Center for Mathematics and Theoretical Physics

Quantum Field Theory Aspects of Condensed Matter Physics
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Laboratori Nazionali di Frascati -- INFN

The KZ Anomaly in Topological Insulators: An Introduction

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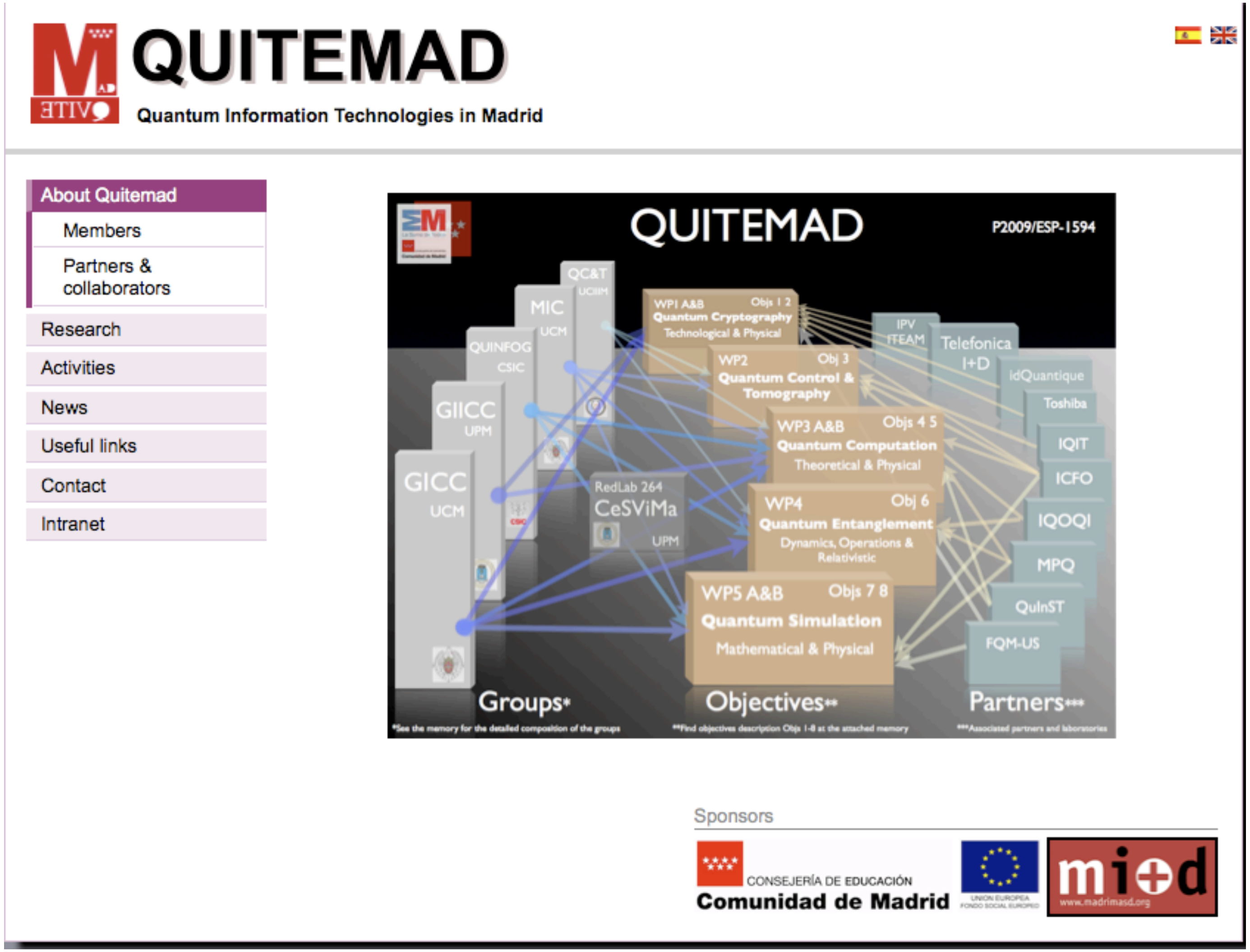


Outline of the Seminar

I. Why Topological Effects in Quantum Physics

II. Quantum Insulating States: Review

III. Topological Insulators: Definition and some Examples

IV. The KZ Anomaly in Topological Insulators

References

"Topology induced anomalous defect production by crossing a quantum critical point",
A. Bermudez, D. Patanè, L. Amico, M. A. Martin-Delgado
arXiv:0811.3843 **(2009). Phys. Rev. Lett. 102, 135702, (2009)**

"Dynamical delocalization of Majorana edge states by sweeping across a quantum critical point",
A. Bermudez, L. Amico, M. A. Martin-Delgado
arXiv:0907.3134 **(2009). New J. of Phys. (2010)**

This year curiosity

- Year 2011 is amazingly prime:
- 2011 = 157 + 163 + 167 + 173 + 179 + 181 + 191 + 193 + 197 + 199 + 211

is prime and sum of 11 successive primes (ending in 211… similar to 2011)

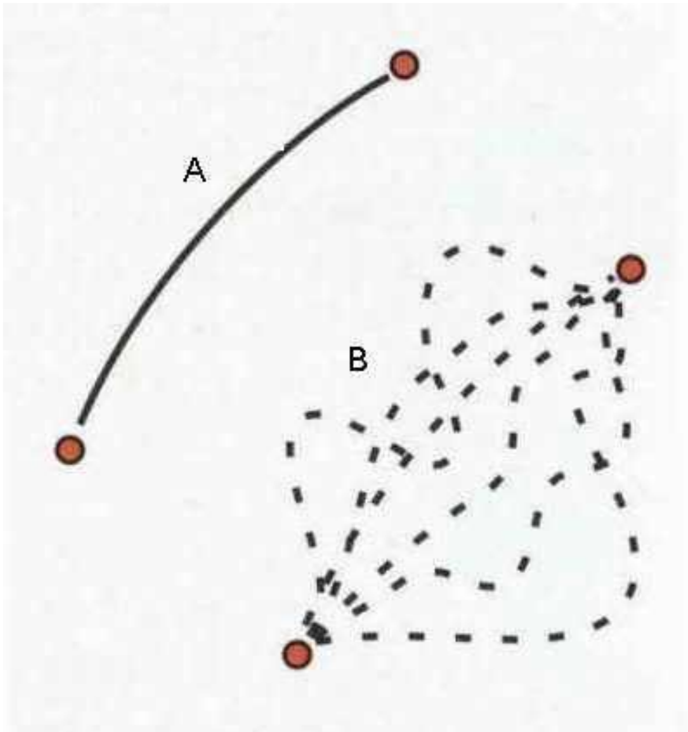
No gaps! (prime syzygy, alignment)

mathematicians should substitute ‘eclipses’ by ‘prime syzygys’

I. Why Topological Effects in Quantum Physics

Classical vs. Quantum

Path Integral Picture:



I. Why Topological Effects in Quantum Physics

Classical vs. Quantum

Operator Picture: Superposition Principle

$$|\Psi(t)\rangle = |x_1;t\rangle + |x_2;t\rangle + \ldots |x_N;t\rangle$$ **Space representation**

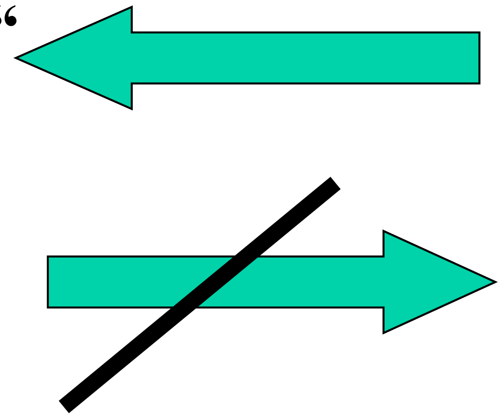
$$= |p_1;t\rangle + |p_2;t\rangle + \ldots |p_N;t\rangle$$ **Momentum**

$$= |\gamma_1;t\rangle + |\gamma_2;t\rangle + \ldots |\gamma_N;t\rangle$$ **Internal Parameters**

I. Why Topological Effects in Quantum Physics

Classical vs. Quantum

Quantum “ ” Topological Effect



II. Quantum Insulating States: Review

Insulating State: a state that cannot propagate a certain type of degrees of freedom, i.e., no current.

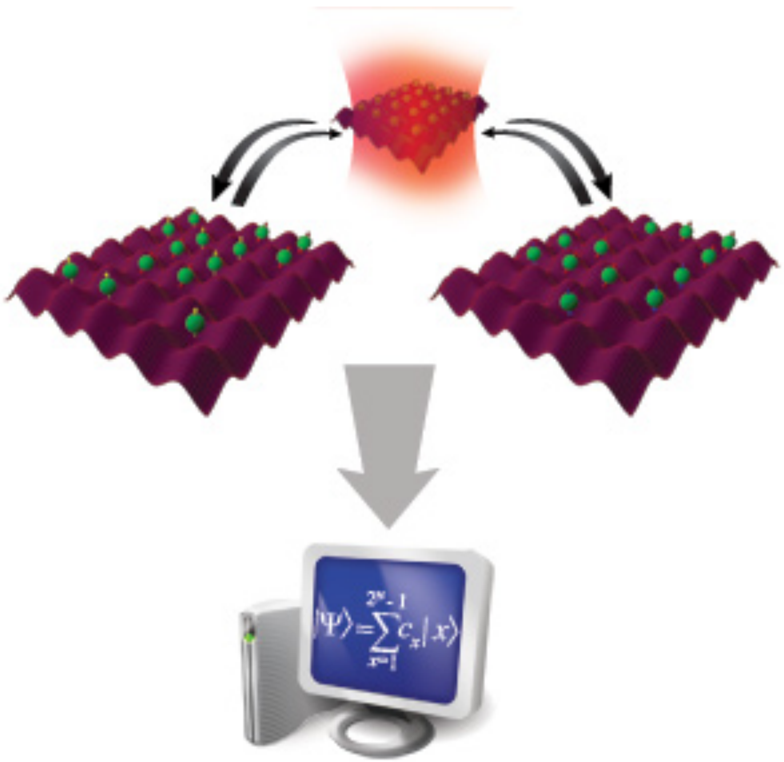
Ex: no transport of electric current

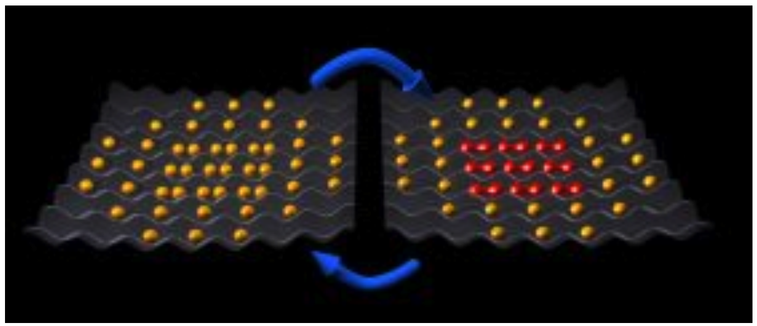
Signature: the quantum state presents LOCALIZATION as opposed to scattering states.

II. Quantum Insulating States: Review

Very popular now in Quantum Information:

Mott Insulators





II. Quantum Insulating States: Review

CLASSES OF INSULATORS:

For every different localization mechanism, a possible insulator

- **Band Insulators (Normal Insulators)**
- **Mott Insulators**
- **Anderson Insulators**
- **Peierls Insulators**

…..

Yet, there are more → **Topological Insulators**

II. Quantum Insulating States: Review

CLASSES OF INSULATORS:

- **Band Insulators (Normal Insulators)**

Mechanism: Fermi level at the gap of a conduction band
Gap + Pauli Principle

- **Mott Insulators**
- **Peierls Insulators**
- **Anderson Insulators**

…..

II. Quantum Insulating States: Review

- **Band Insulators (Normal Insulators)**

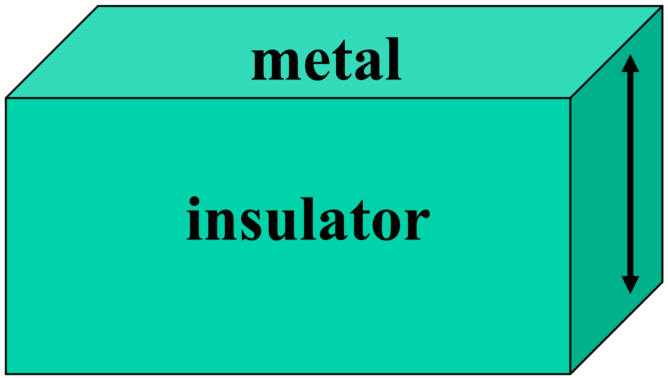
Mechanism: Fermi level at the gap of a conduction band
Gap + Pauli Principle

III. Topological Insulators: Definition and some Examples

1) Bulk: it is an insulator (gapped modes), due to some kind of localization mechanism (band, Mott, Peierls, Anderson

2) Boundary: it is a metal (gapless modes)

3) Characterized by some Topological Number (stable against local pertubations)



Quantum Phase Transition:
From bulk to boundary

III. Topological Insulators: Definition and some Examples

→ **We can talk about several types of topological insulators**

• Topological Band Insulators = "Topological Insulators", but also …

• Topological Mott Insulators

• Topological Peierls Insulators

• Topological Anderson Insulators etc…

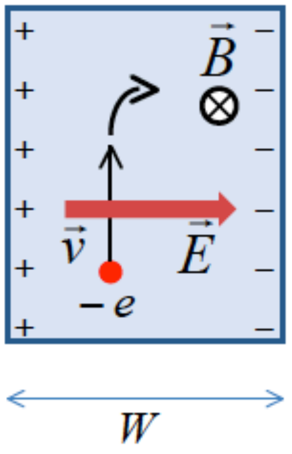
UNEXPLORED!

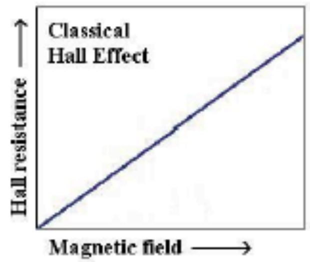
III. Topological Insulators: Definition and some Examples

We shall focus on Topological Band Insulators
= TOPOLOGIAL INSULATORS

Prominent example: quantum Hall effect

- Classical Hall effect





n : electron density

Electric current $I = -nevW$

Electric field $E = \frac{v}{c} B$

Hall voltage $V_H = EW = \frac{B}{-ne} I$

Hall resistance $R_H = \frac{B}{-ne}$

Hall conductance $\sigma_{xy} = \frac{1}{R_H}$

Lorentz force

$\vec{F} = -e\vec{v} \times \vec{B}$

Topological Orders in Condensed Matter

Characteristics of the topological degeneracy

(i) Degeneracy (# of g.s.) depending on the topology of the system (sphere, torus….)

(ii) Absence of the local order parameter

Emblematic Example: Fractional Quantum Hall Liquids (FQH)

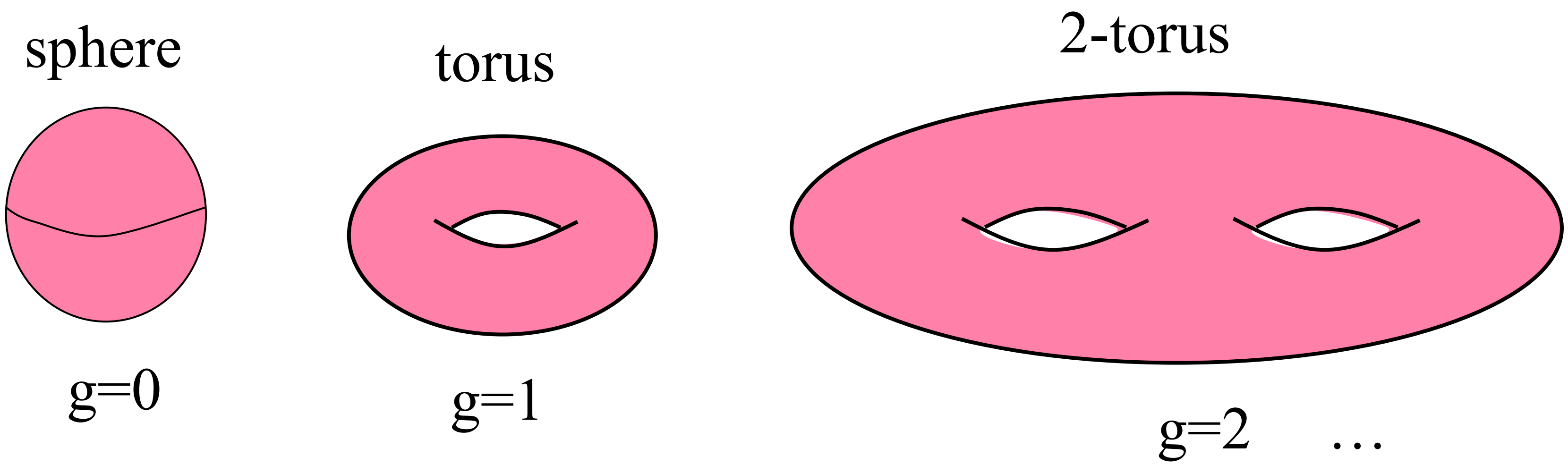
FQH systems contain many different phases at T=0 which have the same symmetry

filling fraction $\nu = \frac{\text{density of electrons}}{\text{density of magnetic flux quanta}}$

Those phases cannot be distinguished by symmetries

→ Cannot be described by Landau's SSB

Topological Orders in Condensed Matter



FQH liquid (Laughlin)

$$\Psi_m = \left[\prod_{i\neq j}(z_i - z_j)^m\right] e^{-\frac{1}{4l_B^2}\sum_i |z_i|^2}$$

Fractional Charge $e^* = e/m$

G.S. Degeneracy $D(\Psi_0) = m^g$

Proposal: FQH states contain a new kind of order, Topological Order

Topological order is new because it cannot be described by a Symmetry Breaking or Local Order Parameter

None of the usual tools that we have used to characterize a phase apply to Topological Order

Challenges for New States of Matter…

Particle Physics

The Standard Model

Quantum Gauge Theory (Continuous)

SU(3)xSU(2)xU(1)

Strongly Correlated Systems

The Standard Model

Fermi Liquid Theory
+
SSB = Spontaneous Symmetry Breaking
+
RG = Renormalization Group

IN STRONGLY CORRELATED SYSTEMS,
BEYOND THE STANDARD MODEL IS POSSIBLE

==> NEW PHYSICS

EXAMPLES:

- **LUTTINGER LIQUID**
- **SPIN-CHARGE SEPARATION**
- **HIGH-TC SUPERCONDUCTIVITY**
- **TOPOLOGICAL ORDERS (Fractional QHE, etc.)**

STRONGLY CORRELATED SYSTEMS

HAMILTONIAN SYMMETRIES

Global Symmetries

Ej: SU(2) rotation
Heisenberg like

Continuous Symmetries

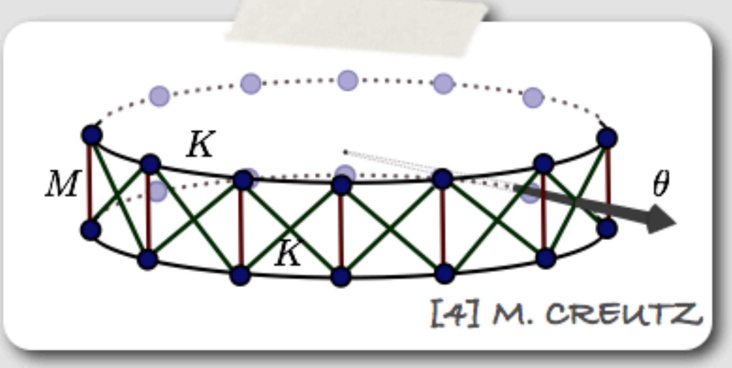
Local Symmetries
(Quantum Gauge Theories)

Ej: Z_2, Z_2xZ_2
Kitaev, Color Codes

Discrete Symmetries

III. Topological Insulators: Definition and some Examples

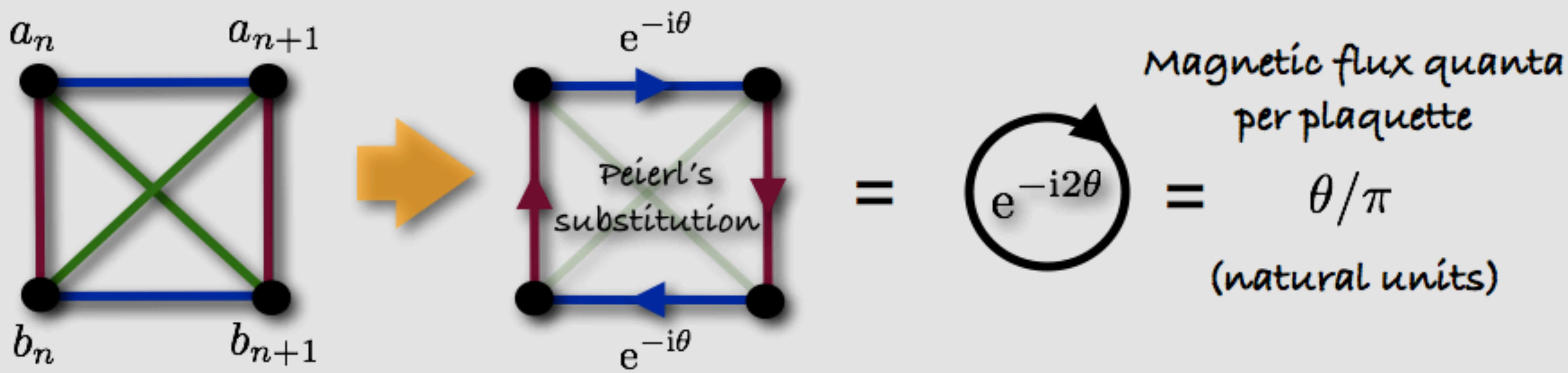
Creutz Ladder: Topological Edge States



We consider a system of **spinless fermions hopping** in a ladder along **horizontal**, **vertical**, and **diagonal** links, and subjected to an external **magnetic field**

horizontal hopping — diagonal hopping — vertical hopping

$$H = -\sum_n \Big[K\left(e^{-i\theta} a^\dagger_{n+1} a_n + e^{i\theta} b^\dagger_{n+1} b_n\right) + K\left(b^\dagger_{n+1} a_n + a^\dagger_{n+1} b_n\right) + M a^\dagger_n b_n + \text{h.c.} \Big]$$



III. Topological Insulators: Definition and some Examples

We shall first study the ladder with **periodic boundary conditions**, where one finds a **two-band insulator** $\epsilon = E/2K$

$$\epsilon(q) = \cos q \cos\theta \pm \sqrt{\sin^2 q \sin^2\theta + (m + \cos q)^2}$$

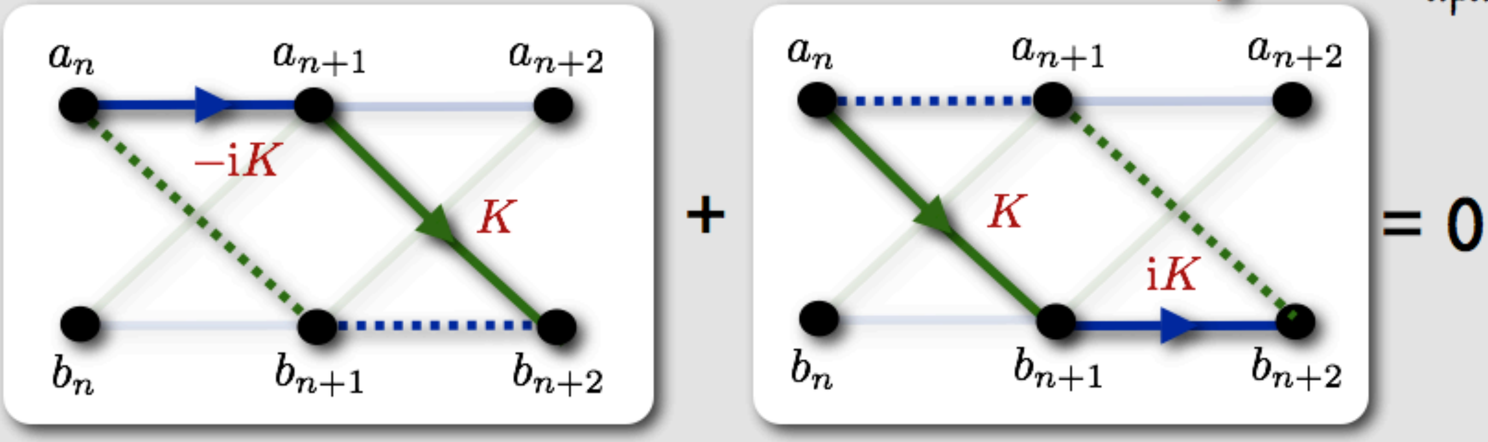
delocalised solutions with well-defined momentum

We shall focus on the case of **vanishing vertical hopping**

In the limit of '<u>half a quantum flux per plaquette</u>' $\theta = \pm\pi/2$, one gets <u>flat bands</u> $E = \pm 2K$, and thus nothing moves

$$v_g \sim \partial_q E = 0$$

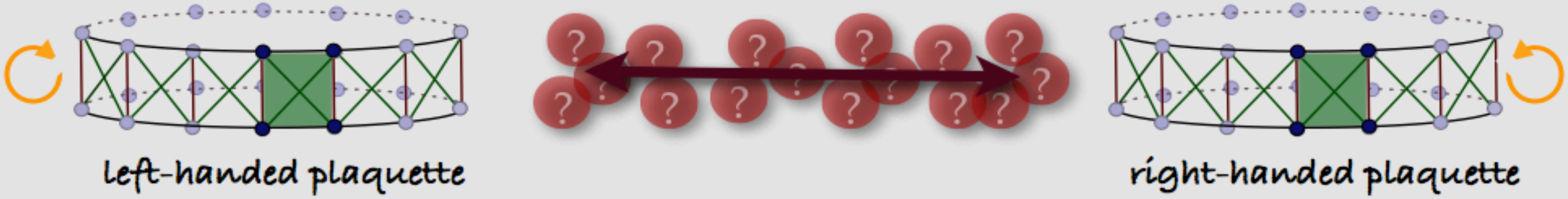
Fermions cannot tunnel two-sites apart!



<u>Solutions will be localised within a single plaquette</u>

III. Topological Insulators: Definition and some Examples

Now, we study the effects of a varying flux $\theta \in [-\pi/2, \pi/2]$



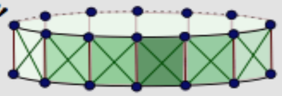
Fermions become delocalised away from the half-flux regime,
and there is a critical point at $\theta_c = 0$.

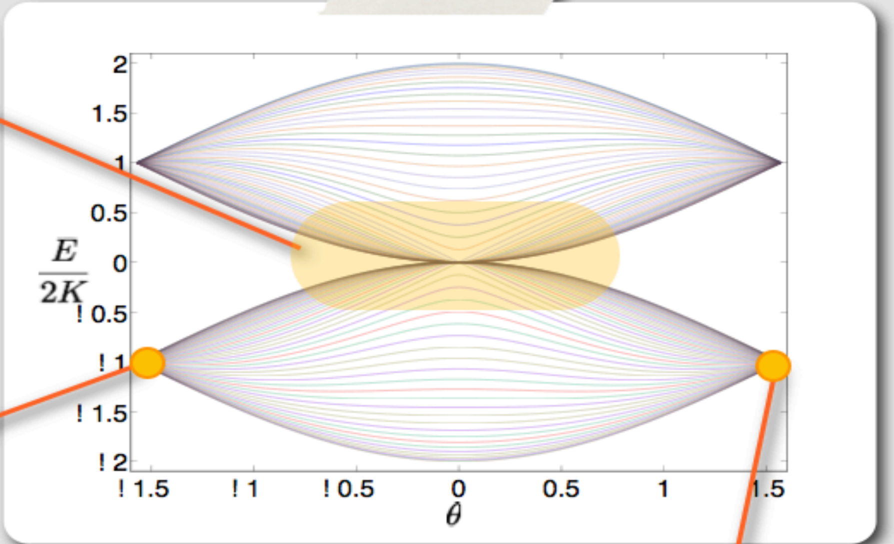
III. Topological Insulators: Definition and some Examples

We shall study the defect produced by the **adiabatic flux quench** across the critical point, and compare the two **topology-inequivalent cases (** plaquette-like states and edge states)

a) Periodic Ladder: Initial ground state corresponds to a plaquette-fermion

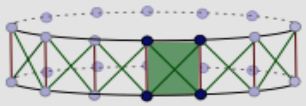
In the critical region, delocalised defects are created





The underlying excitation mechanism is a collection of L - Landau-Zener processes

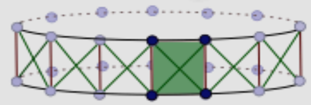
Initial plaquette fermion $\theta = -\pi/2$,



$$|-2K\rangle_n \sim (-ia_n^\dagger + b_n^\dagger + a_{n+1}^\dagger - ib_{n+1}^\dagger)|0\rangle$$

This plaquette g.s. populates every level in the lower band as $\theta(t) > -\pi/2$.

Adiabatically connected plaquette fermion $\theta = \pi/2$,

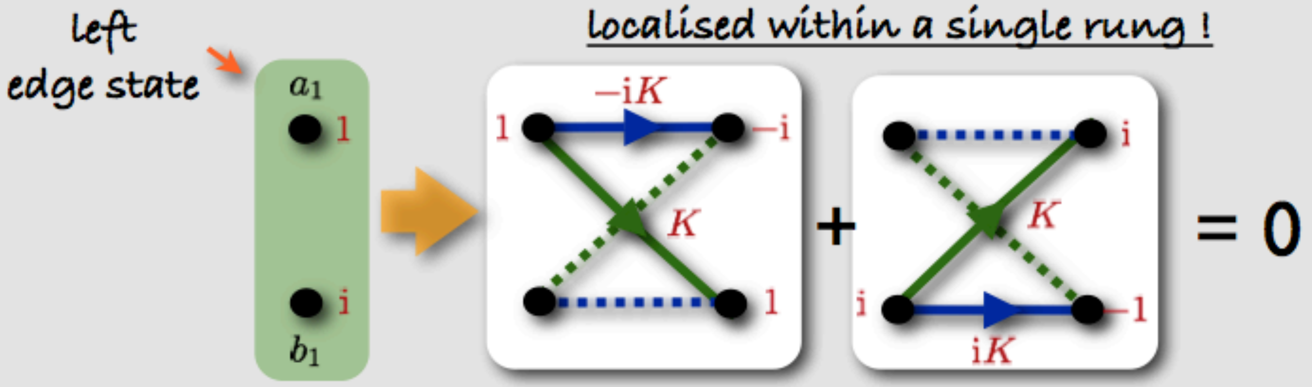


$$|-2K\rangle_n \sim (+ia_n^\dagger + b_n^\dagger + a_{n+1}^\dagger + ib_{n+1}^\dagger)|0\rangle$$

III. Topological Insulators: Definition and some Examples

If we now change the topology of the system by **opening the ladder**, we find two additional solutions pinned at the boundaries, the so called **edge states**

In the limit of '<u>half a quantum flux per plaquette</u>' $\theta = \pm\pi/2$,



These edge states have **zero-energy** and are somehow linked to the ladder topology (they are indeed a hallmark of topological order)

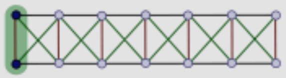
In following sections, we shall study the production of defects across the zero-flux critical point, and present deviations from the KZ scaling

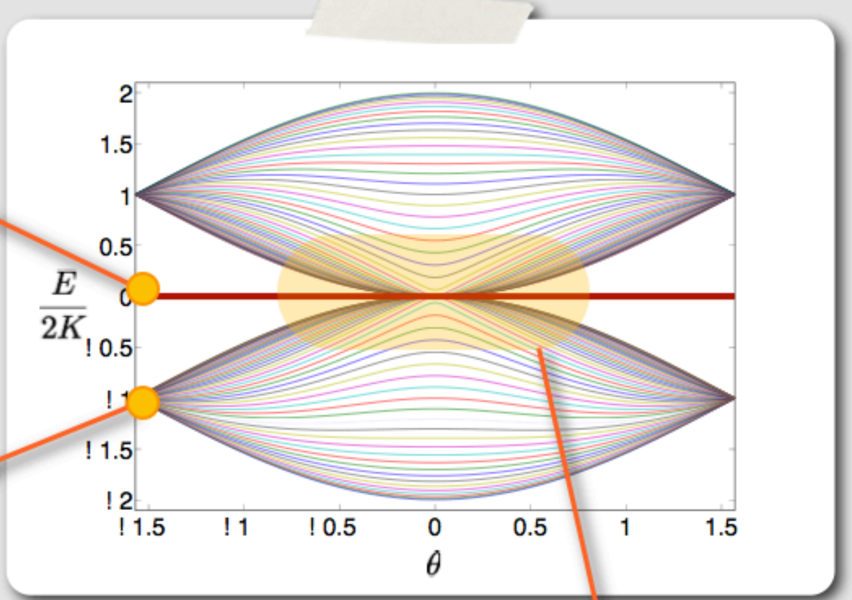
III. Topological Insulators: Definition and some Examples

b) Open Ladder: In this case, the quantum phase transition resembles the corresponding periodic ladder, but two additional zero-energy edge states appear. Hence, we have two types of localised initial states

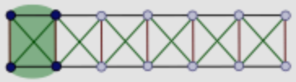
1 Topological

In-gap edge states pinned at the ladder boundaries $|l\rangle \sim (a_1^\dagger - ib_1^\dagger)|0\rangle$

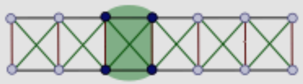




2 Non-topological



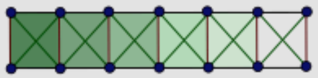
boundary plaquettes



bulk plaquettes

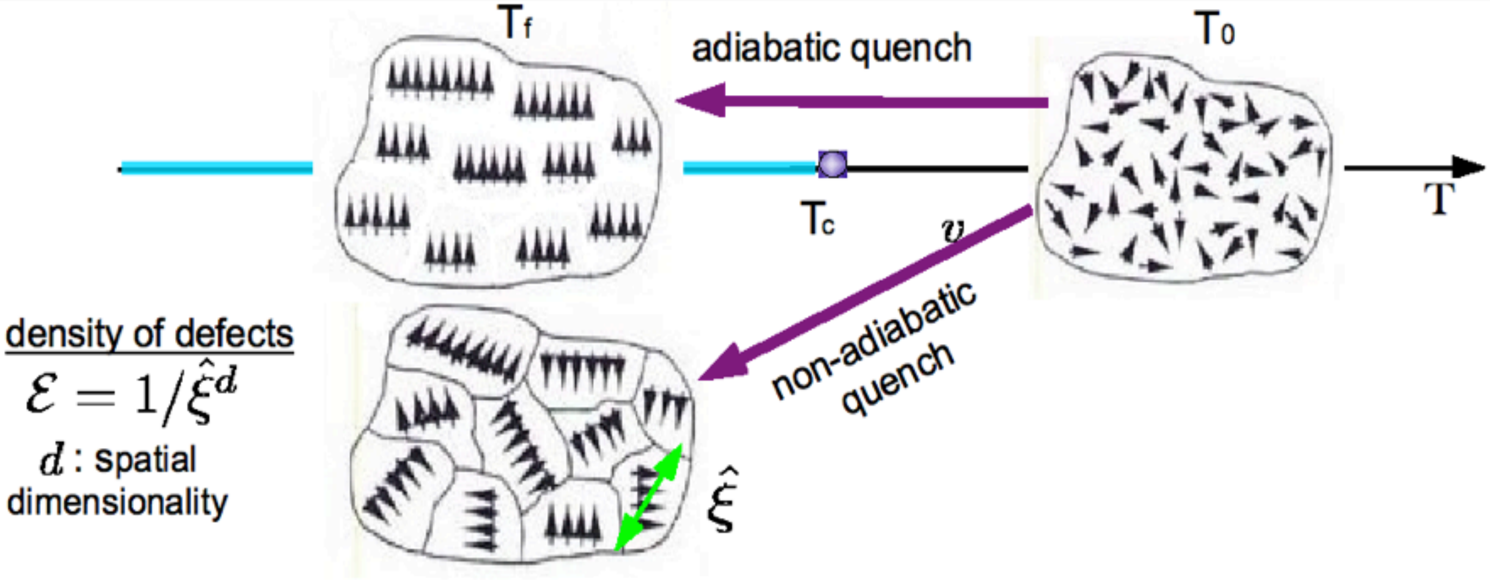
The excitation mechanism are no longer uncoupled Landau-Zener processes, but rather a collective effect in the whole energy band

In the critical region, delocalised defects are created



IV. The KZ Anomaly in Topological Insulators

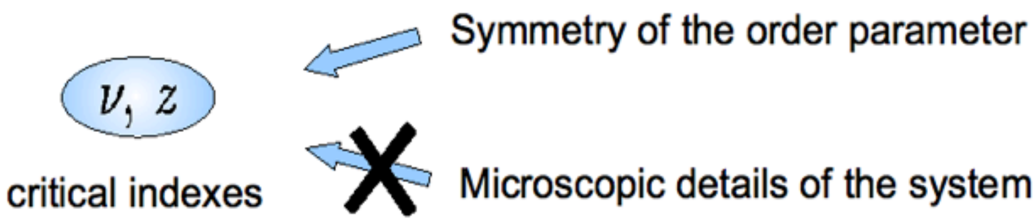
Dynamics of phase transitions: Kibble-Zurek mechanism



Kibble, *J. Phys A* (1976)
Zurek, *Nature* (1985)

$$\mathcal{E}_{KZ} \propto v^{\frac{d\nu}{1+\nu z}}$$

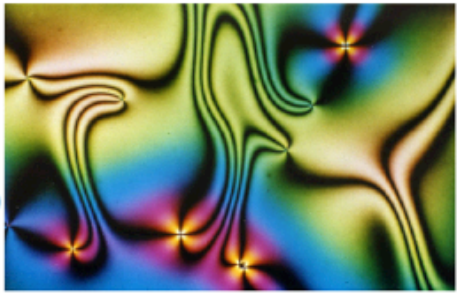
Universality

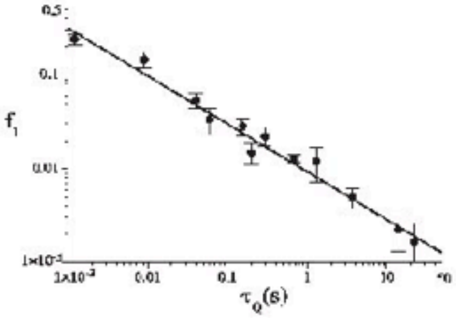


Experimental Tests of the KZ scaling thermal phase transitions

Liquid cristals

Chuang et al., Science (1991)
Bowick et al., Science (1994)



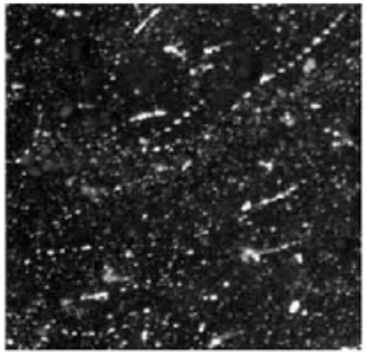


Superconducting rings

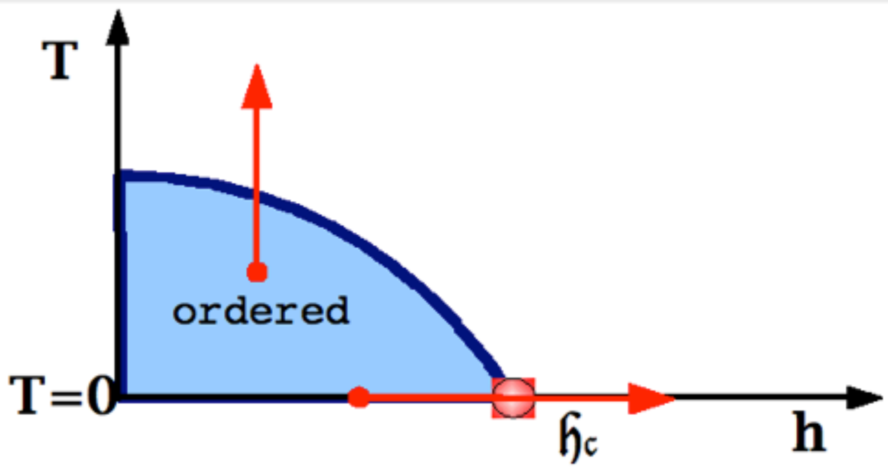
Monaco et al. PRL (2006)
Carmi et al. PRL (2000)
Maniv et al. PRL (2003)

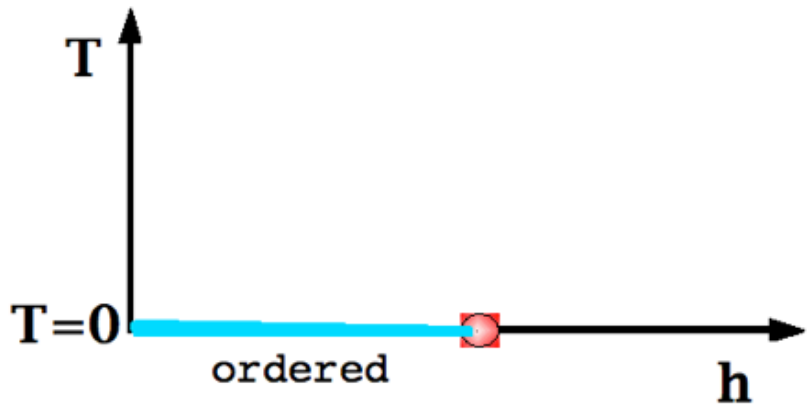
Superfluid systems

Hendry et al., Nature (1994)
Dodd et al., PRL (1998)
Ruutu et al. Nature (1996)
Baurle et al., Nature (1996)



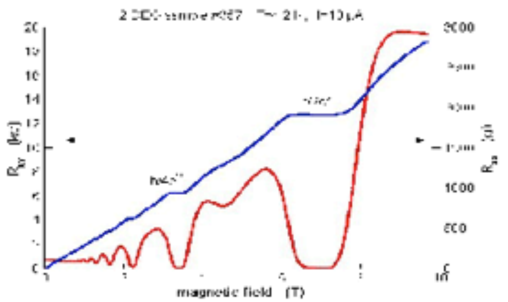
What is a Quantum Phase Transition?



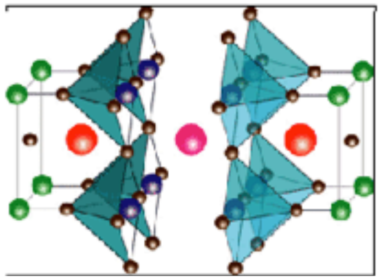


	"thermal" PT	QPT
control parameter	temperature	Pressure, doping, magnetic field, ...
fluctuations	thermal	quantum (Heisenberg principle)

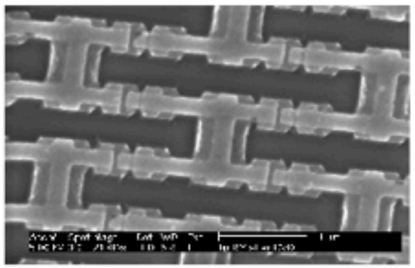
Quantum Hall effect



Antiferromagnetism in cuprate High Tc superconductors

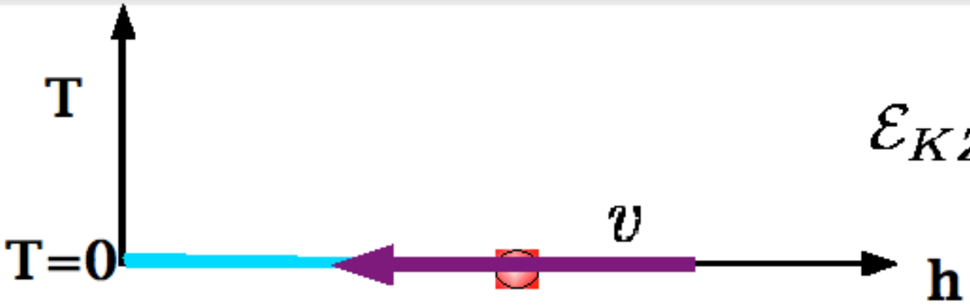


Josephson Junctions Array



IV. The KZ Anomaly in Topological Insulators

Tests of the KZ scaling — quantum phase transitions

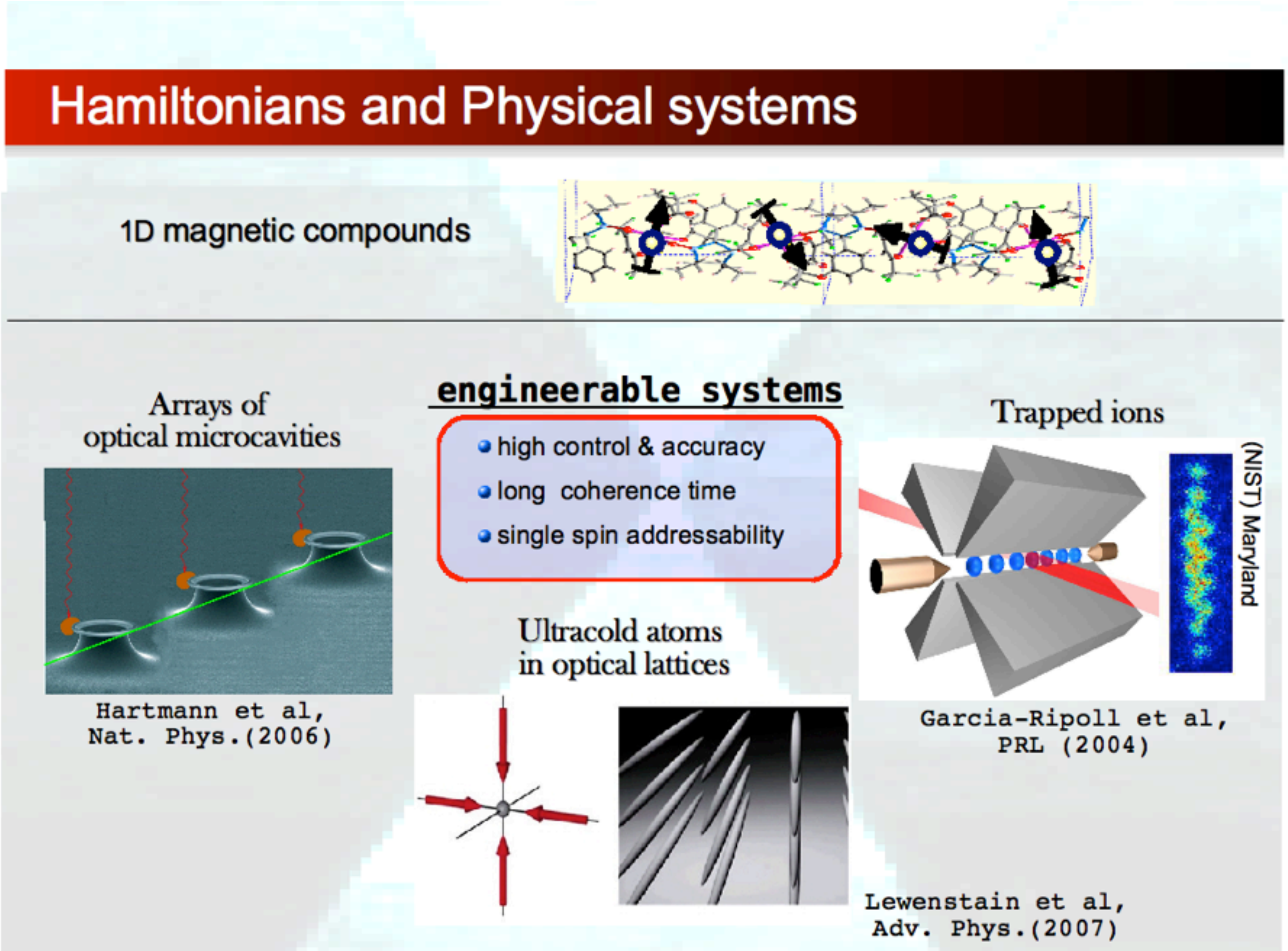


$$\mathcal{E}_{KZ} \propto v^{\frac{d\nu}{1-\nu z}}$$

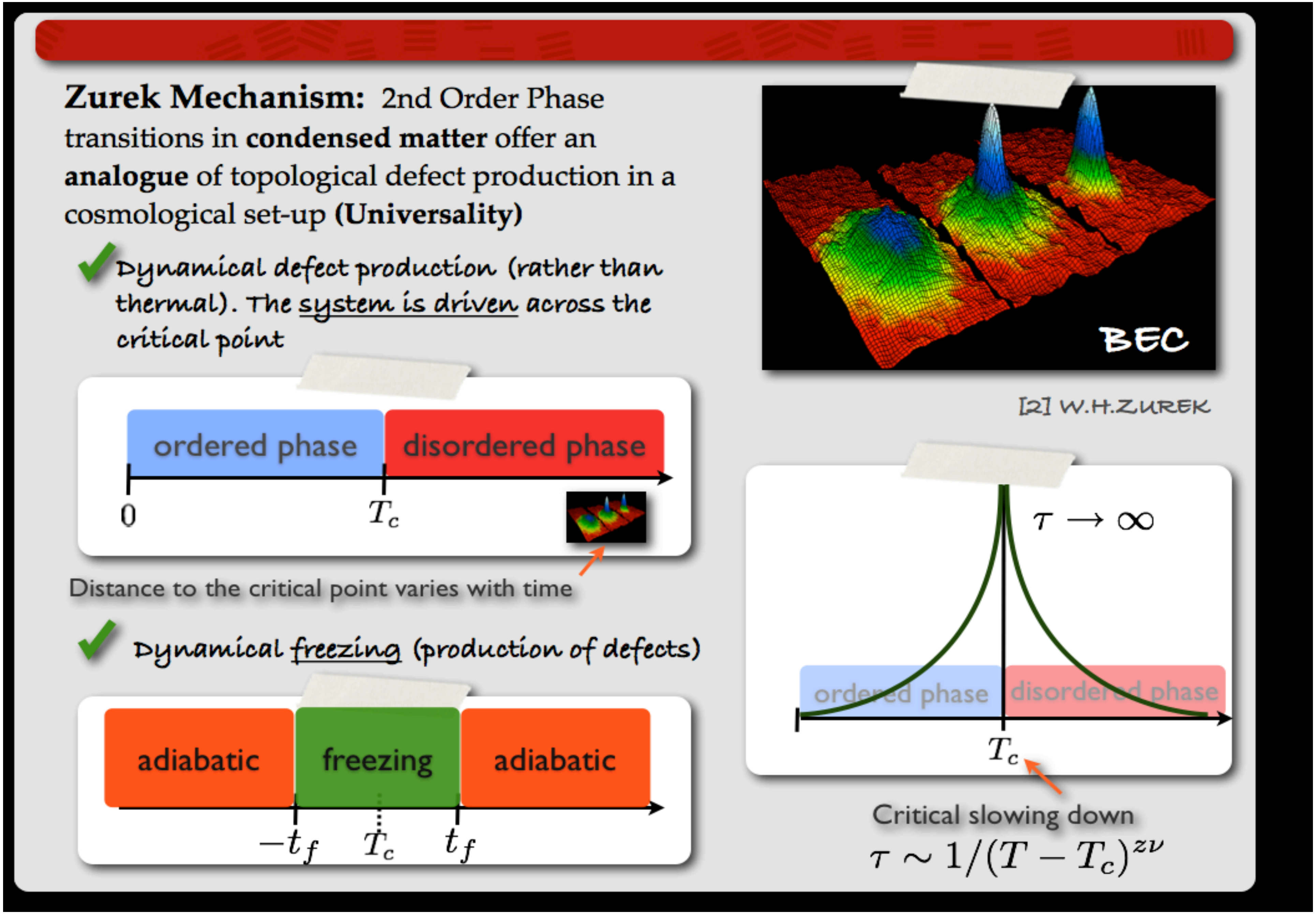
Theory

Spin models	Polkovnikov, PRB (2005); Zurek et. Al, PRL (2005); Dziarmaga, PRL (2005); Sengupta et al, PRL (2008).
Cold atoms in optical lattices	Lamacraft, PRL (2007); Damski et al, PRL (2007); Cucchietti et al, PRA (2007).

IV. The KZ Anomaly in Topological Insulators



IV. The KZ Anomaly in Topological Insulators



IV. The KZ Anomaly in Topological Insulators

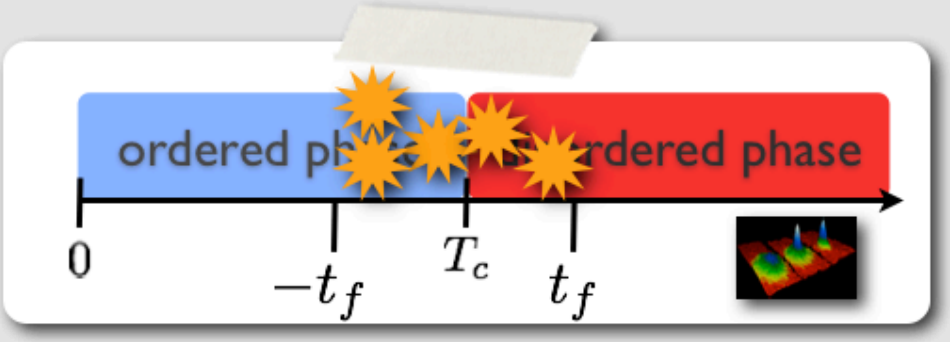
- Linear quench $T - T_c \sim v_q t$
- Defects are produced when the system 'freezes' $t_f = \tau \sim 1/(T-T_c)^{z\nu}$
- Density of defects scales as $n \sim 1/\xi^d \quad \xi \sim 1/(T-T_c)^{\nu}$

The density of defects witnesses the symmetry breaking phase transition

non-equilibrium effects predicted from equilibrium critical scaling

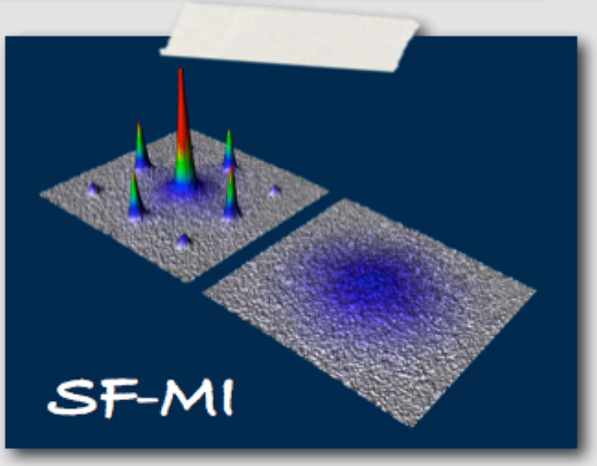
$$n_d \sim v_q^{\frac{d\nu}{1+z\nu}}$$

Production of defects depends upon the phase transition universality class



IV. The KZ Anomaly in Topological Insulators

KZ scaling was initially tested on a variety of thermal phase transitions (mean-field, Landau-Ginzburg 1D, 2D, 3D, BEC, Ising 2D...).



What happens at T=0 ?

Quantum phase transitions describe the abrupt change of the system's ground state as some parameters of the Hamiltonian are modified.

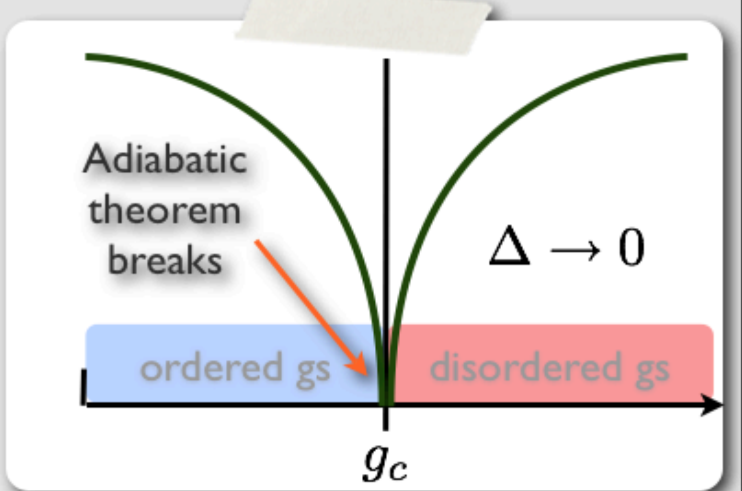
At T=0, the critical fluctuations (long wavelengths) must be treated quantum mechanically, and are characterized by a **vanishing energy gap**

$$\Delta \sim |g - g_c|^{z\nu}$$

$$\xi \sim \frac{1}{\Delta^{1/z}} \qquad \tau \sim \frac{1}{\Delta}$$

diverging lengths

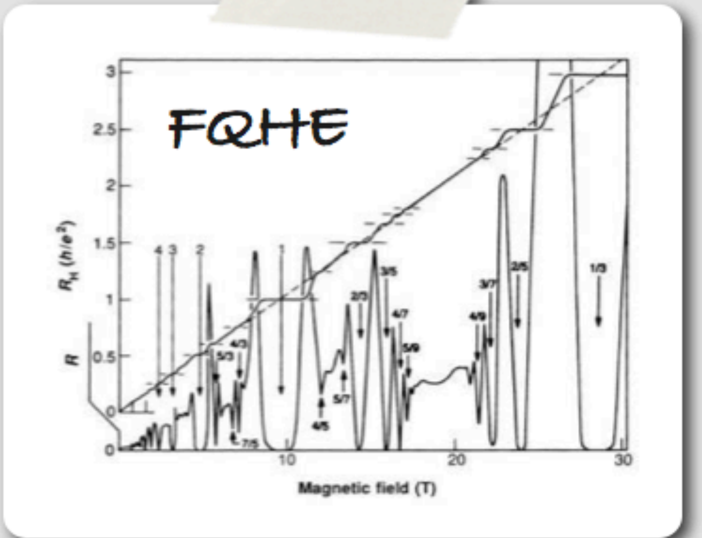
$$n_d \sim v_q^{\frac{d\nu}{1+z\nu}}$$



[3] ZUREK, DORNER, ZOLLER

IV. The KZ Anomaly in Topological Insulators

KZ scaling has been tested on a wide variety of 2nd order quantum phase transitions (Quantum Ising model, Bose-Hubbard...). It is rooted in:



Symmetry breaking

Local order parameter

Finite propagation speed

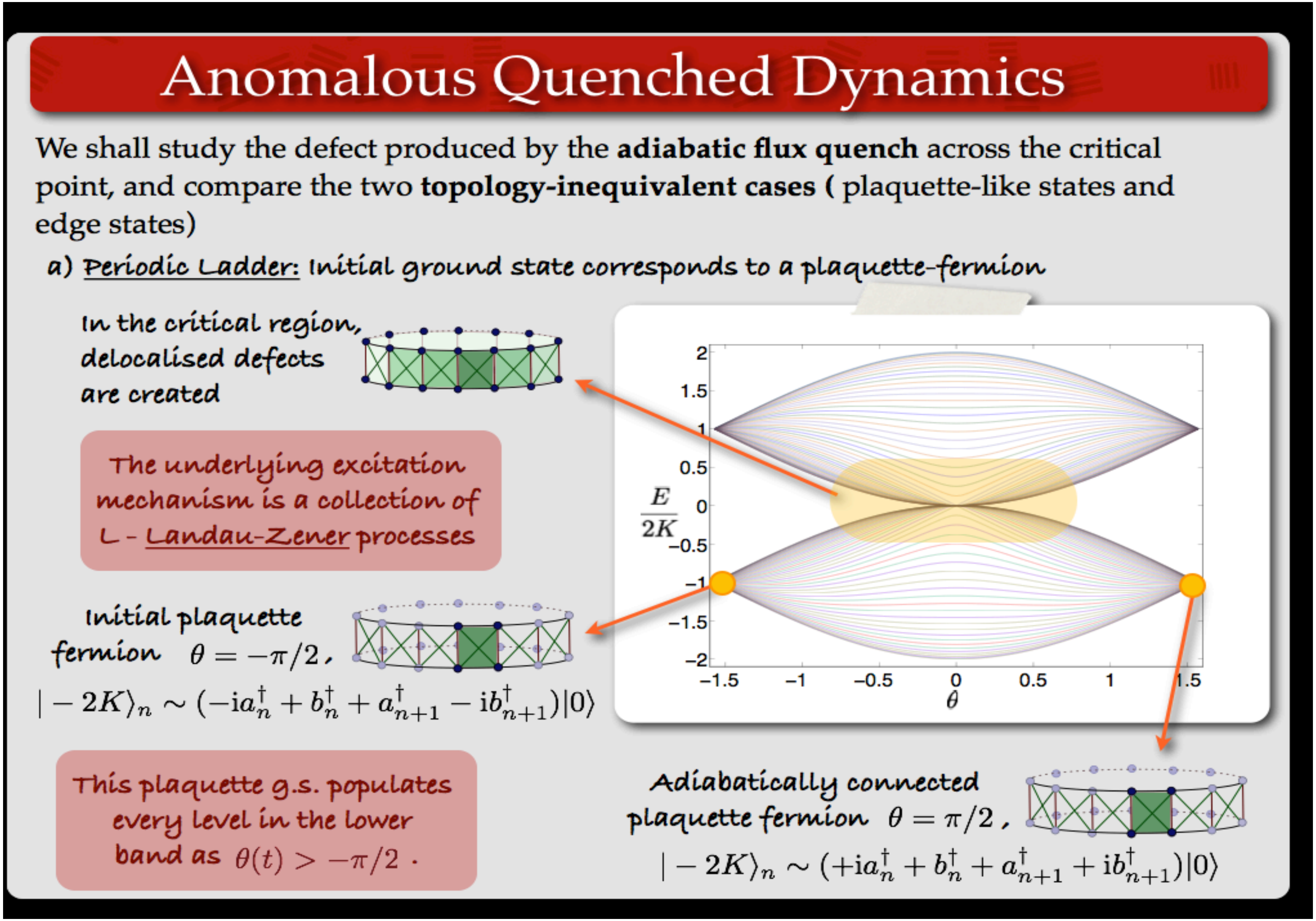
Causality

What happens beyond the Landau symmetry-breaking description ?

Beyond the Landau paradigm, phases can be characterised by **non-local order parameters.** Accordingly, KZ arguments based on causality and selection of 'local vacua' should not work here. Fingerprints of this new kind of order are

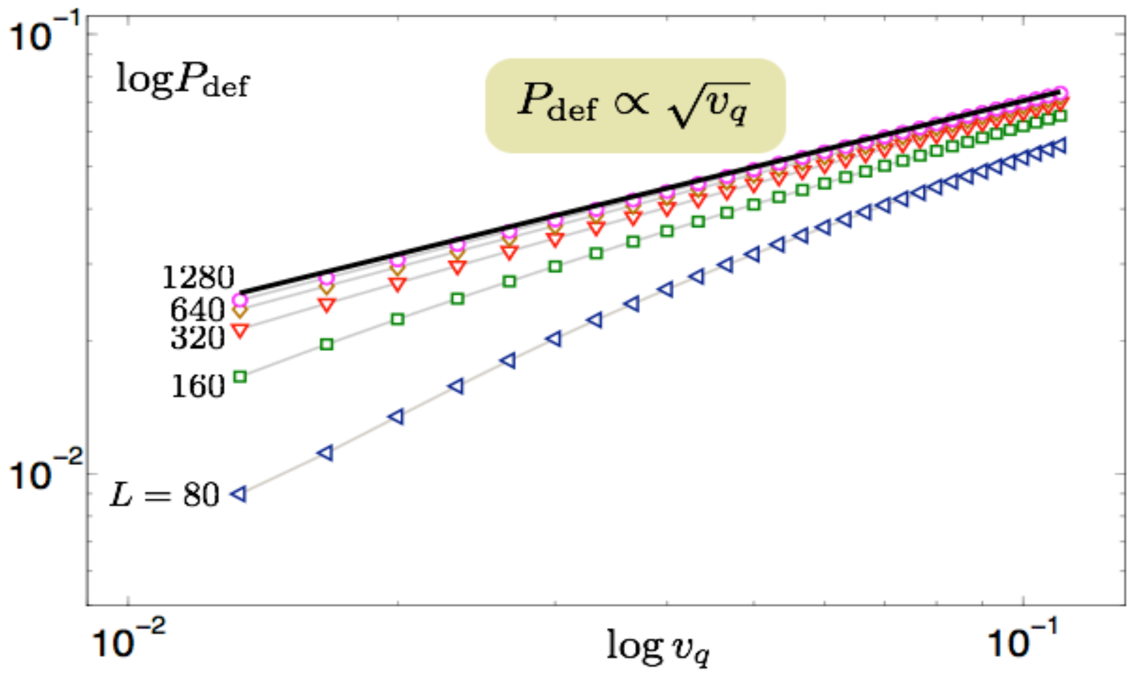
Topological order ⇄ Ground state degeneracy / edge states / anyonic excitations

IV. The KZ Anomaly in Topological Insulators



IV. The KZ Anomaly in Topological Insulators

Computing the density of produced defects, we get $P_{\rm def} = \sum_{E>0} |\langle E|\Psi(t_{\rm f})\rangle|^2$



The density of delocalised defects scales according to the KZ mechanism

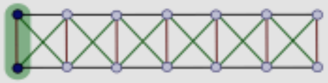
$$\nu = z = 1 \qquad P_{\rm def}^{\rm KZ} \propto v_q^{\frac{d\nu}{1+z\nu}} \propto v_q^{1/2}$$

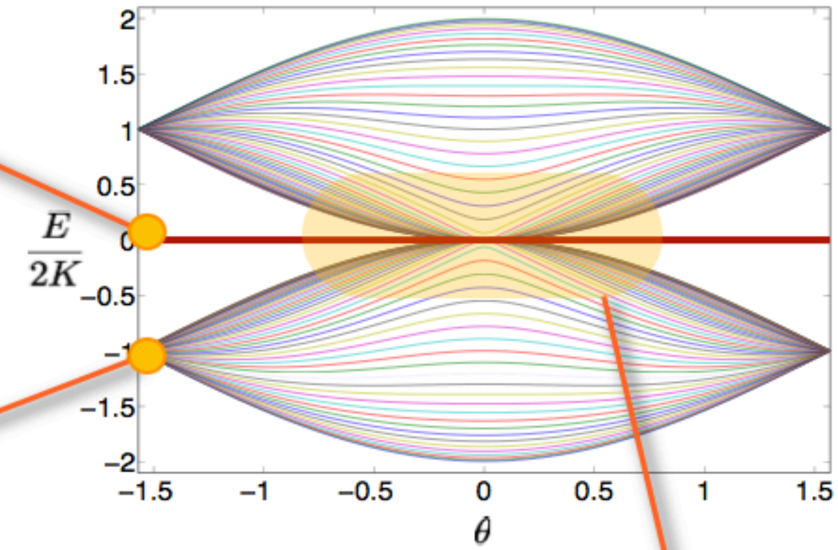
IV. The KZ Anomaly in Topological Insulators

b) Open Ladder: In this case, the quantum phase transition resembles the corresponding periodic ladder, but two additional zero-energy edge states appear. Hence, we have two types of localised initial states

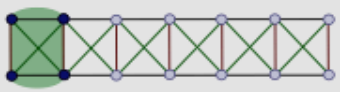
1 Topological

In-gap edge states pinned at the ladder boundaries $|l\rangle \sim (a_1^\dagger - ib_1^\dagger)|0\rangle$

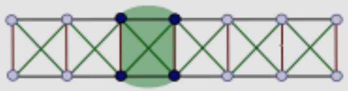




2 Non-topological

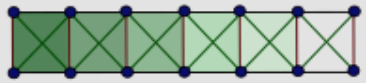


boundary plaquettes



bulk plaquettes

In the critical region, delocalised defects are created

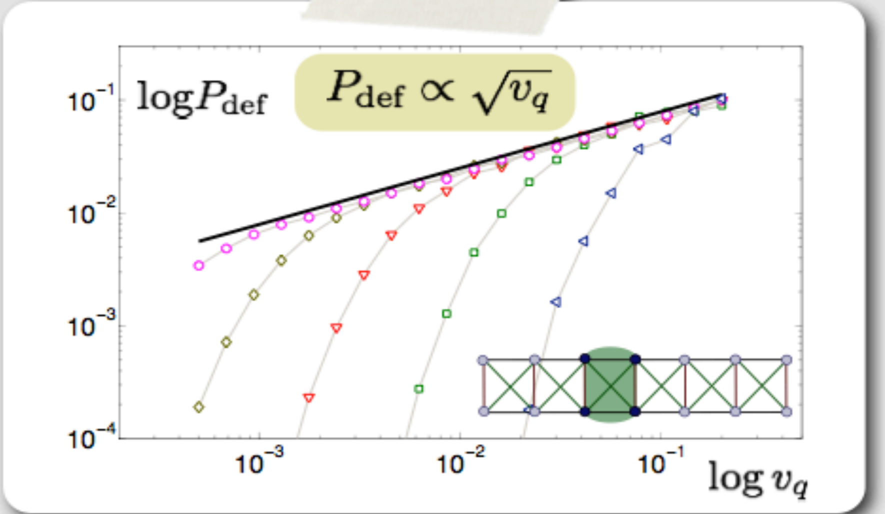


The excitation mechanism are no longer uncoupled Landau-Zener processes, but rather a collective effect in the whole energy band

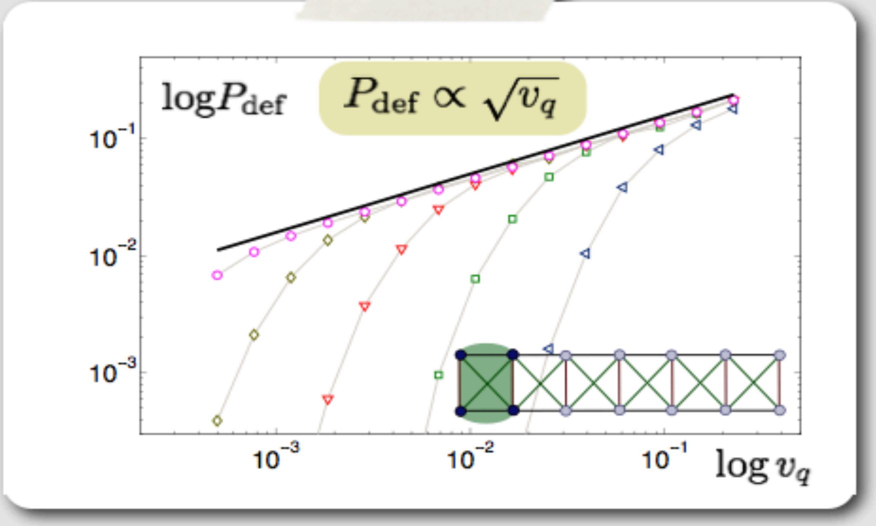
IV. The KZ Anomaly in Topological Insulators

Computing the density of produced defects, we get $P_{\rm def} = \sum_{E>0} |\langle E|\Psi(t_{\rm f})\rangle|^2$

bulk plaquettes



boundary plaquettes

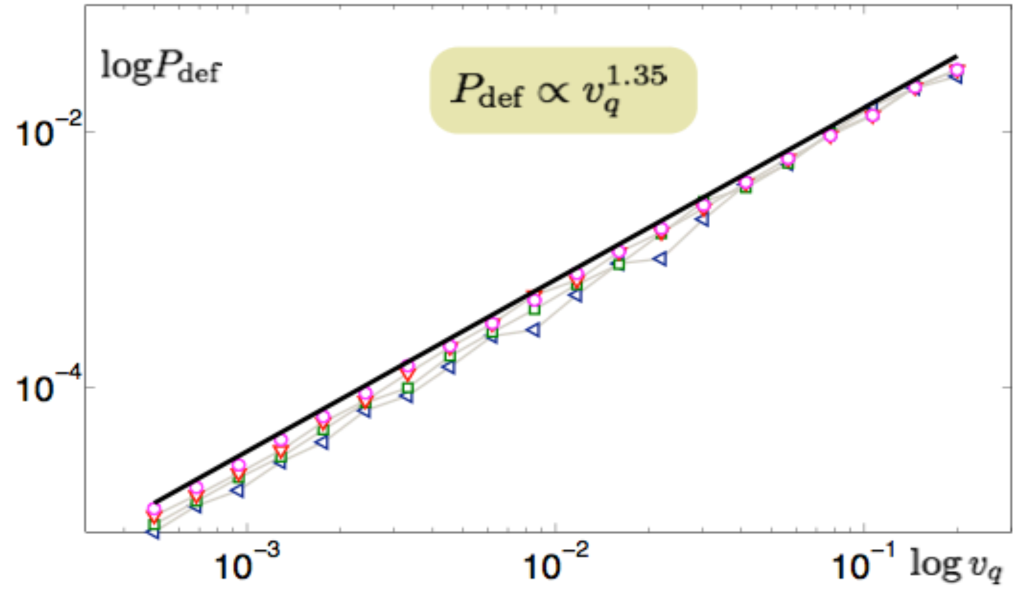


Non-topological plaquette states follow KZ mechanism regardless of their position within the ladder

$$\nu = z = 1 \qquad P_{\rm def}^{\rm KZ} \propto v_q^{\frac{d\nu}{1+z\nu}} \propto v_q^{1/2}$$

IV. The KZ Anomaly in Topological Insulators

Conversely, if the initial state is an edge state, we get the following amount of delocalised defects



Non-universal quench dynamics

Breaking of the KZ mechanism

$$P_{\mathrm{def}} \propto v_q^{1.35} \neq P_{\mathrm{def}}^{\mathrm{KZ}}$$

topology induced anomalous defect production

Edge states are more robust against defect production ($v_q \ll 1$)

edge states are decoupled from the low-energy excitations (gapless mode)

IV. The KZ Anomaly in Topological Insulators

Some Conclusions

- ☐ EDGE STATES HAVE ANOMALOUS DYNAMICS
- ☐ TOPOLOGICAL PROTECTION
- ☐ ROBUST IN A RANGE OF PARAMETERS -> CANDIDATE FOR A PROTECTED QUANTUM MEMORY (USING EDGE STATES AS QUBITS)
- ☐ NOT A SIMPLE SURFACE EFFECT (E.G. ATTRACTION)

Experimental Proposals

- THEY ARE WELCOME FOR THE CREUTZ LADDER

- ONE POSSIBILITY: SPINOR FERMI GASES IN OPTICAL LATTICES

  THE 2 SPECIES OF FERMIONIC OPERATORS IN THE LADDER MAY CORRESPOND TO THE FERMIONIC OPERATORS ASSOCIATED TO DIFFERENT SPIN COMPONENTS;

  WHEREAS THE NON-INTERACTING REGIME IS REACHED BY MEANS OF FESBACH RESONANCES.

Open Questions

Many, ...almost Any

Open Questions

i/ Role of topological charges of edges on the scaling

ii/ Extension to Non-Abelian Fields

iii/ Role of Interactions (Fermion Picture)

iv/ Models in 2D: edge states are currents (QHE)

v/ Models in 3D: edge states are Dirac Fermions (Graphene)

...

MAIN RESULT

NOVEL SIGNATURE FOR TOPOLOGICAL INSULATORS

THE KZ ANOMALY

References

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arXiv:1105.0932**. New J. of Phys. (2011)**

For now, this is it!

MANY THANKS FOR YOUR ATTENTION

THE END