1. 试证明进态和出态满足如下Lippmann-Schwinger方程:

$$\Psi_\alpha^\pm = \Phi_\alpha + \int d\beta \frac{T_{\beta\alpha}^\pm \Phi_\beta}{E_\alpha - E_\beta \pm i\epsilon}$$

   其中: $T_{\beta\alpha}^\pm = (\Phi_\beta, V\Psi_\alpha^\pm)$.

2. 试证明上题定义的$T_{\beta\alpha}^\pm$满足:

$$(T_{\alpha\beta}^\pm)^* - T_{\beta\alpha}^\pm = -\int d\gamma (T_{\gamma\beta}^\pm)^* T_{\gamma\alpha}^\pm [\frac{1}{E_\alpha - E_\gamma \pm i\epsilon} - \frac{1}{E_\beta - E_\gamma \mp i\epsilon}]$$

3. 试利用上题结论及作业1给出的Lippmann-Schwinger方程证明进出态之间的正交性$(\Psi_\beta^\pm, \Psi_\alpha^\pm) = \delta(\beta - \alpha)$.

4. 试证明S矩阵元与作业1给出的$T_{\beta\alpha}^\pm$的关系为:

$$S_{\beta\alpha} = \delta(\beta - \alpha) - 2\pi i \delta(E_\alpha - E_\beta) T_{\beta\alpha}^+$$

5. 试利用上题结果和作业2给出的关系证明S矩阵的幺正性

$$\int d\gamma \, S_{\gamma\beta}^* S_{\gamma\alpha} = \delta(\beta - \alpha)$$

6. 试从

$$\begin{aligned} U(\tau, \tau_0) &= 1 - i \int_{\tau_0}^{\tau} dt_1 V(t_1) + (-i)^2 \int_{\tau_0}^{\tau} dt_1 \int_{\tau_0}^{t_1} dt_2 V(t_1) V(t_2) \\ &\quad + (-i)^3 \int_{\tau_0}^{\tau} dt_1 \int_{\tau_0}^{t_1} dt_2 \int_{\tau_0}^{t_2} dt_3 V(t_1) V(t_2) V(t_3) + \cdots \end{aligned}$$

   推导出

$$U(\tau, \tau_0) = \mathbf{T} \, e^{-i \int_{\tau_0}^{\tau} dt \, V(t)}$$

7. 试导出S矩阵元的传统微扰论展开式：

$$\begin{aligned} S_{\beta\alpha} &= \delta(\beta - \alpha) - 2\pi i \delta(E_\beta - E_\alpha) \Bigg[V_{\beta\alpha} + \int d\gamma \, \frac{V_{\beta\gamma} V_{\gamma\alpha}}{E_\alpha - E_\gamma + i\epsilon} \\ &\quad + \int d\gamma d\gamma' \, \frac{V_{\beta\gamma} V_{\gamma\gamma'} V_{\gamma'\alpha}}{(E_\alpha - E_\gamma + i\epsilon)(E_\alpha - E_{\gamma'} + i\epsilon)} + \cdots \Bigg] \end{aligned}$$

8. 试利用

$$a(q) \Phi_{q_1 \cdots q_N} = \sum_{r=1}^{N} \delta_{r1} \delta(q - q_r) \Phi_{q_1 \cdots q_{r-1} q_{r+1} \cdots q_N} \qquad N \geq 1$$

$$a(q) \Phi_0 = 0$$

   证明:

$$a(q') a^\dagger(q) \mp a^\dagger(q) a(q') = \delta(q' - q)$$

$$a(q') a(q) \mp a(q) a(q') = 0 \qquad a^\dagger(q') a^\dagger(q) \mp a^\dagger(q) a^\dagger(q') = 0$$

9. 试利用

$$N \equiv \int d\vec{q}\, a^{\dagger}(q)a(q)$$

证明关系

$$[N, a^{\dagger}(q)] = a^{\dagger}(q) \qquad\qquad [N, a(q)] = a(q)$$

10. 试证明下式定义的$\Delta_{+}(x-y)$函数

$$\Delta_{+}(m, x-y) \quad \equiv \quad \int \frac{d\vec{p}}{(2\pi)^3 2p^0} e^{-ip\cdot x}$$

在类空区间可以表达为汉克尔函数

$$\Delta_{+}(M, x) = \frac{M}{4\pi^2\sqrt{-x^2}} K_1(M\sqrt{-x^2})$$

11. 试证明下式定义的$\Delta(M, x)$

$$\Delta(M, x) \equiv \int \frac{d\vec{p}}{2p^0(2\pi)^3}[e^{-ip\cdot x} - e^{ip\cdot x}]$$

满足

$$\dot{\Delta}(M, x) = -i\cos(p^0 x^0)\delta(\vec{x})$$

12. 试证明为了保证空间反射变换后的场$\phi_P = \eta^*\phi^+ + \eta^c\phi^{+c\dagger}$及其共轭$\phi_P^{\dagger}$ 作为基本元素来构造$\mathcal{H}(x)$能够使因果性条件

$$[\mathcal{H}(x), \mathcal{H}(x')] = 0 \quad x - x' \text{ 类空间隔}$$

成立,并可以实现要求$[Q^a, \mathcal{H}(x)] = 0$, 则必须要求$\eta^c = \eta^*$.

13. 试利用

$$\phi(x) \quad = \quad \int \frac{d\vec{p}}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{\dagger}(\vec{p})]$$

和作业8及作业11的结果，证明

$$[\phi(\vec{x}, t), \pi(\vec{y}, t)]_- = i\delta(\vec{x} - \vec{y})$$
$$[\phi(\vec{x}, t), \phi(\vec{y}, t)]_- = [\pi(\vec{x}, t), \pi(\vec{y}, t)]_- = 0$$

14. 对带电标量场，试导出其哈密顿量和作用量及正则对易关系。

15. 试证明量子情形下自由标量场的正则场方程

$$\dot{\phi}(\vec{x}, t) = i[H_0, \phi(\vec{x}, t)] = \frac{\delta H_0}{\delta\pi(\vec{x}, t)} \qquad \dot{\pi}(\vec{x}, t) = i[H_0, \pi(\vec{x}, t)] = -\frac{\delta H_0}{\delta\phi(\vec{x}, t)}$$

16. 试证明

$$-u^\dagger(\vec{p},\sigma)\beta u(\vec{p},\sigma') = v^\dagger(\vec{p},\sigma)\beta v(\vec{p},\sigma') = \delta_{\sigma\sigma'}$$

17. 试证明

$$H_0 = \mathrm{Re}\int d\vec{x}\ :[\psi^\dagger(x)\beta(i\vec{\gamma}\cdot\nabla + M)\psi(x)]:$$

18. 试证明

$$u^\dagger(\vec{p},\sigma)u(\vec{p},\sigma') = v^\dagger(\vec{p},\sigma)v(\vec{p},\sigma') = \frac{2p^0}{M}\delta_{\sigma\sigma'}$$

19. 试证明正则反对易关系

$$\{\psi_l(\vec{x},t),\pi_{\bar{l}}(\vec{y},t)\} = i\delta_{l\bar{l}}\delta(\vec{x}-\vec{y})$$
$$\{\psi_l(\vec{x},t),\psi_{\bar{l}}(\vec{y},t)\} = \{\pi_l(\vec{x},t),\pi_{\bar{l}}(\vec{y},t)\} = 0$$

20. 试证明量子情形下的自由旋量场正则场方程

$$\dot{\psi}(\vec{x},t) = i[H_0,\psi(\vec{x},t)] = \frac{\delta H_0}{\delta\pi(\vec{x},t)} \qquad \dot{\pi}(\vec{x},t) = i[H_0,\pi(\vec{x},t)] = -\frac{\delta H_0}{\delta\psi(\vec{x},t)}$$

21. 试推导$\overline{\psi}(x)\psi(x)$、$\overline{\psi}(x)\gamma_5\psi(x)$、$\overline{\psi}(x)\gamma^\mu\psi(x)$、$\overline{\psi}(x)\gamma^\mu\gamma_5\psi(x)$和$\overline{\psi}(x)[\gamma^\mu,\gamma^\nu]\psi(x)$在空间反射变换下的变换性质.

22. 试推导$\overline{\psi}(x)\psi(x)$、$\overline{\psi}(x)\gamma_5\psi(x)$、$\overline{\psi}(x)\gamma^\mu\psi(x)$、$\overline{\psi}(x)\gamma^\mu\gamma_5\psi(x)$和$\overline{\psi}(x)[\gamma^\mu,\gamma^\nu]\psi(x)$在时间反演变换下的变换性质.

23. 试推导$\overline{\psi}(x)\psi(x)$、$\overline{\psi}(x)\gamma_5\psi(x)$、$\overline{\psi}(x)\gamma^\mu\psi(x)$、$\overline{\psi}(x)\gamma^\mu\gamma_5\psi(x)$和$\overline{\psi}(x)[\gamma^\mu,\gamma^\nu]\psi(x)$在电荷共轭变换下的变换性质.

24. 试利用

$$\sum_{\bar{\sigma}} u^\mu(0,\bar{\sigma})\vec{J}^{(j)}_{\bar{\sigma}\sigma} = \vec{\mathcal{J}}^\mu{}_\nu u^\nu(0,\sigma)$$
$$\sum_{\bar{\sigma}} v^\mu(0,\bar{\sigma})\vec{J}^{(j)*}_{\bar{\sigma}\sigma} = -\vec{\mathcal{J}}^\mu{}_\nu v^\nu(0,\sigma)$$
$$(\mathcal{J}_k)^0{}_0 = (\mathcal{J}_k)^0{}_i = (\mathcal{J}_k)^i{}_0 = 0 \qquad (\mathcal{J}_k)^i{}_j = -i\epsilon_{ijk}$$
$$-(J^{(j)}_{23} \pm iJ^{(j)}_{31})_{\sigma'\sigma} = (J^{(j),1} \pm iJ^{(j),2})_{\sigma'\sigma} = \delta_{\sigma',\sigma\pm1}\sqrt{(j\mp\sigma)(j\pm\sigma+1)}$$
$$-(J^{(j)}_{12})_{\sigma'\sigma} = (J^{(j),3})_{\sigma'\sigma} = \sigma\delta_{\sigma'\sigma}$$

证明

$$u^\mu(0,0) = v^\mu(0,0) = (2M)^{-1/2}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$u^\mu(0,1) = -v^\mu(0,-1) = -\frac{(2M)^{-1/2}}{\sqrt{2}}\begin{bmatrix}0\\1\\i\\0\end{bmatrix}$$

$$u^\mu(0,-1) = -v^\mu(0,1) = \frac{(2M)^{-1/2}}{\sqrt{2}}\begin{bmatrix}0\\1\\-i\\0\end{bmatrix}$$

25. 试证明

$$\Pi^{\mu\nu}(\vec{p}) \equiv \sum_{\sigma} e^{\mu}(\vec{p},\sigma)e^{\nu *}(\vec{p},\sigma) = g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{M^2}$$

26. 试证明

$$[v^{\mu}(x), v^{\nu}(y)]_{-} = [v^{\mu}(x), v^{\nu\dagger}(y)]_{-} = [g^{\mu\nu} + \frac{\partial^{\mu}\partial^{\nu}}{M^2}]\Delta(M, x-y)$$

27. 对带电矢量场，试导出其哈密顿量和作用量及正则对易关系。

28. 试对有质量和无质量的矢量场分别证明

$$e^{\mu}(\vec{p},\sigma)e_{\mu}(\vec{p},\sigma') = \delta_{\sigma\sigma'}$$

29. 试利用

$$\begin{aligned}
&[v^{\mu}(x), v^{\nu}(y)]_{-} = [v^{\mu}(x), v^{\nu\dagger}(y)]_{-} = [g^{\mu\nu} + \frac{\partial^{\mu}\partial^{\nu}}{M^2}]\Delta(M, x-y) \\
&\Delta(M, x) \equiv \int \frac{d\vec{p}}{2p^0(2\pi)^3}[e^{-ip\cdot x} - e^{ip\cdot x}] \\
&\vec{\pi}(x) = -\dot{\vec{v}}(x) + \nabla v_0(x)
\end{aligned}$$

证明对易关系

$$\begin{aligned}
&[v^i(\vec{x},t), \pi^j(\vec{y},t)]_{-} = i\delta^{ij}\delta(\vec{x}-\vec{y}) \\
&[v^i(\vec{x},t), v^j(\vec{y},t)]_{-} = [\pi^i(\vec{x},t), \pi^j(\vec{y},t)]_{-} = 0
\end{aligned}$$

30. 试证明量子情形下自由矢量场的正则场方程

$$\dot{\vec{v}}(\vec{x},t) = i[H_0, \vec{v}(\vec{x},t)] = \frac{\delta H_0}{\delta\vec{\pi}(\vec{x},t)} \qquad \dot{\vec{\pi}}(\vec{x},t) = i[H_0, \vec{\pi}(\vec{x},t)] = -\frac{\delta H_0}{\delta\vec{v}(\vec{x},t)}$$

31. 试证明下式给出的旋量场的哈密顿量的定义就是时间平移变换的生成元

$$H_0' = \sum_{\sigma}\int d\vec{p}\ [a^{\dagger}(\vec{p},\sigma)a(\vec{p},\sigma) + a^{c\dagger}(\vec{p},\sigma)a^{c}(\vec{p},\sigma)]\sqrt{\vec{p}^2 + M^2}$$

32. 试证明下式给出的矢量场的哈密顿量的定义就是时间平移变换的生成元

$$H_0' = \sum_{\sigma}\int d\vec{p}\ a^{\dagger}(\vec{p},\sigma)a(\vec{p},\sigma)\sqrt{\vec{p}^2 + M^2}$$