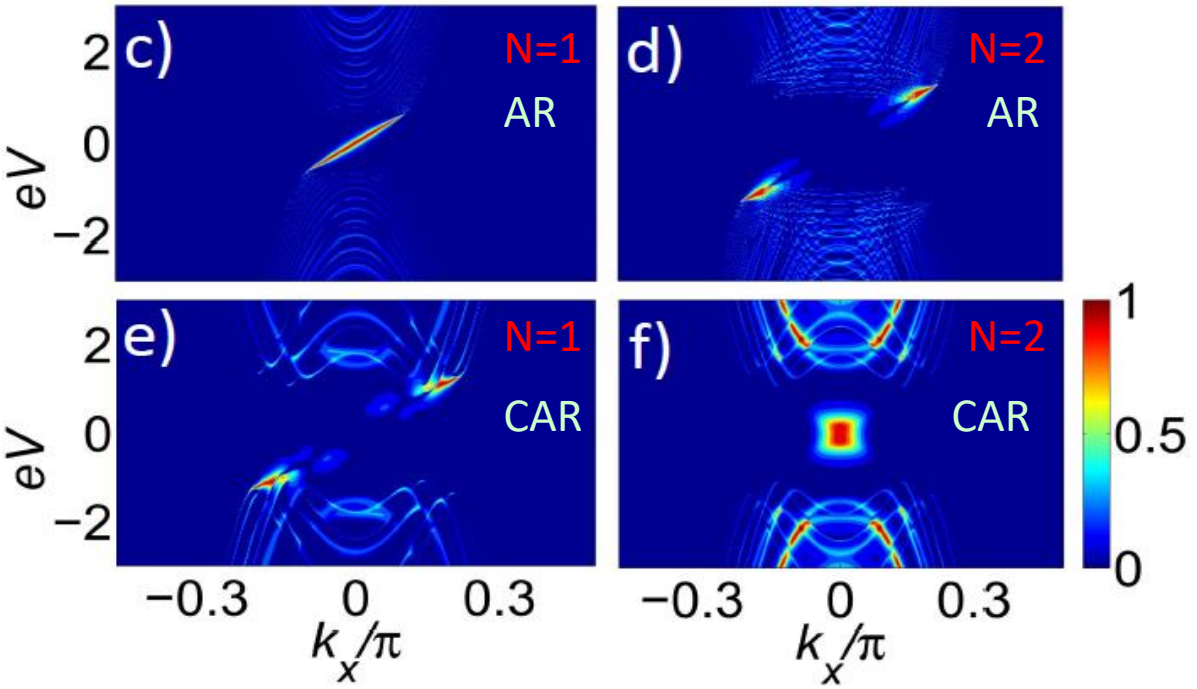


Qi et al. 2010 PRB









감사합니다