量子场论　量子电动力学

本章介绍和讨论现实中存在的一种具体 量子场–电磁场

王青

清华大学

2024年9月2日-12月20日

理论基础
- 拉格朗日量
- 生成泛函与微扰论
- 格林函数的相互关联

一圈图
- 光子传播子
- 费米子传播子
- 相互作用顶角

重整化
- 关于重整化的一般分析
- 一圈图的重整化
- 跑动耦合常数

基本物理过程与物理量
- 基本物理过程
- $e^+e^- \to \mu^+\mu^-$
- 电子磁矩

经典电动力学

$$\mathcal{L}_{\text{Maxwell}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - J_\mu(x)A^\mu(x) \qquad -J_\mu A^\mu = \vec{J}\cdot\vec{A} - \rho\phi$$

$$\mathcal{L}_{\text{free vector}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M^2A^2(x) \qquad F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

- 电磁场是零质量的矢量场
- 物质场以电磁流的方式进入电磁场的拉格朗日量

规范对称性

- 零质量矢量量子场具有规范对称性: $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\Omega(x)$
- 规范对称性导致流守恒： $\partial^\mu J_\mu = 0$

加入费米子　最小耦合： $J_\mu(x) = e\overline{\psi}(x)\gamma_\mu\psi(x) = J^\dagger_\mu(x)$

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)$$

费米子的规范变换： $\psi(x) \to \psi'(x) = e^{-ie\Omega(x)}\psi(x)$ 　 $[Q, \psi(x)] = -e\psi(x)$

量子电动力学

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

量子化 $A_0(x) = 0$ 略去$i0^+$项

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ \delta[A_0] e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

$$\Rightarrow \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \int \mathcal{D}[\partial_0\Omega]\ e^{-i\int d^4x\ \frac{\lambda}{2}[\partial_0\Omega]^2} \delta[A_0+\partial_0\Omega] e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}A_0^2+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

不同规范的选择 $F[A_\mu(x)] = 0$

- 洛伦兹规范 **(Lorentz gauge):** $F[A_\mu(x)] = \partial_\mu A^\mu(x)$
- 库伦规范 **(Coulomb gauge):** $F[A_\mu(x)] = \nabla\cdot\vec{A}(x)$
- 瞬时规范 **(Temporal gauge):** $F[A_\mu(x)] = A^0(x)$
- 轴规范 **(Axial gauge):** $F[A_\mu(x)] = A^3(x)$
- 幺正规范 **(Unitary gauge):** $F[A_\mu(x)] = \phi(x) - \phi^\dagger(x)$

量子电动力学

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

量子化准备: $\delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|}\delta(x-x_i) \Rightarrow \int_{-\infty}^{\infty} dx\, \delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|}$

$$\Delta(A) \equiv \int \mathcal{D}\Omega\, \delta[F(A_\mu + \partial_\mu\Omega)] = {\rm Det}^{-1}\mathcal{M}(A) \qquad F[A_\mu(x)] = 0$$

只考虑 $F[A_\mu + B_\mu] = F[A_\mu] + F[B_\mu]$

$$\mathcal{M}(A;x,y) \equiv \frac{\delta F[A_\mu(x) + \partial_\mu\Omega(x)]}{\delta\Omega(y)} = \frac{\delta F[\partial_\mu\Omega(x)]}{\delta\Omega(y)} = \text{与}A_\mu\text{无关!} \qquad \Delta^{-1}\int \mathcal{D}\Omega\, \delta[F(A_\mu + \partial_\mu\Omega)] = 1$$

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

略去$i0^+$项

$$\xlongequal{A'_\mu = A_\mu + \partial_\mu\Omega} \Delta^{-1}\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \int \mathcal{D}\Omega\, \delta\{F[A'_\mu]\}\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

$$\xlongequal{\psi' = e^{-ie\Omega}\psi} \Delta^{-1}\int \mathcal{D}A'_\mu \mathcal{D}\overline{\psi}'\mathcal{D}\psi' \int \mathcal{D}\Omega\, \delta\{F[A'_\mu]\}\, e^{i\int d^4x\, \{-\frac{1}{4}F'_{\mu\nu}(x)F^{\mu\nu\prime}(x) + \overline{\psi}'(x)[i\partial\!\!\!/ - eA\!\!\!/'(x) - M]\psi'(x)\}}$$

$$= \Delta^{-1}\int \mathcal{D}\Omega \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\, \delta\{F[A_\mu]\} e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

量子电动力学

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

量子化 $F[A_\mu(x)] = 0$ 略去$i0^+$项

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ \delta\{F[A_\mu]\} e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

$$\Rightarrow \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \int \mathcal{D}[F[\partial_\mu\Omega]] e^{-i\int d^4x \frac{\lambda}{2}F^2[\partial_\mu\Omega]} \delta\{F[A_\mu] + F[\partial_\mu\Omega]\} e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

$$= \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}F^2[A_\mu(x)] + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

不同规范的选择 $F[A_\mu(x)] = 0$

- 洛伦兹规范 **(Lorentz gauge):** $F[A_\mu(x)] = \partial_\mu A^\mu(x)$
- 库伦规范 **(Coulomb gauge):** $F[A_\mu(x)] = \nabla \cdot \vec{A}(x)$
- 瞬时规范 **(Temporal gauge):** $F[A_\mu(x)] = A^0(x)$
- 轴规范 **(Axial gauge):** $F[A_\mu(x)] = A^3(x)$
- 幺正规范 **(Unitary gauge):** $F[A_\mu(x)] = \phi(x) - \phi^\dagger(x)$

量子电动力学的变形： 纯旋量场理论

略去$i0^+$项

$$\int d^4x\,\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)\}$$

$$=\int d^4x\,\{-\frac{1}{2}(\partial_\mu A_\nu-\partial_\nu A_\mu)\partial^\mu A^\nu-\frac{\lambda}{2}[\partial^\mu A_\mu]^2-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}[i\partial\!\!\!/-M]\psi\}$$

$$=\int d^4x\,\{\frac{1}{2}A_\mu[g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu]A_\nu-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}[i\partial\!\!\!/-M]\psi\}$$

$$=\int d^4x\,\{\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu-A_\mu J^\mu+\overline{\psi}[i\partial\!\!\!/-M]\psi\}\quad iD_0^{-1,\mu\nu}=g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu\quad J^\mu=e\overline{\psi}\gamma^\mu\psi$$

$$\int \mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{i\int d^4x\,\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)\}}$$

$$=\int \mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{i\int d^4x\,\{\frac{1}{2}(A+iJD_0)_\mu iD_0^{-1,\mu\nu}(A+iD_0J)_\nu+\frac{i}{2}J_\mu D_0^{\mu\nu}J_\nu+\overline{\psi}[i\partial\!\!\!/-M]\psi\}}$$

$$=C\times\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{-\frac{e^2}{2}\int d^4xd^4y\,[\overline{\psi}(x)\gamma_\mu\psi(x)]D_0^{\mu\nu}(x,y)[\overline{\psi}(y)\gamma_\nu\psi(y)]+i\int d^4x\,\overline{\psi}(x)[i\partial\!\!\!/-M]\psi(x)}$$

$$D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)\qquad C=\int\mathcal{D}A_\mu\, e^{\frac{i}{2}\int d^4x\,A_\mu iD_0^{-1,\mu\nu}A_\nu}$$

量子电动力学的变形： 纯旋量场理论

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/ -eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= C\times\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{-\frac{e^2}{2}\int d^4x d^4y\ [\overline{\psi}(x)\gamma_\mu\psi(x)]D_0^{\mu\nu}(x,y)[\overline{\psi}(y)\gamma_\nu\psi(y)]\ +i\int d^4x\ \overline{\psi}(x)[i\partial\!\!\!/ -M]\psi(x)}$$

$$D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y) \qquad C=\int \mathcal{D}A_\mu\ e^{\frac{i}{2}\int d^4x\ A_\mu iD_0^{-1,\mu\nu}A_\nu}$$

Fierz 变换

$$\delta^{\xi\xi'}\delta^{\zeta\zeta'}=a^{\zeta\xi}\delta^{\xi'\zeta'}+b^{\zeta\xi}(\gamma_5)^{\xi'\zeta'}+c_\mu^{\zeta\xi}(\gamma^\mu)^{\xi'\zeta'}+d_\mu^{\zeta\xi}(\gamma^\mu\gamma_5)^{\xi'\zeta'}+f_{\mu\nu}^{\zeta\xi}(\sigma^{\mu\nu})^{\xi'\zeta'}$$

两边同时乘$\delta^{\zeta'\xi'}$, $(\gamma_5)^{\zeta'\xi'}$, $(\gamma_{\mu'})^{\zeta'\xi'}$ $(\gamma_{\mu'}\gamma_5)^{\zeta'\xi'}$ $(\sigma_{\mu'\nu'})^{\zeta'\xi'}=\frac{i}{2}[\gamma_{\mu'},\gamma_{\nu'}]^{\zeta'\xi'}$

$$\delta^{\zeta\xi}=4a^{\zeta\xi} \quad (\gamma_5)^{\zeta\xi}=4b^{\zeta\xi} \quad (\gamma_{\mu'})^{\zeta\xi}=4c_{\mu'}^{\zeta\xi} \quad (\gamma_{\mu'}\gamma_5)^{\zeta\xi}=-4d_{\mu'}^{\zeta\xi} \quad (\sigma_{\mu'\nu'})^{\zeta\xi}=-8f_{\mu'\nu'}^{\zeta\xi}$$

$$\delta^{\xi\xi'}\delta^{\zeta\zeta'}=\frac{1}{4}\left[\delta^{\zeta\xi}\delta^{\xi'\zeta'}+(\gamma_5)^{\zeta\xi}(\gamma_5)^{\xi'\zeta'}+(\gamma_\mu)^{\zeta\xi}(\gamma^\mu)^{\xi'\zeta'}-(\gamma_\mu\gamma_5)^{\zeta\xi}(\gamma^\mu\gamma_5)^{\xi'\zeta'}+\frac{1}{2}(\sigma_{\mu\nu})^{\zeta\xi}(\sigma^{\mu\nu})^{\xi'\zeta'}\right]$$

量子电动力学的变形： 纯旋量场理论

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/ -eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= C\times\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{-\frac{e^2}{2}\int d^4x d^4y\,[\overline{\psi}(x)\gamma_\mu\psi(x)]D_0^{\mu\nu}(x,y)[\overline{\psi}(y)\gamma_\nu\psi(y)]\,+i\int d^4x\,\overline{\psi}(x)[i\partial\!\!\!/ -M]\psi(x)}$$

$$D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y) \qquad C=\int \mathcal{D}A_\mu\, e^{\frac{i}{2}\int d^4x\, A_\mu iD_0^{-1,\mu\nu}A_\nu}$$

$$\delta^{\xi\xi'}\delta^{\zeta\zeta'}=\frac{1}{4}\left[\delta^{\zeta\xi}\delta^{\xi'\zeta'}+(\gamma_5)^{\zeta\xi}(\gamma_5)^{\xi'\zeta'}+(\gamma_\mu)^{\zeta\xi}(\gamma^\mu)^{\xi'\zeta'}-(\gamma_\mu\gamma_5)^{\zeta\xi}(\gamma^\mu\gamma_5)^{\xi'\zeta'}+\frac{1}{2}(\sigma_{\mu\nu})^{\zeta\xi}(\sigma^{\mu\nu})^{\xi'\zeta'}\right]$$

$$[\overline{\psi}(x)\gamma_\mu\psi(x)][\overline{\psi}(y)\gamma_\nu\psi(y)]=\overline{\psi}_{\xi'}(x)[\gamma_\mu\psi(x)]_\xi\overline{\psi}_\zeta(y)[\gamma_\nu\psi(y)]_{\zeta'}\delta^{\xi\xi'}\delta^{\zeta\zeta'}$$

$$=\frac{1}{4}\Big[[\overline{\psi}(x)\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma_\mu\psi(x)]+[\overline{\psi}(x)\gamma_5\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma_5\gamma_\mu\psi(x)]+[\overline{\psi}(x)\gamma_\sigma\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma^\sigma\gamma_\mu\psi(x)]$$

$$-[\overline{\psi}(x)\gamma_5\gamma_\sigma\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma_5\gamma^\sigma\gamma_\mu\psi(x)]+\frac{1}{2}[\overline{\psi}(x)\sigma_{\sigma\rho}\gamma_\nu\psi(y)][\overline{\psi}(y)\sigma^{\sigma\rho}\gamma_\mu\psi(x)]\Big]$$

$$=\cdots\cdots$$

量子电动力学的变形： 纯矢量场理论

略去$i0^+$项

$$\int \prod_i^n [d\overline{\theta}_i d\theta_i]\, e^{\sum_{ij}^n \overline{\theta}_i \mathcal{M}_{ij}\theta_j} = \int \prod_i^n [d\overline{\theta}_i d\theta_i]\, e^{\sum_{ijk}^n \overline{\theta}_i U^*_{ik} a_k U_{kj}\theta_j} \qquad \mathcal{M}_{ij} = \sum_k^n U^*_{ik} a_k U_{kj}$$

$$= \int \prod_i^n [d\overline{\theta}'_i d\theta'_i]\, e^{\sum_k^n \overline{\theta}'_k a_k \theta'_k} \qquad \theta'_k = \sum_j^n U_{kj}\theta_j \quad \overline{\theta}'_k = \sum_i^n U^*_{ik}\overline{\theta}_i \quad \prod_i^n [d\overline{\theta}_i d\theta_i] = \prod_i^n [d\overline{\theta}'_i d\theta'_i]$$

$$= \int \prod_i^n [d\overline{\theta}'_i d\theta'_i] \prod_k^n e^{\overline{\theta}'_k a_k \theta'_k} = \int \prod_i^n [d\overline{\theta}'_i d\theta'_i] \prod_k^n [1 + \overline{\theta}'_k a_k \theta'_k] = \int \prod_i^n [d\overline{\theta}'_i d\theta'_i] \prod_k^n [\overline{\theta}'_k a_k \theta'_k]$$

$$= \int \prod_i^n [-d\overline{\theta}'_i\, \overline{\theta}'_i\, d\theta'_i\, \theta'_i\, a_i] = \prod_i (-a_i) = \mathrm{Det}(-\mathcal{M}) = e^{\mathrm{Tr}\,\ln(-\mathcal{M})}$$

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2 + \overline{\psi}(x)[i\not\partial - e\not A(x) - M]\psi(x)\}}$$

$$= \int \mathcal{D}A_\mu\, e^{\mathrm{Tr}\,\ln[i\not\partial - e\not A(x) - M] + i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2\}}$$

需发展计算行列式的方法！

1+1维QED（Schwinger模型）精确可解就是因为其行列式可以严格求出，并且结果对光子场是二次依赖！

格林函数生成泛函

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\ \{\frac{1}{2}A_\mu[g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu-i0^+]A_\nu-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}[i\partial\!\!\!/-M+i0^+]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\ \{\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}iS_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$iD_0^{-1,\mu\nu}=g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu-i0^+\qquad iS_0^{-1}=i\partial\!\!\!/-M+i0^+$$

$$=e^{-i\int d^4x\ e\frac{\delta}{-i\delta J^\mu(x)}\frac{\delta}{-i\delta I(x)}\gamma^\mu\frac{\delta}{i\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\ \{\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=e^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{\frac{1}{2}(A+iJD_0)_\mu iD_0^{-1,\mu\nu}(A+iD_0J)_\nu+(\overline{\psi}-i\bar{I}S_0)iS_0^{-1}(\psi-iS_0I)+\frac{i}{2}J_\mu D_0^{\mu\nu}J_\nu+i\bar{I}S_0I\}}$$

$$=C\times e^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\ e^{i\int d^4y d^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]}$$

$$C=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\ [\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0^{-1}\psi]}$$

$$D_0^{\mu\nu}(y,z)=\frac{i}{\partial_y^2-i0^+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)\qquad S_0(y,z)=\frac{i}{i\partial\!\!\!/_y-M+i0^+}\delta(y-z)$$

顶角生成泛函

$$\frac{\delta W[J,I,\bar{I}]}{\delta J_\mu(x)} = -A_c^\mu(x) \qquad \frac{\delta W[J,I,\bar{I}]}{\delta \bar{I}_l(x)} = \psi_{c,l}(x) \qquad \frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)} = -\overline{\psi}_{c,l}(x)$$

$$\Gamma[A_c,\psi_c,\overline{\psi}_c] \equiv W[J,I,\bar{I}] - \int d^4x\, [-J_\mu(x)A_c^\mu(x) + \bar{I}_l(x)\psi_{c,l}(x) + \overline{\psi}_{c,l}(x)I_l(x)]$$

$$\frac{\delta\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta A_c^\mu(x)} = J_\mu(x) \qquad \frac{\delta\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta \psi_{c,l}(x)} = \bar{I}_l(x) \qquad \frac{\delta\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta \overline{\psi}_{c,l}(x)} = -I_l(x)$$

$$\int d^4y\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta J_{\nu'}(y)}\bigg|_{J=I=\bar{I}=0} \frac{\delta^2\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta A_c^{\nu'}(y)\delta A_c^{\nu}(x')}\bigg|_{A_c=A_0,\ \psi_c=\psi_0,\ \overline{\psi}_c=\overline{\psi}_0} = -\delta(x-x')g^\mu_{\ \nu}$$

$$\int d^4y\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_l(x)\delta I_{l''}(y)}\bigg|_{J=I=\bar{I}=0} \frac{\delta^2\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta \overline{\psi}_{c,l''}(y)\delta\psi_{c,l'}(x')}\bigg|_{A_c=A_0,\ \psi_c=\psi_0,\ \overline{\psi}_c=\overline{\psi}_0} = -\delta(x-x')\delta_{ll'}$$

$$\frac{\delta^3 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J=I=\bar{I}=0} = -\int d^4x' d^4y' d^4z'\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta J_{\mu'}(x')} \frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_l(y)\delta I_{l_1}(y')} \frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_{l'}(z)\delta I_{l_2}(z')}\bigg|_{J=I=\bar{I}=0}$$

$$\times \frac{\delta^3\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta A_c^{\mu'}(x')\delta\overline{\psi}_{c,l_1}(y')\delta\psi_{c,l_2}(z')}\bigg|_{A_c=A_0,\ \psi_c=\psi_0,\ \overline{\psi}_c=\overline{\psi}_0}$$

证明见后

格林函数与顶角函数

$$Z[J,I,\bar{I}] = e^{iW[J,I,\bar{I}]} = C \times e^{e\int d^4x \frac{\delta}{\delta J^{\mu}(x)} \frac{\delta}{\delta I(x)} \gamma^{\mu} \frac{\delta}{\delta \bar{I}(x)}} e^{i\int d^4y d^4z [\frac{i}{2} J_{\mu}(y) D_0^{\mu\nu}(y,z) J_{\nu}(z) + i\bar{I}(y) S_0(y,z) I(z)]}$$

$$D_0^{\mu\nu}(x,y) = \frac{i}{\partial_x^2 - i0+}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_x^{\mu}\partial_x^{\nu}}{\partial_x^2}]\delta(x-y) \qquad S_0(x,y) = \frac{i}{i\partial\!\!\!/_x - M + i0+}\delta(x-y)$$

格林函数:

$$\left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J_{\mu}(x)\delta J_{\nu}(y)}\right|_{J=I=\bar{I}=0} \equiv iD^{\mu\nu}(x,y) = iD_0^{\mu\nu}(x,y) + \text{高阶修正}$$

$$\left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_l(x)\delta I_{l'}(y)}\right|_{J=I=\bar{I}=0} \equiv -iS(x,y) = -iS_0(x,y) + \text{高阶修正}$$

$$\left.\frac{\delta^3 W[J,I,\bar{I}]}{\delta J^{\mu}(x)\delta \bar{I}_l(y)\delta I_{l'}(z)}\right|_{J=I=\bar{I}=0} \equiv G_{\mu}(x,y,z) = \int d^4x' D_{0,\mu\nu}(x,x')[S_0(y,x') ie\gamma^{\nu} S_0(x',z)]_{ll'} + \text{高阶修正}$$

$$\int d^4z \frac{\delta^2\Gamma[A,\psi,\overline{\psi}]}{\delta\psi_{l''}(x')\delta\overline{\psi}_{l'}(z)} \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l'}(z)\delta \bar{I}_l(y)}\right|_{I=\bar{I}=0} = -\delta_{l''l}\delta(x'-y)$$

$$\int d^4z \left[\frac{\delta^2\Gamma[A,\psi,\overline{\psi}]}{\delta\psi_{l''}(x')\delta\overline{\psi}_{l'}(z)} \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^{\mu}(x)\delta I_{l'}(z)\delta \bar{I}_l(y)} - \int d^4y' \frac{\delta W[J,I,\bar{I}]}{\delta J^{\mu}(x)\delta J^{\nu}(y')} \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A_{\nu}(y')\delta\psi_{l''}(x')\delta\overline{\psi}_{l'}(z)} \frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l'}(z)\delta \bar{I}_l(y)}\right]_{I=\bar{I}=J^{\mu}=0} = 0$$

$$\left.\frac{\delta^3 W[J,I,\bar{I}]}{\delta J^{\mu}(x)\delta \bar{I}_l(y)\delta I_{l'}(z)}\right|_{J=I=\bar{I}=0} = -\int d^4x' d^4y' d^4z' \frac{\delta W[J,I,\bar{I}]}{\delta J^{\mu}(x)\delta J^{\nu}(x')} \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A_{\nu}(x')\delta\psi_{l_1'}(z')\delta\overline{\psi}_{l_1}(y')} \frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l_1}(y')\delta \bar{I}_l(y)} \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l'}(z)\delta \bar{I}_{l_1'}(z')}\right|_{J=I=\bar{I}=0}$$

格林函数与顶角函数

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=C\times e^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\;e^{i\int d^4yd^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]}$$

$$D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2-i0^+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)\qquad S_0(x,y)=\frac{i}{i\partial\!\!\!/_x-M+i0^+}\delta(x-y)$$

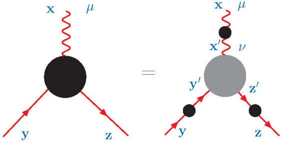
格林函数:

$$\left.\frac{\delta^2W[J,I,\bar{I}]}{\delta J_\mu(x)\delta J_\nu(y)}\right|_{J=I=\bar{I}=0}\equiv iD^{\mu\nu}(x,y)=iD_0^{\mu\nu}(x,y)+\text{高阶修正}$$

$$\left.\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\right|_{J=I=\bar{I}=0}\equiv -iS(x,y)=-iS_0(x,y)+\text{高阶修正}$$

$$\left.\frac{\delta^3W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\right|_{J=I=\bar{I}=0}\equiv G_\mu(x,y,z)=\int d^4x'D_{0,\mu\nu}(x,x')[S_0(y,x')ie\gamma^\nu S_0(x',z)]_{ll'}+\text{高阶修正}$$

$$G_\mu(x,y,z)=\int d^4x'd^4y'd^4z'D_{\mu\nu}(x,x')[S(y,y')i\Gamma^\nu(x',y',z')S(z',z)]_{ll'}$$



$$\frac{\delta^3\Gamma[A,\psi,\bar{\psi}]}{\delta A_\mu(x)\delta\bar{\psi}_l(y)\delta\psi_{l'}(z)}\equiv-\Gamma^\mu_{ll'}(x,y,z)=-e\gamma^\mu_{ll'}\delta(z-x)\delta(y-x)+\text{高阶修正}$$

$$\Gamma[A,\psi,\bar{\psi}]=\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\bar{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi\}+\text{高阶修正}$$

微扰展开

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$= C\times e^{e\int d^4x \frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta \bar{I}(x)}}\ e^{i\int d^4yd^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]}$$

$$C=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ [\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0\psi]}$$

$$D_0^{\mu\nu}(y,z)=\frac{i}{\partial_y^2-i0^+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)\qquad S_0(y,z)=\frac{i}{i\partial\!\!\!/_y-M+i0^+}\delta(y-z)$$

$$Z[J,I,\bar{I}]=Ce^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}e^{i\int d^4yd^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]+i\int d^4x[-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$=Ce^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}e^{i\int d^4yd^4z[-\frac{i}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+i\frac{\delta}{\delta\psi(y)}S_0(y,z)\frac{\delta}{\delta\overline{\psi}(z)}]}e^{i\int d^4x[-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$=Ce^{i\int d^4yd^4z[-\frac{i}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+i\frac{\delta}{\delta\psi(y)}S_0(y,z)\frac{\delta}{\delta\overline{\psi}(z)}]}e^{i\int d^4x[-eA_\mu\overline{\psi}\gamma^\mu\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

微扰展开(续)

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$= Ce^{i\int d^4yd^4z[-\frac{i}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+i\frac{\delta}{\delta\psi(y)}S_0(y,z)\frac{\delta}{\delta\overline{\psi}(z)}]}e^{i\int d^4x[-eA_\mu\overline{\psi}\gamma^\mu\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$C=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ [\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0^{-1}\psi]}$$

$$D_0^{\mu\nu}(y,z)=\frac{i}{\partial_y^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)\qquad S_0(y,z)=\frac{i}{i\partial\!\!\!/_y-M+i0^+}\delta(y-z)$$

$$(\Psi_0^-,\Psi_0^+)=C'\times e^{\int d^4yd^4z\ [\frac{1}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$(\Psi_0^-,\mathbf{T}\ [\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ \Psi_0^+)$$

$$=C\times e^{\int d^4yd^4z\ [\frac{1}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ [\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')$$

$$\times A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

坐标空间费曼规则

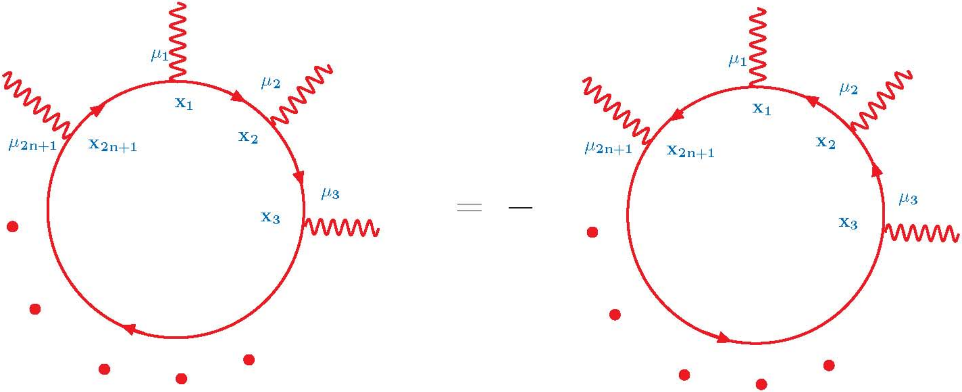
$$(\Psi_0^-, \mathbf{T}\,[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ \Psi_0^+)$$

$$= C \times e^{\int d^4y d^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ [\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')$$

$$\times A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

- 对$(\Psi_0^-, \mathbf{T}\,[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ \Psi_0^+)_C$,标出$n$个时空点$x_1\cdots x_n$,每点引出一条出费米线, 另外$n$个时空点$x_1'\cdots x_n'$, 每点引出一条进费米线，$m$个时空点$y_1\cdots y_m$,每点引出一条光子线
- 对k阶相互作用,分别在k个时空点$z_1,\cdots,z_k$引入相互作用顶点,每个顶点引出一条光子线、一条出费米线和一条进费米线
- 将引出的$n+k$条出费米线与$n+k$条进费米线两两相连,并将$m+k$条光子线两两相连，得到所有可能的拓扑不等价的连接图
- 每对费米子连线代表$S_{0,ll'}(x,x')$,每对光子连线代表$D_0^{\mu\nu}(x,x')$, 每个顶点代表$-ie\gamma^\mu_{ll'}$, 需要对所有顶点的时空坐标求积分
- 如图中出现对称的连线,须乘以对称因子. 封闭费米圈产生额外-1因子

法雷(Furry)定理



应用：**QED中奇数个光子场的格林函数为零！**

$$\left(\Psi_0^-, \mathbf{T}\, A_{\mu_1}(x_1) A_{\mu_2}(x_2) \cdots A_{\mu_{2n+1}}(x_{2n+1}) \Psi_0^+\right)_C = 0$$

法雷(Furry)定理

格林函数中包含奇数个顶角的封闭费米子圈图相互抵消。

证明:

$$\mathcal{C}=i\gamma^2\gamma^0=-\mathcal{C}^{-1}\qquad \mathcal{C}\gamma_\mu\mathcal{C}^{-1}=-\gamma_\mu^T\qquad S_0(x,y)=\frac{i}{i\partial\!\!\!/_x-M+i0^+}\delta(x-y)$$

$$\mathcal{C}S_0(x,y)\mathcal{C}^{-1}=\frac{i}{-i\partial\!\!\!/_x^T-M+i0^+}\delta(x-y)=\frac{i}{i\partial\!\!\!/_y^T-M+i0^+}\delta(x-y)=S_0^T(y,x)$$

$$\begin{aligned}
&\mathrm{tr}[\gamma_{\mu_1}S_0(x_1,x_2)\gamma_{\mu_2}S_0(x_2,x_3)\gamma_{\mu_3}\cdots\gamma_{\mu_n}S_0(x_n,x_1)]\\
&=\mathrm{tr}[\mathcal{C}\gamma_{\mu_1}\mathcal{C}^{-1}\mathcal{C}S_0(x_1,x_2)\mathcal{C}^{-1}\mathcal{C}\gamma_{\mu_2}\mathcal{C}^{-1}\mathcal{C}S_0(x_2,x_3)\mathcal{C}^{-1}\mathcal{C}\gamma_{\mu_3}\cdots\gamma_{\mu_n}\mathcal{C}^{-1}\mathcal{C}S_0(x_n,x_1)\mathcal{C}^{-1}]\\
&=(-1)^n\mathrm{tr}[\gamma_{\mu_1}^TS_0^T(x_2,x_1)\gamma_{\mu_2}^TS_0^T(x_3,x_2)\gamma_{\mu_3}^T\cdots\gamma_{\mu_n}^TS_0^T(x_1,x_n)]\\
&=(-1)^n\mathrm{tr}[S_0(x_1,x_n)\gamma_{\mu_n}\cdots\gamma_{\mu_3}S_0(x_3,x_2)\gamma_{\mu_2}S_0(x_2,x_1)\gamma_{\mu_1}]
\end{aligned}$$

$\mathrm{tr}[\gamma_{\mu_1}S_0(x_1,x_2)\gamma_{\mu_2}S_0(x_2,x_3)\gamma_{\mu_3}\cdots\gamma_{\mu_{2n+1}}S_0(x_{2n+1},x_1)]$和
$\mathrm{tr}[S_0(x_1,x_{2n+1})\gamma_{\mu_{2n+1}}\cdots\gamma_{\mu_3}S_0(x_3,x_2)\gamma_{\mu_2}S_0(x_2,x_1)\gamma_{\mu_1}]$

大小相等，符号相反，相互抵消！

Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

费米子场的平移

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ \{[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)+I(x)\}\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\left[[i\partial\!\!\!/_x-ie\gamma^\mu\frac{\delta}{\delta J^\mu(x)}-M]\frac{\delta}{\delta i\bar{I}(x)}+I(x)\right]Z[J,I,\bar{I}]=0$$

$$i(i\partial\!\!\!/_x-M)\frac{\delta Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta\bar{I}(x)}+e\gamma^\mu\frac{\delta^2 Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta J^\mu(x)\delta\bar{I}(x)}-I(x)=0$$

$$(i\partial\!\!\!/_x-M)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}-ie\gamma^\mu\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}(x)}+e\gamma^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}+I(x)=0$$

$$(i\partial\!\!\!/_x-M)_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}+\delta_{ll''}\delta(x-y)=0$$

费米子的Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$(i\partial\!\!\!/_x-M)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}-ie\gamma^\mu\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}(x)}+e\gamma^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}+I(x)=0$$

$$(i\partial\!\!\!/_x-M)_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\Big|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\Big|_{J^\mu=\bar{I}=I=0}+\delta_{ll''}\delta(x-y)=0$$

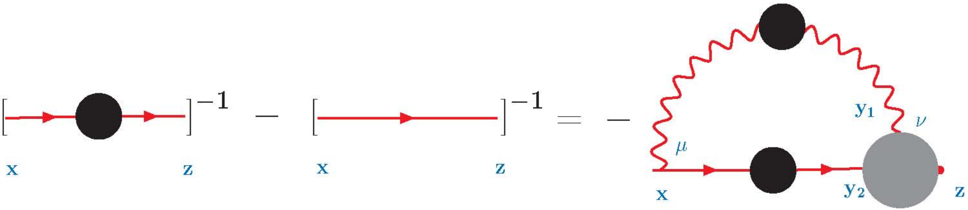
$$W_0=i\int d^4xd^4y\ \bar{I}(x)S_0(x,y)I(y)\qquad S_0(x,y)=i(i\partial\!\!\!/_x-M+i0^+)^{-1}\delta(x-y)$$

$$\int d^4z\ (i\partial\!\!\!/_x-M)_{ll'}\delta(x-z)\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(z)\delta I_{l''}(y)}\Big|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_{l'}(x)\delta I_{l''}(y)}\Big|_{J^\mu=\bar{I}=I=0}-\delta_{ll''}\delta(x-y)=0$$

$$\int d^4z\ S^{-1}_{0,ll'}(x,z)S_{l'l''}(z,y)-ie\gamma^\mu_{ll'}G_{\mu,l'l''}(x,x,y)-\delta_{ll''}\delta(x-y)=0$$

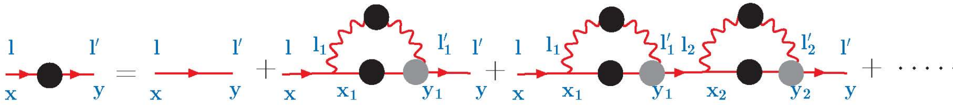
$$S^{-1}(x,z)-S_0^{-1}(x,z)=-ie\gamma^\mu\int d^4y\ G_\mu(x,x,y)S^{-1}(y,z)=e\gamma^\mu\int d^4y_1d^4y_2D_{\mu\nu}(x,y_1)S(x,y_2)\Gamma^\nu(y_1,y_2,z)$$

费米子的Schwinger-Dyson方程



$$S^{-1}(x,z)-S_0^{-1}(x,z)=e\gamma^\mu\int d^4y_1d^4y_2D_{\mu\nu}(x,y_1)S(x,y_2)\Gamma^\nu(y_1,y_2,z)\equiv\Sigma(x,z)$$

$$S=(S_0^{-1}+\Sigma)^{-1}=S_0(1+\Sigma S_0)^{-1}=S_0-S_0\Sigma S_0+S_0\Sigma S_0\Sigma S_0+\cdots$$



Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

光子场的平移

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\{[g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu]A_\nu-e\overline{\psi}\gamma^\mu\psi-J^\mu\}\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\{-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta}{\delta iJ^\nu(x)}-e\gamma^\mu_{ll'}\frac{\delta^2}{\delta I_l(x)\delta\bar{I}_{l'}(x)}-J^\mu(x)\}Z[J,I,\bar{I}]=0$$

$$i[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta J^\nu(x)}-e\gamma^\mu_{ll'}\frac{\delta^2 Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta I_l(x)\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta W[J,I,\bar{I}]}{\delta J^\nu(x)}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x)\delta\bar{I}_{l'}(x)}+e\gamma^\mu_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(x)\delta J^\sigma(y)}\bigg|_{J^\mu=\bar{I}=I=0}+ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\sigma(y)\delta I_l(x)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}+g^\mu_\sigma\delta(x-y)=0$$

光子的Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta W[J,I,\bar{I}]}{\delta J^\nu(x)}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x)\delta\bar{I}_{l'}(x)}+e\gamma^\mu_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(x)\delta J^\sigma(y)}\bigg|_{J^\mu=\bar{I}=I=0}+ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\sigma(y)\delta I_l(x)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}+g^\mu_\sigma\delta(x-y)=0$$

$$W_0=\frac{1}{2}\int d^4x d^4y\; J_\mu(x)iD_0^{\mu\nu}(x,y)J_\nu(y)\qquad iD_0^{-1,\mu\nu}(x,z)=[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-z)$$

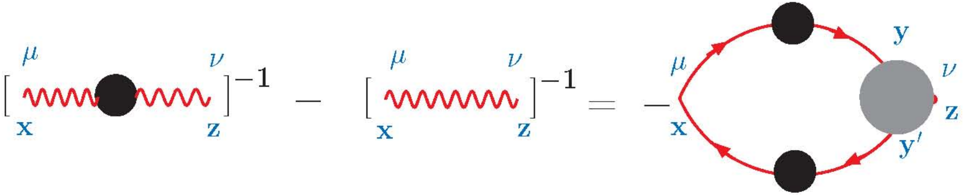
$$\int d^4z[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-z)\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(z)\delta J^\sigma(y)}\bigg|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\sigma(y)\delta\bar{I}_{l'}(x)\delta I_l(x)}\bigg|_{J^\mu=\bar{I}=I=0}+g^\mu_\sigma\delta(x-y)=0$$

$$-\int d^4z\, D_0^{-1,\mu\nu}(x,z)D_{\nu\sigma}(z,y)-ie\gamma^\mu_{ll'}G_{\sigma,l'l}(y,x,x)+g^\mu_\sigma\delta(x-y)=0$$

$$D^{-1,\mu\nu}(x,z)-D_0^{-1,\mu\nu}(x,z)=ie\int d^4y\,\mathrm{tr}[\gamma^\mu G_\sigma(y,x,x)]D^{-1,\sigma\nu}(y,z)$$

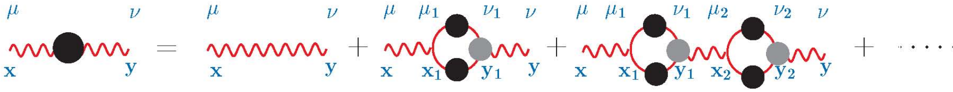
$$=-e\int d^4y d^4y'\;\mathrm{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)]$$

光子的Schwinger-Dyson方程



$$D^{-1,\mu\nu}(x,z)-D_0^{-1,\mu\nu}(x,z)=-e\int d^4y\,d^4y'\,\underbrace{\mathrm{tr}[\gamma^{\mu}S(x,y)\Gamma^{\nu}(z,y,y')S(y',x)]}_{\text{费米子圈和顶角产生的两负号相互抵消了！}}=\Pi^{\mu\nu}(x,z)$$

$$D=(D_0^{-1}+\Pi)^{-1}=D_0(1+\Pi D_0)^{-1}=D_0-D_0\Pi D_0+D_0\Pi D_0\Pi D_0+\cdots$$



光子的Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$D^{-1,\mu\nu}(x,z)-D_0^{-1,\mu\nu}(x,z)=-e\int d^4yd^4y'\ \mathrm{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)]=\Pi^{\mu\nu}(x,z)$$

真空极化张量：只考虑自能（oblique）

$$S(x,y)=\langle\psi(x)\overline{\psi}(y)\rangle \qquad \Gamma^\mu(z,y,y')\approx e\gamma^\mu\delta(y-z)\delta(y'-z)$$

$$\Pi^{\mu\nu}(x,z)\equiv-e\int d^4y\ \mathrm{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)]$$

$$\xRightarrow{\text{oblique}}-e^2\mathrm{tr}[\gamma^\mu\langle\psi(x)\overline{\psi}(z)\rangle\gamma^\nu\langle\psi(z)\overline{\psi}(x)\rangle]=\langle\,\overline{\psi}(x)e\gamma^\mu\psi(x)\overline{\psi}(z)e\gamma^\nu\psi(z)\,\rangle$$

$$=\langle\,J^\mu_{\text{em}}(x)J^\nu_{\text{em}}(z)\,\rangle \qquad J^\mu_{\text{em}}(x)\equiv-\overline{\psi}(x)e\gamma^\mu\psi(x)$$

光子场与费米场的联系

$$-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta W[J,I,\bar I]}{\delta J^\nu(x)}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar I]}{\delta I_l(x)\delta\bar I_{l'}(x)}+e\gamma^\mu_{ll'}\frac{\delta W[J,I,\bar I]}{\delta I_l(x)}\frac{\delta W[J,I,\bar I]}{\delta\bar I_{l'}(x)}-J^\mu(x)=0$$

$$-\int d^4z\; iD_0^{-1,\mu'\nu}(x',z)\frac{\delta W[J,I,\bar I]}{\delta J^\nu(z)}-ie\gamma^{\mu'}_{ll'}\frac{\delta^2 W[J,I,\bar I]}{\delta I_l(x')\delta\bar I_{l'}(x')}+e\gamma^{\mu'}_{ll'}\frac{\delta W[J,I,\bar I]}{\delta I_l(x')}\frac{\delta W[J,I,\bar I]}{\delta\bar I_{l'}(x')}-J^{\mu'}(x')=0$$

$$W_0=\frac{1}{2}\int d^4xd^4y\; J_\mu(x)iD_0^{\mu\nu}(x,y)J_\nu(y)\qquad iD_0^{-1,\mu\nu}(x,z)=[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-z)$$

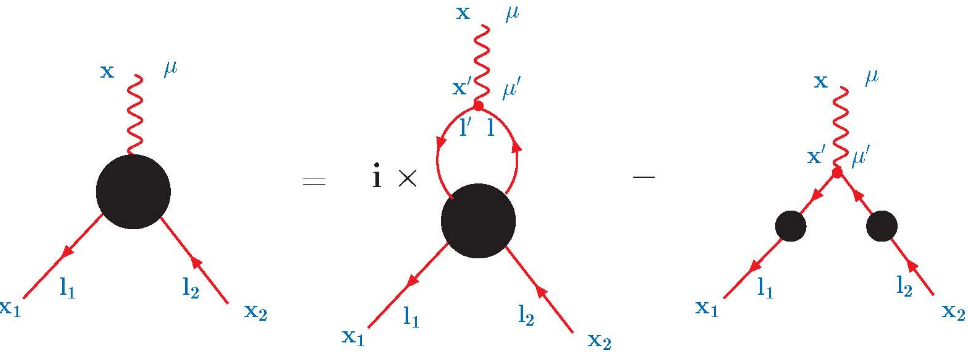
$$\frac{\delta W[J,I,\bar I]}{\delta J^\mu(x)}+\int d^4z\; D_{0,\mu\mu'}(x,x')\left[e\gamma^{\mu'}_{ll'}\frac{\delta^2 W[J,I,\bar I]}{\delta I_l(x')\delta\bar I_{l'}(x')}+ie\gamma^{\mu'}_{ll'}\frac{\delta W[J,I,\bar I]}{\delta I_l(x')}\frac{\delta W[J,I,\bar I]}{\delta\bar I_{l'}(x')}-iJ^{\mu'}(x')\right]=0$$

$$\left.\frac{\delta^2 W[J,I,\bar I]}{\delta J^\nu(y)\delta J^\mu(x)}\right|_{J^\mu=I=\bar I=0}+e\int d^4z\; D_{0,\mu\mu'}(x,x')\gamma^{\mu'}_{ll'}\left.\frac{\delta^3 W[J,I,\bar I]}{\delta J^\nu(y)\delta I_l(x')\delta\bar I_{l'}(x')}\right|_{J^\mu=I=\bar I=0}-iD_{0,\mu\nu}(x,y)=0$$

$$\frac{\delta^3 W[J,I,\bar I]}{\delta I_{l_1}(x_1)\delta\bar I_{l_2}(x_2)\delta J^\mu(x)}+e\gamma^{\mu'}_{ll'}\int d^4z D_{0,\mu\mu'}(x,x')\left[\frac{\delta^4 W[J,I,\bar I]}{\delta I_{l_1}(x_1)\delta\bar I_{l_2}(x_2)\delta I_l(x')\delta\bar I_{l'}(x')}+\frac{i\delta^2 W[J,I,\bar I]}{\delta\bar I_{l_2}(x_2)\delta I_l(x')}\frac{\delta^2 W[J,I,\bar I]}{\delta I_{l_1}(x_1)\delta\bar I_{l'}(x')}\right]$$

$$\overset{J^\mu=I=\bar I=0}{=\!=\!=\!=\!=\!=\!=}0$$

光子场与费米场的联系



$$\frac{\delta^3 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta J^{\mu}(x)}=ie\gamma^{\mu'}_{ll'}\int d^4z\, D_{0,\mu\mu'}(x,x')\left[i\frac{\delta^4 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x')\delta\bar{I}_{l'}(x')}-\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x')}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x')}\right]$$

Ward-Takahashi-Taylor恒等式

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

规范变换: $A_\mu\to A_\mu+\partial_\mu\Omega\quad \psi\to e^{-ie\Omega}\psi$

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\{-\lambda[\partial^2\Omega][\partial^\mu A_\mu]-J^\mu[\partial_\mu\Omega]+ie\Omega(\overline{\psi}I-\bar{I}\psi)\}e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\{-\lambda\partial^2\partial^\mu A_\mu+\partial_\mu J^\mu+ie(\overline{\psi}I-\bar{I}\psi)\}e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\{\lambda\partial_x^2\partial_x^\mu\frac{\delta}{i\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta}{i\delta I(x)}-\bar{I}(x)\frac{\delta}{i\delta\bar{I}(x)}]\}Z[J,I,\bar{I}]=0$$

$$\lambda\partial_x^2\partial_x^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta W[J,I,\bar{I}]}{\delta I(x)}-\bar{I}(x)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}]=0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0}=\partial_{\nu,x}\delta(x-y)\;\Rightarrow\;\partial_x^2\partial_x^\mu\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^{\mu_1}(x_1)\cdots\delta J^{\mu_n}(x_n)}\bigg|_{J^\mu=\bar{I}=I=0}\overset{n\geq2}{=\!=\!=}0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^3W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}=ie\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

Ward-Takahashi-Taylor恒等式

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\bar{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\bar{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\bar{\psi}I+i0^+\ \text{term}\}}$$

$$\lambda\partial_x^2\partial_x^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta W[J,I,\bar{I}]}{\delta I(x)}-\bar{I}(x)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}]=0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0}=\partial_{\nu,x}\delta(x-y)$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^3W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}=ie\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

光子场纵向分量无量子修正: $D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)$

$$\partial_{\mu,x}iD^{\mu\nu}(x,y)=-\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y)=\partial_{\mu,x}iD_0^{\mu\nu}(x,y)$$

相互作用顶角:

$$-\lambda\int d^4x'd^4y'd^4z'\partial_x^2\partial_x^\mu D_{\mu\nu}(x,x')S(y,y')i\Gamma^\nu(x',y',z')S(z',z)=eS(y,z)[\delta(x-z)-\delta(x-y)]$$

$$\partial_{\mu,x}\Gamma^\mu(x,y,z)=eS^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$$

Ward-Takahashi-Taylor恒等式

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$\lambda\partial_x^2\partial_x^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta W[J,I,\bar{I}]}{\delta I(x)}-\bar{I}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}]=0$$

光子场纵向分量无量子修正: $D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)$

$$\partial_{\mu,x}iD^{\mu\nu}(x,y)=-\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y)=\partial_{\mu,x}iD_0^{\mu\nu}(x,y)$$

相互作用顶角: $\partial_{\mu,x}\Gamma^\mu(x,y,z)=eS^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$

$\Gamma_0^\mu(x,y,z)=e\gamma^\mu\delta(y-x)\delta(x-z)$ $\qquad iS_0^{-1}(y,z)=(i\partial\!\!\!/_y-M+i0^+)\delta(y-z)$

$$eS_0^{-1}(y,z)[\delta(y-x)-\delta(z-x)]=-ie[\delta(y-x)-\delta(z-x)](i\partial\!\!\!/_y-M+i0^+)\delta(y-z)$$
$$=e[\delta(y-x)-\delta(z-x)]\partial\!\!\!/_y\delta(y-z)=-e\partial\!\!\!/_z\delta(y-x)\delta(y-z)-e\partial\!\!\!/_y\delta(z-x)\delta(y-z)$$
$$=-e\delta(y-x)\partial\!\!\!/_z\delta(x-z)-e\delta(z-x)\partial\!\!\!/_y\delta(y-x)=e\delta(y-x)\partial\!\!\!/_x\delta(x-z)+e\delta(z-x)\partial\!\!\!/_x\delta(y-x)$$
$$=e\partial\!\!\!/_x[\delta(y-x)\delta(x-z)]=\partial_{\mu,x}\Gamma_0^\mu(x,y,z)$$

三点顶角Ward-Takahashi-Taylor恒等式的变形表达

光子场纵向分量无量子修正: $D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)$

$$\partial_{\mu,x}iD^{\mu\nu}(x,y)=-\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y)=\partial_{\mu,x}iD_0^{\mu\nu}(x,y)$$

$$\frac{-\delta W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta J^\mu(x)}+e\gamma_{ll'}^{\mu'}\int d^4zD_{0,\mu\mu'}^{-1}(x,x')\left[\frac{i\delta^2W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x')\delta\bar{I}_{l'}(x')}+\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x')}\frac{\delta W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x')}\right]$$

$$\overset{J^\mu=I=\bar{I}=0}{=\!=\!=\!=\!=\!=\!=}0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^3W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}=ie\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

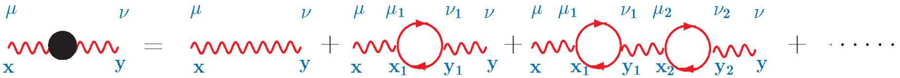
$$e\gamma_{ll'}^\mu\partial_\mu\left[-\frac{\delta^2W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x)\delta\bar{I}_{l'}(x)}+i\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x)}\right]_{J^\mu=\bar{I}=I=0}$$

$$=e\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y)$$

$$= i\frac{e^{\int d^4yd^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ A_\mu(x)A_\nu(y)\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4yd^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}\Bigg|_{A_\mu=\overline{\psi}=\psi=0}$$



$$= iD_{0,\mu\nu}(x,y) + i\int d^4x_1d^4y_1D_{0,\mu\mu_1}(x,x_1)(-)\Pi^{\mu_1\nu_1}(x_1,y_1)D_{0,\nu_1\nu}(y_1,y)$$

$$+i\int d^4x_1d^4y_1d^4x_2d^4y_2D_{0,\mu\mu_1}(x,x_1)\Pi^{\mu_1\nu_1}(x_1,y_1)D_{0,\nu_1\mu_2}(y_1,x_2)\Pi^{\mu_2\nu_2}(x_2,y_2)D_{0,\nu_2\nu}(y_2,y)+\cdots$$

$$=i[D_0(1-\Pi D_0+\Pi D_0\Pi D_0+\cdots)]_{\mu\nu}(x,y)=i[D_0(1+\Pi D_0)^{-1}]_{\mu\nu}(x,y)=[i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

SD方程： $D^{-1}-D_0^{-1}=\Pi$

$$\Pi^{\mu\nu}(x,y) = (-1)\mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$= -e^2\mathrm{tr}[\gamma^\mu(i\partial\!\!\!/_x - M + i0^+)^{-1}\delta(x-y)\gamma^\nu(i\partial\!\!\!/_y - M + i0^+)\delta(y-x)] + O(e^4)$$

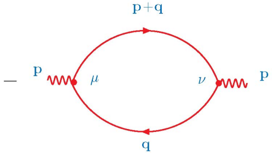
光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$= -e^2\mathrm{tr}[\gamma^\mu(i\partial\!\!\!/_x - M + i0^+)^{-1}\delta(x-y)\gamma^\nu(i\partial\!\!\!/_y - M + i0^+)\delta(y-x)] + O(e^4)$$

$$= -e^2\int\frac{d^4p\,d^4q}{(2\pi)^8}e^{-i(p-q)\cdot(x-y)}\mathrm{tr}[\gamma^\mu(p\!\!\!/ - M + i0^+)^{-1}\gamma^\nu(q\!\!\!/ - M + i0^+)] + O(e^4)$$



$$\Pi^{\mu\nu}(p) = -e^2\int\frac{d^4q}{(2\pi)^4}\ \mathrm{tr}[\gamma^\mu(p\!\!\!/ + q\!\!\!/ - M + i0^+)^{-1}\gamma^\nu(q\!\!\!/ - M + i0^+)] + O(e^4)$$

$$= -e^2\int\frac{d^4q}{(2\pi)^4}\ \mathrm{tr}[\gamma^\mu\frac{p\!\!\!/ + q\!\!\!/ + M}{(p+q)^2 - M^2 + i0^+}\gamma^\nu\frac{q\!\!\!/ + M}{q^2 - M^2 + i0^+}] + O(e^4)$$

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = -e^2\int\frac{d^4q}{(2\pi)^4}\ \mathrm{tr}[\gamma^\mu\frac{\not p+\not q+M}{(p+q)^2-M^2+i0^+}\gamma^\nu\frac{\not q+M}{q^2-M^2+i0^+}] + O(e^4)$$

$$\frac{1}{AB} = \int_0^1 \frac{dx}{[(1-x)A+xB]^2} \qquad \mathrm{tr}\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^\rho = 4(g^{\mu\sigma}g^{\nu\rho} - g^{\mu\nu}g^{\sigma\rho} + g^{\mu\rho}g^{\nu\sigma})$$

$$\begin{aligned}\frac{1}{[(p+q)^2-M^2+i0^+](q^2-M^2+i0^+)} &= \int_0^1\frac{dx}{\{x[(p+q)^2-M^2+i0^+]+(1-x)(q^2-M^2+i0^+)\}^2}\\ &= \int_0^1\frac{dx}{[(q+xp)^2-M^2+i0^++p^2x(1-x)]^2}\end{aligned}$$

$$\mathrm{tr}[\gamma^\mu[\not p+\not q+M]\gamma^\nu(\not q+M)] = 4[(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu - g^{\mu\nu}(p+q)\cdot q + M^2 g^{\mu\nu}]$$

$$\Pi^{\mu\nu}(p) = -4e^2\int_0^1 dx\int\frac{d^4q}{(2\pi)^4}\ \frac{(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu-g^{\mu\nu}(p+q)\cdot q+M^2g^{\mu\nu}}{[(q+xp)^2-M^2+i0^++p^2x(1-x)]^2} + O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = D^{-1}_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = -4e^2\int_0^1 dx \int \frac{d^4q}{(2\pi)^4}\frac{(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu - g^{\mu\nu}(p+q)\cdot q + M^2 g^{\mu\nu}}{[(q+xp)^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

$$\xlongequal{\text{维数正规化}} -4e^2\int_0^1 dx\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu - g^{\mu\nu}(p+q)\cdot q + M^2 g^{\mu\nu}}{[(q+xp)^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

$$\xlongequal{q\to q-xp} -4e^2\int_0^1 dx\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\ [q^2-M^2+i0^+ + p^2x(1-x)]^{-2}\{[(1-x)p^\mu+q^\mu](q^\nu-xp^\nu)$$

讨论见后！ $$+[(1-x)p^\nu+q^\nu](q^\mu-xp^\mu) - g^{\mu\nu}[p(1-x)+q]\cdot(q-xp) + M^2g^{\mu\nu}\} + O(e^4)$$

$$= -4e^2\int_0^1 dx\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{2[(x-1)xp^\mu p^\nu + q^\mu q^\nu] - g^{\mu\nu}[p^2(x-1)x+q^2] + M^2g^{\mu\nu}}{[q^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

$$\xlongequal{q^\mu q^\nu \to \frac{g^{\mu\nu}}{D}q^2} 4e^2\int_0^1 dx\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[q^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\Big|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = 4e^2\int_0^1 dx\ \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[q^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

Wick转动： $q^0 = iq_E^D \quad \vec{q} = \vec{q}_E$

$$\Pi^{\mu\nu}(p) = 4e^2\int_0^1 dx\mu^{4-D}\int \frac{d\vec{q}}{(2\pi)^D}\int_{-\infty}^{\infty}dq^0\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[(q^0)^2 - \vec{q}\cdot\vec{q} - M^2 + i0^+ + p^2x(1-x)]^2}$$

$$= -4e^2\int_0^1 dx\mu^{4-D}\int \frac{d\vec{q}}{(2\pi)^D}\int_{i\infty}^{-i\infty}dq^0\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[(q^0)^2 - \vec{q}\cdot\vec{q} - M^2 + i0^+ + p^2x(1-x)]^2}$$

$$= 4ie^2\int_0^1 dx\mu^{4-D}\int \frac{d\vec{q}_E}{(2\pi)^D}\int_{-\infty}^{\infty}dq_E^D\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2g^{\mu\nu}}{[-q_E^2 - M^2 + i0^+ + p^2x(1-x)]^2}$$

$$= 4ie^2\int_0^1 dx\ \mu^{4-D}\int \frac{d^Dq_E}{(2\pi)^D}\ \frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2g^{\mu\nu}}{[q_E^2 + M^2 - i0^+ - p^2x(1-x)]^2} + O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = 4ie^2\int_0^1 dx\ \mu^{4-D}\int\frac{d^Dq_E}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2 g^{\mu\nu}}{[q_E^2 + M^2 - i0^+ - p^2x(1-x)]^2} + O(e^4)$$

$$\int d^Dq_E = \Omega_D\int_0^\infty \kappa^{D-1}d\kappa \qquad \int_0^1 dx\ x^{\alpha-1}(1-x)^{\beta-1} \equiv B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$(\sqrt{\pi})^D = [\int_{-\infty}^{\infty} dx e^{-x^2}]^D = \int d^Dq_E\ e^{-q_E^2} = \Omega_D\int_0^\infty d\kappa\ \kappa^{D-1}e^{-\kappa^2} = \frac{\Omega_D}{2}\int_0^\infty d\kappa^2\ (\kappa^2)^{\frac{D}{2}-1}e^{-\kappa^2} = \frac{1}{2}\Omega_D\Gamma(D/2)$$

$$\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}\int_0^\infty d\kappa^2\frac{(\kappa^2)^{\frac{D}{2}-1}}{(\kappa^2+\nu^2)^2} \overset{x=\frac{\nu^2}{\kappa^2+\nu^2}}{=\!=\!=\!=\!=} \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\int_0^1 dx\ x^{1-\frac{D}{2}}(1-x)^{\frac{D}{2}-1} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\int_0^\infty d\kappa\frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}\int_0^\infty d\kappa^2\frac{(\kappa^2)^{\frac{D}{2}}}{(\kappa^2+\nu^2)^2} \overset{x=\frac{\nu^2}{\kappa^2+\nu^2}}{=\!=\!=\!=\!=} \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\int_0^1 dx\ x^{-\frac{D}{2}}(1-x)^{\frac{D}{2}} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})$$

光子传播子

$$i(\Psi_0^-,\mathbf{T}\,A_\mu(x)A_\nu(y)\;\Psi_0^+)_C=\frac{\delta^2W[J,I,\bar I]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar I=I=0}=iD_{\mu\nu}(x,y)=[i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y)=\mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)]+O(e^4)=\int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p)=4ie^2\int_0^1 dx\;\mu^{4-D}\int\frac{d^Dq_E}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu-p^2g^{\mu\nu})-(1-\frac{2}{D})g^{\mu\nu}q_E^2-M^2g^{\mu\nu}}{[q_E^2+M^2-i0^+-p^2x(1-x)]^2}+O(e^4)$$

$$\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2}=\frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})\qquad\int_0^\infty d\kappa\frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^2}=\frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})$$

$$\Pi^{\mu\nu}(p)=4ie^2\int_0^1 dx\;\Omega_D\mu^{4-D}\int_0^\infty\frac{\kappa^{D-1}d\kappa}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu-p^2g^{\mu\nu})-(1-\frac{2}{D})g^{\mu\nu}\kappa^2-M^2g^{\mu\nu}}{[\kappa^2+M^2-i0^+-p^2x(1-x)]^2}$$

$$\overset{\Omega_D=\frac{2\pi^{\frac{D}{2}}}{\Gamma(D/2)}}{=\!=\!=\!=\!=\!=}\frac{4ie^2\pi^{\frac{D}{2}}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\int_0^1 dx\{[(1-x)x(2p^\mu p^\nu-p^2g^{\mu\nu})-M^2g^{\mu\nu}][M^2-i0^+-p^2x(1-x)]^{\frac{D}{2}-2}$$
$$\times\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})-g^{\mu\nu}(1-\frac{2}{D})[M^2-i0^+-p^2x(1-x)]^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})\}$$

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\Big|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = \frac{4ie^2\pi^{D/2}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\int_0^1 dx\ \{[(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - M^2 g^{\mu\nu}][M^2 - i0^+ - p^2 x(1-x)]^{\frac{D}{2}-2}$$
$$\times\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2}) - g^{\mu\nu}(1-\frac{2}{D})[M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})\} + O(e^4)$$

$$(1-\tfrac{2}{D})\Gamma(1+\tfrac{D}{2})\Gamma(1-\tfrac{D}{2}) = -\Gamma(\tfrac{D}{2})\Gamma(2-\tfrac{D}{2})$$

$$= \frac{4ie^2\pi^{D/2}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})\int_0^1 dx\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2}$$
$$\times\{[(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - M^2 g^{\mu\nu}] + g^{\mu\nu}[M^2 - i0^+ - p^2x(1-x)]\} + O(e^4)$$
$$= (p^2 g^{\mu\nu} - p^\mu p^\nu)\frac{(-i)e^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\ (1-x)x\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2} + O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar I]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar I=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\frac{(-i)e^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\ (1-x)x\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2} + O(e^4)$$

$$\Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\pi(p^2) \qquad p_\mu \Pi^{\mu\nu}(p) = 0$$

$$\pi(p^2) = -\frac{ie^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\ (1-x)x\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2} + O(e^4)$$

$$\Pi^{\mu\nu}(x,y) = (-g^{\mu\nu}\partial_x^2 + \partial_x^\mu\partial_x^\nu)\pi(-\partial_x^2)\delta(x-y) \qquad iD_0^{-1,\mu\nu}(x,y) = [g^{\mu\nu}\partial_x^2 - (1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-y)$$

$$(D_0^{-1}+\Pi)^{\mu\nu}(x,y) = -i\{(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu)[1 - i\pi(-\partial_x^2)] + \lambda\partial_x^\mu\partial_x^\nu\}\delta(x-y)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = D_{\mu\nu}(x,y) = \{\frac{i}{\partial_x^2[1-i\pi(-\partial_x^2)]}[g^{\mu\nu} - \frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}] + \frac{1}{\lambda}\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^4}\}\delta(x-y)$$

光子传播子

$$(\Psi_0^-, \mathbf{T}A_\mu(x)A_\nu(y)\Psi_0^+)_C = -i\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\Big|_{J^\mu=\bar{I}=I=0} = \{\frac{i}{\partial_x^2[1-i\pi(-\partial_x^2)]}[g^{\mu\nu}-\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]+\frac{1}{\lambda}\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^4}\}\delta(x-y)$$

$$\pi(p^2) = -\frac{ie^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\,(1-x)x\,[M^2-i0^+-p^2x(1-x)]^{\frac{D}{2}-2}+O(e^4)$$

- $\Gamma(2-\frac{D}{2}) \xRightarrow{D\to 4} \frac{1}{2-D/2}-\gamma+O(2-D/2) \qquad \gamma = 0.5772157$
  $\pi(p^2)$在$1+3$维时空产生发散!
- $\pi(p^2)$无极点 $\Rightarrow$ 辐射修正不影响光子质量 1+1维：$M=0$使光子获得质量！
- 辐射修正使光子的波函数前多了一个因子 $\frac{1}{1+i\pi(0)}$

$$A^{\frac{D}{2}-2} = e^{(\frac{D}{2}-2)\ln A} \xRightarrow{D\to 4} 1+(\frac{D}{2}-2)\ln A+O((\frac{D}{2}-2)^2)$$

$$\pi(p^2) = -\frac{ie^2\mu^{4-D}}{2\pi^2}[\frac{1}{2-D/2}-\gamma+\cdots]\int_0^1 dx\,(1-x)x\left[\frac{[M^2-i0^+-p^2x(1-x)]}{4\pi}\right]^{\frac{D}{2}-2}+O(e^4)$$

$$= -\frac{ie^2}{12\pi^2}\left[\frac{1}{2-D/2}-\gamma-6\int_0^1 dx\,(1-x)x\,\ln\frac{[M^2-i0^+-p^2x(1-x)]}{4\pi\mu^2}\right]+O(e^4)$$

常用的正规化

维数正规化 1999年诺贝尔物理奖

$$\int \frac{d^4q}{(2\pi)^4} \Rightarrow \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}$$ 协变，但只停留在圈积分的层次，若扩充到γ矩阵，γ_5有问题

维数正规化中无幂次发散！

$$\int d^Dq_E = \Omega_D \int_0^\infty \kappa^{D-1}d\kappa \qquad \int_0^1 dx\, x^{\alpha-1}(1-x)^{\beta-1} \equiv B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\int_0^\infty d\kappa^2 \frac{(\kappa^2)^{\frac{D}{2}-1+n}}{\kappa^2+\nu^2} \overset{x=\frac{\nu^2}{\kappa^2+\nu^2}}{=\!=\!=\!=\!=} -(\nu^2)^{\frac{D}{2}-1+n}\int_1^0 dx\, x^{-\frac{D}{2}-n}(1-x)^{\frac{D}{2}-1+n} = (\nu^2)^{\frac{D}{2}-1+n}\Gamma(\frac{D}{2}+n)\Gamma(1-\frac{D}{2}-n)$$

$$d\kappa^2 = -\frac{\nu^2}{x^2}dx \qquad \frac{1}{\kappa^2+\nu^2} = \frac{x}{\nu^2} \qquad \kappa^2 = \nu^2 x^{-1}(1-x)$$

$$\int_0^\infty d\kappa^2\,(\kappa^2)^{\frac{D}{2}-2+n} = \lim_{\nu\to 0}\int_0^\infty d\kappa^2 \frac{(\kappa^2)^{\frac{D}{2}-1+n}}{\kappa^2+\nu^2} = \lim_{\nu\to 0}(\nu^2)^{\frac{D}{2}-1+n}\Gamma(\frac{D}{2}+n)\Gamma(1-\frac{D}{2}-n) \overset{\frac{D}{2}-2+n>-1}{=\!=\!=\!=\!=\!=} 0$$

常用的正规化

维数正规化 1999年诺贝尔物理奖

$$\int \frac{d^4q}{(2\pi)^4} \Rightarrow \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}$$ 协变, 但只停留在圈积分的层次, 若扩充到γ矩阵, γ_5有问题

动量截断正规化

$$\int \frac{d^4q_E}{(2\pi)^4} \Rightarrow \frac{\Omega_4}{(2\pi)^4}\int_0^\Lambda dq_E\, q_E^3$$ 不协变

Pauli-Villars正规化

$$\frac{1}{(p+q)^2-M^2+i0^+} \Rightarrow \frac{1}{(p+q)^2-M^2+i0^+} - \frac{1}{(p+q)^2-\Lambda^2+i0^+}$$

维数正规化与动量截断的互换: $\frac{1}{2-D/2}-\gamma+\ln 4\pi\mu^2 \Leftrightarrow \ln\Lambda^2$

$$\frac{2\pi^{D/2}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{16\pi^2}\left(\frac{\nu^2}{4\pi\mu^2}\right)^{\frac{D}{2}-2}\Gamma(2-\frac{D}{2}) = \frac{1}{16\pi^2}\left[\frac{1}{2-D/2}-\gamma+\ln\frac{4\pi\mu^2}{\nu^2}\right]$$

$$\frac{2\pi^2}{(2\pi)^4}\int_0^\Lambda d\kappa\frac{\kappa^3}{(\kappa^2+\nu^2)^2} = \frac{1}{16\pi^2}\int_0^{\Lambda^2} d\kappa^2\frac{\kappa^2}{(\kappa^2+\nu^2)^2} = \frac{1}{16\pi^2}[\ln\frac{\Lambda^2}{\nu^2}+1]$$

关于圈动量积分中的动量平移

- 收敛积分和对数发散积分可以进行动量平移
- 线性发散和二次发散积分动量平移后相差一有限项
- 三次以上发散动量平移后相差低至少二阶的偶次发散项

对数发散： $\frac{1}{A^2B^2} = 6\int_0^1 dx \frac{x(1-x)}{[xA+(1-x)B]^4}$

$$\int d^4q\left[\frac{1}{[(q+p)^2-m^2]^2}-\frac{1}{[q^2-m^2]^2}\right]=\int d^4q\frac{[q^2-m^2]^2-[(q+p)^2-m^2]^2}{[(q+p)^2-m^2]^2[q^2-m^2]^2}$$

$$=6\int d^4q\int_0^1 dx\, x(1-x)\frac{-2(q^2-m^2)(p^2+2q\cdot p)-(p^2+2q\cdot p)^2}{[(xp+q)^2-m^2+p^2x(1-x)]^4}$$

$$=6\int d^4q\int_0^1 dx\, x(1-x)\frac{-2[(q-xp)^2-m^2][p^2+2(q-xp)\cdot p]-[p^2+2(q-xp)\cdot p]^2}{[q^2-m^2+p^2x(1-x)]^4}$$

$\overset{x\leftrightarrow 1-x}{=\!=\!=\!=\!=\!=} 0$ 其中利用了 $x(1-x) \overset{x\leftrightarrow 1-x}{=\!=\!=\!=\!=\!=} x(1-x)$ $2x-1 \overset{x\leftrightarrow 1-x}{=\!=\!=\!=\!=\!=} 1-2x$

被积函数分子 $=-2q^2(1-2x)p^2+(8x-4)(q\cdot p)^2-2(x^2p^2-m^2)(1-2x)p^2-p^4(1-2x)^2$

$\sim [-2(1-2x)x^2-(1-2x)^2]p^4=(2x-1)(2x^2-2x+1)p^4=(2x-1)[2x(x-1)-1]p^4$

线性发散： $\frac{1}{A^2B^2} = 6\int_0^1 dx \frac{x(1-x)}{[xA+(1-x)B]^4}$

$$\int d^4q\left[\frac{q\cdot p}{[(q+p)^2-m^2]^2} - \frac{(q-p)\cdot p}{[q^2-m^2]^2}\right] = \int d^4q\left[\frac{q\cdot p\{[q^2-m^2]^2 - [(q+p)^2-m^2]^2\}}{[(q+p)^2 - m^2]^2[q^2 - m^2]^2} + \frac{p^2}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx \frac{6x(1-x)q\cdot p[-2(q^2-m^2)(p^2+2q\cdot p) - (p^2+2q\cdot p)^2]}{[(xp+q)^2 - m^2 + p^2x(1-x)]^4} + \frac{p^2}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx \frac{6x(1-x)(q-xp)\cdot p\{-2[(q-xp)^2 - m^2][p^2 + 2(q-xp)\cdot p] - [p^2 + 2(q-xp)\cdot p]^2\}}{[q^2 - m^2 + p^2x(1-x)]^4} + \right.$$

被积函数分子 $= 6x(1-x)q\cdot p[-4q^2q\cdot p + 4q\cdot p(1-2x)p^2 - 4(x^2p^2-m^2)q\cdot p - 4q\cdot p(1-2x)p^2]$
$-6(1-x)x^2p^2\{-2q^2(1-2x)p^2 + (8x-4)(q\cdot p)^2 - 2(x^2p^2-m^2)(1-2x)p^2 - p^4(1-2x)^2\}$

$\sim 6x(1-x)[-q^4p^2 - (x^2p^2-m^2)q^2p^2] - 6(1-x)x^2p^2\{3q^2p^2 - 2m^2p^2 - p^4[x^2(1-2x) + (1-2x)^2]\}$

对数发散项 $= \int d^4q\left[\int_0^1 dx \frac{-6x(1-x)[q^2 - m^2 + p^2x(1-x) + m^2 - p^2x(1-x)]^2p^2}{[q^2 - m^2 + p^2x(1-x)]^4} + \frac{p^2}{[q^2-m^2]^2}\right]$

$$\xLongrightarrow{\text{准到发散项}} \int d^4q\left[\int_0^1 dx \frac{-p^2}{[q^2 - m^2 + p^2x(1-x)]^2} + \frac{p^2}{[q^2-m^2]^2}\right] = 0$$

二次发散：$\frac{1}{A^2B^2} = 6\int_0^1 dx \frac{x(1-x)}{[xA+(1-x)B]^4}$

$$\int d^4q\left[\frac{q^\mu q^\nu}{[(q+p)^2-m^2]^2} - \frac{(q^\mu-p^\mu)(q^\nu-p^\nu)}{[q^2-m^2]^2}\right] = \int d^4q\left[\frac{q^\mu q^\nu}{[(q+p)^2-m^2]^2} - \frac{q^\mu q^\nu + p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx\, 6x(1-x)\frac{q^\mu q^\nu[-2(q^2-m^2)(p^2+2q\cdot p)-(p^2+2q\cdot p)^2]}{[(xp+q)^2-m^2+p^2x(1-x)]^4} - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx\, \frac{\{-2[(q-xp)^2-m^2][p^2+2(q-xp)\cdot p]-[p^2+2(q-xp)\cdot p]^2\}}{[q^2-m^2+p^2x(1-x)]^4}\right.$$

$$\left.\times 6x(1-x)(q^\mu q^\nu - xq^\mu p^\nu - xp^\mu q^\nu + x^2p^\mu p^\nu) - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$\xRightarrow{\text{准到发散项}} \int d^4q\left[\int_0^1 dx\, \frac{24q^2 q\cdot p x^2(1-x)(q^\mu p^\nu + p^\mu q^\nu)}{[q^2-m^2+p^2x(1-x)]^4} - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx\, \frac{12q^4x^2(1-x)p^\mu p^\nu}{[q^2-m^2+p^2x(1-x)]^4} - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= p^\mu p^\nu \int d^4q\left[\int_0^1 dx\, \frac{6q^4x(1-x)}{[q^2-m^2+p^2x(1-x)]^4} - \frac{1}{[q^2-m^2]^2}\right] \xRightarrow{\text{准到发散项}} 0$$

二次发散与规范对称性

$$\Pi^{\mu\nu}(p) = -2ie^2\int_0^1 dx\,\Omega_4\int_0^{\Lambda^2}\frac{\kappa^2 d\kappa^2}{(2\pi)^4}\,\frac{(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - \frac{1}{2}g^{\mu\nu}\kappa^2 - M^2 g^{\mu\nu}}{[\kappa^2 + M^2 - i0^+ - p^2x(1-x)]^2}$$

$$= -\frac{2ie^2\Omega_4}{(2\pi)^4}\int_0^1 dx\Big[-\frac{1}{2}g^{\mu\nu}\Lambda^2 + 2(1-x)x(p^\mu p^\nu - p^2 g^{\mu\nu})\ln[\frac{\Lambda^2}{M^2 - p^2x(1-x)} + 1]$$
$$+[\frac{3}{2}M^2 g^{\mu\nu} - (1-x)x(2p^\mu p^\nu - \frac{1}{2}p^2 g^{\mu\nu})\frac{\Lambda^2}{\Lambda^2 + M^2 - p^2x(1-x)}]\Big]$$

Ward-Takahashi-Taylor恒等式： $\partial_{\mu,x}D^{\mu\nu}(x,y) = \partial_{\mu,x}D_0^{\mu\nu}(x,y)$

$$D_0^{\mu\nu}(x,y) = \int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}D_0^{\mu\nu}(p) \qquad D_0^{\mu\nu}(p) = \frac{-i}{p^2}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{p^\mu p^\nu}{p^2}]$$

$$D^{\mu\nu}(p) = [D_0^{-1}(p) + \Pi(p)]^{-1\mu\nu}(p) \qquad \Pi^{\mu\nu}(p) = p^2 g^{\mu\nu}\pi(p^2) - p^\mu p^\nu\pi_1(p^2)$$

$$iD^{-1,\mu\nu}(p) = -p^2[1 - i\pi(p^2)]g^{\mu\nu} + [1 - \lambda - i\pi_1(p^2)]p^\mu p^\nu \overset{\pi=\pi_1=0}{=\!=\!=\!=\!=\!=} iD_0^{-1,\mu\nu}(p)$$

$$D^{\mu\nu}(p) = \frac{-i}{p^2[1 - i\pi(p^2)]}\Big[g^{\mu\nu} + \frac{p^\mu p^\nu[1 - \lambda - i\pi_1(p^2)]}{p^2[\lambda - i\pi(p^2) + i\pi_1(p^2)]}\Big]$$

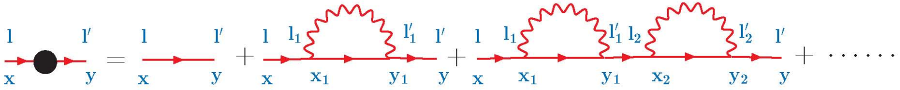
规范不变性禁戒二次发散！

$$p_\mu D^{\mu\nu}(p) = p_\mu D_0^{\mu\nu}(p) = -i\frac{p^\nu}{\lambda p^2} \;\Rightarrow\; \pi_1(p^2) = \pi(p^2) \qquad \Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\pi(p^2)$$

费米子传播子

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_l(x)\delta I_{l'}(y)}\bigg|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y)$$

$$= -i\frac{e^{\int d^4y d^4z\,[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ \psi_l(x)\overline{\psi}_{l'}(y)\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\,[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}\Bigg|_{A_\mu=\overline{\psi}=\psi=0}$$



$$= -iS_{0,ll'}(x,y) - i\int d^4x_1 d^4y_1 S_{0,ll_1}(x,x_1)(-1)\Sigma_{l_1l_1'}(x_1,y_1)S_{0,l_1'l}(y_1,y)$$

$$-i\int d^4x_1 d^4y_1 d^4x_2 d^4y_2 S_{0,ll_1}(x,x_1)\Sigma_{l_1l_1'}(x_1,y_1)S_{0,l_1'l_2}(y_1,x_2)\Sigma_{l_2l_2'}(x_2,y_2)S_{0,l_2'l'}(y_2,y) + \cdots$$

$$= -i[S_0(1-\Sigma S_0+\Sigma S_0\Sigma S_0+\cdots)]_{ll'}(x,y) = -i[S_0(1+\Sigma S_0)^{-1}]_{ll'}(x,y) = -i[(S_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

SD方程： $S^{-1}-S_0^{-1}=\Sigma$

$$\Sigma_{ll'}(x,y) = [(-i)e\gamma^\mu S_0(x,y)D_{0,\mu\nu}(x,y)(-i)e\gamma^\nu]_{ll'} + O(e^4) = \int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$= e^2\{\gamma^\mu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-y)\gamma^\nu\frac{1}{\partial_y^2-i0^+}[g_{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_{\mu,y}\partial_{\nu,y}}{\partial_y^2}]\}_{ll'}\delta(y-x) + O(e^4)$$

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\Big|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y) = -i[(S_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

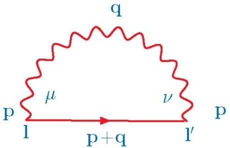
$$\Sigma_{ll'}(x,y) = [(-i)e\gamma^\mu S_0(x,y)D_{0,\mu\nu}(x,y)(-i)e\gamma^\nu]_{ll'} + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$= e^2\gamma^\mu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-y)\gamma^\nu \frac{1}{\partial_y^2 - i0^+}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-x) + O(e^4)$$

$$= e^2\int\frac{d^4pd^4q}{(2\pi)^8}e^{-i(p-q)\cdot(x-y)}\{\gamma^\mu\frac{1}{p\!\!\!/ - M - i0^+}\gamma^\nu\frac{-1}{q^2+i0^+}[g_{\mu\nu} - (1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}]\}_{ll'} + O(e^4)$$

费曼规范: $\lambda = 1$ $\qquad \frac{1}{AB} = \int_0^1 \frac{dx}{[(1-x)A+xB]^2}$

$$\Sigma(p) = -e^2\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\frac{p\!\!\!/ + q\!\!\!/ + M}{(p+q)^2 - M^2 + i0^+}\gamma_\mu\frac{1}{q^2+i0^+} + O(e^4) \quad -$$



$$= -e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{\gamma^\mu(p\!\!\!/ + q\!\!\!/ + M)\gamma_\mu}{\{x[(p+q)^2 - M^2 + i0^+] + (1-x)(q^2+i0^+)\}^2} + O(e^4)$$

$$= 2e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{p\!\!\!/ + q\!\!\!/ - 2M}{\{x[(p+q)^2 - M^2 + i0^+] + (1-x)(q^2+i0^+)\}^2} + O(e^4)$$

费米子传播子

$$-i(\Psi_0^-,\mathbf{T}\,\psi_l(x)\overline{\psi}_{l'}(y)\,\Psi_0^+)_C=\left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\right|_{J^\mu=\bar{I}=I=0}=-iS_{ll'}(x,y)=-i[(S_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

$$\Sigma_{ll'}(x,y)=[(-i)e\gamma^\mu S_0(x,y)D_0(x,y)(-i)e\gamma^\nu]_{ll'}+O(e^4)=\int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$\Sigma(p)=2e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{\not{p}+\not{q}-2M}{\{x[(p+q)^2-M^2+i0^+]+(1-x)(q^2+i0^+)\}^2}+O(e^4)$$

$$=2e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{\not{p}+\not{q}-2M}{[(q+xp)^2+x(1-x)p^2-xM^2+i0^+]^2}+O(e^4)$$

$$=2e^2\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\int_0^1 dx\frac{\not{p}+\not{q}-2M}{[(q+xp)^2+x(1-x)p^2-xM^2+i0^+]^2}+O(e^4)$$

$$=2e^2\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\int_0^1 dx\frac{(1-x)\not{p}+\not{q}-2M}{[q^2+x(1-x)p^2-xM^2+i0^+]^2}+O(e^4)$$

Wick转动： $\int d^Dq\to i\int d^Dq_E=i\Omega_D\int_0^\infty\kappa^{D-1}d\kappa\quad q^2\to-\kappa^2\quad\Omega_D=\frac{2\pi^{D/2}}{\Gamma(D/2)}$

$$\Sigma(p)=\frac{4i\mu^{4-D}e^2\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^\infty\kappa^{D-1}d\kappa\int_0^1 dx\frac{(1-x)\not{p}-2M}{[-\kappa^2+x(1-x)p^2-xM^2+i0^+]^2}+O(e^4)$$

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = [i\partial\!\!\!/_x - M - i0^+ + \Sigma(i\partial_x)]^{-1}\delta(x-y)$$

$$\Sigma(p) = \frac{4i\mu^{4-D}e^2\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^\infty \kappa^{D-1}d\kappa \int_0^1 dx \frac{(1-x)p\!\!\!/ - 2M}{[\kappa^2 - x(1-x)p^2 + xM^2 - i0^+]^2} + O(e^4)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Sigma(p) = \frac{2ie^2\mu^{4-D}\Gamma(2-\frac{D}{2})}{(4\pi)^{D/2}}\int_0^1 dx[(1-x)p\!\!\!/ - 2M][-x(1-x)p^2 + xM^2 - i0^+]^{\frac{D}{2}-2} + O(e^4)$$

$$\Gamma(2-\frac{D}{2}) \xRightarrow{D\to 4} \frac{1}{2-D/2} - \gamma + O(2-D/2) \qquad A^{\frac{D}{2}-2} \xRightarrow{D\to 4} 1 + (\frac{D}{2}-2)\ln A + O((\frac{D}{2}-2)^2)$$

$$\Sigma(p) = \frac{ie^2}{8\pi^2}\left[(\frac{1}{2-D/2}-\gamma)(\frac{1}{2}p\!\!\!/ - 2M) - \int_0^1 dx\ [(1-x)p\!\!\!/ - 2M]\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \frac{e^{\int d^4y d^4z\,[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)e^{-i\int d^4x\, eA_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\,[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\, eA_\mu\overline{\psi}\gamma^\mu\psi}}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$= \int d^4x'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,x')ie\gamma^\nu_{l_1l_2}S_{0,l_2l'}(x',z) + \int d^4x'd^4y'd^4z'\ D_{0,\mu\nu}(x,x')$$

$$\times S_{0,ll_1}(y,y')ie\gamma^{\mu'}_{l_1l_2}S_{0,l_2l_3}(y',x')(-i)e\gamma^\nu_{l_3l_4}S_{0,l_4l_5}(x',z')(-i)e\gamma^{\nu'}_{l_5l_6}S_{0,l_5l'}(z',z)D_{0,\mu'\nu'}(y',z') + O(e^5)$$

$$= \int d^4x'd^4y'd^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

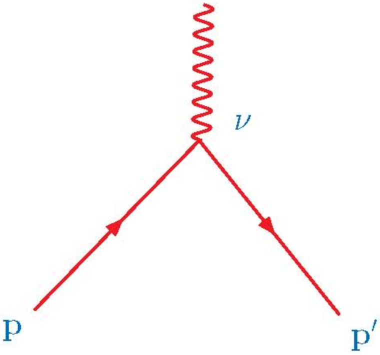
三线顶角

$$\Gamma^\nu(x,y,z) = e\gamma^\nu\delta(y-x)\delta(x-z) - e^3\gamma^{\mu'}S_0(y,x)\gamma^\nu S_0(x,z)\gamma^{\nu'}D_{0,\mu'\nu'}(y,z) + O(e^5)$$

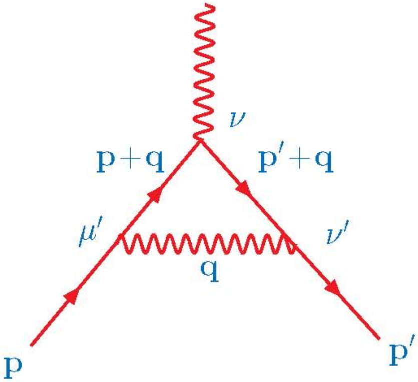
$$= e\gamma^\nu\delta(y-x)\delta(x-z) + ie^3\gamma^{\mu'}(i\partial\!\!\!/_y - M - i0^+)^{-1}\delta(y-x)\gamma^\nu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-z)\gamma^{\nu'}$$

$$\times\frac{1}{\partial_y^2 - i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{\partial_{\mu',y}\partial_{\nu',y}}{\partial_y^2}]\delta(y-z) + O(e^5)$$

三线顶角



$+$



相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \int d^4x' d^4y' d^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

$$\Gamma^\nu(x,y,z) = e\gamma^\nu\delta(y-x)\delta(x-z) + ie^3\gamma^{\mu'}(i\partial\!\!\!/_y - M - i0^+)^{-1}\delta(y-x)\gamma^\nu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-z)\gamma^{\nu'}$$
$$\times\frac{1}{\partial_y^2 - i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{\partial_{\mu',y}\partial_{\nu',y}}{\partial_y^2}]\delta(y-z) + O(e^5)$$
$$= e\gamma^\nu\int\frac{d^4pd^4p'}{(2\pi)^8}e^{-ip\cdot(y-x)-ip'\cdot(x-z)} + ie^3\int\frac{d^4pd^4p'd^4q}{(2\pi)^{12}}e^{-ip\cdot(y-x)-ip'\cdot(x-z)-iq\cdot(y-z)}$$
$$\times\gamma^{\mu'}(p\!\!\!/ - M - i0^+)^{-1}\gamma^\nu(p\!\!\!/' - M - i0^+)^{-1}\gamma^{\nu'}\frac{-1}{q^2+i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{q_{\mu'}q_{\nu'}}{q^2}] + O(e^5)$$
$$= \int\frac{d^4pd^4p'}{(2\pi)^8}e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + ie^3\int\frac{d^4q}{(2\pi)^4}\gamma^{\mu'}\frac{1}{p\!\!\!/ + q\!\!\!/ - M - i0+}\gamma^\nu\frac{1}{p\!\!\!/' + q\!\!\!/ - M - i0+}\gamma^{\nu'}\frac{-1}{q^2+i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{q_{\mu'}q_{\nu'}}{q^2}] + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \int d^4x' d^4y' d^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + ie^3\int\frac{d^4q}{(2\pi)^4}\gamma^{\mu'}\frac{1}{\not p+\not q-M-i0^+}\gamma^\nu\frac{1}{\not p'+\not q-M-i0^+}\gamma^{\nu'}\frac{-1}{q^2+i0^+}[g_{\mu'\nu'}-(1-\frac{1}{\lambda})\frac{q_{\mu'}q_{\nu'}}{q^2}]+O(e^5)$$

费曼规范： $\lambda = 1$ 　　 $\frac{1}{ABC} = 2\int_0^1 dx \int_0^x dy \frac{1}{[Ay+B(x-y)+C(1-x)]^3}$

$$= e\gamma^\nu - ie^3\int\frac{d^4q}{(2\pi)^4}\frac{\gamma^\mu(\not p+\not q+M)\gamma^\nu(\not p'+\not q+M)\gamma_\mu}{[(p+q)^2-M^2+i0^+][(p'+q)^2-M^2+i0^+](q^2+i0^+)} + O(e^5)$$

$$= e\gamma^\nu - 2ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{\gamma^\mu[(\not p+\not q)\gamma^\nu(\not p'+\not q)+M(\not p+\not q)\gamma^\nu+M\gamma^\nu(\not p'+\not q)+M^2\gamma^\nu]\gamma_\mu}{\{y[(p+q)^2-M^2+i0^+]+(x-y)[(p'+q)^2-M^2+i0^+]+(1-x)(q^2+i0^+)\}^3}$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{(\not p'+\not q)\gamma^\nu(\not p+\not q)-2M(p^\nu+q^\nu)-2M(p'^\nu+q^\nu)+M^2\gamma^\nu}{[q^2+2yp\cdot q+2(x-y)p'\cdot q+yp^2+(x-y)p'^2-xM^2+i0^+]^3} + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \int d^4x' d^4y' d^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4pd^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{(\not{p}'+\not{q})\gamma^\nu(\not{p}+\not{q}) - 2M(p^\nu+q^\nu) - 2M(p'^\nu+q^\nu)+M^2\gamma^\nu}{[q^2+2yp\cdot q+2(x-y)p'\cdot q+yp^2+(x-y)p'^2-xM^2+i0^+]^3}+O(e^5)$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{(\not{p}'+\not{q})\gamma^\nu(\not{p}+\not{q}) - 2M(p^\nu+p'^\nu+2q^\nu)+M^2\gamma^\nu}{\{[q+yp+(x-y)p']^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^3}+O(e^5)$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^Dq}{(2\pi)^D}\frac{\mu^{4-D}\ (\not{p}'+\not{q})\gamma^\nu(\not{p}+\not{q}) - 2M(p^\nu+p'^\nu+2q^\nu)+M^2\gamma^\nu}{\{[q+yp+(x-y)p']^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^3}+O(e^5)$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\Big[[(1-x+y)\not{p}'-y\not{p}+\not{q}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'+\not{q}]-2M[(1-2y)$$

$$\times p^\nu + (1-2x+2y)p'^\nu+2q^\nu]+M^2\gamma^\nu\Big]\{q^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^{-3}+O(e^5)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p') \qquad \Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\Big[[(1-x+y)\not{p}'-y\not{p}+\not{q}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'+\not{q}]+M^2\gamma^\nu$$

$$-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu+2q^\nu]\Big]\{q^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^{-3}+O(e^5)$$

$$= e\gamma^\nu - \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']+(M^2+\frac{2}{D}\kappa^2)\gamma^\nu$$

$$-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]\Big]\{-\kappa^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^{-3}+O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + (M^2 + \frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-2}\Gamma(1+\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p') \qquad \Delta^2 \equiv [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+$$

$$\Gamma^\nu_{\rm div}(p,p') = \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2+1)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D+1}d\kappa\gamma^\nu[\kappa^2+\Delta^2]^{-3} + O(e^5)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{\frac{D}{2}}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + (M^2 + \frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-2}\Gamma(1+\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p') \qquad \Delta^2 \equiv [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+$$

$$\Gamma^\nu_{\rm div}(p,p') = \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2+1)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D+1}d\kappa\gamma^\nu[\kappa^2+\Delta^2]^{-3} + O(e^5)$$

$$= \gamma^\nu \frac{e^3\Gamma(2-\frac{D}{2})}{8\pi^2}\int_0^1 dx\int_0^x dy\left[\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right]^{\frac{D}{2}-2} + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{\frac{D}{2}}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + (M^2 + \frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-2}\Gamma(1+\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p') \qquad \Gamma(2-\frac{D}{2}) \xRightarrow{D\to 4} \frac{1}{2-D/2} - \gamma + O(2-D/2)$$

$$\Gamma^\nu_{\rm div}(p,p') = \gamma^\nu \frac{e^3\Gamma(2-\frac{D}{2})}{8\pi^2}\int_0^1 dx\int_0^x dy\left[\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right]^{\frac{D}{2}-2} + O(e^5)$$

$$= \gamma^\nu \frac{e^3}{8\pi^2}\left[\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy\ \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^5)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{\frac{D}{2}}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + (M^2 + \frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p')$$

$$\Gamma^\nu_{\rm div}(p,p') = \gamma^\nu\frac{e^3}{8\pi^2}\left[\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy\ \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^5)$$

$$\Gamma^\nu_{\rm fin}(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p') \qquad \Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p')$$

$$\Gamma^\nu_{\rm div}(p,p') = \gamma^\nu \frac{e^3}{8\pi^2}\left[\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx \int_0^x dy\ \ln \frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^5)$$

$$\Gamma^\nu_{\rm fin}(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx \int_0^x dy \int_0^\infty \kappa^{D-1} d\kappa \Big[[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}']$$

$$+M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^3}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{-1}$$

$$\Gamma^\nu_{\rm fin}(p,p') = e\gamma^\nu + \frac{2e^3\pi^2}{(2\pi)^4}\int_0^1 dx \int_0^x dy \Big[[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}']$$

$$+M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-1} + O(e^5)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{e^3}{8\pi^2}\Big[\gamma^\nu\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx \int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\}$$

$$+\int_0^1 dx \int_0^x dy \frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\Big] + O(e^5)$$

$$y \to x - y \qquad \int_0^x dy \to \int_0^x dy$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{e^3}{8\pi^2}\Big[\gamma^\nu\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx \int_0^x dy\ \ln\frac{[(x-y)p+yp']^2 - (x-y)p^2 - yp'^2 + xM^2 - i0^+}{4\pi\mu^2}\}$$

$$+\int_0^1 dx \int_0^x dy \frac{[(1-y)\not{p}' + (y-x)\not{p}]\gamma^\nu[(1-x+y)\not{p} - y\not{p}'] + M^2\gamma^\nu - 2M[(1-2x+2y)p^\nu + (1-2y)p'^\nu]}{[(x-y)p+yp']^2 - (x-y)p^2 - yp'^2 + xM^2 - i0^+}\Big] + O(e^5)$$

面临的问题与对策

问题：

- 光子和费米子传播子及光子与费米子的相互作用顶角的一圈辐射修正有紫外发散
- 紫外发散通过时空维数的延拓变成在$D=4$时的极点
- 理论是否已经无意义了？

对策：

- 格林函数的发散并不意味着物理可观测量发散，它只意味用拉格朗日量中的量表达的物理量发散
- 实验上观测的是物理量与物理量之间的关系，只要这种关系中没有发散，理论就是有意义的！
- 拉格朗日量中有限的量导致发散的物理量 $\Rightarrow$探索拉格朗日量中发散的量能否导致有限的物理量
- 拉格朗日量中发散的量能否导致有限的物理量$\Rightarrow$物理量与物理量之间的关系没有发散

重整拉格朗日量

拉格朗日量中发散的量?

- 拉格朗日量中发散的量叫“裸量” 加下标B:
  裸场， 裸参数（质量、耦合常数、...）
- 裸量是发散的！ 用有限的量“重整化量”表达裸量

$$\psi_B = Z_2^{1/2}\psi \qquad A_B^\mu = Z_3^{1/2}A^\mu \qquad e_B Z_2 Z_3^{1/2} = Z_1 e \qquad M_B = M + \delta M \qquad \lambda_B = Z_\lambda \lambda$$

用重整化量表达的拉格朗日量与抵消项

$$\begin{aligned}
\mathcal{L}_{\rm QED} &= -\frac{1}{4}F_{B,\mu\nu}(x)F_B^{\mu\nu}(x) - \frac{\lambda_B}{2}[\partial_\mu A_B^\mu(x)]^2 + \overline{\psi}_B(x)[i\partial\!\!\!/ - e_B A\!\!\!/_B(x) - M_B]\psi_B(x) \\
&= -\frac{Z_3}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - Z_3 Z_\lambda \frac{\lambda}{2}[\partial_\mu A^\mu(x)]^2 + Z_2\overline{\psi}(x)[i\partial\!\!\!/ - Z_1 Z_2^{-1} e A\!\!\!/(x) - M - \delta M]\psi(x) \\
&= -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial_\mu A^\mu(x)]^2 + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) - \frac{1}{4}(Z_3-1)F_{\mu\nu}(x)F^{\mu\nu}(x) \\
&\quad -\frac{\lambda}{2}(Z_3 Z_\lambda - 1)[\partial_\mu A^\mu(x)]^2 + \overline{\psi}(x)[(Z_2-1)(i\partial\!\!\!/ - M) - (Z_1-1)eA\!\!\!/(x) - Z_2\delta M]\psi(x)
\end{aligned}$$

对数发散、二次发散的重整相消与理论的自然性

以粒子质量为例

$$M_{\text{phy}}^2 = (u_0 + u_1 + u_2 + \cdots)\Lambda^2 + (l_0 + l_1 + l_2 + \cdots)M_r^2 \ln(\Lambda^2/M_r^2) + (f_0 + f_1 + f_2 + \cdots)M_r^2$$

- 二次发散对高能区物理依赖十分明显
- 对数发散对高能区物理依赖不太明显

三种典型的物理能量尺度： $M_{\text{phy}}^2 \sim M_r^2 \sim (100GeV)^2$

$\Lambda = 10^3\text{GeV}$	$u_0 + u_1 + u_2 + \cdots \sim 10^{-2}$	$l_0 + l_1 + l_2 + \cdots \sim 1$
$\Lambda = 10^{16}\text{GeV}$	$u_0 + u_1 + u_2 + \cdots \sim 10^{-28}$	$l_0 + l_1 + l_2 + \cdots \sim 10^{-1}$
$\Lambda = 10^{19}\text{GeV}$	$u_0 + u_1 + u_2 + \cdots \sim 10^{-34}$	$l_0 + l_1 + l_2 + \cdots \sim 10^{-1}$

自然性：

- 二次发散的存在要求计算的精细调节，对数发散不需要
- 精细调节对计算是不稳定的，不自然！
- 在标量场、矢量场和旋量场中只有标量场和有质量的矢量场具有二次发散，因而它们是不自然的！

重整格林函数和顶角函数

$$\psi_B = Z_2^{1/2}\psi \qquad A_B^\mu = Z_3^{1/2}A^\mu \qquad e_B Z_2 Z_3^{1/2} = Z_1 e \qquad M_B = M + \delta M \qquad \lambda_B = Z_\lambda \lambda$$

重整化的格林函数

$$(\Psi_0^-, \mathbf{T}\,[\psi_{B,l_1}(x_1)\overline{\psi}_{B,l_1'}(x_1')\cdots\psi_{B,l_n}(x_n)\overline{\psi}_{B,l_n'}(x_n')A_{B,\mu_1}(y_1)\cdots A_{B,\mu_m}(y_m)]\ \Psi_0^+)_C$$

$$= Z_2^n Z_3^{m/2}(\Psi_0^-, \mathbf{T}\,[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ \Psi_0^+)$$

重整化的顶角函数

$$\frac{\delta^{2n+m}\Gamma[A_B,\psi_B,\overline{\psi}_B]}{\delta\overline{\psi}_{B,l_1}(x_1)\delta\psi_{B,l_1'}(x_1')\cdots\delta\overline{\psi}_{B,l_n}(x_n)\delta\psi_{B,l_n'}(x_n')\delta A_B^{\mu_1}(y_1)\cdots\delta A_B^{\mu_m}(y_m)}\Bigg|_{A_B=A_0,\ \psi_B=\psi_0,\ \overline{\psi}_B=\overline{\psi}_0}$$

$$= Z_2^{-n}Z_3^{-m/2}\frac{\delta^{2n+m}\Gamma[A_B,\psi_B,\overline{\psi}_B]}{\delta\overline{\psi}_{l_1}(x_1)\delta\psi_{l_1'}(x_1')\cdots\delta\overline{\psi}_{l_n}(x_n)\delta\psi_{l_n'}(x_n')\delta A^{\mu_1}(y_1)\cdots\delta A^{\mu_m}(y_m)}\Bigg|_{A_B=A_0,\ \psi_B=\psi_0,\ \overline{\psi}_B=\overline{\psi}_0}$$

Ward-Takahashi-Taylor恒等式

$$\partial_{\mu,x} i D_B^{\mu\nu}(x,y) = -\frac{\partial_x^\nu}{\lambda_B \partial_x^2}\delta(x-y) \qquad \partial_{\mu,x}\Gamma_B^\mu(x,y,z) = e_B S_B^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$$

$$Z_3\partial_{\mu,x} iD^{\mu\nu}(x,y) = -Z_\lambda^{-1}\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y) \quad Z_2^{-1}Z_3^{-\frac{1}{2}}\partial_{\mu,x}\Gamma^\mu(x,y,z) = iZ_1 Z_2^{-1}Z_3^{-\frac{1}{2}} e Z_2^{-1} S^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$$

$Z_\lambda = Z_3^{-1}$ $\qquad$ $Z_1 = Z_2$ 顶角重整化完全由光子的重整化决定；**Abel**近似

重整化格林函数的生成泛函

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{Z_3}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+Z_2\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/(x)-M-\delta M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{Z_3}{2}A_\mu[g^{\mu\nu}\partial^2-(1-Z_3^{-1}\lambda)\partial^\mu\partial^\nu-i0^+]A_\nu-Z_2eA_\mu\overline{\psi}\gamma^\mu\psi+Z_2\overline{\psi}[i\partial\!\!\!/-M-\delta M-i0^+]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu i\tilde{D}_0^{-1,\mu\nu}A_\nu-Z_2eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}i\tilde{S}_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$i\tilde{D}_0^{-1,\mu\nu}=Z_3g^{\mu\nu}\partial^2-(Z_3-\lambda)\partial^\mu\partial^\nu-i0^+ \qquad i\tilde{S}_0^{-1}=Z_2[i\partial\!\!\!/-M-\delta M-i0^+]$$

$$=e^{i\int d^4x\ Z_2e\frac{\delta}{-i\delta J^\mu(x)}\frac{\delta}{-i\delta I(x)}\gamma^\mu\frac{\delta}{i\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu i\tilde{D}_0^{-1,\mu\nu}A_\nu+\overline{\psi}i\tilde{S}_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=e^{-Z_2e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{\frac{1}{2}(A+iJ\tilde{D}_0)_\mu i\tilde{D}_0^{-1,\mu\nu}(A+i\tilde{D}_0J)_\nu+(\overline{\psi}-i\bar{I}S_0)iS_0^{-1}(\psi-iS_0I)+\frac{i}{2}J_\mu\tilde{D}_0^{\mu\nu}J_\nu+i\bar{I}\tilde{S}_0I\}}$$

$$=C\times e^{-Z_2e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\ e^{\int d^4yd^4z[-\frac{1}{2}J_\mu(y)\tilde{D}_0^{\mu\nu}(y,z)J_\nu(z)-\bar{I}(y)\tilde{S}_0(y,z)I(z)]}$$

$$C=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ [\frac{1}{2}A_\mu i\tilde{D}_0^{-1,\mu\nu}A_\nu+\overline{\psi}\ i\tilde{S}_0^{-1}\psi]}$$

$$\underline{D_0\to\tilde{D}_0\qquad S_0\to\tilde{S}_0\qquad e\to Z_2e}$$

$$\tilde{D}_0^{\mu\nu}(y,z)=\frac{i}{Z_3\partial_y^2-i0^+}[g^{\mu\nu}-(1-\frac{Z_3}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)\qquad \tilde{S}_0(y,z)=\frac{iZ_2^{-1}}{i\partial\!\!\!/_y-M-\delta M-i0^+}\delta(y-z)$$

S矩阵的重整化不变性

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0,\mathbf{T}\psi_{H,B,1}(x_1)\cdots\psi_{H,B,n+l}(x_{n+l})\Psi_0)$$

$$=\int\frac{d^4q_1\cdots d^4q_{n+l}}{(i\pi)^{n+l}}e^{-iq_1x_1\cdots-iq_nx_n+iq_{n+1}x_{n+1}\cdots+iq_{n+l}x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}q_1^0(\Psi_0^-,\psi_{B,1}(0)\Psi^-_{\vec{q}_1,\sigma_1})\cdots q_n^0(\Psi_0^-,\psi_{B,n}(0)\Psi^-_{\vec{q}_n\sigma_n})$$

$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n},\Psi^+_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}})_B\, q^0_{n+1}(\Psi^+_{\vec{q}_{n+1},\sigma_{n+1}},\psi_{B,n+1}(0)\Psi_0^+)\cdots q^0_{n+l}(\Psi^+_{\vec{q}_{n+l},\sigma_{n+l}},\psi_{B,n+l}(0)\Psi_0^+)$$

$$\psi_{H,B,i}(x)=Z_i^{1/2}\psi_{H,i}(x)\qquad \psi_{B,i}=Z_i^{1/2}\psi_i(x)$$

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0,\mathbf{T}\psi_{H,1}(x_1)\cdots\psi_{H,n+l}(x_{n+l})\Psi_0)$$

$$=\int\frac{d^4q_1\cdots d^4q_{n+l}}{(i\pi)^{n+l}}e^{-iq_1\cdot x_1\cdots-iq_n\cdot x_n+iq_{n+1}\cdot x_{n+1}\cdots+iq_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}q_1^0(\Psi_0^-,\psi_1(0)\Psi^-_{\vec{q}_1,\sigma_1})\cdots q_n^0(\Psi_0^-,\psi_n(0)\Psi^-_{\vec{q}_n\sigma_n})$$

$$\times(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n},\Psi^+_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}})\, q^0_{n+1}(\Psi^+_{\vec{q}_{n+1},\sigma_{n+1}},\psi_{n+1}(0)\Psi_0^+)\cdots q^0_{n+l}(\Psi^+_{\vec{q}_{n+l},\sigma_{n+l}},\psi_{n+l}(0)\Psi_0^+)$$

$$(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n},\Psi^+_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}})_B=(\Psi^-_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n},\Psi^+_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}})$$

发散度

考虑一个有L个圈的费曼图，假设其分别有E_f, E_b条费米子和玻色子外线，I_f, I_b条费米子和玻色子内线，v_i个含d_i个微商，f_i条费米子线和b_i玻色子线的第i种类型顶角。考虑这个费曼图的发散度ω.则

$$\omega = 4L - 2I_b - I_f + \sum_i d_i v_i$$

$$L = I_f + I_b + 1 - \sum_i v_i \qquad E_f + 2I_f = \sum_i f_i v_i \qquad E_b + 2I_b = \sum_i b_i v_i$$

$$\omega = 4 - \frac{3}{2}E_f - E_b + \sum_i (\frac{3}{2}f_i + b_i + d_i - 4)v_i$$

设第i型顶角的耦合常数的量纲按质量量纲计为g_i

$$g_i + \frac{3}{2}f_i + b_i + d_i = 4 \quad \Rightarrow \quad \omega = 4 - \frac{3}{2}E_f - E_b - \sum_i g_i v_i$$

理论按可重整性分类

考虑一个有L个圈的费曼图，假设其分别有E_f, E_b条费米子和玻色子外线，v_i个含 d_i个微商，f_i条费米子线 和 b_i玻色子线的第i种类型顶角，第i型顶角的耦合常数的量纲按质量量纲计为g_i。此费曼图的发散度ω.

$$\omega = 4 - \frac{3}{2}E_f - E_b - \sum_i g_i v_i \qquad g_i = 4 - \frac{3}{2}f_i - b_i - d_i$$

理论和算符分类

- $g_i < 0$对应的顶角叫**irrelevant**算符。它的存在导致随着v_i的增加将出现无穷多发散类型的费曼图，因而理论是不可重整的。
- $g_i = 0$对应的顶角叫**marginal**算符；$g_i > 0$对应的顶角叫**relevant**算符。它们的存在导致理论有望可重整。满足此条件的相互作用顶角类型为：

Marginal		$\bar\psi\partial\!\!\!/\psi$	$\bar\psi\phi\psi, \bar\psi A\!\!\!/\psi$				$\phi\partial^2\phi, \partial^2A^2$		$\partial A\phi^2, \partial A^3$	ϕ^4, A^4, ϕ^2A^2
Relevant	$\bar\psi\psi$			ϕ	ϕ^2, A^2	$\phi\partial A$		$\phi^3, A^2\phi$		
f_i	2	2	2	0	0	0	0	0	0	0
b_i	0	0	1	1	2	2	2	3	3	4
d_i	0	1	0	0	0	1	2	0	1	0

其中考虑了协变性的要求。