量子场论 相互作用、格林函数与路径积分

本章建立 量子场论 中量子“相互作用场”的正则和路径积分描述

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S矩阵与正规乘积

S矩阵与场的产生湮灭算符展开

$$\Phi_\alpha = a^\dagger \cdots b^\dagger \Phi_0 \qquad \Phi_\beta = d^\dagger \cdots f^\dagger \Phi_0$$

$$S_{\beta\alpha} = (\Phi_\beta, S\Phi_\alpha) = (\Phi_\beta, \mathbf{T}\, e^{-i\int_{-\infty}^{\infty} dt\ V(\psi,\pi)}\Phi_\alpha) = (\Phi_\beta, U(\infty,-\infty)\Phi_\alpha)$$

$$\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [\underbrace{e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)}_{\psi^+} + \underbrace{e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)}_{\psi^-}] \qquad \pi(x) = \pi^+(x) + \pi^-(x)$$

需要处理很多个产生湮灭算符的真空期望值的方法！

正规乘积： $\mathbf{N}(UVW\cdots Z) = \delta_P U'V'W'\cdots Z'$ 上章用的符号是 ：$\cdots\cdots$：

其中$U'V'W'\cdots Z'$中的算符排列顺序是在原来$UVW\cdots Z$的算符排列顺序基础之上，将所有的消灭算符排在所有产生算符的左边面对大家，也就是纸面上的右边，而消灭算符之间的相对顺序和产生算符之间的相对顺序则保持不变。

从$UVW\cdots Z$调换到$U'V'W'\cdots Z'$，费米子交换偶数次$\delta_P = 1$,否则$\delta_P = -1$。

算符的收缩： $\overbrace{\psi_l(x)\psi_{l'}(x')} \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

$$\overbrace{\psi_l(x)\psi_{l'}(x')} = (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)$$

证明： 不妨设$t > t'$

$$\begin{aligned}
\mathbf{T}(\psi_l(x)\psi_{l'}(x')) &= \mathbf{T}(\psi_l(x))\psi_{l'}(x') = \mathbf{N}(\psi_l(x))(\psi_{l'}^+(x') + \psi_{l'}^-(x')) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) + \psi_l^+(x)\psi_{l'}^-(x') \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) \pm \psi_{l'}^-(x')\psi_l^+(x) + [\psi_l^+(x), \psi_{l'}^-(x')]_\mp \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) + \mathbf{N}(\psi_l^+(x)\psi_{l'}^-(x')) + [\psi_l^+(x), \psi_{l'}^-(x')]_\mp \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l(x)\psi_{l'}^-(x')) + (\Phi_0, [\psi_l^+(x), \psi_{l'}^-(x')]_\mp \Phi_0) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \psi_l^+(x)\psi_{l'}^-(x')\Phi_0) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \psi_l(x)\psi_{l'}(x')\Phi_0) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)
\end{aligned}$$

算符的收缩：$\overbracket{\psi_l(x)\psi_{l'}}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

$$\overbracket{\psi_l(x)\psi_{l'}}(x') = (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)$$

证明： 不妨设$t > t'$

$$\begin{aligned}
\mathbf{T}(\psi_l(x)\psi_{l'}(x')) = \mathbf{T}(\psi_l(x))\psi_{l'}(x') &= \mathbf{N}(\psi_l(x))(\psi_{l'}^{+}(x') + \psi_{l'}^{-}(x')) \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + [\psi_l^{+}(x), \psi_{l'}^{-}(x')]_{\mp} \\
&= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)
\end{aligned}$$

推论：相互之间的对易或反对易性质导致

- 相邻的不同场的场算符之间的收缩等于零
- 相邻的消灭算符之间,产生算符之间的收缩等于零

算符的收缩: $\overbracket{\psi_l(x)\psi_{l'}}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值
相邻的不同场之间的收缩等于零; 消灭算符之间,产生算符之间的收缩等于零

定义不相邻的场算符的收缩: $\mathbf{N}(U\,\overbracket{VW\cdots YZ}) \equiv \delta_P\,\overbracket{VZ}\,\mathbf{N}(UW\cdots Y)$

其中的上划线是指将两端的算符V和Z进行收缩操作,对上划线中间的算符$W\cdots Y$不发生作用,只是考虑因子$\delta_P=(-1)^n$, n是将V和Z调到一起所交换的费米子算符的数目.

推论: $\mathbf{N}(U\,VW\cdots XY\,Z) \equiv \delta_P\,\overbracket{VZ}\,\overbracket{WY}\,\mathbf{N}(U\cdots X)$

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(\overbracket{UV\cdots XYZ}) + \mathbf{N}(U\overbracket{V\cdots XYZ}) + \cdots + \mathbf{N}(UV\cdots X\overbracket{YZ})$$

证明: 如果Z是湮灭算符

$\overbracket{UZ} = \mathbf{T}(UZ) - \mathbf{N}(UZ) = UZ - UZ = 0$ $\Rightarrow$ 预备定理恒成立, 只需讨论Z是产生算符的情况!

Wick定理

算符的收缩：$\overbracket{\psi_l(x)\psi_{l'}(x')} \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

定义不相邻的场算符的收缩：$\mathbf{N}(U\,VW\cdots YZ) \equiv \delta_P\; VZ\, \mathbf{N}(UW\cdots Y)$

$\delta_P = (-1)^n$, n是将V和Z调到一起所交换的费米子算符的数目.

推论：$\mathbf{N}(U\;V\;W\cdots XY\;Z) \equiv \delta_P\; VZ\;WY\;\mathbf{N}(U\cdots X)$

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ)$$

Z是产生算符; 若$UV\cdots XY$是产生算符，则因它们的收缩为零，结果恒成立

证明: Z是产生算符; 若$UV\cdots XY$是湮灭算符的情形成立，则对一产生算符P

$$P\mathbf{N}(UV\cdots XY)Z = P\mathbf{N}(UV\cdots XYZ) + P\mathbf{N}(UV\cdots XYZ) + P\mathbf{N}(UV\cdots XYZ) + \cdots + P\mathbf{N}(UV\cdots XYZ)$$

利用$PZ = 0 \quad\Rightarrow\quad \mathbf{N}(PUV\cdots XYZ) = 0$ 上式等号左右两边的P都可以移入正规乘积，加上本行式子得到下面结果

$$\mathbf{N}(PUV\cdots XY)Z = \mathbf{N}(PUV\cdots XYZ) + \mathbf{N}(PUV\cdots XYZ) + \mathbf{N}(PUV\cdots XYZ) + \cdots + \mathbf{N}(PUV\cdots XYZ)$$

$\Rightarrow$ 进一步可以假设U是产生算符，$V\cdots XY$是湮灭算符 重复一遍上面证明，结果仍然成立

$\Rightarrow$ 预备定理成立,这时$UV\cdots XY$产生湮灭算符都有 只需讨论$UV\cdots XY$是湮灭算符的情况！

Wick定理

算符的收缩: $\psi_l(x)\psi_{l'}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

相邻的不同场之间的收缩等于零; 消灭算符之间,产生算符之间的收缩等于零

定义不相邻的场算符的收缩: $\mathbf{N}(U\ VW\cdots YZ) \ \equiv\ \delta_P\ VZ\ \mathbf{N}(UW\cdots Y)$

$\delta_P = (-1)^n$, n是将V和Z调到一起所交换的费米子算符的数目.

推论: $\mathbf{N}(U\ V\ W\cdots XY\ Z) \ \equiv\ \delta_P\ VZ\ WY\ \mathbf{N}(U\cdots X)$

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z = \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ)$$

证明: Z是产生算符; $UV\cdots XY$湮灭算符: 归纳法

$$n=1: \quad \mathbf{N}(Y)Z = YZ = \mathbf{T}(YZ) = \mathbf{N}(YZ) + YZ$$

$\Rightarrow$ 预备定理成立, 假设对n成立, 只需推出对$n+1$也成立! 对湮灭算符R:

$$RN(UV\cdots XY)Z = \underbrace{RN(UV\cdots XYZ)}_{\text{计算见下行}} + RN(UV\cdots XYZ) + RN(UV\cdots XYZ) + \cdots + RN(UV\cdots XYZ)$$

$$RN(UV\cdots XYZ) = \delta_P \mathbf{T}(RZ)UV\cdots XY = \delta_P(\mathbf{N}(RZ) + RZ)UV\cdots XY = \mathbf{N}(RUV\cdots XYZ) + \mathbf{N}(RUV\cdots XYZ)$$

$$\mathbf{N}(RUV\cdots XY)Z = \mathbf{N}(RUV\cdots XYZ) + \mathbf{N}(RUV\cdots XYZ) + \mathbf{N}(RUV\cdots XYZ) + \cdots + \mathbf{N}(RUV\cdots XYZ)$$

Wick定理:

预备定理:如果算符Z的时间值比所有算符都早,则有

$$\mathbf{N}(UV\cdots XY)Z=\mathbf{N}(UV\cdots XYZ)+\mathbf{N}(UV\cdots XYZ)+\mathbf{N}(UV\cdots XYZ)+\cdots+\mathbf{N}(UV\cdots XYZ)$$

定理1:

$$\begin{aligned}\mathbf{T}(UV\cdots XYZ) &= \mathbf{N}(UV\cdots XYZ) && \text{无收缩项}\\ &+\mathbf{N}(UV\cdots XYZ)+\cdots+\mathbf{N}(UV\cdots XYZ)+ && \text{一次收缩项}\\ &+\mathbf{N}(UV\cdots XYZ)+\cdots && \text{二次收缩项}\\ &+\cdots\\ &+\mathbf{N}(UVW\cdots XYZ)+\cdots && \text{全部收缩项}\end{aligned}$$

证明: 归纳法

$n=2$阶成立! 假设对n阶已经成立. 右乘时间早于所有其它算符时间的算符R

$$\mathbf{T}(UV\cdots XYZ)R=\mathbf{T}(UV\cdots XYZR)$$

$$\mathbf{N}(UV\cdots XYZ)R=\mathbf{N}(UV\cdots XYZR)+\mathbf{N}(UV\cdots XYZR)+\mathbf{N}(UV\cdots XYZR)+\cdots$$

$$\mathbf{N}(UV\cdots XYZ)R=\delta_P UV\mathbf{N}(W\cdots XYZ)R=\mathbf{N}(UV\cdots XYZR)+\mathbf{N}(UV\ W\cdots XYZR)+\cdots$$

$\cdots\cdots\quad\Rightarrow\quad n+1$阶也成立!

Wick定理:

定理1:

$$
\begin{aligned}
\mathbf{T}(UV\cdots XYZ) \quad = \quad & \mathbf{N}(UV\cdots XYZ) && \text{无收缩项} \\
& + \mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ) + && \text{一次收缩项} \\
& + \mathbf{N}(UV\cdots XYZ) + \cdots && \text{二次收缩项} \\
& + \cdots \\
& + \mathbf{N}(UVW\cdots XYZ) + \cdots && \text{全部收缩项}
\end{aligned}
$$

定理2:

若T乘积中含N乘积,展开仍成立,但需将各N乘积内部各算符之间的收缩略去.
此时若N乘积内部是纯湮灭或纯产生算符,本来就不存在内部的收缩。
若N乘积是一对湮灭产生算符,N乘积就等于T乘积减去收缩。再多用归纳法。

$$\Phi_\alpha = a^\dagger \cdots b^\dagger \Phi_0 \qquad\qquad \Phi_\beta = d^\dagger \cdots f^\dagger \Phi_0$$

$$S_{\beta\alpha} = (\Phi_\beta, S\Phi_\alpha) = (\Phi_\beta, \mathbf{T}\ e^{-i\int_{-\infty}^{\infty} dt\ V(\psi,\pi)}\Phi_\alpha)$$

$$\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [\underbrace{e^{-ip\cdot x} u(\vec{p},\sigma) a(\vec{p},\sigma)}_{\psi^+} + \underbrace{e^{ip\cdot x} v(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma)}_{\psi^-}] \qquad \pi(x) = \pi^+(x) + \pi^-(x)$$

场的收缩：

$$\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [e^{-ip\cdot x} u(\vec{p},\sigma) a(\vec{p},\sigma) + e^{ip\cdot x} v(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma)]$$

$$\overline{\psi_l(x)\psi_{l'}(x')} = (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0) = \theta(t-t')[\psi_l^+(x), \psi_{l'}^-(x')]_\mp \pm \theta(t'-t)[\psi_{l'}^+(x'), \psi_l^-(x)]_\mp$$

标量场: $u(\vec{p}) = v(\vec{p}) = \frac{1}{\sqrt{2}(\vec{p}^2+M^2)^{1/4}}$ $\qquad f(z) = \frac{1}{2\pi i}\oint_{\text{逆时针}} dw \frac{f(w)}{w-z}$

$$\overline{\phi(x)\phi^\dagger(y)} = \int \frac{d^3p d^3p' \,\{[e^{-ip\cdot x}a(\vec{p}),\ e^{ip'\cdot y}a^\dagger(\vec{p}')]\theta(x^0-y^0) + [e^{-ip'\cdot y}a^c(\vec{p}'),\ e^{ip\cdot x}a^{c\dagger}(\vec{p})]\theta(y^0-x^0)\}}{(2\pi)^3 2(\vec{p}^2+M^2)^{1/4}(\vec{p}'^2+M^2)^{1/4}}$$

$$= \int \frac{d^3p\, e^{i\vec{p}\cdot(\vec{x}-\vec{y})}}{2(2\pi)^3\sqrt{\vec{p}^2+M^2}} [e^{-i\sqrt{\vec{p}^2+M^2}(x^0-y^0)}\theta(x^0-y^0) + e^{i\sqrt{\vec{p}^2+M^2}(x^0-y^0)}\theta(y^0-x^0)]$$

$$= \int \frac{d^3p\, e^{i\vec{p}\cdot(\vec{x}-\vec{y})}}{2(2\pi)^3} \int_{-\infty}^{\infty} \frac{dp^0 \quad e^{-ip^0(x^0-y^0)}}{2i\pi\sqrt{\vec{p}^2+M^2}} [-\frac{1}{p^0-\sqrt{\vec{p}^2+M^2}+i0^\dagger} + \frac{1}{p^0+\sqrt{\vec{p}^2+M^2}-i0^\dagger}]$$

$$= \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)} \frac{i}{p^2-M^2+i0^\dagger} =$$

Wick定理

场的收缩：

$$\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec p,\sigma)a(\vec p,\sigma)+e^{ip\cdot x}v(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]$$

$$\overbrace{\psi_l(x)\psi_{l'}(x')}=(\Phi_0,\mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)=\theta(t-t')[\psi_l^+(x),\psi_{l'}^-(x')]_\mp\pm\theta(t'-t)[\psi_{l'}^+(x'),\psi_l^-(x)]_\mp$$

旋量场: $u(\vec p,\sigma)=\sqrt{M/p^0}D(L(p))u(0,\sigma)\qquad v(\vec p,\sigma)=\sqrt{M/p^0}D(L(p))v(0,\sigma)$

$$\overbrace{\psi(x)_l\overline{\psi}_{l'}(y)}=\int\frac{d^3pd^3p'}{(2\pi)^3}\sum_{\sigma\sigma'}\{[e^{-ip\cdot x}u_l(\vec p,\sigma)a(\vec p,\sigma),\ e^{ip'\cdot y}\bar u_{l'}(\vec p',\sigma')a^\dagger(\vec p',\sigma')]_+\theta(x^0-y^0)$$

$$-[e^{-ip'\cdot y}\bar v_{l'}(\vec p',\sigma')a^c(\vec p',\sigma'),\ e^{ip\cdot x}v_l(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]_+\theta(y^0-x^0)\}$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{(2\pi)^3}\sum_\sigma[e^{-i\sqrt{\vec p^2+M^2}(x^0-y^0)}u_l(\vec p,\sigma)\bar u_{l'}(\vec p,\sigma)\theta(x^0-y^0)-e^{i\sqrt{\vec p^2+M^2}(x^0-y^0)}\bar v_{l'}(-\vec p,\sigma)v_l(-\vec p,\sigma)\theta(y^0-x^0)]$$

$$N_{l\bar l}(\vec p)\equiv\sum_\sigma u_l(\vec p,\sigma)u_{\bar l}^*(\vec p,\sigma)=\sum_\sigma u_l(\vec p,\sigma)\bar u_{\bar l}(\vec p,\sigma)\beta=\frac{1}{2p^0}[p^\mu\gamma_\mu+M]\beta\qquad M_{l\bar l}(\vec p)\equiv\sum_\sigma v_l(\vec p,\sigma)v_{\bar l}^*(\vec p,\sigma)=\sum_\sigma v_l(\vec p,\sigma)\bar v_{\bar l}(\vec p,\sigma)\beta=\frac{1}{2p^0}[p^\mu\gamma_\mu-M]\beta$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0\quad e^{-ip^0(x^0-y^0)}}{2i\pi2\sqrt{\vec p^2+M^2}}\left[-\frac{p^0\gamma^0-\vec p\cdot\vec\gamma+M}{p^0-\sqrt{\vec p^2+M^2}+i0^\dagger}-\frac{-p^0\gamma^0+\vec p\cdot\vec\gamma-M}{p^0+\sqrt{\vec p^2+M^2}-i0^\dagger}\right]_{ll'}$$

$$=\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\left[\frac{i(p_\mu\gamma^\mu+M)}{p^2-M^2+i0^\dagger}\right]_{ll'}=\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\left[\frac{i}{p_\mu\gamma^\mu-M+i0^\dagger}\right]_{ll'}=$$

场的收缩：

$$\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec p,\sigma)a(\vec p,\sigma)+e^{ip\cdot x}v(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]$$

$$\overline{\psi_l(x)\psi_{l'}(x')}=(\Phi_0,\mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)=\theta(t-t')[\psi_l^+(x),\psi_{l'}^-(x')]_\mp\pm\theta(t'-t)[\psi_{l'}^+(x'),\psi_l^-(x)]_\mp$$

有质矢量场: $u^\mu(\vec p,\sigma)=v^{\mu*}(\vec p,\sigma)=(2p^0)^{-1/2}e^\mu(\vec p,\sigma)$ $\quad e^\mu(\vec p,\sigma)\equiv L^\mu{}_\nu(\vec p)e^\nu(0,\sigma)$

$$\overline{v^\mu(x)v^{\nu\dagger}(y)}=\int\frac{d^3pd^3p'}{2(2\pi)^3(\vec p^2+M^2)^{1/4}(\vec p'^2+M^2)^{1/4}}\sum_{\sigma\sigma'}\{[e^{-ip\cdot x}e^\mu(\vec p,\sigma)a(\vec p,\sigma),\ e^{ip'\cdot y}e^{\nu*}(\vec p',\sigma')a^\dagger(\vec p',\sigma')]$$

$$+[e^{-ip'\cdot y}e^\nu(\vec p',\sigma')a^c(\vec p',\sigma'),\ e^{ip\cdot x}e^\mu(\vec p,\sigma)a^{c\dagger}(\vec p,\sigma)]\theta(y^0-x^0)\}$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{2(2\pi)^3\sqrt{\vec p^2+M^2}}\sum_\sigma[e^{-i\sqrt{\vec p^2+M^2}(x^0-y^0)}e^\mu(\vec p,\sigma)e^{\nu*}(\vec p,\sigma)\theta(x^0-y^0)+e^{i\sqrt{\vec p^2+M^2}(x^0-y^0)}e^\nu(-\vec p,\sigma)e^{\mu*}(-\vec p,\sigma)\theta(y^0-x^0)]$$

$$=\int\frac{d^3p\ e^{i\vec p\cdot(\vec x-\vec y)}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0\quad e^{-ip^0(x^0-y^0)}}{2i\pi2\sqrt{\vec p^2+M^2}}[-\frac{-g^{\mu\nu}+p^\mu p^\nu/M^2}{p^0-\sqrt{\vec p^2+M^2}+i0^\dagger}+\frac{-g^{\mu\nu}+p^\mu p^\nu/M^2}{p^0+\sqrt{\vec p^2+M^2}-i0^\dagger}]$$

$$=\int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\,\frac{i(-g^{\mu\nu}+p^\mu p^\nu/M^2)}{p^2-M^2+i0^\dagger}\quad=$$

态之间的编时乘积的格林函数 $(\Psi^-_\beta, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$

$U(t,t') = \Omega^\dagger(t)\Omega(t')$ $\quad \psi_H(x) = \Omega(t)\psi(x)\Omega^\dagger(t)$ $\quad \Psi^\pm_\alpha = \Omega(\mp\infty)\Phi_\alpha$

$$S_{\beta\alpha} = (\Phi_\beta, U(\infty,-\infty)\Phi_\alpha) = (\Phi_\beta, \mathbf{T}\, e^{-i\int d^4x\, V(\psi,\pi)}\Phi_\alpha)$$

$$S_{\beta\alpha}[J] \equiv (\Phi_\beta, \mathbf{T}\, e^{-i\int d^4x\, V_J(\psi,\pi)}\Phi_\alpha) \Leftarrow V_J(\psi,\pi) \equiv V(\psi,\pi) + \psi_l(x)J_l(x)$$

$$\left.\frac{\delta^n S_{\beta\alpha}[J]}{\delta J_{l_1}(x_1)\delta J_{l_2}(x_2)\cdots\delta J_{l_n}(x_n)}\right|_{J_l(x)=0} = (-i)^n(\Phi_\beta, \mathbf{T}\, \psi_{l_1}(x_1)\psi_{l_2}(x_2)\cdots\psi_{l_n}(x_n)\; e^{-i\int d^4x\, V(\psi,\pi)}\Phi_\alpha)$$

$= (-i)^n(\Phi_\beta, \mathbf{T}\, \psi_{l_1}(x_1)\psi_{l_2}(x_2)\cdots\psi_{l_n}(x_n)\; U(\infty,-\infty)\Phi_\alpha)$ Wick定理 所有可能的全部收缩项之和

$$\overset{t_1\geq t_2\geq\cdots\geq t_n}{=\!=\!=\!=\!=\!=} (-i)^n(\Phi_\beta, U(\infty,t_1)\psi_{l_1}(x_1)U(t_1,t_2)\psi_{l_2}(x_2)U(t_2,t_3)\cdots U(t_{n-1},t_n)\psi_{l_n}(x_n)U(t_n,-\infty)\Phi_\alpha)$$

$$= (-i)^n(\Phi_\beta, \Omega^\dagger(\infty)\Omega(t_1)\psi_{l_1}(x_1)\Omega^\dagger(t_1)\Omega(t_2)\psi_{l_2}(x_2)\Omega^\dagger(t_2)\cdots\Omega(t_n)\psi_{l_n}(x_n)\Omega^\dagger(t_n)\Omega(-\infty)\Phi_\alpha)$$

$$= (-i)^n(\Psi^-_\beta, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

$$= (-i)^n(\Psi^-_\beta, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

编时乘积的格林函数$(\Psi_0^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$

以后会证明: 若已知所有的格林函数,则就可以得到体系的作用量!

对前n个粒子在无穷将来$t_1,\ldots,t_n\to\infty$，后l个粒子在无穷过去$t_{n+1},\ldots,t_{n+l}\to-\infty$的情况:

$$
\begin{aligned}
&(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}\\
&=\theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)\\
&\quad\times\sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l}\theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)\\
&\quad\times(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)
\end{aligned}
$$

- 编时因子$\theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$是 不可以 被去掉 的!
- $(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+) = (\Psi_0^-, \psi_{H,l_1}(x_1+a)\cdots\psi_{H,l_n}(x_n+a)\psi_{H,l_{n+1}}(x_{n+1}+a)\cdots\psi_{H,l_{n+l}}(x_{n+l}+a)\Psi_0^+)$

编时乘积的格林函数$(\Psi_0^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$

讨论前n个粒子在无穷将来，后l个粒子在无穷过去的情况:

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times\sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l}\theta(x_1^0 - x_2^0)\cdots\theta(x_{n-1}^0 - x_n^0)\theta(x_{n+1}^0 - x_{n+2}^0)\cdots\theta(x_{n+l-1}^0 - x_{n+l}^0)$$

$$\times(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times\sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l}\theta(x_1^0 - x_2^0)\cdots\theta(x_{n-1}^0 - x_n^0)\theta(x_{n+1}^0 - x_{n+2}^0)\cdots\theta(x_{n+l-1}^0 - x_{n+l}^0)$$

$$\times\int d\gamma d\gamma'(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\gamma^-)(\Psi_\gamma^-, \Psi_{\gamma'}^+)(\Psi_{\gamma'}^+, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times \sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l} \theta(x_1^0-x_2^0)\cdots\theta(x_{n-1}^0-x_n^0)\theta(x_{n+1}^0-x_{n+2}^0)\cdots\theta(x_{n+l-1}^0-x_{n+l}^0)$$

$$\times \sum_{\sigma_1\cdots\sigma_{n+l}} \int d\vec{p}_1\cdots d\vec{p}_{n+l}\, (\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$\Psi_0^\pm = \Psi_0 \quad \Psi^\pm_{\vec{p},\sigma} = \Psi_{\vec{p},\sigma}$ 进态和出态的粒子无穷分离 $\psi_{H,l_i}(x_i) = e^{iP\cdot x_i}\psi_H(0)e^{-iP\cdot x_i}$

$$(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}) = (\Psi_0, \psi_{H,l_1}(x_1)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(x_n)\Psi_{\vec{p}_n,\sigma_n})$$

$$= e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n}(\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= (\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(x_{n+1})\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(x_{n+l})\Psi_0)$$

$$= e^{ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(0)\Psi_0)$$

为什么只考虑n和l粒子的中间态？

$$\int d\gamma d\gamma' (\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\gamma^-)(\Psi_\gamma^-, \Psi_{\gamma'}^+)(\Psi_{\gamma'}^+, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= \sum_{\sigma_1\cdots\sigma_{n+l}} \int d\vec{p}_1\cdots d\vec{p}_{n+l}\, (\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$(\Psi_0^-, \psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}) = (\Psi_0, \psi_{H,l_1}(x_1)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(x_n)\Psi_{\vec{p}_n,\sigma_n})$$

$$(\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$= (\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(x_{n+1})\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(x_{n+l})\Psi_0)$$

- ♣ 如果考虑$m > n$粒子的中间态：因子化后会出现真空态与单粒子态的内积,因而为零
- ♠ 如果考虑$m < n$粒子的中间态：无穷分离时因子化后的会出现单个场的真空期望值，因而为零

多粒子态的归一化：记$\Phi_{p_1,\sigma_1,n;p_2,\sigma_2,n_2;\cdots} = \Phi_{p_1,p_2,\cdots}$

真空态Φ_0和单粒子态Φ_q

$$(\Phi_0, \Phi_0) = 1 \qquad (\Phi_{q'}, \Phi_q) = \delta(q' - q) \equiv \delta^3(\vec{q}' - \vec{q})\delta_{\sigma'\sigma}\delta_{n'n}$$

对两粒子态$\Phi_{q_1q_2}$
$$(\Phi_{q_1'q_2'}, \Phi_{q_1q_2}) = \frac{1}{2!}[\delta(q_1' - q_1)\delta(q_2' - q_2) \pm \delta(q_2' - q_1)\delta(q_1' - q_2)]$$

负号对两个粒子都是费米子的情形,正号对其它情形(两个粒子都是玻色子或一个是玻色子一个是费米子).

一般情况：
$$(\Phi_{q_1'q_2'\cdots q_M'}, \Phi_{q_1q_2\cdots q_N}) = \frac{\delta_{MN}}{N!}\sum_{\mathcal{P}}\delta_{\mathcal{P}}\prod_i \delta(q_i - q'_{\mathcal{P}i})$$

求和对所有可能的对指标$1, 2, \cdots, N$的交换排序$\mathcal{P}$实行.对交换排序中涉及奇数次费米子交换时,$\delta_{\mathcal{P}} = -1$,其它情况的交换排序$\delta_{\mathcal{P}} = 1$.

约定：
$$(\Phi_{\alpha'}, \Phi_\alpha) = \delta(\alpha' - \alpha) \qquad \int d\alpha \cdots \equiv \sum_{n_1\sigma_1n_2\sigma_2\cdots}\int d\vec{p}_1 \int d\vec{p}_2 \cdots$$

$$\Phi = \int d\alpha\, \Phi_\alpha(\Phi_\alpha, \Phi) \qquad 1 = \int d\alpha\, \Phi_\alpha)(\Phi_\alpha$$

多粒子态构成完备集！

约化公式

$$\left(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+\right)\Big|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times \sum_{\mathcal{P}\ 1,\cdots,n}\ \sum_{\mathcal{P}\ n+1,\cdots,n+l} \theta(x_1^0 - x_2^0)\cdots\theta(x_{n-1}^0 - x_n^0)\theta(x_{n+1}^0 - x_{n+2}^0)\cdots\theta(x_{n+l-1}^0 - x_{n+l}^0)$$

$$\times \sum_{\sigma_1\cdots\sigma_{n+l}} \int d\vec{p}_1\cdots d\vec{p}_{n+l} e^{-ip_1x_1-\cdots-ip_nx_n+ip_{n+1}x_{n+1}+\cdots+ip_{n+l}x_{n+l}} (\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times\left(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}\right)(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\sum_{\mathcal{P}\ 1,\cdots,n} \theta(x_1^0 - x_2^0)\cdots\theta(x_{n-1}^0 - x_n^0) = 1 \qquad \sum_{\mathcal{P}\ n+1,\cdots,n+l} \theta(x_{n+1}^0 - x_{n+2}^0)\cdots\theta(x_{n+l-1}^0 - x_{n+l}^0) = 1$$

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times \sum_{\sigma_1\cdots\sigma_{n+l}} \int d\vec{p}_1\cdots d\vec{p}_{n+l} e^{-ip_1x_1-\cdots-ip_nx_n+ip_{n+1}x_{n+1}+\cdots+ip_{n+l}x_{n+l}} (\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times\left(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}}\right)(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \theta(x_1^0)\cdots\theta(x_n^0)\theta(-x_{n+1}^0)\cdots\theta(-x_{n+l}^0)$$

$$\times\sum_{\sigma_1\cdots\sigma_{n+l}}\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1x_1-\cdots-ip_nx_n+ip_{n+1}x_{n+1}+\cdots+ip_{n+l}x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\theta(x_i^0-x_{i+1}^0) = -\frac{1}{2i\pi}\int_{-\infty}^{\infty}d\omega_i\frac{e^{-i\omega_i(x_i^0-x_{i+1}^0)}}{\omega_i+i0+}$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}d\omega_1\cdots d\omega_{n+l}\frac{e^{-i\omega_1x_1^0-\cdots-i\omega_nx_n^0+i\omega_{n+1}x_{n+1}^0+\cdots+i\omega_{n+l}x_{n+l}^0}}{(\omega_1+i0+)\cdots(\omega_{n+l}+i0+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}\frac{d\omega_1\cdots d\omega_{n+l}e^{-i\omega_1 x_1^0\cdots-i\omega_n x_n^0+i\omega_{n+1}x_{n+1}^0\cdots+i\omega_{n+l}x_{n+l}^0}}{(\omega_1+i0^+)\cdots(\omega_{n+l}+i0^+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\int_{-\infty}^{\infty}\frac{d\omega_1\cdots d\omega_{n+l}}{(\omega_1+i0^+)\cdots(\omega_{n+l}+i0^+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l}e^{-ip'_1\cdot x_1-\cdots-ip'_n\cdot x_n+ip'_{n+1}\cdot x_{n+1}+\cdots+ip'_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\vec{p}'_i=\vec{p}_i\quad i=1,2,\cdots,n+l$$

$$p_i^{0\prime}=\sqrt{\vec{p}_i^2+M_i^2}+\omega_i\quad i=1,2,\cdots,n+l$$

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \int_{-\infty}^{\infty} \frac{d\omega_1\cdots d\omega_{n+l}}{(\omega_1+i0^+)\cdots(\omega_{n+l}+i0^+)}$$

$$\times\int d\vec{p}_1\cdots d\vec{p}_{n+l} e^{-ip_1'\cdot x_1-\cdots-ip_n'\cdot x_n+ip_{n+1}'\cdot x_{n+1}+\cdots+ip_{n+l}'\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)$$

$$\vec{p}_i' = \vec{p}_i \quad i = 1, 2, \cdots, n+l$$

$$p_i^{0\prime} = \sqrt{\vec{p}_i^2 + M_i^2} + \omega_i \quad i = 1, 2, \cdots, n+l$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \int d^4p_1\cdots d^4p_{n+l} e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}}(\Psi_0,\psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})$$

$$\cdots(\Psi_0,\psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)$$

$$\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)(p_1^0-\sqrt{\vec{p}_1^2+M_1^2}+i0^+)^{-1}\cdots(p_{n+l}^0-\sqrt{\vec{p}_{n+l}^2+M_{n+l}^2}+i0^+)^{-1}$$

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \int d^4p_1\cdots d^4p_{n+l} e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}} (\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})$$

$$\cdots(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}\psi_{H,l_{n+1}}(0)\Psi_0)$$

$$\cdots(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}\psi_{H,l_{n+l}}(0)\Psi_0)(p_1^0-\sqrt{\vec{p}_1^2+M_1^2}+i0^+)^{-1}\cdots(p_{n+l}^0-\sqrt{\vec{p}_{n+l}^2+M_{n+l}^2}+i0^+)^{-1}$$

$$\frac{1}{(p^0-\sqrt{\vec{p}^2+M^2}+i0^+)} = \frac{(p^0+\sqrt{\vec{p}^2+M^2}-i0^+)}{(p^0)^2-(\vec{p}^2+M^2-i0^+)} = \frac{2\sqrt{\vec{p}^2+M^2}}{p^2-M^2+i0^+} + \text{非极点项}$$

$$= \int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4} e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \Big\{ \frac{2i(2\pi)^3 p_1^0(\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2-M_1^2+i0^+}$$

$$\cdots\frac{2i(2\pi)^3 p_n^0(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2-M_n^2+i0^+}(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})$$

$$\times\frac{2i(2\pi)^3 p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(0)\Psi_0)}{p_{n+1}^2-M_{n+1}^2+i0^+}\cdots\frac{2i(2\pi)^3 p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(0)\Psi_0)}{p_{n+l}^2-M_{n+l}^2+i0^+} + \text{非物理极点项}\Big\}$$

$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|$前$n$个粒子在无穷将来，后$l$个粒子在无穷过去的情况

$$= \int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4} e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \Bigg\{ \frac{2i(2\pi)^3 p_1^0(\Psi_0, \psi_{H,l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2 - M_1^2 + i0^+}$$

$$\cdots \frac{2i(2\pi)^3 p_n^0(\Psi_0, \psi_{H,l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})$$

$$\times \frac{2i(2\pi)^3 p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{H,l_{n+1}}(0)\Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{H,l_{n+l}}(0)\Psi_0)}{p_{n+l}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \Bigg\}$$

$$= \int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4} e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \Bigg\{ \frac{2i(2\pi)^3 p_1^0(\Psi_0, \psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2 - M_1^2 + i0^+}$$

$$\cdots \frac{2i(2\pi)^3 p_n^0(\Psi_0, \psi_{l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})$$

去掉了下标H！

$$\times \frac{2i(2\pi)^3 p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}}, \psi_{l_{n+1}}(0)\Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}}, \psi_{n+l}(0)\Psi_0)}{p_{l_{n+l}}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \Bigg\}$$

约化公式

$$\left(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+\right)\Big|_{\text{前}n\text{个粒子在无穷将来，后}l\text{个粒子在无穷过去的情况}}$$

$$= \int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{n+l}}{(2\pi)^4} e^{-ip_1\cdot x_1-\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}+\cdots+ip_{n+l}\cdot x_{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \left\{ \frac{2i(2\pi)^3 p_1^0(\Psi_0, \psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})}{p_1^2 - M_1^2 + i0^+} \right.$$

$$\cdots \frac{2i(2\pi)^3 p_n^0(\Psi_0, \psi_{l_n}(0)\Psi_{\vec{p}_n,\sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots,\vec{p}_{n+l},\sigma_{n+l}})$$

$$\left. \times \frac{2i(2\pi)^3 p_{n+1}^0(\Psi_{\vec{p}_{n+1}\sigma_{n+1}}, \psi_{l_{n+1}}(0)\Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0(\Psi_{\vec{p}_{n+l}\sigma_{n+l}}, \psi_{l_{n+l}}(0)\Psi_0)}{p_{n+l}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \right\}$$

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)\left(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+\right)\Big|_{\text{前}n\text{个粒子在无穷将来,后}l\text{个粒子在无穷过去的情况}}$$

$$= \int \frac{d^4p_1\cdots d^4p_{n+l}}{(i\pi)^{n+l}} e^{-ip_1\cdot x_1\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}\cdots+ip_{n+l}\cdot x_{n+l}} \sum_{\sigma_1\cdots\sigma_{n+l}} \left\{ p_1^0(\Psi_0, \psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0, \psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n}) \right.$$

$$\times (\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n}, \Psi^+_{\vec{p}_{n+1},\sigma_{n+1},\cdots\vec{p}_{n+l},\sigma_{n+l}}) p_{n+1}^0(\Psi_{\vec{p}_{n+1}\sigma_{n+1}}, \psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l}\sigma_{n+l}}, \psi_{l_{n+l}}(0)\Psi_0)$$

$$\left. + \text{物理零点项} \right\}$$

约化公式

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)|_{\text{前}n\text{个粒子在无穷将来,后}l\text{个粒子在无穷过去的情况}}$$

$$=\int\frac{d^4p_1\cdots d^4p_{n+l}}{(i\pi)^{n+l}}e^{-ip_1\cdot x_1\cdots-ip_n\cdot x_n+ip_{n+1}\cdot x_{n+1}\cdots+ip_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}\Big\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec p_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec p_n\sigma_n})$$

$$\times(\Psi^-_{\vec p_1,\sigma_1,\cdots,\vec p_n,\sigma_n},\Psi^+_{\vec p_{n+1}\sigma_{n+1},\cdots\vec p_{n+l}\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec p_{n+1}\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec p_{n+l}\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

$$+\text{物理零点项}\Big\}$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

不再要求 前n个粒子在无穷将来，后l个粒子在无穷过去

$$\equiv\int\frac{d^4p_1\cdots d^4p_{n+l}}{(2\pi)^{4n+4l}}e^{-ip_1\cdot x_1\cdots-ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}G_{00}(p_1,\cdots,p_n,p_{n+1},\cdots,p_{n+l})$$

$$(-p_1^2+M_1^2)\cdots(-p_{n+l}^2+M_{n+l}^2)G_{00}(p_1,\cdots,p_n,-p_{n+1},\cdots,-p_{n+l})$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}\Big\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec p_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec p_n\sigma_n})(\Psi^-_{\vec p_1,\sigma_1,\cdots,\vec p_n,\sigma_n},\Psi^+_{\vec p_{n+1}\sigma_{n+1},\cdots\vec p_{n+l}\sigma_{n+l}})$$

$$\times p_{n+1}^0(\Psi_{\vec p_{n+1}\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec p_{n+l}\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)+\text{物理零点项}\Big\}$$

约化公式

$$(-p_1^2+M_1^2)\cdots(-p_{n+l}^2+M_{n+l}^2)G_{00}(p_1,\cdots,p_n,-p_{n+1},\cdots,-p_{n+l})$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}\Big\{p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n})(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1}\sigma_{n+1},\cdots\vec{p}_{n+l}\sigma_{n+l}})$$

$$\times p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)+\text{物理零点项}\Big\}$$

$$=\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$\lim_{p_i^2\to M_i^2}\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n,\sigma_n},\Psi^+_{\vec{p}_{n+1}\sigma_{n+1},\cdots\vec{p}_{n+l}\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec{p}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip_n\cdot x_n-ip_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}p_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{p}_1,\sigma_1})\cdots p_n^0(\Psi_0,\psi_{l_n}(0)\Psi_{\vec{p}_n\sigma_n})$$

$$\times(\Psi^-_{\vec{p}_1,\sigma_1,\cdots,\vec{p}_n\sigma_n},\Psi^+_{\vec{p}_{n+1}\sigma_{n+1},\cdots\vec{p}_{n+l}\sigma_{n+l}})p_{n+1}^0(\Psi_{\vec{p}_{n+1}\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0)\cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l}\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0)$$

实标量场: $\phi(x)=\int\frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p})+e^{ip\cdot x}a^\dagger(\vec{p})]$ $\quad a(\vec{p})\Phi_{\vec{q}}=\delta(p-q)\Phi_0$

$a(\vec{p})\Psi_{\vec{q}}=Z_\phi\delta(p-q)\Psi_0$ $\quad(2\pi)^{3/2}\sqrt{2q^0}(\Psi_0,\phi(0)\Psi_{\vec{q}})=Z_\phi$ $\quad Z_\phi=0$ 束缚态

$$(\Psi^-_{\vec{q}_1,\cdots,\vec{q}_n},\Psi^+_{\vec{q}_{n+1},\cdots,\vec{q}_{n+l}})$$

$$=\frac{i^{n+l}}{(2\pi)^{3(n+l)/2}Z_\phi^{n+l}\sqrt{2q_1^0\cdots2q_{n+l}^0}}\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n\cdot x_n-iq_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_{n+l})\Psi_0^+)$$

约化公式

复标量场: $\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{c\dagger}(\vec{p})]$

$$\int d^4x_1\cdots d^4x_{n+l}e^{ip_1\cdot x_1\cdots+ip^c_n\cdot x_n-ip^c_{n+1}\cdot x_{n+1}\cdots-ip_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial^2_{n+l}+M^2_{n+l})(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots$$

$$\phi_H(x_{n'})\phi_H^\dagger(x_{n'+1})\cdots\phi_H^\dagger(x_n)\phi_H(x_{n+1})\cdots\phi_H(x_{n+l'})\phi_H^\dagger(x_{n+l'+1})\cdots\phi_H^\dagger(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_{n+l}}p_1^0(\Psi_0,\phi(0)\Psi_{\vec{p}_1})\cdots p^0_{n'}(\Psi_0,\phi(0)\Psi_{\vec{p}_{n'}})p^{c0}_{n'+1}(\Psi_0,\phi^\dagger(0)\Psi_{\vec{p}^c_{n'+1}})\cdots p_n^{c0}(\Psi_0,\phi^\dagger(0)\Psi_{\vec{p}^c_n})$$

$$\times(\Psi^-_{\vec{p}_1,\cdots,\vec{p}_{n'},\vec{p}^c_{n'+1},\cdots,\vec{p}^c_n},\Psi^+_{\vec{p}^c_{n+1},\cdots,\vec{p}^c_{n+l'},\vec{p}_{n+l'+1}\cdots,\vec{p}_{n+l}})p^{c0}_{n+1}(\Psi_{\vec{p}^c_{n+1}},\phi(0)\Psi_0)\cdots p^{c0}_{n+l'}(\Psi_{\vec{p}^c_{n+l'}},\phi(0)\Psi_0)$$

$$\times p^0_{n+l'+1}(\Psi_{\vec{p}_{n+l'+1}},\phi^\dagger(0)\Psi_0)\cdots p^0_{n+l}(\Psi_{\vec{p}_{n+l},\sigma_{n+l}},\phi^\dagger(0)\Psi_0)$$

$$a(\vec{p})\Psi_{\vec{q}}=Z_\phi\delta(p-q)\Psi_0 \qquad (2\pi)^{3/2}\sqrt{2q^0}(\Psi_0,\phi(0)\Psi_{\vec{q}})=(2\pi)^{3/2}\sqrt{2q^0}(\Psi_{\vec{q}},\phi^\dagger(0)\Psi_0)=Z_\phi$$

$$a^c(\vec{p})\Psi_{\vec{q}^c}=Z_\phi\delta(p-q^c)\Psi_0 \qquad (2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_0,\phi^\dagger(0)\Psi_{\vec{q}^c})=(2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_{\vec{q}^c},\phi(0)\Psi_0)=Z_\phi$$

$$(\Psi^-_{\vec{q}_1,\cdots,\vec{q}_{n'},\vec{q}^c_{n'+1}\cdots,\vec{q}^c_n},\Psi^+_{\vec{q}^c_{n+1},\cdots,\vec{q}^c_{n+l'},\vec{q}_{n+l'+1},\cdots,\vec{q}_{n+l}})=\frac{[i/(2\pi)^{3/2}\sqrt{2}]^{n+l}}{Z_\phi^{n+l}\sqrt{q_1^0\cdots q_n^{c0}q^{c0}_{n+1}\cdots q^0_{n+l}}}\int d^4x_1\cdots d^4x_{n+l}$$

$$\times e^{iq_1\cdot x_1\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial^2_{n+l}+M^2_{n+l})(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H^\dagger(x_n)\phi_H(x_{n+1})\cdots\phi_H^\dagger(x_{n+l})\Psi_0^+)$$

$$\int d^4x_1\cdots d^4x_{n+l}\; e^{iq_1\cdot x_1+\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots$$
$$\psi_{H,l_{n'}}(x_{n'})\overline{\psi}_{H,l_{n'+1}}(x_{n'+1})\cdots\overline{\psi}_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\psi_{H,l_{n+l'}}(x_{n+l'})\overline{\psi}_{H,l_{n+l'+1}}(x_{n+l'+1})\cdots\overline{\psi}_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$
$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_n^c\sigma_{n+1}^c\cdots\sigma_{n+l}} q_1^0(\Psi_0,\psi_{l_1}(0)\Psi_{\vec{q}_1,\sigma_1})\cdots q_n^{c0}(\Psi_0,\overline{\psi}_{l_n}(0)\Psi_{\vec{q}_n^c,\sigma_n^c})$$
$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n^c,\sigma_n^c},\Psi^+_{\vec{q}_{n+1}^c,\sigma_{n+1}^c,\cdots,\vec{q}_{n+l},\sigma_{n+l}})q_{n+1}^{c0}(\Psi_{\vec{q}_{n+1}^c,\sigma_{n+1}^c},\psi_{n+1}(0)\Psi_0)\cdots q_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}},\overline{\psi}_{l_{n+l}}(0)\Psi_0)$$

旋量场: $\psi_l(x)=(2\pi)^{-3/2}\sum_\sigma\int d\vec{p}\,[u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma)+v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)]$

$$\overline{\psi}_l(x)=(2\pi)^{-3/2}\sum_\sigma\int d\vec{p}\,[\overline{u}_l(\vec{p},\sigma)e^{ip\cdot x}a^\dagger(\vec{p},\sigma)+\overline{v}_l(\vec{p},\sigma)e^{-ip\cdot x}a^c(\vec{p},\sigma)]$$

$a(\vec{p})\Psi_{\vec{q}}=Z_\psi\delta(p-q)\Psi_0$ $\qquad$ $a^c(\vec{p})\Psi_{\vec{q}^c}=Z_\psi\delta(p-q^c)\Psi_0$

$(2\pi)^{3/2}(\Psi_0,\psi_l(0)\Psi_{\vec{q},\sigma})=Z_\psi u_l(\vec{q},\sigma)$ $\qquad$ $(2\pi)^{3/2}(\Psi_{\vec{q},\sigma},\overline{\psi}_l(0)\Psi_0)=Z_\psi\overline{u}_l(\vec{q},\sigma)$

$(2\pi)^{3/2}(\Psi_0,\overline{\psi}_l(0)\Psi_{\vec{q}^c,\sigma^c})=Z_\psi\overline{v}_l(\vec{q}^c,\sigma^c)$ $\qquad$ $(2\pi)^{3/2}(\Psi_{\vec{q}^c,\sigma^c},\psi_l(0)\Psi_0)=Z_\psi v_l(\vec{q}^c,\sigma^c)$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\overline{\psi}_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=(-2^{5/2}i\pi^{3/2})^{n+l}Z_\psi^{n+l}q_1^0\cdots q_n^{c0}q_{n+1}^{c0}\cdots q_{n+l}^0\sum_{\sigma_1\cdots\sigma_n^c\sigma_{n+1}^c\cdots\sigma_{n+l}}u_{l_1}(\vec{q}_1,\sigma_1)\cdots\bar{v}_{l_n}(\vec{q}_n^c,\sigma_n^c)$$

$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n^c,\sigma_n^c},\Psi^+_{\vec{q}_{n+1}^c,\sigma_{n+1}^c\cdots,\vec{q}_{n+l},\sigma_{n+l}})v_{l_{n+1}}(\vec{q}_{n+1}^c,\sigma_{n+1}^c)\cdots\bar{u}_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})$$

$$u(\vec{p},\sigma)=\sqrt{M/p^0}D(L(p))u(0,\sigma)\qquad v(\vec{p},\sigma)=\sqrt{M/p^0}D(L(p))v(0,\sigma)\qquad D(L(p))\beta D^\dagger(L(p))=\beta$$

$$\beta=\gamma^0=\begin{pmatrix}0&-I\\-I&0\end{pmatrix}\quad u(0,\tfrac{1}{2})=\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{-1}{\sqrt{2}}\\0\end{bmatrix}\quad u(0,-\tfrac{1}{2})=\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\\0\\\frac{-1}{\sqrt{2}}\end{bmatrix}\quad v(0,\tfrac{1}{2})=\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix}\quad v(0,-\tfrac{1}{2})=\begin{bmatrix}\frac{-1}{\sqrt{2}}\\0\\\frac{-1}{\sqrt{2}}\\0\end{bmatrix}$$

$$\bar{u}(\vec{p},\sigma)u(\vec{p},\sigma')=\frac{M}{p^0}\bar{u}(0,\sigma)\beta D^\dagger(L(p))\beta D(L(p))u(0,\sigma')=\frac{M}{p^0}\bar{u}(0,\sigma)u(0,\sigma')=\frac{M}{p^0}\delta_{\sigma\sigma'}$$

$$\bar{v}(\vec{p},\sigma)v(\vec{p},\sigma')=\frac{M}{p^0}\bar{v}(0,\sigma)\beta D^\dagger(L(p))\beta D(L(p))v(0,\sigma')=\frac{M}{p^0}\bar{v}(0,\sigma)v(0,\sigma')=-\frac{M}{p^0}\delta_{\sigma\sigma'}$$

约化公式

$$\left(\Psi^{-}_{\vec{q}_1,\sigma_1,\cdots,\vec{q}^{\,c}_n,\sigma^c_n},\ \Psi^{+}_{\vec{q}^{\,c}_{n+1},\sigma^c_{n+1},\cdots,\vec{q}_{n+l},\sigma_{n+l}}\right)$$

$$\begin{aligned}
&= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots v_{l_n}(\vec{q}^{\,c}_n,\sigma^c_n)\\
&\quad\times\bar{v}_{l_{n+1}}(\vec{q}^{\,c}_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\\
&\quad\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)\left(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n}(x_n)\psi_{H,l_{n+1}}(x_{n+1})\cdots\overline{\psi}_{H,l_{n+l}}(x_{n+l})\Psi_0^+\right)
\end{aligned}$$

$$\begin{aligned}
&= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots v_{l_n}(\vec{q}^{\,c}_n,\sigma^c_n)\\
&\quad\times\bar{v}_{l_{n+1}}(\vec{q}^{\,c}_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\\
&\quad\times[(-i\partial\!\!\!/_{x_1}-M_1)(i\partial\!\!\!/_{x_1}-M_1)]_{l_1\bar{l}_1}\cdots[(-i\partial\!\!\!/_{x_n}-M_n)(i\partial\!\!\!/_{x_n}-M_n)]_{\bar{l}_n l_n}[(-i\partial\!\!\!/_{x_{n+1}}-M_{n+1})(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})]_{l_{n+1}\bar{l}_{n+1}}\\
&\quad\cdots[(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})(i\partial\!\!\!/_{x_{n+l}}-M_{n+l})]_{\bar{l}_{n+l}l_{n+l}}\left(\Psi_0^-,\mathbf{T}\psi_{H,\bar{l}_1}(x_1)\cdots\overline{\psi}_{H,\bar{l}_n}(x_n)\psi_{H,\bar{l}_{n+1}}(x_{n+1})\cdots\psi_{H,\bar{l}_{n+l}}(x_{n+l})\Psi_0^+\right)
\end{aligned}$$

约化公式

$$\left(\Psi^{-}_{\vec q_1,\sigma_1,\cdots,\vec q^c_n,\sigma^c_n},\ \Psi^{+}_{\vec q^c_{n+1},\sigma^c_{n+1},\cdots,\vec q_{n+l},\sigma_{n+l}}\right)$$

$$= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar u_{l_1}(\vec q_1,\sigma_1)\cdots v_{l_n}(\vec q^c_n,\sigma^c_n)$$

$$\times \bar v_{l_{n+1}}(\vec q^c_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec q_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times[(-i\partial\!\!\!/_{x_1}-M_1)(i\partial\!\!\!/_{x_1}-M_1)]_{l_1\bar l_1}\cdots[(-i\partial\!\!\!/_{x_n}-M_n)(i\partial\!\!\!/_{x_n}-M_n)]_{\bar l_n l_n}[(-i\partial\!\!\!/_{x_{n+1}}-M_{n+1})(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})]_{l_{n+1}\bar l_{n+1}}$$

$$\cdots[(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})(i\partial\!\!\!/_{x_{n+l}}-M_{n+l})]_{\bar l_{n+l}l_{n+l}}\left(\Psi^-_0,\mathbf{T}\psi_{H,\bar l_1}(x_1)\cdots\overline{\psi}_{H,\bar l_n}(x_n)\psi_{H,\bar l_{n+1}}(x_{n+1})\cdots\psi_{H,\bar l_{n+l}}(x_{n+l})\Psi^+_0\right)$$

$$= (-1)^{-n'+l'-l}(2^{5/2}i\pi^{3/2})^{-n-l}Z_\psi^{-n-l}(M_1\cdots M_{n+l})^{-1}\bar u_{l_1}(\vec q_1,\sigma_1)\cdots v_{l_n}(\vec q^c_n,\sigma^c_n)$$

$$\times \bar v_{l_{n+1}}(\vec q^c_{n+1},\sigma^c_{n+1})\cdots u_{l_{n+l}}(\vec q_{n+l},\sigma_{n+l})\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times[(-q\!\!\!/_1-M_1)(i\partial\!\!\!/_{x_1}-M_1)]_{l_1\bar l_1}\cdots[(-i\partial\!\!\!/_{x_n}-M_n)(q\!\!\!/^c_n-M_n)]_{\bar l_n l_n}[(q\!\!\!/^c_{n+1}-M_{n+1})(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})]_{l_{n+1}\bar l_{n+1}}$$

$$\cdots[(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})(-q\!\!\!/_{n+l}-M_{n+l})]_{\bar l_{n+l}l_{n+l}}\left(\Psi^-_0,\mathbf{T}\psi_{H,\bar l_1}(x_1)\cdots\overline{\psi}_{H,\bar l_n}(x_n)\psi_{H,\bar l_{n+1}}(x_{n+1})\cdots\psi_{H,\bar l_{n+l}}(x_{n+l})\Psi^+_0\right)$$

$$(q\!\!\!/_{n+l}-M_{n+l})u(\vec q_{n+l},\sigma_{n+l})=0\quad \bar u(\vec q_1,\sigma)(q\!\!\!/_1-M_1)=0\quad (q\!\!\!/^c_n+M_n)v(\vec q^c_n,\sigma^c_n)=0\quad \bar v(\vec q^c_{n+1},\sigma^c_{n+1})(q\!\!\!/^c_{n+1}+M_{n+1})=0$$

约化公式

$$\left(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_{n'},\sigma_{n'},\vec{q}^c_{n'+1},\sigma^c_{n'+1},\cdots,\vec{q}^c_n,\sigma^c_n},\ \Psi^+_{\vec{q}^c_{n+1},\sigma^c_{n+1},\cdots,\vec{q}^c_{n+l'},\sigma^c_{n+l'},\vec{q}_{n+l'+1},\sigma_{n+l'+1},\cdots,\vec{q}_{n+l},\sigma_{n+l}}\right)$$

$$=(-1)^{-n'+l'-l}[(2\pi)^{\frac{3}{2}}i]^{-n-l}Z_\psi^{-n-l}\bar{u}_{l_1}(\vec{q}_1,\sigma_1)\cdots\bar{u}_{l_{n'}}(\vec{q}_{n'},\sigma_{n'})v_{l_{n'+1}}(\vec{q}^c_{n'+1},\sigma^c_{n'+1})\cdots v_{l_n}(\vec{q}^c_n,\sigma^c_n)$$

$$\times\bar{v}_{l_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots\bar{v}_{l_{n+l'}}(\vec{q}^c_{n+l'},\sigma^c_{n+l'})u_{l_{n+l'+1}}(\vec{q}_{n+l'+1},\sigma_{n+l'+1})\cdots u_{l_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})$$

$$\times\int d^4x_1\cdots d^4x_{n+l}\,e^{iq_1\cdot x_1+\cdots+iq_{n'}\cdot x_{n'}+iq^c_{n'+1}\cdot x_{n'+1}+\cdots+iq^c_n\cdot x_n-iq^c_{n+1}\cdot x_{n+1}-\cdots-iq^c_{n+l'}\cdot x_{n+l'}-iq_{n+l'+1}\cdot x_{n+l'+1}-\cdots-iq_{n+l}\cdot x_{n+l}}$$

$$\times(i\partial\!\!\!/_{x_1}-M_1)_{l_1\bar{l}_1}\cdots(i\partial\!\!\!/_{x_{n'}}-M_{n'})_{l_{n'}\bar{l}_{n'}}(-i\partial\!\!\!/_{x_{n'+1}}-M_{n'+1})_{\bar{l}_{n'+1}l_{n'+1}}\cdots(-i\partial\!\!\!/_{x_n}-M_n)_{\bar{l}_nl_n}$$

$$\times(i\partial\!\!\!/_{x_{n+1}}-M_{n+1})_{l_{n+1}\bar{l}_{n+1}}\cdots(i\partial\!\!\!/_{x_{n+l'}}-M_{n+l'})_{l_{n+l'}\bar{l}_{n+l'}}(-i\partial\!\!\!/_{x_{n+l'+1}}-M_{n+l'+1})_{\bar{l}_{n+l'+1}l_{n+l'+1}}\cdots(-i\partial\!\!\!/_{x_{n+l}}-M_{n+l})_{\bar{l}_{n+l}l_{n+l}}$$

$$\times\left(\Psi^-_0,\mathbf{T}\psi_{H,\bar{l}_1}(x_1)\cdots\overline{\psi}_{H,\bar{l}_n}(x_n)\psi_{H,\bar{l}_{n+1}}(x_{n+1})\cdots\overline{\psi}_{H,\bar{l}_{n+l}}(x_{n+l})\Psi^+_0\right)$$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)$$

$$\times(\Psi_0^-,\mathbf{T}v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}^\dagger(x_n)v_{H,\mu_{n+1}}(x_{n+1})\cdots v_{H,\mu_{n+l}}^\dagger(x_{n+l})\Psi_0^+)$$

$$=(-16i\pi^3)^{n+l}\sum_{\sigma_1\cdots\sigma_n^c\sigma_{n+1}\cdots\sigma_{n+l}^c}q_1^0(\Psi_0,v_{\mu_1}(0)\Psi_{\vec{q}_1,\sigma_1})\cdots q_n^{c0}(\Psi_0,v_{\mu_n}(0)\Psi_{\vec{q}_n^c,\sigma_n^c})$$

$$\times(\Psi_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n^c,\sigma_n^c}^-,\Psi_{\vec{q}_{n+1}^c,\sigma_{n+1}^c,\cdots\vec{q}_{n+l},\sigma_{n+l}}^+)q_{n+1}^{c0}(\Psi_{\vec{q}_{n+1}^c,\sigma_{n+1}^c},v_{\mu_{n+1}}(0)\Psi_0)\cdots q_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}},v_{\mu_{n+l}}(0)\Psi_0)$$

矢量场: $v^\mu(x)=(2\pi)^{-3/2}\sum_{\sigma=\pm1}\int\frac{d\vec{p}}{\sqrt{2p^0}}[e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x}+e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$

$(2\pi)^{3/2}\sqrt{2q^0}(\Psi_0,v^\mu(0)\Psi_{\vec{q},\sigma})=Z_ve^\mu(\vec{q},\sigma)$ $\quad$ $(2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_{\vec{q}^c,\sigma^c},v^\mu(0)\Psi_0)=Z_ve^{\mu*}(\vec{q}^c,\sigma^c)$

$(2\pi)^{3/2}\sqrt{2q^{c0}}(\Psi_0,v^{\mu\dagger}(0)\Psi_{\vec{q}^c,\sigma^c})=Z_ve^\mu(\vec{q}^c,\sigma^c)$ $\quad$ $(2\pi)^{3/2}\sqrt{2q^0}(\Psi_{\vec{q},\sigma},v^{\mu\dagger}(0)\Psi_0)=Z_ve^{\mu*}(\vec{q},\sigma)$

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0^-,\mathbf{T}v_{H\mu_1}(x_1)\cdots v_{H\mu_{n+l}}^\dagger(x_{n+l})\Psi_0^+)$$

$$=(-4i\pi^{3/2})^{n+l}Z_v^{n+l}\sqrt{q_1^0\cdots q_{n+l}^0}\sum_{\sigma_1\cdots\sigma_{n+l}}e_{\mu_1}(\vec{q}_1,\sigma_1)\cdots e_{\mu_n}(\vec{q}_n^c,\sigma_n^c)(\Psi_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n^c,\sigma_n^c}^-,\Psi_{\vec{q}_{n+1}^c,\sigma_{n+1}^c,\cdots\vec{q}_{n+l},\sigma_{n+l}}^+)$$

$$\times e_{\mu_{n+1}}^*(\vec{q}_{n+1}^c,\sigma_{n+1}^c)\cdots e_{\mu_{n+l}}^*(\vec{q}_{n+l},\sigma_{n+l})$$

约化公式

$$\int d^4x_1\cdots d^4x_{n+l}e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}\times(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)\left(\Psi_0^-,\mathbf{T}v_{H\mu_1}(x_1)\cdots v^\dagger_{H\mu_{n+l}}(x_{n+l})\Psi_0^+\right)$$

$$=(-4i\pi^{3/2})^{n+l}Z_v^{n+l}\sum_{\sigma_1\cdots\sigma^c_{n+l}}\sqrt{q_1^0}e_{\mu_1}(\vec{q}_1,\sigma_1)\cdots\sqrt{q_n^{c0}}e_{\mu_n}(\vec{q}^c_n,\sigma^c_n)\left(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}^c_n,\sigma^c_n},\Psi^+_{\vec{q}^c_{n+1},\sigma^c_{n+1},\cdots\vec{q}_{n+l},\sigma_{n+l}}\right)$$

$$\times\sqrt{q^{c0}_{n+1}}e^*_{\mu_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots\sqrt{q^0_{n+l}}e^*_{\mu_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})$$

$$\left(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}^c_n,\sigma^c_n},\Psi^+_{\vec{q}^c_{n+1},\sigma^c_{n+1},\cdots,\vec{q}_{n+l},\sigma_{n+l}}\right)$$

$$e^\mu(\vec{p},\sigma)e_\mu(\vec{p},\sigma')=-\delta_{\sigma\sigma'}$$

作业28

$$=\frac{e^{\mu_1}(\vec{q}_1,\sigma_1)\cdots e^{\mu_n}(\vec{q}^c_n,\sigma^c_n)e^{*\mu_{n+1}}(\vec{q}^c_{n+1},\sigma^c_{n+1})\cdots e^{*\mu_{n+l}}(\vec{q}_{n+l},\sigma_{n+l})}{(4i\pi^{3/2})^{n+l}Z_v^{n+l}\sqrt{q_1^0}\cdots\sqrt{q_n^{c0}}\sqrt{q^{c0}_{n+1}}\cdots\sqrt{q^0_{n+l}}}\int d^4x_1\cdots d^4x_{n+l}$$

$$\times e^{iq_1\cdot x_1\cdots+iq_n^c\cdot x_n-iq_{n+1}^c\cdot x_{n+1}\cdots-iq_{n+l}\cdot x_{n+l}}(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)\left(\Psi_0^-,\mathbf{T}v_{H,\mu_1}(x_1)\cdots v^\dagger_{H,\mu_{n+l}}(x_{n+l})\Psi_0^+\right)$$

态之间的编时乘积的格林函数$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,1}(x_1)\psi_{H,2}(x_2)\cdots\psi_{H,n}(x_n)\Psi_\alpha^+)$

Wick定理:

$$\mathbf{T}(UV\cdots XYZ) = \mathbf{N}(UV\cdots XYZ)$$ 无收缩项

$$+\,\mathbf{N}(UV\cdots XYZ) + \cdots + \mathbf{N}(UV\cdots XYZ) +$$ 一次收缩项

$$+\,\mathbf{N}(UV\cdots XYZ) + \cdots$$ 二次收缩项

$$+\ \cdots$$

$$+\,\mathbf{N}(UVW\cdots XYZ) + \cdots$$ 全部收缩项

若T乘积中含N乘积,展开仍成立,但需将各N乘积内部各算符之间的收缩略去.

应用Wick定理: $U(t,t') = \Omega^\dagger(t)\Omega(t')$ $\quad \Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha$

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$$= (\Omega(\infty)\Phi_\beta, \mathbf{T}\ \Omega(t_1)\psi_{l_1}(x_1)U(t_1,t_2)\psi_2(x_2)U(t_2,t_3)\cdots U(t_{n-1},t_n)\psi_{l_n}(x_n)\Omega^\dagger(t_n)\Omega(-\infty)\Phi_\alpha)$$

$$= (\Phi_\beta, \mathbf{T}\ \psi_{l_1}(x_1)\psi_2(x_2)\cdots\psi_{l_n}(x_n)U(\infty,-\infty)\Phi_\alpha) = \underline{\text{所有可能的特别收缩项之和}}$$

保留下来的正规乘积中的湮灭算符必须正好湮灭Φ_α态到某个态，产生算符的厄米共扼必须正好湮灭Φ_β态到同样的态! 特别地，

$$(\Psi_\beta^-, \Psi_\alpha^+) = (\Phi_\beta, U(\infty,-\infty)\Phi_\alpha)$$

$$(\Psi_0^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = (\Phi_0, \mathbf{T}\ \psi_{l_1}(x_1)\psi_2(x_2)\cdots\psi_{l_n}(x_n)U(\infty,-\infty)\Phi_0)$$

编时乘积的格林函数$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$

考虑 $x_1^0, \cdots, x_r^0 > 0, \quad x_{r+1}^0, \cdots, x_n^0 < 0$

$$(\Psi_\beta^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$$= \theta(\min[x_1^0\cdots x_r^0] - \max[x_{r+1}^0\cdots x_n^0]) \quad \Leftarrow$$

这里已把前r个场晚于后n-r个场的要求通过θ函数施加进去了

$$\times(\Psi_\beta^-, \mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi_\alpha^+)$$

$$= \theta(\min[x_1^0\cdots x_r^0] - \max[x_{r+1}^0\cdots x_n^0])$$

$$\times\int d\gamma(\Psi_\beta^-, \mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\Psi_\gamma)(\Psi_\gamma, \mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi_\alpha^+)$$

$$= \text{单粒子中间态项} + \text{多粒子中间态项}$$

Ψ_γ 是一组多粒子态完备集；是 H 的本征态

单粒子中间态项： $x_1^0,\cdots,x_r^0>0$, $x_{r+1}^0,\cdots,x_n^0<0$

$$\theta(\min[x_1^0\cdots x_r^0]-\max[x_{r+1}^0\cdots x_n^0])$$

$$\times\sum_\sigma\int d\vec{p}\,(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi_\alpha^+)$$

$$\psi_{H,l_i}(x_i)=e^{iP\cdot x_i}\psi_{H,l_i}(0)e^{-iP\cdot x_i}\qquad\qquad \psi_{H,l_i}(x_i-x)=e^{-iP\cdot x}\psi_{H,l_i}(x_i)e^{iP\cdot x}$$

$$\psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_r}(x_r)=e^{iP\cdot x_1}\psi_{H,l_1}(0)\psi_{H,l_2}(x_2-x_1)\cdots\psi_{H,l_r}(x_r-x_1)e^{-iP\cdot x_1}$$

$$\psi_{H,l_{r+1}}(x_{r+1})\psi_{H,l_{r+2}}(x_{r+2})\cdots\psi_{H,l_n}(x_n)=e^{iP\cdot x_{r+1}}\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(x_{r+2}-x_{r+1})\cdots\psi_{H,l_n}(x_n-x_{r+1})e^{-iP\cdot x_{r+1}}$$

$$e^{-iP\cdot x_1}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}e^{iP\cdot x_{r+1}}=e^{-iP\cdot x_1}\Psi_{\vec{p},\sigma})(e^{-iP\cdot x_{r+1}}\Psi_{\vec{p},\sigma}=e^{-ip\cdot(x_1-x_{r+1})}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}$$

$$=\theta(\min[x_1^0\cdots x_r^0]-\max[x_{r+1}^0\cdots x_n^0])$$

$$\times\sum_\sigma\int d\vec{p}\;e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}}(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(x_2-x_1)\cdots\psi_{H,l_r}(x_r-x_1)\}\Psi_{\vec{p},\sigma})$$

$$\times(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(x_{r+2}-x_{r+1})\cdots\psi_{H,l_n}(x_n-x_{r+1})\}\Psi_\alpha^+)$$

单粒子中间态项：$x_i = x_1 + y_i\ (i = 2, 3, \cdots, r)\quad x_i = x_{r+1} + y_i\ (i = r+2, \cdots, n)$

$$\theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0])$$

$$\times \sum_\sigma \int d\vec{p}\ (\Psi_\beta^-, \mathbf{T}\{\psi_{H,l_1}(x_1)\cdots\psi_{H,l_r}(x_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(x_{r+1})\cdots\psi_{H,l_n}(x_n)\}\Psi_\alpha^+)$$

$$= \theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0])$$

$$\times \sum_\sigma \int d\vec{p}\ e^{i(p_\beta - p)x_1 + i(p - p_\alpha)x_{r+1}} (\Psi_\beta^-, \mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(x_2 - x_1)\cdots\psi_{H,l_r}(x_r - x_1)\}\Psi_{\vec{p},\sigma})$$

$$\times (\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(x_{r+2} - x_{r+1})\cdots\psi_{H,l_n}(x_n - x_{r+1})\}\Psi_\alpha^+)$$

$$\min[x_1^0, \cdots, x_r^0] - \max[x_{r+1}^0, \cdots, x_n^0] = x_1^0 - x_{r+1}^0 + \min[0, y_2^0, \cdots, y_r^0] - \max[0, y_{r+1}^0, \cdots, y_n^0]$$

$$= \frac{i}{2\pi}\int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \cdots, y_r^0] - \max[0, y_{r+1}^0, \cdots, y_n^0])} \qquad \theta(\tau) = -\int_{-\infty}^{\infty} \frac{d\omega}{2i\pi}\frac{e^{-i\omega\tau}}{\omega + i0^+}$$

$$\times \sum_\sigma \int d\vec{p}(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$\times e^{i(p_\beta - p)x_1 + i(p - p_\alpha)x_{r+1} - i\omega(x_1^0 - x_{r+1}^0)}$$

格林函数的极点

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) \qquad x_1^0,\cdots,x_r^0>0,\quad x_{r+1}^0,\cdots,x_n^0<0$$

$$=\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

$$\times\sum_\sigma\int d\vec{p}(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p}\sigma})(\Psi_{\vec{p}\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$x_i=x_1+y_i\ (i=2,3,\cdots,r)\quad x_i=x_{r+1}+y_i\ (i=r+2,\cdots,n)$$

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)=\int\frac{d^4q_1\cdots d^4q_n}{(2\pi)^{4n}}\ e^{-iq_1\cdot x_1-\cdots-iq_n\cdot x_n}G_{\beta\alpha}(q_1,\cdots,q_n)$$

$$-i(q_1\cdot x_1+\cdots+q_n\cdot x_n)$$

$$=-i[(q_1+\cdots+q_r)\cdot x_1+(q_{r+1}+\cdots+q_n)\cdot x_{r+1}+q_2\cdot y_2+\cdots+q_r\cdot y_r+q_{r+2}\cdot y_{r+2}+\cdots+q_n\cdot y_n]$$

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) \qquad \tilde{q}_1\equiv q_1+\cdots+q_r\quad \tilde{q}_{r+1}\equiv q_{r+1}+\cdots+q_n$$

$$=\int\frac{d^4\tilde{q}_1d^4\tilde{q}_{r+1}d^4q_2\cdots d^4q_rd^4q_{r+2}\cdots d^4q_n}{(2\pi)^{4n}}\ e^{-i\tilde{q}_1\cdot x_1-i\tilde{q}_{r+1}\cdot x_{r+1}-iq_2\cdot y_2-\cdots-iq_r\cdot y_r-iq_{r+2}\cdot y_{r+2}-\cdots-iq_n\cdot y_n}$$

$$\times G_{\beta\alpha}(q_1,\cdots,q_n)$$

格林函数的极点

$$(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)|_{\text{单粒子中间态}} \qquad x_1^0,\cdots,x_r^0>0,\quad x_{r+1}^0,\cdots,x_n^0<0$$

$$= \frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}\int d\vec{p}\; e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

$$\times\sum_\sigma(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+) \qquad \tilde{q}_1\equiv q_1+\cdots+q_r \quad \tilde{q}_{r+1}\equiv q_{r+1}+\cdots+q_n$$

$$=\int\frac{d^4\tilde{q}_1 d^4\tilde{q}_{r+1}d^4q_2\cdots d^4q_r d^4q_{r+2}\cdots d^4q_n}{(2\pi)^{4n}}\, e^{-i\tilde{q}_1\cdot x_1-i\tilde{q}_{r+1}\cdot x_{r+1}-iq_2\cdot y_2-\cdots-iq_r\cdot y_r-iq_{r+2}\cdot y_{r+2}-\cdots-iq_n\cdot y_n}$$

$$\times G_{\beta\alpha}(q_1,\cdots,q_n) \qquad y_i=x_i-y_1\ (i=2,3,\cdots,r) \qquad y_i=x_i-x_{r+1}\ (i=r+2,\cdots,n)$$

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\; e^{i\tilde{q}_1\cdot x_1+i\tilde{q}_{r+1}\cdot x_{r+1}+iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

$$\times(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$$=\int d^4x_1 d^4x_{r+1}d^4y_2\cdots d^4y_r d^4y_{r+2}\cdots d^4y_n\,(\Psi_\beta^-, \mathbf{T}\, \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+$$

$$\times\, e^{i\tilde{q}_1\cdot x_1+i\tilde{q}_{r+1}\cdot x_{r+1}+iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

格林函数的极点

$$(\Psi_\beta^-, \mathbf{T}\ \psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)|_{\text{单粒子中间态}} \qquad x_1^0,\cdots,x_r^0>0,\quad x_{r+1}^0,\cdots,x_n<0$$

$$=\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}\int d\vec{p}\ e^{i(p_\beta-p)x_1+i(p-p_\alpha)x_{r+1}-i\omega(x_1^0-x_{r+1}^0)}$$

$$\times\sum_\sigma(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$$\tilde{q}_1\equiv q_1+\cdots+q_r\quad \tilde{q}_{r+1}\equiv q_{r+1}+\cdots+q_n\quad y_i=x_i-y_1\ (i=2,\cdots,r)\quad y_i=x_i-x_{r+1}\ (i=r+2,\cdots,n)$$

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1d^4x_{r+1}d^4y_2\cdots d^4y_rd^4y_{r+2}\cdots d^4y_n(\Psi_\beta^-,\mathbf{T}\psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+$$

$$\times\ e^{i\tilde{q}_1\cdot x_1+i\tilde{q}_{r+1}\cdot x_{r+1}+iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

$\Downarrow$ 下面只考虑了前**r**个场晚于后**n-r**个场的情形 $\Downarrow$

$$G_{\beta\alpha}(q_1,\cdots,q_n)|_{\text{单粒子中间态}}=\int d^4y_2\cdots d^4y_rd^4y_{r+2}\cdots d^4y_n\ e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}$$

$$\times\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}\int d\vec{p}\ (2\pi)^8\delta(\vec{p}-\vec{p}_\beta-\vec{\tilde{q}}_1)$$

$$\times\delta(-\sqrt{\vec{p}^2+M^2}-\omega+p_\beta^0+\tilde{q}_1^0)\delta(-\vec{\tilde{q}}_{r+1}+\vec{p}_\alpha-\vec{p})\delta(\tilde{q}_{r+1}^0-p_\alpha^0+\sqrt{\vec{p}^2+M^2}+\omega$$

$$\times\sum_\sigma(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

$G_{\alpha\beta}(q_1,\cdots,q_n)|_{\text{单粒子中间态}}$

$$=\int d^4y_2\cdots d^4y_r d^4y_{r+2}\cdots d^4y_n e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}$$

$$\times\sum_\sigma\int d\vec{p}(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi$$

$$\times(2\pi)^8\delta(\vec{p}-\vec{p}_\beta-\vec{q}_1-\cdots-\vec{q}_r)\delta(-\sqrt{\vec{p}^2+M^2}-\omega+p_\beta^0+q_1^0+\cdots+q_r^0)$$

$$\times\delta(-\vec{q}_{r+1}-\cdots-\vec{q}_n+\vec{p}_\alpha-\vec{p})\delta(q_{r+1}^0+\cdots+q_n^0-p_\alpha^0+\sqrt{\vec{p}^2+M^2}+\omega)$$

$q\equiv q_1+\cdots+q_r+p_\beta=-q_{r+1}-\cdots-q_n+p_\alpha$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)\frac{\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)}{q^0-\sqrt{\vec{q}^2+M^2}+i0^+}$$

$$\frac{1}{q^0-\sqrt{\vec{q}^2+M^2}+i0^+}=\frac{q^0+\sqrt{\vec{q}^2+M^2}-i0^+}{(q^0)^2-(\vec{q}^2+M^2-i0^+)}=\frac{2\sqrt{\vec{q}^2+M^2}}{q^2-M^2+i0^+}+\text{非极点项}$$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)\frac{2q^0\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)}{q^2-M^2+i0^+}$$

格林函数的极点

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\, e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}(\Psi_\beta^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_\alpha^+)$$

$G_{\beta\alpha}(q_1,\cdots,q_n)|$单粒子中间态

$$=\int d^4y_2\cdots d^4y_r d^4y_{r+2}\cdots d^4y_n e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r+iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega+i0^+}e^{-i\omega(\min[0,y_2^0,\cdots,y_r^0]-\max[0,y_{r+1}^0,\cdots,y_n^0])}$$

$$\times\sum_\sigma\int d\vec{p}\,(\Psi_\beta^-\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{p},\sigma})(\Psi_{\vec{p},\sigma}\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi$$

$$\times(2\pi)^8\delta(\vec{p}-\vec{p}_\beta-\vec{q}_1-\cdots-\vec{q}_r)\delta(-\sqrt{\vec{p}^2+M^2}-\omega+p_\beta^0+q_1^0+\cdots+q_r^0)$$

$$\times\delta(-\vec{q}_{r+1}-\cdots-\vec{q}_n+\vec{p}_\alpha-\vec{p})\delta(q_{r+1}^0+\cdots+q_n^0-p_\alpha^0+\sqrt{\vec{p}^2+M^2}+\omega)$$

$q\equiv q_1+\cdots+q_r+p_\beta=-q_{r+1}-\cdots-q_n+p_\alpha$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)\frac{2q^0\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)}{q^2-M^2+i0^+}+\text{非极点项}$$

$$M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)=\int d^4y_2\cdots d^4y_r e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r}(\Psi_\beta^-,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{q},\sigma})$$

$$M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)=\int d^4y_{r+2}\cdots d^4y_n e^{iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}(\Psi_{\vec{q},\sigma},\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi_\alpha^+$$

格林函数的极点

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\; e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}(\Psi^-_\beta,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

$$M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)=\int d^4y_2\cdots d^4y_r e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r}(\Psi^-_\beta,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{q},\sigma})$$

$$M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)=\int d^4y_{r+2}\cdots d^4y_n e^{iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}(\Psi_{\vec{q},\sigma},\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi^+_\alpha$$

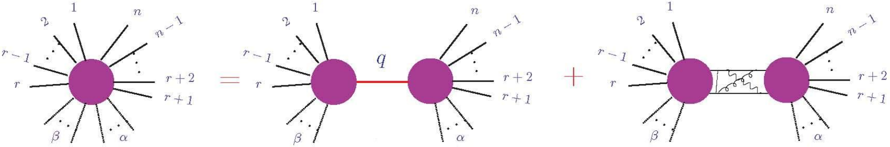
$$G_{\beta\alpha}(q_1,\cdots,q_n)=\sum_\sigma\int\frac{d^4q}{(2\pi)^4}\left[(2\pi)^4\delta(q_1+\cdots+q_r+p_\beta-q)(2\pi)^{\frac{3}{2}}(2q^0)^{\frac{1}{2}}M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)\right]$$

$$\times\frac{i}{q^2-M^2+i0^+}$$

通常单粒子态的贡献是最主要的！ $\Rightarrow$ 单粒子为主 有效拉氏量

$$\times\left[(2\pi)^4\delta(q+q_{r+1}+\cdots+q_n-p_\alpha)(2\pi)^{\frac{3}{2}}(2q^0)^{\frac{1}{2}}M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)\right]$$

$+$多粒子中间态项 $+$ 非极点项



格林函数的极点

$$G_{\beta\alpha}(q_1,\cdots,q_n)=\int d^4x_1\cdots d^4x_n\, e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}(\Psi^-_\beta,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi^+_\alpha)$$

$$=i(2\pi)^7\delta^4(q_1+\cdots q_n+p_\beta-p_\alpha)(q^2-M^2+i0^+)^{-1}2q^0\sum_\sigma M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)$$

$$+\text{多粒子中间态项}+\text{非极点项}$$

$$M_{\beta|\vec{q},\sigma}(q_2,\cdots,q_r)=\int d^4y_2\cdots d^4y_r e^{iq_2\cdot y_2+\cdots+iq_r\cdot y_r}(\Psi^-_\beta,\mathbf{T}\{\psi_{H,l_1}(0)\psi_{H,l_2}(y_2)\cdots\psi_{H,l_r}(y_r)\}\Psi_{\vec{q},\sigma})$$

$$M_{-\vec{q},\sigma|\alpha}(q_{r+2},\cdots,q_n)=\int d^4y_{r+2}\cdots d^4y_n e^{iq_{r+2}\cdot y_{r+2}+\cdots+iq_n\cdot y_n}(\Psi_{\vec{q},\sigma},\mathbf{T}\{\psi_{H,l_{r+1}}(0)\psi_{H,l_{r+2}}(y_{r+2})\cdots\psi_{H,l_n}(y_n)\}\Psi^+_\alpha$$

$$\int d^4xd^4ye^{iq\cdot x+iq'\cdot y}(\Psi^-_0,\mathbf{T}\psi_{H,l_1}(x)\psi_{H,l_2}(y)\Psi^+_0)=\int d^4xd^4ye^{iq\cdot(x-y)+i(q+q')\cdot y}(\Psi^-_0,\mathbf{T}\psi_{H,l_1}(x)\psi_{H,l_2}(y)\Psi^+_0$$

$$=i(2\pi)^7\delta^4(q+q')\frac{2q^0\sum_\sigma(\Psi^-_0,\psi_{H,l_1}(0)\Psi_{\vec{q},\sigma})(\Psi_{\vec{q},\sigma},\psi_{H,l_2}(0)\Psi^+_0)}{q^2-M^2+i0^+}+\text{其它项}$$

$$(\Psi^-_0,\mathbf{T}\psi_{H,l_1}(x)\psi_{H,l_2}(y)\Psi^+_0)=\int\frac{d^4q}{2\pi}e^{-iq\cdot(x-y)}\frac{2iq^0\sum_\sigma(\Psi^-_0,\psi_{H,l_1}(0)\Psi_{\vec{q},\sigma})(\Psi_{\vec{q},\sigma},\psi_{H,l_2}(0)\Psi^+_0)}{q^2-M^2+i0^+}+\cdot$$

两点函数的谱函数: X:标量 $\psi(x)=e^{iP\cdot x}\psi(0)e^{-iP\cdot x}$ Ψ_0完全可替换为Ψ_α

$$(\Psi_0,X(x)X^\dagger(y)\Psi_0)=\sum_n(\Psi_0,X(x)\Psi_n)(\Psi_n,X^\dagger(y)\Psi_0)=\sum_n e^{-ip_n\cdot(x-y)}|(\Psi_0,X(0)\Psi_n)|^2$$

$$=\int d^4p\, e^{-ip\cdot(x-y)}\underbrace{\sum_n\delta^4(p-p_n)|(\Psi_0,X(0)\Psi_n)|^2}_{(2\pi)^{-3}\theta(p^0)\rho(p^2)\qquad p_n^0\geq 0}=\int_0^\infty d\mu^2\rho(\mu^2)\underbrace{\int\frac{d^4p}{(2\pi)^3}\,e^{-ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)}_{\Delta_+(x-y,\mu^2)}$$

$$(\Psi_0,X^\dagger(y)X(x)\Psi_0)=\sum_n(\Psi_0,X^\dagger(y)\Psi_n)(\Psi_n,X(x)\Psi_0)=\sum_n e^{ip_n\cdot(x-y)}|(\Psi_n,X(0)\Psi_0)|^2$$

$$=\int d^4p\, e^{ip\cdot(x-y)}\underbrace{\sum_n\delta^4(p-p_n)|(\Psi_n,X(0)\Psi_0)|^2}_{(2\pi)^{-3}\theta(p^0)\tilde\rho(p^2)\qquad p_n^0\geq 0}=\int_0^\infty d\mu^2\tilde\rho(\mu^2)\underbrace{\int\frac{d^4p}{(2\pi)^3}\,e^{ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)}_{\Delta_+(y-x,\mu^2)}$$

$$(\Psi_0,[X(x),X^\dagger(y)]\Psi_0)=\int_0^\infty d\mu^2[\rho(\mu^2)\Delta_+(x-y,\mu^2)-\tilde\rho(\mu^2)\Delta_+(y-x,\mu^2)]\xRightarrow{\text{类空；求导}}\rho(\mu^2)=\tilde\rho(\mu^2)$$

$$(\Psi_0,\mathbf{T}\,X(x)X^\dagger(y)\Psi_0)=\int_0^\infty d\mu^2\rho(\mu^2)\,\underbrace{[\theta(x^0-y^0)\Delta_+(x-y,\mu^2)+\theta(y^0-x^0)\Delta_+(y-x,\mu^2)]}_{\Delta_F(x-y,\mu^2)}$$

格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2)\Delta_F(x-y,\mu^2) \quad \Delta_+(x-y,\mu^2) \equiv \int \frac{d^4p}{(2\pi)^3} e^{-ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)$$

$$(\Psi_0, [X(x), X^\dagger(y)]\Psi_0) = \int_0^\infty d\mu^2\ \rho(\mu^2)[\Delta_+(x-y,\mu^2) - \Delta_+(y-x,\mu^2)]$$

$$\frac{\partial}{\partial x^0}\Delta_+(x-y,\mu^2)\Big|_{x^0=y^0} = \int \frac{d^4p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\theta(p^0)(-ip^0)\delta((p^0)^2 - \vec{p}^2 - \mu^2) = -\frac{i}{2}\delta(\vec{x}-\vec{y})$$

$$\frac{\partial}{\partial x^0}\Delta_+(y-x,\mu^2)\Big|_{x^0=y^0} = \int \frac{d^4p}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\theta(p^0)(ip^0)\delta((p^0)^2 - \vec{p}^2 - \mu^2) = \frac{i}{2}\delta(\vec{x}-\vec{y})$$

$$(\Psi_0, [\dot{X}(x), X^\dagger(y)]\Big|_{x^0=y^0}\Psi_0) = -i\delta(\vec{x}-\vec{y})\int_0^\infty d\mu^2\ \rho(\mu^2)$$

如果 $\dot{X}\dot{X}^\dagger = -\ddot{X}X^\dagger$ 是动能项，$\dot{X}$对应的广义动量就是 $-X^\dagger$。 $\dot{\phi} = \frac{\delta H}{\delta \pi}$

$$[\dot{X}(x), X^\dagger(y)]\Big|_{x^0=y^0} = -i\delta(\vec{x}-\vec{y}) \Rightarrow \int_0^\infty d\mu^2\ \rho(\mu^2) = 1$$

只对基本场成立！

格林函数的极点

$$\rho(p^2)\equiv(2\pi)^3\sum_n\delta^4(p-p_n)|(\Psi_0,X(0)\Psi_n)|^2=(2\pi)^3\sum_n\delta^4(p-p_n)|(\Psi_n,X(0)\Psi_0)|^2$$

$$(\Psi_0,\mathbf{T}\,X(x)X^\dagger(y)\Psi_0)=\int_0^\infty d\mu^2\rho(\mu^2)\Delta_F(x-y,\mu^2)\qquad \Delta_+(x-y,\mu^2)\equiv\int\frac{d^4p}{(2\pi)^3}e^{-ip\cdot(x-y)}\theta(p^0)\delta(p^2-\mu^2)$$

$$\Delta_F(x-y,\mu^2)\equiv[\theta(x^0-y^0)\Delta_+(x-y,\mu^2)+\theta(y^0-x^0)\Delta_+(y-x,\mu^2)]$$

$$=\int\frac{d^4p}{(2\pi)^3}\,\delta((p^0)^2-(\vec p)^2-\mu^2)\theta(p^0)[e^{-ip\cdot(x-y)}\theta(x^0-y^0)+e^{ip\cdot(x-y)}\theta(y^0-x^0)]$$

$$=\int\frac{d^4p}{2p^0(2\pi)^3}\,\delta(p^0-\sqrt{\vec p^2+\mu^2})\theta(p^0)[e^{-ip\cdot(x-y)}\theta(x^0-y^0)+e^{ip\cdot(x-y)}\theta(y^0-x^0)]$$

$$=\int\frac{d\vec p}{2\sqrt{(\vec p)^2+\mu^2}(2\pi)^3}\,e^{i\vec p\cdot(\vec x-\vec y)}[e^{-i\sqrt{\vec p^2+\mu^2}(x^0-y^0)}\theta(x^0-y^0)+e^{i\sqrt{\vec p^2+\mu^2}(x^0-y^0)}\theta(y^0-x^0)]$$

$$=\int\frac{d\vec p}{2\sqrt{(\vec p)^2+\mu^2}(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp^0}{2i\pi}\,e^{-ip^0(x^0-y^0)+i\vec p\cdot(\vec x-\vec y)}\left[-\frac{1}{p^0-\sqrt{\vec p^2+\mu^2}+i0^+}+\frac{1}{p^0+\sqrt{\vec p^2+\mu^2}-i0^+}\right]$$

$$=\int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\frac{i}{p^2-\mu^2+i0^+}\qquad (\Psi_0,\mathbf{T}\,X(x)X^\dagger(y)\Psi_0)=\int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\int_0^\infty d\mu^2\frac{i\rho(\mu^2)}{p^2-\mu^2+i0^+}$$

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, \Delta_X(p^2+i0^+) \qquad \Delta_X(z) = \int_0^\infty d\mu^2 \frac{i\rho(\mu^2)}{z-\mu^2}$$

若中间态谱中最低质量M的单粒子态$\Psi_{\vec p_\alpha}$ $(\Psi_0, X(0)\Psi_{\vec p_\alpha}) \neq 0$，其对谱函数贡献：

$$(2\pi)^3 \int d\vec p_\alpha \delta(p^0 - \sqrt{\vec p_\alpha^2 + M^2})\delta(\vec p - \vec p_\alpha)|(\Psi_0, X(0)\Psi_{\vec p_\alpha})|^2$$

$$= (2\pi)^3\delta(p^0 - \sqrt{\vec p^2 + M^2})|(\Psi_0, X(0)\Psi_{\vec p})|^2 = (2\pi)^3\theta(p^0)2p^0\delta((p^0)^2 - \vec p^2 - M^2)|(\Psi_0, X(0)\Psi_{\vec p})|^2$$

$$= Z_\alpha\delta(p^2 - M^2) \qquad Z_\alpha \equiv (2\pi)^3\theta(p^0)2p^0|(\Psi_0, X(0)\Psi_{\vec p})|^2 \geq 0$$

$$\rho(\mu^2) = Z_\alpha\delta(\mu^2 - M^2) + \sigma(\mu^2) \qquad 1 = Z_\alpha + \int_{M_0^2}^\infty d\mu^2\sigma(\mu^2)$$ $M_0 \geq M$为其它粒子或连续谱出现的下限 $Z_\alpha \leq 1$

$$\Delta_X(z) = \frac{iZ_\alpha}{z - M^2} + \int_{M_0^2}^\infty d\mu^2 \frac{i\rho(\mu^2)}{z-\mu^2}$$ 可能有割线！ $-i\Delta_X(M^2\pm 0^+) = \pm\infty \quad -i\Delta_X(M^2\pm i0^+) = \mp i\infty$

格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, \Delta_X(p^2+i0^+) \qquad \Delta_X(z) = \int_0^\infty d\mu^2\, \frac{i\rho(\mu^2)}{z-\mu^2}$$

对复数z，引入 $\Pi(z) \equiv z - i\Delta_X^{-1}(z)$ $\quad \Delta_X(z) = \frac{i}{z-\Pi(z)}$

$\Delta_X(z), \Pi(z)$ 在除正实轴外的 全复z平面 解析

在正实轴上下两边，对实$x \geq 0$

$$\frac{i}{y+i0^+} = \mathbf{P}\frac{i}{y} + \pi\delta(y) \qquad \frac{i}{y-i0^+} = \frac{-i}{-y+i0^+} = -\left[\mathbf{P}\frac{i}{-y} + \pi\delta(-y)\right] = \mathbf{P}\frac{i}{y} - \pi\delta(y)$$

$$\Delta_X(x+i0^+) - \Delta_X(x-i0^+) = \int_0^\infty d\mu^2\rho(\mu^2)\left[\frac{i}{x+i0^+-\mu^2} - \frac{i}{x-i0^+-\mu^2}\right] = 2\pi\rho(x)$$

$\Pi(x \pm i0^+) \equiv u(x) \mp iv(x)$ 若有单粒子态极点: $\Pi(M^2) = M^2 \Rightarrow u(M^2) = M^2 \quad v(M^2) = 0$

$$v(x) = \frac{i}{2}[\Pi(x+i0^+) - \Pi(x-i0^+)] = \frac{i}{2}[-i\Delta_X^{-1}(x+i0^+) + i\Delta_X^{-1}(x-i0^+)]$$

$$= \frac{\Delta_X(x-i0^+)-\Delta_X(x+i0^+)}{2\Delta_X(x+i0^+)\Delta_X(x-i0^+)} = \frac{-\pi\rho(x)}{\Delta_X(x+i0^+)\Delta_X(x-i0^+)} \xlongequal{\Delta_X^{-1}=i(\Pi-z)} \pi\rho(x)\left[[x-u(x)]^2+v^2(x)\right] \geq 0$$

格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n)|(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T}\, X(x)X^\dagger(y)\Psi_0) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, \Delta_X(p^2+i0^+) \qquad \Pi(x\pm i0^+) \equiv u(x) \mp iv(x) \qquad v(x) \geq 0$$

$$\Delta_X(z) = \frac{i}{z-\Pi(z)} = \int_0^\infty d\mu^2\, \frac{i\rho(\mu^2)}{z-\mu^2} \qquad \Delta_X(x+i0^+) - \Delta_X(x-i0^+) = 2\pi\rho(x)$$

定义复平面上的 $\Delta_X^{\mathrm{II}}(z) \equiv \frac{i}{z-u(z)+iv(z)}$ 若 $u(x\pm i0^+) = u(x),\ v(x\pm i0^+) = v(x)$

$\Delta_X^{\mathrm{II}}(x+i0^+) \xlongequal{\text{只在上缘上等，下缘不等}} \Delta_X(x+i0^+) \Rightarrow \Delta_X^{\mathrm{II}}(z)$可看成是$\Delta_X(z)$在第二叶黎曼面上的解析延拓

第二叶黎曼面上的极点z_0满足： $\underline{z_0 - u(z_0) + iv(z_0) = 0}$ 除$z_0 = M^2$外，可能有或无其它解！

若有解； $z_0 = re^{-i\theta} = (\tilde{M} - \frac{i}{2}\Gamma)^2$ 总可以这样选！ $\tilde{M} \equiv \sqrt{r}\cos\frac{\theta}{2},\quad \Gamma \equiv 2\sqrt{r}\sin\frac{\theta}{2} \geq 0$

$$\tilde{M}^2 - \frac{\Gamma^2}{4} - i\tilde{M}\Gamma - u(z_0) + iv(z_0) = 0 \qquad \tilde{M}^2 = \frac{\Gamma^2}{4} + \mathrm{Re}u(z_0) + \mathrm{Im}v(z_0) \qquad \tilde{M}\Gamma = \mathrm{Re}v(z_0) - \mathrm{Im}u(z_0)$$

$$\tilde{M}\Gamma \xRightarrow{\mathrm{Im}z_0 \ll \mathrm{Re}z_0} \mathrm{Re}v(\mathrm{Re}z_0) - \underbrace{\mathrm{Im}u(\mathrm{Re}z_0)}_{=0} + \underbrace{\mathrm{Re}[v'(\mathrm{Re}z_0)i\mathrm{Im}z_0]}_{=0} - \mathrm{Im}[u'(\mathrm{Re}z_0)i\mathrm{Im}z_0]$$

$$= v(\mathrm{Re}z_0) + u'(\mathrm{Re}z_0)\tilde{M}\Gamma \Rightarrow \tilde{M} = \frac{v(\mathrm{Re}z_0)}{\Gamma[1-u'(\mathrm{Re}z_0)]} \xRightarrow{\text{上页末：}v(x)\geq 0 \quad u'(\mathrm{Re}z_0)<1} \geq 0$$

格林函数的极点

$$\rho(p^2)\equiv(2\pi)^3\sum_n\delta^4(p-p_n)|(\Psi_0,X(0)\Psi_n)|^2=(2\pi)^3\sum_n\delta^4(p-p_n)|(\Psi_n,X(0)\Psi_0)|^2\geq 0$$

$$(\Psi_0,\mathbf{T}\,X(x)X^\dagger(y)\Psi_0)=\int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\,\Delta_X(p^2+i0^+)\qquad \Pi(x\pm i0^+)\equiv u(x)\mp iv(x)\qquad v(x)\geq 0$$

$$\Delta_X(z)=\frac{i}{z-\Pi(z)}=\int_0^\infty d\mu^2\,\frac{i\rho(\mu^2)}{z-\mu^2}\qquad \Delta_X(x+i0^+)-\Delta_X(x-i0^+)=2\pi\rho(x)$$

$\Delta_X^{\rm II}(z)\equiv\frac{i}{z-u(z)+iv(z)}$ 可看成是$\Delta_X(z)$ 在第二叶黎曼面上的解析延拓

第二叶黎曼面上的极点z_0满足： $z_0-u(z_0)+iv(z_0)=0$ 除$z_0=M^2$外，可能有或无其它解！

如果有解：可选 $z_0=(\tilde M-\frac{i}{2}\Gamma)^2$。 极点附近：

$$\Delta_X^{\rm II}(z)\equiv\frac{i}{z-u(z)+iv(z)}\approx\frac{i}{(z-z_0)[1-u'(z_0)+iv'(z_0)]}\approx\frac{i}{1-u'(z_0)+iv'(z_0)}\,\frac{1}{z-(\tilde M-\frac{i}{2}\Gamma)^2}$$ Breit-Winger公式

$$\Gamma\geq 0\quad \tilde M\Big|_{{\rm Im}z_0\ll{\rm Re}z_0,\ u'({\rm Re}z_0)<1}\geq 0\qquad e^{-iHt}\Rightarrow e^{-iMt}\Rightarrow e^{-i\tilde Mt-\frac{\Gamma}{2}t}\Rightarrow e^{-\Gamma t}$$ 在实轴上，极点被 共振峰 取代！

$$\left|\frac{i}{x-(\tilde M-\frac{i}{2}\Gamma)^2}\right|^2=\frac{1}{(x-\tilde M^2+\frac{\Gamma^2}{4})^2+\tilde M^2\Gamma^2}\xLeftarrow{\text{最高 } x=\tilde M^2}\frac{1}{\Gamma^2(\tilde M^2+\frac{\Gamma^2}{16})}\xLeftarrow{\text{半高 } x\simeq(\tilde M\pm\frac{\Gamma}{2})^2}\frac{1}{2\Gamma^2(\tilde M^2+\frac{\Gamma^2}{8}\pm\tilde M\frac{\Gamma}{2})}$$

关于格林函数对于动量变量的依赖关系:

♣ 极点位置是粒子的物理质量! 极点位于实轴! 实轴上还可能会有割线!

♢ 除实轴外，格林函数在首叶动量全平面 解析！

♡ 解析函数实部与虚部存在强烈的关联色散关系！

♠ 还存在其它动量叶，在其上可能会在非实轴上存在极点！ 导致动量依赖多值

¶ 第二叶上极点可表为 $\tilde{M}-i\Gamma/2$ 沿第一叶割线上方的积分通过围道可包括进其贡献

✠ Γ产生态演化几率的衰减 $e^{-\Gamma t}$；它是共振峰的 半宽度！

S矩阵与格林函数 $\psi_S(x) = e^{-iH_0 t}\psi(x)e^{iH_0 t}$ $\pi_S(x) = e^{-iH_0 t}\pi(x)e^{iH_0 t}$

$$\psi_H(\vec{x},t) \equiv \Omega(t)\psi(\vec{x},t)\Omega^\dagger(t) = e^{iHt}\psi_S(\vec{x},t)e^{-iHt} \qquad \Omega(t) = e^{iHt}e^{-iH_0 t}$$

$$(\partial_1^2 + M_1^2)\cdots(\partial_{n+l}^2 + M_{n+l}^2)(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_{n+l}}(x_{n+l})\Psi_0^+)$$

$$=\int \frac{d^4q_1\cdots d^4q_{n+l}}{(i\pi)^{n+l}} e^{-iq_1\cdot x_1\cdots-iq_n\cdot x_n+iq_{n+1}\cdot x_{n+1}\cdots+iq_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}} q_1^0(\Psi_0^-,\psi_{l_1}(0)\Psi^-_{\vec{q}_1,\sigma_1})\cdots q_n^0(\Psi_0^-,\psi_{l_n}(0)\Psi^-_{\vec{q}_n,\sigma_n})$$

$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots\vec{q}_n,\sigma_n},\Psi^+_{\vec{q}_{n+1},\sigma_{n+1},\cdots\vec{q}_{n+l},\sigma_{n+l}})q_{n+1}^0(\Psi^+_{\vec{q}_{n+1},\sigma_{n+1}},\psi_{l_{n+1}}(0)\Psi_0^+)\cdots q_{n+l}^0(\Psi^+_{\vec{q}_{n+l},\sigma_{n+l}},\psi_{l_{n+l}}(0)\Psi_0^+)$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) \xlongequal{t_1>\cdots>t_n} (\Psi_0^-,\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)$$

$$=(\Psi_0^-,e^{iHt_1}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iHt_n}\Psi_0^+)$$

$$=(\Psi_0^-,e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$$

需要研究:

$$(\Psi_0^-,e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$$

量子场

$$\psi_S(x)=e^{-iH_0t}\psi(x)e^{iH_0t} \quad \psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)+e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]$$

$$\dot{\psi}_S(x)=\dot{\pi}_S(x)=0 \quad [\psi_{S,l}(\vec{x},t),\psi_{S,l'}(\vec{y},t)]_\mp=[\pi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp=0 \quad [\psi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp=i\delta(\vec{x}-\vec{y})$$

$$(\Psi_0^-,e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$$

固定空间点量子场算符的本征态: 费米子对应grassman数

$$\psi_{S,l}(x)|\psi(\vec{x})\rangle=\psi_l(\vec{x})|\psi(\vec{x})\rangle \quad \int d\psi(\vec{x})\,|\psi(\vec{x})\rangle\langle\psi(\vec{x})|=I \quad \langle\psi(\vec{x})|\psi'(\vec{x})\rangle=\delta(\psi(\vec{x})-\psi'(\vec{x}))$$

$$\pi_{S,l}(x)|\pi(\vec{x})\rangle=\pi_l(\vec{x})|\pi(\vec{x})\rangle \quad \int d\pi(\vec{x})\,|\pi(\vec{x})\rangle\langle\pi(\vec{x})|=I \quad \langle\pi(\vec{x})|\pi'(\vec{x})\rangle=\delta(\pi(\vec{x})-\pi'(\vec{x}))$$

- 在有限维空间中,厄米算符的全部本征态矢量构成完备集
- 任意一个算符总可以分解为一个厄米算符和一个反厄米算符的叠加

所有空间点量子场算符本征态: $|\psi\rangle\equiv\prod_{\vec{x}}|\psi(\vec{x})\rangle$ $|\pi\rangle\equiv\prod_{\vec{x}}|\pi(\vec{x})\rangle$ 固定点场算符只作用对应点的态

$$\int\mathcal{D}_s\psi\,|\psi\rangle\langle\psi|=I \quad \langle\psi|\psi'\rangle=\delta(\psi-\psi')\equiv\prod_{\vec{x}}\delta(\psi(\vec{x})-\psi'(\vec{x})) \quad \mathcal{D}_s\psi\equiv\prod_{\vec{x}}d\psi(\vec{x})$$

Grassmann数⇔反对易数: $\theta\eta = -\eta\theta$

单变量: $$\theta^2 = 0 \qquad f(\theta) = A + B\theta \qquad \frac{\partial f(\theta)}{\partial\theta} = B \qquad \frac{\partial^2}{\partial\theta^2} = 0$$

$$\int d\theta\, f(\theta) = \int d\theta(A + B\theta) = A\int d\theta + B\int d\theta\;\theta$$

要求平移不变性: $$\int d\theta\, f(\theta) = \int d\theta\, f(\theta+\eta) = (A - B\eta)\int d\theta + B\int d\theta\;\theta \;\Rightarrow\; \eta\int d\theta = 0$$

$$\int d\theta = 0$$ 约定: $$\int d\theta\;\theta = 1 \;\Rightarrow\; \int d\theta\, f(\theta) = \frac{\partial f(\theta)}{\partial\theta} = B$$

$$1 = \int d\theta\;\delta(\theta - \eta) = \int d\theta\;\delta(\theta) \;\Rightarrow\; \delta(\theta) = \theta$$ 但是: $\delta(-\theta) = -\delta(\theta) \neq \delta(\theta)$

多变量: $\theta_i\theta_j + \theta_j\theta_i = 0$

$$\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j} + \frac{\partial}{\partial\theta_j}\frac{\partial}{\partial\theta_i} = 0$$ 左微商

$$f(\theta) = \sum_{k=0}^{n}\sum_{i_1,\cdots,i_k}^{n} C^{(k)}_{i_1,\cdots,i_k}\theta_{i_1}\theta_{i_k}\cdots\theta_{i_k} \qquad \delta(\theta) = \theta_1\cdots\theta_n$$

构造实标量场本征态:

$$\phi_S(x) = e^{-iH_0 t}\int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^\dagger(\vec{p})]e^{iH_0 t}$$

$$\phi_S(x)|\phi(\vec{x})\rangle = \phi(\vec{x})|\phi(\vec{x})\rangle \qquad \int d\phi(\vec{x})\ |\phi(\vec{x})\rangle\langle\phi(\vec{x})| = I \quad \langle\phi(\vec{x})|\phi'(\vec{x}')\rangle = \delta(\phi(\vec{x}) - \phi'(\vec{x}'))$$

$$|\phi(\vec{x})\rangle = \int d^3q\, f(\vec{q})e^{-iH_0 t}\exp\{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q}) - \tfrac{1}{2}\int d^3q' e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')\}\Phi_0$$

$$\phi_S(\vec{x})|\phi(\vec{x})\rangle = \int d^3q\, f(\vec{q})e^{-iH_0 t}\int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^\dagger(\vec{p})]$$

$$\times e^{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q}) - \frac{1}{2}\int d^3q' e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')}\Phi_0$$

$$= \int d^3q\, f(\vec{q})e^{-iH_0 t}\int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}e^{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q}) - \frac{1}{2}\int d^3q' e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')}$$

$$\times\left[e^{ip\cdot x}a^\dagger(\vec{p}) - [\{(2\pi)^{3/2}\sqrt{2q^0}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q}) - \frac{1}{2}\int d^3q' e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')\}, e^{-ip\cdot x}a(\vec{p})]\right]\Phi_0$$

$$= \int d^3q\, f(\vec{q})e^{-iH_0 t}\int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}e^{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q}) - \frac{1}{2}\int d^3q' e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')}$$

$$\times\left[e^{ip\cdot x}a^\dagger(\vec{p}) + e^{-ip\cdot x}\{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})\delta(q-p) - \int d^3q' e^{2iq'\cdot x}a^\dagger(\vec{q}')\delta(q'-p)\}\right]\Phi_0$$

构造实标量场本征态: $$\phi_S(x)=e^{-iH_0t}\int\frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p})+e^{ip\cdot x}a^\dagger(\vec{p})]e^{iH_0t}$$

$$\phi_S(x)|\phi(\vec{x})\rangle=\phi(\vec{x})|\phi(\vec{x})\rangle \qquad \int d\phi(\vec{x})\ |\phi(\vec{x})\rangle\langle\phi(\vec{x})|=I \quad \langle\phi(\vec{x})|\phi'(\vec{x}')\rangle=\delta(\phi(\vec{x})-\phi'(\vec{x}'))$$

$$|\phi(\vec{x})\rangle=\int d^3q\, f(\vec{q})e^{-iH_0t}\exp\{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q})-\tfrac{1}{2}\int d^3q'\, e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')\}\Phi_0$$

$$\phi_S(\vec{x})|\phi(\vec{x})\rangle=\int d^3q\, f(\vec{q})e^{-iH_0t}\int\frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}e^{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q})-\frac{1}{2}\int d^3q'\, e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')}$$

$$\times\left[e^{ip\cdot x}a^\dagger(\vec{p})+e^{-ip\cdot x}\{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})\delta(q-p)-\int d^3q'\, e^{2iq'\cdot x}a^\dagger(\vec{q}')\delta(q'-p)\}\right]\Phi_0$$

$$=\phi(\vec{x})\int d^3q\, f(\vec{q})e^{-iH_0t}e^{(2\pi)^{3/2}\sqrt{2q^0}e^{iq\cdot x}\phi(\vec{x})a^\dagger(\vec{q})-\frac{1}{2}\int d^3q'\, e^{2iq'\cdot x}a^{\dagger 2}(\vec{q}')}\Phi_0$$

研究性问题: 如何调节$f(\vec{q})$,使得

$$\int d\phi(\vec{x})\ |\phi(\vec{x})\rangle\langle\phi(\vec{x})|=I \qquad \langle\phi(\vec{x})|\phi'(\vec{x}')\rangle=\delta(\phi(\vec{x})-\phi'(\vec{x}'))$$

场及其动量

$$[\psi_{S,l}(\vec{x},t),\psi_{S,l'}(\vec{y},t)]_{\mp}=[\pi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_{\mp}=0 \qquad [\psi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_{\mp}=i\delta(\vec{x}-\vec{y})$$

$$\pi_{S,l}(x)e^{i\int d\vec{y}\xi(\vec{y})\psi_S(y)}|\pi\rangle=e^{i\int d\vec{y}\xi(\vec{y})\psi_S(y)}\{\pi_{S,l}(x)+(-i)\int d\vec{z}\xi(\vec{z})[\psi_S(z),\pi_{S,l}(x)]_{\mp}\}|\pi\rangle \xlongequal{e^A B e^{-A}=e^{Ad(A)}B}$$

$$=(\pi_l(\vec{x})+\xi_l(\vec{x}))e^{i\int d\vec{y}\xi(\vec{y})\psi_S(y)}|\pi\rangle\Rightarrow e^{i\int d\vec{y}\xi(\vec{y})\psi_S(y)}|\pi\rangle=|\pi+\xi\rangle\Rightarrow|\pi\rangle=e^{i\int d\vec{y}\pi(\vec{y})\psi_S(y)}|0\rangle_\pi$$

$$\psi_{S,l}(x)e^{-i\int d\vec{y}\zeta(\vec{y})\pi_S(y)}|\psi\rangle=e^{-i\int d\vec{y}\zeta(\vec{y})\pi_S(y)}\{\psi_{S,l}(x)+i\int d\vec{z}\zeta(\vec{z})[\pi_S(z),\psi_{S,l}(x)]_{\mp}\}|\psi\rangle \xlongequal{e^A B e^{-A}=e^{Ad(A)}B}$$

$$=(\psi_l(\vec{x})+\zeta_l(\vec{x}))e^{-i\int d\vec{y}\zeta(\vec{y})\pi_S(y)}|\psi\rangle\Rightarrow e^{-i\int d\vec{y}\zeta(\vec{y})\pi_S(y)}|\psi\rangle=|\psi+\zeta\rangle\Rightarrow|\psi\rangle=e^{-i\int d\vec{y}\psi(\vec{y})\pi_S(y)}|0\rangle_\psi$$

$$\begin{aligned}\langle\psi|\pi\rangle&=\langle\psi|e^{i\int d\vec{y}\pi(\vec{y})\psi_S(y)}|0\rangle_\pi=e^{i\int d\vec{y}\pi(\vec{y})\psi(\vec{y})}\langle\psi|0\rangle_\pi=e^{i\int d\vec{y}\pi(\vec{y})\psi(\vec{y})}\;{}_\psi\langle 0|e^{i\int d\vec{z}\psi(\vec{z})\pi_S(z)}|0\rangle_\pi\\&=e^{i\int d\vec{y}\pi(\vec{y})\psi(\vec{y})}\;{}_\psi\langle 0|0\rangle_\pi\end{aligned}$$

$$\delta(\pi'-\pi)=\int\mathcal{D}_s\psi\;\langle\pi'|\psi\rangle\langle\psi|\pi\rangle=\int\mathcal{D}_s\psi\;e^{i\int d\vec{y}[-\pi'(\vec{y})+\pi(\vec{y})]\psi(\vec{y})}|{}_\psi\langle 0|0\rangle_\pi|^2=\delta(\pi'-\pi)|{}_\psi\langle 0|0\rangle_\pi|^2\prod_{\vec{x}}(2\pi)$$

$$\langle\psi|\pi\rangle=e^{i\int d\vec{y}\;\pi(\vec{y})\psi(\vec{y})}/\sqrt{\prod_{\vec{x}}(2\pi)}$$ 如果是费米场π, ψ是Grassman变量

$$\pi_{S,l}(\vec{x},t)=-i\frac{\delta}{\delta\psi_{S,l}(\vec{x},t)}$$

泛函微商

$$[\psi_{s,l}(\vec{x},t),\psi_{s,l'}(\vec{y},t)]_{\mp}=[\pi_{s,l}(\vec{x},t),\pi_{s,l'}(\vec{y},t)]_{\mp}=0 \qquad [\psi_{s,l}(\vec{x},t),\pi_{s,l'}(\vec{y},t)]_{\mp}=i\delta(\vec{x}-\vec{y})$$

$$\langle\psi|\pi\rangle=e^{i\int d\vec{y}\ \pi(\vec{y})\psi(\vec{y})}/\prod_{\vec{x}}\sqrt{2\pi} \qquad \pi_{s,l}(\vec{x},t)=-i\frac{\delta}{\delta\psi_{s,l}(\vec{x},t)}$$

三维空间： $\frac{\delta}{\delta\psi(\vec{x},t)}\psi(\vec{y},t)=\delta(\vec{x}-\vec{y})$

$$\frac{\delta}{\delta\psi(\vec{x},t)}f[\psi(\vec{y},t)]\equiv\lim_{\Delta\to 0}\frac{f[\psi(\vec{y},t)+\delta(\vec{x}-\vec{y})\Delta]-f[\psi(\vec{y},t)]}{\Delta}=f'[\psi(\vec{y},t)]\delta(\vec{x}-\vec{y})$$

$$\frac{\delta}{\delta\psi(\vec{x},t)}\int d^3y\, f[\psi(\vec{y},t)]=\int d^3y\, f'[\psi(\vec{y},t)]\frac{\delta}{\delta\psi(\vec{x},t)}\psi(\vec{y},t)=f'[\psi(\vec{x},t)]$$

量子场 $\psi_S(x)=e^{-iH_0t}\psi(x)e^{iH_0t}$ $\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)+e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]$

$\dot{\psi}_S(x)=\dot{\pi}_S(x)=0$ $[\psi_{S,l}(\vec{x},t),\psi_{S,l'}(\vec{y},t)]_\mp=[\pi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp=0$ $[\psi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp=i\delta(\vec{x}-\vec{y})$

$(\Psi_0^-,e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$

所有空间点量子场算符本征态: $|\psi\rangle\equiv\prod_{\vec{x}}|\psi(\vec{x})\rangle$ $|\pi\rangle\equiv\prod_{\vec{x}}|\pi(\vec{x})\rangle$

$\psi_{S,l}(x)|\psi\rangle=\psi_l(\vec{x})|\psi\rangle$ $\int\mathcal{D}_s\psi\,|\psi\rangle\langle\psi|=I$ $\langle\psi|\psi'\rangle=\delta(\psi-\psi')$ $\mathcal{D}_s\psi\equiv\prod_{\vec{x}}d\psi(\vec{x})$

$\pi_{S,l}(x)|\pi\rangle=\pi_l(\vec{x})|\pi\rangle$ $\int\mathcal{D}_s\pi\,|\pi\rangle\langle\pi|=I$ $\langle\pi|\pi'\rangle=\delta(\pi-\pi')$ $\mathcal{D}_s\pi\equiv\prod_{\vec{x}}d\pi(\vec{x})$

$$(\Psi_0^-,e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$$

$$=\int\mathcal{D}_s\psi'\mathcal{D}_s\psi_1\cdots\mathcal{D}_s\psi_n\mathcal{D}_s\psi''(\Psi_0^-|\psi'\rangle\langle\psi'|e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)|\psi_1\rangle\langle\psi_1|e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)|\psi_2\rangle\langle\psi_2|\cdots$$

$$\cdots\psi_{S,l_{n-1}}(x_{n-1})|\psi_{n-1}\rangle\langle\psi_{n-1}|e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)|\psi_n\rangle\langle\psi_n|e^{-iH(t_n-t'')}|\psi''\rangle\langle\psi''|\Psi_0^+)$$

$$=\int\mathcal{D}_s\psi'\cdots\mathcal{D}_s\psi''\psi_{l_1}(\vec{x}_1)\psi_{l_2}(\vec{x}_2)\cdots\psi_{l_{n-1}}(\vec{x}_{n-1})\psi_{l_n}(\vec{x}_n)(\Psi_0^-|\psi'\rangle\langle\psi'|e^{-iH(t'-t_1)}|\psi_1\rangle\langle\psi_1|e^{-iH(t_1-t_2)}|\psi_2\rangle$$

$$\cdots\langle\psi_{n-1}|e^{-iH(t_{n-1}-t_n)}|\psi_n\rangle\langle\psi_n|e^{-iH(t_n-t'')}|\psi''\rangle\langle\psi''|\Psi_0^+)\Rightarrow$$ 需要计算 $\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle$

时间演化算符的矩阵元

量子场 $\psi_S(x)=e^{-iH_0t}\psi(x)e^{iH_0t}$ $\psi(x)=\sum_\sigma\int\frac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)+e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]$

$\dot{\psi}_S(x)=\dot{\pi}_S(x)=0$ $[\psi_{S,l}(\vec{x},t),\psi_{S,l'}(\vec{y},t)]_\mp=[\pi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp=0$ $[\psi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp=i\delta(\vec{x}-\vec{y})$

$(\Psi_0^-,e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$

所有空间点量子场算符本征态: $|\psi\rangle\equiv\prod_{\vec{x}}|\psi(\vec{x})\rangle$ $|\pi\rangle\equiv\prod_{\vec{x}}|\pi(\vec{x})\rangle$

$\psi_{S,l}(x)|\psi\rangle=\psi_l(\vec{x})|\psi\rangle$ $\int\mathcal{D}_s\psi\,|\psi\rangle\langle\psi|=I$ $\langle\psi|\psi'\rangle=\delta(\psi-\psi')$ $\mathcal{D}_s\psi\equiv\prod_{\vec{x}}d\psi(\vec{x})$

$\pi_{S,l}(x)|\pi\rangle=\pi_l(\vec{x})|\pi\rangle$ $\int\mathcal{D}_s\pi\,|\pi\rangle\langle\pi|=I$ $\langle\pi|\pi'\rangle=\delta(\pi-\pi')$ $\mathcal{D}_s\pi\equiv\prod_{\vec{x}}d\pi(\vec{x})$

需要计算 $\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle$

将t',t''中等间隔地分为$m+1$份，从左向右分别标记为$t_m,t_{m-1},\cdots,t_2,t_1$

m选择的足够大$\Delta t=t_{i+1}-t_i=(t'-t'')/(m+1)\xrightarrow{m\to\infty}0$

$$\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle=\int\mathcal{D}_s\psi_m\cdots\mathcal{D}_s\psi_1\,\langle\psi'|e^{-iH(t'-t_m)}|\psi_m\rangle\langle\psi_m|e^{-iH(t_m-t_{m-1})}|\psi_{m-1}\rangle\cdots$$

$$\cdots\langle\psi_2|e^{-iH(t_2-t_1)}|\psi_1\rangle\langle\psi_1|e^{-iH(t_1-t'')}|\psi''\rangle$$ $\Rightarrow$ 需要计算 $\langle\psi_{i+1}|e^{-iH\Delta t}|\psi_i\rangle$

无穷小演化算符： $e^{-iH\Delta t}$ $\Delta t=(t'-t'')/(m+1)\to 0$

$$\langle\psi|\pi\rangle=e^{i\int d\vec{y}\ \pi(\vec{y})\psi(\vec{y})}/\prod_{\vec{x}}\sqrt{2\pi}$$

$$\begin{aligned}
&\langle\psi_{i+1}|e^{-iH\Delta t}|\psi_i\rangle=\langle\psi_{i+1}|e^{-i\Delta tH(\psi_S,\pi_S)}|\psi_i\rangle\\
&=\delta(\psi_{i+1}-\psi_i)-i\Delta t\langle\psi_{i+1}|H(\psi_S,\pi_S)|\psi_i\rangle\\
&=\delta(\psi_{i+1}-\psi_i)-i\Delta t\int[\mathcal{D}_s\pi_{i+1}]\langle\psi_{i+1}|\pi_{i+1}\rangle\langle\pi_{i+1}|H(\psi_S,\pi_S)|\psi_i\rangle\\
&=\delta(\psi_{i+1}-\psi_i)-i\Delta t\int[\mathcal{D}_s\pi_{i+1}]H(\psi_i,\pi_{i+1})\langle\psi_{i+1}|\pi_{i+1}\rangle\langle\pi_{i+1}|\psi_i\rangle\\
&=\delta(\psi_{i+1}-\psi_i)-i\Delta t\int[\mathcal{D}_s\pi_{i+1}]H(\psi_i,\pi_{i+1})e^{i\int d\vec{y}\ \pi_{i+1}(\vec{y})[\psi_{i+1}(\vec{y})-\psi_i(\vec{y})]}/\prod_{\vec{x}}(2\pi)\\
&=\int[\mathcal{D}_s\pi_{i+1}][1-i\Delta tH(\psi_i,\pi_{i+1})]e^{i\int d\vec{y}\ \pi_{i+1}(\vec{y})[\psi_{i+1}(\vec{y})-\psi_i(\vec{y})]}/\prod_{\vec{x}}(2\pi)\\
&=\int[\mathcal{D}_s\pi_{i+1}]e^{i\int d\vec{y}\ \pi_{i+1}(\vec{y})[\psi_{i+1}(\vec{y})-\psi_i(\vec{y})]-i\Delta tH(\psi_i,\pi_{i+1})}/\prod_{\vec{x}}(2\pi)
\end{aligned}$$

无穷小演化算符： $e^{-iH\Delta t}$ $\Delta t = (t' - t'')/(m+1) \to 0$

$$\langle\psi_{i+1}|e^{-iH\Delta t}|\psi_i\rangle = \int [\mathcal{D}_s\pi_{i+1}] e^{i\int d\vec{y}\ \pi_{i+1}(\vec{y})[\psi_{i+1}(\vec{y})-\psi_i(\vec{y})]-i\Delta t H(\psi_i,\pi_{i+1})} / \prod_{\vec{x}}(2\pi)$$

$$\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle = \int \mathcal{D}_s\psi_m\cdots\mathcal{D}_s\psi_1\ \langle\psi'|e^{-iH(t'-t_m)}|\psi_m\rangle\langle\psi_m|e^{-iH(t_m-t_{m-1})}|\psi_{m-1}\rangle\cdots$$

$$\cdots\langle\psi_2|e^{-iH(t_2-t_1)}|\psi_1\rangle\langle\psi_1|e^{-iH(t_1-t'')}|\psi''\rangle$$

$\psi_{m+1} = \psi'$ $\psi_0 = \psi''$

$$= [\prod_{\vec{x}}(2\pi)]^{-(m+1)}\int[\prod_{i=1}^{m}\mathcal{D}_s\psi_i]\int[\prod_{i=0}^{m}\mathcal{D}_s\pi_{i+1}]$$

$$\times e^{\sum_{i=0}^{m}\{i\int d\vec{x}\ \pi_{i+1}(\vec{x})[\psi_{i+1}(\vec{x})-\psi_i(\vec{x})]-i\Delta t\ H(\psi_i,\pi_{i+1})]\}}$$

$\psi_i \Rightarrow \psi_{i+1}$

$H = \int d\vec{x}\,\mathcal{H}$

$$= \lim_{m\to\infty}[\prod_{\vec{x}}(2\pi)]^{-(m+1)}\int_{\psi''}^{\psi'}\mathcal{D}\psi\int\mathcal{D}\pi\ e^{i\int_{t''}^{t'}d^4x\ \{\pi(x)\dot{\psi}(x)-\mathcal{H}[\psi(x),\pi(x)]\}}$$

其中我们定义 $\psi_i(\vec{x}) \equiv \psi(\vec{x}, t_i)$，$\pi_i(\vec{x}) \equiv \pi(\vec{x}, t_i)$ $\int_{\psi''}^{\psi'}\mathcal{D}\psi = \int[\prod_{i=1}^{m}\mathcal{D}_s\psi_i]$

因此 $\psi_{i+1}(\vec{x}) - \psi_i(\vec{x}) = \psi(\vec{x}, t_{i+1}) - \psi(\vec{x}, t_i) = \dot{\psi}(\vec{x}, t_i)\Delta t$

时间演化算符的矩阵元

$$\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle = \lim_{n\to\infty}[\prod_{\vec{x}}(2\pi)]^{-(n+1)} \int_{\psi''}^{\psi'} \mathcal{D}\psi\mathcal{D}\pi\; e^{i\int_{t''}^{t'} d^4x\,\{\pi(x)\dot\psi(x)-\mathcal{H}[\psi(x),\pi(x)]\}}$$

双线性π: $\mathcal{H}[\psi(x),\pi(x)] = c_{lm}\pi_l(x)\pi_m(x) + \mathcal{V}[\psi(x)]$

$$\int \mathcal{D}\pi\; e^{i\int_{t''}^{t'} d^4x\,\{\pi^T(x)\dot\psi(x)-\mathcal{H}[\psi(x),\pi(x)]\}} = \int \mathcal{D}\pi\; e^{i\int_{t''}^{t'} d^4x\,\{\pi^T(x)\dot\psi(x)-\pi^T(x)C\pi(x)-\mathcal{V}[\psi(x)]\}}$$

$$= \int \mathcal{D}\pi\; e^{i\int_{t''}^{t'} d^4x\,\{-[\pi^T(x)-\frac{1}{2}\dot\psi^T(x)C^{-1}]C[\pi(x)-\frac{1}{2}C^{-1}\dot\psi(x)]+\frac{1}{4}\dot\psi^T(x)C^{-1}\dot\psi(x)-\mathcal{V}[\psi(x)]\}}$$

$$= Const \times e^{i\int_{t''}^{t'} d^4x\,\{\frac{1}{4}\dot\psi^T(x)C^{-1}\dot\psi(x)-\mathcal{V}[\psi(x)]\}} \qquad Const \equiv \int \mathcal{D}\pi\; e^{-i\int_{t''}^{t'} d^4x\,\pi^T(x)C\pi(x)}$$

将$\mathcal{H}[\psi(x),\pi(x)]$看成是一个经典场体系的哈密顿量密度

$$\dot\psi_l(x) = \frac{\partial\mathcal{H}[\psi(x),\pi(x)]}{\partial\pi_l(x)} = 2c_{lm}\pi_m(x) \qquad \pi(x) = \frac{1}{2}C^{-1}\dot\psi(x)$$

$$\mathcal{L}[\psi(x),\dot\psi(x)] = \pi(x)\dot\psi(x) - \mathcal{H}[\psi(x),\pi(x)] = \frac{1}{4}\dot\psi^T(x)C^{-1}\dot\psi(x) - \mathcal{V}[\psi(x)]$$

$$\int \mathcal{D}\pi\; e^{i\int_{t''}^{t'} d^4x\,\{\pi(x)\dot\psi(x)-\mathcal{H}[\psi(x),\pi(x)]\}} = Const \times e^{i\int_{t''}^{t'} d^4x\,\mathcal{L}[\psi(x),\dot\psi(x)]}$$

将$\mathcal{H}[\psi(x),\pi(x)]$看成是一个经典场体系的哈密顿量密度

$$\dot{\psi}_l(x) = \frac{\partial\mathcal{H}[\psi(x),\pi(x)]}{\partial\pi_l(x)} = 2c_{lm}\pi_m(x) \qquad \pi(x) = \frac{1}{2}C^{-1}\dot{\psi}(x)$$

$$\mathcal{L}[\psi(x),\dot{\psi}(x)] = \pi(x)\dot{\psi}(x) - \mathcal{H}[\psi(x),\pi(x)] = \frac{1}{4}\dot{\psi}^T(x)C^{-1}\dot{\psi}(x) - \mathcal{V}[\psi(x)]$$

$$\int \mathcal{D}\pi\ e^{i\int_{t''}^{t'} d^4x\ \{\pi(x)\dot{\psi}(x)-\mathcal{H}[\psi(x),\pi(x)]\}} = Const \times e^{i\int_{t''}^{t'} d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)]}$$

正规乘积$\xRightarrow{\text{算符}\to\text{路径积分}}$普通乘积

$$\mathcal{L}_0 = \begin{cases} \frac{1}{2}[(\partial_\mu\phi(x))^2 - M^2\phi^2(x)] & \text{实自由标量场} \\ \mathrm{Re}\ \overline{\psi}(x)(i\gamma^\mu\partial_\mu - M)\psi(x) & \text{自由旋量场} \\ -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M^2v^2(x) & \text{实自由矢量场} \end{cases}$$

时间演化算符的矩阵元

$$\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle = \lim_{m\to\infty}[\prod_{\vec{x}}(2\pi)]^{-(m+1)}\int_{\psi''}^{\psi'}\mathcal{D}\psi\mathcal{D}\pi\ e^{i\int_{t''}^{t'}d^4x\ \{\pi(x)\dot\psi(x)-\mathcal{H}[\psi(x),\pi(x)]\}}$$

将$\mathcal{H}[\psi(x),\pi(x)]$看成是一个经典场体系的哈密顿量密度

$$\int\mathcal{D}\pi\ e^{i\int_{t''}^{t'}d^4x\ \{\pi(x)\dot\psi(x)-\mathcal{H}[\psi(x),\pi(x)]\}} = Const\times e^{i\int_{t''}^{t'}d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)]}$$

$$\langle\psi'|e^{-iH(t'-t'')}|\psi''\rangle = const\times\int_{\psi''}^{\psi'}\mathcal{D}\psi\ e^{i\int_{t''}^{t'}d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)]}\qquad const = Const\times\lim_{m\to\infty}[\prod_{\vec{x}}(2\pi)]^{-(m+1)}$$

$$(\Psi_0^-, e^{-iH(t'-t_1)}\psi_{S,l_1}(x_1)e^{-iH(t_1-t_2)}\psi_{S,l_2}(x_2)\cdots\psi_{S,l_{n-1}}(x_{n-1})e^{-iH(t_{n-1}-t_n)}\psi_{S,l_n}(x_n)e^{-iH(t_n-t'')}\Psi_0^+)$$

$$=\int\mathcal{D}_s\psi'\cdots\mathcal{D}_s\psi''\psi_{l_1}(\vec{x}_1)\psi_{l_2}(\vec{x}_2)\cdots\psi_{l_{n-1}}(\vec{x}_{n-1})\psi_{l_n}(\vec{x}_n)(\Psi_0^-|\psi'\rangle\langle\psi'|e^{-iH(t'-t_1)}|\psi_1\rangle\langle\psi_1|e^{-iH(t_1-t_2)}|\psi_2\rangle$$

$$\cdots\langle\psi_{n-1}|e^{-iH(t_{n-1}-t_n)}|\psi_n\rangle\langle\psi_n|e^{-iH(t_n-t'')}|\psi''\rangle\langle\psi''|\Psi_0^+)$$

$$=\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\psi_{l_2}(x_2)\cdots\psi_{l_{n-1}}(x_{n-1})\psi_{l_n}(x_n)\ (\Psi_0^-|\psi'\rangle\ e^{i\int_{t''}^{t'}d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)]}\ \langle\psi''|\Psi_0^+)$$

$$\int\mathcal{D}\psi\equiv\int\mathcal{D}_s\psi'\mathcal{D}_s\psi''\int_{\psi''}^{\psi'}\mathcal{D}\psi$$

实标量场的波函数： 取$\psi_{H,i}=\psi_{H,l_i}$, $\psi_i=\psi_{l_i}$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)=\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{i\int_{t''}^{t'}d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)]}\ (\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+)$$

$$\psi(x)=\int\frac{d^3p}{\sqrt{2E(2\pi)^3}}[e^{-ip\cdot x}a(\vec{p})+e^{ip\cdot x}a^\dagger(\vec{p})]\qquad\int d^3x\ e^{-i\vec{p}\cdot\vec{x}}\psi(x)=\frac{1}{\sqrt{2E}}[e^{-iEt}a(\vec{p})+e^{iEt}a^\dagger(-\vec{p})]$$

$$\pi(x)=\dot{\psi}(x)=-i\int\frac{d^3p}{(2\pi)^3}\sqrt{\frac{E}{2}}[e^{-ip\cdot x}a(\vec{p})-e^{ip\cdot x}a^\dagger(\vec{p})]\qquad\int d^3x e^{-i\vec{p}\cdot\vec{x}}\pi(x)=-i\sqrt{\frac{E}{2}}[e^{-iEt}a(\vec{p})-e^{iEt}a^\dagger(-\vec{p})]$$

$$\int d^3x\ e^{-i\vec{p}\cdot\vec{x}+iEt}[\sqrt{\frac{E}{2}}\psi(\vec{x},t)+\frac{i}{\sqrt{2E}}\pi(\vec{x},t)]=a(\vec{p})\qquad a(\vec{p})\Phi_0=0$$

$$\Phi_0=\Omega^\dagger(\mp\infty)\Psi_0^\pm\quad\Omega(t)=e^{iHt}e^{-iH_0t}\quad\psi_S(x)=e^{-iH_0t}\psi(x)e^{iH_0t}\quad\pi_S(x)=e^{-iH_0t}\pi(x)e^{iH_0t}$$

$$\int d^3x\ e^{-i\vec{p}\cdot\vec{x}+iE\tau}[\sqrt{\frac{E}{2}}\psi(\vec{x},\tau)+\frac{i}{\sqrt{2E}}\pi(\vec{x},\tau)]\Omega^\dagger(\mp\infty)\Psi_0^\pm=0$$

$$\Rightarrow\int d^3x\ e^{-i\vec{p}\cdot\vec{x}+iE\tau}[\sqrt{\frac{E}{2}}\psi(\vec{x},\tau)+\frac{i}{\sqrt{2E}}\pi(\vec{x},\tau)]e^{iH_0\tau}\Psi_0^\pm\overset{\tau\to\mp\infty}{=\!=\!=\!=}0$$

$$\Rightarrow\int d^3x\ e^{-i\vec{p}\cdot\vec{x}+iE\tau}[\sqrt{\frac{E}{2}}\psi_S(\vec{x},\tau)+\frac{i}{\sqrt{2E}}\pi_S(\vec{x},\tau)]\Psi_0^\pm\overset{\tau\to\mp\infty}{=\!=\!=\!=}0$$

实标量场的波函数1

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = \int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{i\int_{t''}^{t'} d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)]}\ (\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+)$$

$$\Phi_0 = \Omega^\dagger(\mp\infty)\Psi_0^\pm \quad \Omega(t) = e^{iHt}e^{-iH_0t} \quad \psi_S(x) = e^{-iH_0t}\psi(x)e^{iH_0t} \quad \pi_S(x) = e^{-iH_0t}\pi(x)e^{iH_0t}$$

$$\int d^3x\, e^{-i\vec{p}\cdot\vec{x}+iE\tau}[\sqrt{\frac{E}{2}}\psi_S(\vec{x},\tau) + \frac{i}{\sqrt{2E}}\pi_S(\vec{x},\tau)]\Psi_0^\pm \xlongequal{\tau\to\mp\infty} 0$$

$$\int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[\sqrt{\frac{E}{2}}\psi_S(x) + \frac{i}{\sqrt{2E}}\pi_S(x)]\Psi_0^\pm = 0$$

$$[\psi_{S,l}(\vec{x},t), \psi_{S,l'}(\vec{y},t)] = [\pi_{S,l}(\vec{x},t), \pi_{S,l'}(\vec{y},t)] = 0 \quad [\psi_{S,l}(\vec{x},t), \pi_{S,l'}(\vec{y},t)] = i\delta(\vec{x}-\vec{y}) \quad \pi_{S,l}(\vec{x},t) = \frac{-i\delta}{\delta\psi_{S,l}(\vec{x},t)}$$

$$\psi_{S,l}(x)|\psi\rangle = \psi_l(\vec{x})|\psi\rangle \qquad \int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[\sqrt{\frac{E}{2}}\psi(\vec{x}) + \frac{1}{\sqrt{2E}}\frac{\delta}{\delta\psi(\vec{x})}]\langle\psi|\Psi_0^\pm) = 0$$

$$\langle\psi|\Psi_0^\pm) = \mathcal{N}e^{-\frac{1}{2}\int d^3x d^3y\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x})\psi(\vec{y})} \xRightarrow{\mathcal{E}(\vec{x},\vec{y})=\mathcal{E}(\vec{y},\vec{x})} \int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[E\psi(\vec{x}) - \int d^3y\, \mathcal{E}(\vec{x},\vec{y})\psi(\vec{y})] = 0$$

$$e^{-i\vec{p}\cdot\vec{x}}E = \int d^3y\, e^{-i\vec{p}\cdot\vec{y}}\mathcal{E}(\vec{y},\vec{x}) \xRightarrow{E=\sqrt{\vec{P}^2+M^2}} \mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\sqrt{\vec{p}^2+M^2}$$

实标量场的波函数2

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = \int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{i\int_{t''}^{t'} d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)]}\ (\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+)$$

$$\langle\psi|\Psi_0^\pm) = \mathcal{N}e^{-\frac{1}{2}\int d^3x d^3y\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x})\psi(\vec{y})} \xRightarrow{\mathcal{E}(\vec{x},\vec{y})=\mathcal{E}(\vec{y},\vec{x})} \mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\sqrt{\vec{p}^2+M^2}$$

$$t'' = -\infty,\quad \psi''(\vec{x}) \equiv \psi(\vec{x},-\infty) \qquad t' = \infty,\quad \psi'(\vec{x}) \equiv \psi(\vec{x},\infty)$$

$$(\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+) = |\mathcal{N}|^2 e^{-\frac{1}{2}\int d^3x d^3y\ \mathcal{E}(\vec{x},\vec{y})[\psi(\vec{x},\infty)\psi(\vec{y},\infty)+\psi(\vec{x},-\infty)\psi(\vec{y},-\infty)]}$$

$$= \lim_{\epsilon\to 0+} |\mathcal{N}|^2 e^{-\frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x},\tau)\psi(\vec{y},\tau)e^{-\epsilon|\tau|}}$$

$$f(\infty)+f(-\infty) = \int_{-\infty}^{\infty} d[\frac{\tau}{|\tau|}f(\tau)]\ \lim_{\epsilon\to 0+} e^{-\epsilon|\tau|} = \lim_{\epsilon\to 0+}\int_{-\infty}^{\infty}[d\{\frac{\tau}{|\tau|}f(\tau)\ e^{-\epsilon|\tau|}\} - \frac{\tau}{|\tau|}f(\tau)de^{-\epsilon|\tau|}]$$

$$= \lim_{\epsilon\to 0+} \epsilon\int_{-\infty}^{\infty}\frac{\tau}{|\tau|}f(\tau)e^{-\epsilon|\tau|}d|\tau| = \lim_{\epsilon\to 0+}\epsilon\int_{-\infty}^{\infty} d\tau\, f(\tau)e^{-\epsilon|\tau|}$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = |\mathcal{N}|^2\lim_{\epsilon\to 0+}\int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)$$

$$\times \exp\left\{i\int d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)] - \frac{\epsilon}{2}\int d^3x d^3y\int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x},\tau)\psi(\vec{y},\tau)e^{-\epsilon|\tau|}\right\}$$

实标量场的波函数3

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+} \int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)$$
$$\times \exp\left\{ i\int d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)] - \frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x},\tau)\psi(\vec{y},\tau)e^{-\epsilon|\tau|}\right\}$$

$$(\Psi_0^-,\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+}\int \mathcal{D}\psi \exp\left\{ i\int d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)] - \frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x},\tau)\psi(\vec{y},\tau)e^{-\epsilon|\tau|}\right\}$$

$$\mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\sqrt{\vec{p}^2+M^2}$$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_H(x_1)\cdots\psi_H(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int \mathcal{D}\psi\ \psi(x_1)\cdots\psi(x_n)\ e^{iS[\psi]}}{\int \mathcal{D}\psi\ e^{iS[\psi]}}$$

$$S[\psi] \equiv \int d^4x\ \ \mathcal{L}[\psi(x),\dot{\psi}(x)] + \lim_{\epsilon\to 0+}\frac{i\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi(\vec{x},\tau)\psi(\vec{y},\tau)\ e^{-\epsilon|\tau|}$$

波函数

旋量场的波函数： 取$\psi_{H,i}=\psi_{H,l_i}$，$\psi_i=\psi_{l_i}$

$$\psi(x)=(2\pi)^{-\frac{3}{2}}\sum_\sigma\int d\vec{p}\,[u(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma)+v(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)]\quad \pi(x)=i\psi^*(x)$$

$$(p_\mu\gamma^\mu-M)u(\vec{p},\sigma)=0\;\Rightarrow\;\gamma^0\frac{\vec{p}\cdot\vec{\gamma}+M}{E}u(\vec{p},\sigma)=u(\vec{p},\sigma)$$

$$(p_\mu\gamma^\mu+M)v(\vec{p},\sigma)=0\;\Rightarrow\;\gamma^0\frac{\vec{p}\cdot\vec{\gamma}+M}{E}v(-\vec{p},\sigma)=-v(-\vec{p},\sigma)$$

$$\int d^3x\,e^{-i\vec{p}\cdot\vec{x}}\gamma^0\frac{\vec{p}\cdot\vec{\gamma}+M}{E}\psi(x)=(2\pi)^{\frac{3}{2}}\sum_\sigma[e^{-iEt}u(\vec{p},\sigma)a(\vec{p},\sigma)-e^{iEt}v(-\vec{p},\sigma)a^{c\dagger}(-\vec{p},\sigma)]$$

$$i\int d^3x\,e^{-i\vec{p}\cdot\vec{x}}\pi^*(x)=\int d^3x\,e^{-i\vec{p}\cdot\vec{x}}\psi(x)=(2\pi)^{\frac{3}{2}}\sum_\sigma[e^{-iEt}u(\vec{p},\sigma)a(\vec{p},\sigma)+e^{iEt}v(-\vec{p},\sigma)a^{c\dagger}(-\vec{p},\sigma)]$$

$\Phi_0=\Omega^\dagger(\mp\infty)\Psi_0^\pm$ $\quad\Omega(t)=e^{iHt}e^{-iH_0t}$ $\quad\psi_S(x)=e^{-iH_0t}\psi(x)e^{iH_0t}$ $\quad\pi_S(x)=e^{-iH_0t}\pi(x)e^{iH_0t}$

$$\int d^3x\,e^{-i\vec{p}\cdot\vec{x}+iE\tau}[\gamma^0\frac{\vec{p}\cdot\vec{\gamma}+M}{E}\psi(\vec{x},\tau)+i\pi^*(\vec{x},\tau)]\Omega^\dagger(\mp\infty)\Psi_0^\pm=0$$

$$\Rightarrow\;\int d^3x\,e^{-i\vec{p}\cdot\vec{x}+iE\tau}[\gamma^0\frac{\vec{p}\cdot\vec{\gamma}+M}{E}\psi_S(\vec{x},\tau)+i\pi_S^*(\vec{x},\tau)]\Psi_0^\pm\overset{\tau\to\mp\infty}{=\!=\!=\!=}0$$

旋量场的波函数1

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = \int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{i\int_{t''}^{t'} d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)]}\ (\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+)$$

$$\Phi_0 = \Omega^\dagger(\mp\infty)\Psi_0^\pm \quad \Omega(t) = e^{iHt}e^{-iH_0t} \quad \psi_S(x) = e^{-iH_0t}\psi(x)e^{iH_0t} \quad \pi_S(x) = e^{-iH_0t}\pi(x)e^{iH_0t}$$

$$\int d^3x\ e^{-i\vec p\cdot\vec x+iE\tau}[\gamma^0\frac{\vec p\cdot\vec\gamma+M}{E}\psi_S(\vec x,\tau) + i\pi_S^*(\vec x,\tau)]\Psi_0^\pm \xlongequal{\tau\to\mp\infty} 0$$

$$\int d^3x\ e^{-i\vec p\cdot\vec x}[\gamma^0\frac{\vec p\cdot\vec\gamma+M}{E}\psi_S(x) + i\pi_S^*(x)]\Psi_0^\pm = 0$$

$$\{\psi_{S,l}(\vec x,t),\psi_{S,l'}(\vec y,t)\} = \{\pi_{S,l}(\vec x,t),\pi_{S,l'}(\vec y,t)\} = 0 \quad \{\psi_{S,l}(\vec x,t),\pi_{S,l'}(\vec y,t)\} = i\delta(\vec x-\vec y) \quad \pi_{S,l}(\vec x,t) = i\frac{\delta}{\delta\psi_l(\vec x,t)}$$

$$\psi_{S,l}(x)|\psi\rangle = \psi_l(\vec x)|\psi\rangle \qquad \int d^3x\ e^{-i\vec p\cdot\vec x}[\gamma^0\frac{\vec p\cdot\vec\gamma+M}{E}\psi(\vec x) + \frac{\delta}{\delta\psi^*(\vec x)}]\langle\psi|\Psi_0^\pm) = 0$$

$$\langle\psi|\Psi_0^\pm) = \mathcal{N}e^{\int d^3x d^3y\ \overline{\psi}(\vec x)\mathcal{E}(\vec x,\vec y)\psi(\vec y)} \Rightarrow \int d^3x\ e^{-i\vec p\cdot\vec x}\gamma^0[\frac{\vec p\cdot\vec\gamma+M}{E}\psi(\vec x) + \int d^3y\ \mathcal{E}(\vec x,\vec y)\psi(\vec y)] = 0$$

$$e^{-i\vec p\cdot\vec x}\frac{\vec p\cdot\vec\gamma+M}{E} = -\int d^3y\ e^{-i\vec p\cdot\vec y}\mathcal{E}(\vec y,\vec x) \xRightarrow{E=\sqrt{\vec p^2+M^2}} \mathcal{E}(\vec x,\vec y) = \int\frac{d^3p}{(2\pi)^3}\ e^{i\vec p\cdot(\vec x-\vec y)}\frac{-\vec p\cdot\vec\gamma-M}{\sqrt{\vec p^2+M^2}}$$

旋量场的波函数2

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = \int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{i\int_{t''}^{t'} d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)]}\ (\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+)$$

$$\langle\psi|\Psi_0^\pm) = \mathcal{N} e^{\int d^3x d^3y\ \overline{\psi}(\vec{x})\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y})} \Rightarrow \mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\ \frac{-\vec{p}\cdot\vec{\gamma}-M}{\sqrt{\vec{p}^2+M^2}}$$

$$t'' = -\infty,\quad \psi''(\vec{x}) \equiv \psi(\vec{x},-\infty) \qquad t' = \infty,\quad \psi'(\vec{x}) \equiv \psi(\vec{x},\infty)$$

$$(\Psi_0^-|\psi'\rangle\langle\psi''|\Psi_0^+) = |\mathcal{N}|^2 e^{\int d^3x d^3y\ [\overline{\psi}(\vec{x},\infty)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},\infty)+\overline{\psi}(\vec{x},-\infty)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},-\infty)]}$$

$$= \lim_{\epsilon\to 0+} |\mathcal{N}|^2 e^{\epsilon\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},\tau)e^{-\epsilon|\tau|}}$$

$$f(\infty)+f(-\infty) = \int_{-\infty}^{\infty} d[\frac{\tau}{|\tau|}f(\tau)]\ \lim_{\epsilon\to 0+} e^{-\epsilon|\tau|} = \lim_{\epsilon\to 0+}\int_{-\infty}^{\infty}[d\{\frac{\tau}{|\tau|}f(\tau)\ e^{-\epsilon|\tau|}\} - \frac{\tau}{|\tau|}f(\tau)de^{-\epsilon|\tau|}]$$

$$= \lim_{\epsilon\to 0+} \epsilon\int_{-\infty}^{\infty} \frac{\tau}{|\tau|}f(\tau)e^{-\epsilon|\tau|}d|\tau| = \lim_{\epsilon\to 0+} \epsilon\int_{-\infty}^{\infty} d\tau\, f(\tau)e^{-\epsilon|\tau|}$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+}\int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)$$

$$\times \exp\left\{ i\int d^4x\ \mathcal{L}[\psi(x),\dot{\psi}(x)] + \epsilon\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},\tau)e^{-\epsilon|\tau|} \right\}$$

旋量场的波函数3

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+}\int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)$$
$$\times\exp\left\{i\int d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)] + \epsilon\int d^3x d^3y\int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},\tau)e^{-\epsilon|\tau|}\right\}$$

$$(\Psi_0^-,\Psi_0^+) = |\mathcal{N}|^2\lim_{\epsilon\to 0+}\int\mathcal{D}\psi\exp\left\{i\int d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)] + \epsilon\int d^3x d^3y\int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},\tau)e^{-\epsilon|\tau|}\right\}$$

$$\mathcal{E}(\vec{x},\vec{y}) = \int\frac{d^3p}{(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\frac{-\vec{p}\cdot\vec{\gamma}-M}{\sqrt{\vec{p}^2+M^2}}$$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1)\cdots\psi_{H,n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int\mathcal{D}\psi\ \psi(x_1)\cdots\psi(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}}$$

$$S[\psi] \equiv \int d^4x\ \ \mathcal{L}[\psi(x),\dot\psi(x)] - \lim_{\epsilon\to 0+} i\epsilon\int d^3x d^3y\int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{y})\psi(\vec{y},\tau)\ e^{-\epsilon|\tau|}$$

实无质量矢量场的波函数：取$\psi_{H,i} = v_{H,\mu_i}$，$\psi_i = v_{\mu_i}$

$$(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+) = \int \mathcal{D}v \, v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)\, e^{i\int_{t''}^{t'} d^4x\, \mathcal{L}[v(x),\dot{v}(x)]}\, (\Psi_0^-|v'\rangle\langle v''|\Psi_0^+)$$

$$v^\mu(x) = \sum_{\sigma=\pm 1}\int \frac{d^3p}{\sqrt{2E}(2\pi)^{3/2}}[e^{-ip\cdot x}e^\mu(\vec{p},\sigma)a(\vec{p},\sigma) + e^{ip\cdot x}e^{\mu *}(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)] \quad e^0(\vec{p},\sigma) = 0$$

$$\int d^3x\, e^{-i\vec{p}\cdot\vec{x}}v^\mu(x) = \frac{(2\pi)^{3/2}}{\sqrt{2E}}\sum_{\sigma=\pm 1}[e^{-iEt}e^\mu(\vec{p},\sigma)a(\vec{p},\sigma) + e^{iEt}e^{\mu *}(-\vec{p},\sigma)a^\dagger(-\vec{p},\sigma)]$$

$$\pi^\mu(x) = \dot{v}^\mu(x) = -i\sum_{\sigma=\pm 1}\int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{E}{2}}[e^{-ip\cdot x}e^\mu(\vec{p},\sigma)a(\vec{p},\sigma) - e^{ip\cdot x}e^{\mu *}(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)] \quad \pi^0 = 0$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x}}\pi^\mu(x) = -i(2\pi)^{3/2}\sqrt{\frac{E}{2}}\sum_{\sigma=\pm 1}[e^{-iEt}e^\mu(\vec{p},\sigma)a(\vec{p},\sigma) - e^{iEt}e^{\mu *}(-\vec{p},\sigma)a^\dagger(-\vec{p},\sigma)]$$

$$(2\pi)^{-3/2}\int d^3x\, e^{-i\vec{p}\cdot\vec{x}+iEt}[\sqrt{\frac{E}{2}}v^\mu(\vec{x},t) + \frac{i}{\sqrt{2E}}\pi^\mu(\vec{x},t)] = \sum_{\sigma=\pm 1} e^\mu(\vec{p},\sigma)a(\vec{p},\sigma) \qquad a(\vec{p},\sigma)\Phi_0 = 0$$

实无质量矢量场的波函数1：取$\psi_{H,i}=v_{H,\mu_i}$，$\psi_i=v_{\mu_i}$

$$(\Psi_0^-,\mathbf{T}v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+)=\int\mathcal{D}v\;v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)\;e^{i\int_{t''}^{t'}d^4x\,\mathcal{L}[v(x),\dot v(x)]}\;(\Psi_0^-|v'\rangle\langle v''|\Psi_0^+)$$

$$(2\pi)^{-3/2}\int d^3x\,e^{-i\vec p\cdot\vec x+iEt}[\sqrt{\frac{E}{2}}v^\mu(\vec x,t)+\frac{i}{\sqrt{2E}}\pi^\mu(\vec x,t)]=\sum_{\sigma=\pm1}e^\mu(\vec p,\sigma)a(\vec p,\sigma)\qquad a(\vec p,\sigma)\Phi_0=0$$

$$\Phi_0=\Omega^\dagger(\mp\infty)\Psi_0^\pm\quad\Omega(t)=e^{iHt}e^{-iH_0t}\quad v_S^\nu(x)=e^{-iH_0t}v^\mu(x)e^{iH_0t}\quad\pi_S^i(x)=e^{-iH_0t}\pi^i(x)e^{iH_0t}$$

$$\int d^3x\,e^{-i\vec p\cdot\vec x+iE\tau}[\sqrt{\frac{E}{2}}v^\mu(\vec x,\tau)+\frac{i}{\sqrt{2E}}\pi^\mu(\vec x,\tau)]\Omega^\dagger(\mp\infty)\Psi_0^\pm=0$$

$$\Rightarrow\int d^3x\,e^{-i\vec p\cdot\vec x+iE\tau}[\sqrt{\frac{E}{2}}v^\mu(\vec x,\tau)+\frac{i}{\sqrt{2E}}\pi^\mu(\vec x,\tau)]e^{iH_0\tau}\Psi_0^\pm\overset{\tau\to\mp\infty}{=\!=\!=\!=}0$$

$$\Rightarrow\int d^3x\,e^{-i\vec p\cdot\vec x+iE\tau}[\sqrt{\frac{E}{2}}v_S^\mu(\vec x,\tau)+\frac{i}{\sqrt{2E}}\pi_S^\mu(\vec x,\tau)]\Psi_0^\pm\overset{\tau\to\mp\infty}{=\!=\!=\!=}0$$

实无质量矢量场的波函数2

$$(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+) = \int \mathcal{D}v\; v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)\; e^{i\int_{t''}^{t'} d^4x\, \mathcal{L}[v(x),\dot v(x)]}\; (\Psi_0^-|v'\rangle\langle v''|\Psi_0^+)$$

$$\Phi_0 = \Omega^\dagger(\mp\infty)\Psi_0^\pm \qquad \Omega(t) = e^{iHt}e^{-iH_0t} \qquad v_S(x) = e^{-iH_0t}v(x)e^{iH_0t} \qquad \pi_S(x) = e^{-iH_0t}\pi(x)e^{iH_0t}$$

$$\int d^3x\, e^{-i\vec p\cdot\vec x + iE\tau}[\sqrt{\frac{E}{2}}v_S^\mu(\vec x,\tau) + \frac{i}{\sqrt{2E}}\pi_S^\mu(\vec x,\tau)]\Psi_0^\pm \xlongequal{\tau\to\mp\infty} 0$$

$$\int d^3x\, e^{-i\vec p\cdot\vec x}[\sqrt{\frac{E}{2}}v_S^\mu(x) + \frac{i}{\sqrt{2E}}\pi_S^\mu(x)]\Psi_0^\pm = 0$$

$$[v_S^i(\vec x,t), v_S^j(\vec y,t)] = [\pi_S^i(\vec x,t), \pi_S^j(\vec y,t)] = 0 \quad [\psi_S^i(\vec x,t), \pi_S^j(\vec y,t)] = i\delta^{ij}\delta(\vec x-\vec y) \quad \pi_S^i(\vec x,t) = -i\frac{\delta}{\delta v^i(\vec x,t)}$$

$$v_S^\mu(x)|v\rangle = v^\mu(\vec x)|v\rangle \qquad v^0(\vec x) = 0 \qquad \int d^3x\, e^{-i\vec p\cdot\vec x}[\sqrt{\frac{E}{2}}v^i(\vec x) + \frac{1}{\sqrt{2E}}\frac{\delta}{\delta v^i(\vec x)}]\langle v|\Psi_0^\pm) = 0$$

$$\langle v|\Psi_0^\pm) = \mathcal{N}e^{\frac12\int d^3x d^3y\, \mathcal{E}(\vec x,\vec y)v_\mu(\vec x)v^\mu(\vec y)} \xLongrightarrow{\mathcal{E}(\vec x,\vec y)=\mathcal{E}(\vec y,\vec x)} \int d^3x\, e^{-i\vec p\cdot\vec x}[Ev^\mu(\vec x) - \int d^3y\, \mathcal{E}(\vec x,\vec y)v^\mu(\vec y)] = 0$$

$$e^{-i\vec p\cdot\vec x}E = \int d^3y\, e^{-i\vec p\cdot\vec y}\mathcal{E}(\vec y,\vec x) \xLongrightarrow{E=\sqrt{\vec p^2}} \mathcal{E}(\vec x,\vec y) = \int\frac{d^3p}{(2\pi)^3}\, e^{i\vec p\cdot(\vec x-\vec y)}\sqrt{\vec p^2}$$

实无质量矢量场的波函数3

$$(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+) = \int \mathcal{D}v\ v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)\ e^{i\int_{t''}^{t'} d^4x\ \mathcal{L}[v(x),\dot{v}(x)]}\ (\Psi_0^-|v'\rangle\langle v''|\Psi_0^+)$$

$$\langle v|\Psi_0^\pm\rangle = \mathcal{N} e^{\frac{1}{2}\int d^3x d^3y\ \mathcal{E}(\vec{x},\vec{y}) v_\mu(\vec{x}) v^\mu(\vec{y})} \xRightarrow{\mathcal{E}(\vec{x},\vec{y})=\mathcal{E}(\vec{y},\vec{x})} \mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\sqrt{\vec{p}^2}$$

$$t'' = -\infty,\quad v''(\vec{x}) \equiv v(\vec{x},-\infty) \qquad t' = \infty,\quad v'(\vec{x}) \equiv v(\vec{x},\infty)$$

$$(\Psi_0^-|v'\rangle\langle v''|\Psi_0^+) = |\mathcal{N}|^2 e^{\frac{1}{2}\int d^3x d^3y\ \mathcal{E}(\vec{x},\vec{y})[v_\mu(\vec{x},\infty)v^\mu(\vec{y},\infty)+v_\mu(\vec{x},-\infty)v^\mu(\vec{y},-\infty)]}$$

$$= \lim_{\epsilon\to 0+} |\mathcal{N}|^2 e^{\frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y}) v_\mu(\vec{x},\tau) v^\mu(\vec{y},\tau) e^{-\epsilon|\tau|}}$$

$$f(\infty)+f(-\infty) = \int_{-\infty}^{\infty} d[\frac{\tau}{|\tau|}f(\tau)] \lim_{\epsilon\to 0+} e^{-\epsilon|\tau|} = \lim_{\epsilon\to 0+}\int_{-\infty}^{\infty}[d\{\frac{\tau}{|\tau|}f(\tau)\ e^{-\epsilon|\tau|}\} - \frac{\tau}{|\tau|}f(\tau) d e^{-\epsilon|\tau|}]$$

$$= \lim_{\epsilon\to 0+} \epsilon \int_{-\infty}^{\infty} \frac{\tau}{|\tau|} f(\tau) e^{-\epsilon|\tau|} d|\tau| = \lim_{\epsilon\to 0+} \epsilon \int_{-\infty}^{\infty} d\tau\, f(\tau) e^{-\epsilon|\tau|}$$

$$(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+}\int \mathcal{D}v\ v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)$$

$$\times \exp\left\{ i\int d^4x\ \mathcal{L}[v(x),\dot{v}(x)] + \frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y}) v_\mu(\vec{x},\tau) v^\mu(\vec{y},\tau) e^{-\epsilon|\tau|} \right\}$$

实无质量矢量场的波函数4

$$(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+} \int \mathcal{D}v\, v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)$$

$$\times \exp\left\{ i\int d^4x\, \mathcal{L}[v(x),\dot{v}(x)] + \frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\, \mathcal{E}(\vec{x},\vec{y}) v_\mu(\vec{x},\tau) v^\mu(\vec{y},\tau) e^{-\epsilon|\tau|} \right\}$$

$$(\Psi_0^-,\Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon\to 0+} \int \mathcal{D}v \exp\left\{ i\int d^4x\, \mathcal{L}[v(x),\dot{v}(x)] + \frac{\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\, \mathcal{E}(\vec{x},\vec{y}) v_\mu(\vec{x},\tau) v^\mu(\vec{y},\tau) e^{-\epsilon|\tau|} \right\}$$

$$\mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \sqrt{\vec{p}^2}$$

$$\frac{(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1)\cdots v_{H,\mu_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int \mathcal{D}v\, v_{\mu_1}(x_1)\cdots v_{\mu_n}(x_n)\, e^{iS[v]}}{\int \mathcal{D}v\, e^{iS[v]}}$$

$$S[v] \equiv \int d^4x\;\; \mathcal{L}[v(x),\dot{v}(x)] - \lim_{\epsilon\to 0+} \frac{i\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\, \mathcal{E}(\vec{x},\vec{y}) v_\mu(\vec{x},\tau) v^\mu(\vec{y},\tau)\; e^{-\epsilon|\tau|}$$

关于格林函数的路径积分表达关系:

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{l_1,H}(x_1)\cdots\psi_{l_n,H}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int \mathcal{D}\psi\ e^{iS[\psi]}}$$

♣ 量子场的格林函数可以用泛函平均来表达！

♡ 平均的权重是体系的经典作用量！

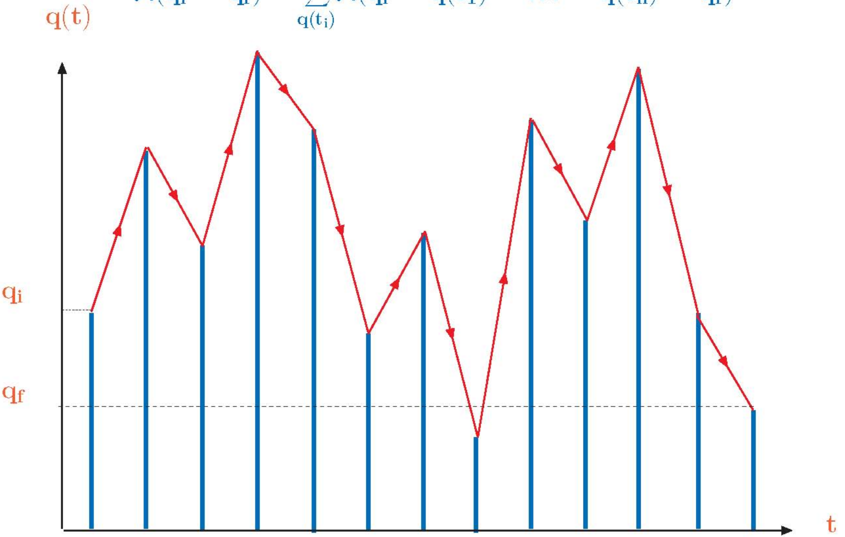
♠ 平均的空间是经典场的所有位形空间，它包括:

- 由经典场方程决定的“经典轨道” 通常它的贡献最大（因为震荡最弱）！
- 场的连续位形构成的“连续轨道”
- 最一般的“折线型轨道”

♢ 费米场用Grassmann变量描述！ 它实际上没有大小的概念，“轨道”？

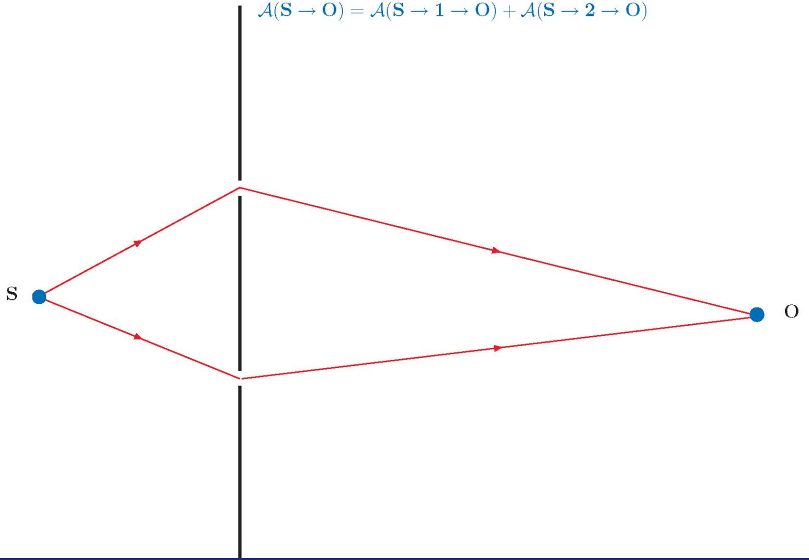
波函数

$$\mathcal{A}(q_i \to q_f) = \sum_{q(t_i)} \mathcal{A}(q_i \to q(t_1) \to \ldots \to q(t_n) \to q_f)$$



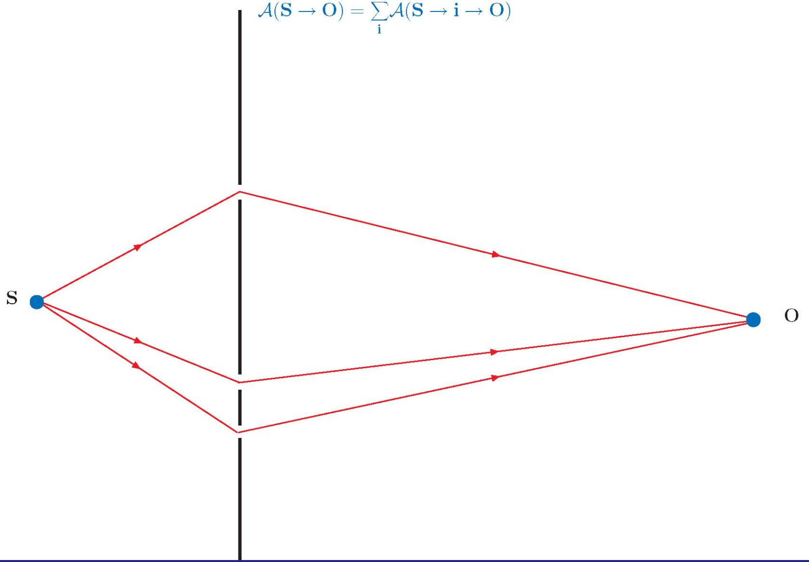
波函数

$$\mathcal{A}(\mathbf{S} \to \mathbf{O}) = \mathcal{A}(\mathbf{S} \to \mathbf{1} \to \mathbf{O}) + \mathcal{A}(\mathbf{S} \to \mathbf{2} \to \mathbf{O})$$



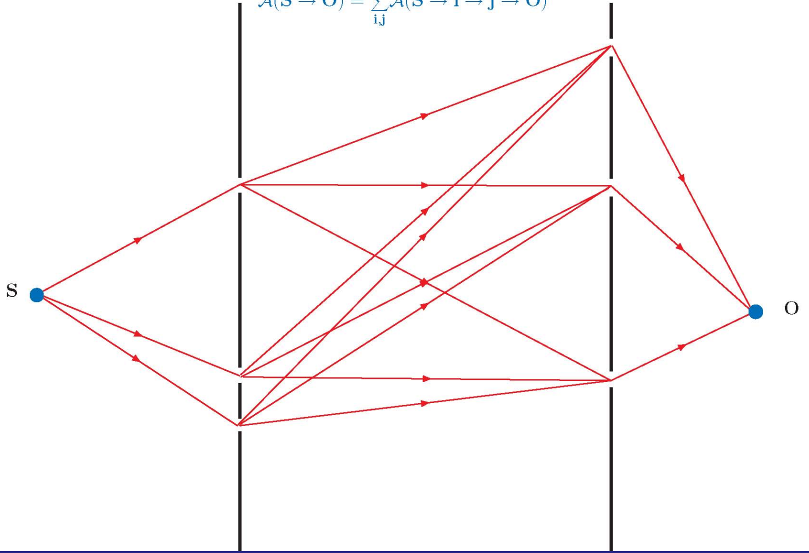
波函数

$$\mathcal{A}(\mathrm{S} \to \mathrm{O}) = \sum_{\mathrm{i}} \mathcal{A}(\mathrm{S} \to \mathrm{i} \to \mathrm{O})$$



波函数

$$\mathcal{A}(S \to O) = \sum_{i,j} \mathcal{A}(S \to i \to j \to O)$$



玻色场的生成泛函

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}}\qquad(\Psi_0^-,\Psi_0^+)=|\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]}$$

$$S[\psi]\equiv\int d^4x\ \mathcal{L}[\psi(x),\dot\psi(x)]\pm\lim_{\epsilon\to0^+}\frac{i\epsilon}{2}\int d^3x d^3y\int_{-\infty}^{\infty}d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi_l(\vec{x},\tau)\psi_l(\vec{y},\tau)\ e^{-\epsilon|\tau|}$$

$$Z[J]\equiv\int\mathcal{D}\psi\ e^{iS_J[\psi]}=e^{iW[J]}=|\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J\qquad S_J[\psi]=S[\psi]+\int d^4xJ_l(x)\psi_l(x)$$

$$(\Psi_0^-,\Psi_0^+)_J=(\Phi_0,U_J(+\infty,-\infty)\Phi_0)\xlongequal{U(t,t')=e^{iH_0t}e^{-iH(t-t')}e^{-iH_0t'}}e^{-iE_JT}\Big|_{T\to\infty}$$

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\delta^nZ[J]}{Z[0]i^n\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\Big|_{J=0}$$

$$\frac{Z[J]}{Z[0]}=1+\sum_{n=1}^{\infty}\frac{i^n}{n!}\int d^4x_1\cdots d^4x_n\ \frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}\ J_{l_1}(x_1)\cdots J_{l_n}(x_n)$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)_C\equiv\frac{\delta^nW[J]}{i^{n-1}\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\Big|_{J=0}$$

$$W[J]=W[0]+\sum_{n=1}^{\infty}\frac{i^{n-1}}{n!}\int d^4x_1\cdots d^4x_n\ (\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)_C\ J_{l_1}(x_1)\cdots J_{l_n}(x_n)$$

连接图的生成泛函与作用量

$$e^{iW[J]} = \int \mathcal{D}\phi\ e^{iS[\phi]} e^{i\int d^4x\ J(x)\phi(x)}$$

泛函傅立叶反变换:

$$\begin{aligned} e^{iS[\phi]} &= \int \mathcal{D}J\ e^{iW[J]} e^{-i\int d^4x\ J(x)\phi(x)} \\ &= \int \mathcal{D}J \int \mathcal{D}\phi'\ e^{iS[\phi']} e^{i\int d^4x\ J(x)[\phi'(x)-\phi(x)]} = \textbf{const} \int \mathcal{D}\phi'\ e^{iS[\phi']}\delta(\phi'-\phi) \end{aligned}$$

作用量和联接图的生成泛函互为泛函傅立叶变换!

知道了所有连接格林函数, 也就知道了作用量, 反之亦然!

有效作用量与顶角生成泛函

有效作用量: $\psi_{c,l}(x) \equiv \dfrac{\delta W[J]}{\delta J_l(x)}$ $\quad\quad \Gamma[\psi_c] \equiv W[J] - \int d^4x\, J_l(x)\psi_{c,l}(x)$

$$\frac{\delta\Gamma[\psi_c]}{\delta\psi_{c,l}(x)} = \int d^4y\, \frac{\delta W[J]}{\delta J_{l'}(y)}\frac{\delta J_{l'}(y)}{\delta\psi_{c,l}(x)} - \int d^4y\, \frac{\delta J_{l'}(y)}{\delta\psi_{c,l}(x)}\psi_{c,l'}(y) - J_l(x) = -J_l(x)$$

$$S_J[\psi] \equiv S[\psi] + \int d^4x\, J_l(x)\psi_l(x) \qquad \frac{\delta S_J[\psi]}{\delta\psi_l(x)} = 0 \quad \Rightarrow \quad \frac{\delta S[\psi]}{\delta\psi_l(x)} = -J_l(x)$$

顶角生成泛函

$$\Gamma_{l_1\cdots l_n}(x_1,\cdots,x_n) \equiv \left.\frac{\delta^n\Gamma[\psi_c]}{\delta\psi_{c,l_1}(x_1)\cdots\delta\psi_{c,l_n}(x_n)}\right|_{\psi_c=\psi_0} \qquad \psi_{0,l}(x) = \left.\frac{\delta W[J]}{\delta J_l(x)}\right|_{J=0}$$

$$\Gamma[\psi_c]=\Gamma[\psi_0]+\sum_{n=1}\frac{1}{n!}\int d^4x_1\cdots d^4x_n\,\Gamma_{l_1\cdots l_n}(x_1,\cdots,x_n)[\psi_{c,l_1}(x_1)-\psi_{0,l_1}(x_1)]\cdots[\psi_{c,l_n}(x_n)-\psi_{0,l_n}(x_n)]$$

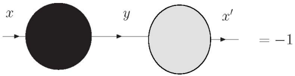
$$\delta(x-x')\delta_{ll'} = \frac{\delta\psi_{c,l}(x)}{\delta\psi_{c,l'}(x')} = \int d^4y\frac{\delta\psi_{c,l}(x)}{\delta J_{l''}(y)}\frac{\delta J_{l''}(y)}{\delta\psi_{c,l'}(x')} = -\int d^4y\,\frac{\delta^2 W[J]}{\delta J_l(x)\delta J_{l''}(y)}\frac{\delta^2\Gamma[\psi_c]}{\delta\psi_{c,l''}(y)\delta\psi_{c,l'}(x')}$$

$$\int d^4y\,(\Psi_0^-, \psi_{H,l}(x)\psi_{H,l''}(y)\Psi_0^+)_C\,\Gamma_{l''l'}(y,x') = i\delta(x-x')\delta_{ll'}$$

两线顶角:

$$\delta(x-x')\delta_{ll'} = \frac{\delta\psi_{c,l}(x)}{\delta\psi_{c,l'}(x')} = \int d^4y \frac{\delta\psi_{c,l}(x)}{\delta J_{l''}(y)}\frac{\delta J_{l''}(y)}{\delta\psi_{c,l'}(x')} = -\int d^4y \frac{\delta^2 W[J]}{\delta J_l(x)\delta J_{l''}(y)}\frac{\delta^2\Gamma[\psi_c]}{\delta\psi_{c,l''}(y)\delta\psi_{c,l'}(x')}$$

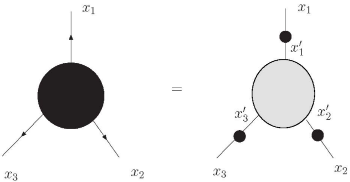
三线顶角:



$$0 = \frac{\delta}{\delta J(x_3)}\int d^4x_2 \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)}\frac{\delta\Gamma[\psi_c]}{\delta\psi_c(x_2)\delta\psi_c(x')}$$

$$= \int d^4x_2 \left[\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)}\frac{\delta^2\Gamma[\psi_c]}{\delta\psi_c(x_2)\delta\psi_c(x')} + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)}\int d^4y \frac{\delta^3\Gamma[\psi_c]}{\delta\psi_c(y)\delta\psi_c(x_2)\delta\psi_c(x')}\frac{\delta\psi_c(y)}{\delta J(x_3)}\right.$$

$$\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)} = \int d^4x_1' d^4x_2' d^4x_3' \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\frac{\delta^3\Gamma[\psi_c]}{\delta\psi_c(x_1')\delta\psi_c(x_2')\delta\psi_c(x_3')}$$



四线顶角： $\dfrac{\delta^4 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)}$

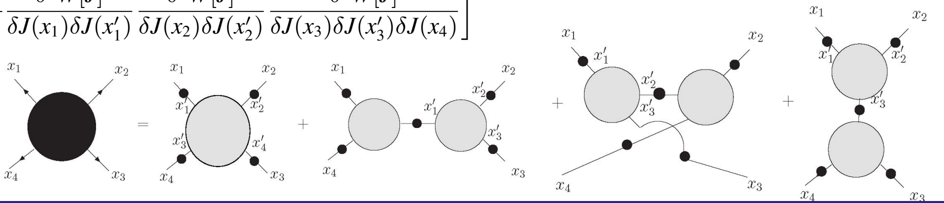
$$
= \frac{\delta}{\delta J(x_4)}\left[\int d^4x_1' d^4x_2' d^4x_3' \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\frac{\delta^3 \Gamma[\psi_c]}{\delta\psi_c(x_1')\delta\psi_c(x_2')\delta\psi_c(x_3')}\right]
$$

$$
= \int d^4x_1' d^4x_2' d^4x_3' d^4x_4' \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\frac{\delta^2 W[J]}{\delta J(x_4)\delta J(x_4')}
$$

$$
\times \frac{\delta^4 \Gamma[\psi_c]}{\delta\psi_c(x_1')\delta\psi_c(x_2')\delta\phi_c(x_3')\delta\phi_c(x_4')} + \int d^4x_1' d^4x_2' d^4x_3' \frac{\delta^3 \Gamma[\psi_c]}{\delta\psi_c(x_1')\delta\psi_c(x_2')\delta\psi_c(x_3')}
$$

$$
\times\left[\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_1')\delta J(x_4)}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')} + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^3 W[J]}{\delta J(x_2)\delta J(x_2')\delta J(x_4)}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\right.
$$

$$
\left. + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^3 W[J]}{\delta J(x_3)\delta J(x_3')\delta J(x_4)}\right]
$$



自由玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}} \qquad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]}$$

$$S[\psi] \equiv \int d^4x\ \mathcal{L}[\psi(x), \dot{\psi}(x)] \pm \lim_{\epsilon\to 0+}\frac{i\epsilon}{2}\int d^3x d^3y \int_{-\infty}^{\infty} d\tau\ \mathcal{E}(\vec{x},\vec{y})\psi_l(\vec{x},\tau)\psi_l(\vec{y},\tau)\ e^{-\epsilon|\tau|} \quad \begin{matrix} +\ \text{标量} \\ -\ \text{矢量}\end{matrix}$$

$$Z[J] \equiv \int\mathcal{D}\psi\ e^{iS_J[\psi]} = |\mathcal{N}|^{-2}(\Psi_0^-, \Psi_0^+)_J \qquad S_J[\psi] = S[\psi] + \int d^4x\ J_l(x)\psi_l(x)$$

$$S_0 = \frac{1}{2}\int d^4x d^4x'\ [i\mathcal{D}_{ll'}^{-1}(x,x')\psi_l(x)\psi_{l'}(x') \pm \lim_{\epsilon\to 0+} i\epsilon\ \mathcal{E}(\vec{x},\vec{x}')\psi_l(x)\psi_l(x')\delta(t-t')\ e^{-\epsilon|t+t'|/2}]$$

$$Z_0[J] = \int\mathcal{D}\psi\ \exp i\{\frac{1}{2}\int d^4x d^4x'\ i\mathcal{D}_{ll'}^{\prime;-1}(x,x')\psi_l(x)\psi_{l'}(x') + \int d^4x\ J_l(x)\psi_l(x)\}$$

$$= \int\mathcal{D}\psi\ \exp i\mathrm{Tr}(\frac{1}{2}\psi i\mathcal{D}^{-1}\psi + J\psi) = \int\mathcal{D}\psi\ \exp i\{\frac{i}{2}\mathrm{Tr}[(\psi - iJ\mathcal{D})\mathcal{D}^{-1}(\psi - i\mathcal{D}J) + J\mathcal{D}J]\}$$

$$= C\times\exp[\frac{-1}{2}\int d^4x d^4x'\mathcal{D}_{ll'}'(x,x')J_l(x)J_{l'}(x')] \quad C = \int\mathcal{D}\psi\ \exp[\frac{-1}{2}\int d^4x d^4x'\mathcal{D}_{ll'}^{\prime;-1}(x,x')\psi_l(x)\psi_{l'}(x')]$$

$$W_0[J] = \frac{i}{2}\int d^4x d^4x'\mathcal{D}_{ll'}'(x,x')J_l(x)J_{l'}(x') \quad \mathcal{D}_{ll'}^{\prime;-1}(x,x') = \mathcal{D}_{ll'}^{-1}(x,x') \pm \lim_{\epsilon\to 0+}\epsilon\mathcal{E}(\vec{x},\vec{x}')\delta(t-t')e^{-\epsilon|t+t'|/2}$$

自由玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}} \qquad (\Psi_0^-,\Psi_0^+) = |\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]}$$

$$S_0 = \frac{1}{2}\int d^4x d^4x'\ [i\mathcal{D}_{ll'}^{-1}(x,x')\psi_l(x)\psi_{l'}(x') \pm \lim_{\epsilon\to 0+} i\epsilon\ \mathcal{E}(\vec{x},\vec{x}')\psi_l(x)\psi_l(x')\delta(t-t')\ e^{-\epsilon|t+t'|/2}]$$

$$Z_0[J] = C\times\exp[\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x')] \quad C=\int\mathcal{D}\psi\exp[\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'^{,-1}_{ll'}(x,x')\psi_l(x)\psi_{l'}(x')]$$

$$W_0[J] = \frac{i}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x') \qquad \mathcal{D}'^{,-1}_{ll'}(x,x') = \mathcal{D}^{-1}_{ll'}(x,x') \pm \lim_{\epsilon\to 0+}\epsilon\mathcal{E}(\vec{x},\vec{x}')\delta(t-t')e^{-\epsilon|t+t'|/2}$$

$$\psi_{c,l}(x) = \frac{\delta W_0[J]}{\delta J_l(x)} = \int d^4x'\ i\mathcal{D}'_{ll'}(x,x')J_{l'}(x') \qquad J_l(x) = -i\int d^4x'\mathcal{D}'^{,-1}_{ll'}(x,x')\psi_{c,l'}(x')$$

$$\Gamma_0[\psi_c] = W_0[J] - \int d^4xJ_l(x)\psi_{c,l}(x) = \frac{i}{2}\int d^4xd^4x'\mathcal{D}'^{,-1}_{ll'}(x,x')\psi_{c,l}(x)\psi_{c,l'}(x') = S_0[\psi_c]$$

自由玻色场只有两线顶角： $\Gamma_{0,ll'}(x,x') = \dfrac{\delta^2\Gamma_0[\phi_c]}{\delta\psi_{c,l}(x)\delta\psi_{c,l'}(x')} = i\mathcal{D}'^{,-1}_{ll'}(x,x')$

玻色场传播子 (两线顶角的逆)

$$S_0 = \frac{1}{2}\int d^4x d^4x' \,[i\mathcal{D}^{-1}_{ll'}(x,x')\psi_l(x)\psi_{l'}(x') \pm \lim_{\epsilon\to 0+} i\epsilon\,\mathcal{E}(\vec{x},\vec{x}')\psi_l(x)\psi_l(x')\delta(t-t')\,e^{-\epsilon|t+t'|/2}]$$

$$W_0[J] = \frac{i}{2}\int d^4x d^4x' \mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x') \qquad \mathcal{D}'^{,-1}_{ll'}(x,x') = \mathcal{D}^{-1}_{ll'}(x,x') \pm \lim_{\epsilon\to 0+}\epsilon\,\mathcal{E}(\vec{x},\vec{x}')\delta(t-t')e^{-\epsilon|t+t'|/2}$$

$$W[J] = W[0] + \sum_{n=1}^{\infty}\frac{i^{n-1}}{n!}\int d^4x_1\cdots d^4x_n\,(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)_C\, J_{l_1}(x_1)\cdots J_{l_n}(x_n)$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)_C \overset{\text{自由场}}{=\!=\!=\!=\!=} \begin{cases} 0 & n\neq 2 \\ \mathcal{D}'_{l_1 l_2}(x_1,x_2) = \psi_{l_1}(x_1)\psi_{l_2}(x_2) & n=2 \end{cases}$$

实自由标量场： $S_0[\phi] = \int d^4x\,\frac{1}{2}[(\partial_\mu\phi(x))^2 - M^2\phi^2(x)]$

$$i\mathcal{D}^{-1}(x,x') = -(\partial_x^2 + M^2)\delta(x-x') = \int\frac{d^4p}{(2\pi)^4}(p^2-M^2)e^{-ip\cdot(x-x')} \qquad \mathcal{E}(\vec{x},\vec{y}) = \int\frac{d^3p}{(2\pi)^3}\,e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\sqrt{\vec{p}^2+M^2}$$

$$\mathcal{D}'^{,-1}(x,x') = \int\frac{d^4p}{(2\pi)^4}e^{-ip(x-x')}[-i(p^2-M^2) + \epsilon\sqrt{\vec{p}^2+M^2}e^{-\epsilon\frac{|t+t'|}{2}}] = -i\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-x')}(p^2-M^2+i0^+)$$

$$\mathcal{D}'(x,x') = \int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-x')}\,\frac{i}{p^2-M^2+i0^+}$$

玻色场传播子 (两线顶角的逆)

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)_C \overset{\text{自由场}}{=\!=\!=\!=\!=} \begin{cases} 0 & n \neq 2 \\ \mathcal{D}'_{l_1l_2}(x_1,x_2) = \psi_{l_1}(x_1)\psi_{l_2}(x_2) & n = 2 \end{cases}$$

对完全格林函数: $\left.\frac{\delta^2 W[J]}{\delta J_l(x)\delta J_{l'}(x')}\right|_{J_l=0} \overset{\text{自由场}}{=\!=\!=\!=\!=} i\mathcal{D}_{ll'}(x,x')$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \left.\frac{\delta^n Z[J]}{i^n Z[J]\delta J_{l_1}(x_1)\cdots\delta J_n(x_n)}\right|_{J_l=0} = e^{-iW[J]}\left.\frac{\delta^n e^{iW[J]}}{i^n\delta J_{l_1}(x_1)\cdots\delta J_n(x_n)}\right|_{J_l=0}$$

$$\overset{\text{自由场}}{=\!=\!=\!=\!=} \begin{cases} 0 & n = \text{奇数} \\ \sum\limits_{\mathcal{P}\ 2,3,\ldots,n} \underbrace{\mathcal{D}'_{l_1l_2}(x_1,x_2)\cdots\mathcal{D}'_{l_{n-1}l_n}(x_{n-1},x_n)}_{\text{不连接图}} & n = \text{偶数} \end{cases}$$

不计传播子内部的顺序差别，及传播子的整体交换

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\psi_{H,l_3}(x_3)\psi_{H,l_4}(x_4)\Psi_0^+)$$
$$= \mathcal{D}'_{l_1l_2}(x_1,x_2)\mathcal{D}'_{l_3l_4}(x_3,x_4) + \mathcal{D}'_{l_1l_3}(x_1,x_3)\mathcal{D}'_{l_2l_4}(x_2,x_4) + \mathcal{D}'_{l_1l_4}(x_1,x_4)\mathcal{D}'_{l_2l_3}(x_2,x_3)$$

实自由标量场的粒子与相互作用力诠释

$$S_0[\phi] = \int d^4x \, \tfrac{1}{2}[(\partial_\mu\phi(x))^2 - M^2\phi^2(x)]$$

$$W_0[J] = \frac{i}{2}\int d^4x d^4x' \mathcal{D}'(x,x')J(x)J(x') \qquad \mathcal{D}'(x,x') = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-x')}\frac{i}{p^2-M^2+i0^+}$$

$$J(p) \equiv \int d^4x\, e^{-ip\cdot x}J(x) = J^*(-p) \qquad W_0[J] = -\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}J^*(p)\frac{1}{p^2-M^2+i0^+}J(p)$$ $J^*(p)$来自$J(-p)$

由实自由标量场传递两个相互分离局域源 $J(x) = J_1(x) + J_2(x)$ 之间的相互作用

$$W_{0,\rm int}[J] = -\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\left[J_1^*(p)\frac{1}{p^2-M^2+i0^+}J_2(p) + J_2^*(p)\frac{1}{p^2-M^2+i0^+}J_1(p)\right]$$

相互作用在 $p^2 = M^2$ 处最强！

$$J_i(x) = q_i\delta(\vec{x}-\vec{x}_i) \qquad J_i(p) = 2\pi q_i\delta(p^0)e^{i\vec{p}\cdot\vec{x}_i} \qquad 2\pi\delta(p^0)\Big|_{p^0=0} = T \qquad W_{0,\rm int}[J] = -E_J T$$

$$E_J = -q_1q_2\int\frac{d\vec{p}}{(2\pi)^3}\frac{e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}^2+M^2} = \frac{-q_1q_2}{(2\pi)^2}\int_0^\infty dp\, p^2\int_{-1}^1 d\cos\theta\frac{e^{ip|\vec{x}_1-\vec{x}_2|\cos\theta}}{p^2+M^2} = \frac{-q_1q_2}{4\pi|\vec{x}_1-\vec{x}_2|}e^{-m|\vec{x}_1-\vec{x}_2|}$$

$$\frac{dE_J}{d|\vec{x}_1-\vec{x}_2|}\Bigg|_{q_1q_2>0} > 0 \quad\Rightarrow$$ 吸引势！ 同号相吸：库伦定律,比萨定律(见后); 万有引力定律

玻色场传播子 (两线顶角的逆)

$$S_0=\frac{1}{2}\int d^4xd^4x'\ [i\mathcal{D}_{ll'}^{-1}(x,x')\psi_l(x)\psi_{l'}(x')\pm\lim_{\epsilon\to 0+} i\epsilon\,\mathcal{E}(\vec{x},\vec{x}')\psi_l(x)\psi_l(x')\delta(t-t')\ e^{-\epsilon|t+t'|/2}]$$

$$W_0[J]=\frac{i}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x')\qquad \mathcal{D}'^{,-1}_{ll'}(x,x')=\mathcal{D}'_{ll'}(x,x')\pm\lim_{\epsilon\to 0+}\epsilon\mathcal{E}(\vec{x},\vec{x}')\delta(t-t')e^{-\epsilon|t+t'|/2}$$

$$W[J]=W[0]+\sum_{n=1}^{\infty}\frac{i^{n-1}}{n!}\int d^4x_1\cdots d^4x_n\ (\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)_C\ J_{l_1}(x_1)\cdots J_{l_n}(x_n)$$

实零质量自由矢量场： $S_0[\phi]=\int d^4x\ \frac{1}{2}\{-[\partial_\mu v_\nu(x)][\partial^\mu v^\nu(x)]\}$

$$i\mathcal{D}^{\mu\nu,-1}(x,x')=g^{\mu\nu}\partial_x^2\delta(x-x')=\int\frac{d^4p}{(2\pi)^4}(-p^2)g^{\mu\nu}\ e^{-ip\cdot(x-x')}\qquad \mathcal{E}(\vec{x},\vec{y})=\int\frac{d^3p}{(2\pi)^3}\ e^{i\vec{p}\cdot(\vec{x}-\vec{y})}\sqrt{\vec{p}^2}$$

$$\mathcal{D}^{\mu\nu\prime,-1}(x,x')=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-x')}g^{\mu\nu}(ip^2-\epsilon\sqrt{\vec{p}^2}e^{-\epsilon|t+t'|/2})=i\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-x')}g^{\mu\nu}(p^2+i0^+)$$

$$\mathcal{D}^{\mu\nu\prime}(x,x')=\int\frac{d^4p}{(2\pi)^4}\frac{-ig^{\mu\nu}e^{-ip\cdot(x-x')}}{p^2+i0^+}\Rightarrow W_0[J]=\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-x')}\frac{J_\mu^*(p)J^\mu(p)}{p^2+i0^+}$$

$$\xRightarrow{J^\mu(x)=\delta_0^\mu[q_1\delta(\vec{x}+\vec{x}_1)+q_2\delta(\vec{x}-\vec{x}_2)]}\ E_J=\frac{q_1q_2}{4\pi|\vec{x}_1-\vec{x}_2|}$$

平方反比: <u>同号电荷相斥，同向电流相吸</u>！

相互作用玻色场

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}}=\frac{\delta^n Z[J]}{i^n Z[J]\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\bigg|_{J=0}$$

$$Z[J]\equiv\int\mathcal{D}\psi e^{iS_J[\psi]}=|\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J\quad(\Psi_0^-,\Psi_0^+)=|\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS_J[\psi]}\quad S_J[\psi]=S[\psi]+\int d^4xJ_l(x)\psi_l(x)$$

$$S_{0,J}=S_0[\psi]+\int d^4xJ_l(x)\psi_l(x)\quad S=S_0+\int d^4x\ \mathcal{L}_I[\psi(x)]\quad S_0=\frac{1}{2}\int d^4xd^4x'\ i\mathcal{D}'^{-1}_{ll'}(x,x')\psi_l(x)\psi_{l'}(x')$$

$$Z_0[J]\equiv\int\mathcal{D}\psi\ e^{iS_{0,J}[\psi]}=C\times\exp[\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x')]\quad V=-\int d\vec{x}\ \mathcal{L}_I\quad \mathcal{H}=-\mathcal{L}_I[\psi]$$

微扰展开:

$$\frac{\delta\int d^4xJ_l(x)\psi_l(x)}{\delta J_l(y)}=\psi_l(y)\quad \frac{\delta e^{i\int d^4xJ_l(x)\psi_l(x)}}{i\delta J_l(y)}=e^{i\int d^4xJ_l(x)\psi_l(x)}\frac{\delta i\int d^4xJ_l(x)\psi_l(x)}{i\delta J_l(y)}=\psi_l(y)e^{i\int d^4xJ_l(x)\psi_l(x)}$$

$$Z[J]=\int\mathcal{D}\psi e^{i\int d^4x\ \mathcal{L}_I[\psi(x)]}e^{iS_{0,J}[\psi]}=\int\mathcal{D}\psi e^{i\int d^4x\ \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]}e^{iS_{0,J}[\psi]}=e^{i\int d^4x\ \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]}\int\mathcal{D}\psi e^{iS_{0,J}[\psi]}$$

$$=e^{i\int d^4x\ \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]}Z_0[J]=C\times e^{i\int d^4x\ \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]}\exp[\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x')]$$

相互作用玻色场

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}}=\left.\frac{\delta^n Z[J]}{i^n Z[J]\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\right|_{J=0}$$

$$Z[J]\equiv\int\mathcal{D}\psi e^{iS_J[\psi]}=|\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J\quad(\Psi_0^-,\Psi_0^+)=|\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]}\quad S_J[\psi]=S[\psi]+\int d^4xJ_l(x)\psi_l(x)$$

$$S_{0,J}=S[\psi]+\int d^4xJ_l(x)\psi_l(x)\quad S=S_0+\int d^4x\ \mathcal{L}_I[\psi(x)]\quad S_0=\frac{1}{2}\int d^4xd^4x'\ i\mathcal{D}'^{,-1}_{ll'}(x,x')\psi_l(x)\psi_{l'}(x')$$

$$Z_0[J]\equiv\int\mathcal{D}\psi\ e^{iS_{0,J}[\psi]}=C\times\exp[\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x')]\quad V=-\int d\vec{x}\ \mathcal{L}_I\quad\mathcal{H}=-\mathcal{L}_I[\psi]$$

微扰展开(变形):

$$Z[J]=C\times e^{i\int d^4x\ \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]}e^{\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')J_l(x)J_{l'}(x')}e^{i\int d^4xJ_l(x)\psi_l(x)}\Big|_{\psi(x)=0}$$

$$=C\times e^{i\int d^4x\ \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]}e^{\frac{-1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')\frac{\delta^2}{i^2\delta\psi_l(x)\delta\psi_{l'}(x')}}e^{i\int d^4xJ_l(x)\psi_l(x)}\Big|_{\psi(x)=0}$$

$$=C\times e^{\frac{1}{2}\int d^4xd^4x'\mathcal{D}'_{ll'}(x,x')\frac{\delta^2}{\delta\psi_l(x)\delta\psi_{l'}(x')}}e^{i\int d^4x\ [\mathcal{L}_I[\psi]+J_l(x)\psi_l(x)]}\Big|_{\psi(x)=0}$$

相互作用玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}} = \frac{\delta^n Z[J]}{i^n Z[J]\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\Big|_{J=0}$$

$$Z[J]\equiv\int\mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J \quad (\Psi_0^-,\Psi_0^+) = |\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]} \quad S_J[\psi]=S[\psi]+\int d^4x J_l(x)\psi_l(x)$$

$$S_{0,J}=S[\psi]+\int d^4x J_l(x)\psi_l(x) \quad S=S_0+\int d^4x\ \mathcal{L}_I[\psi(x)] \quad S_0=\frac{1}{2}\int d^4x d^4x'\ i\mathcal{D}'^{,-1}_{ll'}(x,x')\psi_l(x)\psi_{l'}(x')$$

微扰展开(变形):

$$Z[J] = C\times e^{\frac{1}{2}\int d^4x d^4x'\mathcal{D}'_{ll'}(x,x')\frac{\delta^2}{\delta\psi_i(x)\delta\psi_{l'}(x')}} e^{i\int d^4x\ [\mathcal{L}_I[\psi]+J_l(x)\psi_l(x)]}\Big|_{\psi(x)=0}$$

$$(\Psi_0^-, \Psi_0^+) = C'\times e^{\frac{1}{2}\int d^4x d^4x'\mathcal{D}'_{ll'}(x,x')\frac{\delta^2}{\delta\psi_i(x)\delta\psi_{l'}(x')}} e^{i\int d^4x\ \mathcal{L}_I[\psi]}\Big|_{\psi(x)=0}$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = C' e^{\frac{1}{2}\int d^4x d^4x'\mathcal{D}'_{ll'}(x,x')\frac{\delta^2}{\delta\psi_i(x)\delta\psi_{l'}(x')}}\psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)e^{i\int d^4x\ \mathcal{L}_I[\psi]}\Big|_{\psi(x)=0}$$

玻色场的费曼图与费曼规则

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+) = (\Phi_0, \mathbf{T}\psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)e^{i\int d^4x\ \mathcal{L}_I[\psi]}\Phi_0)$$ 所有可能的收缩

$$= C'e^{\frac{1}{2}\int d^4x d^4x' \mathcal{D}'_{ll'}(x,x')\frac{\delta^2}{\delta\psi_l(x)\delta\psi_{l'}(x')}}\psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)e^{i\int d^4x\ \mathcal{L}_I[\psi]}\Big|_{\psi(x)=0}$$

- 每一个传播子必须微掉两个场(两个场的收缩得到传播子) 微商产生两δ函数抵消传播子的两个$\int d^4x$积分, 并将传播子的坐标取为被微掉的两个场的坐标 $\Rightarrow$每一个传播子$D'(x,x')$连接两个场 $\frac{1}{2}$用与抵消两个场的交换,x, x'分别是这两个场的坐标 如果总共有l个传播子,本应有因子$\frac{1}{l!}$,但考虑到这l个传播子共有$l!$个安排方式,它们给出同样结果, 因此可略去因子$\frac{1}{l!}$,同时不再计入不同的安排方式
- 相互作用顶角贡献位于同一时空点的若干个场$i\int d^4x\ \mathcal{L}_I[\psi(x)]$, 外线贡献位于不同时空点的n个场$\psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)$
- 所有场都必须被全部微掉(收缩掉)$\Rightarrow$传播子止于顶角或外线
- 微商对所有的场是平权的,任何一个微商对每个场都要操作一次
- 如果总共有k个顶角,应该有因子$\frac{1}{k!}$

存在一个用图形方式(顶角,传播子, 外线)按一定规则表达结果的方式: 费曼图

费米场的生成泛函

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')\ e^{iS[\psi,\overline{\psi}]}}{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]}}$$

$$(\Psi_0^-,\Psi_0^+)=|\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]}\qquad S[\psi,\overline{\psi}]\equiv\int d^4x\ \mathcal{L}[\psi(x),\overline{\psi}(x)]+\epsilon\text{ 项}$$

$$Z[I,\bar{I}]\equiv\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{iS_I[\psi,\overline{\psi}]}=e^{iW[I,\bar{I}]}=|\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_I\quad S_I[\psi,\overline{\psi}]=S[\psi,\overline{\psi}]+\int d^4x[\bar{I}_l(x)\psi_l(x)+\overline{\psi}_l(x)I_l(x)]$$

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\delta^{2n}Z[J]}{Z[0]\delta\bar{I}_{l_1}(x_1)\delta I_{l_1'}(x_1')\cdots\delta\bar{I}_{l_n}(x_n)\delta I_{l_n'}(x_n')}\Big|_{I=\bar{I}=0}$$

$$\frac{Z[I,\bar{I}]}{Z[0]}=\sum_{n=0}^{\infty}\frac{1}{n!}\int d^4x_1\cdots d^4x_n'\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}\bar{I}_{l_1}(x_1)I_{l_1'}(x_1')\cdots\bar{I}_{l_n}(x_n)I_{l_n'}(x_n')$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)_C\equiv\frac{i\delta^{2n}W[I,\bar{I}]}{\delta\bar{I}_{l_1}(x_1)\delta I_{l_1'}(x_1')\cdots\delta\bar{I}_{l_n}(x_n)\delta I_{l_n'}(x_n')}\Big|_{I=\bar{I}=0}$$

$$W[I,\bar{I}]=\sum_{n=1}^{\infty}\frac{-i}{n!}\int d^4x_1\cdots d^4x_n'(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)_C\bar{I}_{l_1}(x_1)I_{l_1'}(x_1')\cdots\bar{I}_{l_n}(x_n)I_{l_n'}(x_n')$$

有效作用量

$$\psi_{c,l}(x) \equiv \frac{\delta W[I,\bar{I}]}{\delta \bar{I}_l(x)} \qquad \overline{\psi}_{c,l}(x) \equiv -\frac{\delta W[I,\bar{I}]}{\delta I_l(x)}$$

$$\Gamma[\psi_c,\overline{\psi}_c] \equiv W[I,\bar{I}] - \int d^4x\,[\bar{I}_l(x)\psi_{c,l}(x) + \overline{\psi}_{c,l}(x)I_l(x)]$$

$$\frac{\delta\Gamma[\psi_c,\overline{\psi}_c]}{\delta\psi_{c,l}(x)} = \int d^4y\left[\frac{\delta\bar{I}_{l'}(y)}{\delta\psi_{c,l}(x)}\frac{\delta W[I,\bar{I}]}{\delta\bar{I}_{l'}(y)} + \frac{\delta I_{l'}(y)}{\delta\psi_{c,l}(x)}\frac{\delta W[I,\bar{I}]}{\delta I_{l'}(y)} - \frac{\delta\bar{I}_{l'}(y)}{\delta\psi_{c,l}(x)}\psi_{c,l'}(y) + \overline{\psi}_{c,l'}(y)\frac{\delta I_{l'}(y)}{\delta\psi_{c,l}(x)} + \bar{I}_l(x)\right]$$
$$= \bar{I}_l(x)$$

$$\frac{\delta\Gamma[\psi_c,\overline{\psi}_c]}{\delta\overline{\psi}_{c,l}(x)} = \int d^4y\left[\frac{\delta\bar{I}_{l'}(y)}{\delta\overline{\psi}_{c,l}(x)}\frac{\delta W[I,\bar{I}]}{\delta\bar{I}_{l'}(y)} + \frac{\delta I_{l'}(y)}{\delta\overline{\psi}_{c,l}(x)}\frac{\delta W[I,\bar{I}]}{\delta I_{l'}(y)} + \overline{\psi}_{c,l'}(y)\frac{\delta I_{l'}(y)}{\delta\overline{\psi}_{c,l}(x)} - \frac{\delta\bar{I}_{l'}(y)}{\delta\overline{\psi}_{c,l}(x)}\psi_{c,l'}(y) - I_l(x)\right]$$
$$= -I_l(x)$$

$$S_I[\psi,\overline{\psi}] = S[\psi,\overline{\psi}] + \int d^4x[\bar{I}_l(x)\psi_l(x) + \overline{\psi}_l(x)I_l(x)]$$

$$\frac{\delta S_I[\psi,\overline{\psi}]}{\delta\psi_l(x)} = \frac{\delta S_I[\psi,\overline{\psi}]}{\delta\overline{\psi}_l(x)} = 0 \quad\Rightarrow\quad \frac{\delta S[\psi,\overline{\psi}]}{\delta\psi_l(x)} = \bar{I}_l(x) \qquad \frac{\delta S[\psi,\overline{\psi}]}{\delta\overline{\psi}_l(x)} = -I_l(x)$$

顶角生成泛函

$$\psi_{c,l}(x)\equiv\frac{\delta W[I,\bar{I}]}{\delta\bar{I}_l(x)}\quad \bar{\psi}_{c,l}(x)\equiv-\frac{\delta W[I,\bar{I}]}{\delta I_l(x)}\qquad \Gamma[\psi_c,\bar{\psi}_c]\equiv W[I,\bar{I}]-\int d^4x[\bar{I}_l(x)\psi_{c,l}(x)+\bar{\psi}_{c,l}(x)I_l(x)]$$

$$\psi_{0,l}(x)=\frac{\delta W[I,\bar{I}]}{\delta\bar{I}_l(x)}\bigg|_{I=\bar{I}=0}\quad \bar{\psi}_{0,l}(x)=-\frac{\delta W[I,\bar{I}]}{\delta I_l(x)}\bigg|_{I=\bar{I}=0}\quad \frac{\delta\Gamma[\psi_c,\bar{\psi}_c]}{\delta\psi_{c,l}(x)}=\bar{I}_l(x)\quad \frac{\delta\Gamma[\psi_c,\bar{\psi}_c]}{\delta\bar{\psi}_{c,l}(x)}=-I_l(x)$$

$$\Gamma_{l_1l_1'\cdots l_nl_n'}(x_1,x_1'\cdots,x_n,x_n')\equiv\frac{\delta^{2n}\Gamma[\psi_c,\bar{\psi}_c]}{\delta\bar{\psi}_{c,l_1}(x_1)\delta\psi_{c,l_1'}(x_1')\cdots\delta\bar{\psi}_{c,l_n}(x_n)\delta\psi_{c,l_n'}(x_n')}\bigg|_{\psi_c=0,\ \bar{\psi}_c=0}$$

$$\Gamma[\psi_c,\bar{\psi}_c]=\Gamma[0,0]+\sum_{n=1}\frac{(-1)^n}{(2n)!}\int d^4x_1d^4x_1'\cdots d^4x_nd^4x_n'\Gamma_{l_1l_1'\cdots l_nl_n'}(x_1,x_1',\cdots,x_n,x_n')\bar{\psi}_{c,l_1}(x_1)\psi_{c,l_1'}(x_1')\cdots\bar{\psi}_{c,l_n}(x_n)\psi_{c,l_n'}(x_n')$$

$$\delta(x-x')\delta_{ll'}=\frac{\delta\psi_{c,l}(x)}{\delta\psi_{c,l'}(x')}=\int d^4y\left[\frac{\delta I_{l''}(y)}{\delta\psi_{c,l'}(x')}\frac{\delta\psi_{c,l}(x)}{\delta I_{l''}(y)}+\frac{\delta\bar{I}_{l''}(y)}{\delta\psi_{c,l'}(x')}\frac{\delta\psi_{c,l}(x)}{\delta\bar{I}_{l''}(y)}\right]$$

$$=\int d^4y\left[-\frac{\delta^2\Gamma[\psi_c,\bar{\psi}_c]}{\delta\psi_{c,l'}(x')\delta\bar{\psi}_{c,l''}(y)}\frac{\delta^2W[I,\bar{I}]}{\delta I_{l''}(y)\delta\bar{I}_l(x)}+\frac{\delta^2\Gamma[\psi_c,\bar{\psi}_c]}{\delta\psi_{c,l'}(x')\delta\psi_{c,l''}(y)}\frac{\delta^2W[I,\bar{I}]}{\delta\bar{I}_{l''}(y)\delta\bar{I}_l(x)}\right]$$

$$\int d^4y\ (\Psi_0^-,\psi_{H,l}(x)\psi_{H,l''}(y)\Psi_0^+)_C\ \Gamma_{l''l'}(y,x')=-i\delta(x-x')\delta_{ll'}$$

自由费米场

$$S_0 = \int d^4x d^4x' iS_{ll'}^{-1}(x,x')\overline{\psi}_l(x)\psi_{l'}(x') - \lim_{\epsilon\to 0+} i\epsilon \int d^3x d^3x' \int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{x}')\psi(\vec{x}',\tau)\ e^{-\epsilon|\tau|}$$

$$= \int d^4x d^4x'\ iS_{ll'}^{\prime,-1}(x,x')\overline{\psi}_l(x)\psi_{l'}(x') \qquad iS_{ll'}^{-1}(x,x') = (i\gamma^\mu\partial_{\mu,x} - M)_{ll'}\delta(x-x')$$

$$S_{ll'}^{\prime,-1}(x,x') = -i(i\gamma^\mu\partial_{\mu,x} - M)\delta(x-x') - \epsilon\mathcal{E}(\vec{x},\vec{x}')e^{-\epsilon\frac{|t+t'|}{2}}\delta(t-t') \quad \mathcal{E}(\vec{x},\vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \frac{-\vec{p}\cdot\vec{\gamma} - M}{\sqrt{\vec{p}^2+M^2}}$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')}[-i(\not{p}-M) + \epsilon e^{-\epsilon\frac{|t+t'|}{2}} \frac{\vec{p}\cdot\vec{\gamma}+M}{\sqrt{\vec{p}^2+M^2}}] = -i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')}[\not{p} - M + i0^+]$$

注意$\not{p}$中的微小移动不影响p^0的极点: $(p^0)^2 - \vec{p}^2(1-i0^+)^2 - (M-i0^+)^2 = (p^0)^2 - \vec{p}^2 - M^2 + i0^+ = p^2 + i0^+$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\ \psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')\ e^{iS[\psi,\overline{\psi}]}}{\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]}}$$

$$(\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]}$$

$$S[\psi,\overline{\psi}] \equiv \int d^4x \mathcal{L}[\psi(x),\overline{\psi}(x)] - \lim_{\epsilon\to 0+} i\epsilon \int d^3x d^3x' \int_{-\infty}^{\infty} d\tau\ \overline{\psi}(\vec{x},\tau)\mathcal{E}(\vec{x},\vec{x}')\psi(\vec{x}',\tau)\ e^{-\epsilon|\tau|}$$

自由费米场 $S_0 = \int d^4xd^4x'\ i\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')\ e^{iS[\psi,\overline{\psi}]}}{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]}}$$

$$(\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]} \qquad S[\psi,\overline{\psi}] \equiv \int d^4x\ \mathcal{L}[\psi(x),\overline{\psi}(x)]\Big|_{\mathcal{S}\to\mathcal{S}'}$$

$$Z_0[I,\bar{I}] = \int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \exp i\{\int d^4xd^4x'\ i\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x') + \int d^4x[\bar{I}_l(x)\psi_l(x) + \overline{\psi}_l(x)I_l(x)]\}$$

$$= \int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \exp i\mathrm{Tr}[\overline{\psi}i\mathcal{S}'^{,-1}\psi + \bar{I}\psi + \overline{\psi}I]$$

$$= \int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \exp i[\mathrm{Tr}[(\overline{\psi} - i\bar{I}\mathcal{S}')i\mathcal{S}'^{,-1}(\psi - i\mathcal{S}'I) + \bar{I}i\mathcal{S}'I]$$

$$= C \times \exp[-\int d^4xd^4x'\mathcal{S}'_{ll'}(x,x')\bar{I}_l(x)I_{l'}(x')]$$

$$W_0[I,\bar{I}] = i\int d^4xd^4x'\mathcal{S}'_{ll'}(x,x')\bar{I}_l(x)I_{l'}(x') \qquad C = \int\mathcal{D}\overline{\psi}\mathcal{D}'\psi\exp[-\int d^4xd^4x'\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')]$$

$$\mathcal{S}'^{,-1}(x,x') = -i(i\gamma^\mu\partial_{\mu,x} - M + i0^+)\delta(x-x') = \int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-x')}(-i)(\gamma^\mu p_\mu - M + i0^+)$$

费米场的生成泛函

自由费米场 $S_0 = \int d^4x d^4x'\ i\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')\ e^{iS[\psi,\overline{\psi}]}}{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]}}$$

$$(\Psi_0^-,\Psi_0^+) = |\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi,\overline{\psi}]} \qquad S[\psi,\overline{\psi}] \equiv \int d^4x\ \mathcal{L}[\psi(x),\overline{\psi}(x)]\Big|_{\mathcal{S}\to\mathcal{S}'}$$

$$Z_0[I,\bar{I}] = C\times\exp[-\int d^4xd^4x'\mathcal{S}'_{ll'}(x,x')\bar{I}_l(x)I_{l'}(x')] \quad \mathcal{S}'^{,-1}(x,x') = -i(i\gamma^\mu\partial_{\mu,x}-M+i0^+)\delta(x-x')$$

$$W_0[I,\bar{I}] = i\int d^4xd^4x'\mathcal{S}'_{ll'}(x,x')\bar{I}_l(x)I_{l'}(x') \qquad C=\int\mathcal{D}\overline{\psi}\mathcal{D}'\psi\exp[-\int d^4xd^4x'\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')]$$

$$\psi_{c,l}(x)\equiv\frac{\delta W_0[I,\bar{I}]}{\delta\bar{I}_l(x)} = i\int d^4x'\mathcal{S}'_{ll'}(x,x')I_{l'}(x') \quad \overline{\psi}_{c,l}(x)\equiv-\frac{\delta W_0[I,\bar{I}]}{\delta I_l(x)} = i\int d^4x'\bar{I}_{l'}(x')\mathcal{S}'_{l'l}(x',x)$$

$$\Gamma_0[\psi_c,\overline{\psi}_c]\equiv W_0[I,\bar{I}]-\int d^4x[\bar{I}_l(x)\psi_{c,l}(x)+\overline{\psi}_{c,l}(x)I_l(x)] = \int d^4xd^4x'i\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_{c,l}(x)\psi_{c,l'}(x') = S_0[\psi_c,\overline{\psi}_c]$$

自由费米场只有两线顶角： $\Gamma_{0,ll'}(x,x') = \dfrac{\delta^2\Gamma_0[\phi_c]}{\delta\overline{\psi}_{c,l}(x)\delta\psi_{c,l'}(x')} = -i\mathcal{S}'^{,-1}_{ll'}(x,x')$

相互作用费米场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\,\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n}(x_n)e^{iS[\psi,\overline{\psi}]}}{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS[\psi,\overline{\psi}]}} = \frac{\delta^n Z[I,\bar{I}]}{Z[I,\bar{I}]\delta\bar{I}_{l_1}(x_1)\cdots\delta I_{l_n'}(x_n')}\Bigg|_{I=\bar{I}=0}$$

$$Z[I,\bar{I}] \equiv \int\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{iS_I[\psi]} = |\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J \quad S_I[\psi,\overline{\psi}] = S[\psi,\overline{\psi}] + \int d^4x[\bar{I}_l(x)\psi_l(x)+\overline{\psi}_l(x)I_l(x)]$$

$$S_{0,I} = S_0[\psi,\overline{\psi}] + \int d^4x(\bar{I}\psi+\overline{\psi}I) \quad S = S_0 + \int d^4x\;\mathcal{L}_I[\psi(x),\overline{\psi}] \quad S_0 = \int d^4x d^4x'\; i\mathcal{S}'^{,-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')$$

$$Z_0[I,\bar{I}] \equiv \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS_{0,I}[\psi,\overline{\psi}]} = C\times\exp[-\int d^4x d^4x' \mathcal{S}'_{ll'}(x,x')\bar{I}_l(x)I_{l'}(x')]$$

微扰展开:

$$Z[I,\bar{I}] = \int\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{i\int d^4x\;\mathcal{L}_I[\psi(x),\overline{\psi}(x)]}e^{iS_{0,I}[\psi,\overline{\psi}]} = \int\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{i\int d^4x\;\mathcal{L}_I[\frac{\delta}{i\delta\bar{I}(x)},\frac{\delta}{-i\delta I(x)}]}e^{iS_{0,I}[\psi,\overline{\psi}]}$$

$$= e^{i\int d^4x\;\mathcal{L}_I[\frac{\delta}{i\delta\bar{I}(x)},\frac{\delta}{-i\delta I(x)}]}\int\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{iS_{0,I}[\psi,\overline{\psi}]} = e^{i\int d^4x\;\mathcal{L}_I[\frac{\delta}{i\delta\bar{I}(x)},\frac{\delta}{-i\delta I(x)}]}Z_0[I,\bar{I}]$$

$$= C\times e^{i\int d^4x\;\mathcal{L}_I[\frac{\delta}{i\delta\bar{I}(x)},\frac{\delta}{-i\delta I(x)}]}\exp[-\int d^4x d^4x'\mathcal{S}'_{ll'}(x,x')\bar{I}_l(x)I_{l'}(x')]$$

相互作用费米场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n}(x_n)e^{iS[\psi,\overline{\psi}]}}{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS[\psi,\overline{\psi}]}} = \frac{\delta^n Z[I,\overline{I}]}{Z[I,\overline{I}]\delta\overline{I}_{l_1}(x_1)\cdots\delta I_{l_n'}(x_n')}\Big|_{I=\overline{I}=0}$$

$$Z[I,\overline{I}] \equiv \int\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{iS_I[\psi]} = |\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J \qquad S_I[\psi,\overline{\psi}] = S[\psi,\overline{\psi}] + \int d^4x[\overline{I}_l(x)\psi_l(x) + \overline{\psi}_l(x)I_l(x)]$$

$$S_{0,I} = S[\psi,\overline{\psi}] + \int d^4x(\overline{I}\psi + \overline{\psi}I) \quad S = S_0 + \int d^4x\,\mathcal{L}_I[\psi(x),\overline{\psi}] \quad S_0 = \int d^4x d^4x'\; i\mathcal{S}'^{-1}_{ll'}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')$$

$$Z_0[I,\overline{I}] \equiv \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS_{0,I}[\psi,\overline{\psi}]} = C\times\exp[-\int d^4x d^4x'\mathcal{S}'_{ll'}(x,x')\overline{I}_l(x)I_{l'}(x')]$$

微扰展开(变形):

$$Z[I,\overline{I}] = C\times e^{i\int d^4x\;\mathcal{L}_I[\frac{\delta}{i\delta\overline{I}(x)},\frac{\delta}{-i\delta I(x)}]}\exp[-\int d^4x d^4x'\mathcal{S}'_{ll'}(x,x')\overline{I}_l(x)I_{l'}(x')]$$

$$= C\times e^{i\int d^4x\;\mathcal{L}_I[\frac{\delta}{i\delta\overline{I}(x)},\frac{\delta}{-i\delta I(x)}]} e^{-\int d^4xd^4x'\mathcal{S}'_{ll'}(x,x')\frac{\delta^2}{\delta\psi_l(x)\delta\overline{\psi}_{l'}(x')}} e^{i\int d^4x[\overline{I}_l(x)\psi_l(x)+\overline{\psi}_l(x)I_l(x)]}\Big|_{\overline{\psi}=\psi=0}$$

$$= C\times e^{\int d^4xd^4x'\mathcal{S}_{ll'}(x,x')\frac{\delta^2}{\delta\overline{\psi}_{l'}(x')\delta\psi_l(x)}} e^{i\int d^4x[\mathcal{L}_I[\psi,\overline{\psi}]+\overline{I}_l(x)\psi_l(x)+\overline{\psi}_l(x)I_l(x)]}\Big|_{\overline{\psi}=\psi=0}$$

相互作用费米场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)} = \frac{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n'}(x_n')e^{iS[\psi,\overline{\psi}]}}{\int\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS[\psi,\overline{\psi}]}} = \frac{\delta^n Z[I,\bar{I}]}{Z[I,\bar{I}]\delta\bar{I}_{l_1}(x_1)\cdots\delta I_{l_n'}(x_n')}\Bigg|_{I=\bar{I}=0}$$

$$Z[I,\bar{I}] \equiv \int\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{iS_I[\psi]} = |\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J \qquad S_I[\psi,\overline{\psi}] = S[\psi,\overline{\psi}] + \int d^4x[\bar{I}_l(x)\psi_l(x)+\overline{\psi}_l(x)I_l(x)]$$

$$S_{0,I} = S[\psi,\overline{\psi}] + \int d^4x(\bar{I}\psi+\overline{\psi}I) \quad S = S_0 + \int d^4x\; \mathcal{L}_I[\psi,\overline{\psi}] \quad S_0 = \int d^4x d^4x'\; i\mathcal{S}_{ll'}'^{-1}(x,x')\overline{\psi}_l(x)\psi_{l'}(x')$$

微扰展开(变形):

$$Z[I,\bar{I}] = C \times e^{\int d^4xd^4x'\mathcal{S}_{ll'}'(x,x')\frac{\delta^2}{\delta\overline{\psi}_{l'}(x')\delta\psi_l(x)}} e^{i\int d^4x[\mathcal{L}_I[\psi,\overline{\psi}]+\bar{I}_l(x)\psi_l(x)+\overline{\psi}_l(x)I_l(x)]}\Bigg|_{\overline{\psi}=\psi=0}$$

$$(\Psi_0^-,\Psi_0^+) = C' \times e^{\int d^4xd^4x'\mathcal{S}_{ll'}'(x,x')\frac{\delta^2}{\delta\overline{\psi}_{l'}(x')\delta\psi_l(x)}} e^{i\int d^4x\;\mathcal{L}_I[\psi,\overline{\psi}]}\Bigg|_{\overline{\psi}=\psi=0}$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+) = C\times e^{\int d^4xd^4x'\mathcal{S}_{ll'}'(x,x')\frac{\delta^2}{\delta\overline{\psi}_{l'}(x')\delta\psi_l(x)}}\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n'}(x_n')\; e^{i\int d^4x\;\mathcal{L}_I[\psi,\overline{\psi}]}\Bigg|_{\overline{\psi}=\psi=0}$$

费米场的费曼图与费曼规则

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+) = (\Phi_0, \mathbf{T}\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n'}(x_n')\ e^{i\int d^4x\mathcal{L}_I[\psi,\overline{\psi}]}\Phi_0)$$ 所有可能的收缩

$$= C\times e^{\int d^4x d^4x' S'_{ll'}(x,x')\frac{\delta^2}{\delta\overline{\psi}_{l'}(x')\delta\psi_l(x)}}\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n'}(x_n')\ e^{i\int d^4x\mathcal{L}_I[\psi,\overline{\psi}]}\Big|_{\overline{\psi}=\psi=0}$$

- 每一个传播子必须微掉ψ_l和$\overline{\psi}_{l'}$场各一个(两个场的收缩得到传播子) 微商产生两δ函数抵消传播子的两个$\int d^4x$积分, 并将传播子的坐标取为被微掉的两个场的坐标 $\Rightarrow$每一个传播子$S'_{ll'}(x,x')$连接两个场,x,x'分别是ψ和$\overline{\psi}$两个场的坐标
- 相互作用顶角贡献位于同一时空点的若干个场$i\int d^4x\ \mathcal{L}_I[\psi(x),\overline{\psi}(x)]$, 外线贡献位于不同时空点的2n个场$\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')$
- 所有场都必须被全部微掉(收缩掉)$\Rightarrow$传播子止于顶角或外线
- 微商对所有的场是平权的
- 如果总共有k个顶角,应该有因子$\frac{1}{k!}$
- 封闭的费米子圈产生额外的-1 tr$[\underline{\psi_1\overline{\psi}_2}\ \underline{\psi_2\overline{\psi}_3}\cdots\underline{\psi_{n-1}\overline{\psi}_n}\ \underline{\psi_n\overline{\psi}_1}]$

存在一个用图形方式(顶角,传播子, 外线)按一定规则表达结果的方式: 费曼图

量子场论与经典拉格朗日量

$$\frac{(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}=\frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}}=\frac{\delta^n Z[J]}{i^n Z[J]\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\Big|_{J=0}$$

$$Z[J]\equiv\int\mathcal{D}\psi e^{iS_J[\psi]}=|\mathcal{N}|^{-2}(\Psi_0^-,\Psi_0^+)_J \qquad (\Psi_0^-,\Psi_0^+)=|\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]}$$

$$S_J=S[\psi]+\int d^4xJ_l(x)\psi_l(x) \qquad S=S_0+\int d^4x\ \mathcal{L}_I[\psi(x)] \qquad S_0=\frac{1}{2}\int d^4xd^4x'\ i\mathcal{D}'^{,-1}_{ll'}(x,x')\psi_l(x)\psi_{l'}(x')$$

经典拉格朗日体系:

$$S_0=\int d^4x\ \mathcal{L}_0 \qquad \mathcal{L}_0=\pi_l\dot{\psi}_l-\mathcal{H}_0 \qquad H_0=\int d\vec{x}\ \mathcal{H}_0 \qquad \pi_l=\frac{\partial\mathcal{L}_0}{\partial\dot{\psi}_l}$$

$$S=\int d^4x\mathcal{L} \qquad \mathcal{L}=\mathcal{L}_0-\mathcal{H} \qquad H=H_0+V=\int d\vec{x}\ [\pi_l\dot{\psi}_l-\mathcal{L}] \qquad V=\int d\vec{x}\ \mathcal{H} \qquad \pi_l\equiv\frac{\partial\mathcal{L}}{\partial\dot{\psi}_l}=\frac{\partial\mathcal{L}_0}{\partial\dot{\psi}_l}$$

$$0=\delta S=\int d^4x\ [\frac{\partial\mathcal{L}}{\partial\psi_l(x)}\delta\psi_l(x)+\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\delta\partial_\mu\psi_l(x)]=\int d^4x\ [\frac{\partial\mathcal{L}}{\partial\psi_l(x)}-\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}]\delta\psi_l(x)$$

$$\frac{\partial\mathcal{L}}{\partial\psi_l(x)}-\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}=0$$

经典拉格朗日场方程

量子场论与经典拉格朗日量

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\cdots\psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int\mathcal{D}\psi\ \psi_{l_1}(x_1)\cdots\psi_{l_n}(x_n)\ e^{iS[\psi]}}{\int\mathcal{D}\psi\ e^{iS[\psi]}} = \frac{\delta^n Z[J]}{i^n Z[J]\delta J_{l_1}(x_1)\cdots\delta J_{l_n}(x_n)}\bigg|_{J=0}$$

$$Z[J] \equiv \int\mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2}(\Psi_0^-, \Psi_0^+)_J \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2\int\mathcal{D}\psi\ e^{iS[\psi]} \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x)\psi_l(x)$$

$$0 = \delta S \quad \Rightarrow \quad \frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)} = 0$$

经典拉格朗日场方程

量子拉格朗日场方程:

$$0 = \int\mathcal{D}\psi e^{iS_J[\psi+\delta\psi]} - \int\mathcal{D}\psi e^{iS_J[\psi]} = i\int\mathcal{D}\psi\left[\delta S[\psi] + \int d^4x J_l(x)\delta\psi_l(x)\right] e^{iS_J[\psi]}$$

$$\delta S[\psi] = \int d^4x\left[\frac{\partial\mathcal{L}}{\partial\psi_l(x)}\delta\psi_l(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\delta\partial_\mu\psi_l(x)\right] = \int d^4x\ \delta\psi_l(x)\left[\frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\right]$$

$$\int\mathcal{D}\psi\left[\frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)} + J_l(x)\right]e^{iS_J[\psi]} = 0$$

Schwinger-Dyson方程

$$\int\mathcal{D}\psi O[\psi_l(y)]\left[\frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\right]e^{iS[\psi]} \xlongequal{[O(\frac{\partial}{\partial x}),x]=O'(\frac{\partial}{\partial x})} -\int\mathcal{D}\psi\ O'[\psi_l(y)]\delta(x-y)e^{iS[\psi]}$$

经典场的Noether定理：

拉格朗日量密度具有整体连续对称性是指它在场量的如下变换下保持不变：

$$\psi_i(x) \to \psi_l'(x) = \psi_l(x) + i\epsilon\mathcal{F}_l(x) \qquad \epsilon\text{是无穷小的常数}$$

$$\Rightarrow \quad \frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu\mathcal{F}_l(x) = 0$$

将变换参数取为依赖于时空坐标的参数$\epsilon(x)$

$$\delta S = \int d^4x\left[\frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x)i\epsilon(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu(\mathcal{F}_l(x)i\epsilon(x))\right] = -\int d^4x\mathcal{J}^\mu(x)\partial_\mu\epsilon(x) = \int d^4x\epsilon(x)\partial_\mu\mathcal{J}^\mu(x)$$

$$\mathcal{J}^\mu(x) \equiv -i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\mathcal{F}_l(x)$$ 整体连续对称性变换对应的"流" $$\frac{\delta S}{\delta\partial_\mu\epsilon(x)} = -\mathcal{J}^\mu(x)$$

若场满足拉格朗日方程

$$\frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)} = 0 \qquad \Rightarrow \qquad \partial_\mu\mathcal{J}^\mu(x) = 0$$

经典场的Noether定理：

拉格朗日量密度具有整体连续对称性是指它在场量的如下变换下保持不变：

$$\psi_i(x) \to \psi_l'(x) = \psi_l(x) + i\epsilon\mathcal{F}_l(x)$$ ϵ是无穷小的常数

$$\Rightarrow \quad \frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu\mathcal{F}_l(x) = 0$$ 若变换前后的差不为零，而是等于时空的全微商，则称 反常

$$\mathcal{J}^\mu(x) \equiv -i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\mathcal{F}_l(x)$$ 整体连续对称性变换对应的"流" $$\frac{\delta S}{\delta\partial_\mu\epsilon(x)} = -\mathcal{J}^\mu(x)$$

若场满足拉格朗日方程: $$\frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)} = 0 \Rightarrow \partial_\mu\mathcal{J}^\mu(x) = 0$$

拉格朗日量密度若不具有 下面整体连续对称性但场满足拉格朗日方程

$$\psi_i(x) \to \psi_l'(x) = \psi_l(x) + i\epsilon\mathcal{F}_l(x)$$ 仍可定义相应的流: $$\mathcal{J}^\mu(x) \equiv -i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\mathcal{F}_l(x)$$

$$\delta\mathcal{L} = \epsilon[\frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu\mathcal{F}_l(x)] = \epsilon\partial_\mu\mathcal{J}^\mu(x)$$

量子场的Noether定理：

拉格朗日量密度具有整体连续对称性是指它在场量的如下变换下保持不变：

$$\psi_l(x) \to \psi_l'(x) = \psi_l(x) + i\epsilon\mathcal{F}_l(x) \qquad \epsilon\text{是无穷小的常数}$$

$$\Rightarrow \quad \frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu\mathcal{F}_l(x) = 0 \xRightarrow{\epsilon\text{依赖}x\text{或不要求对称性但满足场方程}} \delta S = \int d^4x\epsilon(x)\partial_\mu\mathcal{J}^\mu(x)$$

$$\mathcal{J}^\mu(x) \equiv i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\mathcal{F}_l(x) \qquad \text{整体连续对称性变换对应的"流"} \qquad \frac{\delta S}{\delta\partial_\mu\epsilon(x)} = -\mathcal{J}^\mu(x)$$

$$0 = \int \mathcal{D}\psi[\delta S + \int d^4x\, J_l(x)\delta\psi_l(x)]e^{iS_J[\psi]} = \int \mathcal{D}\psi \int d^4x\,[\epsilon(x)\partial_\mu\mathcal{J}^\mu + J_l(x)i\epsilon(x)\mathcal{F}_l(x)]e^{iS_J[\psi]}$$

$$\Rightarrow \quad \int \mathcal{D}\psi\,[\partial_\mu\mathcal{J}^\mu + iJ_l(x)\mathcal{F}_l(x)]\,e^{iS_J[\psi]} = 0 \qquad \underline{\textbf{Ward-Takahashi-Taylor 恒等式}}$$

对称性"荷"： $$Q(t) \equiv \int d\vec{x}\, J^0(x) = -i\int d\vec{x}\,\pi_l(\vec{x},t)\mathcal{F}_l(\vec{x},t) \qquad \pi_l \equiv \frac{\partial\mathcal{L}}{\partial\dot{\psi}_l}$$

经典: $\partial_\mu\mathcal{J}^\mu = 0 \;\Rightarrow\; \dot{Q}(t) = 0$ 量子: $\int \mathcal{D}\psi\,[\dot{Q}(t) + i\int d^3x\, J_l(\vec{x},t)\mathcal{F}_l(\vec{x},t)]\,e^{iS_J[\psi]} = 0$

$$[\psi_l(\vec{x},t), \pi_{\bar{l}}(\vec{x},t)] = i\delta(\vec{x}-\vec{y})\delta_{l\bar{l}} \quad \Rightarrow \quad [Q(t), \psi_l(\vec{x},t)] = \mathcal{F}(\vec{x},t) \qquad \text{起场变换生成元的作用}$$

$$\psi_l(x) \to \psi_l'(x') = \psi_l(x) \qquad \psi_l'(x) = \psi_l(x-\epsilon) = \psi_l(x) - \epsilon^\mu \partial_\mu \psi_l(x)$$

能动量守恒与时空平移 $x \to x' = x + \epsilon$

$$\delta S = \int d^4x \mathcal{L}[\psi'(x')] - \int d^4x \mathcal{L}[\psi(x)] = \int d^4x \{\mathcal{L}[\psi'(x')] - \mathcal{L}[\psi'(x)] + \mathcal{L}[\psi'(x)] - \mathcal{L}[\psi(x)]\}$$

$$= \int d^4x \, [\frac{\partial \mathcal{L}}{\partial x^\mu}\epsilon^\mu - \frac{\partial \mathcal{L}}{\partial \psi_l(x)}\epsilon^\mu \partial_\mu \psi_l(x) - \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)}\epsilon^\mu \partial_\nu \partial_\mu \psi_l(x)]$$

$$= \int d^4x \, \{\frac{\partial \mathcal{L}}{\partial x^\mu}\epsilon^\mu - \partial_\nu [\frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)}]\epsilon^\mu \partial_\mu \psi_l(x) - \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)}\epsilon^\mu \partial_\nu \partial_\mu \psi_l(x)\}$$

$$= -\epsilon^\mu \int d^4x \, \partial_\nu [-g^\nu_\mu \mathcal{L} + \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)}\partial_\mu \psi_l(x)] \qquad T^\mu_\nu \equiv -\mathcal{L} g^\mu_\nu + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)}\partial_\nu \psi_l(x)$$

$$P_\mu(t) = \int d\vec{x} \, T^0{}_\mu = \int d\vec{x} \, [-\mathcal{L} g^0_\mu + \pi_l(x)\partial_\mu \psi_l(x)]$$

$H \equiv P^0 = \int d\vec{x} \, [-\mathcal{L} + \pi_l(x)\dot{\psi}_l(x)]$ $\qquad \vec{P} \equiv \int d\vec{x} \, \pi_l(x) \nabla \psi_l(x)$ 平移生成元的场算符表示 $\qquad \mathcal{F}_{l,\mu}(x) = i\partial_\mu \psi_l(x)$

$$\Rightarrow [P^\mu(t), \psi_l(\vec{x},t)] = -i\partial^\mu \psi_l(\vec{x},t) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} p^\mu [-e^{-ip\cdot x} u(\vec{p},\sigma) a(\vec{p},\sigma) + e^{ip\cdot x} v(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma)]$$

$$\int \mathcal{D}\psi \, [\dot{P}^\mu(t) - \int d^3x J_l(\vec{x},t)\partial_\mu \psi_l(\vec{x},t)] e^{iS_J[\psi]} = 0$$

角动量守恒与时空转动 $x^\mu \to x^{\mu\prime} = \Lambda^\mu{}_\nu x^\nu$

$$\Lambda^\mu{}_\nu = g^\mu{}_\nu + \omega^\mu{}_\nu \quad \omega_{\mu\nu} = -\omega_{\nu\mu} \quad \psi^l(x) \to \psi_l'(x') = \psi_l(x) + \frac{i}{2}\omega^{\mu\nu}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x)$$

$$\psi_l'(x) = \psi_l(\Lambda^{-1}x) + \frac{i}{2}\omega^{\mu\nu}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x) = \psi_l(x) - \omega^{\mu\nu}x_\nu\partial_\mu\psi_l(x) + \frac{i}{2}\omega^{\mu\nu}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x)$$

$$\delta S = \int d^4x\mathcal{L}[\psi'(x')] - \int d^4x\mathcal{L}[\psi(x)] = \int d^4x\{\mathcal{L}[\psi'(x')] - \mathcal{L}[\psi'(x)] + \mathcal{L}[\psi'(x)] - \mathcal{L}[\psi(x)]\}$$

$$= \int d^4x\,\{\frac{\partial\mathcal{L}}{\partial x^\mu}\omega^{\mu\nu}x_\nu + \frac{\partial\mathcal{L}}{\partial\psi_l(x)}\omega^{\sigma\rho}[-x_\rho\partial_\sigma\psi_l(x) + \frac{i}{2}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)]$$

$$+ \frac{\partial\mathcal{L}}{\partial\partial_\nu\psi_l(x)}\omega^{\sigma\rho}\partial_\nu[-x_\rho\partial_\sigma\psi_l(x) + \frac{i}{2}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)]\}$$

$$= \int d^4x\,\{\partial_\mu[x_\nu\mathcal{L}]\omega^{\mu\nu} + \partial_\nu[\frac{\partial\mathcal{L}}{\partial\partial_\nu\psi_l(x)}]\omega^{\sigma\rho}[-x_\rho\partial_\sigma\psi_l(x) + \frac{i}{2}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)]$$

$$+ \frac{\partial\mathcal{L}}{\partial\partial_\nu\psi_l(x)}\omega^{\sigma\rho}\partial_\nu[-x_\rho\partial_\sigma\psi_l(x) + \frac{i}{2}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)]\}$$

$$= \omega^{\sigma\rho}\int d^4x\,\partial_\nu\{x_\rho\mathcal{L}g^\nu{}_\sigma + \frac{\partial\mathcal{L}}{\partial\partial_\nu\psi_l(x)}[-x_\rho\partial_\sigma\psi_l(x) + \frac{i}{2}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)]\}$$

拉格朗日体系

$$x^\mu \to x^{\mu\prime} = \Lambda^\mu{}_\nu x^\nu \quad \Lambda^\mu{}_\nu = g^\mu{}_\nu + \omega^\mu{}_\nu \quad \omega_{\mu\nu} = -\omega_{\nu\mu} \quad \psi_l(x) \to \psi_l'(x') = \psi_l(x) + \frac{i}{2}\omega^{\mu\nu}(\mathcal{J}_{\mu\nu})_l{}^m\psi^m(x)$$

$$\psi_l'(x) = \psi_l(\Lambda^{-1}x) + \frac{i}{2}\omega^{\mu\nu}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x) = \psi_l(x) - \omega^{\mu\nu}x_\nu\partial_\mu\psi_l(x) + \frac{i}{2}\omega^{\mu\nu}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x)$$

$$\delta S = \omega^{\sigma\rho}\int d^4x\,\partial_\nu\{x_\rho\mathcal{L}g^\nu{}_\sigma + \frac{\partial\mathcal{L}}{\partial\partial_\nu\psi_l(x)}[-x_\rho\partial_\sigma\psi_l(x) + \frac{i}{2}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)]\}$$

$$= \frac{1}{2}\omega^{\sigma\rho}\int d^4x\,\partial_\nu\{x_\sigma T^\nu{}_\rho - x_\rho T^\nu{}_\sigma + i\frac{\partial\mathcal{L}}{\partial\partial_\nu\psi_l(x)}(\mathcal{J}_{\sigma\rho})_l{}^m\psi_m(x)\}$$

$$\mathcal{M}^{\mu\sigma\rho} \equiv x^\sigma T^{\mu\rho} - x^\rho T^{\mu\sigma} + i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}(\mathcal{J}^{\sigma\rho})_l{}^m\psi_m(x) \qquad T^\mu_\nu \equiv -\mathcal{L}g^\mu_\nu + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\nu\psi_l(x)$$

$$J^{\mu\nu}(t) = \int d\vec{x}\,\mathcal{M}^{0\mu\nu} = \int d\vec{x}\,[x^\mu T^{0\nu} - x^\nu T^{0\mu} + i\pi_l(x)(\mathcal{J}^{\mu\nu})_l{}^m\psi_m(x)]$$ 转动生成元的场算符表示

$$J^{\mu\nu} = -J^{\nu\mu} \quad J^\mu{}_\mu = 0 \qquad J^{ij} = L^{ij} + S^{ij} \qquad L^{ij} = \int d\vec{x}\,(x^iP^j - x^jP^i) \qquad S^{ij} = i\int d\vec{x}\,\pi_l(x)(\mathcal{J}^{ij})_l{}^m\psi_m(x)$$

无穷小独立变换参数: $1/2\omega^{\mu\nu} \;\Rightarrow\; \mathcal{F}_{l,\mu\nu}(x) = \frac{i}{2}(x_\nu\partial_\mu - x_\mu\partial_\nu)\psi_l(x) + \frac{1}{2}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x)$

$$[J_{\mu\nu}(t), \psi_l(\vec{x}, t)] = \frac{i}{2}(x_\nu\partial_\mu - x_\mu\partial_\nu)\psi_l(x) + \frac{1}{2}(\mathcal{J}_{\mu\nu})_l{}^m\psi_m(x)$$

$\lambda\phi^4$理论概况

经典理论:

$$\mathcal{L}[\phi(x)] = \frac{1}{2}[\partial_\mu\phi(x)][\partial^\mu\phi(x)] - \frac{1}{2}M^2\phi^2(x) - \frac{\lambda}{4!}\phi^4(x)$$

- 正常: M为实数; $\lambda \geq 0$
- M为虚数 $\Rightarrow$ 非稳定系统 自发破缺!
- $\lambda = 0 \Rightarrow$ 自由场!
- $\lambda < 0 \Rightarrow$ 系统无定义(能量无下界)!

量子理论:

- 理论上最简单的$3+1$维量子场理论
- 现实中最不清楚的$3+1$维量子场理论
- 高于和等于$3+1$维的理论平庸(有限大λ导致零观测耦合强度) (2,0)理论?
- 低于$3+1$维的理论是非平庸的. 构造场论的研究对象!

$\lambda\phi^4$理论的强耦合极限

格点化: D维欧氏空间；N个分量

$$\mathcal{L}_{\lambda\phi^4}[\vec{\phi}(x)] = -\frac{1}{2}[\partial_\mu\vec{\phi}(x)]\cdot[\partial^\mu\vec{\phi}(x)] - \frac{1}{2}M^2\vec{\phi}^2(x) - \frac{\lambda}{4!}[\vec{\phi}^2(x)]^2$$

$$= -\frac{1}{2}\lim_{\Delta\to 0}\sum_\mu\left[\frac{\vec{\phi}(x_\mu+\Delta)-\vec{\phi}(x_\mu)}{\Delta}\right]^2 - \frac{1}{2}M^2\vec{\phi}^2(x) - \frac{\lambda}{4!}[\vec{\phi}^2(x)]^2$$

$$= -\frac{1}{2}\lim_{\Delta\to 0}\sum_\mu\frac{\vec{\phi}^2(x_\mu+\Delta)+\vec{\phi}^2(x_\mu)-2\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2} - \frac{1}{2}M^2\vec{\phi}^2(x) - \frac{\lambda}{4!}[\vec{\phi}^2(x)]^2$$

强耦合极限:

$$e^{-W_{\lambda\phi^4}[\vec{J}]} \equiv \int[\mathcal{D}\vec{\phi}]\, e^{-\int d^Dx\{\mathcal{L}_{\lambda\phi^4}[\vec{\phi}(x)]+\vec{J}(x)\cdot\vec{\phi}(x)\}}$$

$$= \int[\mathcal{D}\vec{\phi}]\, e^{-\int d^Dx\{-\frac{1}{2}\sum_\mu\frac{\vec{\phi}^2(x_\mu+\Delta)+\vec{\phi}^2(x_\mu)-2\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2}-\frac{1}{2}M^2\vec{\phi}^2(x)-\frac{\lambda}{4!}[\vec{\phi}^2(x)]^2+\vec{J}(x)\cdot\vec{\phi}(x)\}}$$

$$= \int[\mathcal{D}\vec{\phi}]\, e^{-\int d^Dx\{\sum_\mu\frac{\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2}-\frac{1}{2}(\frac{2}{\Delta^2}+M^2)\vec{\phi}^2(x)-\frac{\lambda}{4!}[\vec{\phi}^2(x)]^2+\vec{J}(x)\cdot\vec{\phi}(x)\}}$$

$\lambda\phi^4$理论的费曼图

强耦合极限：

$$e^{-W_{\lambda\phi^4}[\vec{J}]}=\int[\mathcal{D}\vec{\phi}]\,e^{-\int d^Dx\{\sum_\mu\frac{\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2}-\frac{1}{2}(\frac{2}{\Delta^2}+M^2)\vec{\phi}^2(x)-\frac{\lambda}{4!}[\vec{\phi}^2(x)]^2+\vec{J}(x)\cdot\vec{\phi}(x)\}}$$

$$=\int[\mathcal{D}\vec{\phi}]\,e^{-\sum_i\{\sum_{i'\,\mathrm{nearest}}\Delta^{D-2}\vec{\phi}_{i'}\cdot\vec{\phi}_i-\frac{1}{2}(2\Delta^{D-2}+\Delta^DM^2)\vec{\phi}_i^2-\Delta^D\frac{\lambda}{4!}[\vec{\phi}_i^2]^2+\Delta^D\vec{J}_i\cdot\vec{\phi}_i\}}$$

$$\overset{\vec{S}_i\equiv\Delta^{D/2-1}\vec{\phi}_i}{=\!=\!=\!=\!=\!=\!=}\int[\mathcal{D}\vec{S}]\,e^{-\sum_i\{\sum_{i'\,\mathrm{nearest}}\vec{S}_{i'}\cdot\vec{S}_i-\frac{1}{2}(2+\Delta^2M^2)\vec{S}_i^2-\Delta^{4-D}\frac{\lambda}{4!}[\vec{S}_i^2]^2+\Delta^{1+D/2}\vec{J}_i\cdot\vec{S}_i\}}$$

$$=\int[\mathcal{D}\vec{S}]\,e^{-\sum_i\{\sum_{i'\,\mathrm{nearest}}\vec{S}_{i'}\cdot\vec{S}_i-\Delta^{4-D}\frac{\lambda}{4!}[\vec{S}_i^2+\frac{6}{\lambda}\Delta^{D-4}(2+\Delta^2M^2)]^2+\Delta^{1+D/2}\vec{J}_i\cdot\vec{S}_i\}}$$

$$\delta(x)=\lim_{\lambda\to\infty}\sqrt{\frac{\lambda}{4!\pi}}e^{-\frac{\lambda}{4!}x^2}\quad\overset{\lambda\to\infty}{=\!=\!=\!=}\int[\mathcal{D}\vec{S}]\left[\prod_i\delta[\vec{S}_i^2+\frac{6}{\lambda}\Delta^{D-4}(2+\Delta^2M^2)]\right]e^{-\sum_i\{\sum_{i'\,\mathrm{nearest}}\vec{S}_{i'}\cdot\vec{S}_i+\Delta^{1+D/2}\vec{J}_i\cdot\vec{S}_i\}}$$

$$\overset{N\equiv2s+1,\quad M^2=-\frac{\lambda}{6}s(s-1)\Delta^{2-D}-2\Delta^{-2}}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=}\int[\mathcal{D}\vec{S}]\left[\prod_i\delta[\vec{S}_i^2-s(s+1)]\right]e^{-\sum_i\left\{\sum_{i'\,\mathrm{nearest}}\vec{S}_{i'}\cdot\vec{S}_i+\Delta^{1+D/2}\vec{J}_i\cdot\vec{S}_i\right\}}$$

D维Ising模型

$\lambda\phi^4$理论的费曼规则

$$\mathcal{L}[\phi(x)]=\frac{1}{2}[\partial_\mu\phi(x)][\partial^\mu\phi(x)]-\frac{1}{2}M^2\phi^2(x)-\frac{\lambda}{4!}\phi^4(x) \quad \mathcal{D}'(x,x')=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-x')}\frac{i}{p^2-M^2+i0^+}$$

$$(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_n)\Psi_0^+)=C'e^{\frac{1}{2}\int d^4xd^4x'\mathcal{D}'(x,x')\frac{\delta^2}{\delta\phi(x)\delta\phi(x')}}\phi(x_1)\cdots\phi(x_n)e^{\frac{-i\lambda}{4!}\int d^4x\ \phi^4(x)}\Big|_{\phi(x)=0}$$

$$=(\Phi_0,\mathbf{T}[\phi(x_1)\cdots\phi(x_n)e^{\frac{-i\lambda}{4!}\int d^4x\ \phi^4(x)}]\Phi_0)$$

坐标空间:

- 对n点连通格林函数$(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_n)\Psi_0^+)_C$,标出$n$个时空点$x_1,\cdots,x_n$,每个点引出一条线
- 对m阶相互作用,分别在m个时空点$y_1,\cdots,y_m$引入相互作用顶点,每个顶点引出四条线
- 将引出的$n+4m$条线两两相连,得到所有可能的拓扑不等价的连接图
- 每条连线代表$\mathcal{D}'(x,x')=\phi(x)\phi(x')\equiv\Delta(x,x')$,每个顶点代表$-i\lambda$,需要对所有顶点的时空坐标求积分
- 若图中出现对称连线,必须乘以对称因子$1/S$

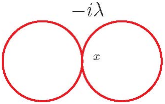
(S: 每个单个圈有因子2;连接两点的n条线有因子$n!$;真空图有镜像对称加因子2)

坐标空间$\lambda\phi^4$理论的费曼图——连接真空图

$$\frac{1}{2!}\int d^4xd^4x'd^4yd^4y'\frac{1}{2}\mathcal{D}'(x,x')\frac{1}{2}\mathcal{D}'(y,y')\frac{\delta^2}{\delta\phi(x)\delta\phi(x')}\frac{\delta^2}{\delta\phi(y)\delta\phi(y')}\frac{-i\lambda}{4!}\int d^4z\,\phi^4(z)$$

$$=(\Phi_0,\mathbf{T}[\frac{-i\lambda}{4!}\int d^4x\,\phi^4(x)]\Phi_0)=3\frac{-i\lambda}{4!}\int d^4x\,\phi(x)\phi(x)\,\phi(x)\phi(x)$$

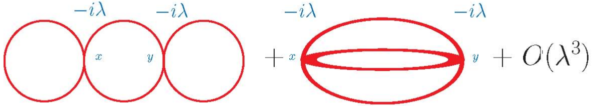
$$=\frac{-i\lambda}{8}\int d^4x\,\Delta^2(x,x)$$



$$\frac{1}{4!}\int d^4x_1d^4x_1'd^4x_2d^4x_2'd^4x_3d^4x_3'd^4x_4d^4x_4'\frac{1}{2}\mathcal{D}'(x_1,x_1')\frac{1}{2}\mathcal{D}'(x_2,x_2')\frac{1}{2}\mathcal{D}'(x_3,x_3')\frac{1}{2}\mathcal{D}'(x_4,x_4')$$

$$\times\frac{\delta^2}{\delta\phi(x_1)\delta\phi(x_1')}\frac{\delta^2}{\delta\phi(x_2)\delta\phi(x_2')}\frac{\delta^2}{\delta\phi(x_3)\delta\phi(x_3')}\frac{\delta^2}{\delta\phi(x_4)\delta\phi(x_4')}\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2\int d^4zd^4z'\,\phi^4(z)\phi^4(z')$$

$$=\frac{(-i\lambda)^2}{16}\int d^4xd^4y\,\Delta(x,x)\Delta^2(x,y)\Delta(y,y)+\frac{(-i\lambda)^2}{48}\int d^4xd^4y\,\Delta^4(x,y)+O(\lambda^3)$$

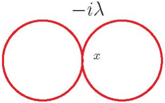


坐标空间$\lambda\phi^4$理论的费曼图——连接真空图

$$\frac{1}{2!}\int d^4xd^4x'd^4yd^4y'\frac{1}{2}\mathcal{D}'(x,x')\frac{1}{2}\mathcal{D}'(y,y')\frac{\delta^2}{\delta\phi(x)\delta\phi(x')}\frac{\delta^2}{\delta\phi(y)\delta\phi(y')}\frac{-i\lambda}{4!}\int d^4z\,\phi^4(z)$$

$$=(\Phi_0,\mathbf{T}[\frac{-i\lambda}{4!}\int d^4x\,\phi^4(x)]\Phi_0)=3\frac{-i\lambda}{4!}\int d^4x\,\overline{\phi(x)\phi(x)}\;\overline{\phi(x)\phi(x)}$$

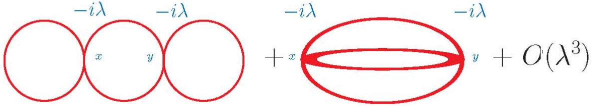
$$=\frac{-i\lambda}{8}\int d^4x\,\Delta^2(x,x)$$



$$(\Phi_0,\mathbf{T}[\frac{1}{2!}(\frac{-i\lambda}{4!})^2\int d^4xd^4y\,\phi^4(x)\phi^4(y)]\Phi_0)+O(\lambda^3)$$

$$=\frac{1}{2!}(\frac{-i\lambda}{4!})^2\int d^4xd^4y\Big[(2*3)^2*2\,\overline{\phi(x)\phi(x)}\,[\overline{\phi(x)\phi(y)}]^2\,\overline{\phi(y)\phi(y)}+4!\,[\overline{\phi(x)\phi(y)}]^4\Big]+O(\lambda^3)$$

$$=\frac{(-i\lambda)^2}{16}\int d^4xd^4y\,\Delta(x,x)\Delta^2(x,y)\Delta(y,y)+\frac{(-i\lambda)^2}{48}\int d^4xd^4y\,\Delta^4(x,y)+O(\lambda^3)$$



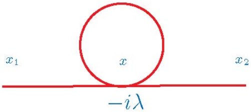
坐标空间$\lambda\phi^4$理论的费曼图——连接2点格林函数图

$$\frac{1}{3!}\int d^4x d^4x' d^4y d^4y' d^4z d^4z' \frac{1}{2}\mathcal{D}'(x,x')\frac{1}{2}\mathcal{D}'(y,y')\frac{1}{2}\mathcal{D}'(z,z')\frac{\delta^2}{\delta\phi(x)\delta\phi(x')}\frac{\delta^2}{\delta\phi(y)\delta\phi(y')}\frac{\delta^2}{\delta\phi(z)\delta\phi(z')}$$

$$\times\frac{-i\lambda}{4!}\int d^4y_1\ \phi(x_1)\phi(x_2)\phi^4(y_1)$$

$$=(\Phi_0,\mathbf{T}[\frac{-i\lambda}{4!}\int d^4x\ \phi(x_1)\phi(x_2)\phi^4(x)]\Phi_0)=4*3\ \frac{-i\lambda}{4!}\int d^4x\ \phi(x_1)\phi(x)\ \phi(x)\phi(x)\ \phi(x)\phi(x_2)$$

$$=\frac{-i\lambda}{2}\int d^4x\Delta(x_1,x)\Delta(x,x)\Delta(x,x_2)$$



坐标空间$\lambda\phi^4$理论的费曼图——连接2点格林函数图

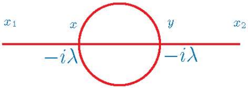
$$\frac{1}{5!}\int d^4y_1 d^4y_1' d^4y_2 d^4y_2' d^4y_3 d^4y_3' d^4y_4 d^4y_4' d^4y_5 d^4y_5' \frac{1}{2}\mathcal{D}'(y_1,y_1')\frac{1}{2}\mathcal{D}'(y_2,y_2')\frac{1}{2}\mathcal{D}'(y_3,y_3')\frac{1}{2}\mathcal{D}'(y_4,y_4')$$

$$\times\frac{1}{2}\mathcal{D}'(y_5,y_5')\frac{\delta^2}{\delta\phi(y_1)\delta\phi(y_1')}\frac{\delta^2}{\delta\phi(y_2)\delta\phi(y_2')}\frac{\delta^2}{\delta\phi(y_3)\delta\phi(y_3')}\frac{\delta^2}{\delta\phi(y_4)\delta\phi(y_4')}\frac{\delta^2}{\delta\phi(y_5)\delta\phi(y_5')}$$

$$\times\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2\int d^4z_1 d^4z_2\ \phi(x_1)\phi(x_2)\phi^4(z_1)\phi^4(z_2)$$

$$=\int d^4x d^4y\ \left[\frac{(-i\lambda)^2}{6}\Delta(x_1,x)\Delta^3(x,y)\Delta(y,x_2)+\frac{(-i\lambda)^2}{4}\Delta(x_1,x)\Delta(x,x)\Delta(x,y)\Delta(y,y)\Delta(y,x_2)\right.$$

$$\left.+\frac{(-i\lambda)^2}{4}\Delta(x_1,x)\Delta(x,y)\Delta(y,y)\Delta(y,x)\Delta(x,x_2)\right]+O(\lambda^3)$$



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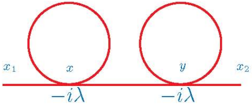
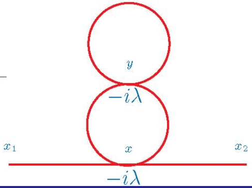


$+$



$\lambda\phi^4$理论的费曼图

$$(\Phi_0, \mathbf{T}[\frac{1}{2!}(\frac{-i\lambda}{4!})^2 \int d^4x d^4y\, \phi(x_1)\phi(x_2)\phi^4(x)\phi^4(y)]\Phi_0)$$

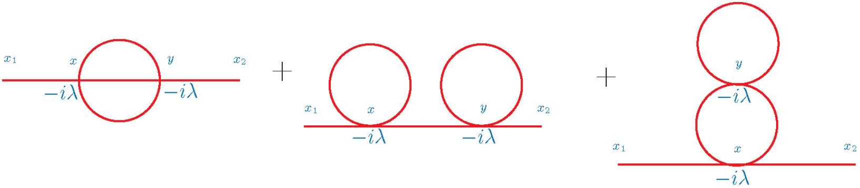
$$= \frac{1}{2!}\,(\frac{-i\lambda}{4!})^2 \int d^4x d^4y \Big[\, 8*4*3*2\ \overline{\phi(x_1)\phi(x)}\ [\overline{\phi(x)\phi(y)}]^3\ \overline{\phi(y)\phi(x_2)}$$

$$+\, 8*4*3*3\ \overline{\phi(x_1)\phi(x)}\ \overline{\phi(x)\phi(x)}\ \overline{\phi(x)\phi(y)}\ \overline{\phi(y)\phi(y)}\ \overline{\phi(y)\phi(x_2)}$$

$$+\, 8*3*4*3\ \overline{\phi(x_1)\phi(x)}\ \overline{\phi(x)\phi(y)}\ \overline{\phi(y)\phi(y)}\ \overline{\phi(y)\phi(x)}\ \overline{\phi(x)\phi(x_2)}\ \Big]$$

$$= \int d^4x d^4y\, [\frac{(-i\lambda)^2}{6}\Delta(x_1,x)\Delta^3(x,y)\Delta(y,x_2) + \frac{(-i\lambda)^2}{4}\Delta(x_1,x)\Delta(x,x)\Delta(x,y)\Delta(y,y)\Delta(y,x_2)$$

$$+\, \frac{(-i\lambda)^2}{\;}\Delta(x_1,x)\Delta(x,y)\Delta(y,y)\Delta(y,x)\Delta(x,x_2)] + O(\lambda^3)$$



坐标空间$\lambda\phi^4$理论的费曼图——连接4点格林函数图

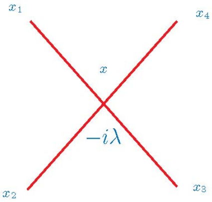
$$\frac{1}{4!}\int d^4y_1d^4y_1'd^4y_2d^4y_2'd^4y_3d^4y_3'd^4y_4d^4y_4'\frac{1}{2}\mathcal{D}'(y_1,y_1')\frac{1}{2}\mathcal{D}'(y_2,y_2')\frac{1}{2}\mathcal{D}'(y_3,y_3')\frac{1}{2}\mathcal{D}'(y_4,y_4')$$

$$\times\frac{\delta^2}{\delta\phi(y_1)\delta\phi(y_1')}\frac{\delta^2}{\delta\phi(y_2)\delta\phi(y_2')}\frac{\delta^2}{\delta\phi(y_3)\delta\phi(y_3')}\frac{\delta^2}{\delta\phi(y_4)\delta\phi(y_4')}\frac{-i\lambda}{4!}\int d^4z\,\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\phi^4(z)$$

$$=(\Phi_0,\mathbf{T}[\frac{-i\lambda}{4!}\int d^4x\,\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\phi^4(x)]\Phi_0)$$

$$=\frac{-i\lambda}{4!}\int d^4x\,4!\,\phi(x_1)\phi(x)\ \phi(x_2)\phi(x)\ \phi(x)\phi(x_3)\ \phi(x)\phi(x_4)$$

$$=-i\lambda\int d^4x\,\Delta(x_1,x)\Delta(x_2,x)\Delta(x,x_3)\Delta(x,x_4)$$



坐标空间$\lambda\phi^4$理论的费曼图——连接4点格林函数图

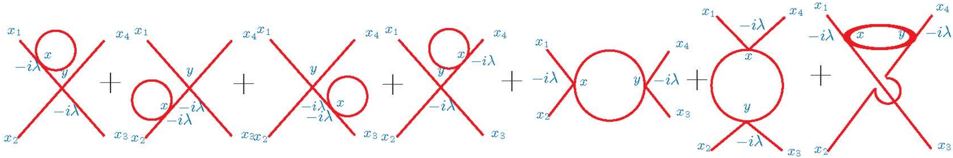
$$\frac{1}{6!}\int d^4y_1d^4y_1'd^4y_2d^4y_2'd^4y_3d^4y_3'd^4y_4d^4y_4'd^4y_5d^4y_5'd^4y_6d^4y_6'\frac{1}{2}\mathcal{D}'(y_1,y_1')\frac{1}{2}\mathcal{D}'(y_2,y_2')\frac{1}{2}\mathcal{D}'(y_3,y_3')$$

$$\times\frac{1}{2}\mathcal{D}'(y_4,y_4')\frac{1}{2}\mathcal{D}'(y_5,y_5')\frac{1}{2}\mathcal{D}'(y_6,y_6')\frac{\delta^2}{\delta\phi(y_1)\delta\phi(y_1')}\frac{\delta^2}{\delta\phi(y_2)\delta\phi(y_2')}\frac{\delta^2}{\delta\phi(y_3)\delta\phi(y_3')}\frac{\delta^2}{\delta\phi(y_4)\delta\phi(y_4')}$$

$$\times\frac{\delta^2}{\delta\phi(y_5)\delta\phi(y_5')}\frac{\delta^2}{\delta\phi(y_6)\delta\phi(y_6')}\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2\int d^4zd^4z'\ \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\phi^4(z)\phi^4(z')+O(\lambda^3)$$

$$=\frac{(-i\lambda)^2}{2}\int d^4xd^4y\ \Delta(x_1,x)\Delta(x,x)\Delta(x,y)\Delta(x_2,y)\Delta(y,x_3)\Delta(y,x_4)+x_1\Leftrightarrow x_2,\ x_3,\ x_4$$

$$+\frac{(-i\lambda)^2}{2}\int d^4xd^4y\ \Delta(x_1,x)\Delta(x_2,x)\Delta^2(x,y)\Delta(y,x_3)\Delta(y,x_4)+x_2\Leftrightarrow x_3,\ x_4+O(\lambda^3)$$



$\lambda\phi^4$理论的费曼图

$$= (\Phi_0, \mathbf{T}[\frac{1}{2!}(\frac{-i\lambda}{4!})^2 \int d^4x d^4y\, \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\phi^4(x)\phi^4(y)]\Phi_0) + O(\lambda^3)$$

$$= \frac{1}{2!}(\frac{-i\lambda}{4!})^2 \int d^4x d^4y [8*3*2*4*3\, \phi(x_1)\phi(x)\ \phi(x)\phi(x)\ \phi(x)\phi(y)\ \phi(x_2)\phi(y)\ \phi(y)\phi(x_3)\ \phi(y)\phi(x_4)$$

$$+ 8*3*2*4*3\ \phi(x_1)\phi(y)\ \phi(x_2)\phi(x)\ \phi(x)\phi(x)\ \phi(x)\phi(y)\ \phi(y)\phi(x_3)\ \phi(y)\phi(x_4)$$

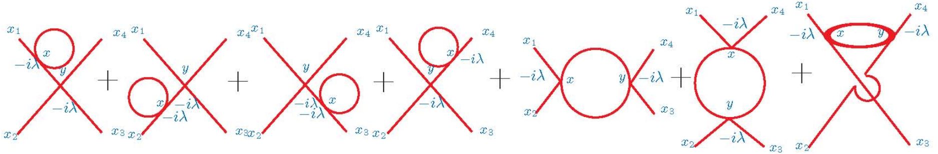
$$+ 8*3*2*4*3\ \phi(x_1)\phi(y)\ \phi(x_2)\phi(y)\ \phi(x)\phi(x_3)\ \phi(x)\phi(x)\ \phi(x)\phi(y)\ \phi(y)\phi(x_4)$$

$$+ 8*3*2*4*3\ \phi(x_1)\phi(y)\ \phi(x_2)\phi(y)\ \phi(y)\phi(x_3)\ \phi(x)\phi(x)\ \phi(x)\phi(y)\ \phi(x)\phi(x_4)$$

$$+ 8*3*4*3*2\ \phi(x_1)\phi(x)\ \phi(x_2)\phi(x)\ [\phi(x)\phi(y)]^2\ \phi(y)\phi(x_3)\ \phi(y)\phi(x_4)\ ] + O(\lambda^3)$$

$$= \frac{(-i\lambda)^2}{2} \int d^4x d^4y\, \Delta(x_1,x)\Delta(x,x)\Delta(x,y)\Delta(x_2,y)\Delta(y,x_3)\Delta(y,x_4) + x_1 \Leftrightarrow x_2, x_3, x_4$$

$$+(-i\lambda)^2/2 \int d^4x d^4y\, \Delta(x_1,x)\Delta(x_2,x)\Delta^2(x,y)\Delta(y,x_3)\Delta(y,x_4) + x_2 \Leftrightarrow x_3,\ x_4 + O(\lambda^3)$$



$\lambda\phi^4$理论的费曼规则

$$\mathcal{L}[\phi(x)]=\frac{1}{2}[\partial_\mu\phi(x)][\partial^\mu\phi(x)]-\frac{1}{2}M^2\phi^2(x)-\frac{\lambda}{4!}\phi^4(x) \quad \mathcal{D}'(x,x')=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-x')}\frac{i}{p^2-M^2+i0+}$$

$$G(q_1,\cdots,q_n)(2\pi)^4\delta(q_1+\cdots q_n)=\int d^4x_1\cdots d^4x_n\;e^{iq_1\cdot x_1+\cdots+iq_n\cdot x_n}(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_n)\Psi_0^+)$$

动量空间:

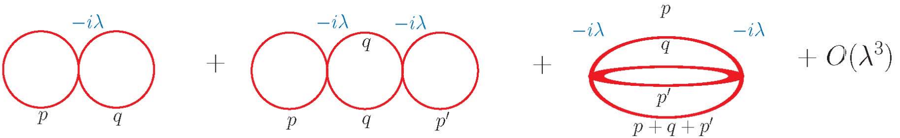
- 对n点连通格林函数$G(q_1,\cdots,q_n)_C$,标出n个外点,每个点引出一条线对应流入动量$q_1,\cdots,q_n$
- 对m阶相互作用,分别在m个点引入相互作用顶点,每个顶点引出四条线
- 将引出的$n+4m$条线两两相连,每条线具有一个四动量，每个顶点有一δ函数$(2\pi)^4\delta(p_1+p_2+p_3+p_4)$,其中$p_i$是流入顶角的四动量。得到所有可能的拓扑不等价的连接图
- 每条连线代表$\frac{i}{p^2-M^2+i0+}$,每个顶点代表$-i\lambda$,需对所有动量求积分$\int\frac{d^4p}{(2\pi)^4}$
- 如果图中出现对称的连线,必须乘以对称因子$1/S$

(S: 每个单个圈有因子2；连接两点的n条线有因子$n!$；真空图有镜像对称加因子2)

动量空间$\lambda\phi^4$理论的费曼图——连接真空图

$$\Delta(p) \equiv i/(p^2 - M^2 + i0^+) \qquad (2\pi)^4\delta(p) \overset{p=0}{=\!=\!=} \int d^4x$$

$$\int d^4x \left[\frac{-i\lambda}{8}\int\frac{d^4p d^4q}{(2\pi)^8}\Delta(p)\Delta(q) + \frac{(-i\lambda)^2}{16}\int\frac{d^4p d^4q d^4p'}{(2\pi)^{12}}\Delta(p)\Delta^2(q)\Delta(p') + \frac{(-i\lambda)^2}{48}\int\frac{d^4p d^4q d^4p'}{(2\pi)^{12}}\Delta(p)\Delta(q)\Delta(p')\Delta(p+q+p')\right] + O(\lambda^3)$$



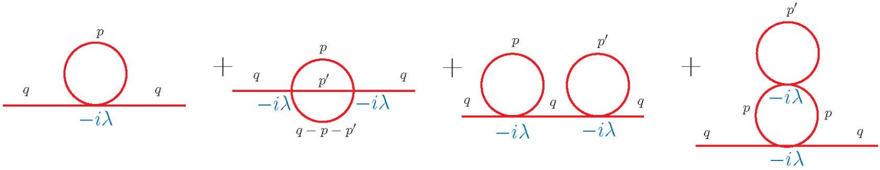
动量空间$\lambda\phi^4$理论的费曼图——连接2点格林函数图

$$\Delta(p) \equiv i/(p^2 - M^2 + i0^+) \qquad (2\pi)^4\delta(p) \overset{p=0}{=\!=\!=} \int d^4x$$

$$\frac{-i\lambda}{2}\int\frac{d^4p}{(2\pi)^4}\Delta(q)\Delta(p)\Delta(q) + \frac{(-i\lambda)^2}{6}\int\frac{d^4pd^4p'}{(2\pi)^8}\Delta(q)\Delta(p)\Delta(p')\Delta(q-p-p')\Delta(q)$$

$$+\frac{(-i\lambda)^2}{4}\int\frac{d^4pd^4p'}{(2\pi)^8}\Delta(q)\Delta(p)\Delta(q)\Delta(p')\Delta(q)$$

$$+\frac{(-i\lambda)^2}{4}\int\frac{d^4pd^4p'}{(2\pi)^8}\Delta(q)\Delta^2(p)\Delta(p')\Delta(q) + O(\lambda^3)$$



动量空间$\lambda\phi^4$理论的费曼图——连接4点格林函数图

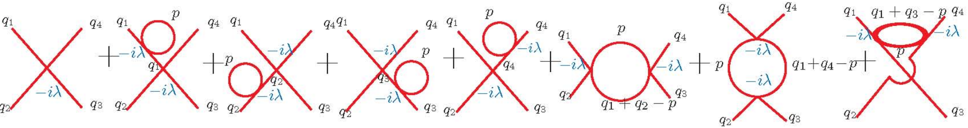
$$\Delta(p) \equiv i/(p^2 - M^2 + i0^+) \qquad (2\pi)^4\delta(p) \overset{p=0}{=\!=\!=} \int d^4x$$

$$-i\lambda\, \Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(q_4) \qquad\qquad q_1+q_2+q_3+q_4=0$$

$$+\frac{(-i\lambda)^2}{2}\int\frac{d^4p}{(2\pi)^4}[\Delta(q_1)\Delta(p)\Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(q_4)+\Delta(q_1)\Delta(q_2)\Delta(p)\Delta(q_2)\Delta(q_3)\Delta(q_4)$$

$$+\Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(p)\Delta(q_3)\Delta(q_4)+\Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(q_4)\Delta(p)\Delta(q_4)]$$

$$+\frac{(-i\lambda)^2}{2}\int\frac{d^4p}{(2\pi)^4}\,\Delta(q_1)\Delta(q_2)\Delta(p)[\Delta(q_1+q_2-p)+\Delta(q_1+q_4-p)+\Delta(q_1+q_3-p)]\Delta(q_3)\Delta(q_4)$$



在$\lambda\phi^4$理论中用连接费曼图构造完全的费曼图

n点格林函数: $G_n(x_1,\cdots,x_n)\equiv(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_n)\Psi_0^+)_C$

$$\frac{(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}$$

$$=\sum_{\nu_1+\cdots+n\nu_n=n}\frac{n!}{\nu_1!\cdots\nu_n!(1!)^{\nu_1}\cdots(n!)^{\nu_n}}\underbrace{G_1(x_1)\cdots G_1(x_{\nu_1})}_{\nu_1\ \text{terms}}\underbrace{G_2(x_{\nu_1+1},x_{\nu_1+2})\cdots G_2(x_{\nu_1+2\nu_2-1},x_{\nu_1+2\nu_2})}_{\nu_2\ \text{terms}}\cdots$$

完全格林函数的生成泛函:

$$\frac{Z[J]}{Z[0]}=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int d^4x_1\cdots d^4x_n\,\frac{(\Psi_0^-,\mathbf{T}\phi_H(x_1)\cdots\phi_H(x_n)\Psi_0^+)}{(\Psi_0^-,\Psi_0^+)}J(x_1)\cdots J(x_n)$$

$$=\sum_{n=0}^{\infty}\sum_{\nu_1+\cdots+n\nu_n=n}\frac{i^n}{n!}\frac{n!}{\nu_1!\cdots\nu_n!(1!)^{\nu_1}\cdots(n!)^{\nu_n}}\int d^4x_1\cdots d^4x_n\ J(x_1)\cdots J(x_n)$$

$$\times\underbrace{G_1(x_1)\cdots G_1(x_{\nu_1})}_{\nu_1\ \text{terms}}\underbrace{G_2(x_{\nu_1+1},x_{\nu_1+2})\cdots G_2(x_{\nu_1+2\nu_2-1},x_{\nu_1+2\nu_2})}_{\nu_2\ \text{terms}}\cdots$$

$\lambda\phi^4$理论的费曼图

在$\lambda\phi^4$理论中用连接费曼图构造完全的费曼图

完全格林函数的生成泛函:

$$\frac{Z[J]}{Z[0]} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \cdots d^4x_n \, \frac{(\Psi_0^-, \mathbf{T}\psi_H(x_1)\cdots\psi_H(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} J(x_1)\cdots J(x_n)$$

$$= \sum_{n=0}^{\infty} \sum_{\nu_1+\cdots+n\nu_n=n} \frac{i^{\nu_1+\cdots+n\nu_n}}{n!} \frac{n!}{\nu_1!\cdots\nu_n!(1!)^{\nu_1}\cdots(n!)^{\nu_n}} \int d^4x_1\cdots d^4x_n \;\; J(x_1)\cdots J(x_n)$$

$$\times \underbrace{G_1(x_1)\cdots G_1(x_{\nu_1})}_{\nu_1 \text{ terms}} \underbrace{G_2(x_{\nu_1+1}, x_{\nu_1+2})\cdots G_2(x_{\nu_1+2\nu_2-1}, x_{\nu_1+2\nu_2})}_{\nu_2 \text{ terms}} \cdots$$

$$= \sum_{\nu_1=0}^{\infty} \frac{i^{\nu_1}}{\nu_1!(1!)^{\nu_1}} \left[\int d^4x_1 \; J(x_1)G_1(x_1)\right]^{\nu_1} \sum_{\nu_2=0}^{\infty} \frac{i^{\nu_2}}{\nu_2!(2!)^{\nu_2}} \left[\int d^4x_2 d^4x_3 \; J(x_2)J(x_3)G_2(x_2,x_3)\right]^{\nu_2} \times \cdots$$

$$= \exp\left[\frac{i}{1!}\int d^4x_1 \; J(x_1)G_1(x_1) + \frac{i}{2!}\int d^4x_2 d^4x_3 \; J(x_2)J(x_3)G_2(x_2,x_3) + \cdots\right] = e^{iW[J]-iW[0]}$$

4费米理论概况

经典理论:

$$\mathcal{L}[\psi(x)] = \overline{\psi}(x)(i\gamma^\mu\partial_\mu - M)\psi(x) - \frac{g}{4}[\overline{\psi}(x)\Gamma\psi(x)]^2$$

- 正常： M为实数； $g \geq 0$； $\Gamma \subset \gamma$矩阵
- $M = 0 \Rightarrow$ 有可能动力学产生质量依赖Γ!
- $g = 0 \Rightarrow$ 自由场!
- $g < 0 \Rightarrow$ 系统无定义(能量无下界)!

量子理论:

- 理论上最低阶的$3+1$维纯费米子相互作用量子场理论
- 现实中最实用的的$3+1$维有效纯费米子量子场理论
- $3+1$维的理论是不可重整的
- $1+1$维和$2+1$的理论是可重整的

粒子理论与超导BCS理论对应 $\xleftarrow{\text{08年诺贝尔物理奖}}\!\!\rightarrow$ 质量（能隙）产生与费米子对凝聚

NJL模型 Physical Review 122,345(1960): $\mathcal{L}[\psi(x)] = \bar\psi i\gamma^\mu\partial_\mu\psi - g[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2]$

场方程： $[i\gamma^\mu\partial_\mu - 2g(\bar\psi\psi - \gamma_5\bar\psi\gamma_5\psi)]\psi = 0$

$$\xRightarrow{\text{平均场近似}} [i\gamma^\mu\partial_\mu - 2g(\langle\bar\psi\psi\rangle - \gamma_5\underbrace{\langle\bar\psi\gamma_5\psi\rangle}_{\text{宇称守恒}\Rightarrow\ 0})]\psi = 0 \xRightarrow{\text{质量产生}} M = 2g\langle\bar\psi\psi\rangle$$

费米子对凝聚: $\psi = \psi_R + \psi_L \qquad \bar\psi_{\gamma_5}^{1}\psi = \bar\psi_R\psi_L \pm \bar\psi_L\psi_R \Leftarrow$ 玻色型集体激发 !

$$[i\gamma^\mu\partial_\mu - 2g\langle\bar\psi\psi\rangle]\psi = 0 \qquad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \qquad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\begin{pmatrix} -2g\langle\bar\psi\psi\rangle & -i\partial^0 - i\vec\sigma\cdot\vec\partial \\ -i\partial^0 + i\vec\sigma\cdot\vec\partial & -2g\langle\bar\psi\psi\rangle \end{pmatrix}\begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = 0 \qquad \psi_{\substack{R\\L}}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-iEt + i\vec p\cdot\vec r}\psi_{\substack{R\\L}}(p)$$

$$E = \pm\sqrt{\vec p^2 + (2g\langle\bar\psi\psi\rangle)^2} \xLeftrightarrow{\text{费米子质量产生 NJL 超导能隙产生}} E = \pm\sqrt{\epsilon_p^2 + \Delta^2}$$

$$\left\{\begin{array}{l} E\psi_L(p) = \vec\sigma\cdot\vec p\ \psi_L(p) - 2g\langle\bar\psi\psi\rangle\psi_R(p) \\ \text{左右手场相干混合成Dirac场} \\ E\psi_R(p) = -\vec\sigma\cdot\vec p\ \psi_R(p) - 2g\langle\bar\psi\psi\rangle\psi_L(p) \end{array}\right. \xLeftrightarrow{\text{左右手态混合 NJL 电子空穴混合}} \left\{\begin{array}{l} E\psi_{p+} = \epsilon_p\psi_{p+} + \Delta\psi^*_{-p-} \\ \text{电子空穴相干混合成费米型集体激发} \\ E\psi^*_{-p-} = -\epsilon_p\psi^*_{-p-} + \Delta\psi_{p+} \end{array}\right.$$

4费米理论的费曼规则

$$\mathcal{L}[\psi(x),\overline{\psi}(x)]=\overline{\psi}(x)(i\partial\!\!\!/-M)\psi(x)-\frac{g}{4}[\overline{\psi}(x)\Gamma\psi(x)]^2\quad \mathcal{S}'(x,x')=\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-x')}i(p\!\!\!/-M+i0^+)^{-1}$$

$$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\cdots\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)=C\times e^{\int d^4xd^4x'\,\mathcal{S}'_{ll'}(x,x')\frac{\delta^2}{\delta\overline{\psi}_{l'}(x')\delta\psi_l(x)}}\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n'}(x_n')\,e^{-\frac{ig}{4}\int d^4x\,(\overline{\psi}\Gamma\psi)^2}\Big|_{\overline{\psi}=\psi=0}$$

$$=(\Phi_0,\mathbf{T}\{\psi_{l_1}(x_1)\cdots\overline{\psi}_{l_n'}(x_n')e^{-\frac{ig}{4}\int d^4x\,(\overline{\psi}\Gamma\psi)^2}\}\Psi_0)$$

坐标空间:

- 对$2n$点连通格林函数$(\Psi_0^-,\mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\overline{\psi}_{H,l_n'}(x_n')\Psi_0^+)_C$,标出$n$个时空点$x_1,\cdots,x_n$,每个点引出一条出线, 另外$n$个时空点$x_1',\cdots,x_n'$,每个点引出一条进线,
- 对m阶相互作用,分别在m个时空点$y_1,\cdots,y_m$引入相互作用顶点,每个顶点引出两条出线、两条进线
- 将引出的$n+2m$条出线与$n+2m$条出线两两相连,得到所有可能的拓扑不等价的连接图
- 每条连线代表$\mathcal{S}'(x,x')=\psi(x)\overline{\psi}(x')$,每个顶点代表$-ig\Gamma_{l_1l_1'}\Gamma_{l_2l_2'}$, 需要对所有顶点的时空坐标求积分
- 若图中出现对称连线,必须乘以对称因子;封闭费米子圈产生额外的-1

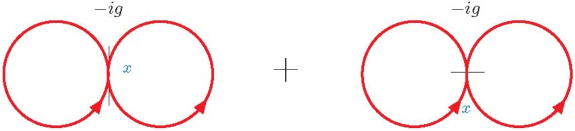
坐标空间4费米理论的费曼图——连接真空图

$$\frac{1}{2!}\int d^4x d^4x' d^4y d^4y'\ S_{l_1 l_1'}(x,x')S_{l_2 l_2'}(y,y')\frac{\delta^2}{\delta\overline{\psi}_{l_1'}(x')\delta\psi_{l_1}(x)}\frac{\delta^2}{\delta\overline{\psi}_{l_2'}(y')\delta\psi_{l_2}(y)}(-\frac{ig}{4})\int d^4x\ (\overline{\psi}\Gamma\psi)^2$$

$$= (\Phi_0, \mathbf{T}[(-\frac{ig}{4})\int d^4x\ (\overline{\psi}\Gamma\psi)^2]\Psi_0)$$

$$= (-\frac{ig}{4})\int d^4x\left[\overline{\psi}(x)\Gamma\psi(x)\ \overline{\psi}(x)\Gamma\psi(x)\ -\ \mathrm{tr}[\Gamma\psi(x)\overline{\psi}(x)\Gamma\psi(x)\overline{\psi}(x)]\right]$$

$$= \frac{-ig}{4}\int d^4x\ \{[\mathrm{tr}(S(x,x)\Gamma)]^2 - \mathrm{tr}[S(x,x)\Gamma]^2\}$$



坐标空间4费米理论的费曼图——连接真空图

$$\frac{1}{4!}\int d^4x_1 d^4x_1' d^4x_2 d^4x_2' d^4x_3 d^4x_3' d^4x_4 d^4x_4'\ S_{l_1l_1'}(x_1,x_1')S_{l_2l_2'}(x_2,x_2')S_{l_3l_3'}(x_3,x_3')S_{l_4l_4'}(x_4,x_4')$$

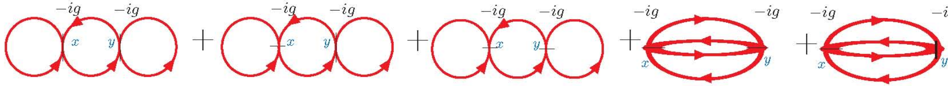
$$\times\frac{\delta^2}{\delta\overline{\psi}_{l_1'}(x_1')\delta\psi_{l_1}(x_1)}\frac{\delta^2}{\delta\overline{\psi}_{l_2'}(x_2')\delta\psi_{l_2}(x_2)}\frac{\delta^2}{\delta\overline{\psi}_{l_3'}(x_3')\delta\psi_{l_3}(x_3)}\frac{\delta^2}{\delta\overline{\psi}_{l_4'}(x_4')\delta\psi_{l_4}(x_4)}$$

$$\times\frac{1}{2!}(-\frac{ig}{4})^2\int d^4xd^4y\ [\overline{\psi}(x)\Gamma\psi(x)]^2[\overline{\psi}(y)\Gamma\psi(y)]^2$$

$$=\frac{(-ig)^2}{8}\int d^4xd^4y\{-\mathrm{tr}[\Gamma S(x,x)]\mathrm{tr}[\Gamma S(x,y)\Gamma S(y,x)]\mathrm{tr}[\Gamma S(y,y)]$$

$$+2\mathrm{tr}[\Gamma S(x,x)\Gamma S(x,y)\Gamma S(y,x)]\mathrm{tr}[\Gamma S(y,y)]-\mathrm{tr}[\Gamma S(x,x)\Gamma S(x,y)\Gamma S(y,y)\Gamma S(y,x)]\}$$

$$+\frac{(-ig)^2}{16}\int d^4xd^4y\ \Big[\{\mathrm{tr}[\Gamma S(x,y)\Gamma S(y,x)]\}^2-\mathrm{tr}[\Gamma S(x,y)\Gamma S(y,x)\Gamma S(x,y)\Gamma S(y,x)]\Big]+O(g^3)$$



坐标空间4费米理论的费曼图——连接真空图

$$= (\Phi_0, \mathbf{T}\{\ \frac{1}{2!}(-\frac{ig}{4})^2 \int d^4x d^4y\ [\overline{\psi}(x)\Gamma\psi(x)]^2 [\overline{\psi}(y)\Gamma\psi(y)]^2\}\ \Psi_0)$$

$$= \frac{1}{2!}(-\frac{ig}{4})^2 \int d^4x \Big[-2*2\ \overline{\psi}(x)\Gamma\psi(x)\ \mathrm{tr}[\Gamma\psi(x)\overline{\psi}(y)\Gamma\psi(y)\overline{\psi}(x)]\ \overline{\psi}(y)\Gamma\psi(y)$$

$$-4*2\ \mathrm{tr}[\Gamma\psi(x)\overline{\psi}(x)\Gamma\psi(x)\overline{\psi}(y)\Gamma\psi(y)\overline{\psi}(x)]\ \overline{\psi}(y)\Gamma\psi(y)$$

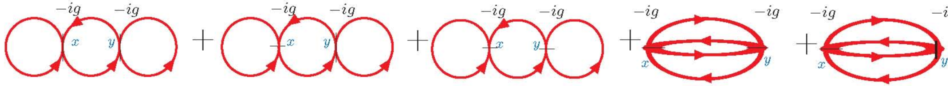
$$-2*2\ \mathrm{tr}[\Gamma\psi(x)\overline{\psi}(x)\Gamma\psi(x)\overline{\psi}(y)\Gamma\psi(y)\overline{\psi}(y)\Gamma\psi(y)\overline{\psi}(x)]$$

$$+2\ \mathrm{tr}[\Gamma\psi(x)\overline{\psi}(y)\Gamma\psi(y)\overline{\psi}(x)]\ \mathrm{tr}[\Gamma\psi(x)\overline{\psi}(y)\Gamma\psi(y)\overline{\psi}(x)]\ \Big]$$

$$= \frac{(-ig)^2}{8} \int d^4x d^4y \{-\mathrm{tr}[\Gamma S(x,x)]\mathrm{tr}[\Gamma S(x,y)\Gamma S(y,x)]\mathrm{tr}[\Gamma S(y,y)]$$

$$+2\mathrm{tr}[\Gamma S(x,x)\Gamma S(x,y)\Gamma S(y,x)]\mathrm{tr}[\Gamma S(y,y)] - \mathrm{tr}[\Gamma S(x,x)\Gamma S(x,y)\Gamma S(y,y)\Gamma S(y,x)]\}$$

$$+\frac{(-ig)^2}{16} \int d^4x d^4y\ \Big[\{\mathrm{tr}[\Gamma S(x,y)\Gamma S(x,y)]\}^2 - \mathrm{tr}[\Gamma S(x,y)\Gamma S(y,x)\Gamma S(x,y)\Gamma S(y,x)]\Big] + O(g^3)$$



坐标空间4费米理论的费曼图——连接2点格林函数图

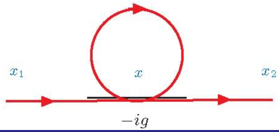
$$\frac{1}{3!}\int d^4x d^4x' d^4y d^4y' d^4y_1 d^4y_1'\ S_{l_1 l_1'}(x,x')S_{l_2 l_2'}(y,y')S_{l_3 l_3'}(y_1,y_1')\frac{\delta^2}{\delta\overline{\psi}_{l_1'}(x')\delta\psi_{l_1}(x)}\frac{\delta^2}{\delta\overline{\psi}_{l_2'}(y')\delta\psi_{l_2}(y)}$$

$$\times\frac{\delta^2}{\delta\overline{\psi}_{l_3'}(y_1')\delta\psi_{l_3}(y_1)}\psi_l(x_1)\overline{\psi}_{l'}(x_2)(-\frac{ig}{4})\int d^4z\ [\overline{\psi}(z)\Gamma\psi(z)]^2$$

$$=(\Phi_0,\mathbf{T}\{\ \psi_l(x_1)\overline{\psi}_{l'}(x_2)\ (-\frac{ig}{4})\int d^4x\ [\overline{\psi}(x)\Gamma\psi(x)]^2\}\ \Psi_0)$$

$$=-\frac{ig}{4}\int d^4x\ [\ 2\psi_l(x_1)\overline{\psi}(x)\ \overline{\psi}(x)\Gamma\psi(x)\Gamma\psi(x)\overline{\psi}_{l'}(x_2)-2\psi_l(x_1)\overline{\psi}(x)\Gamma\psi(x)\overline{\psi}(x)\Gamma\psi(x)\overline{\psi}_{l'}(x_2)\ ]$$

$$=\frac{ig}{2}\int d^4x\ \{\ S_{ll_1}(x_1,x)\Gamma_{l_1l_2}\mathrm{tr}[S(x,x)\Gamma]S_{l_2l'}(x,x_2)-S_{ll_1}(x_1,x)\Gamma_{l_1l_2}S_{l_2l_3}(x,x)\Gamma_{l_3l_4}S_{l_4l'}(x,x_2)\ \}$$



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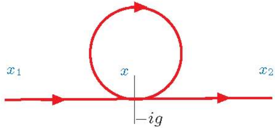
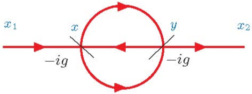


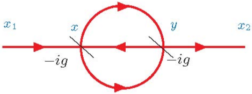
4费米理论的费曼图

坐标空间4费米理论的费曼图——连接2点格林函数图

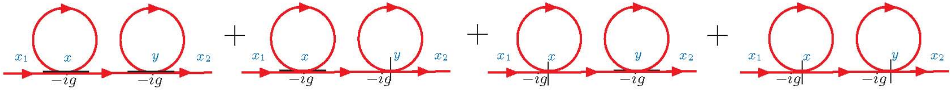
$$-\frac{(-ig)^2}{4}\int d^4x d^4y\; S_{ll_1}(x_1,x)\Gamma_{l_1l_2}\mathrm{tr}[S(x,y)\Gamma]^2 S_{l_2l_3}(x,y)\Gamma_{l_3l_4}S_{l_4l'}(y,x_2)$$



$$+\frac{(-ig)^2}{4}\int d^4x d^4y\; S_{ll_1}(x_1,x)\Gamma_{l_1l_2}S_{l_2l_3}(x,y)\Gamma_{l_3l_4}S_{l_3l_4}(y,x)\Gamma_{l_4l_5}S_{l_4l_5}(x,y)\Gamma_{l_5l_6}S_{l_6l'}(y,x_2)$$



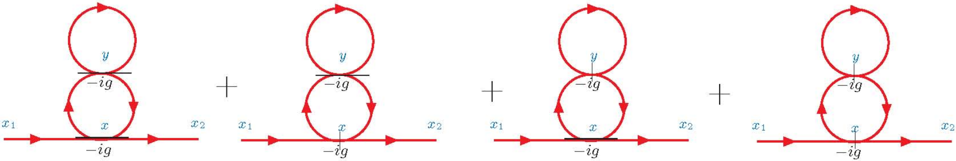
$$+\frac{(-ig)^2}{4}\int d^4x d^4y\; S(x_1,x)\Gamma\{\mathrm{tr}[S(x,x)\Gamma]S(x,y)\Gamma\mathrm{tr}[S(y,y)\Gamma]-\mathrm{tr}[S(x,x)\Gamma]S(x,y)\Gamma S(y,y)\Gamma$$
$$-S(x,x)\Gamma S(x,y)\Gamma\mathrm{tr}[S(y,y)\Gamma]+S(x,x)\Gamma S(x,y)\Gamma S(y,y)\Gamma\}S(y,x_2)$$



坐标空间4费米理论的费曼图——连接2点格林函数图

$$+\frac{(-ig)^2}{4}\int d^4x d^4y\; S(x_1,x)\Gamma\Big[\mathrm{tr}\{\Gamma S(x,y)\mathrm{tr}[\Gamma S(y,y)]\Gamma S(y,x)\} - \Gamma\mathrm{tr}[S(x,y)\Gamma S(y,y)\Gamma S(y,x)]$$

$$-\Gamma S(x,y)\mathrm{tr}[\Gamma S(y,y)]\Gamma S(y,x) + \Gamma S(x,y)\Gamma S(y,y)\Gamma S(y,x)\Big]S(x,x_1) + O(g^3)$$



坐标空间4费米理论的费曼图——连接4点格林函数图

$$\frac{1}{4!}\int d^4y_1d^4y_1'd^4y_2d^4y_2'd^4y_3d^4y_3'd^4y_4d^4y_4'\ S_{l_1''l_1'}(y_1,y_1')S_{l_2''l_2'}(y_2,y_2')S_{l_3''l_3'}(y_3,y_3')S_{l_4''l_4'}(y_4,y_4')$$

$$\times\frac{\delta^2}{\delta\overline{\psi}_{l_1'}(y_1')\delta\psi_{l_1''}(y_1)}\frac{\delta^2}{\delta\overline{\psi}_{l_2'}(y_2')\delta\psi_{l_2''}(y_2)}\frac{\delta^2}{\delta\overline{\psi}_{l_3'}(y_3')\delta\psi_{l_3''}(y_3)}\frac{\delta^2}{\delta\overline{\psi}_{l_4'}(y_4')\delta\psi_{l_4''}(y_4)}$$

$$\times(-\frac{ig}{4})\int d^4z\ \psi_{l_1}(x_1)\overline{\psi}_{l_2}(x_2)\psi_{l_3}(x_3)\overline{\psi}_{l_4}(x_4)[\overline{\psi}(z)\Gamma\psi(z)]^2$$

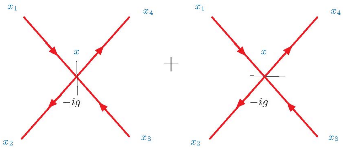
$$=(\Phi_0,\mathbf{T}\{\ \psi_{l_1}(x_1)\overline{\psi}_{l_2}(x_2)\psi_{l_3}(x_3)\overline{\psi}_{l_4}(x_4)\ (-\frac{ig}{4})\int d^4x\ [\overline{\psi}(x)\Gamma\psi(x)]^2\}\ \Psi_0)$$

$$=-\frac{ig}{4}\int d^4x\ [\ 2\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x)\Gamma_{l_1'l_2'}\psi_{l_2'}(x)\overline{\psi}_{l_2}(x_2)\ \psi_{l_3}(x_3)\overline{\psi}_{l_3'}(x)\Gamma_{l_3'l_4'}\psi_{l_4'}(x)\overline{\psi}_{l_4}(x_4)$$

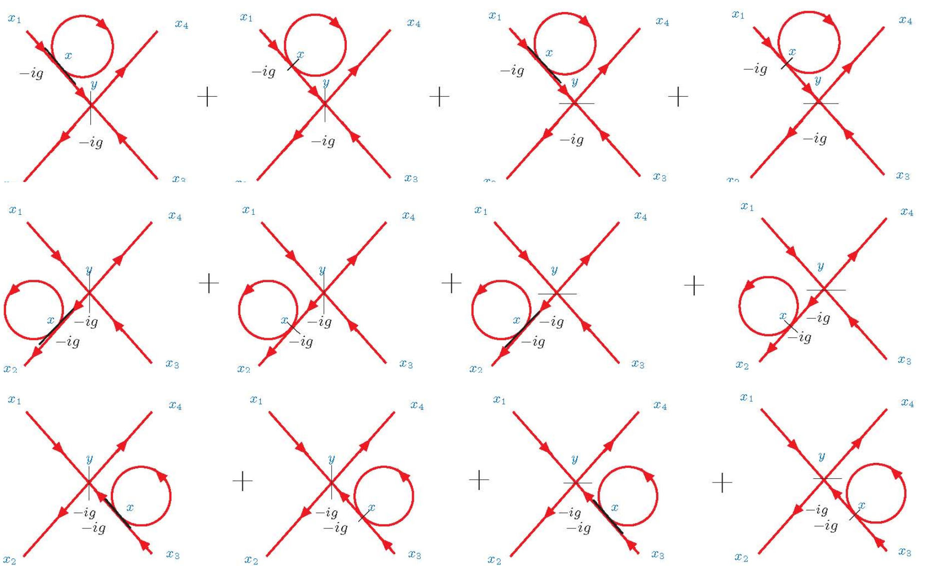
$$-2\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x)\Gamma_{l_1'l_4'}\psi_{l_4'}(x)\overline{\psi}_{l_4}(x_4)\ \psi_{l_3}(x_3)\overline{\psi}_{l_3'}(x)\Gamma_{l_3'l_2'}\psi_{l_2'}(x)\overline{\psi}_{l_2}(x_2)\ ]$$

$$=\frac{-ig}{2}\int d^4x\ [S_{l_1l_1'}(x_1,x)\Gamma_{l_1'l_2'}S_{l_2'l_2}(x,x_2)S_{l_3l_3'}(x_3,x)\Gamma_{l_3'l_4'}S_{l_4'l_4}(x,x_4)$$

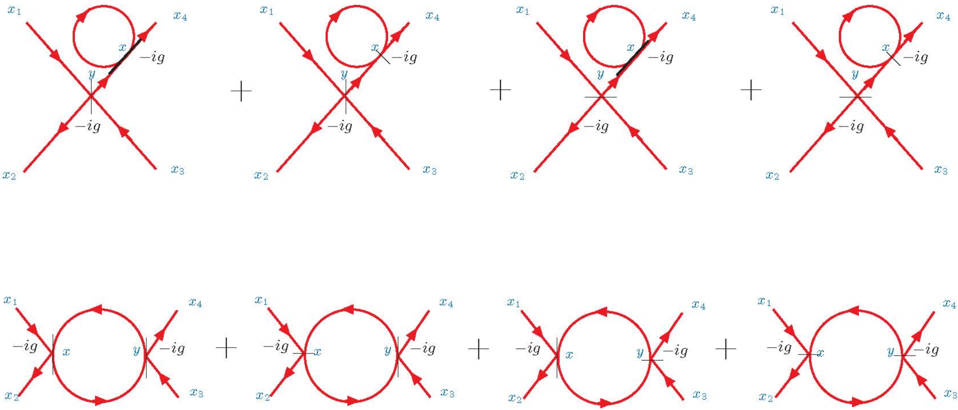
$$-S_{l_1l_1'}(x_1,x)\Gamma_{l_1'l_4'}S_{l_4'l_4}(x,x_4)S_{l_3l_3'}(x_3,x)\Gamma_{l_3'l_2'}S_{l_2'l_2}(x,x_2)]$$



4费米理论的费曼图



4费米理论的费曼图



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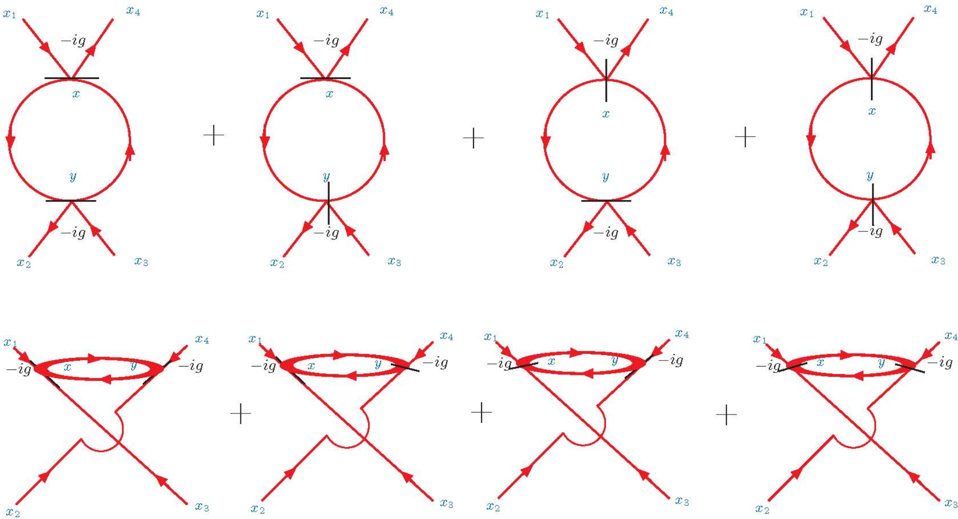
x_1
x_4
$-ig$
x
y
$-ig$
x_2
x_3
+
x_1
x_4
$-ig$
x
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$-ig$
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4费米理论的费曼图

$$\mathcal{L}[\phi(x)] = \overline{\psi}(x)(i\gamma^\mu\partial_\mu - M)\psi(x) - \frac{g}{4}[\overline{\psi}(x)\Gamma\psi(x)]^2 \quad \mathcal{S}'(x,x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')} \frac{1}{\not p - M + i0^+}$$

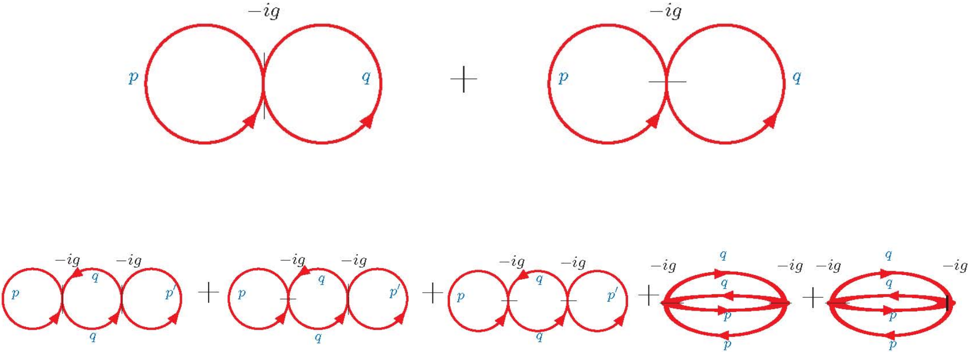
$$G_{l_1 l_1' \cdots l_n l_n'}(q_1, p_1 \cdots, q_n, p_n)(2\pi)^4\delta(q_1 - p_1 + \cdots q_n - p_n)$$

$$= \int d^4x_1 d^4x_1' \cdots d^4x_n d^4x_n' e^{iq_1\cdot x_1 - ip_1\cdot x_1' + \cdots + iq_n\cdot x_n - ip_n\cdot x_n'} (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\overline{\psi}_{H,l_1'}(x_1')\cdots\psi_{H,l_n}(x_n)\psi_{H,l_n'}(x_n')\Psi_0^+)$$

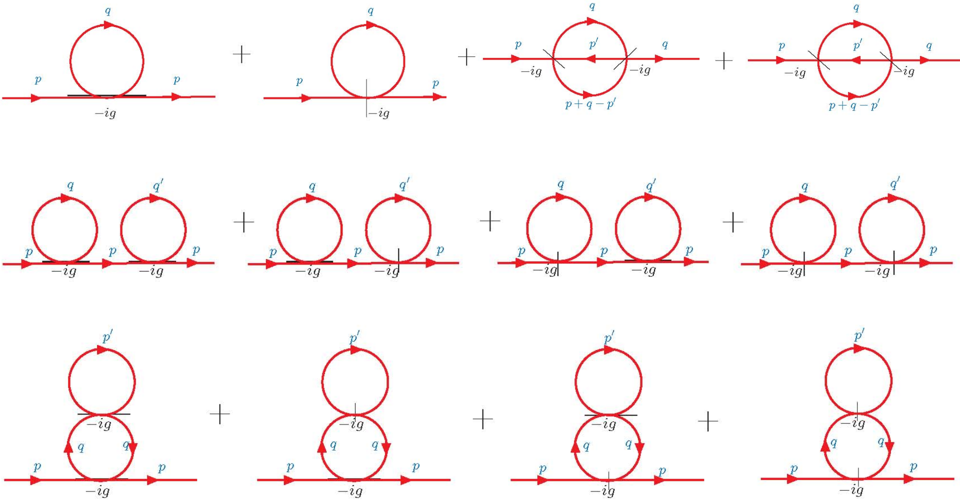
动量空间

- 对$2n$点连通格林函数$G(q_1, p_1 \cdots, q_n, p_n)_C$,标出$2n$个外点,其中的$n$个点各引一条对应流入动量$q_1, \cdots, q_n$的进线，另外$n$个点各引一条对应流出动量$p_1, \cdots, p_n$的出线
- 对m阶相互作用,分别在m个点引入相互作用顶点,每个顶点引出两条出线，两条进线
- 将引出的$n+2m$条进线和$n+2m$条出线两两相连,每条线具有一个四动量从进线向出线方向流动。每个顶点有一δ函数$(2\pi)^4\delta(p_1+p_2+p_3+p_4)$,其中$p_i$是流入顶角的四动量。得到所有可能的拓扑不等价的连接图
- 连线代表$\frac{i}{\not p - M + i0^+}$,顶点代表$-ig\Gamma_{l_1 l_1'}\Gamma_{l_2 l_2'}$,需对所有动量求积分$\int\frac{d^4p}{(2\pi)^4}$
- 如果图中出现对称连线,须乘对称因子.封闭的费米子圈产生额外的-1

4费米理论的费曼图

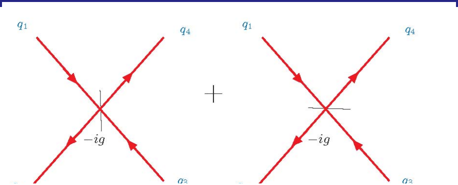


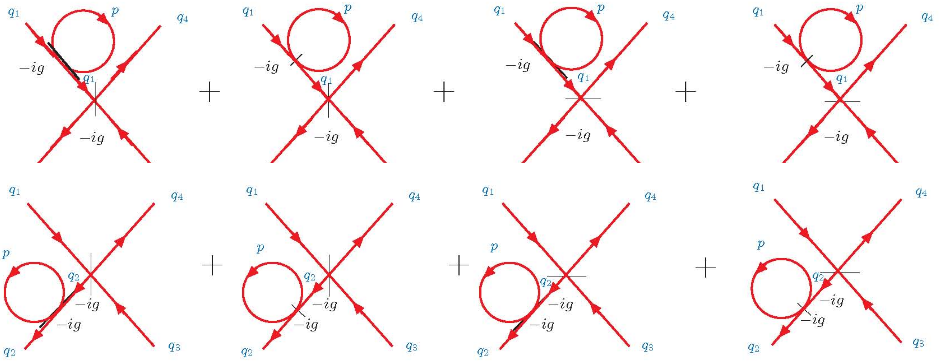
4费米理论的费曼图



4费米理论的费曼图

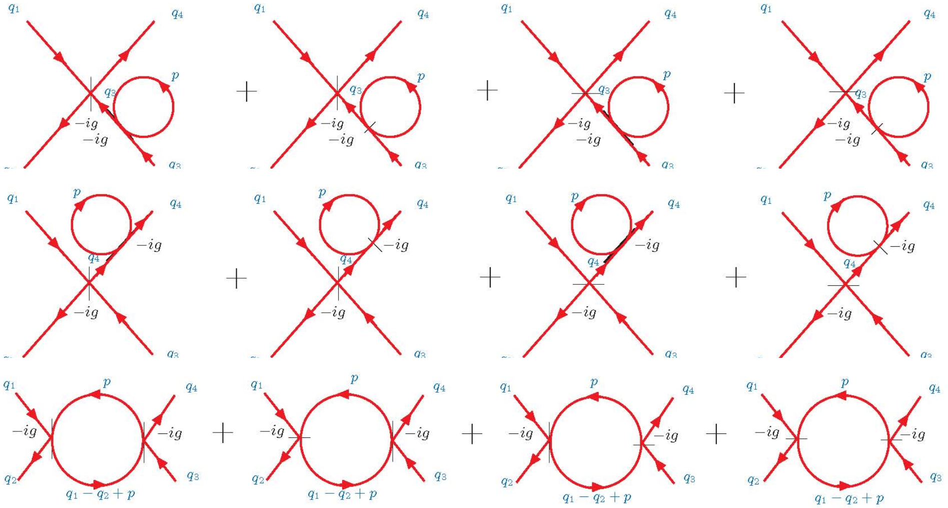
$$q_1 - q_2 + q_3 - q_4 = 0$$





4费米理论的费曼图

$$q_1 - q_2 + q_3 - q_4 = 0$$



4费米理论的费曼图

$$q_1 - q_2 + q_3 - q_4 = 0$$