①

Introduction to electron-phonon interaction

Dielectric function: density-density fluctuations:

① $\varepsilon_e(q,\omega) = 1 + \frac{4\pi e^2}{q^2}\chi(q,\omega)$

$$\chi_{FL}^{\circ}(q,\omega) = \frac{\chi_0(q,\omega)}{1+F_0^s}$$

static limit: $s = \frac{\omega}{q v_F} \ll 1$ $\quad \chi_{FL}^{\circ} = \frac{N_0}{1+F_0^s}$

$$V(q) = \frac{4\pi e^2}{q^2} \xrightarrow{\varepsilon(q,0)} V_{sc}(q) = \frac{4\pi e^2}{q^2 + k_{TF}^2}$$

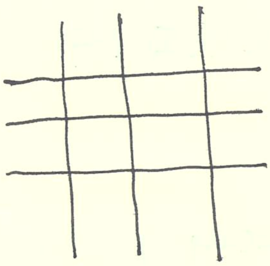
$$k_{TF}^2 = \frac{4\pi e^2 N_0}{1+F_0^s}$$

↓ Fourier

$$\frac{e^2}{r} e^{-r/k_F}$$

长波近似下： $q \to \infty$ $\quad$ static screening

②



phonon : energy scale

atom :

$M_{Nucleon} \propto \frac{1}{\sim 1850} \sim 10^5 \text{ or } 10^6$

continuum model

$$\omega_e^2 = \frac{4\pi N e^2}{m}$$

↑ 振动频肯定与 m 有关

electron density variation:

ii. $\delta\rho_{el} = -e\chi_0(\phi_{ex}+\phi_{ind})$

正是江南好风景，落花时节又逢君

$$\Omega_p^2 = \frac{4\pi n (Ze)^2}{M} \quad : \qquad Cu: 31$$

$$\approx \frac{m_e}{M} Z^2 \omega_e^2 \qquad \Omega_p^2 \sim 10^{-2}\omega_e^2$$

$$\downarrow$$

$$\frac{1}{10^4} \times 10^{-2} \qquad \Omega_p \sim 10\,meV$$

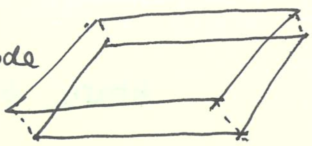
$\sim$ Dybeye temperature

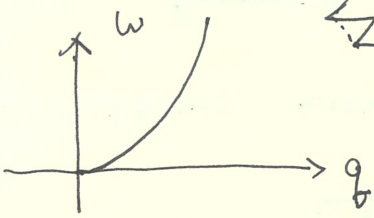
10^{12} Hz $\sim$ 30meV : 电子的能量 10^{15}

因为我们考虑正电荷plasmon，没有管负电荷

$\downarrow$

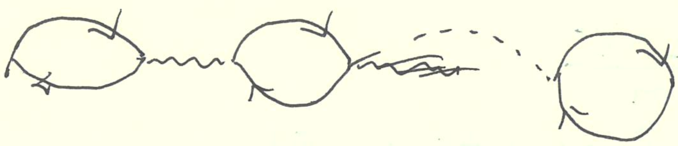
组合成一个eigonmode





plasmon 没有 gap

RPA 上面的 dielectric function



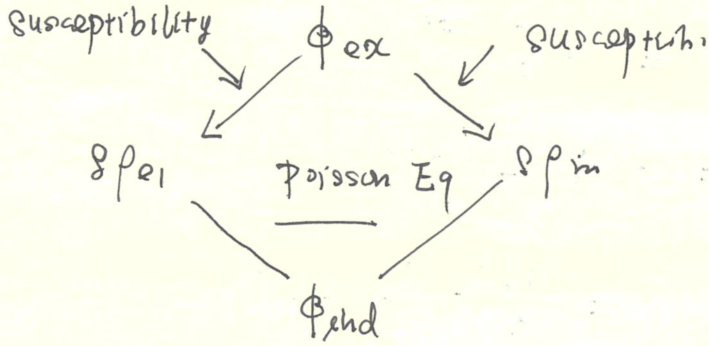
electron density variation:

1. $\delta\rho_{el} = -e^2 \chi^0_{el} (\phi_{ex} + \phi_{ind})$

Galois theory: 外部的 ↓ 安稳如山   内，， ↓ 各地反E

2. $\delta\rho_{ion} = -Ze\, \chi_{ion}(Ze)(\phi_{ex} + \phi_{ind})$

3. $\nabla^2 \phi_{ind} = -4\pi(\delta\rho_{el} + \delta\rho_{ion})$



Fourier transformation:

$$\delta\rho_{el}(q,\omega) = -e^2 \chi_{el}(q,\omega)\left(\phi_{ex} - \frac{4\pi}{q^2}(\delta\rho_{el}(q,\omega) + \delta\rho_{ion}(q,\omega))\right)$$

$$\delta\rho_{ion}(q,\omega) = -(Ze)^2 \chi_{ion}(q,\omega)\left(\phi_{ex} - \frac{4\pi^2}{q^2}(\quad)\right)$$

$$\Rightarrow \begin{cases} \delta\rho_{el}(q,\omega) = -e^2 \chi_{el}(q,\omega)\, \phi_{ex}(q,\omega) / \varepsilon(q,\omega) \\ \delta\rho_{ion}(q,\omega) = -(Ze)^2 \chi^0_{ion}\, \phi_{ex} / \varepsilon(q,\omega) \end{cases}$$

$$\Rightarrow \varepsilon(q,\omega) = 1 + \frac{4\pi e^2}{q^2}(\chi_{el}(q,\omega) + Z^2 \chi_{ion}(q,\omega))$$

$\gamma = \frac{}{v_F q} \ll 1$

Hydrodynamics:

$$\begin{cases} \frac{\partial \rho_{ion}}{\partial t} = -\nabla \cdot \vec{j}_{ion} \\ \frac{\partial \vec{p}_{in}}{\partial t} = -\frac{n}{M} \nabla V_{ex} \end{cases}$$

↓

Newton second law

$$\begin{cases} -i\omega \delta\rho_{ion}(q,\omega) = -\vec{q} \cdot \vec{J}(q,\omega) \\ -i\omega \vec{J}(q,\omega) = -\frac{n}{M} i q V_{ex}(q,\omega) \end{cases}$$

$$\Rightarrow \delta\rho_{ion}(q,\omega) = \frac{nq^2}{M\omega^2} V_{ex}(q,\omega)$$

$\Downarrow$ def

$$-\frac{nq^2}{m\omega^2}$$

$$\Rightarrow \varepsilon(q,\omega) = 1 + \frac{k_{TF}^2}{q^2} - \frac{nq^2}{m\omega^2}$$

↓ 2-com ↓ 为什么保留 ω?

$$\boxed{\varepsilon(q,\omega) = 1 + \frac{4\pi e^2}{q^2}\left(\chi_{el}(q,\omega) + \chi_{ion}^{0}(q,\omega)\right)}$$

↓ 就是这

§: Renormalized phonon spectra

$$\chi_{ion} = \frac{\chi^0}{\varepsilon(q,\omega)} \approx \frac{-nq^2/M\omega^2}{k_{TF}^2/q^2 - \frac{\Omega^2}{\omega^2}} \simeq -\frac{q^2}{k_{TF}^2}\cdot\frac{nq^2/M}{\omega^2 - c^2q^2}$$

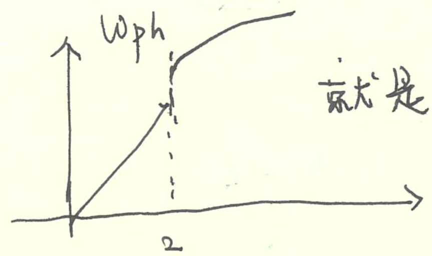
analogous to $\wedge$: $\varepsilon = 1 - \frac{\Omega^2}{\omega^2}$ $\quad \omega >> \Omega$

$$c/v_F = \frac{\Omega_p}{v_F k_{TF}} = \frac{4\pi e^2 Z^2 n_{ion}}{M} = \frac{1+F_0^s}{4\pi e^2 N_0 v_F^2}$$

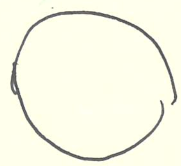
$$\approx \left(\frac{m^*(1+F_0^s)Z}{3m}\right)^{0.5} \sim \underline{10^{-2}}$$

光子动量小，能量大。

① $q \to 2k_F$, $\omega \to 0$: $\varepsilon(q,\omega) = 1 + \frac{k_{TF}^2}{8k_F^2}\left(1 - (1-x)\,\mathrm{Log}\left|\frac{1-x}{2}\right|\right)$

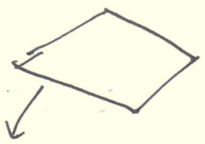


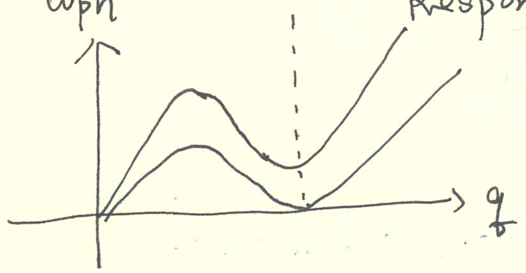
就是



大于 $2k_F$ 后够不上

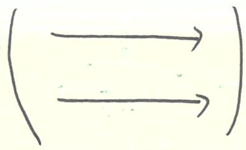
② if Fs nested

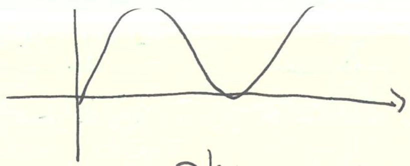




Respon is diverge

频率会掉下来





3: Effective electron-electron interaction

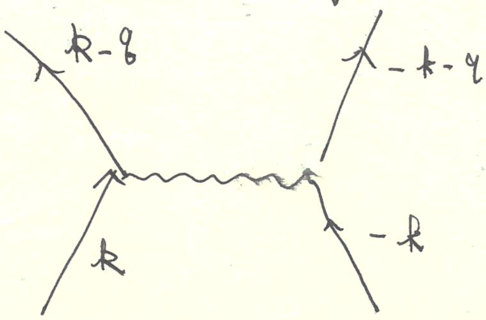
$$\frac{e^2}{r} \xrightarrow[\varepsilon(q,\omega)]{RPA} \frac{4\pi e^2}{q^2+k_{TF}^2}$$ （还没有考虑声子对于 screening 的影响）

$\Downarrow$ $\omega=0$

$$\downarrow$$

$$\frac{4\pi e^2}{1+\frac{k_{TF}^2}{q^2}-\frac{\Omega_p^2}{\omega^2}} \approx \frac{4\pi e^2}{q^2+k_{TF}^2}\left(1+\frac{\omega_{ph}^2}{\omega^2-\omega_{ph}^2}\right)$$

$\downarrow$ phonon induce



① $\langle f|H_{e\text{-}ph}|i\rangle \equiv \langle f|H_{e\text{-}ph}|m\rangle\langle m|H_{ep}|i\rangle$

$$= \frac{1}{2}\left(\frac{1}{E_i-E_m}+\frac{1}{E_f-E_m}\right)$$

$$\swarrow$$

$$\frac{1}{\varepsilon_k-\varepsilon_{k+q}-\hbar\omega_{-q}} + \frac{1}{\varepsilon_{-k-q}-(\varepsilon_{-k}+\hbar\omega_q)}$$

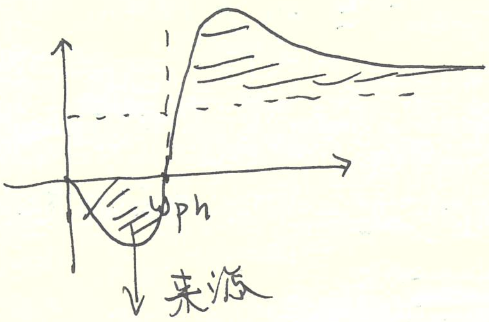
$$\frac{2\hbar\omega}{(\hbar\omega)^2 - (\varepsilon_k - \varepsilon_{k-q})^2} = |g(k,q)|^2$$

计算 3 - T. <f| 1)

$$2|g(k,q)|^2 \cdot \hbar\omega_q = \frac{4\pi e^2}{q^2 k_{TF}^2}\,\omega^2(q)$$

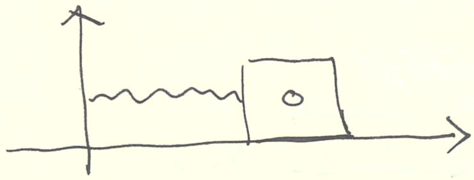
$$\Rightarrow ?? \; |g(k,q)|^2 \propto \frac{1}{2}\left(\frac{\partial n}{\partial \mu}\right)^{-1} \hbar\omega(q) \propto q$$

Goldsten mode   Longitude



吃去去，吐出来

4: overscreening



$$\begin{cases} \ddot{x} + \omega_0^2 x + \gamma\dot{x} = \frac{ZeE}{m} e^{-i\omega t} \\ P = Zenx \qquad E_{ind} = -4\pi P \end{cases}$$

$$E = E_0 + E_{ind} = E_0 - 4\pi ZenX$$

$$X(\omega) = \frac{ZeE}{m}\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma} \equiv \frac{P(\omega)}{E(\omega)}$$

1. $\chi_0(\omega) = \frac{\Omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma} \cdot \frac{1}{4\pi}$

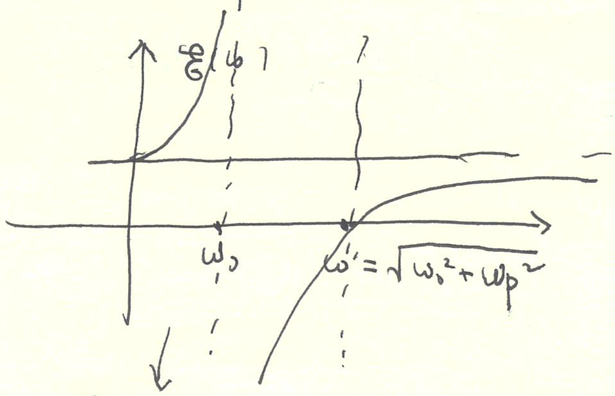
2. $\varepsilon(\omega) = 1 + 4\pi\chi_0 = 1 + \frac{\Omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma}$

3. $E_{ind} = \left(1 - \frac{1}{\varepsilon(\omega)}\right) E_0$

$\varepsilon(\omega)$ zero points: $\omega'^2 = \omega_0^2 + \Omega^2$

↓ shift

(驱动和校仪)



A