量子场论 自由量子场，粒子与反粒子

本章建立和描述 量子场论 中的量子“自由场”

王青

清华大学

2024年9月2日-12月20日

多粒子态
多粒子态
玻色子与费米子
进态、出态与S矩阵
进态和出态
S矩阵
S矩阵的微扰展开
反应率与碰撞截面
量子场
产生与湮灭算符
产生和湮灭算符在坐标空间的表达:
平移
推进与转动
标量量子场
反粒子的引入
标量场的分立对称性变换性质
自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系
旋量量子场
γ矩阵
费米统计
旋量场的分立对称性变换性质
自由旋量场的场方程、哈密顿量和作用量、正则对易关系
矢量量子场
按自旋分类
矢量场的分立对称性变换性质
自由矢量场的场方程、哈密顿量和作用量、正则对易关系
任意自旋量子场
非奇次洛伦兹群的一般不可约表示
有质量的任意自旋量子场
无质量的任意自旋量子场

单粒子态：

$$U(\Lambda,a)\Psi_{p,\sigma,n}=e^{ia_\mu(\Lambda p)^\mu}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\sigma'}D_{\sigma'\sigma}(W(\Lambda,p))\Psi_{\Lambda p,\sigma',n}$$

- 自旋为 $j,\sigma=-j,\ldots,j$ 的有质量态: $D_{\sigma'\sigma}(W)=(e^{\frac{i}{2}\Theta_{ik}(\Lambda,p)J_{ik}^{(j)}})_{\sigma'\sigma}$ p转成$\Lambda(p-k/\alpha)$的转动
- 自旋、 螺旋度为 σ 的无质量态: $D_{\sigma'\sigma}(W)=\delta_{\sigma'\sigma}e^{i\theta\sigma}$ 和p垂直矢量在Λ空间转动后与Λp垂直矢量的夹角

无相互作用的多粒子态: 单粒子态的直乘 能否从本征值鉴别是单粒子态还是多粒子态?

- 讨论不同种类的粒子，引入分立的指标n来代表粒子所属的种类
- 每个粒子用其能动量和参考动量系角动量第三分量及粒子种类来标记

一个一般的无相互作用的多粒子态可以写为 $\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$
它的能动量、自旋的第三分量(只对基本参考动量)和可能的$U(1)$本征值为:

$$P_0^\mu\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=(p_1^\mu+p_2^\mu+\cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$J_0^3\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots}=(\sigma_1+\sigma_2+\cdots)\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots}$$

$$(Q_0)_a\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=(q_{a1}+q_{a2}+\cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

对称性生成元标记下标0: 没有相互作用的多粒子体系

单粒子态:

$$U(\Lambda,a)\Psi_{p,\sigma,n}=e^{ia_\mu(\Lambda p)^\mu}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\sigma'}D_{\sigma'\sigma}(W(\Lambda,p))\Psi_{\Lambda p,\sigma',n}$$

- 自旋为j的有质量态: $D_{\sigma'\sigma}(W)=D^{(j)}_{\sigma'\sigma}(W)$ p转成$\Lambda(p-k/\alpha)$的转动
- 自旋为j,螺旋度为σ的无质量态: $D_{\sigma'\sigma}(W)=\delta_{\sigma'\sigma}e^{i\theta\sigma}$ 和p垂直矢量在Λ空间转动后与Λp垂直矢量的夹角

无相互作用的多粒子态: 单粒子态的直乘 $\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$

$$P_0^\mu\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=(p_1^\mu+p_2^\mu+\cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$J_0^3\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots}=(\sigma_1+\sigma_2+\cdots)\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots}$$ 为什么? 整体运动

$$(Q_0)_a\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=(q_{a1}+q_{a2}+\cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

在时空平移和转动洛伦兹变换$U_0(\Lambda,a)$下:

$$U_0(\Lambda,a)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=e^{ia_\mu((\Lambda p_1)^\mu+(\Lambda p_2)^\mu+\cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots}{p_1^0p_2^0\cdots}}\sum_{\sigma_1'\sigma_2'\cdots}D_{\sigma_1'\sigma_1}(W(\Lambda,p_1))$$

$$\times D_{\sigma_2'\sigma_2}(W(\Lambda,p_2))\cdots\Phi_{\Lambda p_1,\sigma_1',n_1;\Lambda p_2,\sigma_2',n_2;\cdots}$$

在相互独立的内部$U_0(1)$对称性变换下:

$$U_0(T(\theta))\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=e^{i(q_{a1}+q_{a2}+\cdots)\theta^a}\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

排序问题：

如果所有的粒子都各属于不同的种类，它们之间可以区分，原则上可以规定一种标准的排序方式，例如第一种粒子排在第一位，第二种粒子排在第二位，$\cdots$,等等.

但如果在一个多粒子态中有某两个粒子的种类相同,$\Phi_{\cdots;p,\sigma,n;\cdots;p',\sigma',n;\cdots}$，交换此两个粒子的态$\Phi_{\cdots;p',\sigma',n;\cdots;p,\sigma,n;\cdots}$与原来的态是无法区分的 这似乎只在微观世界才会发生，因而限制了结果的应用范围，需要对同类粒子的排序进行详细的讨论。

交换两个同类粒子无法区分，交换前后的态之间只能相差一个相角:

$$\Phi_{\cdots;p,\sigma,n;\cdots;p',\sigma',n;\cdots} = \alpha_n(p,\sigma;p',\sigma')\,\Phi_{\cdots;p',\sigma',n;\cdots;p,\sigma,n;\cdots}$$

相角$\alpha_n(p,\sigma;p',\sigma')$与其它粒子的$p,\sigma,n$无关. 我们讨论的是自由粒子的多粒子态，如果交换其中的某两个粒子还要受到其它粒子的影响，意味着其它粒子对这两个参与交换的粒子有相互作用，就不是自由粒子态了。

$$\Phi_{\cdots;p,\sigma,n;\cdots;p',\sigma',n;\cdots} = \alpha_n(p,\sigma;p',\sigma')\ \Phi_{\cdots;p',\sigma',n;\cdots;p,\sigma,n;\cdots}$$

两边实施洛伦兹变换

$$\sum_{\bar\sigma\bar\sigma'\cdots} D_{\bar\sigma\sigma}(W(\Lambda,p))D_{\bar\sigma'\sigma'}(W(\Lambda,p'))\Phi_{\cdots,\Lambda p,\bar\sigma,n;\cdots;\Lambda p',\bar\sigma',n;\cdots}$$

$$= \alpha_n(p,\sigma;p',\sigma')\sum_{\bar\sigma'\bar\sigma} D_{\bar\sigma'\sigma'}(W(\Lambda,p'))D_{\bar\sigma\sigma}(W(\Lambda,p))\Phi_{\cdots,\Lambda p',\bar\sigma',n;\cdots;\Lambda p,\bar\sigma,n;\cdots}$$

$$\Phi_{\cdots,\Lambda p,\bar\sigma,n;\cdots;\Lambda p',\bar\sigma',n;\cdots} = \alpha_n(p,\sigma;p',\sigma')\ \Phi_{\cdots,\Lambda p',\bar\sigma',n;\cdots;\Lambda p,\bar\sigma,n;\cdots}$$

$$\Rightarrow\quad \alpha_n(p,\sigma;p',\sigma') = \alpha_n(\Lambda p,\bar\sigma;\Lambda p',\bar\sigma')$$

$\alpha_n(p,\sigma;p',\sigma')$与$\sigma,\sigma'$无关, 可略去$\alpha_n$中的$\sigma$指标,并且在时空转动下是不变的,

$$\alpha_n(p,\sigma;p',\sigma') = \alpha_n(p,p') \qquad\qquad \alpha_n(p,p') = \alpha_n(\Lambda p,\Lambda p')$$

在1+3维时空由p和p'构造的时空转动不变量只可能为p^2,p'^2和$p^\mu p'_\mu$,
考虑到$p^2 = M^2 = p'^2$并且$p^\mu p'_\mu$对交换p和p'是对称的

1+2维时空中同类粒子交换可出现任意相角 任意子统计

- 交换粒子可等价为绕两粒子中点的转角为 π 的转动，产生相角 $e^{i2\pi\sigma}$
- 1+2维时空只一个非三个J $\Rightarrow$不量子化！ $\Rightarrow$ σ可取连续值 拓扑非平凡可转一圈不回归

$$\Phi_{\cdots;p,\sigma,n;\cdots;p',\sigma',n;\cdots} = \alpha_n(p,p')\ \Phi_{\cdots;p',\sigma',n;\cdots;p,\sigma,n;\cdots}$$

$\alpha_n(p,p') = \alpha_n(\Lambda p,\Lambda p')$ 在四维时空$\alpha_n(p,p')$只能依赖p^2,p'^2和$p^\mu p'_\mu$
结合$p^2{=}M^2{=}p'^2$及$p^\mu p'_\mu$对交换p和p'是对称的$\alpha_n(p,p') = \alpha_n(p',p)$

$$\Phi_{\cdots;p,\sigma,n;\cdots;p',\sigma',n;\cdots} = \alpha_n(p,p')\Phi_{\cdots;p',\sigma',n;\cdots;p,\sigma,n;\cdots}$$
$$\Phi_{\cdots;p',\sigma',n;\cdots;p,\sigma,n;\cdots} = \alpha_n(p',p)\Phi_{\cdots;p,\sigma,n;\cdots;p',\sigma',n;\cdots}$$
$$\Rightarrow \alpha_n(p,p')\alpha_n(p',p) = 1 \quad \Rightarrow \quad \alpha_n(p,p') = \pm 1$$

1+3维时空中同类粒子交换只可能出现两种情况：变号或不变号

- $\alpha_n(p,p') = +1$的粒子叫玻色子。 对玻色子同类粒子交换不变号
- $\alpha_n(p,p') = -1$的粒子叫费米子。 对费米子同类粒子交换一次出一负号

进一步对不同类粒子之间的排序在标准的排序方式基础上作如下的安排：

- 最近邻的费米子与费米子之间交换一次出一负号
- 最近邻的玻色子与玻色子，玻色子与费米子之间交换不变号

关于对全同粒子对称性的评述

- ♣ 我们的量子场论是建筑在粒子具有全同性的要求基础上的
- ♠ 它是假设吗？还是与生俱来，是我们用单粒子态定义量子场论所导致的呢！
- ♡ 核心是要建立粒子态的完备描述！
- ♢ 当粒子态的完备描述不再被需要时，全同性就丧失了！
- ✠ 对全同性丧失的临界尺度的研究还很少！
- ¶ 我们目前涉猎的物理理论居然要跨越全同性有无的边界，十分匪夷所思！

多粒子态的归一化：记$\Phi_{p_1,\sigma_1,n;p_2,\sigma_2,n_2;\cdots} = \Phi_{p_1,p_2,\cdots}$

真空态Φ_0和单粒子态Φ_q

$$(\Phi_0, \Phi_0) = 1 \qquad (\Phi_{q'}, \Phi_q) = \delta(q'-q) \equiv \delta^3(\vec{q}\,'-\vec{q})\delta_{\sigma'\sigma}\delta_{n'n}$$ 非洛伦兹不变！

对两粒子态$\Phi_{q_1q_2}$ $$(\Phi_{q_1'q_2'}, \Phi_{q_1q_2}) = \frac{1}{2!}[\delta(q_1'-q_1)\delta(q_2'-q_2) \pm \delta(q_2'-q_1)\delta(q_1'-q_2)]$$

负号对两个粒子都是费米子的情形,正号对其它情形(两个粒子都是玻色子或一个是玻色子一个是费米子).

一般情况：$$(\Phi_{q_1'q_2'\cdots q_M'}, \Phi_{q_1q_2\cdots q_N}) = \frac{\delta_{MN}}{N!}\sum_{\mathcal{P}}\delta_{\mathcal{P}}\prod_i \delta(q_i - q_{\mathcal{P}i}')$$ N只针对同种粒子

求和对所有可能的对指标$1,2,\cdots,N$的交换排序$\mathcal{P}$实行.对交换排序中涉及奇数次费米子交换时,$\delta_{\mathcal{P}} = -1$,其它情况的交换排序$\delta_{\mathcal{P}} = 1$.

约定：$$(\Phi_{\alpha'}, \Phi_\alpha) = \delta(\alpha'-\alpha) \qquad \int d\alpha\cdots \equiv \sum_{n_1\sigma_1n_2\sigma_2\cdots}\int d\vec{p}_1\int d\vec{p}_2\cdots$$

$$\Phi = \int d\alpha\ \Phi_\alpha(\Phi_\alpha, \Phi) \qquad 1 = \int d\alpha\ \Phi_\alpha(\Phi_\alpha,\ \ )$$ 多粒子态构成完备集！ 只要H厄米、有下界无上界

散射问题:

一组在宏观上相距很远的相互之间没有相互作用的粒子逐渐相互接近到微观上很小的区域发生相互作用，再逐渐互相分离到宏观上相距很远相互之间不再有相互作用的区域。

相互作用发生在粒子逐渐相互接近到微观上很小的区域。记一个有相互作用体系的总体时间平移生成元算符为H,被称为体系的哈密顿量，是体系总四动量的零分量。把这个体系的相互作用撤除得到的无相互作用的自由粒子体系的哈密顿量记为H_0.将两个哈密顿量的差定义为体系的相互作用V：

$$H = H_0 + V$$

将体系中粒子相距很远，相互之间没有相互作用的初始和末了状态分别称为进态和出态，记为: Ψ^+_α和Ψ^-_α

- 进态和出态分别构成完备集
- 相互之间无相互作用⇒进态和出态分别可被看成一组自由多粒子态
- 进态和出态满足与自由多粒子态同样的洛仑兹和内部对称性变换关系
- 过程的持续：无相互作用的无穷将来⇐发生相互作用的现在⇐无相互作用的无穷过去

进态和出态满足的洛仑兹变换: 满足与自由多粒子态同样的有相互作用系统的洛仑兹变换关系

$$U(\Lambda,a)\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=e^{ia_\mu((\Lambda p_1)^\mu+(\Lambda p_2)^\mu+\cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots}{p_1^0p_2^0\cdots}}\sum_{\sigma_1'\sigma_2'\cdots}D_{\sigma_1'\sigma_1}(W(\Lambda,p_1))$$

$$\times D_{\sigma_2'\sigma_2}(W(\Lambda,p_2))\cdots\Psi^{\pm}_{\Lambda p_1,\sigma_1',n_1;\Lambda p_2,\sigma_2',n_2;\cdots}\equiv\int d\alpha'\ (U_0)_{\alpha'\alpha}\Psi^{\pm}_{\alpha'}$$

$$U(T(\theta))\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}=e^{i(q_{a1}+q_{a2}+\cdots)\theta^a}\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$\Lambda=1,\ a=(\epsilon,\vec{0})$$

$$e^{i\epsilon H}\Psi^{\pm}_\alpha=e^{i\epsilon E_\alpha}\Psi^{\pm}_\alpha\qquad H\Psi^{\pm}_\alpha=E_\alpha\Psi^{\pm}_\alpha\qquad E_\alpha=p_1^0+p_2^0+\cdots$$

将H_0选择的使其具有与H完全一样的本征值谱,即

$$H_0\Phi_\alpha=E_\alpha\Phi_\alpha\qquad(\Phi_{\alpha'},\Phi_\alpha)=\delta(\alpha'-\alpha)$$

Φ_α是H_0的本征态。

进态和出态满足的洛仑兹变换: 满足与自由多粒子态同样的有相互作用系统的洛仑兹变换关系

$$U(\Lambda,a)\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{ia_\mu((\Lambda p_1)^\mu+(\Lambda p_2)^\mu+\cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots}{p_1^0 p_2^0\cdots}}\sum_{\sigma_1'\sigma_2'\cdots} D_{\sigma_1'\sigma_1}(W(\Lambda,p_1))$$

$$\times D_{\sigma_2'\sigma_2}(W(\Lambda,p_2))\cdots\Psi^{\pm}_{\Lambda p_1,\sigma_1',n_1;\Lambda p_2,\sigma_2',n_2;\cdots} \equiv \int d\alpha'\ (U_0)_{\alpha'\alpha}\Psi^{\pm}_{\alpha'}$$

$$U(T(\theta))\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{i(q_{a1}+q_{a2}+\cdots)\theta^a}\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$H\Psi^{\pm}_\alpha = E_\alpha\Psi^{\pm}_\alpha \qquad H_0\Phi_\alpha = E_\alpha\Phi_\alpha \qquad E_\alpha = p_1^0+p_2^0+\cdots$$

过程的持续 $\Rightarrow$ 量子态随时间的演化 $\xRightarrow{\text{理论中没定义}}$ 需定义态随时间的演化!

时间的演化

关于时间的附注：

绝对的、真实的和数学的时间，由其特性决定，自身均匀地流逝，与一切外在事务无关，又名延续；

相对的、表象的和普通的时间是可感知和外在的（不论是精确的或是不均匀的）对运动之延续的量度，它常被用以代替真实的时间，如一小时、一天、一个月、一年。

《自然哲学之数学原理–宇宙体系》 伊萨克.牛顿 1686年5月8日

时间的演化是： 均匀流逝

E.Wigner, Ann.Math.40,149(1939) On Unitary Representations of the Inhomogeneous Lorentz Group

If we knew, e.g., the operator K corresponding to the measurement of a physical quantity at the time $t = 0$, we could follow up the change of this quantity throughout time. In order to obtain its value for the time $t = t_1$, we could transform the original wave function ϕ_l by $D(l', l)$ to a coordinate system l' the time scale of which begins a time t_1 later ($t' = t - t_1$, $t \geq t_1$). The measurement of the quantity in question in this coordinate system for the time 0 ($t' = 0$) is given–as in original one–by the operator K. This measurement is identical, however, with the measurement of the quantity at time $t_1 = t$ in the original system. ······

量子态随时间的演化

将态的演化时间翻译为两观测者的观测时间差 就像一个人自己O_τ和他手上戴的手表O

- 观测者O的时钟标记t，观测到的为理论原始假设中的态Ψ
- 观测者O_τ的时钟标记$t' = t - \tau$，感受 态的演化，观测到的态为$\Psi(\tau)$

观测者O_τ自我时间为“现在” $t' = 0$ 时，观测者O的时间纪录为$t = \tau$. $\Psi(\tau)$对τ的依赖就像一个人在看自己的手表。

$$\Psi(\tau) = U(1, -\tau)\Psi = e^{-iH\tau}\Psi \qquad i\frac{\partial}{\partial\tau}\Psi(\tau) = H\,\Psi(\tau)$$ 薛定谔方程！

suppose that a standard observer $\mathcal{O}$ sets his or her clock so that $t = 0$ is at some time during the collision process, while some other observer $\mathcal{O}'$ at rest with respect to the first uses a clock set so that $t' = 0$ is at a time $t = \tau$; that is, the two observers' time coordinates are related by $t' = t - \tau$. Then if $\mathcal{O}$ sees the system to be in a state Ψ, $\mathcal{O}'$ will see the system in a state $U(1, -\tau)\Psi = \exp(-iH\tau)\Psi$. Thus the appearance of the state long before or long after the collision (in whatever basis is used by $\mathcal{O}$) is found by applying a time-translation operator $\exp(-iH\tau)$ with $\tau \to -\infty$ or $\tau \to +\infty$, respectively. Of course, if the state is really

时间的均匀流逝体现为一系列观测者之间的 均匀的时间观测 间隔 ！

进态和出态与H_0的本征态的关系：

进态：

要求在无穷过去$O_{-\infty}$观测的进态与自由粒子态完全相同：

$$\Psi^+(-\infty) = \Phi(-\infty) \qquad U(1,-\tau)|_{\tau=-\infty}\Psi^+ = U_0(1,-\tau)|_{\tau=-\infty}\Phi$$

公式应在波包（不同本征态的叠加）意义下理解，否则带入本征值将导致$\Psi^+_\alpha = \Phi_\alpha$

$\Rightarrow e^{-iH\tau}\Psi^+_\alpha|_{\tau\to-\infty} = e^{-iH_0\tau}\Phi_\alpha|_{\tau\to-\infty}$ 或 $\Psi^+_\alpha = \Omega(-\infty)\Phi_\alpha$先将态自由地演化到无穷将来再相互作用地演化回现在

$$\Omega(\tau) \equiv U^\dagger(1,-\tau)U_0(1,-\tau) = U(1,\tau)U_0^\dagger(1,\tau) = e^{iH\tau}e^{-iH_0\tau}$$

出态：

要求在无穷将来$O_{+\infty}$观测的出态与自由粒子态完全相同：

$$\Psi^-(+\infty) = \Phi(+\infty) \qquad U(1,-\tau)|_{\tau=+\infty}\Psi^- = U_0(1,-\tau)|_{\tau=+\infty}\Phi$$

公式应在波包（不同本征态的叠加）意义下理解，否则带入本征值将导致$\Psi^-_\alpha = \Phi_\alpha$

$\Rightarrow e^{-iH\tau}\Psi^-_\alpha|_{\tau\to+\infty} = e^{-iH_0\tau}\Phi_\alpha|_{\tau\to+\infty}$ 或 $\Psi^-_\alpha = \Omega(+\infty)\Phi_\alpha$先将态自由地演化回无穷过去再相互作用地演化到现在

作业1,2,3

关于进态和出态:

♣ 引入系统的演化: 时间演化≡时间平移!

♢ 自由粒子态与相互作用的混合体?

♡ 相互作用暧昧地引入? 含糊不清 和撤除? 绝热近似

♠ 与自由粒子同样的质量谱?

¶ $e^{-iH\tau}\Psi_{\alpha}^{\pm}|_{\tau\to\mp\infty} = e^{-iH_0\tau}\Phi_{\alpha}|_{\tau\to\mp\infty}$?

♡ 不含时间的 $\Psi_{\alpha}^{\pm}$ 囊括了全部的系统演化!

进态和出态的内积定义为S矩阵元： $S_{\beta\alpha}=(\Psi^-_\beta,\Psi^+_\alpha)$ $\alpha\to\beta$ 几率幅

S矩阵元的性质： 作业4,5 $S_{\beta\alpha}=\delta(\beta-\alpha)-2\pi i\delta(E_\alpha-E_\beta)T^+_{\beta\alpha}$

利用进出态的完备性 是厄米、有下界无上界的H的本征态所张开的态空间的完备性

$$\Psi^+_\alpha=\int d\beta\ \Psi^-_\beta(\Psi^-_\beta,\Psi^+_\alpha)=\int d\beta\ S_{\beta\alpha}\Psi^-_\beta$$

$$\Psi^-_\alpha=\int d\beta\ \Psi^+_\beta(\Psi^+_\beta,\Psi^-_\alpha)=\int d\beta\ S^*_{\alpha\beta}\Psi^+_\beta$$

幺正性质

$$\int d\beta\ S^*_{\beta\gamma}S_{\beta\alpha}=\int d\beta\ (\Psi^+_\gamma,\Psi^-_\beta)(\Psi^-_\beta,\Psi^+_\alpha)=(\Psi^+_\gamma,\Psi^+_\alpha)=\delta(\gamma-\alpha)$$

将S矩阵元建立在自由粒子基上，引入S矩阵算符: $\Psi^\pm_\alpha=\Omega(\mp\infty)\Phi_\alpha$

$$S_{\beta\alpha}\equiv(\Phi_\beta,S\Phi_\alpha)\ \to\ S=\Omega^\dagger(\infty)\Omega(-\infty)=U(+\infty,-\infty)$$

$$U(\tau,\tau_0)=\Omega^\dagger(\tau)\Omega(\tau_0)=e^{iH_0\tau}e^{-iH(\tau-\tau_0)}e^{-iH_0\tau_0}=U_0(1,\tau)U^\dagger(1,\tau-\tau_0)U_0^\dagger(1,\tau_0)$$

$$\Omega(\tau)=e^{iH\tau}e^{-iH_0\tau}=U(1,\tau)U_0^\dagger(1,\tau)$$

注意 $U(\tau,\tau_0)$ 和 $U(\Lambda,a)$ 是不同的量!

S矩阵

定义为进态和出态内积的S矩阵元 $S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha)$

利用洛仑兹变换和内部对称性变换算符的幺正性质

$\Rightarrow$ **S矩阵元在洛仑兹变换和内部对称性变换下是不变的!**

S矩阵元的洛仑兹变换不变性: $S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha) = (U(\Lambda,a)\Psi^-_\beta, U(\Lambda,a)\Psi^+_\alpha)$

$$S_{p_1',\sigma_1',n_1';p_2',\sigma_2',n_2';\cdots p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = \int d\bar\beta d\bar\alpha\ (U_0^*)_{\bar\beta\beta} S_{\bar\beta\bar\alpha} (U_0)_{\bar\alpha\alpha}$$ 见后

$$= e^{ia_\mu((\Lambda p_1)^\mu + (\Lambda p_2)^\mu + \cdots - (\Lambda p_1)^{\mu\prime} - (\Lambda p_2)^{\mu\prime} - \cdots)} \sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots(\Lambda p_1')^0(\Lambda p_2')^0\cdots}{p_1^0 p_2^0\cdots p_1^{0\prime} p_2^{0\prime}\cdots}}$$

$$\times \sum_{\bar\sigma_1,\bar\sigma_2,\cdots} D^{(j_1)}_{\bar\sigma_1\sigma_1}(W(\Lambda,p_1)) D^{(j_2)}_{\bar\sigma_2\sigma_2}(W(\Lambda,p_2))\cdots \sum_{\bar\sigma_1',\bar\sigma_2',\cdots} D^{(j_1')*}_{\bar\sigma_1'\sigma_1'}(W(\Lambda,p_1')) D^{(j_2')*}_{\bar\sigma_2'\sigma_2'}(W(\Lambda,p_2'))\cdots$$

$$\times S_{\Lambda p_1',\bar\sigma_1',n_1';\Lambda p_2',\bar\sigma_2',n_2';\cdots \Lambda p_1,\bar\sigma_1,n_1;\Lambda p_2,\bar\sigma_2,n_2;\cdots}$$

左边与平移参量a无关 $\Rightarrow$ 要求等式右边a无关 $\Rightarrow$ $p_1^\mu + p_2^\mu + \cdots - p_1^{\mu\prime} - p_2^{\mu\prime} - \cdots = 0$

记$p_\alpha = p_\beta$, 四动量是连续变量，动量守恒意味S矩阵元中含因子$\delta(\vec p_\beta - \vec p_\alpha)$

$$S_{\beta\alpha} - \delta(\beta-\alpha) \overset{\text{作业4}}{=\!=\!=} -2\pi i\delta(E_\alpha - E_\beta)T^+_{\beta\alpha} = -2\pi i M_{\beta\alpha}\delta^4(p_\beta - p_\alpha)$$

定义为进态和出态内积的S矩阵元 $S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha)$

利用洛仑兹变换和内部对称性变换算符的幺正性质

$\Rightarrow$ **S矩阵元在洛仑兹变换和内部对称性变换下是不变的!**

S矩阵元的洛仑兹变换不变性: $S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha) = (U(\Lambda, a)\Psi^-_\beta, U(\Lambda, a)\Psi^+_\alpha)$

$$p_\alpha = p_\beta \qquad S_{\beta\alpha} - \delta(\beta - \alpha) = -2\pi i M_{\beta\alpha}\delta^4(p_\beta - p_\alpha)$$

S矩阵元的内部对称性变换不变性: $S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha) = (U(T(\theta))\Psi^-_\beta, U(T(\theta))\Psi^+_\alpha)$

$$S_{p_1',\sigma_1',n_1';p_2',\sigma_2',n_2';\cdots p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$
$$= e^{i(q_{a1}+q_{a2}+\cdots-q_{a1}'-q_{a2}'-\cdots)\theta^a} S_{p_1',\sigma_1',n_1';p_2',\sigma_2',n_2';\cdots p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

左边是与内部转动参量θ^a无关 $\Rightarrow$ 要求等式右边θ^a无关 $q_{a1} + q_{a2} + \cdots - q_{a1}' - q_{a2}' - \cdots = 0$

记 $q_\alpha = q_\beta$

定义为进态和出态内积的S矩阵元 $S_{\beta\alpha}=(\Psi_\beta^-,\Psi_\alpha^+)$

利用洛仑兹变换和内部对称性变换算符的幺正性质

$\Rightarrow$ **S矩阵元在洛仑兹变换和内部对称性变换下是不变的!**

S矩阵元的洛仑兹变换不变性: $S_{\beta\alpha}=(\Psi_\beta^-,\Psi_\alpha^+)=(U(\Lambda,a)\Psi_\beta^-,U(\Lambda,a)\Psi_\alpha^+)$

$$p_\alpha=p_\beta \qquad S_{\beta\alpha}-\delta(\beta-\alpha)=-2\pi i M_{\beta\alpha}\delta^4(p_\beta-p_\alpha)$$

用自由粒子的洛仑兹变换和内部对称性变换: $U_0(\Lambda,a)$ $U_0(T(\theta))$

$$U_0(\Lambda,a)\Phi_\alpha=\int d\bar\alpha(U_0)_{\bar\alpha\alpha}\Phi_{\bar\alpha} \qquad S_{\beta\alpha}\equiv(\Phi_\beta,S\Phi_\alpha)\Rightarrow S=\int d\beta d\alpha\ \Phi_\beta S_{\beta\alpha}\Phi_\alpha$$

$$S_{\beta\alpha}=(U\Psi_\beta^-,U\Psi_\alpha^+)\overset{\text{见}U_0\text{矩阵元最早定义}}{=\!=\!=\!=\!=}\int d\bar\beta d\bar\alpha((U_0)_{\bar\beta\beta}\Psi_{\bar\beta}^-,(U_0)_{\bar\alpha\alpha}\Psi_{\bar\alpha}^+)=\int d\bar\beta d\bar\alpha(U_0^*)_{\bar\beta\beta}S_{\bar\beta\bar\alpha}(U_0)_{\bar\alpha\alpha}$$

$$\Rightarrow\ S=\int d\beta d\alpha\ \Phi_\beta S_{\beta\alpha}\Phi_\alpha=\int d\bar\beta d\bar\alpha d\beta d\alpha\ \Phi_\beta(U_0^*)_{\bar\beta\beta}S_{\bar\beta\bar\alpha}(U_0)_{\bar\alpha\alpha}\Phi_\alpha$$

$$=\int d\bar\beta d\bar\alpha d\beta d\alpha\Phi_\beta(U_0^*)_{\bar\beta\beta}(\Phi_{\bar\beta},S\Phi_{\bar\alpha})(U_0)_{\bar\alpha\alpha}\Phi_\alpha=\int d\beta d\alpha\Phi_\beta(U_0\Phi_\beta,SU_0\Phi_\alpha)\Phi_\alpha=U_0^{-1}(\Lambda,a)SU_0(\Lambda,a)$$

U_0可以是任意一个幺正算符 **S矩阵算符是洛仑兹变换和内部对称性变换下不变的!**

S矩阵的微扰展开

$$S = U(+\infty, -\infty) \qquad U(\tau,\tau_0) = \Omega^\dagger(\tau)\Omega(\tau_0) = e^{iH_0\tau} e^{-iH(\tau-\tau_0)} e^{-iH_0\tau_0}$$

$$i\frac{d}{d\tau}U(\tau,\tau_0) = e^{iH_0\tau}(-H_0+H)e^{-iH(\tau-\tau_0)}e^{-iH_0\tau_0} = V(\tau)U(\tau,\tau_0) \qquad V(\tau) = e^{iH_0\tau}Ve^{-iH_0\tau}$$

$$U(\tau,\tau_0) = 1 - i\int_{\tau_0}^{\tau} dt_1 V(t_1) + (-i)^2\int_{\tau_0}^{\tau} dt_1 \int_{\tau_0}^{t_1} dt_2 V(t_1)V(t_2)$$

$$+(-i)^3\int_{\tau_0}^{\tau} dt_1 \int_{\tau_0}^{t_1} dt_2 \int_{\tau_0}^{t_2} dt_3 V(t_1)V(t_2)V(t_3) + \cdots$$

$$= \mathbf{T}\, e^{-i\int_{\tau_0}^{\tau} dt\; V(t)}$$

$\mathbf{T}$ 是编时乘积，它将时间早的的算符排在右边 作业6,7

$$V(t) = \int d\vec{x}\;\tilde{\mathcal{H}}(\vec{r},t) \qquad \tilde{\mathcal{H}}(x) = \tilde{\mathcal{H}}(\vec{r},t)$$

是局域的相互作用哈密顿量密度 生成元开始直接和时空坐标发生关系！

局域性也可由 **Cluster Decomposition Principle** 得到.

由于V是H中的相互作用部分,不是洛仑兹变换的协变量, 如此引入相互作用哈密顿量密度可以确保它是洛仑兹变换的标量。

S矩阵的洛仑兹和内部对称性不变性要求$\tilde{\mathcal{H}}(x)$是洛仑兹和内部对称性不变量

猜测： $U_0(\Lambda,a)\tilde{\mathcal{H}}(x)U_0^{-1}(\Lambda,a) = \tilde{\mathcal{H}}(\Lambda x + a)$ $\qquad U_0(T(\theta))\tilde{\mathcal{H}}(x)U_0^{-1}(T(\theta)) = \tilde{\mathcal{H}}(x)$

$[\tilde{\mathcal{H}}(x), \tilde{\mathcal{H}}(x')] = 0 \qquad x - x'$ 类空间隔

类时、类光间隔保时序，类空不保，因而要可对易！

反应率

将系统限制在有限的空间中： 周期性边条件 $\Rightarrow e^{i\vec{p}\cdot\vec{L}} = 1$

$$\vec{p} = \frac{2\pi}{L}(n_1, n_2, n_3) \quad \delta^3_V(\vec{p}' - \vec{p}) = \frac{1}{(2\pi)^3}\int_V d^3x\, e^{i(\vec{p}-\vec{p}')\cdot\vec{x}} = \frac{V}{(2\pi)^3}\delta_{\vec{p}',\vec{p}}$$

$$\Psi^{\rm Box}_\alpha \equiv [\frac{(2\pi)^3}{V}]^{N_\alpha/2}\Psi_\alpha \rightarrow (\Psi^{\rm Box}_\beta, \Psi^{\rm Box}_\alpha) = \delta_{\beta\alpha} \quad \textbf{Kronecker} \rightarrow S_{\beta\alpha} = [\frac{V}{(2\pi)^3}]^{\frac{N_\beta+N_\alpha}{2}} S^{\rm Box}_{\beta\alpha}$$

$$\delta_T(E_\alpha - E_\beta) = \frac{1}{2\pi}\int_{-T/2}^{T/2} dt\, e^{i(E_\alpha - E_\beta)t} \qquad d\beta = d\vec{p}_1 d\vec{p}_2 \cdots$$

$$P(\alpha\rightarrow\beta) \equiv |S^{\rm Box}_{\beta\alpha}|^2 = [\frac{(2\pi)^3}{V}]^{N_\alpha+N_\beta}|S_{\beta\alpha}|^2$$

$d\beta$区间态的数目：$d\mathcal{N}_\beta = [\frac{V}{(2\pi)^3}]^{N_\beta} d\beta$ 按第二行的积分，它是归一的

$$dP(\alpha\rightarrow\beta) = P(\alpha\rightarrow\beta)d\mathcal{N}_\beta = [\frac{(2\pi)^3}{V}]^{N_\alpha}|S_{\beta\alpha}|^2 d\beta$$

对有偏转的部分：

$$S_{\beta\alpha} \equiv -2i\pi\delta^3_V(\vec{p}_\beta - \vec{p}_\alpha)\delta_T(E_\beta - E_\alpha)M_{\beta\alpha} \xrightarrow{VT\ \text{large}} -2i\pi\delta^4(p_\beta - p_\alpha)M_{\beta\alpha}$$

$$dP(\alpha\rightarrow\beta) = (2\pi)^2[\frac{(2\pi)^3}{V}]^{N_\alpha-1}\frac{T}{2\pi}|M_{\beta\alpha}|^2\delta^3_V(\vec{p}_\beta - \vec{p}_\alpha)\delta_T(E_\beta - E_\alpha)d\beta$$

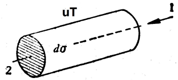
$$d\Gamma(\alpha\rightarrow\beta) \equiv \frac{dP(\alpha\rightarrow\beta)}{T} = (2\pi)^{3N_\alpha-2}V^{1-N_\alpha}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

碰撞截面

$$S_{\beta\alpha} \xrightarrow{\text{connect part}} -2i\pi\delta^4(p_\beta - p_\alpha)M_{\beta\alpha}$$

$$d\Gamma(\alpha\to\beta) \equiv \frac{dP(\alpha\to\beta)}{T} = (2\pi)^{3N_\alpha-2}V^{1-N_\alpha}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

$N_\alpha = 1$:
$$d\Gamma(\alpha\to\beta) = 2\pi|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$



在体积边界面上各处向相对运动方向随机发射一个粒子1，此粒子在全空间V各处等几率出现

$N_\alpha = 2$:
$$\Phi_\alpha = \frac{u_\alpha}{V}$$

$$d\sigma(\alpha\to\beta) \equiv \frac{d\Gamma(\alpha\to\beta)}{\Phi_\alpha} = (2\pi)^4u_\alpha^{-1}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

$$dP(1+2\to\beta) = \frac{d\sigma(1+2\to\beta)u_\alpha T}{V}$$

洛伦兹变换性质：

$$S_{p_1',\sigma_1',n_1';p_2',\sigma_2',n_2';\cdots p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$= e^{ia_\mu((\Lambda p_1)^\mu+(\Lambda p_2)^\mu+\cdots-(\Lambda p_1)^{\mu\prime}-(\Lambda p_2)^{\mu\prime}-\cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots(\Lambda p_1')^0(\Lambda p_2')^0\cdots}{p_1^0 p_2^0\cdots p_1^{0\prime}p_2^{0\prime}\cdots}}$$

$$\times\sum_{\bar\sigma_1,\bar\sigma_2,\cdots} D^{(j_1)}_{\bar\sigma_1\sigma_1}(W(\Lambda,p_1))D^{(j_2)}_{\bar\sigma_2\sigma_2}(W(\Lambda,p_2))\cdots\sum_{\bar\sigma_1',\bar\sigma_2',\cdots} D^{(j_1')*}_{\bar\sigma_1'\sigma_1'}(W(\Lambda,p_1'))D^{(j_2')*}_{\bar\sigma_2'\sigma_2'}(W(\Lambda,p_2'))\cdots$$

$$\times S_{\Lambda p_1',\bar\sigma_1',n_1';\Lambda p_2',\bar\sigma_2',n_2';\cdots\Lambda p_1,\bar\sigma_1,n_1;\Lambda p_2,\bar\sigma_2,n_2;\cdots}$$

$$S_{\beta\alpha}-\delta(\beta-\alpha)=-2\pi i M_{\beta\alpha}\delta^4(p_\beta-p_\alpha)$$

$$R_{\beta\alpha}\equiv\sum_{\text{spins}}|M_{\beta\alpha}|^2\prod_\beta E\prod_\alpha E$$

需对初态σ末态σ'求和以分别消去D和D^*矩阵

is invariant

$$N_\alpha=1:\quad \sum_{\text{spins}} d\Gamma(\alpha\to\beta)=2\pi E_\alpha^{-1}R_{\beta\alpha}\delta^4(p_\beta-p_\alpha)\frac{d\beta}{\prod_\beta E}$$

$$N_\alpha=2:\quad \sum_{\text{spins}} d\sigma(\alpha\to\beta)=(2\pi)^4u_\alpha^{-1}E_1^{-1}E_2^{-1}R_{\beta\alpha}\delta^4(p_\beta-p_\alpha)\frac{d\beta}{\prod_\beta E}$$

洛伦兹变换性质：

$$R_{\beta\alpha} \equiv \sum_{\text{spins}} |M_{\beta\alpha}|^2 \prod_\beta E \prod_\alpha E$$ **is invariant**

$$N_\alpha = 1: \quad \sum_{\text{spins}} d\Gamma(\alpha \to \beta) = 2\pi E_\alpha^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E}$$ 按$1/E_\alpha$变换

$$N_\alpha = 2: \quad \sum_{\text{spins}} d\sigma(\alpha \to \beta) = (2\pi)^4 u_\alpha^{-1} E_1^{-1} E_2^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E}$$ 不变！

$$u_\alpha \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}$$

$$\vec{p}_1 = 0 \to E_1 = m_1 \to p_1 \cdot p_2 = m_1 E_2 \to u_\alpha = \frac{|\vec{p}_2|}{E_2} \qquad \vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - v^2}} \quad E = \frac{m_0}{\sqrt{1 - v^2}}$$

$$p_1 = (\vec{p}, E_1) \;\; p_2 = (-\vec{p}, E_2) \;\; E = E_1 + E_2 \to u_\alpha = \frac{|\vec{p}|E}{E_1 E_2} = \left|\frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2}\right|$$

$$(|\vec{p}|^2 + E_1 E_2)^2 - (E_1^2 - |\vec{p}|^2)(E_2^2 - |\vec{p}|^2) = |\vec{p}|^2 (E_1 + E_2)^2$$

相空间:

$$N_\alpha = 1: \quad \sum_{\text{spins}} d\Gamma(\alpha \to \beta) = 2\pi E_\alpha^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E}$$

按$1/E_\alpha$变换

$$\vec{p}_\alpha = 0 \qquad \vec{p}_1' = -\vec{p}_2' - \vec{p}_3' - \cdots$$

$$\delta^4(p_\beta - p_\alpha) d\beta = \delta^3(\vec{p}_1' + \vec{p}_2' + \cdots)\delta(E_1' + E_2' + \cdots - E) d\vec{p}_1' d\vec{p}_2' \cdots$$

$$N_\alpha = 2: \quad \sum_{\text{spins}} d\sigma(\alpha \to \beta) = (2\pi)^4 u_\alpha^{-1} E_1^{-1} E_2^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E} \qquad \vec{p}_1 + \vec{p}_2 = 0$$

$$N_\beta = 2: \quad \delta^4(p_\beta - p_\alpha) d\beta = \delta(E_1' + E_2' - E) d\vec{p}_1' \qquad 1 \to 1' + 2' \qquad 1 + 2 \to 1' + 2'$$

$$= \delta\left(\sqrt{|\vec{p}_1'|^2 + m_1'^2} + \sqrt{|\vec{p}_1'|^2 + m_2'^2} - E\right) |\vec{p}_1'|^2 d|\vec{p}_1'| d\Omega = \frac{|\vec{p}_1'| E_1' E_2'}{E} d\Omega$$

$$(E - E_1')^2 = E_1'^2 + m_2'^2 - m_1'^2 \qquad (E - E_2')^2 = E_2'^2 + m_1'^2 - m_2'^2$$

$$E_1' = \sqrt{|\vec{p}_1'|^2 + m_1'^2} = \frac{E^2 - m_2'^2 + m_1'^2}{2E} \qquad E_2' = \sqrt{|\vec{p}_1'|^2 + m_2'^2} = \frac{E^2 - m_1'^2 + m_2'^2}{2E}$$

$$|\vec{p}_1'| = \frac{\sqrt{(E^2 - m_1'^2 - m_2'^2)^2 - 4m_1'^2 m_2'^2}}{2E} \qquad \left.\frac{d\Gamma(\alpha \to \beta)}{d\Omega}\right|_{N_\alpha = 1} \xlongequal{d\Gamma = 2\pi |M_{\beta\alpha}|^2 \delta^4(p_\beta - p_\alpha) d\beta} \frac{2\pi |\vec{p}_1'| E_1' E_2'}{E} |M_{\beta\alpha}|^2$$

相空间：

$$N_\alpha = 2: \quad \sum_{\text{spins}} d\sigma(\alpha\to\beta) = (2\pi)^4 u_\alpha^{-1} E_1^{-1} E_2^{-1} R_{\beta\alpha}\delta^4(p_\beta - p_\alpha)\frac{d\beta}{\prod_\beta E}$$

不变！

$$N_\beta = 2: \qquad 1+2\to 1'+2' \qquad \vec{p}_1 + \vec{p}_2 = 0$$

$$\frac{d\sigma(\alpha\to\beta)}{d\Omega} = \frac{(2\pi)^4|\vec{p}_1'|E_1'E_2'}{Eu_\alpha}|M_{\beta\alpha}|^2 = \frac{(2\pi)^4|\vec{p}_1'|E_1'E_2'E_1E_2}{E^2|\vec{p}_1|}|M_{\beta\alpha}|^2$$

$$N_\beta = 3:$$

$$\delta^4(p_\beta - p_\alpha)d\beta = d\vec{p}_2'd\vec{p}_3'\delta\left(\sqrt{(\vec{p}_2'+\vec{p}_3')^2 + m_1'^2} + \sqrt{\vec{p}_2'^2 + m_2'^2} + \sqrt{\vec{p}_3'^2 + m_3'^2} - E\right)$$

$$d\vec{p}_2'd\vec{p}_3' = |\vec{p}_2'|^2 d|\vec{p}_2'||\vec{p}_3'|^2 d|\vec{p}_3'|d\Omega_3 d\phi_{23} d\cos\theta_{23} \qquad \frac{\partial E_1'}{\partial\cos\theta_{23}} = \frac{|\vec{p}_2'||\vec{p}_3'|}{E_1'}$$

$$\delta^4(p_\beta - p_\alpha)d\beta = |\vec{p}_2'|d|\vec{p}_2'|\ |\vec{p}_3'|d|\vec{p}_3'|\ E_1'd\Omega_3 d\phi_{23} = E_1'E_2'E_3'dE_2'dE_3'd\Omega_3 d\phi_{23}$$

integrate out $\cos\theta_{23}$

关于量子场理论，目前我们已经：

- 建立了自由粒子态
- 引入时空平移和转动及内部对称性的生成元算符
- 用H建立了S矩阵理论直接描述散射实验 相互作用通过e^{-iHt}演化引入

只需知道$V(t)$对自由粒子态的作用： $S_{\alpha\beta} = (\Phi_\beta, S\Phi_\alpha) = \delta(\beta-\alpha) - 2\pi i M_{\beta\alpha}\delta^4(p_\beta - p_\alpha)$

$$S = \mathbf{T}e^{-i\int_{-\infty}^{\infty}dt\ V(t)} \qquad V(t) = e^{iH_0t}Ve^{-iH_0t} = \int d\vec{x}\mathcal{H}(x) \qquad [\mathcal{H}(\mathbf{x}), \mathcal{H}(\mathbf{x}')]_{类空} = \mathbf{0} \qquad H = H_0 + V$$

$N_\alpha = 1:$ $\quad d\Gamma(\alpha\to\beta) = 2\pi|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$ 按 $1/E_\alpha$ 变换

$N_\alpha = 2:$ $\quad d\sigma(\alpha\to\beta) \equiv \dfrac{d\Gamma(\alpha\to\beta)}{\Phi_\alpha} = (2\pi)^4 u_\alpha^{-1}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$ 洛伦兹变换不变量

建立局域的$\mathcal{H}(x)$对自由粒子态的作用? 本章

- 引入联系不同粒子态的算符：产生与湮灭算符
- 将H_0和V及其它生成元算符用产生与湮灭算符局域地表达出来

建立完整的计算体系： 下章 Wick定理； 约化公式； 路径积分

产生算符作用在某个多粒子态上定义为在此态上多加入一个粒子

$$a^\dagger(q)\Phi_{q_1q_2\cdots q_N} \equiv \Phi_{qq_1q_2\cdots q_N} \qquad \Phi_{q_1q_2\cdots q_N} = a^\dagger(q_1)a^\dagger(q_2)\cdots a^\dagger(q_N)\Phi_0$$

产生算符的厄米共轭算符: $(\Phi_{q'_1q'_2\cdots q'_M}, \Phi_{q_1q_2\cdots q_N}) = \frac{\delta_{MN}}{N!}\sum_{\mathcal{P}}\delta_{\mathcal{P}}\prod_i \delta(q_i - q'_{\mathcal{P}i})$

$$(\Phi_{q'_1\cdots q'_M}, a(q)\Phi_{q_1\cdots q_N}) = (a^\dagger(q)\Phi_{q'_1\cdots q'_M}, \Phi_{q_1\cdots q_N}) = (\Phi_{qq'_1\cdots q'_M}, \Phi_{q_1\cdots q_N})$$

对指标$1,2,\cdots,N$交换$\mathcal{P}$的求和可写成对一特殊的要被置换到首位$(\mathcal{P}r=1)$的指标r的求和,再加上对剩余的指标$1,\cdots,r-1,r+1,\cdots,N$的所有置换$\bar{\mathcal{P}}$求和$\sum_{\mathcal{P}} = \sum_{r=1}^N\sum_{\bar{\mathcal{P}}}$. 利用$\delta_{\mathcal{P}} = \delta_{r1}\delta_{\bar{\mathcal{P}}}$($\delta_{r1}$是将换到首位所贡献的$\delta_{\mathcal{P}}$),

$$(\Phi_{q'_1\cdots q'_M}, a(q)\Phi_{q_1\cdots q_N}) = \frac{\delta_{N,M+1}}{N!}\sum_{r=1}^{N}\sum_{\bar{\mathcal{P}}}\delta_{r1}\delta_{\bar{\mathcal{P}}}\delta(q-q_r)\prod_{i=1}^{M}\delta(q'_i - q_{\bar{\mathcal{P}}i})$$

$$= \begin{cases} \sum_{r=1}^{N}\delta_{r1}\delta(q-q_r)(\Phi_{q'_1\cdots q'_M}, \Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N}) & N\geq 1 \\ 0 & N=0 \end{cases}$$

湮灭算符: $a(q)\Phi_{q_1\cdots q_N} = \sum_{r=1}^{N}\delta_{r1}\delta(q-q_r)\Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N} \qquad N\geq 1 \qquad a(q)\Phi_0 = 0$

产生算符： $a^\dagger(q)\Phi_{q_1q_2\cdots q_N} \equiv \Phi_{qq_1q_2\cdots q_N}$ $\quad \Phi_{q_1q_2\cdots q_N} = a^\dagger(q_1)a^\dagger(q_2)\cdots a^\dagger(q_N)\Phi_0$

湮灭算符： $a(q)\Phi_{q_1\cdots q_N} = \sum_{r=1}^{N}\delta_{r1}\delta(q-q_r)\Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N}$ $\quad N \geq 1 \quad a(q)\Phi_0 = 0$

产生与湮灭算符的性质：作业8

$$a(q')a^\dagger(q) \mp a^\dagger(q)a(q') = \delta(q'-q)$$

$$a(q')a(q) \mp a(q)a(q') = 0 \qquad a^\dagger(q')a^\dagger(q) \mp a^\dagger(q)a^\dagger(q') = 0$$

- 负号对应两个粒子都是玻色子或一个是玻色子一个是费米子的情况
- 正号对应两个粒子都是费米子的情况

粒子数算符：作业9

$$N \equiv \int d\vec{q}\; a^\dagger(q)a(q) \qquad [N, a^\dagger(q)] = a^\dagger(q) \qquad [N, a(q)] = -a(q)$$

产生湮灭算符在洛伦兹变换下的行为： $a^\dagger(p)\Phi_{p_1p_2\cdots p_N} \equiv \Phi_{pp_1p_2\cdots p_N}$

$$U_0(\Lambda,b)\Phi_{p,\sigma,n;p_1,\sigma_1,n_1;\cdots} = e^{ib_\mu((\Lambda p)^\mu+(\Lambda p_1)^\mu+\cdots)}\sqrt{\frac{(\Lambda p)^0(\Lambda p_1)^0\cdots}{p^0p_1^0\cdots}}\sum_{\bar\sigma\bar\sigma_1\cdots}D_{\bar\sigma\sigma}(W(\Lambda,p))$$

$$= U_0(\Lambda,b)a^\dagger(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b)U_0(\Lambda,b)\Phi_{p_1,\sigma_1,n_1;\cdots} \qquad \times D_{\bar\sigma_1\sigma_1}(W(\Lambda,p_1))\cdots\Phi_{\Lambda p,\bar\sigma,n;\Lambda p_1,\sigma_1',n_1;\cdots}$$

$$U_0(\Lambda,b)a^\dagger(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b) = e^{i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar\sigma}D_{\bar\sigma\sigma}(W(\Lambda,p))a^\dagger(\vec{p}_\Lambda,\bar\sigma,n)$$

$$= e^{i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar\sigma}D^*_{\sigma\bar\sigma}(W^{-1}(\Lambda,p))a^\dagger(\vec{p}_\Lambda,\bar\sigma,n)$$

$$U_0(\Lambda,b)a(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b) = e^{-i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar\sigma}D^\dagger_{\bar\sigma\sigma}(W(\Lambda,p))a(\vec{p}_\Lambda,\bar\sigma,n)$$

$$= e^{-i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar\sigma}D_{\sigma\bar\sigma}(W^{-1}(\Lambda,p))a(\vec{p}_\Lambda,\bar\sigma,n)$$

产生湮灭算符在空间反射变换下的行为： $a^\dagger(p)\Phi_{p_1p_2\cdots p_N} \equiv \Phi_{pp_1p_2\cdots p_N}$

有质量正能: $P\Psi_{p,\sigma} = \eta\Psi_{\mathcal{P}p,\sigma}$

无质量正能: $P\Psi_{p,\sigma} = \eta_\sigma e^{\mp i\pi\sigma}\Psi_{\mathcal{P}p,-\sigma}$ 负号: $0 \le \phi < \pi$，正号: $\pi \le \phi < 2\pi$ ϕ是$\vec{p}$在xy平面上投影与x轴的夹角

产生湮灭有质量的粒子算符在空间反射变换下的行为：

$$Pa^\dagger(\vec{p},\sigma,n)P^{-1} = \eta a^\dagger(\mathcal{P}\vec{p},\sigma,n) \qquad Pa(\vec{p},\sigma,n)P^{-1} = \eta^* a(\mathcal{P}\vec{p},\sigma,n)$$

产生湮灭无质量的粒子算符在空间反射变换下的行为：

$$Pa^\dagger(\vec{p},\sigma,n)P^{-1} = \eta_\sigma e^{\mp i\pi\sigma} a^\dagger(\mathcal{P}\vec{p},-\sigma,n) \qquad Pa(\vec{p},\sigma,n)P^{-1} = \eta_\sigma^* e^{\pm i\pi\sigma} a(\mathcal{P}\vec{p},-\sigma,n)$$

负号: $0 \le \phi < \pi$，正号: $\pi \le \phi < 2\pi$ （ϕ是$\vec{p}$在xy平面上的投影与x轴的夹角）

产生湮灭算符在时间反演变换下的行为： $a^\dagger(p)\Phi_{p_1p_2\cdots p_N} \equiv \Phi_{pp_1p_2\cdots p_N}$

有质量正能： $T\Psi_{p,\sigma} = (-1)^{j-\sigma}\Psi_{\mathcal{P}p,-\sigma}$

无质量正能： $T\Psi_{p,\sigma} = \zeta_\sigma e^{\pm i\pi\sigma}\Psi_{\mathcal{P}p,\sigma}$

$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ 中: 正号$0 \le \phi < \pi$,负号对应$\pi \le \phi < 2\pi$

产生湮灭有质量的粒子算符在时间反演变换下的行为：

$$Ta^\dagger(\vec{p},\sigma,n)T^{-1} = (-1)^{j-\sigma}a^\dagger(\mathcal{P}\vec{p},-\sigma,n) \qquad Ta(\vec{p},\sigma,n)T^{-1} = (-1)^{j-\sigma}a(\mathcal{P}\vec{p},-\sigma,n)$$

产生湮灭无质量的粒子算符在时间反演变换下的行为：

$$Ta^\dagger(\vec{p},\sigma,n)T^{-1} = \zeta_\sigma e^{\pm i\pi\sigma}a^\dagger(\mathcal{P}\vec{p},\sigma,n) \qquad Ta(\vec{p},\sigma,n)T^{-1} = \zeta_\sigma^* e^{\mp i\pi\sigma}a(\mathcal{P}\vec{p},\sigma,n)$$

$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ 中正号$0 \le \phi < \pi$,负号对应$\pi \le \phi < 2\pi$

产生湮灭算符在内部对称性变换下的行为： $a^\dagger(p)\Phi_{p_1p_2\cdots p_N} \equiv \Phi_{pp_1p_2\cdots p_N}$

$$U_0(T(\theta))\Psi_{q,\sigma} = e^{iq_a\theta^a}\Psi_{q,\sigma} \qquad Q_a\,\Psi_{q,\sigma} = q_a\,\Psi_{q,\sigma}$$

产生湮灭粒子算符在内部对称性$U(1)$生成元Q_a变换下的行为：

$$[Q_a,\ a^\dagger(\vec{q})]\Phi_{q_1q_2\cdots q_n} = Q_a\Phi_{qq_1q_2\cdots q_n} - a^\dagger(\vec{q})(q_{a1}+q_{a2}+\cdots)\Phi_{q_1q_2\cdots q_n}$$

$$= q_a\Phi_{qq_1q_2\cdots q_n} = q_a a^\dagger(\vec{q})\Phi_{q_1q_2\cdots q_n}$$

$$[Q_a,\ a^\dagger(\vec{q})] = q_a a^\dagger(\vec{q}) \qquad [Q_a,\ a(\vec{q})] = -q_a a(\vec{q})$$

产生和湮灭算符在坐标空间的表达:

$$\tilde{\mathcal{H}}(x) = \sum_{NM}\int dp_1\cdots dp_N dp'_1\cdots dp'_M\ \tilde{c}(x,p_1,\cdots,p_N,p'_1,\cdots,p'_M)\ a(p_1)\cdots a(p_N)a^\dagger(p'_1)\cdots a^\dagger(p'_M)$$

$$= \sum_{NM} c_{NM}\ \psi_l^{+,N}(x)\psi_l^{-,M}(x)$$

$$\psi_l^+(x) = \sum_{\sigma,n}\int d^3p\ u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n}\int d^3p\ v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

♣ 局域相互作用哈密顿量的表达需要

$$S = \mathbf{T}\ e^{-i\int_{\tau_0}^{\tau} dt\ V(t)} \qquad V(t) = e^{iH_0t}Ve^{-iH_0t} = \int d\vec{x}\ \tilde{\mathcal{H}}(\vec{r},t)$$

$$U_0(\Lambda,a)\tilde{\mathcal{H}}(x)U_0^{-1}(\Lambda,a) = \tilde{\mathcal{H}}(\Lambda x+a)$$

✠ 态空间的产生与湮灭直接与坐标空间联系 量子场: 产生湮灭算符的集合！

♡ 局域性！ 它导致的结果可满足cluster decomposition原理，但反过来呢 ？

♠ 因果性！ $[\tilde{\mathcal{H}}(x),\tilde{\mathcal{H}}(x')] = 0\quad x-x'$ 类空间隔

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p\ u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p\ v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$[Q_a,\ a^\dagger(\vec{q})] = q_a a^\dagger(\vec{q}) \quad [Q_a,\ a(\vec{q})] = -q_a a(\vec{q})$$

$$[Q_a,\ \psi_l^\pm(x)] = \mp q_a \psi_l^\pm(x)$$

要求$u_l(x;\vec{p},\sigma,n)$和$v_l(x;\vec{p},\sigma,n)$满足: $U_0(\Lambda,a)\tilde{\mathcal{H}}(x)U_0^{-1}(\Lambda,a) = \tilde{\mathcal{H}}(\Lambda x+a)$

$$U_0(\Lambda,a)\psi_l^+(x)U_0^{-1}(\Lambda,a) = \sum_{\bar{l}} D_{l\bar{l}}^+(\Lambda^{-1})\psi_{\bar{l}}^+(\Lambda x + a)$$

$$U_0(\Lambda,a)\psi_l^-(x)U_0^{-1}(\Lambda,a) = \sum_{\bar{l}} D_{l\bar{l}}^-(\Lambda^{-1})\psi_{\bar{l}}^-(\Lambda x + a)$$

$$D^\pm(\Lambda^{-1})D^\pm(\bar{\Lambda}^{-1}) = D^\pm((\bar{\Lambda}\Lambda)^{-1}) \qquad D^\pm(\Lambda_1)D^\pm(\Lambda_2) = D^\pm(\Lambda_1\Lambda_2)$$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x)=\sum_{\sigma,n}\int d^3p\; u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x)=\sum_{\sigma,n}\int d^3p\; v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$[Q_a,\ \psi_l^\pm(x)] = \mp q_a\psi_l^\pm(x) \qquad U_0(\Lambda,a)\psi_l^\pm(x)U_0^{-1}(\Lambda,a) = \sum_{\bar{l}} D^\pm_{l\bar{l}}(\Lambda^{-1})\psi_{\bar{l}}^\pm(\Lambda x+a)$$

$$U_0(e^\omega,\epsilon) = e^{\frac{i}{2}\omega_{\rho\sigma}J^{\rho\sigma}+i\epsilon_\rho P^\rho} \qquad D^\pm(e^\omega) = e^{\frac{i}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma}}$$

算$D^\pm$等价于算$\mathcal{J}^{\sigma\rho}$的矩阵元！场表示没么正性要求

$$i[\mathcal{J}^{\mu\nu},\mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

$$[\frac{i}{2}\omega_{\rho\sigma}J^{\rho\sigma} + i\epsilon_\rho P^\rho, \psi_l^\pm(x)] = -\frac{i}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma}_{l\bar{l}}\psi_{\bar{l}}^\pm(x) + [\omega_{\rho\sigma}x^\sigma + \epsilon_\rho]\partial^\rho\psi_l^\pm(x)$$

$$[P^\rho,\psi_l^\pm(x)] = -i\partial^\rho\psi_l^\pm(x) \qquad [x^\sigma,-i\partial^\rho] = ig^{\sigma\rho}$$

$H\sim -i\partial_t,\ \vec{P}\sim i\nabla$ 作用算符上与通常作用态上相差一个负号！

$$[J^{\rho\sigma},\psi_l^\pm(x)] = -\mathcal{J}^{\rho\sigma}_{l\bar{l}}\psi_{\bar{l}}^\pm(x) + i(x^\rho\partial^\sigma - x^\sigma\partial^\rho)\psi_l^\pm(x) \qquad L^{\rho\sigma} \equiv -i(x^\rho\partial^\sigma - x^\sigma\partial^\rho)$$

$$i[L^{\mu\nu},L^{\rho\sigma}] = g^{\nu\rho}L^{\mu\sigma} - g^{\mu\rho}L^{\nu\sigma} - g^{\sigma\mu}L^{\rho\nu} + g^{\sigma\nu}L^{\rho\mu} \qquad [\mathcal{J}^{\mu\nu},L^{\rho\sigma}] = 0$$

$$i[i\partial^\mu,L^{\rho\sigma}] = g^{\mu\rho}i\partial^\sigma - g^{\mu\sigma}i\partial^\rho \qquad [i\partial^\mu,\mathcal{J}^{\rho\sigma}] = 0 \qquad [i\partial^\mu,i\partial^\rho] = 0$$

场空间的Pauli-Lubanski算符: $W^\mu \equiv -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}i\partial_\nu(L_{\rho\sigma}+\mathcal{J}_{\rho\sigma}) = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}i\partial_\nu\mathcal{J}_{\rho\sigma}$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x)=\sum_{\sigma,n}\int d^3p\, u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x)=\sum_{\sigma,n}\int d^3p\, v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$U_0(\Lambda,a)\psi_l^\pm(x)U_0^{-1}(\Lambda,a)=\sum_{\bar{l}}D_{l\bar{l}}^\pm(\Lambda^{-1})\psi_{\bar{l}}^\pm(\Lambda x+a)$$

$$U_0(\Lambda,b)a(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b)=e^{-i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}}D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda,p))a(\vec{p}_\Lambda,\bar{\sigma},n)$$

$$U_0(\Lambda,b)a^\dagger(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b)=e^{i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}}D_{\sigma\bar{\sigma}}^*(W^{-1}(\Lambda,p))a^\dagger(\vec{p}_\Lambda,\bar{\sigma},n)$$

$$\sum_{\bar{l}}D_{l\bar{l}}^+(\Lambda^{-1})u_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n)=\sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{\sigma}D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda,p))e^{-i(\Lambda p)\cdot b}u_l(x;\vec{p},\sigma,n)$$

$$\sum_{\bar{l}}D_{l\bar{l}}^-(\Lambda^{-1})v_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n)=\sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{\sigma}D_{\sigma\bar{\sigma}}^*(W^{-1}(\Lambda,p))e^{i(\Lambda p)\cdot b}v_l(x;\vec{p},\sigma,n)$$

$$d^3p\sqrt{\frac{(\Lambda p)^0}{p^0}}=\frac{d^3p}{p^0}\sqrt{p^0(\Lambda p)^0}=\frac{d^3(\Lambda p)}{(\Lambda p)^0}\sqrt{p^0(\Lambda p)^0}=d^3(\Lambda p)\sqrt{\frac{p^0}{(\Lambda p)^0}}$$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x)=\sum_{\sigma,n}\int d^3p\; u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x)=\sum_{\sigma,n}\int d^3p\; v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{l}} D^+_{l\bar{l}}(\Lambda^{-1})u_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n) = \sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{\sigma} D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda,p))e^{-i(\Lambda p)\cdot b}u_l(x;\vec{p},\sigma,n)$$

$$\sum_{\bar{l}} D^-_{l\bar{l}}(\Lambda^{-1})v_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n) = \sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{\sigma} D^*_{\sigma\bar{\sigma}}(W^{-1}(\Lambda,p))e^{i(\Lambda p)\cdot b}v_l(x;\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n)D_{\bar{\sigma}\sigma}(W(\Lambda,p))=\sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{l} D^+_{\bar{l}l}(\Lambda)e^{-i(\Lambda p)\cdot b}u_l(x;\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n)D^*_{\bar{\sigma}\sigma}(W(\Lambda,p))=\sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{l} D^-_{\bar{l}l}(\Lambda)e^{i(\Lambda p)\cdot b}v_l(x;\vec{p},\sigma,n)$$

平移

$$\psi_l^+(x)=\sum_{\sigma,n}\int d^3p\ u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n)\quad \psi_l^-(x)=\sum_{\sigma,n}\int d^3p\ v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}u_{\bar l}(\Lambda x+b;\vec{p}_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)e^{-i(\Lambda p)\cdot b}u_l(x;\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}v_{\bar l}(\Lambda x+b;\vec{p}_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)e^{i(\Lambda p)\cdot b}v_l(x;\vec{p},\sigma,n)$$

$\Lambda=1,b$任意,$\Rightarrow D^+(\Lambda)=D^-(\Lambda)=1 \Rightarrow u_l(x+b;\vec{p},\sigma,n)=e^{-ip\cdot b}u_l(x;\vec{p},\sigma,n)\quad v_l(x+b;\vec{p},\sigma,n)=e^{ip\cdot b}v_l(x;\vec{p},\sigma,n)$

$\Rightarrow u_l(x;\vec{p},\sigma,n)=\dfrac{e^{-ip\cdot x}}{(2\pi)^{3/2}}u_l(\vec{p},\sigma,n)$ 注意指数上的正负号！ $v_l(x;\vec{p},\sigma,n)=\dfrac{e^{ip\cdot x}}{(2\pi)^{3/2}}v_l(\vec{p},\sigma,n)$

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec{p},\sigma,n)a(\vec{p},\sigma,n)\qquad \underbrace{\psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)}$$

$$\langle 0|\psi_l^-(x)p^\rho|\Psi\rangle\sim -i\partial^\rho\langle 0|\psi_l^-(x)|\Psi\rangle=\langle 0|[P^\rho,\psi_l^-(x)]|\Psi\rangle=-\langle 0|\psi_l^-(x)P^\rho|\Psi\rangle$$

$$\sum_{\bar\sigma}u_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}v_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)v_l(\vec{p},\sigma,n)$$

有质量情况

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec{p},\sigma,n)a(\vec{p},\sigma,n)\quad \psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}}u_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n)D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar{l}l}(\Lambda)u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}}v_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n)D^*_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar{l}l}(\Lambda)v_l(\vec{p},\sigma,n)$$

推进： 定义$q=\Lambda p$， 取$p=k=(M,0,0,0)$

$\Lambda=L(q)$, $L(q)$由第一章给出是沿$\vec{q}$方向的"推进"： $q=L(q)(M,0,0,0)=\Lambda p$

则$L(p)=1$， 导致$W(\Lambda,p)\equiv L^{-1}(\Lambda p)\Lambda L(p)=L^{-1}(q)L(q)=1$

$$u_{\bar{l}}(\vec{q},\sigma,n)=\sqrt{\frac{M}{q^0}}\sum_l D^+_{\bar{l}l}(L(q))u_l(0,\sigma,n)$$

$$v_{\bar{l}}(\vec{q},\sigma,n)=\sqrt{\frac{M}{q^0}}\sum_l D^-_{\bar{l}l}(L(q))v_l(0,\sigma,n)$$

有质量情况

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec p,\sigma,n)a(\vec p,\sigma,n)\quad \psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec p,\sigma,n)a^\dagger(\vec p,\sigma,n)$$

$$\sum_{\bar\sigma}u_{\bar l}(\vec p_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)u_l(\vec p,\sigma,n)$$

$$\sum_{\bar\sigma}v_{\bar l}(\vec p_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)v_l(\vec p,\sigma,n)$$

转动: $D^+=D^-$ 为了适应未来反粒子的引入!

取$p=k=(M,0,0,0)$,$\Lambda=R$, 导致$\vec p_\Lambda=0$: $q=\Lambda p=p$

则上章给出的$L(p)=1$，导致$W(\Lambda,p)\equiv L^{-1}(\Lambda p)\Lambda L(p)=L^{-1}(p)R=R$

$$\sum_{\bar\sigma}u_{\bar l}(0,\bar\sigma,n)D_{\bar\sigma\sigma}(R)=\sum_l D^+_{\bar l l}(R)u_l(0,\sigma,n)\xrightarrow{\text{非平庸 (平庸是恒等式)}}\sum_{\bar\sigma}u_{\bar l}(0,\bar\sigma,n)\vec J^{(j_n)}_{\bar\sigma\sigma}=\sum_l\vec{\mathcal{J}}_{\bar l l}u_l(0,\sigma,n)$$

⇕ 它们是可以构造出来的! 对固定 j_n' 的 $\vec{\mathcal{J}}_{\bar l l}$ 选择 $j_n\le j_n'$ 的 $\vec J^{(j_n)}$ 进行求解

$$\sum_{\bar\sigma}v_{\bar l}(0,\bar\sigma,n)D^*_{\bar\sigma\sigma}(R)=\sum_l D^-_{\bar l l}(R)v_l(0,\sigma,n)\xrightarrow{\text{非平庸 (平庸是恒等式)}}\sum_{\bar\sigma}v_{\bar l}(0,\bar\sigma,n)\vec J^{(j_n)*}_{\bar\sigma\sigma}=-\sum_l\vec{\mathcal{J}}_{\bar l l}v_l(0,\sigma,n)$$

无质量情况

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}u_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)u_l(\vec{p},\sigma,n) \quad D_{\bar\sigma\sigma}(W(\Lambda,p))=e^{i\theta(\Lambda,p)\sigma}\delta_{\bar\sigma\sigma}$$

$$\sum_{\bar\sigma}v_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)v_l(\vec{p},\sigma,n) \quad D^*_{\bar\sigma\sigma}(W(\Lambda,p))=e^{-i\theta(\Lambda,p)\sigma}\delta_{\bar\sigma\sigma}$$

取$p=k=(\kappa,0,0,\kappa)$,$\Lambda=L(q)$,: $q=L(q)(\kappa,0,0,\kappa)=\Lambda p$

则$L(p)=1$，导致$W(\Lambda,p)\equiv L^{-1}(\Lambda p)\Lambda L(p)=L^{-1}(q)L(q)=1$

$$u_{\bar l}(\vec{q},\sigma,n)=\sqrt{\frac{k^0}{q^0}}\sum_l D^+_{\bar l l}(L(q))u_l(\vec{k},\sigma,n) \quad v_{\bar l}(\vec{q},\sigma,n)=\sqrt{\frac{k^0}{q^0}}\sum_l D^-_{\bar l l}(L(q))v_l(\vec{k},\sigma,n)$$

取：$p=k=(\kappa,0,0,\kappa)$，$\Lambda=W \quad q=\Lambda p=k$

$$u_{\bar l}(\vec{k},\sigma,n)e^{i\theta(W,k)\sigma}=\sum_l D^+_{\bar l l}(W)u_l(\vec{k},\sigma,n) \quad v_{\bar l}(\vec{k},\sigma,n)e^{-i\theta(W,k)\sigma}=\sum_l D^-_{\bar l l}(W)v_l(\vec{p},\sigma,n)$$

无质量情况: $W(\theta,\alpha,\beta)=S(\alpha,\beta)R(\theta)$

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec{p},\sigma,n)a(\vec{p},\sigma,n)\quad \psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}u_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)u_l(\vec{p},\sigma,n)\quad D_{\bar\sigma\sigma}(W(\Lambda,p))=e^{i\theta(\Lambda,p)\sigma}\delta_{\bar\sigma\sigma}$$

$$\sum_{\bar\sigma}v_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)v_l(\vec{p},\sigma,n)\quad D^*_{\bar\sigma\sigma}(W(\Lambda,p))=e^{-i\theta(\Lambda,p)\sigma}\delta_{\bar\sigma\sigma}$$

$$u_{\bar l}(\vec{k},\sigma,n)e^{i\theta(W,k)\sigma}=\sum_l D^+_{\bar l l}(W)u_l(\vec{k},\sigma,n)\quad v_{\bar l}(\vec{k},\sigma,n)e^{-i\theta(W,k)\sigma}=\sum_l D^-_{\bar l l}(W)v_l(\vec{p},\sigma,n)$$

$$u_{\bar l}(\vec{k},\sigma,n)e^{i\theta\sigma}=\sum_l D^+_{\bar l l}(R(\theta))u_l(\vec{k},\sigma,n)\quad v_{\bar l}(\vec{k},\sigma,n)e^{-i\theta\sigma}=\sum_l D^-_{\bar l l}(R(\theta))v_l(\vec{k},\sigma,n)$$

⇕ 它们是可以构造出来的! ⇕

$$u_{\bar l}(\vec{k},\sigma,n)=\sum_l D^+_{\bar l l}(S(\alpha,\beta))u_l(\vec{k},\sigma,n)\quad v_{\bar l}(\vec{k},\sigma,n)=\sum_l D^-_{\bar l l}(S(\alpha,\beta))v_l(\vec{k},\sigma,n)$$

推进与转动

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

有质量： $u_{\bar{l}}(\vec{q},\sigma,n)=\sqrt{\frac{M}{q^0}}\sum_l D^+_{\bar{l}l}(L(q))u_l(0,\sigma,n) \quad v_{\bar{l}}(\vec{q},\sigma,n)=\sqrt{\frac{M}{q^0}}\sum_l D^-_{\bar{l}l}(L(q))v_l(0,\sigma,n)$

$$\sum_{\bar{\sigma}}u_{\bar{l}}(0,\bar{\sigma},n)D_{\bar{\sigma}\sigma}(R)=\sum_l D^+_{\bar{l}l}(R)u_l(0,\sigma,n) \xRightarrow{\text{非平庸 (平庸是恒等式)}} \sum_{\bar{\sigma}}u_{\bar{l}}(0,\bar{\sigma},n)\vec{J}^{(j_n)}_{\bar{\sigma}\sigma}=\sum_l\vec{\mathcal{J}}_{\bar{l}l}u_l(0,\sigma,n)$$

$$\sum_{\bar{\sigma}}v_{\bar{l}}(0,\bar{\sigma},n)D^*_{\bar{\sigma}\sigma}(R)=\sum_l D^-_{\bar{l}l}(R)v_l(0,\sigma,n) \xRightarrow{\text{非平庸 (平庸是恒等式)}} \sum_{\bar{\sigma}}v_{\bar{l}}(0,\bar{\sigma},n)\vec{J}^{(j_n)*}_{\bar{\sigma}\sigma}=-\sum_l\vec{\mathcal{J}}_{\bar{l}l}v_l(0,\sigma,n)$$

无质量： $u_{\bar{l}}(\vec{q},\sigma,n)=\sqrt{\frac{k^0}{q^0}}\sum_l D^+_{\bar{l}l}(L(q))u_l(\vec{k},\sigma,n) \quad v_{\bar{l}}(\vec{q},\sigma,n)=\sqrt{\frac{k^0}{q^0}}\sum_l D^-_{\bar{l}l}(L(q))v_l(\vec{k},\sigma,n)$

$$u_{\bar{l}}(\vec{k},\sigma,n)e^{i\theta\sigma}=\sum_l D^+_{\bar{l}l}(R(\theta))u_l(\vec{k},\sigma,n) \quad v_{\bar{l}}(\vec{k},\sigma,n)e^{-i\theta\sigma}=\sum_l D^-_{\bar{l}l}(R(\theta))v_l(\vec{k},\sigma,n) \quad D^+=D^-=\text{实矩阵}$$

$$u_{\bar{l}}(\vec{k},\sigma,n)=\sum_l D^+_{\bar{l}l}(S(\alpha,\beta))u_l(\vec{k},\sigma,n) \quad v_{\bar{l}}(\vec{k},\sigma,n)=\sum_l D^-_{\bar{l}l}(S(\alpha,\beta))v_l(\vec{k},\sigma,n) \quad v_l=u_l^*$$

后面对自旋0, 1/2的场只讨论有质量的情形（零质量取有质量的零质量极限，且无奇异），对自旋1及以上的单独讨论零质量情形。

$$\psi_l^+(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{-ip\cdot x}u_l(\vec{p},\sigma,n)a(\vec{p},\sigma,n)\quad \psi_l^-(x)=\sum_{\sigma,n}\int\frac{d^3p}{(2\pi)^{3/2}}e^{ip\cdot x}v_l(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}u_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma}v_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)v_l(\vec{p},\sigma,n)$$

标量量子场: $D^\pm(\Lambda)=1\quad D(W)=1,\ u_l(\vec{p}_\Lambda)\sqrt{(\Lambda p)^0}=u_l(\vec{p})\sqrt{p^0}\quad v_l(\vec{p}_\Lambda)\sqrt{(\Lambda p)^0}=v_l(\vec{p})\sqrt{p^0}$

只考虑有质量的标量场: $u(\vec{p})=v(\vec{p})=1/\sqrt{2p^0}$ 无质量可看作质量趋于0的极限

$$\phi^+(x)=\int\frac{d\vec{p}}{(2\pi)^{3/2}\sqrt{2p^0}}e^{-ip\cdot x}a(\vec{p})\quad \phi^-(x)=\int\frac{d\vec{p}}{(2\pi)^{3/2}\sqrt{2p^0}}e^{ip\cdot x}a^\dagger(\vec{p})=(\phi^+(x))^\dagger$$

$$[\phi^+(x),\phi^+(y)]_\mp=0\qquad [\phi^-(x),\phi^-(y)]_\mp=0$$

对易子: 玻色子; 反对易子: 费米子

$$\Delta_+(M,x-y)\equiv[\phi^+(x),\phi^-(y)]_\mp=\int\frac{d\vec{p}\,d\vec{p}'}{(2\pi)^3(2p^0 2p'^0)^{1/2}}e^{-ipx}e^{ip'y}\delta(\vec{p}-\vec{p}')=\int\frac{d\vec{p}}{(2\pi)^3 2p^0}e^{-ip\cdot(x-y)}$$

作业10

♣ 动量空间的产生与湮灭算符:

$$[a(q),a^\dagger(q')]_\mp=\delta(\vec{q}-\vec{q}')\qquad [a(q),a(q')]_\mp=[a^\dagger(q),a^\dagger(q')]_\mp=0\quad a(q)\Phi_0=0$$

$$N\equiv\int d\vec{p}\,N(p)\qquad [N(p),a^\dagger(q)]=\delta(\vec{p}-\vec{q})a^\dagger(q)\qquad [N,a^\dagger(q)]=a^\dagger(q)$$

$$N(p)\equiv a^\dagger(p)a(p)\qquad [N(p),a(q)]=-\delta(\vec{p}-\vec{q})a(q)\qquad [N,a(q)]=-a(q)$$

♠ 坐标空间的产生与湮灭算符:

$$[\phi^+(x),\phi^-(x')]_\mp=\Delta_+(M,x-x')\quad [\phi^+(x),\phi^+(x')]_\mp=[\phi^-(x),\phi^-(x')]_\mp=0\quad \phi^+(x)\Phi_0=0$$

$$\Delta_+(M,x)=\int\frac{d\vec{p}}{(2\pi)^3 2p^0}e^{-ip\cdot x}\overset{x类空}{=\!=\!=}\frac{M}{4\pi^2\sqrt{-x^2}}K_1(M\sqrt{-x^2})\overset{M\to\infty,t=0}{=\!=\!=\!=\!=\!=}\frac{1}{2M}\delta(\vec{x})$$

$$\tilde{N}(t)\equiv\int d\vec{x}\,\phi^-(x)\phi^+(x)\qquad [\tilde{N}(t),\phi^-(\vec{x},t)]=\int d\vec{x}'\phi^-(\vec{x}',t)\Delta_+(M,\vec{x}'-\vec{x},0)$$

$$[\tilde{N}(t),\phi^+(\vec{x},t)]=-\int d\vec{x}'\phi^+(\vec{x}',t)\Delta_+(M,\vec{x}-\vec{x}',0)$$

关于坐标空间的产生与湮灭算符

$$[\phi^+(x),\phi^-(x')]_\mp=\Delta_+(M,x-x') \quad [\phi^+(x),\phi^+(x')]_\mp=[\phi^-(x),\phi^-(x')]_\mp=0 \quad \phi^+(x)\Phi_0=0$$

$$\Delta_+(M,x)=\int\frac{d\vec{p}}{(2\pi)^3 2p^0}e^{-ip\cdot x}\overset{x类空}{=\!=\!=}\frac{M}{4\pi^2\sqrt{-x^2}}K_1(M\sqrt{-x^2})\overset{M\to\infty,t=0}{=\!=\!=\!=\!=\!=}\frac{1}{2M}\delta(\vec{x})$$

$$\tilde{N}(t)\equiv\int d\vec{x}\,\phi^-(x)\phi^+(x) \qquad [\tilde{N}(t),\phi^-(\vec{x},t)]=\int d\vec{x}'\phi^-(\vec{x}',t)\Delta_+(M,\vec{x}'-\vec{x},0)$$

♣ 波包：

$$[\tilde{N}(t),\phi^+(\vec{x},t)]=-\int d\vec{x}'\phi^+(\vec{x}',t)\Delta_+(M,\vec{x}-\vec{x}',0)$$

若存在：$\rho(\vec{x})=2M\int d\vec{x}'\rho(\vec{x}')\Delta_+(M,\vec{x}'-\vec{x},0)$ $\qquad \tilde{\phi}^-(t)\equiv\int d\vec{x}\,\rho(\vec{x})\phi^-(\vec{x},t)$

$$[\tilde{N}(t),\tilde{\phi}^-(t)]=\int d\vec{x}d\vec{x}'\rho(\vec{x}')\Delta_+(M,\vec{x}'-\vec{x},0)\phi^-(\vec{x},t)=\tilde{\phi}^-(t)$$

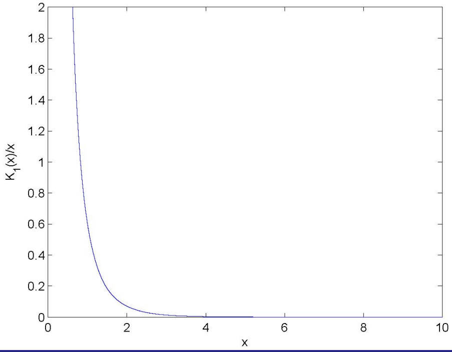
$$[\tilde{N}(t),\tilde{\phi}^+(t)]=-\int d\vec{x}d\vec{x}'\rho(\vec{x}')\Delta_+(M,\vec{x}'-\vec{x},0)\phi^+(\vec{x},t)=-\tilde{\phi}^+(t)$$

坐标空间的产生与湮灭算符: $\Delta_+(M,x) = \frac{M}{4\pi^2\sqrt{-x^2}} K_1(M\sqrt{-x^2})$

$$\tilde{N}(t) \equiv \int d\vec{x}\, \phi^-(x)\phi^+(x)$$

$$[\tilde{N}(t), \phi^-(\vec{x},t)] = \int d\vec{x}'\phi^-(\vec{x}',t)\Delta_+(M,\vec{x}'-\vec{x},0)$$

$$[\tilde{N}(t), \phi^+(\vec{x},t)] = -\int d\vec{x}'\phi^+(\vec{x}',t)\Delta_+(M,\vec{x}-\vec{x}',0)$$



$$K_1(x) = \int_0^\infty dt \frac{t\sin xt}{\sqrt{1+t^2}}$$

ϕ^+和ϕ^-场之间的不对易（或不反对易）将对构造$\tilde{\mathcal{H}}(x)$产生很大的困难。如果$\tilde{\mathcal{H}}(x)$是由ϕ^+和ϕ^-场构造的，$\tilde{\mathcal{H}}(x)$在类空区对易要求ϕ^+和ϕ^-场之间必须是对易或反对易的，否则ϕ^+场和ϕ^-场之间必须达成某种平衡，以使ϕ^+场和ϕ^-场之间的不对易（或不反对易）的影响能够以某种形式被消掉。

为了寻找这种能够消除这种不对易性对$\tilde{\mathcal{H}}(x)$在类空区的对易性的影响的规则，将ϕ^+场和ϕ^-场进行一个线性组合$\phi(x) \equiv \kappa\phi^+(x) + \lambda\phi^-(x)$,看是否能够实现这个线性组合场在类空区间的对易或反对易性质

$$[\phi(x),\phi^\dagger(y)]_\mp = |\kappa|^2\Delta_+(M,x-y) \mp |\lambda|^2\Delta_+(M,y-x) \quad [\phi(x),\phi(y)]_\mp = \kappa\lambda\{\Delta_+(M,x-y) \mp \Delta_+(M,y-x)\}$$

$\Delta_+(M,x-y)$是洛伦兹不变量$\xRightarrow{\text{类空间隔可实现}x-y\to y-x}\Delta_+(M,x-y) = \Delta_+(M,y-x)$

$\Rightarrow$ 选对易子和$|\kappa| = |\lambda|$

产生和湮灭算符还有一个相角的任意性，ϕ前的一个整体常数是无关紧要的.可将κ和λ的值选择为1,即：$\phi(x) = \phi^+(x) + \phi^-(x)$. 以这样一种组合的标量玻色子场$\phi(x)$和其共轭$\phi^\dagger(x)$来构造$\mathcal{H}(x)$可以保证其在类控区间相互对易。

ϕ^+和ϕ^-场之间的不对易（或不反对易）将对构造$\tilde{\mathcal{H}}(x)$产生很大的困难。
改进办法是将ϕ^+场和ϕ^-场进行一个线性组合$\phi(x) \equiv \phi^+(x) + \phi^-(x)$,它使得在类空区间 $[\phi(x), \phi^\dagger(y)] = [\phi(x), \phi(y)] = 0$

引发问题: 如ϕ场带某种守恒内部荷, $\tilde{\mathcal{H}}(x)$与生成此对称性的生成元对易,

$$[Q^a, \tilde{\mathcal{H}}(x)] = 0 \quad \Rightarrow \quad [Q_a, \phi^+(x)]_- = -q_a \phi^+(x) \qquad [Q_a, \phi^-(x)]_- = q_a \phi^-(x)$$

当荷$q_a \neq 0$时，若以ϕ^+和ϕ^-场为基本元素构造$\tilde{\mathcal{H}}(x)$,要保证$[Q^a, \tilde{\mathcal{H}}(x)] = 0$,每一项中含$\phi^+$的数目和含$\phi^-$的数目相等即可.

但若改用以ϕ和$\phi^\dagger$场作为基本元素构造$\tilde{\mathcal{H}}(x)$,不管怎样构造,都不可能保证每一项中含ϕ^+的数目和含ϕ^-的数目相等.无法实现要求$[Q^a, \tilde{\mathcal{H}}(x)] = 0$.

ϕ^+和ϕ^-场之间的不对易（或不反对易）将对构造$\tilde{\mathcal{H}}(x)$产生很大的困难。
改进办法是将ϕ^+场和ϕ^-场进行一个线性组合$\phi(x) \equiv \phi^+(x) + \phi^-(x)$,它使得在类空区间 $[\phi(x), \phi^\dagger(y)] = [\phi(x), \phi(y)] = 0$ 但无法实现$[Q^a, \tilde{\mathcal{H}}(x)] = 0$.

以ϕ和$\phi^\dagger$场作为基本元素构造$\tilde{\mathcal{H}}(x)$造成无法实现要求$[Q^a, \tilde{\mathcal{H}}(x)] = 0$的困难的根本原因是$\phi$场不像$\phi^+$那样有确定的荷,因为它是由带$-q_a$荷的$\phi^+$部分和带$+q_a$荷的$\phi^-$部分叠加而成的.

为了使ϕ场有确定的荷,将ϕ中的ϕ^-部分的粒子换为另外一种能使其带有$-q_a$荷但具有与原来粒子相同质量的标量玻色粒子,$\phi(x) = \phi^+(x) + \phi^{+c\dagger}(x)$

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{-ip\cdot x} a(\vec{p}) \qquad \phi^{+c\dagger}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{ip\cdot x} a^{c\dagger}(\vec{p}) = (\phi^{+c}(x))^\dagger$$

$$[Q_a, \phi^+(x)]_- = -q_a \phi^+(x) \qquad [Q_a, (\phi^{+c}(x))^\dagger]_- = -q_a (\phi^{+c}(x))^\dagger$$

$$[Q_a,\ a(\vec{p})]_- = -q_a a(\vec{p}) \qquad [Q_a,\ a^c(\vec{p})]_- = +q_a a^c(\vec{p})$$

$a(\vec{p})$和$a^c(\vec{p})$是各带相反荷但具有相同质量的两种不同标量玻色子湮灭算符.
上标^c 用于指示电荷共轭,反映它是另一种具有同样质量但带相反荷的粒子.

$$[\phi(x),\phi^\dagger(y)] = [\phi^+(x),\phi^{+\dagger}(y)] - [\phi^{+c}(x),\phi^{+c\dagger}(y)] = \Delta(M,x-y) \overset{\text{类空区间}}{=\!=\!=\!=} 0 \qquad [Q_a,\phi(x)] = -q_a\phi(x)$$

$$\Delta(M,x) = \Delta_+(M,x) - \Delta_+(M,-x) = \int \frac{d\vec{p}}{2p^0(2\pi)^3}[e^{-ip\cdot x} - e^{ip\cdot x}]$$

作业11

$\phi(x)=\phi^+(x)+\phi^{+c\dagger}(x)$ $[Q_a,\phi^+(x)]_-=-q_a\phi^+(x)$ $[Q_a,(\phi^{+c}(x))^\dagger]_-=-q_a(\phi^{+c}(x))^\dagger$

$$\phi^+(x)=\int\frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}e^{-ip\cdot x}a(\vec{p}) \qquad \phi^{+c\dagger}(x)=\int\frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}e^{ip\cdot x}a^{c\dagger}(\vec{p})=(\phi^{+c}(x))^\dagger$$

$$[Q_a,\ a(\vec{p})]_-=-q_a a(\vec{p}) \qquad [Q_a,\ a^c(\vec{p})]_-=+q_a a^c(\vec{p})$$

$a(\vec{p})$和$a^c(\vec{p})$是各带相反荷但具有相同质量的两种不同标量玻色子湮灭算符.
上标^c 用于指示电荷共轭,反映它是另一种具有同样质量但带相反荷的粒子.

$$[\phi(x),\phi^\dagger(y)]=\Delta(M,x-y)\overset{\text{类空区间}}{=\!=\!=\!=}0 \qquad [Q_a,\phi(x)]=-q_a\phi(x)$$

第一式保证用以ϕ和$\phi^\dagger$场构造$\tilde{\mathcal{H}}(x)$可以使其在类空区间对易因而保证因果性。这只有两种粒子的质量相同才能达到!

第二式保证用以ϕ和$\phi^\dagger$场来构造$\tilde{\mathcal{H}}(x)$可以实现要求$[Q^a,\tilde{\mathcal{H}}(x)]=0$.

如果$q_a\neq0$,则$a^c(\vec{p})\neq a(\vec{p})$. 这时体系中具有两种带有相反荷但质量相同的玻色标量粒子. 我们将这种质量相同但荷相反的粒子叫反粒子.

如果$q_a=0$,则可以选择$a^c(\vec{p})=a(\vec{p})$, 此时,粒子本身就是它自己的反粒子, 这样的粒子不带荷,相应的场是自共轭场$\phi(x)=\phi^\dagger(x)$.

复数域为反粒子留出空间

$$\phi(x) = \phi^+(x) + \phi^{+c\dagger}(x) \quad [Q_a, \phi^+(x)]_- = -q_a\phi^+(x) \quad [Q_a, (\phi^{+c}(x))^\dagger]_- = -q_a(\phi^{+c}(x))^\dagger$$

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{-ip\cdot x} a(\vec{p}) \qquad \phi^{+c\dagger}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{ip\cdot x} a^{c\dagger}(\vec{p}) = (\phi^{+c}(x))^\dagger$$

$$[Q_a,\ a(\vec{p})]_- = -q_a a(\vec{p}) \qquad [Q_a,\ a^c(\vec{p})]_- = +q_a a^c(\vec{p})$$

$a(\vec{p})$和$a^c(\vec{p})$是各带相反荷但具有相同质量的两种不同标量玻色子湮灭算符.
上标^c 用于指示电荷共轭,反映它是另一种具有同样质量但带相反荷的粒子.

$$[\phi(x), \phi^\dagger(y)] = \Delta(M, x-y) \overset{\text{类空区间}}{=\!=\!=\!=} 0 \qquad [Q_a, \phi(x)] = -q_a\phi(x)$$

S矩阵的相对论不变性和内部对称性不变性要求在体系中存在反粒子!

理论将粒子湮灭和反粒子产生等权重地分配在场的定义中, 意味物理上把它们看成是"等价"的从荷的角度, 它意味着:

- 产生粒子"等价"于消灭反粒子，或产生反粒子"等价"于消灭粒子
- 粒子反粒子碰到一起将有发生 湮灭反应 的可能性!

这正是狄拉克的空穴理论,空穴现在被反粒子所取代.

关于反粒子的评注：

♣ 我们生存在一个粒子的世界，所有反粒子都是不稳定的!

♢ 可理解为所有反粒子都碰上粒子而湮灭掉了！ 需要相互作用

♡ 但这要求现在世界中 粒子的数目远大于反粒子的数目!

♠ 为什么会有这种 物质反物质的不对称性？

¶ 它要求基本相互作用中有产生物质反物质不对称效应的项！

✠ *C*或*CP*破坏可望达到这个目的！

空间反射变换: $Pa^\dagger(\vec{p},\sigma,n)P^{-1}=\eta a^\dagger(\mathcal{P}\vec{p},\sigma,n)$ $\quad Pa(\vec{p},\sigma,n)P^{-1}=\eta^* a(\mathcal{P}\vec{p},\sigma,n)$

$$Pa(\vec{p})P^{-1}=\eta^* a(-\vec{p}) \qquad Pa^c(\vec{p})P^{-1}=\eta^{c*}a^c(-\vec{p})$$

$$P\phi^+(x)P^{-1}=\eta^*\phi^+(\mathcal{P}x) \qquad P\phi^{+c}(x)P^{-1}=\eta^{c*}\phi^{+c}(\mathcal{P}x)$$

空间反射变换后的场$\phi_P=\eta^*\phi^+ + \eta^c\phi^{+c\dagger}$及其共轭$\phi_P^\dagger$来构造$\tilde{\mathcal{H}}(x)$已能够使其在类空区间相互对易,并可以实现$[Q^a,\tilde{\mathcal{H}}(x)]=0$! 我们进一步可以通过选择$a$与$a^\dagger$及$a^c$与$a^{c\dagger}$之间的相对相角，使得在保证$\phi=\phi^+ + \phi^{+c\dagger}$的同时，还对称地有 作业12

$$\eta^c=\eta^* \qquad \Rightarrow \qquad \eta\eta^c=1$$

无自旋的粒子和反粒子的联合宇称位相为偶. 我们现在拥有统一的对ϕ场的空间反射变换：

$$P\phi(x)P^{-1}=\eta^*\phi(\mathcal{P}x)$$

$\eta^*=1$标量 $\quad \eta^*=-1$赝标量

标量场的分立对称性变换性质

时间反演变换: $Ta^\dagger(\vec{p},\sigma,n)T^{-1}=(-1)^{j-\sigma}a^\dagger(\mathcal{P}\vec{p},-\sigma,n)$ $Ta(\vec{p},\sigma,n)T^{-1}=(-1)^{j-\sigma}a(\mathcal{P}\vec{p},-\sigma,n)$

$$Ta(\vec{p})T^{-1} = (-1)^{j-\sigma}a(-\vec{p}) \qquad Ta^c(\vec{p})T^{-1} = (-1)^{j-\sigma}a^c(-\vec{p})$$

$$T\phi^+(x)T^{-1} = (-1)^{j-\sigma}\phi^+(-\mathcal{P}x) \qquad T\phi^{+c}(x)T^{-1} = (-1)^{j-\sigma}\phi^{+c}(-\mathcal{P}x)$$

无自旋的粒子和反粒子的联合时间宇称位相为偶. 我们现在拥有统一的对ϕ场的时间反演变换:

$$T\phi(x)T^{-1} = (-1)^{j-\sigma}\phi(-\mathcal{P}x)$$

电荷共轭变换C:

$$Ca(\vec{p})C^{-1}=\xi^* a^c(\vec{p}) \Rightarrow C\phi^+(x)C^{-1}=\xi^*\phi^{+c}(\pm x) \qquad Ca^c(\vec{p})C^{-1}=\xi^{c*}a(\vec{p}) \Rightarrow C\phi^{+c}(x)C^{-1}=\xi^{c*}\phi^+(\pm x)$$

不可以连续变形到单位变换的变换,需要判断它是幺正算符,还是反幺正算符

$$CU_0(\Lambda,a)C^{-1}\phi^{+,c}(x)CU_0^{-1}(\Lambda,a) = CU_0(\Lambda,a)C^{-1}\xi C\phi^+(\pm x)C^{-1}CU_0^{-1}(\Lambda,a)$$ +幺正；−反幺正

$$= \xi CU_0(\Lambda,a)\phi^+(\pm x)U_0^{-1}(\Lambda,a) = \xi C\phi^+(\pm(\Lambda x+a)) = \phi^{+,c}(\Lambda x+a)C$$

$$= U_0(\Lambda,a)\phi^{+,c}(x)U_0^{-1}(\Lambda,a)C \quad \Rightarrow \quad CU_0(\Lambda,a)C^{-1}=U_0(\Lambda,a) \qquad U_0(\Lambda,a)C=CU_0(\Lambda,a)$$

取$\Lambda^\mu{}_\nu = g^\mu{}_\nu + \omega^\mu{}_\nu$ 和$a^\mu = \epsilon^\mu$,准到ω和ϵ一阶 $U(1+\omega,\epsilon)=1+\frac{i}{2}\omega_{\rho\sigma}J^{\rho\sigma}+i\epsilon_\rho P^\rho+\cdots$

$$CiJ^{\rho\sigma}C^{-1} = iJ^{\rho\sigma} \qquad CiP^\rho C^{-1} = iP^\rho$$

由于还不能确定C是幺正算符,还是反幺正算符,暂把虚数i保留在了C和C^{-1}算符的中间. 在上式中对四动量的零分量有 $CiHC^{-1} = iH$

如果C是反幺正算符将导致$CHC^{-1} = -H$.它意味如果假设物理体系具有电荷共轭对称性, 则对应能量为E每一个正能态都应有相应的负能态$-E$在物理谱中出现, <u>实验上并没有发现负能态</u>, 因此要求C是反幺正算符是不对的.
应取C为幺正算符.

$$C\vec{J}C^{-1} = \vec{J} \qquad C\vec{K}C^{-1} = -\vec{K} \qquad C\vec{P}C^{-1} = -\vec{P} \qquad CHC^{-1} = H$$

电荷共轭变换C及CPT联合变换:

$$Ca(\vec{p})C^{-1} = \xi^* a^c(\vec{p}) \qquad Ca^c(\vec{p})C^{-1} = \xi^{c*} a(\vec{p})$$

C 幺正算符:

$$C\phi^+(x)C^{-1} = \xi^* \phi^{+c}(x) \qquad C\phi^{+c}(x)C^{-1} = \xi^{c*} \phi^+(x)$$

电荷共轭变换后的场$\phi_C = \xi^* \phi^{c+} + \xi^c \phi^{+\dagger}$ 及其共轭$\phi_C^\dagger$来构造$\tilde{\mathcal{H}}(x)$已能够使其在类空区间相互对易,并可以实现$[Q^a, \tilde{\mathcal{H}}(x)] = 0$! 我们进一步可以通过选择$a$与$a^\dagger$,$a^c$与$a^{c\dagger}$及$a$与$a^c$之间的相对相角，相角可以与$\vec{p}$、c相关，使得在保证$\phi = \phi^+ + \phi^{+c\dagger}$和 $\phi_C = \xi^*(\phi^+ + \phi^{+c\dagger})$的同时，还对称地有

$$\xi^c = \xi^* \qquad \Rightarrow \qquad \xi\xi^c = 1$$

无自旋的粒子和反粒子的联合电荷共轭宇称位相为偶. 我们现在拥有统一的对ϕ场的电荷共轭变换:

$$C\phi(x)C^{-1} = \xi^* \phi^\dagger(x) \qquad T\phi(x)T^{-1} = (-1)^{j-\sigma}\phi(-\mathcal{P}x) \qquad P\phi(x)P^{-1} = \eta^* \phi(\mathcal{P}x)$$

对CPT联合变换: $$CPT\ \phi(x)\ [CPT]^{-1} = \xi^*(-1)^{j-\sigma}\eta^*\phi^\dagger(-x)$$