量子场论　自由量子场，粒子与反粒子

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多粒子态

单粒子态:

$$U(\Lambda, a)\Psi_{p,\sigma,n} = e^{ia_\mu(\Lambda p)^\mu}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\sigma'}D_{\sigma'\sigma}(W(\Lambda, p))\Psi_{\Lambda p,\sigma',n}$$

▶ 自旋为j的有质量态: $D_{\sigma'\sigma}(W) = D^{(j)}_{\sigma'\sigma}(W)$

▶ 自旋为j,螺旋度为σ的无质量态: $D_{\sigma'\sigma}(W) = \delta_{\sigma'\sigma}e^{i\theta\sigma}$

无相互作用的多粒子态: 单粒子态的直乘

▶ 讨论不同种类的粒子,引入分立的指标n来代表粒子所属的种类

▶ 每个粒子用其能动量和参考动量系角动量的第三分量及粒子的种类来标记

一个一般的无相互作用的多粒子态可以写为 $\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$
它的能动量、自旋的第三分量(只对基本参考动量)和可能的$U(1)$本征值为:

$$P_0^\mu\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = (p_1^\mu + p_2^\mu + \cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$J_0^3\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots} = (\sigma_1 + \sigma_2 + \cdots)\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots}$$

$$(Q_0)_a\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = (q_{a1} + q_{a2} + \cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

多粒子态

单粒子态:

$$U(\Lambda, a)\Psi_{p,\sigma,n} = e^{ia_\mu(\Lambda p)^\mu}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\sigma'}D_{\sigma'\sigma}(W(\Lambda, p))\Psi_{\Lambda p,\sigma',n}$$

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无相互作用的多粒子态: 单粒子态的直乘 $\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$

$$P_0^\mu\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = (p_1^\mu + p_2^\mu + \cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$J_0^3\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots} = (\sigma_1 + \sigma_2 + \cdots)\Phi_{k_1,\sigma_1,n_1;k_2,\sigma_2,n_2;\cdots}$$

$$(Q_0)_a\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = (q_{a1} + q_{a2} + \cdots)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

在时空平移和转动洛伦兹变换$U_0(\Lambda, a)$下:

$$U_0(\Lambda, a)\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{ia_\mu((\Lambda p_1)^\mu+(\Lambda p_2)^\mu+\cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots}{p_1^0 p_2^0\cdots}}\sum_{\sigma_1'\sigma_2'\cdots}D_{\sigma_1'\sigma_1}(W(\Lambda, p_1))$$

$$\times D_{\sigma_2'\sigma_2}(W(\Lambda, p_2))\cdots\Phi_{\Lambda p_1,\sigma_1',n_1;\Lambda p_2,\sigma_2',n_2;\cdots}$$

在相互独立的内部$U_0(1)$对称性变换下:

$$U_0(T(\theta))\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{i(q_{a1}+q_{a2}+\cdots)\theta^a}\Phi_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

玻色子与费米子

排序问题：

> 如果所有的粒子都各属于不同的种类，它们之间可以区分，原则上可以规定一种标准的排序方式，例如第一种粒子排在第一位，第二种粒子排在第二位，…,等等.

> 但如果在一个多粒子态中有某两个粒子的种类相同,$\Phi_{\cdots \, ;p,\sigma,n;\cdots \, ;p',\sigma',n;\cdots}$,交换此两个粒子的态$\Phi_{\cdots \, ;p',\sigma',n;\cdots \, ;p,\sigma,n;\cdots}$与原来的态是无法区分的,需要对同类粒子的排序进行详细的讨论。

> 交换两个同类粒子无法区分，交换前后的态之间只能相差一个相角：
>
> $$\Phi_{\cdots \, ;p,\sigma,n;\cdots \, ;p',\sigma',n;\cdots} = \alpha_n(p,\sigma;p',\sigma') \; \Phi_{\cdots \, ;p',\sigma',n;\cdots \, ;p,\sigma,n;\cdots}$$
>
> 相角$\alpha_n(p,\sigma;p',\sigma')$与其它粒子的$p,\sigma,n$无关. 我们讨论的是自由粒子的多粒子态，如果交换其中的某两个粒子还要受到其它粒子的影响，意味着其它粒子对这两个参与交换的粒子有相互作用，就不是自由粒子态了。

玻色子与费米子

$$\Phi_{\cdots ;p,\sigma,n;\cdots ;p',\sigma',n;\cdots} = \alpha_n(p,\sigma;p',\sigma')\,\Phi_{\cdots ;p',\sigma',n;\cdots ;p,\sigma,n;\cdots}$$

两边实施洛伦兹变换

$$\sum_{\bar{\sigma}\bar{\sigma}'\cdots} D_{\bar{\sigma}\sigma}(W(\Lambda,p))D_{\bar{\sigma}'\sigma'}(W(\Lambda,p'))\Phi_{\cdots ,\Lambda p,\bar{\sigma},n;\cdots ;\Lambda p',\bar{\sigma}',n;\cdots}$$

$$= \alpha_n(p,\sigma;p',\sigma')\sum_{\bar{\sigma}'\bar{\sigma}} D_{\bar{\sigma}'\sigma'}(W(\Lambda,p'))D_{\bar{\sigma}\sigma}(W(\Lambda,p))\Phi_{\cdots ,\Lambda p',\bar{\sigma}',n;\cdots ;\Lambda p,\bar{\sigma},n;\cdots}$$

$$\Phi_{\cdots ,\Lambda p,\bar{\sigma},n;\cdots ;\Lambda p',\bar{\sigma}',n;\cdots} = \alpha_n(p,\sigma;p',\sigma')\,\Phi_{\cdots ,\Lambda p',\bar{\sigma}',n;\cdots ;\Lambda p,\bar{\sigma},n;\cdots}$$

$$\Rightarrow \quad \alpha_n(p,\sigma;p',\sigma') = \alpha_n(\Lambda p,\bar{\sigma};\Lambda p',\bar{\sigma}')$$

$\alpha_n(p,\sigma;p',\sigma')$与$\sigma,\sigma'$无关，可略去$\alpha_n$中的$\sigma$指标，并且在时空转动下是不变的，

$$\alpha_n(p,\sigma;p',\sigma') = \alpha_n(p,p') \qquad\qquad \alpha_n(p,p') = \alpha_n(\Lambda p,\Lambda p')$$

在四维时空由p和p'构造的时空转动不变量只可能为p^2,p'^2和$p^\mu p'_\mu$，考虑到$p^2 = M^2 = p'^2$并且$p^\mu p'_\mu$对交换p和p'是对称的

在1+1维时空中，$p - p'$的符号是洛伦兹变换不变但对交换p和p'不是对称而是反对称的。导致除玻色子和费米子外的任意子anyon.

玻色子与费米子

$$\Phi_{\cdots\ ;p,\sigma,n;\cdots\ ;p',\sigma',n;\cdots} = \alpha_n(p,p')\ \Phi_{\cdots\ ;p',\sigma',n;\cdots\ ;p,\sigma,n;\cdots}$$

$\alpha_n(p,p') = \alpha_n(\Lambda p, \Lambda p')$ 在四维时空 $\alpha_n(p,p')$ 只能依赖 p^2, p'^2 和 $p^\mu p'_\mu$

结合 $p^2 = M^2 = p'^2$ 及 $p^\mu p'_\mu$ 对交换 p 和 p' 是对称的 $\alpha_n(p,p') = \alpha_n(p',p)$

$$\Phi_{\cdots\ ;p,\sigma,n;\cdots\ ;p',\sigma',n;\cdots} = \alpha_n(p,p')\Phi_{\cdots\ ;p',\sigma',n;\cdots\ ;p,\sigma,n;\cdots}$$

$$\Phi_{\cdots\ ;p',\sigma',n;\cdots\ ;p,\sigma,n;\cdots} = \alpha_n(p',p)\Phi_{\cdots\ ;p,\sigma,n;\cdots\ ;p',\sigma',n;\cdots}$$

$$\Rightarrow \alpha_n(p,p')\alpha_n(p',p) = 1 \qquad \Rightarrow \quad \alpha_n(p,p') = \pm 1$$

3+1维时空中同类粒子交换只可能出现两种情况：变号或不变号

▶ $\alpha_n(p,p') = +1$的粒子叫玻色子。 对玻色子同类粒子交换不变号

▶ $\alpha_n(p,p') = -1$的粒子叫费米子。 对费米子同类粒子交换一次出一负号

进一步对不同类粒子之间的排序在标准的排序方式基础上作如下的安排：

▶ <u>最近邻的</u>费米子与费米子之间交换一次出一负号

▶ <u>最近邻的</u>玻色子与玻色子，玻色子与费米子之间交换不变号

玻色子与费米子

多粒子态的归一化: 记 $\Phi_{p_1,\sigma_1,n;p_2,\sigma_2,n_2;\cdots} = \Phi_{p_1,p_2,\cdots}$

真空态 Φ_0 和单粒子态 Φ_q

$$(\Phi_0, \Phi_0) = 1 \qquad (\Phi_{q'}, \Phi_q) = \delta(q' - q) \equiv \delta^3(\vec{q}' - \vec{q})\delta_{\sigma'\sigma}\delta_{n'n} \qquad \text{非洛伦兹不变!}$$

对两粒子态 $\Phi_{q_1q_2}$ $\qquad (\Phi_{q_1'q_2'}, \Phi_{q_1q_2}) = \delta(q_1' - q_1)\delta(q_2' - q_2) \pm \delta(q_2' - q_1)\delta(q_1' - q_2)$

负号对两个粒子都是费米子的情形,正号对其它情形(两个粒子都是玻色子或一个是玻色子一个是费米子).

一般情况: $\qquad (\Phi_{q_1'q_2'\cdots q_M'}, \Phi_{q_1q_2\cdots q_N}) = \delta_{MN}\sum_{\mathcal{P}}\delta_{\mathcal{P}}\prod_i\delta(q_i - q'_{\mathcal{P}i})$

求和对所有可能的对指标 $1, 2, \cdots, N$ 的交换排序 $\mathcal{P}$ 实行.对交换排序中涉及奇数次费米子交换时,$\delta_{\mathcal{P}} = -1$,其它情况的交换排序 $\delta_{\mathcal{P}} = 1$.

约定: $\qquad (\Phi_{\alpha'}, \Phi_\alpha) = \delta(\alpha' - \alpha) \qquad \int d\alpha \cdots \equiv \sum_{n_1\sigma_1n_2\sigma_2\cdots}\int d\vec{p}_1\int d\vec{p}_2\cdots$

$$\Phi = \int d\alpha\, \Phi_\alpha(\Phi_\alpha, \Phi) \qquad 1 = \int d\alpha\, \Phi_\alpha)(\Phi_\alpha \qquad \text{多粒子态构成完备集!} \quad 证明?$$

进态和出态

散射问题:

一组在宏观上相距很远的相互之间没有相互作用的粒子逐渐相互接近到微观上很小的区域发生相互作用，再逐渐互相分离到宏观上相距很远相互之间不再有相互作用的区域。

相互作用发生在粒子逐渐相互接近到微观上很小的区域。记一个有相互作用体系的总体时间平移生成元算符为H，被称为体系的哈密顿量，是体系总四动量的零分量。把这个体系的相互作用撤除得到的无相互作用的自由粒子体系的哈密顿量记为H_0。将两个哈密顿量的差定义为体系的相互作用V：

$$H = H_0 + V$$

将体系中粒子相距很远，相互之间没有相互作用的初始和末了状态分别称为进态和出态，记为：Ψ_α^+和Ψ_α^-

► 进态和出态分别构成完备集

► 相互之间无相互作用⇒进态和出态分别可被看成一组自由多粒子态

► 进态和出态满足与自由多粒子态同样的洛仑兹和内部对称性变换关系

► 过程的持续：无相互作用的无穷将来⇐发生相互作用的现在⇐无相互作用的无穷过去

进态和出态

进态和出态满足的洛仑兹变换： 满足与自由多粒子态同样的<u>有相互作用系统的</u>洛仑兹变换关系

$$U(\Lambda, a)\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{ia_\mu((\Lambda p_1)^\mu + (\Lambda p_2)^\mu + \cdots)} \sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots}{p_1^0 p_2^0 \cdots}} \sum_{\sigma_1'\sigma_2'\cdots} D_{\sigma_1'\sigma_1}(W(\Lambda,p_1))$$

$$\times D_{\sigma_2'\sigma_2}(W(\Lambda,p_2))\cdots\Psi^{\pm}_{\Lambda p_1,\sigma_1',n_1;\Lambda p_2,\sigma_2',n_2;\cdots} \equiv \int d\alpha'\,(U_0)_{\alpha'\alpha}\Psi^{\pm}_{\alpha'}$$

$$U(T(\theta))\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{i(q_{a1}+q_{a2}+\cdots)\theta^a}\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$\Lambda = 1,\ a = (\epsilon, \vec{0})$$

$$e^{i\epsilon H}\Psi^{\pm}_\alpha = e^{i\epsilon E_\alpha}\Psi^{\pm}_\alpha \qquad H\Psi^{\pm}_\alpha = E_\alpha\Psi^{\pm}_\alpha \qquad E_\alpha = p_1^0 + p_2^0 + \cdots$$

将H_0选择的使其具有与H完全一样的本征值谱，即

$$H_0\Phi_\alpha = E_\alpha\Phi_\alpha \qquad (\Phi_{\alpha'}, \Phi_\alpha) = \delta(\alpha' - \alpha)$$

Φ_α**是H_0的本征态。**

进态和出态

进态和出态满足的洛仑兹变换: 满足与自由多粒子态同样的<u>有相互作用系统的</u>洛仑兹变换关系

$$U(\Lambda, a)\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{ia_\mu((\Lambda p_1)^\mu + (\Lambda p_2)^\mu + \cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots}{p_1^0 p_2^0 \cdots}}\sum_{\sigma_1'\sigma_2'\cdots} D_{\sigma_1'\sigma_1}(W(\Lambda, p_1))$$

$$\times D_{\sigma_2'\sigma_2}(W(\Lambda, p_2))\cdots\Psi^{\pm}_{\Lambda p_1,\sigma_1',n_1;\Lambda p_2,\sigma_2',n_2;\cdots} \equiv \int d\alpha' \, (U_0)_{\alpha'\alpha}\Psi^{\pm}_{\alpha'}$$

$$U(T(\theta))\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots} = e^{i(q_{a1}+q_{a2}+\cdots)\theta^a}\Psi^{\pm}_{p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$H\Psi^{\pm}_\alpha = E_\alpha\Psi^{\pm}_\alpha \qquad H_0\Phi_\alpha = E_\alpha\Phi_\alpha \qquad E_\alpha = p_1^0 + p_2^0 + \cdots$$

过程的持续 ⇒ 量子态随时间的演化 $\overset{\text{理论中没定义}}{=\!=\!=\!=\!=\!\Rightarrow}$ **需定义态随时间的演化!**

进态和出态

时间的演化

关于时间的附注：

绝对的、真实的和数学的时间，由其特性决定，自身均匀地流逝，与一切外在事务无关，又名延续；

相对的、表象的和普通的时间是可感知和外在的（不论是精确的或是不均匀的）对运动之延续的量度，它常被用以代替真实的时间，如一小时、一天、一个月、一年。

《自然哲学之数学原理–宇宙体系》　　伊萨克.牛顿　1686年5月8日

时间的演化是：　均匀流逝

进态和出态

量子态随时间的演化

将态的演化时间翻译为 **两观测者** 的观测时间差

▶ 观测者O的时钟标记t，观测到的为理论原始假设中的态Ψ

▶ 观测者O_τ的时钟标记$t' = t - \tau$，感受 **态的演化**，观测到的态为$\Psi(\tau)$

$$\Psi(\tau) = U(1, -\tau)\Psi = e^{-iH\tau}\Psi \qquad i\frac{\partial}{\partial\tau}\Psi(\tau) = H\,\Psi(\tau) \quad \text{薛定谔方程！}$$

为什么用$t' = t - \tau$，而不采用$t' = t + \tau$?

▶ $t' = t - \tau$相应于观测者O_τ为现在 $t' = 0$ 时，对应观测者O为$t = \tau$

观测者O_τ自我时间为"现在"时，观测者O的时间纪录为$t = \tau$，观测者O_τ观测到的态

▶ $t' = t + \tau$相应于观测者O_τ为现在$t' = \tau$时，对应观测者O为$t = 0$

观测者O_τ的时间定义为演化的时间

时间的均匀流逝体现为一系列观测者之间的 均匀的时间观测 间隔！

进态和出态

E.Wigner, Ann.Math.40,149(1939) On Unitary Representations of the Inhomogeneous Lorentz Group

If we knew, e.g., the operator K corresponding to the measurement

of a physical quantity at <u>the time $t = 0$</u>, we could follow up the change

of this quantity throughout time. In order to obtain its value for <u>the time</u>

<u>$t = t_1$</u>, we could transform the original wave function ϕ_l by $D(l', l)$ to

a coordinate system l' the time scale of which $\underbrace{\text{begins a time } t_1 \text{ later.}}_{t' = t - t_1, \quad t \geq t_1}$

The measurement of the quantity in question in this coordinate system

for $\underbrace{\text{the time } 0}_{t' = 0}$ is given–as in original one–by the operator K. This

measurement is identical, however, with the measurement of

the quantity at <u>time $t_1 = t$</u> in the original system. $\cdots\cdots$
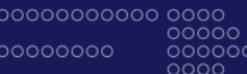
进态和出态

进态和出态与 H_0 的本征态的关系：

进态：

要求在无穷过去 $O_{-\infty}$ 观测的进态与自由粒子态完全相同：

$$\Psi^+(-\infty) = \Phi(-\infty) \qquad U(1,-\tau)|_{\tau=-\infty}\Psi^+ = U_0(1,-\tau)|_{\tau=-\infty}\Phi$$

公式应在波包（不同本征态的叠加）意义下理解，否则带入本征值将导致 $\Psi^+_\alpha = \Phi_\alpha$

$$\Rightarrow e^{-iH\tau}\Psi^+_\alpha|_{\tau\to-\infty} = e^{-iH_0\tau}\Phi_\alpha|_{\tau\to-\infty} \quad \text{或} \quad \Psi^+_\alpha = \Omega(-\infty)\Phi_\alpha$$

$$\Omega(\tau) \equiv U^\dagger(1,-\tau)U_0(1,-\tau) = U(1,\tau)U_0^\dagger(1,\tau) = e^{iH\tau}e^{-iH_0\tau}$$

出态：

要求在无穷将来 $O_{+\infty}$ 观测的出态与自由粒子态完全相同：

$$\Psi^-(+\infty) = \Phi(+\infty) \qquad U(1,-\tau)|_{\tau=+\infty}\Psi^- = U_0(1,-\tau)|_{\tau=+\infty}\Phi$$

公式应在波包（不同本征态的叠加）意义下理解，否则带入本征值将导致 $\Psi^-_\alpha = \Phi_\alpha$

$$\Rightarrow e^{-iH\tau}\Psi^-_\alpha|_{\tau\to+\infty} = e^{-iH_0\tau}\Phi_\alpha|_{\tau\to+\infty} \quad \text{或} \quad \Psi^-_\alpha = \Omega(+\infty)\Phi_\alpha$$

作业1,2,3

进态和出态

关于进态和出态:

♣　**困惑?** 时间演化≡时间平移!

♢　**自由粒子态与相互作用的混合体?**

♡　**相互作用的引入?** 含糊不清

♡　**相互作用的撤除?** 绝热近似

♠　**与自由粒子同样的质量谱?**

¶　$e^{-iH\tau}\Psi^{\pm}_{\alpha}|_{\tau\to\mp\infty} = e^{-iH_0\tau}\Phi_{\alpha}|_{\tau\to\mp\infty}$ **?**

S矩阵

进态和出态的内积定义为S矩阵： $\quad S_{\beta\alpha} = (\Psi_\beta^-, \Psi_\alpha^+)$ $\qquad \alpha \to \beta$ **几率幅**

S矩阵的性质： 作业4,5 $\quad S_{\beta\alpha} = \delta(\beta - \alpha) - 2\pi i \delta(E_\alpha - E_\beta) T_{\beta\alpha}^+$

利用进出态的完备性

$$\Psi_\alpha^+ = \int d\beta \, \Psi_\beta^- (\Psi_\beta^-, \Psi_\alpha^+) = \int d\beta \, S_{\beta\alpha} \Psi_\beta^-$$

$$\Psi_\alpha^- = \int d\beta \, \Psi_\beta^+ (\Psi_\beta^+, \Psi_\alpha^-) = \int d\beta \, S_{\alpha\beta}^* \Psi_\beta^+$$

幺正性质

$$\int d\beta \, S_{\beta\gamma}^* S_{\beta\alpha} = \int d\beta \, (\Psi_\gamma^+, \Psi_\beta^-)(\Psi_\beta^-, \Psi_\alpha^+) = (\Psi_\gamma^+, \Psi_\alpha^+) = \delta(\gamma - \alpha)$$

将S矩阵建立在自由粒子基上，引入S矩阵算符： $\Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha$

$$S_{\beta\alpha} \equiv (\Phi_\beta, S\Phi_\alpha) \; \to \; S = \Omega^\dagger(\infty)\Omega(-\infty) = U(+\infty, -\infty)$$

$$U(\tau, \tau_0) = \Omega^\dagger(\tau)\Omega(\tau_0) = e^{iH_0\tau} e^{-iH(\tau-\tau_0)} e^{-iH_0\tau_0} = U_0(1,\tau)U^\dagger(1,\tau-\tau_0)U_0^\dagger(1,\tau_0)$$

$$\Omega(\tau) = e^{iH\tau} e^{-iH_0\tau} = U(1,\tau)U_0^\dagger(1,\tau)$$

S矩阵

定义为进态和出态内积的S矩阵 $\quad S_{\beta\alpha} = (\Psi_\beta^-, \Psi_\alpha^+)$

利用洛仑兹变换和内部对称性变换算符的幺正性质
⇒ S矩阵在洛仑兹变换和内部对称性变换下是不变的!

S矩阵的洛仑兹变换不变性: $\quad S_{\beta\alpha} = (\Psi_\beta^-, \Psi_\alpha^+) = (U(\Lambda, a)\Psi_\beta^-, U(\Lambda, a)\Psi_\alpha^+)$

$$S_{p_1', \sigma_1', n_1'; p_2', \sigma_2', n_2'; \cdots p_1, \sigma_1, n_1; p_2, \sigma_2, n_2; \cdots} = \int d\bar\beta d\bar\alpha \, (U_0^*)_{\bar\beta\beta} S_{\bar\beta\bar\alpha}(U_0)_{\bar\alpha\alpha} \quad \text{见后}$$

$$= e^{ia_\mu((\Lambda p_1)^\mu + (\Lambda p_2)^\mu + \cdots - (\Lambda p_1)^{\mu'} - (\Lambda p_2)^{\mu'} - \cdots)} \sqrt{\frac{(\Lambda p_1)^0 (\Lambda p_2)^0 \cdots (\Lambda p_1')^0 (\Lambda p_2')^0 \cdots}{p_1^0 p_2^0 \cdots p_1^{0'} p_2^{0'} \cdots}}$$

$$\times \sum_{\bar\sigma_1, \bar\sigma_2, \cdots} D_{\bar\sigma_1 \sigma_1}^{(j_1)}(W(\Lambda, p_1)) D_{\bar\sigma_2 \sigma_2}^{(j_2)}(W(\Lambda, p_2)) \cdots \sum_{\bar\sigma_1', \bar\sigma_2', \cdots} D_{\bar\sigma_1' \sigma_1'}^{(j_1')*}(W(\Lambda, p_1')) D_{\bar\sigma_2' \sigma_2'}^{(j_2')*}(W(\Lambda, p_2')) \cdots$$

$$\times S_{\Lambda p_1', \bar\sigma_1', n_1'; \Lambda p_2', \bar\sigma_2', n_2'; \cdots \Lambda p_1, \bar\sigma_1, n_1; \Lambda p_2, \bar\sigma_2, n_2; \cdots}$$

左边与平移参量a无关 ⇒ 要求等式右边a无关 ⇒ $p_1^\mu + p_2^\mu + \cdots - p_1^{\mu'} - p_2^{\mu'} - \cdots = 0$

记$p_\alpha = p_\beta$，四动量是连续变量，动量守恒意味S矩阵元中含因子$\delta(\vec{p}_\beta - \vec{p}_\alpha)$

$$S_{\beta\alpha} - \delta(\beta - \alpha) \overset{\text{作业4}}{=\!=\!=} -2\pi i \delta(E_\alpha - E_\beta) T_{\beta\alpha}^+ = -2\pi i M_{\beta\alpha} \delta^4(p_\beta - p_\alpha)$$

S矩阵

定义为进态和出态内积的S矩阵　$S_{\beta\alpha} = (\Psi_\beta^-, \Psi_\alpha^+)$

　　利用洛仑兹变换和内部对称性变换算符的幺正性质
　　$\Rightarrow$ S矩阵在洛仑兹变换和内部对称性变换下是不变的!

S矩阵的洛仑兹变换不变性: $S_{\beta\alpha} = (\Psi_\beta^-, \Psi_\alpha^+) = (U(\Lambda, a)\Psi_\beta^-, U(\Lambda, a)\Psi_\alpha^+)$

$$p_\alpha = p_\beta \qquad S_{\beta\alpha} - \delta(\beta - \alpha) = -2\pi i M_{\beta\alpha}\delta^4(p_\beta - p_\alpha)$$

S矩阵的内部对称性变换不变性: $S_{\beta\alpha} = (\Psi_\beta^-, \Psi_\alpha^+) = (U(T(\theta))\Psi_\beta^-, U(T(\theta))\Psi_\alpha^+)$

$S_{p_1', \sigma_1', n_1'; p_2', \sigma_2', n_2'; \cdots p_1, \sigma_1, n_1; p_2, \sigma_2, n_2; \cdots}$
$= e^{i(q_{a1} + q_{a2} + \cdots - q_{a1}' - q_{a2}' - \cdots)\theta^a} S_{p_1', \sigma_1', n_1'; p_2', \sigma_2', n_2'; \cdots p_1, \sigma_1, n_1; p_2, \sigma_2, n_2; \cdots}$

左边是与内部转动参量θ^a无关 $\Rightarrow$ 要求等式右边θ^a无关 $q_{a1} + q_{a2} + \cdots - q_{a1}' - q_{a2}' - \cdots = 0$

记　$q_\alpha = q_\beta$

S矩阵

定义为进态和出态内积的**S矩阵** $\quad S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha)$

利用洛仑兹变换和内部对称性变换算符的幺正性质
⇒ **S**矩阵在洛仑兹变换和内部对称性变换下是不变的!

S矩阵的洛仑兹变换不变性: $S_{\beta\alpha} = (\Psi^-_\beta, \Psi^+_\alpha) = (U(\Lambda,a)\Psi^-_\beta, U(\Lambda,a)\Psi^+_\alpha)$

$p_\alpha = p_\beta \qquad S_{\beta\alpha} - \delta(\beta - \alpha) = -2\pi i M_{\beta\alpha}\delta^4(p_\beta - p_\alpha)$

用自由粒子的洛仑兹变换和内部对称性变换: $U_0(\Lambda,a) \quad U_0(T(\theta))$

$$U_0(\Lambda,a)\Phi_\alpha = \int d\bar{\alpha}(U_0)_{\bar{\alpha}\alpha}\Phi_{\bar{\alpha}} \qquad S_{\beta\alpha} \equiv (\Phi_\beta, S\Phi_\alpha) \Rightarrow S = \int d\beta d\alpha \ \Phi_\beta S_{\beta\alpha}\Phi_\alpha$$

$$S_{\beta\alpha} = (U\Psi^-_\beta, U\Psi^+_\alpha) = \int d\bar{\beta}d\bar{\alpha} \left((U_0)_{\bar{\beta}\beta}\Psi^-_{\bar{\beta}}, (U_0)_{\bar{\alpha}\alpha}\Psi^+_{\bar{\alpha}}\right) = \int d\bar{\beta}d\bar{\alpha}(U_0^*)_{\bar{\beta}\beta}S_{\bar{\beta}\bar{\alpha}}(U_0)_{\bar{\alpha}\alpha}$$

$$\Rightarrow S = \int d\beta d\alpha \ \Phi_\beta S_{\beta\alpha}\Phi_\alpha = \int d\bar{\beta}d\bar{\alpha}d\beta d\alpha \ \Phi_\beta (U_0^*)_{\bar{\beta}\beta}S_{\bar{\beta}\bar{\alpha}}(U_0)_{\bar{\alpha}\alpha}\Phi_\alpha$$

$$= \int d\bar{\beta}d\bar{\alpha}d\beta d\alpha \Phi_\beta (U_0^*)_{\bar{\beta}\beta}(\Phi_{\bar{\beta}}, S\Phi_{\bar{\alpha}})(U_0)_{\bar{\alpha}\alpha}\Phi_\alpha = \int d\beta d\alpha \Phi_\beta (U_0\Phi_\beta, SU_0\Phi_\alpha)\Phi_\alpha = U_0^{-1}(\Lambda,a)SU_0(\Lambda,a)$$

U_0可以是任意一个幺正算符 **S矩阵算符是洛仑兹变换和内部对称性变换下不变的!**

S矩阵的微扰展开

$$S = U(+\infty, -\infty) \qquad U(\tau, \tau_0) = \Omega^\dagger(\tau)\Omega(\tau_0) = e^{iH_0\tau} e^{-iH(\tau-\tau_0)} e^{-iH_0\tau_0}$$

$$i\frac{d}{d\tau}U(\tau, \tau_0) = e^{iH_0\tau}(-H_0 + H)e^{-iH(\tau-\tau_0)}e^{-iH_0\tau_0} = V(\tau)U(\tau, \tau_0) \qquad V(\tau) = e^{iH_0\tau}Ve^{-iH_0\tau}$$

$$U(\tau, \tau_0) = 1 - i\int_{\tau_0}^{\tau} dt_1 V(t_1) + (-i)^2 \int_{\tau_0}^{\tau} dt_1 \int_{\tau_0}^{t_1} dt_2 V(t_1)V(t_2)$$

$$+(-i)^3 \int_{\tau_0}^{\tau} dt_1 \int_{\tau_0}^{t_1} dt_2 \int_{\tau_0}^{t_2} dt_3 V(t_1)V(t_2)V(t_3) + \cdots$$

$$= \mathbf{T}\, e^{-i\int_{\tau_0}^{\tau} dt\, V(t)}$$

$\mathbf{T}$ 是编时乘积，它将时间早的算符排在右边   作业6,7

$$V(t) = \int d\vec{x}\ \tilde{\mathcal{H}}(\vec{r}, t) \qquad \tilde{\mathcal{H}}(x) = \tilde{\mathcal{H}}(\vec{r}, t)$$ 是局域的相互作用哈密顿量密度   生成元开始直接和时空坐标发生关系！

由于V是H中的相互作用部分，不是洛仑兹变换的协变量，如此引入相互作用哈密顿量密度可以确保它是洛仑兹变换的标量.

S矩阵的洛仑兹和内部对称性不变性要求$\tilde{\mathcal{H}}(x)$是洛仑兹和内部对称性不变量

猜测：$U_0(\Lambda, a)\tilde{\mathcal{H}}(x)U_0^{-1}(\Lambda, a) = \tilde{\mathcal{H}}(\Lambda x + a)$ $\qquad U_0(T(\theta))\tilde{\mathcal{H}}(x)U_0^{-1}(T(\theta)) = \tilde{\mathcal{H}}(x)$

$$[\tilde{\mathcal{H}}(x), \tilde{\mathcal{H}}(x')] = 0 \qquad x - x'\ \text{类空间隔} \qquad \textbf{因果性！}$$

反应率与碰撞截面

反应率　　**将系统限制在一个有限的空间中：**

$$\vec{p} = \frac{2\pi}{L}(n_1, n_2, n_3) \quad \delta_V^3(\vec{p}' - \vec{p}) = \frac{1}{(2\pi)^3}\int_V d^3x\, e^{i(\vec{p}-\vec{p}')\cdot\vec{x}} = \frac{V}{(2\pi)^3}\delta_{\vec{p}',\vec{p}}$$

$$\Psi_\alpha^{\text{Box}} \equiv [\frac{(2\pi)^3}{V}]^{N_\alpha/2}\Psi_\alpha \to (\Psi_\beta^{\text{Box}}, \Psi_\alpha^{\text{Box}}) = \delta_{\beta\alpha} \quad \textbf{Kronecker} \to S_{\beta\alpha} = [\frac{V}{(2\pi)^3}]^{\frac{N_\beta+N_\alpha}{2}}S_{\beta\alpha}^{\text{Box}}$$

$$\delta_T(E_\alpha - E_\beta) = \frac{1}{2\pi}\int_{-T/2}^{T/2}dt\, e^{i(E_\alpha - E_\beta)t} \qquad\qquad d\beta = d\vec{p}_1 d\vec{p}_2\cdots$$

$$P(\alpha \to \beta) \equiv |S_{\beta\alpha}^{\text{Box}}|^2 = [\frac{(2\pi)^3}{V}]^{N_\alpha + N_\beta}|S_{\beta\alpha}|^2 \qquad\qquad d\mathcal{N}_\beta = [\frac{V}{(2\pi)^3}]^{N_\beta}d\beta$$

$$dP(\alpha \to \beta) = P(\alpha \to \beta)d\mathcal{N}_\beta = [\frac{(2\pi)^3}{V}]^{N_\alpha}|S_{\beta\alpha}|^2 d\beta \qquad \textbf{对有偏转的部分：}$$

$$S_{\beta\alpha} \equiv -2i\pi\delta_V^3(\vec{p}_\beta - \vec{p}_\alpha)\delta_T(E_\beta - E_\alpha)M_{\beta\alpha} \xrightarrow{\;VT\,\textbf{large}\;} -2i\pi\delta^4(p_\beta - p_\alpha)M_{\beta\alpha}$$

$$dP(\alpha \to \beta) = (2\pi)^2[\frac{(2\pi)^3}{V}]^{N_\alpha - 1}\frac{T}{2\pi}|M_{\beta\alpha}|^2\delta_V^3(\vec{p}_\beta - \vec{p}_\alpha)\delta_T(E_\beta - E_\alpha)d\beta$$

$$d\Gamma(\alpha \to \beta) \equiv \frac{dP(\alpha \to \beta)}{T} = (2\pi)^{3N_\alpha - 2}V^{1-N_\alpha}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

目录 多粒子态 进态、出态与S矩阵 量子场 标det量子场 旋det量子场 矢量量子场 任意自旋量子场 绘景与CPT
○○ ○○○○○○○○ ○○○○○ ○○○○ ○○○○○○○○○○○○○ ○○○○○○○○○○○○○○○ ○○○○○
○○○○ ○○○○○ ○○○○○○ ○○○○ ○○○○○○○○○○○○○○○○○○○ ○○○○○○○○○○○○○○ ○
●○●○○○○○ ○ ○ ○○○
○○○○ ○○○ ○○○○○○ ○○○ ○○○○○○○○○○ ○○○○
○○○○ ○○○○○○○○ ○○○○○○○○○ ○○○○

反应率与碰撞截面

碰撞截面

$$S_{\beta\alpha} - - - \overset{\text{connect part}}{- - - - - -} \to -2i\pi\delta^4(p_\beta - p_\alpha)M_{\beta\alpha}$$

$$d\Gamma(\alpha \to \beta) \equiv \frac{dP(\alpha \to \beta)}{T} = (2\pi)^{3N_\alpha - 2}V^{1-N_\alpha}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

$$N_\alpha = 1: \quad d\Gamma(\alpha \to \beta) = 2\pi|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

$$N_\alpha = 2: \quad \Phi_\alpha = \frac{u_\alpha}{V}$$

$$d\sigma(\alpha \to \beta) \equiv \frac{d\Gamma(\alpha \to \beta)}{\Phi_\alpha} = (2\pi)^4 u_\alpha^{-1}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$$

$$dP(1 + 2 \to \beta) = \frac{d\sigma(1 + 2 \to \beta)u_\alpha T}{V}$$
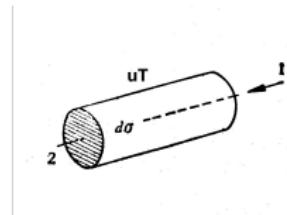
洛伦兹变换性质：

$$S_{p'_1,\sigma'_1,n'_1;p'_2,\sigma'_2,n'_2;\cdots p_1,\sigma_1,n_1;p_2,\sigma_2,n_2;\cdots}$$

$$= e^{ia_\mu((\Lambda p_1)^\mu+(\Lambda p_2)^\mu+\cdots-(\Lambda p_1)^{\mu'}-(\Lambda p_2)^{\mu'}-\cdots)}\sqrt{\frac{(\Lambda p_1)^0(\Lambda p_2)^0\cdots(\Lambda p'_1)^0(\Lambda p'_2)^0\cdots}{p_1^0 p_2^0\cdots p_1^{0'}p_2^{0'}\cdots}}$$

$$\times\sum_{\bar{\sigma}_1,\bar{\sigma}_2,\cdots}D^{(j_1)}_{\bar{\sigma}_1\sigma_1}(W(\Lambda,p_1))D^{(j_2)}_{\bar{\sigma}_2\sigma_2}(W(\Lambda,p_2))\cdots\sum_{\bar{\sigma}'_1,\bar{\sigma}'_2,\cdots}D^{(j'_1)*}_{\bar{\sigma}'_1\sigma'_1}(W(\Lambda,p'_1))D^{(j'_2)*}_{\bar{\sigma}'_2\sigma'_2}(W(\Lambda,p'_2))\cdots$$

$$\times S_{\Lambda p'_1,\bar{\sigma}'_1,n'_1;\Lambda p'_2,\bar{\sigma}'_2,n'_2;\cdots\Lambda p_1,\bar{\sigma}_1,n_1;\Lambda p_2,\bar{\sigma}_2,n_2;\cdots}\qquad \color{red}{S_{\beta\alpha}-\delta(\beta-\alpha)=-2\pi i M_{\beta\alpha}\delta^4(p_\beta-p_\alpha)}$$

$$R_{\beta\alpha}\equiv\sum_{\textbf{spins}}|M_{\beta\alpha}|^2\prod_\beta E\prod_\alpha E\qquad \color{red}{\text{需对初态}\sigma\text{末态}\sigma'\text{求和以分别消去}D\text{和}D^*\text{矩阵}}\qquad \textbf{is invariant}$$

$$N_\alpha=1:\quad\sum_{\textbf{spins}}d\Gamma(\alpha\to\beta)=2\pi E_\alpha^{-1}R_{\beta\alpha}\delta^4(p_\beta-p_\alpha)\frac{d\beta}{\prod_\beta E}$$

$$N_\alpha=2:\quad\sum_{\textbf{spins}}d\sigma(\alpha\to\beta)=(2\pi)^4 u_\alpha^{-1}E_1^{-1}E_2^{-1}R_{\beta\alpha}\delta^4(p_\beta-p_\alpha)\frac{d\beta}{\prod_\beta E}$$

洛伦兹变换性质：

$$R_{\beta\alpha} \equiv \sum_{\text{spins}} |M_{\beta\alpha}|^2 \prod_\beta E \prod_\alpha E \qquad \textbf{is invariant}$$

$$N_\alpha = 1: \quad \sum_{\text{spins}} d\Gamma(\alpha \to \beta) = 2\pi E_\alpha^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E} \qquad \text{按} 1/E_\alpha \text{变换}$$

$$N_\alpha = 2: \quad \sum_{\text{spins}} d\sigma(\alpha \to \beta) = (2\pi)^4 u_\alpha^{-1} E_1^{-1} E_2^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E} \qquad \text{不变！}$$

$$u_\alpha \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}$$

$$\vec{p}_1 = 0 \to E_1 = m_1 \to p_1 \cdot p_2 = m_1 E_2 \to u_\alpha = \frac{|\vec{p}_2|}{E_2} \qquad \vec{p} = \frac{m_0 \vec{v}}{\sqrt{1-v^2}} \quad E = \frac{m_0}{\sqrt{1-v^2}}$$

$$p_1 = (\vec{p}, E_1) \quad p_2 = (-\vec{p}, E_2) \quad E = E_1 + E_2 \to u_\alpha = \frac{|\vec{p}| E}{E_1 E_2} = \left| \frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2} \right|$$

$$(|\vec{p}|^2 + E_1 E_2)^2 - (E_1^2 - |\vec{p}|^2)(E_2^2 - |\vec{p}|^2) = |\vec{p}|^2 (E_1 + E_2)^2$$

反应率与碰撞截面

相空间：

$$N_\alpha = 1 : \sum_{\text{spins}} d\Gamma(\alpha \to \beta) = 2\pi E_\alpha^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E}$$

按 $1/E_\alpha$ 变换

$$\vec{p}_\alpha = 0 \qquad\qquad \vec{p}_1' = -\vec{p}_2' - \vec{p}_3' - \cdots$$

$$\delta^4(p_\beta - p_\alpha)d\beta = \delta^3(\vec{p}_1' + \vec{p}_2' + \cdots)\delta(E_1' + E_2' + \cdots - E)d\vec{p}_1' d\vec{p}_2' \cdots$$

$$N_\alpha = 2 : \sum_{\text{spins}} d\sigma(\alpha \to \beta) = (2\pi)^4 u_\alpha^{-1} E_1^{-1} E_2^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E} \qquad \vec{p}_1 + \vec{p}_2 = 0$$

$$\delta^4(p_\beta - p_\alpha)d\beta = \delta(E_1' + E_2' - E)d\vec{p}_1' \qquad 1 \to 1' + 2' \qquad 1 + 2 \to 1' + 2'$$

$$= \delta(\sqrt{|\vec{p}_1'|^2 + m_1'^2} + \sqrt{|\vec{p}_1'|^2 + m_2'^2} - E)|\vec{p}_1'|^2 d|\vec{p}_1'| d\Omega = \frac{|\vec{p}_1'|E_1'E_2'}{E}d\Omega$$

$$E_1' = \sqrt{|\vec{p}_1'|^2 + m_1'^2} = \frac{E^2 - m_2'^2 + m_1'^2}{2E} \qquad\qquad E_2' = \sqrt{|\vec{p}_1'|^2 + m_2'^2} = \frac{E^2 - m_1'^2 + m_2'^2}{2E}$$

$$|\vec{p}_1'| = \frac{\sqrt{(E^2 - m_1'^2 - m_2'^2)^2 - 4m_1'^2 m_2'^2}}{E} \qquad\qquad \frac{d\Gamma(\alpha \to \beta)}{d\Omega} = \frac{2\pi|\vec{p}_1'|E_1'E_2'}{E}|M_{\beta\alpha}|^2$$

相空间：

$$N_\alpha = 2 : \quad \sum_{\text{spins}} d\sigma(\alpha \to \beta) = (2\pi)^4 u_\alpha^{-1} E_1^{-1} E_2^{-1} R_{\beta\alpha} \delta^4(p_\beta - p_\alpha) \frac{d\beta}{\prod_\beta E} \quad \text{不变!}$$

$$N_\beta = 2 : \qquad 1 + 2 \to 1' + 2' \qquad \vec{p}_1 + \vec{p}_2 = 0$$

$$\frac{d\sigma(\alpha \to \beta)}{d\Omega} = \frac{(2\pi)^4 |\vec{p}_1'| E_1' E_2'}{E u_\alpha} |M_{\beta\alpha}|^2 = \frac{(2\pi)^4 |\vec{p}_1'| E_1' E_2' E_1 E_2}{E^2 |\vec{p}_1|} |M_{\beta\alpha}|^2$$

$$N_\beta = 3 :$$

$$\delta^4(p_\beta - p_\alpha) d\beta = d\vec{p}_2' d\vec{p}_3' \delta\left(\sqrt{(\vec{p}_2' + \vec{p}_3')^2 + m_1'^2} + \sqrt{\vec{p}_2'^2 + m_2'^2} + \sqrt{\vec{p}_3'^2 + m_3'^2} - E\right)$$

$$d\vec{p}_2' d\vec{p}_3' = |\vec{p}_2'|^2 d|\vec{p}_2'| |\vec{p}_3'|^2 d|\vec{p}_3'| d\Omega_3 d\phi_{23} d\cos\theta_{23} \qquad \frac{\partial E_1'}{\partial \cos\theta_{23}} = \frac{|\vec{p}_2'||\vec{p}_3'|}{E_1'}$$

$$\delta^4(p_\beta - p_\alpha) d\beta = |\vec{p}_2'| d|\vec{p}_2'| \, |\vec{p}_3'| d|\vec{p}_3'| \, E_1' d\Omega_3 d\phi_{23} = E_1' E_2' E_3' dE_2' dE_3' d\Omega_3 d\phi_{23}$$

$$\textbf{integrate out } \cos\theta_{23}$$

关于量子场理论，目前我们已经：

- ▶ 建立了自由粒子态

- ▶ 引入时空平移和转动及内部对称性的生成元算符

- ▶ 用H建立了S矩阵理论直接描述散射实验 相互作用通过e^{-iHt}演化引入

只需知道$V(t)$对自由粒子态的作用: $S_{\alpha\beta} = (\Phi_\beta, S\Phi_\alpha) = \delta(\beta - \alpha) - 2\pi i M_{\beta\alpha}\delta^4(p_\beta - p_\alpha)$

$$S = \mathbf{T}e^{-i\int_{-\infty}^{\infty}dt\, V(t)} \qquad V(t) = e^{iH_0 t}Ve^{-iH_0 t} = \int d\vec{x}\mathcal{H}(x) \qquad [\mathcal{H}(\mathbf{x}), \mathcal{H}(\mathbf{x}')]_{类空} = \mathbf{0} \qquad H = H_0 + V$$

$N_\alpha = 1:$ $\qquad d\Gamma(\alpha \to \beta) = 2\pi|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$

$N_\alpha = 2:$ $\qquad d\sigma(\alpha \to \beta) \equiv \dfrac{d\Gamma(\alpha \to \beta)}{\Phi_\alpha} = (2\pi)^4 u_\alpha^{-1}|M_{\beta\alpha}|^2\delta^4(p_\beta - p_\alpha)d\beta$

建立局域的$\mathcal{H}(x)$对自由粒子态的作用? 本章

- ▶ 引入联系不同粒子态的算符: 产生与湮灭算符

- ▶ 将H_0和V及其它生成元算符用产生与湮灭算符**局域地**表达出来

建立完整的计算体系: 下章 **Wick定理; 约化公式; 路径积分**

产生与湮灭算符

产生算符作用在某个多粒子态上定义为在此态上多加入一个粒子

$$a^\dagger(q)\Phi_{q_1q_2\cdots q_n} \equiv \Phi_{qq_1q_2\cdots q_N} \qquad\qquad \Phi_{q_1q_2\cdots q_N} = a^\dagger(q_1)a^\dagger(q_2)\cdots a^\dagger(q_N)\Phi_0$$

产生算符的厄米共轭算符:

$$(\Phi_{q'_1\cdots q'_M}, a(q)\Phi_{q_1\cdots q_N}) = (a^\dagger(q)\Phi_{q'_1\cdots q'_M}, \Phi_{q_1\cdots q_N}) = (\Phi_{qq'_1\cdots q'_M}, \Phi_{q_1\cdots q_N})$$

对指标$1, 2, \cdots, N$交换$\mathcal{P}$的求和可写成对一特殊的要被置换到首位($\mathcal{P}r = 1$)的指标r的求和,再加上对剩余的指标$1, \cdots, r-1, r+1, \cdots, N$的所有置换$\bar{\mathcal{P}}$求和$\sum_{\mathcal{P}} = \sum_{r=1}^{N}\sum_{\bar{\mathcal{P}}}$. 利用$\delta_{\mathcal{P}} = \delta_{r1}\delta_{\bar{\mathcal{P}}}$($\delta_{r1}$是将换到首位所贡献的$\delta_{\mathcal{P}}$),

$$(\Phi_{q'_1\cdots q'_M}, a(q)\Phi_{q_1\cdots q_N}) = \delta_{N,M+1}\sum_{r=1}^{N}\sum_{\bar{\mathcal{P}}}\delta_{r1}\delta_{\bar{\mathcal{P}}}\delta(q-q_r)\prod_{i=1}^{M}\delta(q'_i - q_{\bar{\mathcal{P}}i})$$

$$= \begin{cases} \sum_{r=1}^{N}\delta_{r1}\delta(q-q_r)(\Phi_{q'_1\cdots q'_M}, \Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N}) & N \geq 1 \\ 0 & N = 0 \end{cases}$$

湮灭算符: $a(q)\Phi_{q_1\cdots q_N} = \sum_{r=1}^{N}\delta_{r1}\delta(q-q_r)\Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N}$ \qquad $N \geq 1$ \qquad $a(q)\Phi_0 = 0$

产生与湮灭算符

产生算符: $a^{\dagger}(q)\Phi_{q_1 q_2 \cdots q_n} \equiv \Phi_{q q_1 q_2 \cdots q_N}$ $\quad \Phi_{q_1 q_2 \cdots q_N} = a^{\dagger}(q_1)a^{\dagger}(q_2)\cdots a^{\dagger}(q_N)\Phi_0$

湮灭算符: $a(q)\Phi_{q_1 \cdots q_N} = \sum_{r=1}^{N} \delta_{r1}\delta(q - q_r)\Phi_{q_1 \cdots q_{r-1} q_{r+1} \cdots q_N}$ $\quad N \geq 1$ $\quad a(q)\Phi_0 = 0$

产生与湮灭算符的性质: 作业8

$$a(q')a^{\dagger}(q) \mp a^{\dagger}(q)a(q') = \delta(q' - q)$$
$$a(q')a(q) \mp a(q)a(q') = 0 \qquad a^{\dagger}(q')a^{\dagger}(q) \mp a^{\dagger}(q)a^{\dagger}(q') = 0$$

► 负号对应两个粒子都是玻色子或一个是玻色子一个是费米子的情况

► 正号对应两个粒子都是费米子的情况

粒子数算符: 作业9

$$N \equiv \int d\vec{q}\, a^{\dagger}(q)a(q) \qquad [N, a^{\dagger}(q)] = a^{\dagger}(q) \qquad [N, a(q)] = -a(q)$$

产生与湮灭算符

产生湮灭算符在洛伦兹变换下的行为： $a^\dagger(p)\Phi_{p_1 p_2 \cdots p_n} \equiv \Phi_{p p_1 p_2 \cdots p_N}$

$$U_0(\Lambda, b)\Phi_{p,\sigma,n;p_1,\sigma_1,n_1;\cdots} = e^{ib_\mu((\Lambda p)^\mu + (\Lambda p_1)^\mu + \cdots)} \sqrt{\frac{(\Lambda p)^0 (\Lambda p_1)^0 \cdots}{p^0 p_1^0 \cdots}} \sum_{\bar{\sigma}\bar{\sigma}_1 \cdots} D_{\bar{\sigma}\sigma}(W(\Lambda, p))$$

$$= U_0(\Lambda, b)a^\dagger(\vec{p}, \sigma, n)U_0^{-1}(\Lambda, b)U_0(\Lambda, b)\Phi_{p_1, \sigma_1, n_1; \cdots} \qquad \times D_{\bar{\sigma}_1 \sigma_1}(W(\Lambda, p_1)) \cdots \Phi_{\Lambda p, \bar{\sigma}, n; \Lambda p_1, \sigma'_1, n_1; \cdots}$$

$$U_0(\Lambda, b)a^\dagger(\vec{p}, \sigma, n)U_0^{-1}(\Lambda, b) = e^{i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}} D_{\bar{\sigma}\sigma}(W(\Lambda, p))a^\dagger(\vec{p}_\Lambda, \bar{\sigma}, n)$$

$$= e^{i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}} D^*_{\sigma\bar{\sigma}}(W^{-1}(\Lambda, p))a^\dagger(\vec{p}_\Lambda, \bar{\sigma}, n)$$

$$U_0(\Lambda, b)a(\vec{p}, \sigma, n)U_0^{-1}(\Lambda, b) = e^{-i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}} D^\dagger_{\bar{\sigma}\sigma}(W(\Lambda, p))a(\vec{p}_\Lambda, \bar{\sigma}, n)$$

$$= e^{-i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}} D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda, p))a(\vec{p}_\Lambda, \bar{\sigma}, n)$$

产生与湮灭算符

产生湮灭算符在空间反射变换下的行为： $a^\dagger(p)\Phi_{p_1 p_2 \cdots p_n} \equiv \Phi_{p p_1 p_2 \cdots p_N}$

有质量正能: $P\Psi_{p,\sigma} = \eta \Psi_{\mathcal{P}p,\sigma}$

无质量正能: $P\Psi_{p,\sigma} = \eta_\sigma e^{\mp i\pi\sigma}\Psi_{\mathcal{P}p,-\sigma}$ 负号: $0 \le \phi < \pi$, 正号: $\pi \le \phi < 2\pi$ ϕ是$\vec{p}$在**xy**平面上投影与**x**轴的夹角

产生湮灭**有质量**的粒子算符在空间反射变换下的行为：

$$Pa^\dagger(\vec{p},\sigma,n)P^{-1} = \eta a^\dagger(\mathcal{P}\vec{p},\sigma,n) \qquad Pa(\vec{p},\sigma,n)P^{-1} = \eta^* a(\mathcal{P}\vec{p},\sigma,n)$$

产生湮灭**无质量**的粒子算符在空间反射变换下的行为：

$$Pa^\dagger(\vec{p},\sigma,n)P^{-1} = \eta_\sigma e^{\mp i\pi\sigma} a^\dagger(\mathcal{P}\vec{p},-\sigma,n) \qquad Pa(\vec{p},\sigma,n)P^{-1} = \eta_\sigma^* e^{\pm i\pi\sigma} a(\mathcal{P}\vec{p},-\sigma,n)$$

负号: $0 \le \phi < \pi$, 正号: $\pi \le \phi < 2\pi$ （ϕ是$\vec{p}$在**xy**平面上的投影与**x**轴的夹角）

产生与湮灭算符

产生湮灭算符在时间反演变换下的行为: $a^\dagger(p)\Phi_{p_1 p_2 \cdots p_n} \equiv \Phi_{p p_1 p_2 \cdots p_N}$

有质量正能: $T\Psi_{p,\sigma} = \zeta(-1)^{j-\sigma}\Psi_{\mathcal{P}p,-\sigma}$

无质量正能: $T\Psi_{p,\sigma} = \zeta_\sigma e^{\pm i\pi\sigma}\Psi_{\mathcal{P}p,\sigma}$

$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ 中: 正号 $0 \le \phi < \pi$, 负号对应 $\pi \le \phi < 2\pi$

产生湮灭**有质量**的粒子算符在时间反演变换下的行为:

$$Ta^\dagger(\vec{p},\sigma,n)T^{-1} = \zeta(-1)^{j-\sigma}a^\dagger(\mathcal{P}\vec{p},-\sigma,n) \qquad Ta(\vec{p},\sigma,n)T^{-1} = \zeta^*(-1)^{j-\sigma}a(\mathcal{P}\vec{p},-\sigma,n)$$

产生湮灭**无质量**的粒子算符在时间反演变换下的行为:

$$Ta^\dagger(\vec{p},\sigma,n)T^{-1} = \zeta_\sigma e^{\pm i\pi\sigma}a^\dagger(\mathcal{P}\vec{p},\sigma,n) \qquad Ta(\vec{p},\sigma,n)T^{-1} = \zeta_\sigma^* e^{\mp i\pi\sigma}a(\mathcal{P}\vec{p},\sigma,n)$$

$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ 中正号 $0 \le \phi < \pi$, 负号对应 $\pi \le \phi < 2\pi$

产生与湮灭算符

产生湮灭算符在内部对称性变换下的行为：$a^\dagger(p)\Phi_{p_1 p_2 \cdots p_n} \equiv \Phi_{p p_1 p_2 \cdots p_N}$

$$U_0(T(\theta))\Psi_{q,\sigma} = e^{iq_a\theta^a}\Psi_{q,\sigma} \qquad\qquad Q_a\,\Psi_{q,\sigma} = q_a\,\Psi_{q,\sigma}$$

产生湮灭粒子算符在内部对称性$U(1)$生成元Q_a变换下的行为：

$$[Q_a,\, a^\dagger(\vec{q})]\Phi_{q_1 q_2 \cdots q_n} = Q_a\Phi_{q q_1 q_2 \cdots q_n} - a^\dagger(\vec{q})(q_{a1} + q_{a2} + \cdots)\Phi_{q_1 q_2 \cdots q_n}$$

$$= q\Phi_{q q_1 q_2 \cdots q_n} = q_a a^\dagger(\vec{q})\Phi_{q_1 q_2 \cdots q_n}$$

$$[Q_a,\, a^\dagger(\vec{q})] = q_a a^\dagger(\vec{q}) \qquad\qquad [Q_a,\, a(\vec{q})] = -q_a a(\vec{q})$$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p \; u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p \; v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

♣ **局域相互作用哈密顿量的表达需要**
$$S = \mathbf{T} \; e^{-i\int_{\tau_0}^{\tau} dt \; V(t)} \qquad V(t) = e^{iH_0 t}Ve^{-iH_0 t} = \int d\vec{x} \; \tilde{\mathcal{H}}(\vec{r},t)$$
$$U_0(\Lambda,a)\tilde{\mathcal{H}}(x)U_0^{-1}(\Lambda,a) = \tilde{\mathcal{H}}(\Lambda x + a)$$

✠ **态空间的产生与湮灭直接与坐标空间联系!**

♡ **局域性!**

♠ **因果性!** $\quad [\tilde{\mathcal{H}}(x),\tilde{\mathcal{H}}(x')] = 0 \quad x-x'$ 类空间隔

¶ **它就是 量子场** 产生湮灭算符的集合!

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p \; u_l(x;\vec{p},\sigma,n) a(\vec{p},\sigma,n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p \; v_l(x;\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$[Q_a, \; a^\dagger(\vec{q})] = q_a a^\dagger(\vec{q}) \quad [Q_a, \; a(\vec{q})] = -q_a a(\vec{q})$$

$$[Q_a, \; \psi_l^\pm(x)] = \mp q_a \psi_l^\pm(x)$$

要求$u_l(x;\vec{p},\sigma,n)$**和**$v_l(x;\vec{p},\sigma,n)$**满足:** $U_0(\Lambda,a)\tilde{\mathcal{H}}(x)U_0^{-1}(\Lambda,a) = \tilde{\mathcal{H}}(\Lambda x + a)$

$$U_0(\Lambda,a)\psi_l^+(x)U_0^{-1}(\Lambda,a) = \sum_{\bar{l}} D_{l\bar{l}}^+(\Lambda^{-1})\psi_{\bar{l}}^+(\Lambda x + a)$$

$$U_0(\Lambda,a)\psi_l^-(x)U_0^{-1}(\Lambda,a) = \sum_{\bar{l}} D_{l\bar{l}}^-(\Lambda^{-1})\psi_{\bar{l}}^-(\Lambda x + a)$$

$$D^\pm(\Lambda^{-1})D^\pm(\bar{\Lambda}^{-1}) = D^\pm((\bar{\Lambda}\Lambda)^{-1}) \qquad D^\pm(\Lambda_1)D^\pm(\Lambda_2) = D^\pm(\Lambda_1\Lambda_2)$$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p \; u_l(x; \vec{p}, \sigma, n) a(\vec{p}, \sigma, n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p \; v_l(x; \vec{p}, \sigma, n) a^\dagger(\vec{p}, \sigma, n)$$

$$[Q_a, \; \psi_l^\pm(x)] = \mp q_a \psi_l^\pm(x) \qquad U_0(\Lambda, a)\psi_l^\pm(x)U_0^{-1}(\Lambda, a) = \sum_{\bar{l}} D_{l\bar{l}}^\pm(\Lambda^{-1})\psi_{\bar{l}}^\pm(\Lambda x + a)$$

$$U_0(e^\omega, \epsilon) = e^{\frac{i}{2}\omega_{\rho\sigma}J^{\rho\sigma} + i\epsilon_\rho P^\rho} \qquad D^\pm(e^\omega) = e^{\frac{i}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma}} \quad \text{场的表示没有么正性要求}$$

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

$$[\frac{i}{2}\omega_{\rho\sigma}J^{\rho\sigma} + i\epsilon_\rho P^\rho, \psi_l^\pm(x)] = -\frac{i}{2}\omega_{\rho\sigma}\mathcal{J}_{l\bar{l}}^{\rho\sigma}\psi_{\bar{l}}^\pm(x) + [\omega_{\rho\sigma}x^\sigma + \epsilon_\rho]\partial^\rho\psi_l^\pm(x)$$

$$[P^\rho, \psi_l^\pm(x)] = -i\partial^\rho\psi_l^\pm(x) \qquad [x^\sigma, -i\partial^\rho] = ig^{\sigma\rho}$$

$$[J^{\rho\sigma}, \psi_l^\pm(x)] = -\mathcal{J}_{l\bar{l}}^{\rho\sigma}\psi_{\bar{l}}^\pm(x) + i(x^\rho\partial^\sigma - x^\sigma\partial^\rho)\psi_l^\pm(x) \qquad L^{\rho\sigma} \equiv -i(x^\rho\partial^\sigma - x^\sigma\partial^\rho)$$

$$i[L^{\mu\nu}, L^{\rho\sigma}] = g^{\nu\rho}L^{\mu\sigma} - g^{\mu\rho}L^{\nu\sigma} - g^{\sigma\mu}L^{\rho\nu} + g^{\sigma\nu}L^{\rho\mu} \qquad [\mathcal{J}^{\mu\nu}, L^{\rho\sigma}] = 0$$

$$i[i\partial^\mu, L^{\rho\sigma}] = g^{\mu\rho}i\partial^\sigma - g^{\mu\sigma}i\partial^\rho \qquad [i\partial^\mu, \mathcal{J}^{\rho\sigma}] = 0 \qquad [i\partial^\mu, i\partial^\rho] = 0$$

场空间的Pauli-Lubanski算符: $W^\mu \equiv -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}i\partial_\nu(L_{\rho\sigma} + \mathcal{J}_{\rho\sigma}) = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}i\partial_\nu\mathcal{J}_{\rho\sigma}$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p\, u_l(x;\vec{p},\sigma,n)a(\vec{p},\sigma,n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p\, v_l(x;\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)$$

$$U_0(\Lambda,a)\psi_l^\pm(x)U_0^{-1}(\Lambda,a) = \sum_{\bar{l}} D_{l\bar{l}}^\pm(\Lambda^{-1})\psi_{\bar{l}}^\pm(\Lambda x + a)$$

$$U_0(\Lambda,b)a(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b) = e^{-i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}}D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda,p))a(\vec{p}_\Lambda,\bar{\sigma},n)$$

$$U_0(\Lambda,b)a^\dagger(\vec{p},\sigma,n)U_0^{-1}(\Lambda,b) = e^{i(\Lambda p)\cdot b}\sqrt{\frac{(\Lambda p)^0}{p^0}}\sum_{\bar{\sigma}}D_{\sigma\bar{\sigma}}^*(W^{-1}(\Lambda,p))a^\dagger(\vec{p}_\Lambda,\bar{\sigma},n)$$

$$\sum_{\bar{l}}D_{l\bar{l}}^+(\Lambda^{-1})u_{\bar{l}}(\Lambda x + b;\vec{p}_\Lambda,\sigma,n) = \sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{\bar{\sigma}}D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda,p))e^{-i(\Lambda p)\cdot b}u_l(x;\vec{p},\bar{\sigma},n)$$

$$\sum_{\bar{l}}D_{l\bar{l}}^-(\Lambda^{-1})v_{\bar{l}}(\Lambda x + b;\vec{p}_\Lambda,\sigma,n) = \sqrt{\frac{p^0}{(\Lambda p)^0}}\sum_{\bar{\sigma}}D_{\sigma\bar{\sigma}}^*(W^{-1}(\Lambda,p))e^{i(\Lambda p)\cdot b}v_l(x;\vec{p},\bar{\sigma},n)$$

$$d^3p\sqrt{\frac{(\Lambda p)^0}{p^0}} = \frac{d^3p}{p^0}\sqrt{p^0(\Lambda p)^0} = \frac{d^3(\Lambda p)}{(\Lambda p)^0}\sqrt{p^0(\Lambda p)^0} = d^3(\Lambda p)\sqrt{\frac{p^0}{(\Lambda p)^0}}$$

产生和湮灭算符在坐标空间的表达:

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p \; u_l(x; \vec{p}, \sigma, n) a(\vec{p}, \sigma, n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p \; v_l(x; \vec{p}, \sigma, n) a^\dagger(\vec{p}, \sigma, n)$$

$$\sum_{\bar{l}} D_{l\bar{l}}^+(\Lambda^{-1}) u_{\bar{l}}(\Lambda x + b; \vec{p}_\Lambda, \sigma, n) = \sqrt{\frac{p^0}{(\Lambda p)^0}} \sum_{\bar{\sigma}} D_{\sigma\bar{\sigma}}(W^{-1}(\Lambda, p)) e^{-i(\Lambda p)\cdot b} u_l(x; \vec{p}, \bar{\sigma}, n)$$

$$\sum_{\bar{l}} D_{l\bar{l}}^-(\Lambda^{-1}) v_{\bar{l}}(\Lambda x + b; \vec{p}_\Lambda, \sigma, n) = \sqrt{\frac{p^0}{(\Lambda p)^0}} \sum_{\bar{\sigma}} D_{\sigma\bar{\sigma}}^*(W^{-1}(\Lambda, p)) e^{i(\Lambda p)\cdot b} v_l(x; \vec{p}, \bar{\sigma}, n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\Lambda x + b; \vec{p}_\Lambda, \bar{\sigma}, n) D_{\bar{\sigma}\sigma}(W(\Lambda, p)) = \sqrt{\frac{p^0}{(\Lambda p)^0}} \sum_{l} D_{\bar{l}l}^+(\Lambda) e^{-i(\Lambda p)\cdot b} u_l(x; \vec{p}, \sigma, n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\Lambda x + b; \vec{p}_\Lambda, \bar{\sigma}, n) D_{\bar{\sigma}\sigma}^*(W(\Lambda, p)) = \sqrt{\frac{p^0}{(\Lambda p)^0}} \sum_{l} D_{\bar{l}l}^-(\Lambda) e^{i(\Lambda p)\cdot b} v_l(x; \vec{p}, \sigma, n)$$

平移

$$\psi_l^+(x) = \sum_{\sigma,n} \int d^3p \; u_l(x;\vec{p},\sigma,n) a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int d^3p \; v_l(x;\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \; \sum_l D_{\bar{l}l}^+(\Lambda) e^{-i(\Lambda p)\cdot b} u_l(x;\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\Lambda x+b;\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \; \sum_l D_{\bar{l}l}^-(\Lambda) e^{i(\Lambda p)\cdot b} v_l(x;\vec{p},\sigma,n)$$

$\Lambda=1, b$任意$,\Rightarrow D^+(\Lambda)=D^-(\Lambda)=1 \Rightarrow u_l(x+b;\vec{p},\sigma,n) = e^{-ip\cdot b} u_l(x;\vec{p},\sigma,n) \qquad v_l(x+b;\vec{p},\sigma,n) = e^{ip\cdot b} v_l(x;\vec{p},\sigma,n)$

$$\Rightarrow u_l(x;\vec{p},\sigma,n) = \frac{e^{-ip\cdot x}}{(2\pi)^{3/2}} u_l(\vec{p},\sigma,n) \qquad v_l(x;\vec{p},\sigma,n) = \frac{e^{ip\cdot x}}{(2\pi)^{3/2}} v_l(\vec{p},\sigma,n)$$

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \; \sum_l D_{\bar{l}l}^+(\Lambda) u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \; \sum_l D_{\bar{l}l}^-(\Lambda) v_l(\vec{p},\sigma,n)$$

有质量情况

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma} u_{\bar l}(\vec{p}_\Lambda, \bar\sigma, n) D_{\bar\sigma\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar l l}^+(\Lambda) u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar\sigma} v_{\bar l}(\vec{p}_\Lambda, \bar\sigma, n) D_{\bar\sigma\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar l l}^-(\Lambda) v_l(\vec{p},\sigma,n)$$

推进: 定义 $q = \Lambda p$，取 $p = k = (M,0,0,0)$

$\Lambda = L(q)$, $L(q)$ 是标准的"推进": $q = L(q)(M,0,0,0) = \Lambda p$

则 $L(p) = 1$，导致 $W(\Lambda,p) \equiv L^{-1}(\Lambda p)\Lambda L(p) = L^{-1}(q)L(q) = 1$

$$u_{\bar l}(\vec{q},\sigma,n) = \sqrt{\frac{M}{q^0}} \sum_l D_{\bar l l}^+(L(q)) u_l(0,\sigma,n)$$

$$v_{\bar l}(\vec{q},\sigma,n) = \sqrt{\frac{M}{q^0}} \sum_l D_{\bar l l}^-(L(q)) v_l(0,\sigma,n)$$

推进与转动

有质量情况

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \ \sum_l D_{\bar{l}l}^+(\Lambda) u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \ \sum_l D_{\bar{l}l}^-(\Lambda) v_l(\vec{p},\sigma,n)$$

转动: $D^+ = D^-$ 为了适应未来反粒子的引入!

取$p = k = (M,0,0,0)$，$\Lambda = R$, 导致$\vec{p}_\Lambda = 0$: $q = \Lambda p$

则$L(p) = 1$，导致$W(\Lambda,p) \equiv L^{-1}(\Lambda p)\Lambda L(p) = L^{-1}(p)R = R$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(0,\bar{\sigma},n) D_{\bar{\sigma}\sigma}(R) = \sum_l D_{\bar{l}l}^+(R) u_l(0,\sigma,n) \xRightarrow[\text{平庸是恒等式}]{\text{非平庸}} \sum_{\bar{\sigma}} u_{\bar{l}}(0,\bar{\sigma},n) \vec{\mathcal{J}}_{\bar{\sigma}\sigma}^{(j_n)} = \sum_l \vec{\mathcal{J}}_{\bar{l}l} u_l(0,\sigma,n)$$

对固定 j_n' 的 $\vec{\mathcal{J}}_{\bar{l}l}$ 选择 $j_n \le j_n'$ 的 $\vec{\mathcal{J}}^{(j_n)}$ 进行求解

$$\sum_{\bar{\sigma}} v_{\bar{l}}(0,\bar{\sigma},n) D_{\bar{\sigma}\sigma}^*(R) = \sum_l D_{\bar{l}l}^-(R) v_l(0,\sigma,n) \xRightarrow[\text{平庸是恒等式}]{\text{非平庸}} \sum_{\bar{\sigma}} v_{\bar{l}}(0,\bar{\sigma},n) \vec{\mathcal{J}}_{\bar{\sigma}\sigma}^{(j_n)*} = -\sum_l \vec{\mathcal{J}}_{\bar{l}l} v_l(0,\sigma,n)$$

推进与转动

无质量情况

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^+(\Lambda) u_l(\vec{p},\sigma,n) \qquad D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = e^{i\theta(\Lambda,p)\sigma} \delta_{\bar{\sigma}\sigma}$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^-(\Lambda) v_l(\vec{p},\sigma,n) \qquad D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = e^{-i\theta(\Lambda,p)\sigma} \delta_{\bar{\sigma}\sigma}$$

取 $p = k = (\kappa,0,0,\kappa)$**,** $\Lambda = L(q)$**,**： $q = L(q)(\kappa,0,0,\kappa) = \Lambda p$

则 $L(p) = 1$，**导致** $W(\Lambda,p) \equiv L^{-1}(\Lambda p)\Lambda L(p) = L^{-1}(q)L(q) = 1$

$$u_{\bar{l}}(\vec{q},\sigma,n) = \sqrt{\frac{k^0}{q^0}} \sum_l D_{\bar{l}l}^+(L(q)) u_l(\vec{k},\sigma,n) \qquad v_{\bar{l}}(\vec{q},\sigma,n) = \sqrt{\frac{k^0}{q^0}} \sum_l D_{\bar{l}l}^-(L(q)) v_l(\vec{k},\sigma,n)$$

取： $p = k = (\kappa,0,0,\kappa)$, $\Lambda = W$ $q = \Lambda p = k$

$$u_{\bar{l}}(\vec{k},\sigma,n) e^{i\theta(W,k)\sigma} = \sum_l D_{\bar{l}l}^+(W) u_l(\vec{k},\sigma,n) \qquad v_{\bar{l}}(\vec{k},\sigma,n) e^{-i\theta(W,k)\sigma} = \sum_l D_{\bar{l}l}^-(W) v_l(\vec{p},\sigma,n)$$

推进与转动

无质量情况: $\quad W(\theta, \alpha, \beta) = S(\alpha, \beta)R(\theta)$

$$\psi_l^+(x) = \sum_{\sigma, n} \int \frac{d^3 p}{(2\pi)^{3/2}} e^{-ip \cdot x} u_l(\vec{p}, \sigma, n) a(\vec{p}, \sigma, n) \quad \psi_l^-(x) = \sum_{\sigma, n} \int \frac{d^3 p}{(2\pi)^{3/2}} e^{ip \cdot x} v_l(\vec{p}, \sigma, n) a^\dagger(\vec{p}, \sigma, n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\vec{p}_\Lambda, \bar{\sigma}, n) D_{\bar{\sigma}\sigma}(W(\Lambda, p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^+(\Lambda) u_l(\vec{p}, \sigma, n) \quad D_{\bar{\sigma}\sigma}(W(\Lambda, p)) = e^{i\theta(\Lambda, p)\sigma} \delta_{\bar{\sigma}\sigma}$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\vec{p}_\Lambda, \bar{\sigma}, n) D_{\bar{\sigma}\sigma}^*(W(\Lambda, p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^-(\Lambda) v_l(\vec{p}, \sigma, n) \quad D_{\bar{\sigma}\sigma}^*(W(\Lambda, p)) = e^{-i\theta(\Lambda, p)\sigma} \delta_{\bar{\sigma}\sigma}$$

$$u_{\bar{l}}(\vec{k}, \sigma, n) e^{i\theta(W, k)\sigma} = \sum_l D_{\bar{l}l}^+(W) u_l(\vec{k}, \sigma, n) \quad v_{\bar{l}}(\vec{k}, \sigma, n) e^{-i\theta(W, k)\sigma} = \sum_l D_{\bar{l}l}^-(W) v_l(\vec{p}, \sigma, n)$$

$$u_{\bar{l}}(\vec{k}, \sigma, n) e^{i\theta\sigma} = \sum_l D_{\bar{l}l}^+(R(\theta)) u_l(\vec{k}, \sigma, n) \quad v_{\bar{l}}(\vec{k}, \sigma, n) e^{-i\theta\sigma} = \sum_l D_{\bar{l}l}^-(R(\theta)) v_l(\vec{k}, \sigma, n)$$

$$u_{\bar{l}}(\vec{k}, \sigma, n) = \sum_l D_{\bar{l}l}^+(S(\alpha, \beta)) u_l(\vec{k}, \sigma, n) \quad v_{\bar{l}}(\vec{k}, \sigma, n) = \sum_l D_{\bar{l}l}^-(S(\alpha, \beta)) v_l(\vec{k}, \sigma, n)$$

$$\Rightarrow \quad v_l(\vec{p}, \sigma, n) = u_l^*(\vec{p}, \sigma, n) \quad \text{仍然} D^+ = D^- = \text{实矩阵}$$

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^+(\Lambda) u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\vec{p}_\Lambda,\bar{\sigma},n) D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^-(\Lambda) v_l(\vec{p},\sigma,n)$$

标量量子场: $D^\pm(\Lambda) = 1 \quad D(W) = 1,\ u(\vec{p}_\Lambda)\sqrt{(\Lambda p)^0} = u(\vec{p})\sqrt{p^0} \quad v(\vec{p}_\Lambda)\sqrt{(\Lambda p)^0} = v(\vec{p})\sqrt{p^0}$

只考虑有质量的标量场: $\qquad u(\vec{p}) = v(\vec{p}) = 1/\sqrt{2p^0}$ 无质量可看作质量趋于0的极限

$$\phi^+(x) = \int \frac{d\vec{p}}{(2\pi)^{3/2}\sqrt{2p^0}} e^{-ip\cdot x} a(\vec{p}) \quad \phi^-(x) = \int \frac{d\vec{p}}{(2\pi)^{3/2}\sqrt{2p^0}} e^{ip\cdot x} a^\dagger(\vec{p}) = (\phi^+(x))^\dagger$$

$$[\phi^+(x),\phi^+(y)]_\mp = 0 \qquad [\phi^-(x),\phi^-(y)]_\mp = 0 \quad \text{对易子: 玻色子; 反对易子: 费米子}$$

$$\Delta_+(M,x-y) \equiv [\phi^+(x),\phi^-(y)]_\mp = \int \frac{d\vec{p}\,d\vec{p}'}{(2\pi)^3(2p^0 2p'^0)^{1/2}} e^{-ipx} e^{ip'y} \delta(\vec{p}-\vec{p}') = \int \frac{d\vec{p}}{(2\pi)^3 2p^0} e^{-ip\cdot(x-y)} \text{ 作业10}$$

♣ **动量空间的产生与湮灭算符:**

$$[a(q), a^\dagger(q')] = \delta(\vec{q} - \vec{q}') \qquad [a(q), a(q')] = [a^\dagger(q), a^\dagger(q')] = 0 \qquad a(q)\Phi_0 = 0$$

$$N \equiv \int d\vec{p}\, N(p) \qquad [N(p), a^\dagger(q)] = \delta(\vec{p} - \vec{q})a^\dagger(q) \qquad [N, a^\dagger(q)] = a^\dagger(q)$$

$$N(p) \equiv a^\dagger(p)a(p) \qquad [N(p), a(q)] = -\delta(\vec{p} - \vec{q})a(q) \qquad [N, a(q)] = -a(q)$$

♠ **坐标空间的产生与湮灭算符:**

$$[\phi^+(x), \phi^-(x')] = \Delta_+(M, x - x') \quad [\phi^+(x), \phi^+(x')] = [\phi^-(x), \phi^-(x')] = 0 \quad \phi^+(x)\Phi_0 = 0$$

$$\Delta_+(M, x) = \int \frac{d\vec{p}}{(2\pi)^3 2p^0} e^{-ip \cdot x} \stackrel{x\text{类空}}{===} \frac{M}{4\pi^2 \sqrt{-x^2}} K_1(M\sqrt{-x^2}) \stackrel{M \to \infty, t=0}{======} \frac{1}{2M}\delta(\vec{x})$$

$$\tilde{N}(t) \equiv \int d\vec{x}\, \phi^-(x)\phi^+(x) \qquad [\tilde{N}(t), \phi^-(\vec{x}, t)] = \int d\vec{x}'\phi^-(\vec{x}', t)\Delta_+(M, \vec{x}' - \vec{x}, 0)$$

$$[\tilde{N}(t), \phi^+(\vec{x}, t)] = -\int d\vec{x}'\phi^+(\vec{x}', t)\Delta_+(M, \vec{x} - \vec{x}', 0)$$

关于坐标空间的产生与湮灭算符

$$[\phi^+(x), \phi^-(x')] = \Delta_+(M, x-x') \quad [\phi^+(x), \phi^+(x')] = [\phi^-(x), \phi^-(x')] = 0 \quad \phi^+(x)\Phi_0 = 0$$

$$\Delta_+(M, x) = \int \frac{d\vec{p}}{(2\pi)^3 2p^0} e^{-ip\cdot x} \overset{x\text{类空}}{=\!=\!=} \frac{M}{4\pi^2\sqrt{-x^2}} K_1(M\sqrt{-x^2}) \overset{M\to\infty, t=0}{=\!=\!=\!=\!=} \frac{1}{2M}\delta(\vec{x})$$

$$\tilde{N}(t) \equiv \int d\vec{x} \, \phi^-(x)\phi^+(x) \quad [\tilde{N}(t), \phi^-(\vec{x}, t)] = \int d\vec{x}' \phi^-(\vec{x}', t)\Delta_+(M, \vec{x}'-\vec{x}, 0)$$

♣   波包： $$[\tilde{N}(t), \phi^+(\vec{x}, t)] = -\int d\vec{x}' \phi^+(\vec{x}', t)\Delta_+(M, \vec{x}-\vec{x}', 0)$$

若存在：$\rho(\vec{x}) = 2M \int d\vec{x}' \rho(\vec{x}')\Delta_+(M, \vec{x}'-\vec{x}, 0)$ $\tilde{\phi}^-(t) \equiv \int d\vec{x} \, \rho(\vec{x})\phi^-(\vec{x}, t)$

$$[\tilde{N}(t), \tilde{\phi}^-(t)] = \int d\vec{x} d\vec{x}' \rho(\vec{x}')\Delta_+(M, \vec{x}'-\vec{x}, 0)\phi^-(\vec{x}, t) = \tilde{\phi}^-(t)$$

$$[\tilde{N}(t), \tilde{\phi}^+(t)] = -\int d\vec{x} d\vec{x}' \rho(\vec{x}')\Delta_+(M, \vec{x}'-\vec{x}, 0)\phi^+(\vec{x}, t) = -\tilde{\phi}^+(t)$$

坐标空间的产生与湮灭算符: $\Delta_+(M,x) = \frac{M}{4\pi^2\sqrt{-x^2}}K_1(M\sqrt{-x^2})$

$\tilde{N}(t) \equiv \int d\vec{x}\,\phi^-(x)\phi^+(x)$ 　　$[\tilde{N}(t),\phi^-(\vec{x},t)] = \int d\vec{x}'\phi^-(\vec{x}',t)\Delta_+(M,\vec{x}'-\vec{x},0)$

$$[\tilde{N}(t),\phi^+(\vec{x},t)] = -\int d\vec{x}'\phi^+(\vec{x}',t)\Delta_+(M,\vec{x}-\vec{x}',0)$$
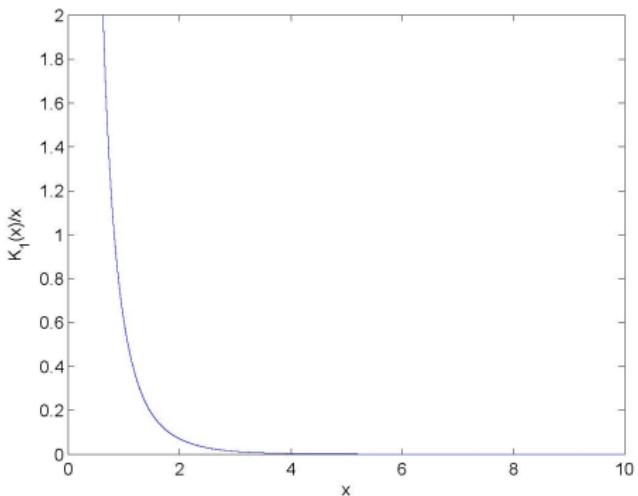
$$K_1(x) = \int_0^\infty dt\,\frac{t\sin xt}{\sqrt{1+t^2}}$$

反粒子的引入

ϕ^+和ϕ^-场之间的不对易（或不反对易）将对构造$\tilde{\mathcal{H}}(x)$产生很大的困难。如果$\tilde{\mathcal{H}}(x)$是由ϕ^+和ϕ^-场构造的，$\tilde{\mathcal{H}}(x)$在类空区对易要求ϕ^+和ϕ^-场之间必须是对易或反对易的，否则ϕ^+场和ϕ^-场之间必须达成某种平衡，以使ϕ^+场和ϕ^-场之间的不对易（或不反对易）的影响能够以某种形式被消掉。

为了寻找这种能够消除这种不对易性对$\tilde{\mathcal{H}}(x)$在类空区的对易性的影响的规则，将ϕ^+场和ϕ^-场进行一个线性组合$\phi(x) \equiv \kappa\phi^+(x) + \lambda\phi^-(x)$,看是否能够实现这个线性组合场在类空区间的对易或反对易性质

$$[\phi(x),\phi^\dagger(y)]_\mp = |\kappa|^2\Delta_+(M,x-y)\mp|\lambda|^2\Delta_+(M,y-x) \quad [\phi(x),\phi(y)]_\mp = \kappa\lambda\{\Delta_+(M,x-y)\mp\Delta_+(M,y-x)\}$$

$\Delta_+(M,x-y)$是洛伦兹不变量$\xRightarrow[====\text{类空间隔可实现}x-y\to y-x====]{}\Delta_+(M,x-y) = \Delta_+(M,y-x)$

$\Rightarrow$ **选对易子和$|\kappa| = |\lambda|$**

产生和湮灭算符还有一个相角的任意性，ϕ前的一个整体常数是无关紧要的.可将κ和λ的值选择为1,即：$\phi(x) = \phi^+(x) + \phi^-(x)$. 以这样一种组合的标量玻色子场$\phi(x)$和其共轭$\phi^\dagger(x)$来构造$\mathcal{H}(x)$可以保证其在类控区间相互对易。

反粒子的引入

ϕ^+和ϕ^-场之间的不对易（或不反对易）将对构造$\tilde{\mathcal{H}}(x)$产生很大的困难。
改进办法是将ϕ^+场和ϕ^-场进行一个线性组合$\phi(x) \equiv \phi^+(x) + \phi^-(x)$,它使得在
类空区间 $[\phi(x), \phi^\dagger(y)] = [\phi(x), \phi(y)] = 0$

引发问题: 如ϕ场带某种守恒内部荷, $\tilde{\mathcal{H}}(x)$与生成此对称性的生成元对易,

$$[Q^a, \tilde{\mathcal{H}}(x)] = 0 \quad \Rightarrow \quad [Q_a, \phi^+(x)]_- = -q_a\phi^+(x) \qquad [Q_a, \phi^-(x)]_- = q_a\phi^-(x)$$

当荷$q_a \neq 0$时，若以ϕ^+和ϕ^-场为基本元素构造$\tilde{\mathcal{H}}(x)$,要保证$[Q^a, \tilde{\mathcal{H}}(x)] = 0$,每
一项中含ϕ^+的数目和含ϕ^-的数目相等即可.

但若改用以ϕ和$\phi^\dagger$场作为基本元素构造$\tilde{\mathcal{H}}(x)$,不管怎样构造,都不可能保证每
一项中含ϕ^+的数目和含ϕ^-的数目相等.无法实现要求$[Q^a, \tilde{\mathcal{H}}(x)] = 0$.

反粒子的引入

ϕ^+和ϕ^-场之间的不对易（或不反对易）将对构造$\tilde{\mathcal{H}}(x)$产生很大的困难。
改进办法是将ϕ^+场和ϕ^-场进行一个线性组合$\phi(x) \equiv \phi^+(x) + \phi^-(x)$,它使得在
类空区间 $[\phi(x), \phi^\dagger(y)] = [\phi(x), \phi(y)] = 0$ 但无法实现$[Q^a, \tilde{\mathcal{H}}(x)] = 0.$

以ϕ和$\phi^\dagger$场作为基本元素构造$\tilde{\mathcal{H}}(x)$造成无法实现要求$[Q^a, \tilde{\mathcal{H}}(x)] = 0$的困难的
根本原因是ϕ场不像ϕ^+那样有确定的荷,因为它是由带$-q_a$荷的ϕ^+部分和
带$+q_a$荷的ϕ^-部分叠加而成的.

为了使ϕ场有确定的荷,将ϕ中的ϕ^-部分的粒子换为另外一种能使其带
有$-q_a$荷但具有与原来粒子相同质量的标量玻色粒子,$\phi(x) = \phi^+(x) + \phi^{+c\dagger}(x)$

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{-ip\cdot x} a(\vec{p}) \qquad \phi^{+c\dagger}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{ip\cdot x} a^{c\dagger}(\vec{p}) = (\phi^{+c}(x))^\dagger$$

$$[Q_a, \phi^+(x)]_- = -q_a\phi^+(x) \qquad\qquad [Q_a, (\phi^{+c}(x))^\dagger]_- = -q_a(\phi^{+c})^\dagger$$

$$[Q_a, a(\vec{p})]_- = -q_a a(\vec{p}) \qquad\qquad [Q_a, a^c(\vec{p})]_- = +q_a a^c(\vec{p})$$

$a(\vec{p})$和$a^c(\vec{p})$是各带相反荷但具有相同质量的两种不同标量玻色子湮灭算符.
上标^{c}用于指示电荷共轭,反映它是另一种具有同样质量但带相反荷的粒子.

$$[\phi(x), \phi^\dagger(y)] = [\phi^+(x), \phi^{+\dagger}(y)] - [\phi^{+c}(x), \phi^{+c\dagger}(y)] = \Delta(M, x-y) \overset{\text{类空区间}}{=\!=\!=} 0 \qquad [Q_a, \phi(x)] = -q_a\phi(x)$$

$$\Delta(M, x) = \Delta_+(M, x) - \Delta_+(M, -x) = \int \frac{d\vec{p}}{2p^0(2\pi)^3}[e^{-ip\cdot x} - e^{ip\cdot x}] \qquad \text{作业11}$$

反粒子的引入

$$\phi(x) = \phi^+(x) + \phi^{+c\dagger}(x) \quad [Q_a, \phi^+(x)]_- = -q_a\phi^+(x) \quad [Q_a, (\phi^{+c}(x))^\dagger]_- = -q_a(\phi^{+c}(x))^\dagger$$

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{-ip\cdot x} a(\vec{p}) \qquad \phi^{+c\dagger}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{ip\cdot x} a^{c\dagger}(\vec{p}) = (\phi^{+c}(x))^\dagger$$

$$[Q_a, a(\vec{p})]_- = -q_a a(\vec{p}) \qquad\qquad [Q_a, a^c(\vec{p})]_- = +q_a a^c(\vec{p})$$

$a(\vec{p})$和$a^c(\vec{p})$是各带相反荷但具有相同质量的两种不同标量玻色子湮灭算符.
上标^{c}用于指示电荷共轭,反映它是另一种具有同样质量但带相反荷的粒子.

$$[\phi(x), \phi^\dagger(y)] = \Delta(M, x-y) \stackrel{\text{类空区间}}{=\!=\!=} 0 \qquad [Q_a, \phi(x)] = -q_a\phi(x)$$

第一式保证用以ϕ和$\phi^\dagger$场构造$\tilde{\mathcal{H}}(x)$可以使其在类空区间对易因而保证因果性。这只有两种粒子的质量相同才能达到!

第二式保证用以ϕ和$\phi^\dagger$场来构造$\tilde{\mathcal{H}}(x)$可以实现要求$[Q^a, \tilde{\mathcal{H}}(x)] = 0$.

如果$q_a \neq 0$,则$a^c(\vec{p}) \neq a(\vec{p})$. 这时体系中具有两种带有相反荷但质量相同的玻色标量粒子. 我们将这种质量相同但荷相反的粒子叫反粒子.

如果$q_a = 0$,则可以选择$a^c(\vec{p}) = a(\vec{p})$,此时,粒子本身就是它自己的反粒子,这样的粒子不带荷,相应的场是自共轭场$\phi(x) = \phi^\dagger(x)$.

反粒子的引入

$$\phi(x) = \phi^+(x) + \phi^{+c\dagger}(x) \quad [Q_a, \phi^+(x)]_- = -q_a\phi^+(x) \quad [Q_a, (\phi^{+c}(x))^\dagger]_- = -q_a(\phi^{+c}(x))^\dagger$$

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{-ip\cdot x} a(\vec{p}) \qquad \phi^{+c\dagger}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} e^{ip\cdot x} a^{c\dagger}(\vec{p}) = (\phi^{+c}(x))^\dagger$$

$$[Q_a, a(\vec{p})]_- = -q_a a(\vec{p}) \qquad\qquad [Q_a, a^c(\vec{p})]_- = +q_a a^c(\vec{p})$$

$a(\vec{p})$和$a^c(\vec{p})$是各带相反荷但具有相同质量的两种不同标量玻色子湮灭算符.
上标^c 用于指示电荷共轭,反映它是另一种具有同样质量但带相反荷的粒子.

$$[\phi(x), \phi^\dagger(y)] = \Delta(M, x-y) \overset{\text{类空区间}}{=\!=\!=\!=} 0 \qquad [Q_a, \phi(x)] = -q_a\phi(x)$$

S矩阵的相对论不变性和内部对称性不变性要求在体系中存在反粒子!

理论将粒子湮灭和反粒子产生等权重地分配在场的定义中, 意味物理上把它们看成是"等价"的从荷的角度, **它意味着:**

▶ 产生粒子"等价"于消灭反粒子, 或产生反粒子"等价"于消灭粒子

▶ 粒子反粒子碰到一起将有发生 湮灭反应 的可能性!

这正是狄拉克的空穴理论,空穴现在被反粒子所取代.

反粒子的引入

关于反粒子的评注：

♣   我们生存在一个粒子的世界，所有反粒子都是不稳定的！

♢   可理解为所有反粒子都碰上粒子而湮灭掉了！ 需要相互作用

♡   但这要求现在世界中 粒子的数目远大于反粒子的数目！

♠   为什么会有这种 物质反物质的不对称性？

¶   它要求基本相互作用中有产生物质反物质不对称效应的项！

✠   C或CP破坏可望达到这个目的！

标量场的分立对称性变换性质

空间反射变换： $Pa^\dagger(\vec{p}, \sigma, n)P^{-1} = \eta a^\dagger(\mathcal{P}\vec{p}, \sigma, n)$　　$Pa(\vec{p}, \sigma, n)P^{-1} = \eta^* a(\mathcal{P}\vec{p}, \sigma, n)$

$Pa(\vec{p})P^{-1} = \eta^* a(-\vec{p})$　　　　$Pa^c(\vec{p})P^{-1} = \eta^{c*} a^c(-\vec{p})$

$P\phi^+(x)P^{-1} = \eta^* \phi^+(\mathcal{P}x)$　　　　$P\phi^{+c}(x)P^{-1} = \eta^{c*}\phi^{+c}(\mathcal{P}x)$

空间反射变换后的场$\phi_P = \eta^*\phi^+ + \eta^c\phi^{+c\dagger}$及其共轭$\phi_P^\dagger$来构造$\tilde{\mathcal{H}}(x)$已能够使其在类空区间相互对易，并可以实现$[Q^a, \tilde{\mathcal{H}}(x)] = 0$！我们进一步可以通过选择$a$与$a^\dagger$及$a^c$与$a^{c\dagger}$之间的相对相角，使得在保证$\phi = \phi^+ + \phi^{+c\dagger}$的同时，还对称地有 作业12

$$\eta^c = \eta^* \qquad \Rightarrow \qquad \eta\eta^c = 1$$

无自旋的粒子和反粒子的联合宇称位相为偶. 我们现在拥有统一的对ϕ场的空间反射变换：

$$P\phi(x)P^{-1} = \eta^*\phi(\mathcal{P}x)$$　　$\eta^* = 1$标量　$\eta^* = -1$赝标量

标量场的分立对称性变换性质

时间反演变换: $Ta^\dagger(\vec{p},\sigma,n)T^{-1}=\zeta(-1)^{j-\sigma}a^\dagger(\mathcal{P}\vec{p},-\sigma,n)$ $Ta(\vec{p},\sigma,n)T^{-1}=\zeta^*(-1)^{j-\sigma}a(\mathcal{P}\vec{p},-\sigma,n)$

$$Ta(\vec{p})T^{-1} = \zeta^* a(-\vec{p}) \qquad\qquad Ta^c(\vec{p})T^{-1} = \zeta^{c*} a^c(-\vec{p})$$

$$T\phi^+(x)T^{-1} = \zeta^* \phi^+(-\mathcal{P}x) \qquad\qquad T\phi^{+c}(x)T^{-1} = \zeta^{c*}\phi^{+c}(-\mathcal{P}x)$$

时间反演变换后的场 $\phi_T = \zeta^*\phi^+ + \zeta^c\phi^{+c\dagger}$ 及其共轭 $\phi_T^\dagger$ 来构造 $\tilde{\mathcal{H}}(x)$ 已能够使其在类空区间相互对易,并可以实现 $[Q^a,\tilde{\mathcal{H}}(x)]=0$! 我们进一步可以通过选择 a 与 $a^\dagger$, a^c 与 $a^{c\dagger}$ 及 a 与 a^c 之间的相对相角, 使得在保证 $\phi = \phi^+ + \phi^{+c\dagger}$ 和 $\phi_P = \eta^*(\phi^+ + \phi^{+c\dagger})$ 的同时, 还对称地有

$$\zeta^c = \zeta^* \qquad\qquad \Rightarrow \qquad\qquad \zeta\zeta^c = 1$$

无自旋的粒子和反粒子的联合时间宇称位相为偶. 我们现在拥有统一的对 ϕ 场的时间反演变换:

$$T\phi(x)T^{-1} = \zeta^*\phi(-\mathcal{P}x)$$

标量场的分立对称性变换性质

电荷共轭变换C及CPT联合变换:

$$Ca(\vec{p})C^{-1} = \xi^* a^c(\vec{p}) \qquad\qquad Ca^c(\vec{p})C^{-1} = \xi^{c*} a(\vec{p})$$

C 线性算符:

$$C\phi^+(x)C^{-1} = \xi^* \phi^{+c}(x) \qquad\qquad C\phi^{+c}(x)C^{-1} = \xi^{c*}\phi^+(x)$$

电荷共轭变换后的场 $\phi_C = \xi^*\phi^{c+} + \xi^c\phi^{+\dagger}$ 及其共轭 $\phi_C^\dagger$ 来构造 $\tilde{\mathcal{H}}(x)$ 已能够使其在类空区间相互对易,并可以实现 $[Q^a, \tilde{\mathcal{H}}(x)] = 0$!我们进一步可以**通过选择**$a$与$a^\dagger$,$a^c$与$a^{c\dagger}$及$a$与$a^c$之间的相对相角,使得在保证 $\phi = \phi^+ + \phi^{+c\dagger}$ 和 $\phi_P = \eta^*(\phi^+ + \phi^{+c\dagger})$ 的同时,还对称地有

$$\xi^c = \xi^* \qquad\Rightarrow\qquad \xi\xi^c = 1$$

无自旋的粒子和反粒子的联合电荷共轭宇称位相为偶. 我们现在拥有统一的对 ϕ 场的电荷共轭变换:

$$C\phi(x)C^{-1} = \xi^*\phi^\dagger(x) \qquad T\phi(x)T^{-1} = \zeta^*\phi(-\mathcal{P}x) \qquad P\phi(x)P^{-1} = \eta^*\phi(\mathcal{P}x)$$

对CPT联合变换: $\quad CPT\ \phi(x)\ [CPT]^{-1} = \xi^*\eta^*\zeta^*\phi^\dagger(-x)$

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{c\dagger}(\vec{p})]$$

$$p^\mu p_\mu = M^2 \quad \Rightarrow \quad (\partial^2 + M^2)\phi(x) = 0 \quad \text{**自由粒子场的Klein-Gordon方程!**}$$

$$\phi^*(x)(\partial^2 + M^2)\phi(x) - \phi(x)(\partial^2 + M^2)\phi^*(x) = 0$$

流矢量: $j_\mu(x) \equiv -iq\{[\partial_\mu\phi^*(x)]\phi(x) - \phi^*(x)[\partial_\mu\phi(x)]\}$ $\qquad \partial^\mu j_\mu(x) = 0$

荷: $\int d^3x\, j^0(x,t) = -iq\int d^3x\,\{[\partial^0\phi^*(x)]\phi(x) - \phi^*(x)\partial^0\phi(x)\} = q\underbrace{\int d^3p[a^\dagger(\vec{p})a(\vec{p}) - a^c(\vec{p})a^{c\dagger}(\vec{p})]}_{\text{对中性粒子为零}}$

$$[\int d^3x\, j^0(x,t), a(\vec{p})] = -qa(\vec{p}) \qquad\qquad [\int d^3x\, j^0(x,t), a^\dagger(\vec{p})] = qa^\dagger(\vec{p})$$

$$[\int d^3x\, j^0(x,t), a^c(\vec{p})] = qa^c(\vec{p}) \qquad\qquad [\int d^3x\, j^0(x,t), a^{c\dagger}(\vec{p})] = -qa^{c\dagger}(\vec{p})$$

$$Q(t) \equiv \int d^3x\, j^0(x,t) = Q^\dagger(t) \qquad [Q, \phi(x)] = -q\phi(x) \qquad [Q, \phi^\dagger(x)] = q\phi^\dagger(x)$$

$$\dot{Q}(t) = \int d^3x\, \nabla\cdot\vec{j}(x,t) = 0 \qquad CQ(t)C^{-1} = q\int d^3p[a^{c\dagger}(\vec{p})a^c(\vec{p}) - a(\vec{p})a^\dagger(\vec{p})] = -Q(t)$$

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{c\dagger}(\vec{p})]$$

$$p^\mu p_\mu = M^2 \quad \Rightarrow \quad (\partial^2 + M^2)\phi(x) = 0 \quad \text{自由粒子场的Klein-Gordon方程！}$$

自共轭自由标量场 $\phi = \phi^+ + \phi^{+\dagger}$ **的哈密顿量就是体系的总能量算符**

$$H'_0 = \int d\vec{p}\, a^\dagger(\vec{p})a(\vec{p})\sqrt{\vec{p}^2+M^2} = \int d\vec{p}\, \frac{1}{2}\sqrt{\vec{p}^2+M^2} : \{a^\dagger(\vec{p})a(\vec{p}) + a(\vec{p})a^\dagger(\vec{p})\} :$$

$: O :$ **代表对算符** O **的正规乘积，即将算符** O **中的湮灭算符排在右边.**
如果去掉正规乘积，将会导致真空的零点能。

验证哈密顿量算符确实是体系平移变换的生成元：

$$a(q)\Phi_{q_1\cdots q_N} = \sum_{r=1}^N \delta(q-q_r)\Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N} \qquad H_0\Phi_{q_1\cdots q_N} = (\sqrt{\vec{q_1}^2+M^2}+\cdots+\sqrt{\vec{q_N}^2+M^2})\Phi_{q_1\cdots q_N}$$

$$H'_0\Phi_{q_1\cdots q_N} = \int d\vec{p}\, a^\dagger(\vec{p})a(\vec{p})\sqrt{\vec{p}^2+M^2}\Phi_{q_1\cdots q_N} = \int d\vec{p}\, a^\dagger(\vec{p})\sum_{r=1}^N \delta(p-q_r)\Phi_{q_1\cdots q_{r-1}q_{r+1}\cdots q_N}\sqrt{\vec{p}^2+M^2}$$

$$= \sum_{r=1}^N \Phi_{q_1\cdots q_{r-1}q_r q_{r+1}\cdots q_N}\sqrt{\vec{q_r}^2+M^2} = H_0\Phi_{q_1\cdots q_N} \quad \Rightarrow \quad H'_0 = H_0 \quad \text{可类似地讨论其它洛伦兹群生成元}$$

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{c\dagger}(\vec{p})]$$

$$p^\mu p_\mu = M^2 \quad \Rightarrow \quad (\partial^2 + M^2)\phi(x) = 0 \quad \text{自由粒子场的Klein-Gordon方程！}$$

自共轭自由标量场 $\phi = \phi^+ + \phi^{+\dagger}$ **的哈密顿量就是体系的总能量算符**

$$H_0 = \int d\vec{p}\, a^\dagger(\vec{p})a(\vec{p})\sqrt{\vec{p}^2+M^2} = \int d\vec{p}\, \frac{1}{2}\sqrt{\vec{p}^2+M^2} : \{a^\dagger(\vec{p})a(\vec{p}) + a(\vec{p})a^\dagger(\vec{p})\} :$$

$: O :$ **代表对算符**O**的正规乘积，即将算符**O**中的湮灭算符排在右边.**
如果去掉正规乘积，将会导致真空的零点能。

哈密顿量的坐标空间表达： $\phi(x) = \int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^\dagger(\vec{p})]$

$$\dot{\phi}(x) = i\int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{p^0}{\sqrt{2p^0}}[-e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^\dagger(\vec{p})] \quad \nabla\phi(x) = i\int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{\vec{p}}{\sqrt{2p^0}}[e^{-ip\cdot x}a(\vec{p}) - e^{ip\cdot x}a^\dagger(\vec{p})]$$

$\dot{O}$**是对**O**的时间依赖求导数。自共轭自由标量场**作业14**哈密顿量还可表达为：**

$$H_0 = \int d\vec{x}\, \frac{1}{2} : [\dot{\phi}^2(x) + (\nabla\phi(x))^2 + M^2\phi^2(x)] :$$

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} [e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{c\dagger}(\vec{p})] \qquad (\partial^2 + M^2)\phi(x) = 0$$

哈密顿量的坐标空间表达： $\phi(x) = \int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} [e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{\dagger}(\vec{p})]$

$$\dot\phi(x) = i\int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{p^0}{\sqrt{2p^0}} [-e^{-ip\cdot x}a(\vec{p}) + e^{ip\cdot x}a^{\dagger}(\vec{p})] \qquad \nabla\phi(x) = i\int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{\vec{p}}{\sqrt{2p^0}} [e^{-ip\cdot x}a(\vec{p}) - e^{ip\cdot x}a^{\dagger}(\vec{p})]$$

$\dot{O}$ **是对** O **的时间依赖求导数。自共轭自由标量场**作业14**哈密顿量还可表达为：**

$$H_0 = \int d\vec{x} \frac{1}{2} : [\dot\phi^2(x) + (\nabla\phi(x))^2 + M^2\phi^2(x)] :$$

将 $\phi(x)$ **取为体系的广义坐标，** H_0 **在广义动量固定情形下对** $\phi(x)$ **的泛函微商定义了体系的广义动量对时间的导数的负值** $-\dot\pi(x)$

$$-\dot\pi(x) \equiv \frac{\delta H_0}{\delta\phi(x)}\bigg|_{\pi固定} = (-\nabla^2 + M^2)\phi(x) = -\ddot\phi(x) \qquad \text{略去边界积分，在微商过程中将} \phi \text{项固定}$$

$$\pi(x) = \dot\phi(x) \qquad S_0 = \int d^4x \ : \dot\phi^2(x) : - \int dt\, H_0 = \int d^4x \frac{1}{2} : [(\partial_\mu\phi(x))^2 - M^2\phi^2(x)] :$$

将作用量 S_0 **取极值就得到场方程！**

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} [e^{-ip\cdot x} a(\vec{p}) + e^{ip\cdot x} a^{c\dagger}(\vec{p})] \qquad (\partial^2 + M^2)\phi(x) = 0$$

哈密顿量的坐标空间表达： $\phi(x) = \int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} [e^{-ip\cdot x} a(\vec{p}) + e^{ip\cdot x} a^\dagger(\vec{p})]$

$$\dot{\phi}(x) = i\int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{p^0}{\sqrt{2p^0}} [-e^{-ip\cdot x} a(\vec{p}) + e^{ip\cdot x} a^\dagger(\vec{p})] \qquad \nabla\phi(x) = i\int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{\vec{p}}{\sqrt{2p^0}} [e^{-ip\cdot x} a(\vec{p}) - e^{ip\cdot x} a^\dagger(\vec{p})]$$

$\dot{O}$是对O的时间依赖求导数。自共轭自由标量场作业14哈密顿量还可表达为：

$$H_0 = \int d\vec{x}\, \frac{1}{2} : [\dot{\phi}^2(x) + (\nabla\phi(x))^2 + M^2\phi^2(x)] : \qquad S_0 = \int d^4x\, \frac{1}{2} : [(\partial_\mu\phi(x))^2 - M^2\phi^2(x)] :$$

等时对易关系：

$$[\phi(\vec{x},t), \pi(\vec{y},t)]_- = i\delta(\vec{x}-\vec{y}) \qquad\qquad \pi(x) = \dot{\phi}(x)$$

$$[\phi(\vec{x},t), \phi(\vec{y},t)]_- = [\pi(\vec{x},t), \pi(\vec{y},t)]_- = 0 \quad \text{作业13}$$

$$\dot{\phi}(\vec{x},t) = i[H_0, \phi(\vec{x},t)] = \frac{\delta H_0}{\delta\pi(\vec{x},t)} \qquad \dot{\pi}(\vec{x},t) = i[H_0, \pi(\vec{x},t)] = -\frac{\delta H_0}{\delta\phi(\vec{x},t)} \quad \text{作业15}$$

扣除作用量中的四度时空体积积分，我们就得到体系的拉格朗日量密度：

$$S_0 = \int d^4x\, \mathcal{L}_0 \qquad \mathcal{L}_0 = \frac{1}{2} : [(\partial_\mu\phi(x))^2 - M^2\phi^2(x)] : \qquad \pi(x) = \frac{\partial\mathcal{L}_0}{\partial\dot{\phi}(x)} = \dot{\phi}(x)$$

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

标量场结果集锦

▶ **场及场方程：**
$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} [e^{-ip\cdot x} a(\vec{p}) + e^{ip\cdot x} a^{c\dagger}(\vec{p})] \qquad (\partial^2 + M^2)\phi(x) = 0$$

▶ **哈密顿量、作用量与广义动量：**
$$H_0 = \int d\vec{x} \frac{1}{2} : [\dot{\phi}^2(x) + (\nabla\phi(x))^2 + M^2\phi^2(x)] :$$

$$S_0 = \int d^4x \, \mathcal{L}_0 \qquad \mathcal{L}_0 = \frac{1}{2} : [(\partial_\mu\phi(x))^2 - M^2\phi^2(x)] : \qquad \pi(x) = \frac{\partial\mathcal{L}_0}{\partial\dot{\phi}(x)} = \dot{\phi}(x)$$

▶ **等时对易关系：** $[\phi(\vec{x},t), \pi(\vec{y},t)]_- = i\delta(\vec{x}-\vec{y})$ $\qquad\qquad \pi(x) = \dot{\phi}(x)$
$$[\phi(\vec{x},t), \phi(\vec{y},t)]_- = [\pi(\vec{x},t), \pi(\vec{y},t)]_- = 0$$

▶ **正则场方程：**
$$\dot{\phi}(\vec{x},t) = i[H_0, \phi(\vec{x},t)] = \frac{\delta H_0}{\delta\pi(\vec{x},t)} \qquad \dot{\pi}(\vec{x},t) = i[H_0, \pi(\vec{x},t)] = -\frac{\delta H_0}{\delta\phi(\vec{x},t)}$$

零质量标量场的结果可在有质量的标量场结果中将质量趋于零得到！

自由标量场的场方程、荷流矢量、哈密顿量和作用量、正则对易关系

标量场的地位:

♣ **是各种量子场中最简单、平庸的场!**

♢ **总被拿来作为例子或玩具模型首先研究!**

♡ **是公理化场论、构造性场论的研究对象!**

♠ **但在现实世界 没有发现 其对应的基本粒子! Higgs?**

¶ **是否它太简单了?**

♰ **发现了很多它不好的性质! 见第四章关于平庸性和不自然性的讨论**

γ矩阵

Clifford 代数与洛伦兹变换生成元

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} \qquad \mathcal{J}^{\mu\nu} \equiv -\frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}] = -\mathcal{J}^{\nu\mu}$$

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = -\frac{i}{16}[\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma} - \gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}\gamma^{\nu} - \mu \Leftrightarrow \nu - \rho \Leftrightarrow \sigma]$$

$$= -\frac{i}{16}[2g^{\nu\rho}\gamma^{\mu}\gamma^{\sigma} - \gamma^{\mu}\gamma^{\rho}\gamma^{\nu}\gamma^{\sigma} - \gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}\gamma^{\nu} - \mu \Leftrightarrow \nu - \rho \Leftrightarrow \sigma]$$

$$= -\frac{i}{16}[2g^{\nu\rho}\gamma^{\mu}\gamma^{\sigma} - 2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + \gamma^{\rho}\gamma^{\mu}\gamma^{\nu}\gamma^{\sigma} - \gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}\gamma^{\nu} - \mu \Leftrightarrow \nu - \rho \Leftrightarrow \sigma]$$

$$= -\frac{i}{16}[2g^{\nu\rho}\gamma^{\mu}\gamma^{\sigma} - 2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + 2\gamma^{\rho}\gamma^{\mu}g^{\nu\sigma} - \gamma^{\rho}\gamma^{\mu}\gamma^{\sigma}\gamma^{\nu} - \gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}\gamma^{\nu} - \mu \Leftrightarrow \nu - \rho \Leftrightarrow \sigma]$$

$$= -\frac{i}{16}[2g^{\nu\rho}\gamma^{\mu}\gamma^{\sigma} - 2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + 2\gamma^{\rho}\gamma^{\mu}g^{\nu\sigma} - 2g^{\mu\sigma}\gamma^{\rho}\gamma^{\nu}$$
$$-2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + 2g^{\nu\rho}\gamma^{\mu}\gamma^{\sigma} - 2\gamma^{\rho}\gamma^{\nu}g^{\mu\sigma} + 2g^{\nu\sigma}\gamma^{\rho}\gamma^{\mu}$$
$$-2g^{\nu\sigma}\gamma^{\mu}\gamma^{\rho} + 2g^{\mu\sigma}\gamma^{\nu}\gamma^{\rho} - 2\gamma^{\rho}\gamma^{\mu}g^{\nu\rho} + 2g^{\mu\rho}\gamma^{\sigma}\gamma^{\nu}$$
$$+2g^{\mu\sigma}\gamma^{\nu}\gamma^{\rho} - 2g^{\nu\sigma}\gamma^{\mu}\gamma^{\rho} + 2\gamma^{\sigma}\gamma^{\nu}g^{\mu\rho} - 2g^{\nu\rho}\gamma^{\sigma}\gamma^{\mu}]$$

$$= \qquad g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

γ矩阵

Clifford 代数的洛伦兹变换

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \mathcal{J}^{\mu\nu} \equiv -\frac{i}{4}[\gamma^\mu, \gamma^\nu] = -\mathcal{J}^{\nu\mu}$$

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

$$\Lambda = 1 + \omega \qquad D(\Lambda) = 1 + \frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu}$$

$$[\mathcal{J}^{\mu\nu}, \gamma^\rho] = -\frac{i}{4}[\gamma^\mu\gamma^\nu\gamma^\rho - \gamma^\rho\gamma^\mu\gamma^\nu - \mu \Leftrightarrow \nu] = -\frac{i}{4}[2\gamma^\mu g^{\nu\rho} - \gamma^\mu\gamma^\rho\gamma^\nu - \gamma^\rho\gamma^\mu\gamma^\nu - \mu \Leftrightarrow \nu]$$

$$= -\frac{i}{4}[2\gamma^\mu g^{\nu\rho} - 2g^{\mu\rho}\gamma^\nu - \mu \Leftrightarrow \nu] = -i(\gamma^\mu g^{\nu\rho} - \gamma^\nu g^{\mu\rho})$$

$$D(\Lambda)\gamma^\rho D^{-1}(\Lambda) = \gamma^\rho + \frac{i}{2}\omega_{\mu\nu}[\mathcal{J}^{\mu\nu}, \gamma^\rho] = \gamma^\rho + \frac{1}{2}\omega_{\mu\nu}(\gamma^\mu g^{\nu\rho} - \gamma^\nu g^{\mu\rho}) = \gamma^\rho + \omega_\mu{}^\rho\gamma^\mu$$

$$= (g_\mu{}^\rho + \omega_\mu{}^\rho)\gamma^\mu = \Lambda_\mu{}^\rho\gamma^\mu$$

$$D(\Lambda)1D^{-1}(\Lambda) = 1 \qquad D(\Lambda)\mathcal{J}^{\rho\sigma}D^{-1}(\Lambda) = \Lambda_\mu{}^\rho\Lambda_\nu{}^\sigma\mathcal{J}^{\mu\nu}$$

γ矩阵

Clifford 代数的完备性

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad\qquad \mathcal{J}^{\mu\nu} \equiv -\frac{i}{4}[\gamma^\mu, \gamma^\nu] = -\mathcal{J}^{\nu\mu}$$

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

任何多于4个的γ^μ的乘积一定可用4个和4个以下的γ^μ的乘积表达

- **多于4个的γ^μ的乘积至少有一对指标重复**
- **将重复指标的这对γ^μ可以去掉，相差±1**
- **剩下的γ^μ的乘积如仍多于4个，重复上面过程**
- **直到剩下的γ^μ的乘积等于或少于4个**

16个独立的γ^μ

1   γ^μ $\mathcal{J}^{\mu\nu}$ $\gamma^\mu\gamma_5$ $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ $\gamma_5^2 = 1$ $\{\gamma_5, \gamma^\mu\} = 0$

若用矩阵构造γ^μ,16个独立自由度意味至少需要在$\sqrt{16} = 4$维空间才能实现！

γ矩阵

Clifford 代数的在Minkovski空间的非厄米性

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \mathcal{J}^{\mu\nu} \equiv -\frac{i}{4}[\gamma^\mu, \gamma^\nu] = -\mathcal{J}^{\nu\mu}$$

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

$$D(\Lambda)\gamma^\rho D^{-1}(\Lambda) = \Lambda_\mu{}^\rho \gamma^\mu \qquad D(\Lambda)1D^{-1}(\Lambda) = 1 \qquad D(\Lambda)\mathcal{J}^{\rho\sigma}D^{-1}(\Lambda) = \Lambda_\mu{}^\rho \Lambda_\nu{}^\sigma \mathcal{J}^{\mu\nu}$$

16个基: $1 \qquad \gamma^\mu \qquad \mathcal{J}^{\mu\nu} \qquad \gamma^\mu\gamma_5 \qquad \gamma_5$

非厄米性: $\gamma^\mu|a^\mu\rangle = a^\mu|a^\mu\rangle$ 不求和 $\Rightarrow 2g^{\mu\mu}|a^\mu\rangle = \{\gamma^\mu, \gamma^\mu\}|a^\mu\rangle = 2a^\mu a^\mu |a^\mu\rangle$

注: γ^μ无共同本征态 $\qquad (a^0)^2 = 1 \qquad (a^1)^2 = (a^2)^2 = (a^3)^2 = -1$

- ▶ 若取$\gamma^\mu = \gamma^{\mu\dagger} \Rightarrow \gamma^\mu$的本征值是实的$\Rightarrow (a^0)^2 = (a^1)^2 = (a^2)^2 = (a^3)^2 = 1$
- ▶ 若取$\gamma^\mu = -\gamma^{\mu\dagger} \Rightarrow \gamma^\mu$的本征值是虚的$\Rightarrow (a^0)^2 = (a^1)^2 = (a^2)^2 = (a^3)^2 = -1$
- ▶ 只有取$\gamma^0 = \gamma^{0\dagger} \qquad \gamma^i = -\gamma^{i\dagger}$
- ▶ 它导致$\mathcal{J}^{\sigma\rho}$不全厄米，在场空间生成的 表示不幺正！ 但不是在态空间
- ▶ 但$\mathcal{J}^{ij}$厄米，在场空间由 纯转动 生成的表示仍幺正！

γ矩阵

Clifford 代数与时空转动的表示矩阵

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \mathcal{J}^{\mu\nu} \equiv -\frac{i}{4}[\gamma^\mu, \gamma^\nu] = -\mathcal{J}^{\nu\mu}$$

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

$$D(\Lambda)\gamma^\rho D^{-1}(\Lambda) = \Lambda_\mu{}^\rho \gamma^\mu \qquad D(\Lambda)1D^{-1}(\Lambda) = 1 \qquad D(\Lambda)\mathcal{J}^{\rho\sigma}D^{-1}(\Lambda) = \Lambda_\mu{}^\rho \Lambda_\nu{}^\sigma \mathcal{J}^{\mu\nu}$$

16个基: $\quad 1 \qquad \gamma^\mu \qquad \mathcal{J}^{\mu\nu} \qquad \gamma^\mu \gamma_5 \qquad \gamma_5$

厄米性的自洽取法: $\quad \gamma^0 = \gamma^{0\dagger} \quad \gamma^i = -\gamma^{i\dagger}$

角动量的平方: $\quad j = \frac{1}{2} \qquad \mathcal{J}^i = -\frac{1}{2}\epsilon_{ijk}\mathcal{J}^{jk} = \frac{i}{8}\epsilon_{ijk}[\gamma^j, \gamma^k]$

$$\mathcal{J}^i \mathcal{J}^i = \frac{-1}{64}\epsilon_{ijk}\epsilon_{ij'k'}[\gamma^j, \gamma^k][\gamma^{j'}, \gamma^{k'}] = \frac{-1}{32}[\gamma^j, \gamma^k][\gamma^j, \gamma^k]$$

$$= \frac{-1}{32}[\gamma^j\gamma^k\gamma^j\gamma^k - \gamma^j\gamma^k\gamma^k\gamma^j - \gamma^k\gamma^j\gamma^j\gamma^k + \gamma^k\gamma^j\gamma^k\gamma^j] = \frac{-1}{16}[\gamma^j\gamma^k\gamma^j\gamma^k - \gamma^j\gamma^k\gamma^k\gamma^j]$$

$$= \frac{-1}{16}[\gamma^j(2g^{kj} - \gamma^j\gamma^k)\gamma^k - 9] = \frac{-1}{16}[6 - 9 - 9] = \frac{3}{4} = \frac{1}{2}\left(\frac{1}{2} + 1\right)$$

γ矩阵

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3 p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p}, \sigma, n) a(\vec{p}, \sigma, n) \quad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3 p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p}, \sigma, n) a^\dagger(\vec{p}, \sigma, n)$$

$$u(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) u(0, \sigma) \qquad v(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) v(0, \sigma)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(0, \bar{\sigma}, n) \vec{J}_{\bar{\sigma}\sigma}^{(j_n)} = \sum_l \vec{\mathcal{J}}_{\bar{l}l} u_l(0, \sigma, n) \qquad [J^i, J^j] = i\epsilon_{ijk} J^k \qquad J^i = -\frac{1}{2}\epsilon_{ijk} J^{jk}$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(0, \bar{\sigma}, n) \vec{J}_{\bar{\sigma}\sigma}^{(j_n)*} = -\sum_l \vec{\mathcal{J}}_{\bar{l}l} v_l(0, \sigma, n) \qquad [\mathcal{J}^i, \mathcal{J}^j] = i\epsilon_{ijk} \mathcal{J}^k \qquad \mathcal{J}^i = -\frac{1}{2}\epsilon_{ijk} \mathcal{J}^{jk}$$

旋量量子场: $\vec{\mathcal{J}} = \frac{1}{2}\vec{\sigma}I \qquad [\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k \qquad \mathcal{J}^2 = \frac{1}{4}\vec{\sigma}\cdot\vec{\sigma}I = \frac{3}{4}I = \frac{1}{2}(\frac{1}{2}+1)I$

只考虑有质量的旋量场： 无质量可以看作质量趋于零的极限

- ► 非平庸的最低表示是 $\vec{J}^{(1/2)} = \frac{1}{2}\vec{\sigma}$ ⇒ **Weyl旋量表示**

- ► $[\sigma_2\sigma^i\sigma_2, \sigma_2\sigma^j\sigma_2] = 2i\epsilon_{ijk}\sigma_2\sigma^k\sigma_2,$
  $\sigma_2\sigma^i\sigma_2 = -\sigma^{i*} \quad \vec{J}^{(1/2)*} = \frac{1}{2}\vec{\sigma}^* = -\frac{1}{2}\sigma_2\vec{\sigma}\sigma_2$
  ⇒**有两个互为共轭的Weyl旋量表示**

- ► **γ矩阵最低4维，考虑两个旋量表示的直和**dim$I = 2$： **Dirac旋量表示**

目录 多粒子态 进态、出态与S矩阵 量子场 标量量子场 **旋量量子场** 矢量量子场 任意自旋量子场 绘景与CPT
○○ ○○○○ ○○○○○○○ ○○○○ ○○○○○○○●○○○○○ ○○○○○○○○○○○○ ○○○○○
○○○○ ○○○○○○○○○ ○○○○ ○○○○ ○○○○○○○ ○○○○
○○○○○ ○ ○○○○ ○○○○ ○○○○○○○○○○○
○○○○○○ ○ ○○○○ ○○○○○○○○○
○○○○○○○

γ矩阵

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0} D(L(p)) u(0,\sigma) \qquad v(\vec{p},\sigma) = \sqrt{M/p^0} D(L(p)) v(0,\sigma)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(0,\bar{\sigma},n) \vec{J}_{\bar{\sigma}\sigma}^{(j_n)} = \sum_l \vec{\mathcal{J}}_{\bar{l}l} u_l(0,\sigma,n) \qquad [J^i, J^j] = i\epsilon_{ijk} J^k$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(0,\bar{\sigma},n) \vec{J}_{\bar{\sigma}\sigma}^{(j_n)*} = -\sum_l \vec{\mathcal{J}}_{\bar{l}l} v_l(0,\sigma,n) \qquad [\mathcal{J}^i, \mathcal{J}^j] = i\epsilon_{ijk} \mathcal{J}^k$$

旋量量子场: $\quad \vec{\mathcal{J}} = \frac{1}{2}\vec{\sigma}I \qquad \vec{J}^{(1/2)} = \frac{1}{2}\vec{\sigma} \qquad \vec{J}^{(1/2)*} = -\frac{1}{2}\sigma_2\vec{\sigma}\sigma_2$

$$\psi_l^+(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p}\, u_l(\vec{p},\sigma) e^{-ip\cdot x} a(\vec{p},\sigma) \qquad \psi_l^{-c}(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p}\, v_l(\vec{p},\sigma) e^{ip\cdot x} a^{c\dagger}(\vec{p},\sigma)$$

$$\sum_{\bar{\sigma}} u_{\bar{m}\bar{a}}(0,\bar{\sigma}) \frac{1}{2}\vec{\sigma}_{\bar{\sigma}\sigma} = \sum_{m,a} \underbrace{\vec{\sigma}_{\bar{m}m} I_{\bar{a}a}}_{\mathcal{J}_{\bar{l}l}} u_{ma}(0,\sigma) \qquad \sum_{\bar{\sigma},\bar{a}} v_{\bar{m}\bar{a}}(0,\bar{\sigma}) \frac{1}{2}(\sigma_2\vec{\sigma}\sigma_2)_{\bar{\sigma}\sigma} = \sum_{m,a} \underbrace{\vec{\sigma}_{\bar{m}m} I_{\bar{a}a}}_{\mathcal{J}_{\bar{l}l}} v_{ma}(0,\sigma)$$

$$\sum_{\bar{\sigma}} u_{\bar{m}\pm}(0,\bar{\sigma}) \frac{1}{2}\vec{\sigma}_{\bar{\sigma}\sigma} = \sum_m \frac{1}{2}\vec{\sigma}_{\bar{m}m} u_{m\pm}(0,\sigma) \qquad \sum_{\bar{\sigma}} v_{\bar{m}\pm}(0,\bar{\sigma}) \frac{1}{2}(\sigma_2\vec{\sigma}\sigma_2)_{\bar{\sigma}\sigma} = \sum_m \frac{1}{2}\vec{\sigma}_{\bar{m}m} v_{m\pm}(0,\sigma)$$

$$u_{m\pm}(0,\sigma) = c_\pm \delta_{m\sigma} \qquad v_{m\pm}(0,\sigma) = -id_\pm(\sigma_2)_{m\sigma} \qquad \pm\ \text{代表}I\text{空间的分量} \quad -i\ \text{消去}\sigma_2\text{中的}i$$

γ矩阵

$$\vec{J}^{(1/2)} = \frac{1}{2}\vec{\sigma} \qquad \vec{J}^{(1/2)*} = -\frac{1}{2}\sigma_2\vec{\sigma}\sigma_2 \qquad \vec{\mathcal{J}} = \frac{1}{2}\vec{\sigma}I$$

旋量量子场:

$$\psi_l^+(x) = (2\pi)^{-3/2}\sum_\sigma\int d\vec{p}\, u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) \qquad \psi_l^{-c}(x) = (2\pi)^{-3/2}\sum_\sigma\int d\vec{p}\, v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))u(0,\sigma) \qquad v(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))v(0,\sigma)$$

$$\sum_{\bar\sigma}u_{\bar l}(0,\bar\sigma,n)\vec{J}_{\bar\sigma\sigma}^{(j_n)} = \sum_l\vec{\mathcal{J}}_{\bar l l}u_l(0,\sigma,n) \qquad \sum_{\bar\sigma}v_{\bar l}(0,\bar\sigma,n)\vec{J}_{\bar\sigma\sigma}^{(j_n)*} = -\sum_l\vec{\mathcal{J}}_{\bar l l}v_l(0,\sigma,n)$$

$$\sum_{\bar\sigma}u_{\bar m\pm}(0,\bar\sigma)\frac{1}{2}\vec{\sigma}_{\bar\sigma\sigma} = \sum_m\frac{1}{2}\vec{\sigma}_{\bar m m}u_{m\pm}(0,\sigma) \qquad \sum_{\bar\sigma}v_{\bar m\pm}(0,\bar\sigma)\frac{1}{2}(\sigma_2\vec{\sigma}\sigma_2)_{\bar\sigma\sigma} = \sum_m\frac{1}{2}\vec{\sigma}_{\bar m m}v_{m\pm}(0,\sigma)$$

$$u_{m\pm}(0,\sigma) = c_\pm\delta_{m\sigma} \qquad v_{m\pm}(0,\sigma) = -id_\pm(\sigma_2)_{m\sigma} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \mathcal{J}^i = -\frac{1}{2}\epsilon_{ijk}\mathcal{J}^{jk} = \begin{pmatrix} \frac{\sigma^i}{2} & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix}$$

原角标 l 现用 $m = \pm 1/2, \pm$;四个分量代表: 选 c_+, d_+ 或 c_-, d_- 为零 $\Rightarrow$ **Weyl旋量**

$$\begin{bmatrix} m=\frac{1}{2},+ \\ m=-\frac{1}{2},+ \\ m=\frac{1}{2},- \\ m=-\frac{1}{2},- \end{bmatrix} \qquad u(0,\frac{1}{2}) = \begin{bmatrix} c_+ \\ 0 \\ c_- \\ 0 \end{bmatrix} \qquad u(0,-\frac{1}{2}) = \begin{bmatrix} 0 \\ c_+ \\ 0 \\ c_- \end{bmatrix} \qquad v(0,\frac{1}{2}) = \begin{bmatrix} 0 \\ d_+ \\ 0 \\ d_- \end{bmatrix} \qquad v(0,-\frac{1}{2}) = -\begin{bmatrix} d_+ \\ 0 \\ d_- \\ 0 \end{bmatrix}$$

γ矩阵

$$\text{旋量量子场:}\quad \mathcal{J}^i = -\frac{1}{2}\epsilon_{ijk}\mathcal{J}^{jk} = \frac{i}{8}\epsilon_{ijk}[\gamma^j, \gamma^k] = \begin{pmatrix} \frac{\sigma^i}{2} & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix}$$

$$\psi_l^+(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\, u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) \quad \psi_l^{-c}(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\, v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))u(0,\sigma) \qquad v(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))v(0,\sigma)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(0,\bar{\sigma},n)\mathcal{J}_{\bar{\sigma}\sigma}^{(j_n)} = \sum_l \vec{\mathcal{J}}_{\bar{l}l}u_l(0,\sigma,n) \quad \sum_{\bar{\sigma}} v_{\bar{l}}(0,\bar{\sigma},n)\mathcal{J}_{\bar{\sigma}\sigma}^{(j_n)*} = -\sum_l \vec{\mathcal{J}}_{\bar{l}l}v_l(0,\sigma,n)$$

$$\begin{bmatrix} m=\frac{1}{2},+ \\ m=-\frac{1}{2},+ \\ m=\frac{1}{2},- \\ m=-\frac{1}{2},- \end{bmatrix} \quad u(0,\tfrac{1}{2}) = \begin{bmatrix} c_+ \\ 0 \\ c_- \\ 0 \end{bmatrix} \quad u(0,-\tfrac{1}{2}) = \begin{bmatrix} 0 \\ c_+ \\ 0 \\ c_- \end{bmatrix} \quad v(0,\tfrac{1}{2}) = \begin{bmatrix} 0 \\ d_+ \\ 0 \\ d_- \end{bmatrix} \quad v(0,-\tfrac{1}{2}) = -\begin{bmatrix} d_+ \\ 0 \\ d_- \\ 0 \end{bmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \text{Itzykson书中的手征表象}$$

$$[\gamma^j,\gamma^k] = -2i\epsilon_{ijk}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} \quad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^j\gamma^k = \begin{pmatrix} -\sigma^j\sigma^k & 0 \\ 0 & -\sigma^j\sigma^k \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

γ矩阵

旋量量子场: $D(\Lambda) = e^{\frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu}}$, $\Lambda = e^{\omega}$, $\mathcal{J}^{\mu\nu} = -\frac{i}{4}[\gamma^\mu, \gamma^\nu]$, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$$\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad u\left(0, \frac{1}{2}\right) = \begin{bmatrix} c_+ \\ 0 \\ c_- \\ 0 \end{bmatrix} \quad u\left(0, -\frac{1}{2}\right) = \begin{bmatrix} 0 \\ c_+ \\ 0 \\ c_- \end{bmatrix} \quad v\left(0, \frac{1}{2}\right) = \begin{bmatrix} 0 \\ d_+ \\ 0 \\ d_- \end{bmatrix} \quad v\left(0, -\frac{1}{2}\right) = -\begin{bmatrix} d_+ \\ 0 \\ d_- \\ 0 \end{bmatrix}$$

$$\psi_l^+(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p}\, u_l(\vec{p}, \sigma) e^{-ip\cdot x} a(\vec{p}, \sigma) \quad \psi_l^{-c}(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p}\, v_l(\vec{p}, \sigma) e^{ip\cdot x} a^c{}^\dagger(\vec{p}, \sigma)$$

$$u(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) u(0, \sigma) \qquad v(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) v(0, \sigma)$$

$\beta = \beta^{-1} = \beta^\dagger = \gamma^0$ γ^0厄米; γ^i反厄米 $\beta\gamma^i\beta^{-1} = -\gamma^i$ $\beta\gamma^0\beta^{-1} = \gamma^0$ $\beta\gamma^{\mu\dagger}\beta = \gamma^\mu$

$\beta\mathcal{J}^{ij}\beta^{-1} = \mathcal{J}^{ij}$ $\beta\mathcal{J}^{i0}\beta^{-1} = -\mathcal{J}^{i0}$ $\Rightarrow$ $\beta\mathcal{J}^{\rho\sigma\dagger}\beta = \mathcal{J}^{\rho\sigma}$

$P\vec{J}P^{-1} = \vec{J}$ $P\vec{K}P^{-1} = -\vec{K}$ $L(p) \equiv e^{\omega}$ $D(L(-\vec{p})) = D(\mathcal{P}L(\vec{p})\mathcal{P}^{-1}) = D(e^{\mathcal{P}\omega\mathcal{P}^{-1}}) = \beta D(L(\vec{p}))\beta$

$$u(-\vec{p}, \sigma) = \sqrt{m/p^0} \beta D(L(\vec{p})) \beta u(0, \sigma) \qquad v(-\vec{p}, \sigma) = \sqrt{m/p^0} \beta D(L(\vec{p})) \beta v(0, \sigma)$$

$\psi^\pm(x) \propto \psi^\pm(\mathcal{P}x)$ $\Rightarrow$ $\beta u(0, \sigma) = b_u u(0, \sigma)$ $\beta v(0, \sigma) = b_v v(0, \sigma)$ $\Rightarrow b_u^2 = b_v^2 = 1$

空间反射对称性要求Dirac旋量: $c_- = -b_u c_+$, $d_- = -b_v d_+$

γ矩阵

旋量量子场: $D(\Lambda) = e^{\frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu}}$, $\Lambda = e^{\omega}$, $\mathcal{J}^{\mu\nu} = -\frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$, $\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$

$$\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad u(0,\tfrac{1}{2}) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-b_u}{\sqrt{2}} \\ 0 \end{bmatrix} \quad u(0,-\tfrac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-b_u}{\sqrt{2}} \end{bmatrix} \quad v(0,\tfrac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-b_v}{\sqrt{2}} \end{bmatrix} \quad v(0,-\tfrac{1}{2}) = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ 0 \\ \frac{b_v}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\psi_l^+(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p}\, u_l(\vec{p},\sigma) e^{-ip\cdot x} a(\vec{p},\sigma) \qquad \psi_l^{-c}(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p}\, v_l(\vec{p},\sigma) e^{ip\cdot x} a^{c\dagger}(\vec{p},\sigma)$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0} D(L(p)) u(0,\sigma) \qquad v(\vec{p},\sigma) = \sqrt{M/p^0} D(L(p)) v(0,\sigma)$$

$$\beta = \beta^{-1} = \beta^{\dagger} = \gamma^0 \quad \gamma^0\text{厄米}; \; \gamma^i\text{反厄米} \quad \beta\gamma^i\beta^{-1} = -\gamma^i \quad \beta\gamma^0\beta^{-1} = \gamma^0 \quad \beta\gamma^{\mu\dagger}\beta = \gamma^{\mu}$$

$$\beta\mathcal{J}^{ij}\beta^{-1} = \mathcal{J}^{ij} \qquad \beta\mathcal{J}^{i0}\beta^{-1} = -\mathcal{J}^{i0} \qquad \Rightarrow \quad \beta\mathcal{J}^{\rho\sigma\dagger}\beta = \mathcal{J}^{\rho\sigma}$$

$$P\vec{J}P^{-1} = \vec{J} \quad P\vec{K}P^{-1} = -\vec{K} \quad L(p) \equiv e^{\omega} \quad D(L(-\vec{p})) = D(\mathcal{P}L(\vec{p})\mathcal{P}^{-1}) = D(e^{\mathcal{P}\omega\mathcal{P}^{-1}}) = \beta D(L(\vec{p}))\beta$$

$$u(-\vec{p},\sigma) = \sqrt{m/p^0} \beta D(L(p)) \beta u(0,\sigma) \qquad v(-\vec{p},\sigma) = \sqrt{m/p^0} \beta D(L(p)) \beta v(0,\sigma)$$

$$\psi^{\pm}(x) \propto \psi^{\pm}(\mathcal{P}x) \Rightarrow \beta u(0,\sigma) = b_u u(0,\sigma) \qquad \beta v(0,\sigma) = b_v v(0,\sigma) \Rightarrow b_u^2 = b_v^2 = 1$$

<u>**空间反射对称性要求Dirac旋量**</u>: $\quad c_- = -b_u c_+, \quad d_- = -b_v d_+$

费米统计

$$\psi_l^+(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\, u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) \quad \psi_l^{-c}(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\, v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)$$

旋量量子场: $D(\Lambda) = e^{\frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu}}$, $\Lambda = e^\omega$, $\mathcal{J}^{\mu\nu} = -\frac{i}{4}[\gamma^\mu,\gamma^\nu]$, $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$

$$\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad u(0,\tfrac{1}{2}) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-b_u}{\sqrt{2}} \\ 0 \end{bmatrix} \quad u(0,-\tfrac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-b_u}{\sqrt{2}} \end{bmatrix} \quad v(0,\tfrac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-b_v}{\sqrt{2}} \end{bmatrix} \quad v(0,-\tfrac{1}{2}) = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ 0 \\ \frac{b_v}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))u(0,\sigma) \quad v(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))v(0,\sigma) \quad \beta\gamma^{\mu\dagger}\beta = \gamma^\mu \quad \beta\mathcal{J}^{\rho\sigma\dagger}\beta = \mathcal{J}^{\rho\sigma}$$

将 $\psi_l^+(x)$ 和 $\psi_l^{-c}(x)$ 进行线性组合来构造 $\tilde{\mathcal{H}}(x)$ 以保证其在类空区间的对易性

$$\psi(x) = \kappa\psi^+(x) + \lambda\psi^{-c}(x) \quad [\psi_l(x),\psi_l^\dagger(y)]_\mp = \int d\vec{p}\,[|\kappa|^2 N_{l\bar{l}}(\vec{p})e^{-ip\cdot(x-y)} \mp |\lambda|^2 M_{l\bar{l}}(\vec{p})e^{ip\cdot(x-y)}]$$

$$N_{l\bar{l}}(\vec{p}) \equiv \sum_\sigma u_l(\vec{p},\sigma)u_l^*(\vec{p},\sigma) \quad M_{l\bar{l}}(\vec{p}) \equiv \sum_\sigma v_l(\vec{p},\sigma)v_l^*(\vec{p},\sigma) \quad N(0) = \frac{1}{2}(1+b_u\beta) \quad M(0) = \frac{1}{2}(1+b_v\beta)$$

$$N(\vec{p}) = M/(2p^0)D(L(p))[1+b_u\beta]D^\dagger(L(p)) \quad M(\vec{p}) = M/(2p^0)D(L(p))[1+b_v\beta]D^\dagger(L(p))$$

费米统计

$$\psi_l^+(x) = (2\pi)^{-3/2}\sum\int d\vec{p}\; u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) \qquad \psi_l^{-c}(x) = (2\pi)^{-3/2}\sum\int d\vec{p}\; v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)$$

将$\psi_l^+(x)$和$\psi_l^{\sigma\,c}(x)$进行线性组合来构造$\tilde{\mathcal{H}}(x)$以保证其在类空区间的对易性

$$\psi(x)=\kappa\psi^+(x)+\lambda\psi^{-c}(x) \qquad [\psi_l(x),\psi_{\bar l}^\dagger(y)]_{\mp}=\int d\vec{p}\,[|\kappa|^2 N_{l\bar l}(\vec{p})e^{-ip\cdot(x-y)}\mp|\lambda|^2 M_{l\bar l}(\vec{p})e^{ip\cdot(x-y)}]$$

$$N_{l\bar l}(\vec{p})\equiv\sum_\sigma u_l(\vec{p},\sigma)u_{\bar l}^*(\vec{p},\sigma) \qquad M_{l\bar l}(\vec{p})\equiv\sum_\sigma v_l(\vec{p},\sigma)v_{\bar l}^*(\vec{p},\sigma) \qquad N(0)=\frac{1}{2}(1+b_u\beta) \qquad M(0)=\frac{1}{2}(1+b_v\beta)$$

$$N(\vec{p})=M/(2p^0)D(L(p))[1+b_u\beta]D^\dagger(L(p)) \qquad\qquad M(\vec{p})=M/(2p^0)D(L(p))[1+b_v\beta]D^\dagger(L(p))$$

$$D(\Lambda)\gamma^\rho D^{-1}(\Lambda)=\Lambda_\sigma^{\ \rho}\gamma^\sigma \qquad [e^{\frac{i}{2}\mathcal{J}^{\mu\nu\dagger}\omega_{\mu\nu}}]=[e^{\frac{i}{2}\mathcal{J}^{\mu\nu}\omega_{\mu\nu}}]^{-1,\dagger} \quad\Rightarrow\quad \beta D(L(p))\beta=D^{\dagger-1}(L(p))$$

$$D(L(p))\beta D^{-1}(L(p))=L_\mu^0(p)\gamma^\mu=p_\mu\gamma^\mu/M \qquad D(L(p))D^\dagger(L(p))=D(L(p))\beta D^{-1}(L(p))\beta=p_\mu\gamma^\mu\beta/M$$

$$N(\vec{p})=\frac{1}{2p^0}[p^\mu\gamma_\mu+b_u M]\beta \qquad\qquad M(\vec{p})=\frac{1}{2p^0}[p^\mu\gamma_\mu+b_v M]\beta$$

$$[\psi_l(x),\psi_{\bar l}^\dagger(y)]_{\mp}=[|\kappa|^2(i\gamma^\mu\partial_{x,\mu}+b_u M)\beta\Delta_+(x-y)\mp|\lambda|^2(-i\gamma^\mu\partial_{x,\mu}+b_v M)\beta\Delta_+(y-x)]_{l\bar l}$$

$$|\kappa|^2=\mp|\lambda|^2 \qquad b_u|\kappa|^2=\pm b_v|\lambda|^2 \quad\Rightarrow\quad \kappa=\lambda=1 \qquad b_u=-b_v=1 \quad\text{与Weinberg书符号相反}$$

$$N(\vec{p})=1/(2p^0)[p^\mu\gamma_\mu+M]\beta \qquad\qquad M(\vec{p})=1/(2p^0)[p^\mu\gamma_\mu-M]\beta$$

费米统计

$$\text{旋量量子场: } D(\Lambda) = e^{\frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu}}, \Lambda = e^{\omega}, \mathcal{J}^{\mu\nu} = -\frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}], \{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$$

$$\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad u(0,\frac{1}{2}) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \quad u(0,-\frac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \quad v(0,\frac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad v(0,-\frac{1}{2}) = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ 0 \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\psi_l^+(x) = (2\pi)^{-3/2}\sum_{\sigma}\int d\vec{p}\, u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) \quad \psi_l^{-c}(x) = (2\pi)^{-3/2}\sum_{\sigma}\int d\vec{p}\, v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))u(0,\sigma) \quad v(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))v(0,\sigma) \quad \beta\gamma^{\mu\dagger}\beta = \gamma^{\mu} \quad \beta\mathcal{J}^{\rho\sigma\dagger}\beta = \mathcal{J}^{\rho\sigma}$$

$$\gamma_{\mu}^T = -\mathcal{C}\gamma_{\mu}\mathcal{C}^{-1} \quad \mathcal{C} \equiv i\gamma^2\beta = -\mathcal{C}^{-1} = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \quad \gamma_{\mu}^* = -\beta\mathcal{C}\gamma_{\mu}\mathcal{C}^{-1}\beta \quad \mathcal{J}_{\mu\nu}^T = -\mathcal{C}\mathcal{J}_{\mu\nu}\mathcal{C}^{-1}$$

$$\gamma^3{}^1 = \mathcal{C}\gamma^3{}^1\mathcal{C}^{-1} \quad \gamma^2{}^0 = -\mathcal{C}\gamma^2{}^0\mathcal{C}^{-1} \quad [e^{\frac{i}{2}\mathcal{J}^{\mu\nu}\omega_{\mu\nu}}]^* = \beta\mathcal{C}[e^{\frac{i}{2}\mathcal{J}^{\mu\nu}\omega_{\mu\nu}}]\mathcal{C}^{-1}\beta \quad D(L(p))^* = \beta\mathcal{C}D(L(p))\mathcal{C}^{-1}\beta$$

$$\mathcal{C}^{-1}\beta u(0,\sigma) = v(0,\sigma) \quad \mathcal{C}^{-1}\beta v(0,\sigma) = u(0,\sigma) \quad u_l^*(\vec{p},\sigma) = \beta\mathcal{C}v_l(\vec{p},\sigma) \quad v_l^*(\vec{p},\sigma) = \beta\mathcal{C}u_l(\vec{p},\sigma)$$

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad \gamma_5^2 = 1 \quad \{\gamma_5, \gamma^{\mu}\} = 0$$

$$\beta\gamma_5\beta^{-1} = -\gamma_5 \quad \mathcal{C}\gamma_5\mathcal{C}^{-1} = \gamma_5^T \quad \beta\mathcal{C}\gamma_5\mathcal{C}^{-1}\beta = -\gamma_5^* \quad [\mathcal{J}^{\mu\nu}, \gamma_5] = 0$$

费米统计

旋量量子场: $D(\Lambda) = e^{\frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu}}$, $\Lambda = e^{\omega}$, $\mathcal{J}^{\mu\nu} = -\frac{i}{4}[\gamma^\mu, \gamma^\nu]$, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$$\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad u(0,\tfrac{1}{2}) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix} \quad u(0,-\tfrac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-1}{\sqrt{2}} \end{bmatrix} \quad v(0,\tfrac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad v(0,-\tfrac{1}{2}) = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ 0 \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\psi_l^+(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\, u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) \quad \psi_l^{-c}(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\, v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)$$

$$u(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))u(0,\sigma) \quad v(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))v(0,\sigma) \quad \{\gamma_5,\gamma^\mu\} = 0 \quad [\mathcal{J}^{\mu\nu},\gamma_5] = 0$$

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad P_L \equiv \frac{1-\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \quad P_R \equiv \frac{1+\gamma_5}{2} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_L + P_R = 1 \quad u_{\substack{L\\R}}(\vec{p},\sigma) \equiv P_{\substack{L\\R}}u(\vec{p},\sigma) \quad v_{\substack{L\\R}}(\vec{p},\sigma) \equiv P_{\substack{L\\R}}v(\vec{p},\sigma) \quad [D(\Lambda), P_{\substack{L\\R}}] = 0$$

$$u_{\substack{L\\R}}(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))u_{\substack{L\\R}}(0,\sigma) \qquad v_{\substack{L\\R}}(\vec{p},\sigma) = \sqrt{M/p^0}D(L(p))v_{\substack{L\\R}}(0,\sigma)$$

费米统计

最后的γ矩阵结果: Itzykson-Zuber书中手征表象 可以相差一个实正交变换

$$\gamma^0 = \beta = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathcal{C} = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}$$

$$u(0, \tfrac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad u(0, -\tfrac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v(0, \tfrac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad v(0, -\tfrac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$u(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) u(0, \sigma) \quad v(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) v(0, \sigma)$$

$$\gamma_5 \mathcal{C}^{-1} u(0, -\sigma) = (-1)^{\frac{1}{2}-\sigma} u(0, \sigma) \qquad \gamma_5 \mathcal{C}^{-1} v(0, -\sigma) = (-1)^{\frac{1}{2}-\sigma} v(0, \sigma)$$

$$\beta = \beta^* \quad \Rightarrow \quad D^*(L(-\vec{p})) = \beta D^*(L(\vec{p}))\beta = \gamma_5 \beta D^*(L(\vec{p}))\beta\gamma_5 = \gamma_5 \mathcal{C} D(L(\vec{p}))\gamma_5 \mathcal{C}^{-1}$$

$$(-1)^{\frac{1}{2}+\sigma} u^*(-\vec{p}, -\sigma) = -\gamma_5 \mathcal{C} u(\vec{p}, \sigma) \qquad (-1)^{\frac{1}{2}+\sigma} v^*(-\vec{p}, -\sigma) = -\gamma_5 \mathcal{C} v(\vec{p}, \sigma)$$

费米统计

计算 $D(L(p))$–证明 $B(|\vec{p}|)$ 是推进变换： $\quad L(p) = R(\hat{p})B(|\vec{p}|)R^{-1}(\hat{p}) \qquad \gamma \equiv \dfrac{\sqrt{p^2 + M^2}}{M}$

$$B(|\vec{p}|) = \begin{pmatrix} \gamma & 0 & 0 & \sqrt{\gamma^2-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{\gamma^2-1} & 0 & 0 & \gamma \end{pmatrix} = \exp \begin{pmatrix} 0 & 0 & 0 & \text{arc cosh}(\gamma) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \text{arc cosh}(\gamma) & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & \text{arc cosh}(\gamma) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \text{arc cosh}(\gamma) & 0 & 0 & 0 \end{pmatrix}^{2n} = \begin{pmatrix} \text{arc cosh}^{2n}(\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{arc cosh}^{2n}(\gamma) \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & \text{arc cosh}(\gamma) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \text{arc cosh}(\gamma) & 0 & 0 & 0 \end{pmatrix}^{2n+1} = \begin{pmatrix} 0 & 0 & 0 & \text{arc cosh}^{2n+1}(\gamma) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \text{arc cosh}^{2n+1}(\gamma) & 0 & 0 & 0 \end{pmatrix}$$

$$\sum_{n=0} \frac{1}{(2n)!} \text{arc cosh}^{2n}(\gamma) = \gamma \qquad \sum_{n=1} \frac{1}{(2n+1)!} \text{arc cosh}^{2n+1}(\gamma) = \sinh(\text{arc cosh}^{2n+1}(\gamma)) = \sqrt{\gamma^2-1}$$

纯推进变换： $\quad \omega_3^0 = \omega_{03} = -\omega_{30} = \omega_0^3 = \text{arc cosh}(\gamma) \qquad \omega_\rho^\sigma \overset{\text{其它}}{=\!=\!=} 0$

费米统计

计算 $D(L(p))$：　　$L(p) = R(\hat{p})B(|\vec{p}|)R^{-1}(\hat{p})$　　　$D(L(\vec{p})) = D(R(\hat{p}))D(B(|\vec{p}|))D(R^{-1}(\hat{p}))$

$B(|\vec{p}|) = e^{\omega}$　　　**纯推进变换：**　　$\omega_3^0 = \omega_{03} = -\omega_{30} = \omega_0^3 = \text{arc}\cosh(\gamma)$　　　$\omega_\rho^\sigma = \underset{\text{其它}}{=====} 0$

$$D(B(|\vec{p}|)) = e^{i\omega_{03}\mathcal{J}^{03}} = \begin{pmatrix} e^{\frac{1}{2}\omega_{03}\sigma^3} & 0 \\ 0 & e^{-\frac{1}{2}\omega_{03}\sigma^3} \end{pmatrix} = \cosh(\frac{1}{2}\omega_{03}) + \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \sinh(\frac{1}{2}\omega_{03})$$

$$\mathcal{J}^{03} = -\frac{i}{4}[\gamma^0, \gamma^3] = -\frac{i}{2} \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix} = -\frac{i}{2} \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}$$

$$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \qquad \mathcal{J}^i = -\frac{1}{2}\epsilon_{ijk}\mathcal{J}^{jk} = \frac{i}{8}\epsilon_{ijk}[\gamma^j, \gamma^k] = \begin{pmatrix} \frac{\sigma^i}{2} & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix}$$

$$D(R(\hat{p})) = e^{-i\phi\mathcal{J}^3}e^{-i\theta\mathcal{J}^2} = [\cos\frac{\phi}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}][\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\sin\frac{\theta}{2}]$$

$$= \cos\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R^{-1}(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

费米统计

计算$D(L(p))$：

$$D(B(|\vec{p}|)) = \cosh(\frac{1}{2}\mathrm{arc\,cosh}(\gamma)) + \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \sinh(\frac{1}{2}\mathrm{arc\,cosh}(\gamma))$$

$$D(R(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R^{-1}(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R(\hat{p}))\begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} D(R^{-1}(\hat{p}))$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\left[\cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}\right.$$

$$\left. \times \left[\cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}\right]\right.$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\left[[\cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}]^2 + \sin^2\frac{\phi}{2}\cos^2\frac{\theta}{2}\right.$$

$$\left. -2i[\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}]\sin\frac{\phi}{2}\cos\frac{\theta}{2}\right]$$

费米统计

计算$D(L(p))$：

$$D(B(|\vec{p}|)) = \cosh(\frac{1}{2}\text{arc}\cosh(\gamma)) + \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \sinh(\frac{1}{2}\text{arc}\cosh(\gamma))$$

$$D(R(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R^{-1}(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R(\hat{p}))\begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}D(R^{-1}(\hat{p}))$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\Bigg[[\cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}]^2 + \sin^2\frac{\phi}{2}\cos^2\frac{\theta}{2}$$

$$-2i[\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}]\sin\frac{\phi}{2}\cos\frac{\theta}{2}\Bigg]$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\Bigg[\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} - 2i[-\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + \begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}]\cos\frac{\phi}{2}\cos\frac{\theta}{2}$$

$$-2i[\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}]\sin\frac{\phi}{2}\cos\frac{\theta}{2}\Bigg]$$

费米统计

计算 $D(L(p))$：
$$D(B(|\vec{p}|)) = \cosh(\frac{1}{2}\text{arc}\cosh(\gamma)) + \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \sinh(\frac{1}{2}\text{arc}\cosh(\gamma))$$

$$D(R(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R^{-1}(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R(\hat{p}))\begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}D(R^{-1}(\hat{p}))$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\left[\cos\theta - i[-\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos^2\frac{\phi}{2}\sin\theta + \frac{1}{2}\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\phi\sin\theta] - \frac{i}{2}\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\phi\sin\theta \right.$$

$$\left. -i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\sin^2\frac{\phi}{2}\sin\theta \right]$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\left[\cos\theta + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\phi\sin\theta - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\phi\sin\theta \right]$$

$$= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\cos\theta + \begin{pmatrix} \sigma^1 & 0 \\ 0 & -\sigma^1 \end{pmatrix}\cos\phi\sin\theta + \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}\sin\phi\sin\theta$$

费米统计

计算$D(L(p))$:

$$L(p) = R(\hat{p})B(|\vec{p}|)R^{-1}(\hat{p}) \qquad D(L(\vec{p})) = D(R(\hat{p}))D(B(|\vec{p}|))D(R^{-1}(\hat{p}))$$

$$D(B(|\vec{p}|)) = \cosh(\frac{1}{2}\text{arc}\cosh(\gamma)) + \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\sinh(\frac{1}{2}\text{arc}\cosh(\gamma))$$

$$D(R(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} - i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R^{-1}(\hat{p})) = \cos\frac{\phi}{2}\cos\frac{\theta}{2} + i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\cos\frac{\phi}{2}\sin\frac{\theta}{2} + i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\sin\frac{\phi}{2}\cos\frac{\theta}{2} - i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}\sin\frac{\phi}{2}\sin\frac{\theta}{2}$$

$$D(R(\hat{p}))\begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}D(R^{-1}(\hat{p})) = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\cos\theta + \begin{pmatrix} \sigma^1 & 0 \\ 0 & -\sigma^1 \end{pmatrix}\cos\phi\sin\theta + \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}\sin\phi\sin\theta$$

$$D(L(\vec{p})) = \cosh(\frac{1}{2}\text{arc}\cosh(\gamma)) + \begin{pmatrix} \Theta(\theta,\phi) & 0 \\ 0 & -\Theta(\theta,\phi) \end{pmatrix}\sinh(\frac{1}{2}\text{arc}\cosh(\gamma))$$

$$\Theta(\theta,\phi) = \sigma^3\cos\theta + \sigma^1\cos\phi\sin\theta + \sigma^2\sin\phi\sin\theta = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix}$$
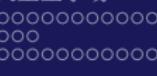
费米统计

计算 $u(\vec{p}, \sigma), v(\vec{p}, \sigma)$: $\quad u(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) u(0, \sigma) \quad\quad v(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) v(0, \sigma)$

$$u(0, \frac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad u(0, -\frac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v(0, \frac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad v(0, -\frac{1}{2}) = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$D(L(\vec{p})) = \cosh(\frac{1}{2} \operatorname{arc cosh}(\gamma)) + \begin{pmatrix} \Theta(\theta, \phi) & 0 \\ 0 & -\Theta(\theta, \phi) \end{pmatrix} \sinh(\frac{1}{2} \operatorname{arc cosh}(\gamma))$$

$$\Theta(\theta, \phi) = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix} \quad \Theta(\theta, \phi) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix} \quad \Theta(\theta, \phi) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} e^{-i\phi}\sin\theta \\ -\cos\theta \end{pmatrix}$$

$$\begin{matrix} u(\vec{p}, \frac{1}{2}) \\ v(\vec{p}, -\frac{1}{2}) \end{matrix} = \frac{1}{\sqrt{2\gamma}} \begin{bmatrix} \pm\cosh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \pm \cos\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \\ \pm e^{i\phi}\sin\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \\ -\cosh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) + \cos\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \\ e^{i\phi}\sin\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \end{bmatrix} \quad \hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\begin{matrix} u(\vec{p}, -\frac{1}{2}) \\ v(\vec{p}, \frac{1}{2}) \end{matrix} = \frac{1}{\sqrt{2\gamma}} \begin{bmatrix} e^{-i\phi}\sin\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \\ \cosh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) - \cos\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \\ \pm e^{-i\phi}\sin\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \\ \mp\cosh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \mp \cos\theta \sinh(\frac{1}{2}\operatorname{arc cosh}(\gamma)) \end{bmatrix} \quad \gamma = \frac{\sqrt{\vec{p}^2 + M^2}}{M}$$

费米统计

$u(\vec{p}, \sigma), v(\vec{p}, \sigma)$**的显式表达:**

$$u(\vec{p}, \frac{1}{2}) = \frac{1}{2\sqrt{\gamma}} \begin{bmatrix} \sqrt{\gamma+1} + \sqrt{\gamma-1}\cos\theta \\ \sqrt{\gamma-1}e^{i\phi}\sin\theta \\ -\sqrt{\gamma+1} + \sqrt{\gamma-1}\cos\theta \\ \sqrt{\gamma-1}e^{i\phi}\sin\theta \end{bmatrix}$$

$$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$u(\vec{p}, -\frac{1}{2}) = \frac{1}{2\sqrt{\gamma}} \begin{bmatrix} \sqrt{\gamma-1}e^{-i\phi}\sin\theta \\ \sqrt{\gamma+1} - \sqrt{\gamma-1}\cos\theta \\ \sqrt{\gamma-1}e^{-i\phi}\sin\theta \\ -\sqrt{\gamma+1} - \sqrt{\gamma-1}\cos\theta \end{bmatrix}$$

$$\gamma = \frac{\sqrt{\vec{p}^2 + M^2}}{M}$$

$$v(\vec{p}, \frac{1}{2}) = \frac{1}{2\sqrt{\gamma}} \begin{bmatrix} \sqrt{\gamma-1}e^{-i\phi}\sin\theta \\ \sqrt{\gamma+1} - \sqrt{\gamma-1}\cos\theta \\ -\sqrt{\gamma-1}e^{-i\phi}\sin\theta \\ \sqrt{\gamma+1} + \sqrt{\gamma-1}\cos\theta \end{bmatrix}$$

$$\cosh(\frac{1}{2}\text{arc}\cosh(\gamma)) = \frac{\sqrt{\gamma+1}}{\sqrt{2}}$$

$$v(\vec{p}, -\frac{1}{2}) = \frac{1}{2\sqrt{\gamma}} \begin{bmatrix} -\sqrt{\gamma+1} - \sqrt{\gamma-1}\cos\theta \\ -\sqrt{\gamma-1}e^{i\phi}\sin\theta \\ -\sqrt{\gamma+1} + \sqrt{\gamma-1}\cos\theta \\ \sqrt{\gamma-1}e^{i\phi}\sin\theta \end{bmatrix}$$

$$\sinh(\frac{1}{2}\text{arc}\cosh(\gamma)) = \frac{\sqrt{\gamma-1}}{\sqrt{2}}$$

旋量场的分立对称性变换性质

空间反射变换: $\quad Pa(\vec{p}, \sigma)P^{-1} = \eta^* a(-\vec{p}, \sigma) \quad Pa^c(\vec{p}, \sigma)P^{-1} = \eta^{c*} a^c(-\vec{p}, \sigma)$

$$u_l(-\vec{p}, \sigma) = b_u \beta u_l(\vec{p}, \sigma) = \beta u_l(\vec{p}, \sigma) \quad v_l(-\vec{p}, \sigma) = b_v \beta v_l(\vec{p}, \sigma) = -\beta v_l(\vec{p}, \sigma)$$

$$\psi_l(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [u_l(\vec{p}, \sigma)e^{-ip\cdot x} a(\vec{p}, \sigma) + v_l(\vec{p}, \sigma)e^{ip\cdot x} a^{c\dagger}(\vec{p}, \sigma)]$$

$$P\psi_l(x)P^{-1} = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [\eta^* u_l(-\vec{p}, \sigma)e^{-ip\cdot \mathcal{P}x} a(\vec{p}, \sigma) + \eta^c v_l(-\vec{p}, \sigma)e^{ip\cdot \mathcal{P}x} a^{c\dagger}(\vec{p}, \sigma)]$$

$$= (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [\eta^* \beta u_l(\vec{p}, \sigma)e^{-ip\cdot \mathcal{P}x} a(\vec{p}, \sigma) - \eta^c \beta v_l(\vec{p}, \sigma)e^{ip\cdot \mathcal{P}x} a^{c\dagger}(\vec{p}, \sigma)]$$

为保证用空间反射态来构造$\tilde{\mathcal{H}}(x)$同样保证能够使其在类空区间相互对易，只能取

$$\eta^c = -\eta^* \qquad \Rightarrow \qquad {\color{red}P\psi(x)P^{-1} = \eta^* \beta \psi(\mathcal{P}x)} \quad \text{作业21}$$

$$P\psi_L(x)P^{-1} = \eta^* \beta \psi_R(\mathcal{P}x) \qquad P\psi_R(x)P^{-1} = \eta^* \beta \psi_L(\mathcal{P}x)$$

旋量场的分立对称性变换性质

时间反演变换: $Ta(\vec{p},\sigma)T^{-1}=\zeta^{*}(-1)^{\frac{1}{2}-\sigma}a(-\vec{p},-\sigma)$ $\quad Ta^{c}(\vec{p},\sigma)T^{-1}=\zeta^{c*}(-1)^{\frac{1}{2}-\sigma}a^{c}(-\vec{p},-\sigma)$

$$(-1)^{\frac{1}{2}+\sigma}u^{*}(-\vec{p},-\sigma)=-\gamma_{5}\mathcal{C}u(\vec{p},\sigma) \quad (-1)^{\frac{1}{2}+\sigma}v^{*}(-\vec{p},-\sigma)=-\gamma_{5}\mathcal{C}v(\vec{p},\sigma)$$

$$\psi_{l}(x)=(2\pi)^{-3/2}\sum_{\sigma}\int d\vec{p}\,[u_{l}(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma)+v_{l}(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)]$$

$$T\psi_{l}(x)T^{-1}=(2\pi)^{-3/2}\sum_{\sigma}\int d\vec{p}\,(-1)^{\frac{1}{2}-\sigma}[\zeta^{*}u_{l}^{*}(\vec{p},\sigma)e^{ip\cdot x}a(-\vec{p},-\sigma)+\zeta^{c}v_{l}^{*}(\vec{p},\sigma)e^{-ip\cdot x}a^{c\dagger}(-\vec{p},\sigma)]$$

$$=(2\pi)^{-3/2}\sum_{\sigma}\int d\vec{p}\,(-1)^{\frac{1}{2}+\sigma}[\zeta^{*}u_{l}^{*}(-\vec{p},-\sigma)e^{ip\cdot\mathcal{P}x}a(\vec{p},\sigma)+\zeta^{c}v_{l}^{*}(-\vec{p},-\sigma)e^{-ip\cdot\mathcal{P}x}a^{c\dagger}(\vec{p},\sigma)]$$

$$=-\gamma_{5}\mathcal{C}(2\pi)^{-3/2}\sum_{\sigma}\int d\vec{p}\,[\zeta^{*}u_{l}(\vec{p},\sigma)e^{ip\cdot\mathcal{P}x}a(\vec{p},\sigma)+\zeta^{c}v_{l}(\vec{p},\sigma)e^{-ip\cdot\mathcal{P}x}a^{c\dagger}(\vec{p},\sigma)]$$

为保证用时间反演态来构造$\tilde{\mathcal{H}}(x)$同样保证能够使其在类空区间相互对易,只能取

$$\zeta^{c}=\zeta^{*} \qquad \Rightarrow \qquad \color{red}{T\psi(x)T^{-1}=-\zeta^{*}\gamma_{5}\mathcal{C}\psi(-\mathcal{P}x)} \quad \text{作业22}$$

旋量场的分立对称性变换性质

电荷共轭与联合CPT变换： $\quad Ca(\vec{p}, \sigma)C^{-1} = \xi^* a^c(\vec{p}, \sigma) \quad Ca^c(\vec{p}, \sigma)C^{-1} = \xi^{c*} a(\vec{p}, \sigma)$

$$u_l^*(\vec{p}, \sigma) = \beta \mathcal{C} v_l(\vec{p}, \sigma) \quad v_l^*(\vec{p}, \sigma) = \beta \mathcal{C} u_l(\vec{p}, \sigma)$$

$$\psi_l(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [u_l(\vec{p}, \sigma) e^{-ip \cdot x} a(\vec{p}, \sigma) + v_l(\vec{p}, \sigma) e^{ip \cdot x} a^\dagger(\vec{p}, \sigma)]$$

$$C\psi_l(x)C^{-1} = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [\xi^* u_l(\vec{p}, \sigma) e^{-ip \cdot x} a^c(\vec{p}, \sigma) + \xi^c v_l(\vec{p}, \sigma) e^{ip \cdot x} a^\dagger(\vec{p}, \sigma)]$$

$$= \beta \mathcal{C} (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [\xi^* v_l^*(\vec{p}, \sigma) e^{-ip \cdot x} a^c(\vec{p}, \sigma) + \xi^c u_l^*(\vec{p}, \sigma) e^{ip \cdot x} a^\dagger(\vec{p}, \sigma)]$$

为保证用电荷共轭态来构造$\tilde{\mathcal{H}}(x)$同样保证能够使其在类空区间相互对易，只能取

$$\xi^c = \xi^* \quad \Rightarrow \quad C\psi(x)C^{-1} = \xi^* \beta \mathcal{C} \psi^*(x)$$

如果$a^c(\vec{p}, \sigma) = a(\vec{p}, \sigma) \Rightarrow C\psi(x)C^{-1} = \xi^* \psi(x)$，此种费米子叫Majorana费米子

Majorana费米子： $\psi(x) = \beta \mathcal{C} \psi^*(x)$ 　作业23

对CPT联合变换： $CPT \, \psi(x) \, [CPT]^{-1} = \xi^* \eta^* \zeta^* \gamma_5 \psi^*(-x)$

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

$$\psi_l(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \, [u_l(\vec{p}, \sigma)e^{-ip\cdot x}a(\vec{p}, \sigma) + v_l(\vec{p}, \sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p}, \sigma)]$$

$$D(L(p))\beta D^{-1}(L(p)) = L^0_\mu(p)\gamma^\mu = p_\mu \gamma^\mu / M$$

$$\frac{1}{M}p_\mu \gamma^\mu \, u(\vec{p}, \sigma) = \frac{1}{M}p_\mu \gamma^\mu \sqrt{\frac{M}{p^0}}D(L(p)) \, u(0, \sigma) = \sqrt{\frac{M}{p^0}}D(L(p))\beta u(0, \sigma) = u(\vec{p}, \sigma)$$

$$\frac{1}{M}p_\mu \gamma^\mu \, v(\vec{p}, \sigma) = \frac{1}{M}p_\mu \gamma^\mu \sqrt{\frac{M}{p^0}}D(L(p)) \, v(0, \sigma) = \sqrt{\frac{M}{p^0}}D(L(p))\beta v(0, \sigma) = -v(\vec{p}, \sigma)$$

$$(p^\mu \gamma_\mu - M)u(\vec{p}, \sigma) = 0 \qquad\qquad (p^\mu \gamma_\mu + M)v(\vec{p}, \sigma) = 0$$

$\Rightarrow$ $(i\gamma^\mu \partial_\mu - M)\psi(x) = 0$ **自由粒子场的Dirac方程！** $i\gamma^\mu \partial_\mu \psi_{_R}^{L}(x) - M\psi_{_L}^{R}(x) = 0$

自由旋量场的哈密顿量是体系的总能量算符

$$H'_0 = \sum_\sigma \int d\vec{p} \, [a^\dagger(\vec{p}, \sigma)a(\vec{p}, \sigma) + a^{c\dagger}(\vec{p}, \sigma)a^c(\vec{p}, \sigma)]\sqrt{\vec{p}^2 + M^2} = H_0$$ 作业31

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

检验**Dirac**方程： $(p^\mu \gamma_\mu - M)u(\vec{p}, \sigma) = 0$ $(p^\mu \gamma_\mu + M)v(\vec{p}, \sigma) = 0$

$$u(\vec{p}, \tfrac{1}{2}) \atop v(\vec{p}, -\tfrac{1}{2})} = \frac{1}{2\sqrt{\gamma}} \begin{bmatrix} \pm\sqrt{\gamma+1} \pm \sqrt{\gamma-1}\cos\theta \\ \pm\sqrt{\gamma-1}e^{i\phi}\sin\theta \\ -\sqrt{\gamma+1} + \sqrt{\gamma-1}\cos\theta \\ \sqrt{\gamma-1}e^{i\phi}\sin\theta \end{bmatrix}$$

$$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$u(\vec{p}, -\tfrac{1}{2}) \atop v(\vec{p}, \tfrac{1}{2})} = \frac{1}{2\sqrt{\gamma}} \begin{bmatrix} \sqrt{\gamma-1}e^{-i\phi}\sin\theta \\ \sqrt{\gamma+1} - \sqrt{\gamma-1}\cos\theta \\ \pm\sqrt{\gamma-1}e^{-i\phi}\sin\theta \\ \mp\sqrt{\gamma+1} \mp \sqrt{\gamma-1}\cos\theta \end{bmatrix}$$

$$\gamma = \frac{\sqrt{\vec{p}^2 + M^2}}{M}$$

$$\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad p_\mu\gamma^\mu = M[\gamma\gamma^0 - \sqrt{\gamma^2-1}(\sin\theta\cos\phi\gamma^1 + \sin\theta\sin\phi\gamma^2 + \cos\theta\gamma^3)]$$

$$p_\mu\gamma^\mu = M\begin{pmatrix} 0 & -\gamma - \sqrt{\gamma^2-1}(\sin\theta\cos\phi\sigma^1 + \sin\theta\sin\phi\sigma^2 + \cos\theta\sigma^3) \\ -\gamma + \sqrt{\gamma^2-1}(\sin\theta\cos\phi\sigma^1 + \sin\theta\sin\phi\sigma^2 + \cos\theta\sigma^3) & 0 \end{pmatrix}$$

$$= M\begin{pmatrix} 0 & 0 & -\gamma - \sqrt{\gamma^2-1}\cos\theta & -\sqrt{\gamma^2-1}e^{-i\phi}\sin\theta \\ 0 & 0 & -\sqrt{\gamma^2-1}e^{i\phi}\sin\theta & -\gamma + \sqrt{\gamma^2-1}\cos\theta \\ -\gamma + \sqrt{\gamma^2-1}\cos\theta & \sqrt{\gamma^2-1}e^{-i\phi}\sin\theta & 0 & 0 \\ \sqrt{\gamma^2-1}e^{i\phi}\sin\theta & -\gamma - \sqrt{\gamma^2-1}\cos\theta & 0 & 0 \end{pmatrix}$$

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

$$\psi_l(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\,[u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) + v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)] \quad (i\gamma^\mu\partial_\mu - M)\psi(x) = 0$$

自由旋量场的哈密顿量是体系的总能量算符

$$H_0 = \sum_\sigma \int d\vec{p}\ [a^\dagger(\vec{p},\sigma)a(\vec{p},\sigma) + a^c(\vec{p},\sigma)a^c(\vec{p},\sigma)]\sqrt{\vec{p}^2 + M^2}$$

哈密顿量的坐标空间表达:

$$\psi^\dagger = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\,[u^\dagger(\vec{p},\sigma)e^{ip\cdot x}a^\dagger(\vec{p},\sigma) + v^\dagger(\vec{p},\sigma)e^{-ip\cdot x}a^c(\vec{p},\sigma)]$$

$$\nabla\psi(x) = (2\pi)^{-3/2}i\sum_\sigma \int d\vec{p}\,\vec{p}[u(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) - v(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)]$$

$$u^\dagger(\vec{p},\sigma)u(\vec{p},\sigma') \overset{\text{作业16}}{====} v^\dagger(\vec{p},\sigma)v(\vec{p},\sigma') = \delta_{\sigma\sigma'} \qquad u^\dagger(\vec{p},\sigma)v(-\vec{p},\sigma') \overset{\text{作业18}}{====} v^\dagger(\vec{p},\sigma)u(-\vec{p},\sigma') = 0$$

$$\overset{\text{作业17}}{===\!\Rightarrow} \quad H_0 = \int d\vec{x}:\,[\psi^\dagger(x)\beta(-i\vec{\gamma}\cdot\nabla + M)\psi(x)]:$$

$$\int d^4x\,[\overline{\psi}i\vec{\gamma}\cdot\nabla\psi(x)]^* = -i\int d^4x[\partial^i\psi^\dagger(x)]\gamma^{i\dagger}\beta\psi(x) = -i\int d^4x[\partial^i\overline{\psi(x)}]\gamma^i\psi(x) = \int d^4x\,\overline{\psi(x)}i\vec{\gamma}\cdot\nabla\psi(x)$$

$$\psi_l(x) = (2\pi)^{-3/2} \sum_\sigma \int d\vec{p} \left[u_l(\vec{p}, \sigma) e^{-ip\cdot x} a(\vec{p}, \sigma) + v_l(\vec{p}, \sigma) e^{ip\cdot x} a^{c\dagger}(\vec{p}, \sigma) \right] \quad (i\gamma^\mu \partial_\mu - M)\psi(x) = 0$$

$$(i\partial_t + i\beta\vec{\gamma}\cdot\nabla - \beta M)\psi(x) = 0 \qquad H_0 = \int d\vec{x} \; : [\psi^\dagger(x)\beta(-i\vec{\gamma}\cdot\nabla + M)\psi(x)] :$$

$\psi(x)$为广义坐标，H_0对$\psi(x)$的泛函微商定义了广义动量对时间导数的负值$-\dot{\pi}(x)$

$$-\dot{\pi}^T(x) \equiv \left. \frac{\delta H_0}{\delta \psi^T(x)} \right|_{\pi \text{固定}} = \beta^*(i\vec{\gamma}^*\cdot\nabla + M)\psi^* = -i\dot{\psi}^*(x) \qquad \text{左泛函微商}$$

$$\pi(x) = i\psi^*(x) = i[\overline{\psi(x)}\beta]^T \qquad \overline{\psi}(x) \equiv \psi^\dagger(x)\beta$$

$$S_0 = \int d^4x \; : \pi^T(x)\cdot\dot{\psi}(x) : - \int dt \, H_0 = \int d^4x \; : \overline{\psi}(x)(i\gamma^\mu\partial_\mu - M)\psi(x) :$$

$$\gamma^\mu\partial_\mu = \beta\partial_t + \vec{\gamma}\cdot\nabla \qquad \text{略去边界积分，将}S_0\text{取极值得到Dirac方程！}$$

$$\int d^4x \, [\overline{\psi} i\gamma^\mu\partial_\mu\psi(x)]^* = -i\int d^4x[\partial_\mu\psi^\dagger(x)]\gamma^{\mu\dagger}\beta\psi(x) = -i\int d^4x[\partial_\mu\overline{\psi}(x)]\gamma^\mu\psi(x) = \int d^4x \, \overline{\psi}(x) i\gamma^\mu\partial_\mu\psi(x)$$

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

$$\psi_l(x) = (2\pi)^{-3/2}\sum_\sigma \int d\vec{p}\,[u_l(\vec{p},\sigma)e^{-ip\cdot x}a(\vec{p},\sigma) + v_l(\vec{p},\sigma)e^{ip\cdot x}a^{c\dagger}(\vec{p},\sigma)] \quad (i\gamma^\mu\partial_\mu - M)\psi(x) = 0$$

$$H_0 = \mathrm{Re}\int d\vec{x} \,:[\psi^\dagger(x)\beta(-i\vec{\gamma}\cdot\nabla + M)\psi(x)]: \quad -\dot{\pi}(x) \equiv \left.\frac{\delta H_0}{\delta\psi(x)}\right|_{\pi\text{固定}} = -i\dot{\psi}^*(x)$$

$$S_0 = \int d^4x \,:\overline{\psi}(x)(i\gamma^\mu\partial_\mu - M)\psi(x): \quad \pi(x) = i\psi^*(x) \quad \overline{\psi}(x) \equiv \psi^\dagger(x)\beta$$

$$\{\psi_l(\vec{x},t),\pi_{\bar{l}}(\vec{y},t)\} = i\delta_{l\bar{l}}\delta(\vec{x}-\vec{y}) \qquad \{\psi_l(\vec{x},t),\psi_{\bar{l}}(\vec{y},t)\} = \{\pi_l(\vec{x},t),\pi_{\bar{l}}(\vec{y},t)\} = 0 \quad \text{作业19}$$

$$\dot{\psi}(\vec{x},t) = i[H_0,\psi(\vec{x},t)] = \frac{\delta H_0}{\delta\pi(\vec{x},t)} \qquad\qquad \dot{\pi}(\vec{x},t) = i[H_0,\pi(\vec{x},t)] = -\frac{\delta H_0}{\delta\psi(\vec{x},t)} \quad \text{作业20}$$

$$S_0 = \int d^4x\,\mathcal{L}_0 \qquad \mathcal{L}_0 =: \overline{\psi}(x)(i\gamma^\mu\partial_\mu - M)\psi(x): \quad \pi^i(x) = \frac{\partial\mathcal{L}_0}{\partial\dot{\psi}(x)} = i\psi^*(x)$$

零质量旋量场的结果可在有质量的旋量场结果中将质量趋于零得到！

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

费米子质量: $\psi_R = P_R \psi$ $\psi_L = P_L \psi$ $\psi^c \equiv C\psi C^{-1} = \xi^* \beta \mathcal{C} \psi^*$ $(\psi_L)^c = \xi^* \beta \mathcal{C} \psi_L^* = P_R \xi^* \beta \mathcal{C} \psi^* = (\psi^c)_R$

$$S_0 = \int d^4x : \overline{\psi}(x)(i\gamma^\mu \partial_\mu - M)\psi(x) :$$

$$= \int d^4x : [\overline{\psi_L}(x)i\gamma^\mu \partial_\mu \psi_L(x) + \overline{\psi_R}(x)i\gamma^\mu \partial_\mu \psi_R(x) - M\overline{\psi_R}(x)\psi_L(x) - M\overline{\psi_L}(x)\psi_R(x)] :$$

♣ 如果费米子质量为零，左手场与右手场不发生作用!

♢ 这时左手场和右手场可以有相互独立的对称性! 手征对称性

♡ 质量负责联系左手场和右手场!

♠ 这时左、右手场不能再有相互独立的对称性! $\psi_R \to R\psi_R$ $\psi_L \to L\psi_L$ $R \neq L$

¶ 若费米子存在手征对称性将<u>禁戒质量项</u>的出现!

✠ 费米子有质量意味手征对称性必须发生破坏!

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

Marjorana费米子_{中微子?} 质量: $\psi^c \equiv C\psi C^{-1} = \xi^* \beta \mathcal{C} \psi^* \xRightarrow{\text{Marjorana}} \xi^* \psi$ 纯中性

$(\psi_L)^c = \xi^* \beta \mathcal{C} \psi_L^* = P_R \xi^* \beta \mathcal{C} \psi^* = (\psi^c)_R \xRightarrow{\text{Marjorana}} \xi^* \psi_R \quad (\psi_R)^c = (\psi^c)_L \xRightarrow{\text{Marjorana}} \xi^* \psi_L$

$\overline{(\psi_L)^c} = \overline{(\psi^c)_R} \xRightarrow{\text{Marjorana}} \xi \overline{\psi_R} \qquad\qquad \overline{(\psi_R)^c} = \overline{(\psi^c)_L} \xRightarrow{\text{Marjorana}} \xi \overline{\psi_L}$

$\overline{\psi} \gamma^\mu \psi = \overline{\psi^c} \gamma^\mu \psi^c = [\beta \mathcal{C} \psi^*]^\dagger \beta \gamma^\mu \beta \mathcal{C} \psi^* = \psi^T \mathcal{C} \gamma^\mu \mathcal{C} \beta \psi^* = \psi^T \gamma^{\mu T} \beta \psi^* = -\overline{\psi} \gamma^\mu \psi = 0$

Dirac mass term $= \int d^4 x : [-D\overline{\psi_R}(x)\psi_L(x) - D\overline{\psi_L}(x)\psi_R(x)] :$

Marjorana mass term $= \int d^4 x : [-A\overline{(\psi_L)^c}(x)\psi_L(x) - A\overline{\psi_L}(x)(\psi_L)^c(x)] :$ **Fermion#violation**

Marjorana mass term $= \int d^4 x : [-B\overline{(\psi_R)^c}(x)\psi_R(x) - B\overline{\psi_R}(x)(\psi_R)^c(x)] :$ **Fermion#violation**

$\chi = \psi_L + (\psi_L)^c \qquad \chi^c = \chi \qquad \omega = \psi_R + (\psi_R)^c \qquad \omega^c = \omega$

$\psi_L = P_L \chi \qquad (\psi_L)^c = P_R \chi \qquad \psi_R = P_R \omega \qquad (\psi_R)^c = P_L \omega$

Marjorana费米子质量: $\psi^c \equiv C\psi C^{-1} = \xi^* \beta \mathcal{C} \psi^* \xrightarrow[\text{Marjorana}]{} \psi$ 纯中性

$$\chi = \psi_L + (\psi_L)^c \qquad \chi^c = \chi \qquad \omega = \psi_R + (\psi_R)^c \qquad \omega^c = \omega$$

$$\psi_L = P_L\chi \qquad (\psi_L)^c = P_R\chi \qquad \psi_R = P_R\omega \qquad (\psi_R)^c = P_L\omega$$

$$D\overline{\psi_R}\psi_L + D\overline{\psi_L}\psi_R + A\overline{(\psi_L)^c}\psi_L + A\overline{\psi_L}(\psi_L)^c + B\overline{(\psi_R)^c}\psi_R + B\overline{\psi_R}(\psi_R)^c$$

$$= \frac{1}{2}D(\overline{\chi}\omega + \overline{\omega}\chi) + A\overline{\chi}\chi + B\overline{\omega}\omega = (\overline{\chi}, \overline{\omega}) \begin{pmatrix} A & \frac{1}{2}D \\ \frac{1}{2}D & B \end{pmatrix} \begin{pmatrix} \chi \\ \omega \end{pmatrix}$$

$$M_{1,2} = \frac{1}{2}\{A + B \pm \sqrt{(A-B)^2 + D^2}\}$$

Eigenstates: $\eta_1 = \chi\cos\theta - \omega\sin\theta \qquad \eta_2 = \chi\sin\theta + \omega\cos\theta \qquad \tan 2\theta = \dfrac{D}{A-B}$

$$D = (M_1 - M_2)\sin 2\theta \qquad A = M_1\cos^2\theta + M_2\sin^2\theta \qquad B = M_1\sin^2\theta + M_2\cos^2\theta$$

自由旋量场的场方程、哈密顿量和作用量、正则对易关系

旋量场的地位:

♣ 是各种非平庸量子场中最简单的场!

♢ 十分复杂!

♡ 没有经典对应!

♠ 现实世界已发现的所有物质型基本粒子都由旋量场描述!

¶ 存在两种的基本旋量场!

✠ 左右手场的不同对称性手征对称性 在现实世界起关键作用

按自旋分类

$$\psi_l^+(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{-ip\cdot x} u_l(\vec{p},\sigma,n) a(\vec{p},\sigma,n) \qquad \psi_l^-(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^{3/2}} e^{ip\cdot x} v_l(\vec{p},\sigma,n) a^\dagger(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} u_{\bar{l}}(\vec{p}_\Lambda, \bar{\sigma}, n) D_{\bar{\sigma}\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^+(\Lambda) u_l(\vec{p},\sigma,n)$$

$$\sum_{\bar{\sigma}} v_{\bar{l}}(\vec{p}_\Lambda, \bar{\sigma}, n) D_{\bar{\sigma}\sigma}^*(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l D_{\bar{l}l}^-(\Lambda) v_l(\vec{p},\sigma,n)$$

矢量量子场: $D(\Lambda)^\mu{}_\nu{}^\pm \equiv \Lambda^\mu{}_\nu$ $\quad (J^1 \pm iJ^2)_{\sigma'\sigma} = \delta_{\sigma'\sigma\pm 1}\sqrt{(j\mp\sigma)(j\pm\sigma+1)}\big|_{j=0,1}$ $\quad J_{\sigma'\sigma}^3 = \sigma\delta_{\sigma'\sigma}$

$$D(1+\omega)^\mu{}_\nu = g^\mu{}_\nu + \omega^\mu{}_\nu = g^\mu{}_\nu + \frac{i}{2}\omega_{\sigma\rho}(\mathcal{J}^{\sigma\rho})^\mu{}_\nu \xRightarrow[\text{不乡正!}]{e^\omega} (\mathcal{J}^{\sigma\rho})^\mu{}_\nu = i(g^\sigma{}_\nu g^{\rho\mu} - g^\rho{}_\nu g^{\sigma\mu})$$

$$\phi^{+\mu}(x) = \sum_\sigma (2\pi)^{-3/2} \int d\vec{p}\, u^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} \qquad \phi^{-\mu}(x) = \sum_\sigma (2\pi)^{-3/2} \int d\vec{p}\, v^\mu(\vec{p},\sigma) a^\dagger(\vec{p},\sigma) e^{ip\cdot x}$$

$$u^\mu(\vec{p},\sigma) = (M/p^0)^{1/2} L(p)^\mu{}_\nu u^\nu(0,\sigma) \qquad v^\mu(\vec{p},\sigma) = (M/p^0)^{1/2} L(p)^\mu{}_\nu v^\nu(0,\sigma)$$

$$\sum_{\bar{\sigma}} u^\mu(0,\bar{\sigma}) \vec{J}_{\bar{\sigma}\sigma}^{(j)} = \vec{\mathcal{J}}^\mu{}_\nu u^\nu(0,\sigma) \qquad \sum_{\bar{\sigma}} v^\mu(0,\bar{\sigma}) \vec{J}_{\bar{\sigma}\sigma}^{(j)*} = -\vec{\mathcal{J}}^\mu{}_\nu v^\nu(0,\sigma)$$

转动生成元 $\vec{\mathcal{J}}^\mu{}_\nu$是$\Lambda^\mu{}_\nu$的纯转动部分的四矢量表达: $(\mathcal{J}_k)^\mu{}_\nu = -\frac{1}{2}\epsilon_{ijk}(\mathcal{J}^{ij})^\mu{}_\nu = i\epsilon_{ijk}g^{i\mu}g^j{}_\nu$

$$(\mathcal{J}_k)^0{}_0 = (\mathcal{J}_k)^0{}_i = (\mathcal{J}_k)^i{}_0 = 0 \qquad (\mathcal{J}_k)^i{}_j = i\epsilon_{ijk} \qquad (\vec{\mathcal{J}}^2)^0{}_0 = (\vec{\mathcal{J}}^2)^0{}_i = (\vec{\mathcal{J}}^2)^i{}_0 = 0 \qquad (\vec{\mathcal{J}}^2)^i{}_j = 2\delta^i{}_j$$
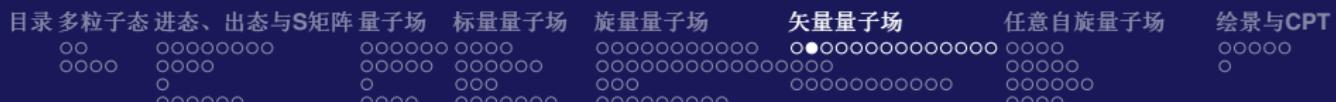
$$\phi^{+\mu}(x)=\sum_\sigma (2\pi)^{-3/2}\int d\vec{p}\, u^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} \qquad \phi^{-\mu}(x)=\sum_\sigma (2\pi)^{-3/2}\int d\vec{p}\, v^\mu(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}$$

$$\sum_{\bar\sigma} u_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^+_{\bar l l}(\Lambda)u_l(\vec{p},\sigma,n) \quad u^\mu(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^\mu{}_\nu u^\nu(0,\sigma)$$

$$\sum_{\bar\sigma} v_{\bar l}(\vec{p}_\Lambda,\bar\sigma,n)D^*_{\bar\sigma\sigma}(W(\Lambda,p))=\sqrt{p^0/(\Lambda p)^0}\sum_l D^-_{\bar l l}(\Lambda)v_l(\vec{p},\sigma,n) \quad v^\mu(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^\mu{}_\nu v^\nu(0,\sigma)$$

$$(\mathcal{J}_k)^0{}_0=(\mathcal{J}_k)^0{}_i=(\mathcal{J}_k)^i{}_0=0 \qquad (\mathcal{J}_k)^i{}_j=i\epsilon_{ijk} \qquad (\vec{\mathcal{J}}^2)^0{}_0=(\vec{\mathcal{J}}^2)^0{}_i=(\vec{\mathcal{J}}^2)^i{}_0=0 \qquad (\vec{\mathcal{J}}^2)^i{}_j=2\delta^i{}_j$$

$$\sum_{\bar\sigma} u^\mu(0,\bar\sigma)\vec{J}^{(j)}_{\bar\sigma\sigma}=\vec{\mathcal{J}}^\mu{}_\nu u^\nu(0,\sigma) \quad \sum_{\bar\sigma} u^0(0,\bar\sigma)(\vec{J}^2)^{(j)}_{\bar\sigma\sigma}=\sum_{\sigma'}\vec{\mathcal{J}}^0{}_\nu u^\nu(0,\sigma')\cdot\vec{J}^{(j)}_{\sigma'\sigma}=(\vec{\mathcal{J}}^2)^0{}_\nu u^\nu(0,\sigma)=0$$

$$\sum_{\bar\sigma} u^i(0,\bar\sigma)(\vec{J}^2)^{(j)}_{\bar\sigma\sigma}=\sum_{\sigma'}\vec{\mathcal{J}}^i{}_\nu u^\nu(0,\sigma')\cdot\vec{J}^{(j)}_{\sigma'\sigma}=(\vec{\mathcal{J}}^2)^i{}_\nu u^\nu(0,\sigma)=2u^i(0,\sigma)$$

$$\sum_{\bar\sigma} v^\mu(0,\bar\sigma)\vec{J}^{(j)*}_{\bar\sigma\sigma}=-\vec{\mathcal{J}}^\mu{}_\nu v^\nu(0,\sigma) \quad \sum_{\bar\sigma} v^0(0,\bar\sigma)(\vec{J}^2)^{(j)*}_{\bar\sigma\sigma}=-\sum_{\sigma'}\vec{\mathcal{J}}^0{}_\nu v^\nu(0,\sigma')\cdot\vec{J}^{(j)*}_{\sigma'\sigma}=(\vec{\mathcal{J}}^2)^0{}_\nu v^\nu(0,\sigma)=0$$

$$\sum_{\bar\sigma} v^i(0,\bar\sigma)(\vec{J}^2)^{(j)*}_{\bar\sigma\sigma}=-\sum_{\sigma'}\vec{\mathcal{J}}^i{}_\nu v^\nu(0,\sigma')\cdot\vec{J}^{(j)*}_{\sigma'\sigma}=(\vec{\mathcal{J}}^2)^i{}_\nu v^\nu(0,\sigma)=2v^i(0,\sigma)$$

按自旋分类

$$\phi^{+\mu}(x)=\sum_\sigma(2\pi)^{-3/2}\int d\vec{p}\ u^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} \qquad \phi^{-\mu}(x)=\sum_\sigma(2\pi)^{-3/2}\int d\vec{p}\ v^\mu(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}$$

$$(\mathcal{J}_k)^0{}_0=(\mathcal{J}_k)^0{}_i=(\mathcal{J}_k)^i{}_0=0 \qquad (\mathcal{J}_k)^i{}_j=i\epsilon_{ijk} \qquad (\vec{\mathcal{J}}^2)^0{}_0=(\vec{\mathcal{J}}^2)^0{}_i=(\vec{\mathcal{J}}^2)^i{}_0=0 \qquad (\vec{\mathcal{J}}^2)^i{}_j=2\delta^i{}_j$$

$$u^\mu(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^\mu{}_\nu u^\nu(0,\sigma) \qquad \sum_{\bar\sigma}u^0(0,\bar\sigma)(\vec{J}^2)^{(j)}_{\bar\sigma\sigma}=0 \qquad \sum_{\bar\sigma}u^i(0,\bar\sigma)(\vec{J}^2)^{(j)}_{\bar\sigma\sigma}=2u^i(0,\sigma)$$

$$v^\mu(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^\mu{}_\nu v^\nu(0,\sigma) \qquad \sum_{\bar\sigma}v^0(0,\bar\sigma)(\vec{J}^2)^{(j)}_{\bar\sigma\sigma}=0 \qquad \sum_{\bar\sigma}v^i(0,\bar\sigma)(\vec{J}^2)^{(j)}_{\bar\sigma\sigma}=2v^i(0,\sigma)$$

$$\sum_{\bar\sigma}u^\mu(0,\bar\sigma)\vec{J}^{(j)}_{\bar\sigma\sigma}=\vec{\mathcal{J}}^\mu{}_\nu u^\nu(0,\sigma) \qquad \sum_{\bar\sigma}v^\mu(0,\bar\sigma)\vec{J}^{(j)*}_{\bar\sigma\sigma}=-\vec{\mathcal{J}}^\mu{}_\nu v^\nu(0,\sigma)$$

自旋为0的情况： $\qquad j=0 \qquad L^\mu{}_0(p)=p^\mu/M$

$$u^i(0)=v^i(0)=0 \qquad u^0(0)=-v^0(0)=i(M/2)^{1/2} \qquad u^\mu(\vec{p})=-v^\mu(\vec{p})=ip^\mu(2p^0)^{-1/2}$$

$\phi^{+\mu}(x)=-\partial^\mu\phi^+(x) \qquad\qquad \phi^{-\mu}(x)=-\partial^\mu\phi^-(x)$

标量场的基础上加上一个微商即可得到现在的矢量场

按自旋分类

$$\phi^{+\mu}(x)=\sum_{\sigma}(2\pi)^{-3/2}\int d\vec{p}\, u^{\mu}(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} \qquad \phi^{-\mu}(x)=\sum_{\sigma}(2\pi)^{-3/2}\int d\vec{p}\, v^{\mu}(\vec{p},\sigma)a^{\dagger}(\vec{p},\sigma)e^{ip\cdot x}$$

$$u^{\mu}(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^{\mu}{}_{\nu}u^{\nu}(0,\sigma) \qquad \sum_{\bar{\sigma}}u^0(0,\bar{\sigma})(\vec{J}^2)^{(j)}_{\bar{\sigma}\sigma}=0 \qquad \sum_{\bar{\sigma}}u^i(0,\bar{\sigma})(\vec{J}^2)^{(j)}_{\bar{\sigma}\sigma}=2u^i(0,\sigma)$$

$$v^{\mu}(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^{\mu}{}_{\nu}v^{\nu}(0,\sigma) \qquad \sum_{\bar{\sigma}}v^0(0,\bar{\sigma})(\vec{J}^2)^{(j)}_{\bar{\sigma}\sigma}=0 \qquad \sum_{\bar{\sigma}}v^i(0,\bar{\sigma})(\vec{J}^2)^{(j)}_{\bar{\sigma}\sigma}=2v^i(0,\sigma)$$

自旋为1有质量的情况: $\quad j=1$

$$\sum_{\bar{\sigma}}u^{\mu}(0,\bar{\sigma})\vec{J}_{\bar{\sigma}\sigma}=\vec{\mathcal{J}}^{\mu}{}_{\nu}u^{\nu}(0,\sigma) \qquad \sum_{\bar{\sigma}}v^{\mu}(0,\bar{\sigma})\vec{J}^*_{\bar{\sigma}\sigma}=-\vec{\mathcal{J}}^{\mu}{}_{\nu}v^{\nu}(0,\sigma)$$

$$J^{\pm}_{\sigma'\sigma}=(J^1\pm iJ^2)_{\sigma'\sigma}=\delta_{\sigma'\sigma\pm1}\sqrt{(1\mp\sigma)(1\pm\sigma+1)} \qquad J^3_{\sigma'\sigma}=\sigma\delta_{\sigma'\sigma}$$

$$(\mathcal{J}^{\sigma\rho})^{\mu}{}_{\nu}=i(g^{\sigma}{}_{\nu}g^{\rho\mu}-g^{\rho}{}_{\nu}g^{\sigma\mu}) \qquad (\mathcal{J}_k)^0{}_0=(\mathcal{J}_k)^0{}_i=(\mathcal{J}_k)^i{}_0=0 \qquad (\mathcal{J}_k)^i{}_j=i\epsilon_{ijk}$$

$$u^0(0,\sigma)=v^0(0,\sigma)=0 \qquad u^i(0,\sigma)\sigma=i\epsilon_{ij3}u^j(0,\sigma) \qquad v^i(0,\sigma)\sigma=-i\epsilon_{ij3}v^j(0,\sigma)$$

$$u^1(0,0)=u^2(0,0)=v^1(0,0)=v^2(0,0)=0 \qquad u^3(0,\pm1)=v^3(0,\pm1)=0$$

$$u^1(0,\sigma)\sigma=iu^2(0,\sigma) \quad u^2(0,\sigma)\sigma=-iu^1(0,\sigma) \quad v^1(0,\sigma)\sigma=-iv^2(0,\sigma) \quad v^2(0,\sigma)\sigma=iv^1(0,\sigma)$$

按自旋分类

$$\phi^{+\mu}(x) = \sum_\sigma (2\pi)^{-3/2} \int d\vec{p}\, u^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} \qquad \phi^{-\mu}(x) = \sum_\sigma (2\pi)^{-3/2} \int d\vec{p}\, v^\mu(\vec{p},\sigma) a^\dagger(\vec{p},\sigma) e^{ip\cdot x}$$

$$\sum_{\bar\sigma} u^\mu(0,\bar\sigma) \vec{\mathcal{J}}_{\bar\sigma\sigma}^\mu = \vec{\mathcal{J}}_\nu^\mu u^\nu(0,\sigma) \qquad\qquad \sum_{\bar\sigma} v^\mu(0,\bar\sigma) \vec{\mathcal{J}}_{\bar\sigma\sigma}^* = -\vec{\mathcal{J}}_\nu^\mu v^\nu(0,\sigma)$$

$$u^0(0,\sigma) = v^0(0,\sigma) = 0 \qquad u^1(0,\sigma)\sigma = iu^2(0,\sigma) \qquad v^1(0,\sigma)\sigma = -iv^2(0,\sigma)$$

$$u^1(0,0) = u^2(0,0) = v^1(0,0) = v^2(0,0) = 0 \qquad u^3(0,\pm1) = v^3(0,\pm1) = 0$$

自旋为1有质量的情况: $\quad j = 1$

$$u^\mu(\vec{p},\sigma) = v^{\mu*}(\vec{p},\sigma) = (2p^0)^{-1/2} e^\mu(\vec{p},\sigma) \qquad u^\mu(0,0) = v^\mu(0,0) = (2M)^{-1/2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

作业24

$$u^\mu(0,1) = -v^\mu(0,-1) = -\frac{(2M)^{-1/2}}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix} \qquad\qquad u^\mu(0,-1) = -v^\mu(0,1) = \frac{(2M)^{-1/2}}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}$$

自旋为1有质量的情况: $u^\mu(\vec{p},\sigma) = v^{\mu*}(\vec{p},\sigma) = (2p^0)^{-1/2}e^\mu(\vec{p},\sigma)$

$$e^\mu(\vec{p},\sigma) \equiv L^\mu_{\ \nu}(\vec{p})e^\nu(0,\sigma) \qquad e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad e^\mu(0,1) = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}, \quad e^\mu(0,-1) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}$$

$$L(\vec{p}) = \begin{pmatrix} \gamma & \sqrt{\gamma^2-1}\,\sin\theta\cos\phi & \sqrt{\gamma^2-1}\,\sin\theta\sin\phi & \sqrt{\gamma^2-1}\,\cos\theta \\ \sqrt{\gamma^2-1}\,\sin\theta\cos\phi & 1+(\gamma-1)\sin^2\theta\cos^2\phi & (\gamma-1)\sin^2\theta\sin\phi\cos\phi & (\gamma-1)\sin\theta\cos\theta\cos\phi \\ \sqrt{\gamma^2-1}\,\sin\theta\sin\phi & (\gamma-1)\sin^2\theta\sin\phi\cos\phi & 1+(\gamma-1)\sin^2\theta\sin^2\phi & (\gamma-1)\sin\theta\cos\theta\sin\phi \\ \sqrt{\gamma^2-1}\,\cos\theta & (\gamma-1)\sin\theta\cos\theta\cos\phi & (\gamma-1)\sin\theta\cos\theta\sin\phi & 1+(\gamma-1)\cos^2\theta \end{pmatrix}$$

$$e^\mu(\vec{p},0) = \begin{bmatrix} \sqrt{\gamma^2-1}\,\cos\theta \\ (\gamma-1)\sin\theta\cos\theta\cos\phi \\ (\gamma-1)\sin\theta\cos\theta\sin\phi \\ 1+(\gamma-1)\cos^2\theta \end{bmatrix} \qquad e^\mu(\vec{p},\pm1) = \mp\frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{\gamma^2-1}e^{\pm i\phi}\sin\theta \\ 1+(\gamma-1)e^{\pm i\phi}\cos\phi\sin^2\theta \\ -i+(\gamma-1)e^{\pm i\phi}\sin\phi\sin^2\theta \\ (\gamma-1)e^{\pm i\phi}\sin\theta\cos\theta \end{bmatrix}$$

按自旋分类

自旋为1有质量的情况: $u^\mu(\vec{p}, \sigma) = v^{\mu*}(\vec{p}, \sigma) = (2p^0)^{-1/2} e^\mu(\vec{p}, \sigma)$

$$e^\mu(\vec{p}, \sigma) \equiv L^\mu_{\ \nu}(\vec{p}) e^\nu(0, \sigma) \qquad e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \ e^\mu(0,1) = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}, \ e^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}$$

$$\phi^{+\mu}(x) = \phi^{-\mu\dagger}(x) = (2\pi)^{-3/2} \sum_\sigma \int \frac{d\vec{p}}{\sqrt{2p^0}} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) e^{-ip\cdot x}$$

$$[\phi^{+\mu}(x), \phi^{-\nu}(y)]_\mp = \int \frac{d\vec{p}}{(2\pi)^3 2p^0} e^{-ip\cdot(x-y)} \Pi^{\mu\nu}(\vec{p}) = [-g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M^2}] \Delta_+(M, x-y) \ {\color{purple}作业25}$$

$$\Pi^{\mu\nu}(\vec{p}) \equiv \sum_\sigma e^\mu(\vec{p}, \sigma) e^{\nu*}(\vec{p}, \sigma) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}$$

为保证因果性，须把矢量粒子看成是玻色子，必须以组合的 v^μ **构造** $\tilde{\mathcal{H}}(x)$

▶ **对不带荷的矢量粒子** $v^\mu(x) \equiv \phi^{+\mu}(x) + \phi^{+\mu\dagger}(x)$

▶ **对带荷的矢量粒子须引入带相反荷的反粒子** $v^\mu(x) \equiv \phi^{+\mu}(x) + \phi^{+c\mu\dagger}(x)$

按自旋分类

自旋为1有质量的情况: $u^\mu(\vec{p},\sigma) = v^{\mu*}(\vec{p},\sigma) = (2p^0)^{-1/2}e^\mu(\vec{p},\sigma)$

$$e^\mu(\vec{p},\sigma) \equiv L^\mu_{\ \nu}(\vec{p})e^\nu(0,\sigma) \qquad e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad e^\mu(0,1) = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}, \quad e^\mu(0,-1) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}$$

$$\phi^{+\mu}(x) = \phi^{-\mu\dagger}(x) = (2\pi)^{-3/2}\sum_\sigma \int \frac{d\vec{p}}{\sqrt{2p^0}}e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x}$$

为保证因果性,须把矢量粒子看成是玻色子,必须以组合的v^μ**构造**$\tilde{\mathcal{H}}(x)$

$$v^\mu(x) = (2\pi)^{-3/2}\sum_\sigma \int \frac{d\vec{p}}{\sqrt{2p^0}}[e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

不带荷的情形相当于$a^c(\vec{p},\sigma) = a(\vec{p},\sigma)$

$$[v^\mu(x), v^\nu(y)]_- = [v^\mu(x), v^{\nu\dagger}(y)]_- = -[g^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{M^2}]\Delta(M, x-y) \quad \text{作业26}$$

按自旋分类

$$\phi^{+\mu}(x)=\sum_\sigma (2\pi)^{-3/2}\int d\vec{p}\, u^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} \qquad \phi^{-\mu}(x)=\sum_\sigma (2\pi)^{-3/2}\int d\vec{p}\, v^\mu(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}$$

$$u^\mu(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^\mu{}_\nu u^\nu(0,\sigma) \qquad u^\mu(\vec{k},\sigma)e^{i\theta\sigma}=\sum_l R^\mu{}_\nu(\theta)u^\nu(\vec{k},\sigma) \qquad u^\mu(\vec{k},\sigma)=S^\mu{}_\nu u^\nu(\vec{k},\sigma)$$

$$v^\mu(\vec{p},\sigma)=(M/p^0)^{1/2}L(p)^\mu{}_\nu v^\nu(0,\sigma) \qquad v^\mu(\vec{k},\sigma)e^{-i\theta\sigma}=R^\mu{}_\nu(\theta)v^\nu(\vec{k},\sigma) \quad v^\mu(\vec{k},\sigma)=S^\mu{}_\nu v^\nu(\vec{k},\sigma)$$

自旋为1无质量的情况: $\quad j=1$有质量的无质量极限产生发散!

$$u^\mu(\vec{p},\sigma)=v^{\mu*}(\vec{p},\sigma)=(2p^0)^{-1/2}e^\mu(\vec{p},\sigma) \qquad e^\mu(\vec{p},\sigma)=L(p)^\mu{}_\nu e^\nu(\vec{k},\sigma)\text{的螺旋度指标取}\pm 1 \qquad p^\mu=L(p)^\mu{}_\nu k^\nu$$

$$e^\mu(\vec{k},\sigma)e^{i\sigma\theta}=R^\mu{}_\nu(\theta)e^\nu(\vec{k},\sigma) \qquad e^\mu(\vec{k},\sigma)=S^\mu{}_\nu(\alpha,\beta)e^\nu(\vec{k},\sigma) \qquad e^\mu(\vec{k},\pm 1)=\frac{1}{\sqrt{2}}(0,1,\pm i,0)$$

$$S^\mu{}_\nu(\alpha,\beta)=\begin{bmatrix} 1+\zeta & \alpha & \beta & -\zeta \\ \alpha & 1 & 0 & -\alpha \\ \beta & 0 & 1 & -\beta \\ \zeta & \alpha & \beta & 1-\zeta \end{bmatrix} \qquad R^\mu{}_\nu(\theta)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \nearrow$$

按自旋分类

自旋为1无质量的情况： $j=1$有质量的无质量极限产生发散！

$$\phi^{+\mu}(x)=\sum_\sigma (2\pi)^{-3/2}\int d\vec{p}\, u^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} \qquad \phi^{-\mu}(x)=\sum_\sigma (2\pi)^{-3/2}\int d\vec{p}\, v^\mu(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}$$

$$u^\mu(\vec{p},\sigma)=v^{\mu*}(\vec{p},\sigma)=(2p^0)^{-1/2}e^\mu(\vec{p},\sigma) \qquad e^\mu(\vec{p},\sigma)=L(p)^\mu_{\ \nu}e^\nu(\vec{k},\sigma)\text{的螺旋度指标取}\pm1 \qquad p^\mu=L(p)^\mu_{\ \nu}k^\nu$$

$$e^\mu(\vec{k},\sigma)e^{i\sigma\theta}=R^\mu_{\ \nu}(\theta)e^\nu(\vec{k},\sigma) \qquad e^\mu(\vec{k},\sigma)=S^\mu_{\ \nu}(\alpha,\beta)e^\nu(\vec{k},\sigma) \qquad e^\mu(\vec{k},\pm1)=\frac{1}{\sqrt{2}}(0,1,\pm i,0)$$

$$S^\mu_{\ \nu}(\alpha,\beta)e^\nu(\vec{k},\pm1)=\frac{1}{\sqrt{2}}\begin{bmatrix} 1+\zeta & \alpha & \beta & -\zeta \\ \alpha & 1 & 0 & -\alpha \\ \beta & 0 & 1 & -\beta \\ \zeta & \alpha & \beta & 1-\zeta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix} \alpha\pm i\beta \\ 1 \\ \pm i \\ \alpha\pm i\beta \end{bmatrix}$$

$$S^\mu_{\ \nu}(\alpha,\beta)e^\nu(\vec{k},\sigma)=e^\mu(\vec{k},\sigma)+(\alpha+i\sigma\beta)\frac{k^\mu}{\sqrt{2}|\vec{k}|} \qquad W(\theta,\alpha,\beta)=S(\alpha,\beta)R(\theta)$$

$$D^\mu_{\ \nu}(W(\theta,\alpha,\beta))e^\nu(\vec{k},\sigma)=S^\mu_{\ \lambda}(\alpha,\beta)R^\lambda_{\ \nu}(\theta)e^\nu(\vec{k},\sigma)=e^{i\sigma\theta}[e^\mu(\vec{k},\sigma)+\frac{\alpha+i\sigma\beta}{\sqrt{2}|\vec{k}|}k^\mu]$$

按自旋分类

自旋为1无质量的情况:

$$\phi^{+\mu}(x) = \sum_{\sigma}(2\pi)^{-3/2}\int d\vec{p}\, u^{\mu}(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} \qquad \phi^{-\mu}(x) = \sum_{\sigma}(2\pi)^{-3/2}\int d\vec{p}\, v^{\mu}(\vec{p},\sigma)a^{\dagger}(\vec{p},\sigma)e^{ip\cdot x}$$

$$u^{\mu}(\vec{p},\sigma) = v^{\mu*}(\vec{p},\sigma) = (2p^0)^{-1/2}e^{\mu}(\vec{p},\sigma) \qquad e^{\mu}(\vec{p},\sigma) = L(p)^{\mu}{}_{\nu}e^{\nu}(\vec{k},\sigma)\text{的螺旋度指标取}\pm 1 \qquad p^{\mu} = L(p)^{\mu}{}_{\nu}k^{\nu}$$

$$e^{\mu}(\vec{k},\sigma)e^{i\sigma\theta} = R^{\mu}{}_{\nu}(\theta)e^{\nu}(\vec{k},\sigma) \qquad e^{\mu}(\vec{k},\sigma) = S^{\mu}{}_{\nu}(\alpha,\beta)e^{\nu}(\vec{k},\sigma) \qquad e^{\mu}(\vec{k},\pm 1) = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$$

$$S^{\mu}{}_{\nu}(\alpha,\beta)e^{\nu}(\vec{k},\sigma) = e^{\mu}(\vec{k},\sigma) + (\alpha + i\sigma\beta)k^{\mu}/(\sqrt{2}|\vec{k}|) \qquad W(\theta,\alpha,\beta) = S(\alpha,\beta)R(\theta)$$

$$D^{\mu}{}_{\nu}(W(\theta,\alpha,\beta))e^{\nu}(\vec{k},\sigma) = S^{\mu}{}_{\lambda}(\alpha,\beta)R^{\lambda}{}_{\nu}(\theta)e^{\nu}(\vec{k},\sigma) = e^{i\sigma\theta}[e^{\mu}(\vec{k},\sigma) + k^{\mu}(\alpha + i\sigma\beta)/(\sqrt{2}|\vec{k}|)]$$

$$D^{\mu}{}_{\nu}(\Lambda)e^{\nu}(\vec{p},\sigma) = D^{\mu}{}_{\nu}(\Lambda L(p))e^{\nu}(\vec{k},\sigma) = D^{\mu}{}_{\nu}(L(\Lambda p)W)e^{\nu}(\vec{k},\sigma) \qquad W \equiv L^{-1}(\Lambda p)\Lambda L(p)$$

$$= D^{\mu}{}_{\nu}(L(\Lambda p))e^{i\sigma\theta(\Lambda,p)}[e^{\nu}(\vec{k},\sigma) + \{\alpha(\Lambda,p) + i\sigma\beta(\Lambda,p)\}k^{\nu}/(\sqrt{2}|\vec{k}|)]$$

$$= e^{i\sigma\theta(\Lambda,p)}[e^{\mu}(\vec{p}_{\Lambda},\sigma) + p^{\mu}_{\Lambda}\{\alpha(\Lambda,p) + i\sigma\beta(\Lambda,p)\}/(\sqrt{2}|\vec{k}|)] \qquad p^{\mu}_{\Lambda} = \Lambda^{\mu}{}_{\nu}p^{\nu}$$

$$e^{-i\sigma\theta(\Lambda,p)}e^{\mu}(\vec{p},\sigma) = D^{\mu}{}_{\nu}(\Lambda^{-1})e^{\nu}(\vec{p}_{\Lambda},\sigma) + p^{\mu}\{\alpha(\Lambda,p) + i\sigma\beta(\Lambda,p)\}/(\sqrt{2}|\vec{k}|)$$

自旋为1无质量的情况:

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma) e^{ip\cdot x}]$$

$$U(\Lambda) a(\vec{p},\sigma) U^{-1}(\Lambda) = \sqrt{\frac{(\Lambda p)^0}{p^0}} e^{-i\sigma\theta(\Lambda p)} a(\vec{p}_\Lambda,\sigma) \qquad U(\Lambda) a^\dagger(\vec{p},\sigma) U^{-1}(\Lambda) = \sqrt{\frac{(\Lambda p)^0}{p^0}} e^{i\sigma\theta(\Lambda p)} a^\dagger(\vec{p}_\Lambda,\sigma)$$

$$e^{-i\sigma\theta(\Lambda,p)} e^\mu(\vec{p},\sigma) = D^\mu_{\ \nu}(\Lambda^{-1}) e^\nu(\vec{p}_\Lambda,\sigma) + p^\mu \{\alpha(\Lambda,p) + i\sigma\beta(\Lambda,p)\}/(\sqrt{2}|\vec{k}|)$$

$$U(\Lambda) v^\mu(x) U^{-1}(\Lambda) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} \sqrt{\frac{(\Lambda p)^0}{p^0}} [e^\mu(\vec{p},\sigma) e^{-i\sigma\theta(\Lambda,p)} a(\vec{p}_\Lambda,\sigma) e^{-ip\cdot x} + \cdots]$$

$$= (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}_\Lambda}{\sqrt{2p^0_\Lambda}} \{[D^\mu_{\ \nu}(\Lambda^{-1}) e^\nu(\vec{p}_\Lambda,\sigma) + p^\mu \frac{\alpha(\Lambda,p) + i\sigma\beta(\Lambda,p)}{\sqrt{2}|\vec{k}|}] a(\vec{p}_\Lambda,\sigma) e^{-ip_\Lambda(\Lambda x)} + \cdots\}$$

$$= D^\mu_{\ \nu}(\Lambda^{-1}) v^\nu(\Lambda x) + \partial^\mu \Omega(x,\Lambda) \qquad D^\mu_{\ \nu}(\Lambda^{-1}) = [D^\mu_{\ \nu}(\Lambda^{-1})]^* = (\Lambda^{-1})^\mu_{\ \nu}$$

$$\Omega(x,\Lambda) \equiv (2\pi)^{-\frac{3}{2}} i \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} \sqrt{\frac{(\Lambda p)^0}{p^0}} [\frac{\alpha(\Lambda,p) + i\sigma\beta(\Lambda,p)}{\sqrt{2}|\vec{k}|} a(\vec{p}_\Lambda,\sigma) e^{-ipx} - \frac{\alpha(\Lambda,p) - i\sigma\beta(\Lambda,p)}{\sqrt{2}|\vec{k}|} a^{c\dagger}(\vec{p}_\Lambda,\sigma) e^{ipx}]$$

按自旋分类

自旋为1无质量的情况:

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$U(\Lambda)a(\vec{p},\sigma)U^{-1}(\Lambda) = \sqrt{\frac{(\Lambda p)^0}{p^0}}e^{-i\sigma\theta(\Lambda p)}a(\vec{p}_\Lambda,\sigma) \qquad U(\Lambda)a^\dagger(\vec{p},\sigma)U^{-1}(\Lambda) = \sqrt{\frac{(\Lambda p)^0}{p^0}}e^{i\sigma\theta(\Lambda p)}a^\dagger(\vec{p}_\Lambda,\sigma)$$

$$U(\Lambda)v^\mu(x)U^{-1}(\Lambda) = D^\mu_\nu(\Lambda^{-1})v^\nu(\Lambda x) + \partial^\mu\Omega(x,\Lambda) \qquad D^\mu_\nu(\Lambda^{-1}) = [D^\mu_\nu(\Lambda^{-1})]^* = (\Lambda^{-1})^\mu_\nu$$

$$\Omega(x,\Lambda) \equiv (2\pi)^{-\frac{3}{2}}i\sum_{\sigma=\pm 1}\int \frac{d\vec{p}}{\sqrt{2p^0}}\sqrt{\frac{(\Lambda p)^0}{p^0}}[\frac{\alpha(\Lambda,p)+i\sigma\beta(\Lambda,p)}{\sqrt{2}|\vec{k}|}a(\vec{p}_\Lambda,\sigma)e^{-ipx} - \frac{\alpha(\Lambda,p)-i\sigma\beta(\Lambda,p)}{\sqrt{2}|\vec{k}|}a^{c\dagger}(\vec{p}_\Lambda,\sigma)e^{ipx}]$$

洛仑兹转动不只给出时空转动结果,还多出全散度项.虽避免了单纯在有质量
矢量场理论中取零质量极限导致的发散,但矢量场洛仑兹变换性质无法实现!
只有矢量场多一个全散度项不产生任何影响的理论能够保证实现理论本该具
有的洛仑兹变换的性质, 它要求理论在如下变换下是不变的:

$$v^\mu(x) \rightarrow v^{\mu'}(x) = v^\mu(x) + \partial^\mu\Omega(x)$$

这个变换叫规范变换. 在规范变换下保持不变的理论叫规范理论.

规范对称性的引入回顾： $v^{\mu+}(x) = v^{\mu-\dagger}(x) = \sum_{\sigma,n} \int \frac{d^3p}{(2\pi)^3} e^{-ip\cdot x} u^\mu(\vec{p},\sigma) a(\vec{p},\sigma)$

$$\sum_{\bar\sigma} u^\mu(\vec{p}_\Lambda, \bar\sigma) D_{\bar\sigma\sigma}(W(\Lambda,p)) = \sqrt{p^0/(\Lambda p)^0} \sum_l \Lambda^\mu_{\ \nu} u^\nu(\vec{p},\sigma) \quad D_{\bar\sigma\sigma}(W(\Lambda,p)) = e^{i\theta(\Lambda,p)\sigma} \delta_{\bar\sigma\sigma}$$

$$u^\mu(\vec{p},\sigma) = v^{\mu*}(\vec{p},\sigma) = (2p^0)^{-1/2} e^\mu(\vec{p},\sigma) \quad \sum_{\bar\sigma} e^\mu(\vec{p}_\Lambda, \bar\sigma) D_{\bar\sigma\sigma}(W(\Lambda,p)) = \sum_l \Lambda^\mu_{\ \nu} e^\nu(\vec{p},\sigma)$$

取 $p = k$, $\quad \Lambda = L(q)$, $\quad q = \Lambda p \Rightarrow L(p) = 1 \Rightarrow W(\Lambda, p) \equiv L^{-1}(\Lambda p)\Lambda L(p) = 1$

$$e^\mu(\vec{q}, \sigma) = \sum_l L^\mu_{\ \nu}(q) e^\nu(\vec{k}, \sigma)$$

取： $p = k$, $\quad \Lambda = S(\alpha, \beta)$, $\quad q = \Lambda p = k \Rightarrow e^\mu(\vec{k}, \sigma) = \sum_l S^\mu_{\ \nu}(\alpha, \beta) e^\nu(\vec{k}, \sigma)$

取： $p = k$, $\quad \Lambda = R(\theta)$, $\quad q = \Lambda p = k \Rightarrow e^\mu(\vec{k}, \sigma) e^{i\theta\sigma} = \sum_l R^\mu_{\ \nu}(\theta) e^\nu(\vec{k}, \sigma)$

$$S^\mu_{\ \nu}(\alpha, \beta) e^\nu(\vec{k}, \sigma) = e^\mu(\vec{k}, \sigma) + (\alpha + i\sigma\beta) \frac{k^\mu}{\sqrt{2}|\vec{k}|} \Rightarrow U(\Lambda) v^\mu(x) U^{-1}(\Lambda) = (\Lambda^{-1})^\mu_{\ \nu} v^\nu(\Lambda x) + \partial^\mu \Omega(x, \Lambda)$$

矢量场的分立对称性变换性质

空间反射变换:

$$v^\mu(x) = (2\pi)^{-3/2} \sum_\sigma \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma) e^{ip\cdot x}]$$

只研究非零质量的情形: $L^\mu_\nu(-\vec{p}) = \mathcal{P}^\mu_\rho L^\rho_\sigma(\vec{p}) \mathcal{P}^\sigma_\nu$ 　　　 $e^\mu(\vec{p},\sigma) = L^\mu_\nu(\vec{p}) e^\nu(0,\sigma)$

$$e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad e^\mu(0,1) = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}, \quad e^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}$$

$$e^\mu(-\vec{p},\sigma) = -\mathcal{P}^\mu_\nu e^\nu(\vec{p},\sigma) \quad Pa(\vec{p},\sigma)P^{-1} = \eta^* a(-\vec{p},\sigma) \quad Pa^c(\vec{p},\sigma)P^{-1} = \eta^{c*} a^c(-\vec{p},\sigma)$$

为保证用空间反射态来构造 $\tilde{\mathcal{H}}(x)$ 同样保证能够使其在类空区间相互对易，只能取

$$\eta^c = \eta^* \qquad \Rightarrow \qquad Pv^\mu(x)(x)P^{-1} = -\eta^* \mathcal{P}^\mu_\nu v^\nu(\mathcal{P}x))$$

矢量场的分立对称性变换性质

时间反演变换:

$$v^\mu(x) = (2\pi)^{-3/2} \sum_\sigma \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p}, \sigma) a^{c\dagger}(\vec{p}, \sigma) e^{ip\cdot x}]$$

只研究非零质量的情形: $L^\mu_{\ \nu}(-\vec{p}) = \mathcal{P}^\mu_{\ \rho} L^\rho_{\ \sigma}(\vec{p}) \mathcal{P}^\sigma_{\ \nu}$ $\qquad e^\mu(\vec{p}, \sigma) = L^\mu_{\ \nu}(\vec{p}) e^\nu(0, \sigma)$

$$e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad e^\mu(0,1) = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}, \quad e^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}$$

$e^{\mu*}(0, \sigma) = (-1)^\sigma e^\mu(0, -\sigma)$ $\qquad e^{\mu*}(\vec{k}, \sigma) = e^\mu(\vec{k}, -\sigma)$ $\qquad {\color{red}(-1)^{1+\sigma} e^{\mu*}(-\vec{p}, -\sigma) = \mathcal{P}^\mu_{\ \nu} e^\nu(\vec{p}, \sigma)}$

$\qquad\qquad {\color{red}Ta(\vec{p}, \sigma)T^{-1} = (-1)^{1-\sigma} \zeta^* a(-\vec{p}, -\sigma)} \qquad\qquad {\color{red}Ta^c(\vec{p}, \sigma)T^{-1} = (-1)^{1-\sigma} \zeta^{c*} a^c(-\vec{p}, -\sigma)}$

为保证用时间反演态来构造$\tilde{\mathcal{H}}(x)$同样保证能够使其在类空区间相互对易，只能取

$$\zeta^c = \zeta^* \qquad\qquad \Rightarrow \qquad\qquad {\color{red}Tv^\mu(x)T^{-1} = \zeta^* \mathcal{P}^\mu_{\ \nu} v^\nu(-\mathcal{P}x)}$$

矢量场的分立对称性变换性质

电荷共轭变换与CPT联合变换:

$$v^\mu(x) = (2\pi)^{-3/2} \sum_\sigma \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma) e^{ip\cdot x}]$$

只研究非零质量的情形: $L^\mu_\nu(-\vec{p}) = \mathcal{P}^\mu_\rho L^\rho_\sigma(\vec{p}) \mathcal{P}^\sigma_\nu$ $\qquad$ $e^\mu(\vec{p},\sigma) = L^\mu_\nu(\vec{p}) e^\nu(0,\sigma)$

$$e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad e^\mu(0,1) = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}, \quad e^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}$$

$$Ca(\vec{p},\sigma)C^{-1} = \xi^* a^c(\vec{p},\sigma) \qquad\qquad Ca^c(\vec{p},\sigma)C^{-1} = \xi^{c*} a(\vec{p},\sigma)$$

为保证用电荷共轭态来构造$\tilde{\mathcal{H}}(x)$同样保证能够使其在类空区间相互对易,只能取

$$\xi^c = \xi^* \qquad \Rightarrow \qquad Cv^\mu(x)(x)C^{-1} = \xi^* v^{\mu\dagger}(x)$$

$$Pv^\mu(x)(x)P^{-1} = -\eta^* \mathcal{P}^\mu_\nu v^\nu(\mathcal{P}x) \qquad Tv^\mu(x)(x)T^{-1} = \zeta^* \mathcal{P}^\mu_\nu v^\nu(-\mathcal{P}x)$$

$$CPTv^\mu(x)[CPT]^{-1} = -\xi^* \eta^* \zeta^* v^{\mu\dagger}(-x)$$

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p}, \sigma) a^{c\dagger}(\vec{p}, \sigma) e^{ip\cdot x}]$$

$$p^\mu p_\mu = M^2 \qquad (\partial^2 + M^2)v^\mu(x) = 0$$

有质量的矢量场:

$$e^\mu(\vec{p}, \sigma) = L^\mu_{\ \nu}(\vec{p}) e^\nu(0, \sigma) \qquad e^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad e^\mu(0,1) = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}, \quad e^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}$$

$$L^i_k(p) = \delta_{ik} + (\gamma - 1)\hat{p}_i \hat{p}_k \qquad L^i_{\ 0}(p) = L^0_{\ i}(p) = \frac{p^i}{M} \qquad L^0_{\ 0}(p) = \gamma \equiv \frac{\sqrt{\vec{p}^2 + M^2}}{M} \qquad \hat{p}_i \equiv \frac{p^i}{|\vec{p}|}$$

$$p_\mu L^\mu_{\ k}(\vec{p}) = p^0 \frac{p^k}{M} - p^i(\delta_{ik} + (\gamma - 1)\frac{p^i p^k}{\vec{p} \cdot \vec{p}}) = p^0 \frac{p^k}{M} - p^k + (1 - \gamma)p^k = 0$$

$$p_\mu e^\mu(\vec{p}, \sigma) = p_\mu L^\mu_{\ k}(\vec{p}) e^k(0, \sigma) + p_\mu L^\mu_{\ 0}(\vec{p}) e^0(0, \sigma) = 0 \quad \Rightarrow \quad \partial_\mu v^\mu(x) = 0$$

$v^\mu(x)$的四个分量不都是独立分量,通常取三个空间分量为独立变量

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma) e^{ip\cdot x}]$$

$$p^\mu p_\mu = M^2 \qquad (\partial^2 + M^2) v^\mu(x) = 0$$

<u>**有质量的矢量场:**</u> $\quad p_\mu e^\mu(\vec{p},\sigma) = 0 \quad \Rightarrow \quad \partial_\mu v^\mu(x) = 0$

$v^\mu(x)$ 四个分量不都是独立的，通常取三个空间分量为独立变量. 三个独立自由度

<u>**无质量的矢量场:**</u> $\quad e^\mu(\vec{p},\sigma) = L(p)^\mu{}_\nu e^\nu(\vec{k},\sigma) \qquad p^\mu = L(p)^\mu{}_\nu k^\nu \quad L(p) = R(\hat{p}) B(\frac{|\vec{p}|}{\kappa})$

$B^\mu{}_\nu(\frac{|\vec{p}|}{\kappa}) e^\nu(\vec{k},\sigma) = e^\mu(\vec{k},\sigma)$ B是沿z轴的boost,不影响只有x,y分量的 $e^\mu(\vec{k},\sigma)$ $\quad e^\mu(\vec{k},\pm 1) = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$

$$e^\mu(\vec{p},\sigma) = [R(\hat{p})B(\frac{|\vec{p}|}{\kappa})]^\mu{}_\nu e^\nu(\vec{k},\sigma) = R^\mu{}_\nu(\hat{p}) e^\nu(\vec{k},\sigma) \qquad R^0{}_0 = 1 \quad R^0{}_i = R^i{}_0 = 0$$

$$e^0(\vec{p},\sigma) = R^0{}_\nu(\hat{p}) e^\nu(\vec{k},\sigma) = e^0(\vec{k},\sigma) = 0 \quad \Rightarrow \quad {\color{red}v^0(x) = 0}$$

$p_\mu e^\mu(\vec{p},\sigma) = (L(p)k)_\mu (L(p)e(\vec{k},\sigma))^\mu = k_\mu e^\mu(\vec{k},\sigma) = 0$ $\Rightarrow \vec{p}\cdot\vec{e}(\vec{p},\sigma) = 0 \Rightarrow {\color{red}\nabla\cdot\vec{v}(x) = 0}$

现在约束条件是两个, 比有质量的矢量场多了一个约束条件. 两个独立自由度

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

自共轭自由矢量场_{作业27}**的哈密顿量和拉格朗日量:**

$$H_0' = \sum_\sigma \int d\vec{p}\ a^\dagger(\vec{p},\sigma)a(\vec{p},\sigma)\sqrt{\vec{p}^2+M^2} = H_0 \quad \text{作业32}$$

$$= \sum_\sigma \int d\vec{p}\ \frac{1}{2}\sqrt{\vec{p}^2+M^2} : \{a^\dagger(\vec{p},\sigma)a(\vec{p},\sigma) + a(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)\} :$$

$$\dot{v}^\mu(x) = i\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^{3/2}}\sqrt{\frac{p^0}{2}}[-e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}]$$

$$\nabla v^\mu(x) = i\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^{3/2}}\frac{\vec{p}}{\sqrt{2p^0}}[e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} - e^{\mu*}(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}]$$

$$H_0 = -\int d\vec{x}\ \frac{1}{2} : [\dot{v}^2(x) + (\nabla v_\mu(x))\cdot(\nabla v^\mu(x)) + M^2 v^2(x)] : \quad {\color{purple}e^\mu(\vec{p},\sigma)e^*_\mu(\vec{p},\sigma') = -\delta_{\sigma\sigma'}} \ \text{作业28}$$

$$= \int d\vec{x}\ \frac{-1}{2} : \{-\dot{\vec{v}}^2(x) + [\nabla\cdot\vec{v}(x)]^2 + [\nabla v_\mu(x)]\cdot[\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

目录 多粒子态 进态、出态与S矩阵 量子场 标量量子场 旋量量子场 **矢量量子场** 任意自旋量子场 绘景与CPT

○○ ○○○○○○○○ ○○○○○○○○ ○○○○○○○○○ ○○○○○○○○○○○○ ○○○○○ ○○○○○
○○○○○○○○ ○○○○ ○○○○○○○○○○○○ ○○○○○○ ○○○○
○○○○○ ○○○○ ○○○○○○○○○ ●○○○○○○○○○ ○○○○

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

自共轭自由矢量场的哈密顿量和拉格朗日量： $(\partial^2 + M^2)v^\mu(x) = 0$

$$\dot{v}^\mu(x) = i\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^{3/2}} \sqrt{\frac{p^0}{2}} [-e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$\nabla v^\mu(x) = i\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} - e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$H_0 = \int d\vec{x} \,(-1/2) : \{-\dot{\vec{v}}^2(x) + [\nabla \cdot \vec{v}(x)]^2 + [\nabla v_\mu(x)] \cdot [\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

$$-\dot{\vec{\pi}}(x) \equiv \frac{\delta H_0}{\delta \vec{v}(x)}\bigg|_{\pi\text{固定}} = \nabla[\nabla \cdot \vec{v}(x)] - (\nabla^2 - M^2)\vec{v}(x) = \nabla[\dot{v}_0(x)] - \ddot{\vec{v}}(x) \Rightarrow \vec{\pi}(x) = \dot{\vec{v}}(x) - \nabla v_0(x)$$

$$S_0 = \int d^4x \, : \vec{\pi}(x) \cdot \dot{\vec{v}}(x) : - \int dt \, H_0 = \int d^4x \, \frac{1}{2} : \{-[\partial_\mu v_\nu(x)][\partial^\mu v^\nu(x)] + M^2 v^2(x)\} :$$

$$= \int d^4x \, : \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}[\partial_\mu v^\mu(x)]^2 + \frac{1}{2}M^2 v^2(x)\} : \quad F_{\mu\nu}(x) \equiv \partial_\mu v_\nu(x) - \partial_\nu v_\mu(x)$$

略去边界积分，将S_0取极值就得到场方程 $(\partial^2 + M^2)v^\mu(x) = 0$

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$\vec{\pi}(x) = \dot{\vec{v}}(x) - \nabla v_0(x)$$

$$H_0 = \int d\vec{x} \, (-1/2) : \{-\dot{\vec{v}}^2(x) + [\nabla \cdot \vec{v}(x)]^2 + [\nabla v_\mu(x)] \cdot [\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

$$S_0 = \int d^4x \, : \vec{\pi}(x) \cdot \dot{\vec{v}}(x) : - \int dt \, H_0$$

$$= \frac{1}{2} \int d^4x \, : \{2\dot{\vec{v}}^2(x) - 2\dot{\vec{v}}(x) \cdot \nabla v_0(x) - \dot{\vec{v}}^2(x) + [\nabla \cdot \vec{v}(x)]^2 + [\nabla v_\mu(x)] \cdot [\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

$$= \frac{1}{2} \int d^4x \, : \{\dot{\vec{v}}^2(x) - 2\nabla \cdot \vec{v}(x) \dot{v}_0(x) + [\nabla \cdot \vec{v}(x)]^2 + [\nabla v_\mu(x)] \cdot [\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

$$= \frac{1}{2} \int d^4x \, : \{\dot{\vec{v}}^2(x) - \dot{v}_0^2(x) + [\nabla v_\mu(x)] \cdot [\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

$$= \frac{1}{2} \int d^4x \, : \{-[\partial_0 v_\mu(x)][\partial_0 v^\mu(x)] + [\nabla v_\mu(x)] \cdot [\nabla v^\mu(x)] + M^2 v^2(x)\} :$$

$$= \frac{1}{2} \int d^4x \, : \{-[\partial_\mu v_\nu(x)][\partial^\mu v^\nu(x)] + M^2 v^2(x)\} :$$

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^{\mu}(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^{\mu}(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu *}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

自共轭自由矢量场的哈密顿量和拉格朗日量：

$$\dot{v}^{\mu}(x) = i\sum_{\sigma} \int \frac{d\vec{p}}{(2\pi)^{3/2}} \sqrt{\frac{p^0}{2}} [-e^{\mu}(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu *}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$\nabla v^{\mu}(x) = i\sum_{\sigma} \int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{\vec{p}}{\sqrt{2p^0}} [e^{\mu}(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} - e^{\mu *}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$S_0 = \int d^4x \; : \{ -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}[\partial_{\mu}v^{\mu}(x)]^2 + \frac{1}{2}M^2 v^2(x) \} : \qquad F_{\mu\nu}(x) \equiv \partial_{\mu}v_{\nu}(x) - \partial_{\nu}v_{\mu}(x)$$

$$S_0 = \int d^4x \; \mathcal{L}_0 \qquad \mathcal{L}_0 =: \{ -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}[\partial_{\mu}v^{\mu}(x)]^2 + \frac{1}{2}M^2 v^2(x) \} :$$

$$\mathcal{L}_0 = 1/2 : \{ [\dot{\vec{v}}^2(x)] - \dot{\vec{v}}(x) \cdot \nabla v_0(x) + \nabla v^0(x) \cdot \nabla v^0(x) - [\partial_i v_j(x)][\partial^i v^j(x)] + M^2 v^2(x) \} :$$

$$\pi^i(x) = \frac{\partial \mathcal{L}_0}{\partial \dot{v}^i(x)} = \dot{v}^i(x) - \partial^i v_0(x)$$

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

自共轭自由矢量场的哈密顿量和拉格朗日量:

$$\dot{v}^\mu(x) = i\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^{3/2}} \sqrt{\frac{p^0}{2}} [-e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$\nabla v^\mu(x) = i\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^{3/2}} \frac{\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} - e^{\mu*}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

$$S_0 = \int d^4x\, \mathcal{L}_0 \qquad \mathcal{L}_0 =: \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}[\partial_\mu v^\mu(x)]^2 + \frac{1}{2}M^2 v^2(x)\} :$$

$$F_{\mu\nu}(x) \equiv \partial_\mu v_\nu(x) - \partial_\nu v_\mu(x) \qquad \pi^i(x) = \frac{\partial \mathcal{L}_0}{\partial \dot{v}^i(x)} = \dot{v}^i(x) - \partial^i v_0(x)$$

$$[v^i(\vec{x},t), \pi^j(\vec{y},t)]_- = i\delta^{ij}\delta(\vec{x}-\vec{y}) \qquad [v^i(\vec{x},t), v^j(\vec{y},t)]_- = [\pi^i(\vec{x},t), \pi^j(\vec{y},t)]_- = 0 \quad \text{作业29}$$

利用对易关系, 可以证明量子情形下的正则场方程 作业30

$$\dot{\vec{v}}(\vec{x},t) = i[H_0, \vec{v}(\vec{x},t)] = \frac{\delta H_0}{\delta \vec{\pi}(\vec{x},t)} \qquad \dot{\vec{\pi}}(\vec{x},t) = i[H_0, \vec{\pi}(\vec{x},t)] = -\frac{\delta H_0}{\delta \vec{v}(\vec{x},t)}$$

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x} + e^{\mu *}(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)e^{ip\cdot x}]$$

自共轭自由矢量场的哈密顿量和拉格朗日量:

$$S_0 = \int d^4x\, \mathcal{L}_0 \qquad \mathcal{L}_0 =: \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}[\partial_\mu v^\mu(x)]^2 + \frac{1}{2}M^2 v^2(x)\} :$$

$$F_{\mu\nu}(x) \equiv \partial_\mu v_\nu(x) - \partial_\nu v_\mu(x) \qquad \pi^i(x) = \frac{\partial \mathcal{L}_0}{\partial \dot{v}^i(x)} = \dot{v}^i(x) - \partial^i v_0(x)$$

关于约束条件 $\partial_\mu v^\mu(x) = 0$ 从原拉格朗日量无法得到 **及** π^0

$$\mathcal{L}_0' =: \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M^2 v^2(x)\} : \qquad \mathcal{L}_0' \text{ 不含 } \dot{v}_0 \ \Rightarrow \ \pi^0 = \frac{\partial \mathcal{L}_0}{\partial \dot{v}^0(x)} = 0 \ {}_{v^0 \text{不是动力学变量!}}$$

$$\frac{\partial \mathcal{L}_0'}{\partial v_\mu(x)} - \partial_\nu \frac{\partial \mathcal{L}_0'}{\partial \partial_\nu v_\mu(x)} = 0 \ \Rightarrow \ M^2 v^\mu + \partial_\nu(\partial^\nu v^\mu - \partial^\mu v^\nu) = 0 \ \Rightarrow \ M^2 \partial_\mu v^\mu = 0$$

$\xRightarrow{\text{非零质量}} \ \partial_\mu v^\mu = 0 \qquad (\partial^2 + M^2)v^\mu = 0 \qquad\qquad {}_{v^\mu \text{ 作为基本场}}$

$\xRightarrow{\text{零质量}} \ \partial_\mu F^{\mu\nu} = 0 \qquad F^{\mu\nu} = \partial^\mu v^\nu - \partial^\nu v^\mu \qquad v^\mu \to v^{\mu\prime} = v^\mu + \partial^\mu \Omega \ {}_{v^\mu \text{不显式出现, } F_{\mu\nu} \text{作为基本场}}$
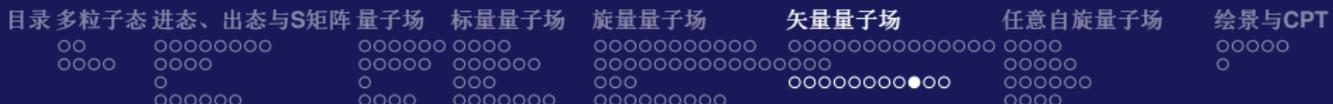
自由矢量场的场方程、哈密顿量和作用量、正则对易关系

$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm 1} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^\mu(\vec{p},\sigma) a(\vec{p},\sigma) e^{-ip\cdot x} + e^{\mu*}(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma) e^{ip\cdot x}]$$

自共轭自由矢量场的哈密顿量和拉格朗日量:

$$S_0 = \int d^4x \, \mathcal{L}_0 \qquad \mathcal{L}_0 =: \{ -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{1}{2}[\partial_\mu v^\mu(x)]^2 + \frac{1}{2} M^2 v^2(x) \} :$$

$$F_{\mu\nu}(x) \equiv \partial_\mu v_\nu(x) - \partial_\nu v_\mu(x) \qquad \pi^i(x) = \frac{\partial \mathcal{L}_0}{\partial \dot{v}^i(x)} = \dot{v}^i(x) - \partial^i v_0(x)$$

推广的处理: 关于约束条件 $\partial_\mu v^\mu(x) = 0$ 从原拉格朗日量无法得到

$$\mathcal{L}_0" =: \{ -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial_\mu v^\mu(x)]^2 + \frac{1}{2} M^2 v^2(x) \} : \qquad \frac{\partial \mathcal{L}_0"}{\partial \lambda} = 0 \Rightarrow \partial_\mu v^\mu(x) = 0$$

$$\frac{\partial \mathcal{L}_0"}{\partial v_\mu(x)} - \partial_\nu \frac{\partial \mathcal{L}_0"}{\partial \partial_\nu v_\mu(x)} = 0 \Rightarrow M^2 v^\mu + \partial_\nu[\partial^\nu v^\mu - (1-\lambda)\partial^\mu v^\nu] = 0 \Rightarrow (\partial^2 + M^2)v^\mu = 0$$

原始导出的拉氏量 $\mathcal{L}_0$ 对应 $\lambda = 1$ 但导不出 $v^0 = 0$; **修改的拉氏量 $\mathcal{L}_0'$ 对应 $\lambda = 0$**

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

矢量场的地位:

♣ 是各种非平庸量子场中有经典对应的最简单的场!

◇ 零质量矢量场要求<u>规范对称性</u>!

♡ 有质矢量场对质量零点连续要求规范对称性的破坏与质量相关!

♠ 现实世界已发现除引力外<u>所有物质间相互作用</u>都由矢量场描述!

¶ 规范对称性是决定物质基本相互作用的对称性!

✠ 时空对称性决定物质的相互作用?

自由矢量场的场方程、哈密顿量和作用量、正则对易关系

如何理解标量、旋量场无质量奇异，而矢量场有质量奇异性？

♣ 因为有无质量的标量、旋量场对应的态的数目无差别！

♠ 而有无质量的矢量场对应的态的数目是有差别的！

✠ 更高自旋的量子场有无质量的态的数目都有差别！

¶ 更高自旋的量子场应该都具有质量的奇异性！

♡ 因此更高自旋的零质量量子场需要<u>规范对称性</u>消除这种奇异性！
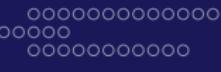
非奇次洛伦兹群的一般不可约表示

$$i[\mathcal{J}^{\mu\nu}, \mathcal{J}^{\rho\sigma}] = g^{\nu\rho}\mathcal{J}^{\mu\sigma} - g^{\mu\rho}\mathcal{J}^{\nu\sigma} - g^{\sigma\mu}\mathcal{J}^{\rho\nu} + g^{\sigma\nu}\mathcal{J}^{\rho\mu}$$

$$\vec{\mathcal{J}} \equiv \{\mathcal{J}^1, \mathcal{J}^2, \mathcal{J}^3\} = \{-\mathcal{J}_1, -\mathcal{J}_2, -\mathcal{J}_3\} = \{-\mathcal{J}^{23}, -\mathcal{J}^{31}, -\mathcal{J}^{12}\}$$

$$\vec{\mathcal{K}} \equiv \{\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3\} = \{-\mathcal{K}_1, -\mathcal{K}_2, -\mathcal{K}_3\} = \{-\mathcal{J}^{10}, -\mathcal{J}^{20}, -\mathcal{J}^{30}\}$$

$$[\mathcal{J}^i, \mathcal{J}^j] = i\epsilon_{ijk}\mathcal{J}^k \qquad [\mathcal{J}^i, \mathcal{K}^j] = i\epsilon_{ijk}\mathcal{K}^k \qquad [\mathcal{K}^i, \mathcal{K}^j] = -i\epsilon_{ijk}\mathcal{J}^k$$

$$\mathcal{A}^i \equiv \frac{1}{2}(\mathcal{J}^i + i\mathcal{K}^i) \qquad \mathcal{B}^i \equiv \frac{1}{2}(\mathcal{J}^i - i\mathcal{K}^i) \qquad \text{若要求} \mathcal{A}, \mathcal{B} \text{是厄米的，则 } \mathcal{J} \text{必须厄米，} \mathcal{K} \text{必须反厄米}$$

$$[\mathcal{A}^i, \mathcal{A}^j] = i\epsilon_{ijk}\mathcal{A}^k \qquad [\mathcal{B}^i, \mathcal{B}^j] = i\epsilon_{ijk}\mathcal{B}^k \qquad [\mathcal{A}^i, \mathcal{B}^j] = 0$$

两个相互独立的自旋分别为A和B整数或半整数 的不耦合的粒子对的直和

$$(\vec{\mathcal{A}})_{a'b',ab} = \delta_{b'b}\vec{J}^{(A)}_{a'a} \qquad (\vec{\mathcal{B}})_{a'b',ab} = \delta_{a'a}\vec{J}^{(B)}_{b'b} \qquad (A,B)\text{表示的维数}(2A+1)(2B+1)$$

$$(J^{(A),3})_{a'a} = a\delta_{a'a} \qquad (J^{(A),1} \pm iJ^{(A),2})_{a'a} = \delta_{a',a\pm1}\sqrt{(A\mp a)(A\pm a+1)} \qquad a = -A, -A+1, \cdots, +A$$

$$(J^{(B),3})_{b'b} = b\delta_{b'b} \qquad (J^{(B),1} \pm iJ^{(B),2})_{b'b} = \delta_{b',b\pm1}\sqrt{(B\mp b)(B\pm b+1)} \qquad b = -B, -B+1, \cdots, +B$$

宇称变换： $(A, B) \overset{\text{宇称变换}}{\Longleftarrow\!=\!=\!=\!\Longrightarrow} (B, A)$

$$\beta\mathcal{J}\beta^{-1} = \mathcal{J} \qquad \beta\mathcal{K}\beta^{-1} = -\mathcal{K} \qquad \Rightarrow \qquad \beta\mathcal{A}\beta^{-1} = \mathcal{B} \qquad \beta\mathcal{B}\beta^{-1} = \mathcal{A}$$

非奇次洛伦兹群的一般不可约表示

$$\mathcal{A}^i \equiv \frac{1}{2}(\mathcal{J}^i + i\mathcal{K}^i) \qquad\qquad \mathcal{B}^i \equiv \frac{1}{2}(\mathcal{J}^i - i\mathcal{K}^i)$$

若要求 $\mathcal{A}, \mathcal{B}$ 是厄米的，则 $\mathcal{J}$ 必须厄米，$\mathcal{K}$ 必须反厄米

$$[\mathcal{A}^i, \mathcal{A}^j] = i\epsilon_{ijk}\mathcal{A}^k \qquad\qquad [\mathcal{B}^i, \mathcal{B}^j] = i\epsilon_{ijk}\mathcal{B}^k \qquad\qquad [\mathcal{A}^i, \mathcal{B}^j] = 0$$

$$(A, B) \xLeftrightarrow{\text{字称变换}} (B, A) \qquad \Leftrightarrow \qquad \beta\mathcal{A}\beta^{-1} = \mathcal{B} \qquad \beta\mathcal{B}\beta^{-1} = \mathcal{A}$$

$$\vec{\mathcal{J}} = \vec{\mathcal{A}} + \vec{\mathcal{B}} \;\Rightarrow\; (A, B) 表示的空间转动表现为具有自旋 \underline{j} 的\underline{场} \quad j = A+B, A+B-1, \cdots, |A-B|$$

$j = 0 \quad (0,0)$场

$j = \frac{1}{2} \quad (\frac{1}{2}, 0)$或$(0, \frac{1}{2})$场 讨论见后 **Dirac**旋量$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$具有确定的字称

$j = 1 \quad (\frac{1}{2}, \frac{1}{2})$ 讨论见后, $(1, 0)$或$(0, 1)$场

- $(\frac{1}{2}, \frac{1}{2})$由$j = 1$和$j = 0$的场组成,分别对应$(\vec{v}, v^0)$；有确定字称
- $(1, 0)$和$(0, 1)$分别对应自对偶和反自对偶的反对称二阶张量
- $(1, 0) \oplus (0, 1)$具有确定的字称

有质量的(A, A)包含$j = 2A, 2A-1, \cdots, 0$的场

非奇次洛伦兹群的一般不可约表示

判断我们讨论过的旋量场：

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \qquad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \qquad \mathcal{J}^{\mu\nu} = -\frac{i}{4}[\gamma^\mu, \gamma^\nu]$$

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad P_L \equiv \frac{1-\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \quad P_R \equiv \frac{1+\gamma_5}{2} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$\gamma^j\gamma^k = \begin{pmatrix} -\sigma^j\sigma^k & 0 \\ 0 & -\sigma^j\sigma^k \end{pmatrix} \Rightarrow \mathcal{J}^i = -\frac{1}{2}\epsilon_{ijk}\mathcal{J}^{jk} = \frac{i}{8}\epsilon_{ijk}[\gamma^j, \gamma^k] = \frac{1}{4}\epsilon_{ijk}\epsilon_{ljk}\begin{pmatrix} \sigma^l & 0 \\ 0 & \sigma^l \end{pmatrix} = \begin{pmatrix} \frac{\sigma^i}{2} & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix}$$

$$[\gamma^0, \gamma^i] = 2\gamma^0\gamma^i = \begin{pmatrix} 2\sigma^i & 0 \\ 0 & -2\sigma^i \end{pmatrix} \qquad \mathcal{K}^i = \mathcal{J}^{0i} = -\frac{i}{4}[\gamma^0, \gamma^i] = \begin{pmatrix} -i\frac{\sigma^i}{2} & 0 \\ 0 & i\frac{\sigma^i}{2} \end{pmatrix}$$

$$\mathcal{A}^i \equiv \frac{1}{2}(\mathcal{J}^i + i\mathcal{K}^i) = \begin{pmatrix} \frac{\sigma^i}{2} & 0 \\ 0 & 0 \end{pmatrix} = P_R\mathcal{J}^i \qquad \mathcal{B}^i \equiv \frac{1}{2}(\mathcal{J}^i - i\mathcal{K}^i) = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix} = P_L\mathcal{J}^i$$

$$\mathcal{J}^2 = \frac{3}{4}I = \mathcal{A}^2 + \mathcal{B}^2 \quad \mathcal{A}^2 = P_R\mathcal{J}^2 = P_R\frac{1}{2}(\frac{1}{2}+1) \quad \mathcal{B}^2 = P_L\mathcal{J}^2 = P_L\frac{1}{2}(\frac{1}{2}+1) \quad \mathcal{A}^i\mathcal{B}^j = 0$$

我们讨论过的旋量场属于$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$场
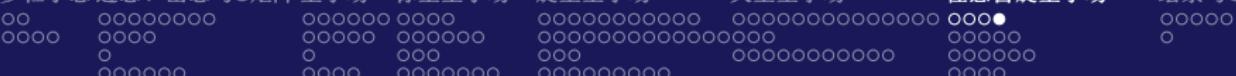
非奇次洛伦兹群的一般不可约表示

判断我们讨论过的矢量场：

$$\begin{matrix} (\mathcal{A}^2)^\mu_{\ \nu} \\ (\mathcal{B}^2)^\mu_{\ \nu} \end{matrix} = \frac{1}{4}(\vec{\mathcal{J}}^2 - \vec{\mathcal{K}}^2 \pm 2i\vec{\mathcal{J}} \cdot \vec{\mathcal{K}})^\mu_{\ \nu} \qquad\qquad (\mathcal{J}^{\sigma\rho})^\mu_{\ \nu} = i(g^\sigma_{\ \nu}g^{\rho\mu} - g^\rho_{\ \nu}g^{\sigma\mu})$$

$$= \frac{1}{4}[(\mathcal{J}^{23})^2 + (\mathcal{J}^{31})^2 + (\mathcal{J}^{12})^2 - (\mathcal{J}^{10})^2 - (\mathcal{J}^{20})^2 - (\mathcal{J}^{30})^2 \pm 2i(\mathcal{J}^{23}\mathcal{J}^{10} + \mathcal{J}^{31}\mathcal{J}^{20} + \mathcal{J}^{12}\mathcal{J}^{30})]^\mu_{\ \nu}$$

$$= \frac{1}{16}\epsilon_{ijk}\epsilon_{ilm}(\mathcal{J}^{jk})^\mu_{\ \lambda}(\mathcal{J}^{lm})^\lambda_{\ \nu} - \frac{1}{4}(\mathcal{J}^{i0})^\mu_{\ \lambda}(\mathcal{J}^{i0})^\lambda_{\ \nu} \pm \frac{i}{4}\epsilon_{ijk}(\mathcal{J}^{i0})^\mu_{\ \lambda}(\mathcal{J}^{jk})^\lambda_{\ \nu}$$

$$= \frac{1}{8}(\mathcal{J}^{jk})^\mu_{\ \lambda}(\mathcal{J}^{jk})^\lambda_{\ \nu} - \frac{1}{4}(\mathcal{J}^{i0})^\mu_{\ \lambda}(\mathcal{J}^{i0})^\lambda_{\ \nu} \pm \frac{i}{4}\epsilon_{ijk}(\mathcal{J}^{i0})^\mu_{\ \lambda}(\mathcal{J}^{jk})^\lambda_{\ \nu}$$

$$= -\frac{1}{8}(g^j_{\ \lambda}g^{k\mu} - g^k_{\ \lambda}g^{j\mu})(g^j_{\ \nu}g^{k\lambda} - g^k_{\ \nu}g^{j\lambda}) + \frac{1}{4}(g^i_{\ \lambda}g^{0\mu} - g^0_{\ \lambda}g^{i\mu})(g^i_{\ \nu}g^{0\lambda} - g^0_{\ \nu}g^{i\lambda})$$

$$\mp i\epsilon_{ijk}(g^i_{\ \lambda}g^{0\mu} - g^0_{\ \lambda}g^{i\mu})(g^j_{\ \nu}g^{k\lambda} - g^k_{\ \nu}g^{j\lambda})$$

$$= \frac{1}{8}[-(g^{jk}g^{k\mu} - g^{kk}g^{j\mu})g^j_{\ \nu} + (g^{jj}g^{k\mu} - g^{kj}g^{j\mu})g^k_{\ \nu} + 2(g^{i0}g^{0\mu} - g^{00}g^{i\mu})g^i_{\ \nu} - 2(g^{ii}g^{0\mu} - g^{0i}g^{i\mu})g^0_{\ \nu}]$$

$$\mp 2i\epsilon_{ijk}[(g^{ik}g^{0\mu} - g^{0k}g^{i\mu})g^j_{\ \nu} - (g^{ij}g^{0\mu} - g^{0j}g^{i\mu})g^k_{\ \nu}]$$

$$= \frac{1}{8}[g^k_{\ \nu}g^{k\mu} - 3g^{j\mu}g^j_{\ \nu} - 3g^{k\mu}g^k_{\ \nu} + g^j_{\ \nu}g^{j\mu} - 2g^{i\mu}g^i_{\ \nu} + 6g^{0\mu}g^0_{\ \nu}] = \frac{3}{4}g^{\rho\mu}g^\rho_{\ \nu} = \frac{1}{2}(\frac{1}{2} + 1)g^\mu_{\ \nu}$$

我们讨论过的矢量场属于$(\frac{1}{2}, \frac{1}{2})$场： $\mathcal{A}^2 = \mathcal{B}^2 = \frac{1}{2}(\frac{1}{2} + 1)I$

有质量的任意自旋量子场

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p [\kappa a(\vec{p}, \sigma) e^{ip\cdot x} u_{ab}(\vec{p}, \sigma) + \lambda a^{c\dagger}(\vec{p}, \sigma) e^{-ip\cdot x} v_{ab}(\vec{p}, \sigma)]$$

$$\sum_a \vec{\mathcal{J}}^{(A)}_{\bar{a}a} u_{a\bar{b}}(0, \sigma) + \sum_b \vec{\mathcal{J}}^{(B)}_{\bar{b}b} u_{a\bar{b}}(0, \sigma) \overset{\vec{\mathcal{J}} = \vec{A} + \vec{B}}{\Longleftarrow\!=\!=\!=\!=\!=} \sum_{a,b} \vec{\mathcal{J}}_{\bar{a}\bar{b},ab} u_{ab}(0, \sigma) = \sum_{\bar{\sigma}} u_{\bar{a}\bar{b}}(0, \bar{\sigma}) \vec{\mathcal{J}}^{(j)}_{\bar{\sigma}\sigma}$$

$$\sum_a \vec{\mathcal{J}}^{(A)}_{\bar{a}a} v_{a\bar{b}}(0, \sigma) + \sum_b \vec{\mathcal{J}}^{(B)}_{\bar{b}b} v_{\bar{a}b}(0, \sigma) \overset{\vec{\mathcal{J}} = \vec{A} + \vec{B}}{\Longleftarrow\!=\!=\!=\!=\!=} \sum_{a,b} \vec{\mathcal{J}}_{\bar{a}\bar{b},ab} v_{ab}(0, \sigma) = -\sum_{\bar{\sigma}} v_{\bar{a}\bar{b}}(0, \bar{\sigma}) \vec{\mathcal{J}}^{(j)*}_{\bar{\sigma}\sigma}$$

CG系数分解: $\Psi^j{}_\sigma \equiv \sum_{ab} C_{AB}(j\sigma; ab)\Psi_{ab}$　**在无穷小空间转动下**

$$\delta\Psi^j{}_\sigma = i\sum_{\bar{\sigma}} \vec{\theta}\cdot\vec{\mathcal{J}}^{(j)}_{\sigma\bar{\sigma}}\Psi^j{}_{\bar{\sigma}} \qquad \delta\Psi_{\bar{a}b} = i\sum_{\bar{a}} \vec{\theta}\cdot\vec{\mathcal{J}}^{(A)}_{\bar{a}a}\Psi_{a\bar{b}} + i\sum_{\bar{b}} \vec{\theta}\cdot\vec{\mathcal{J}}^{(B)}_{\bar{b}b}\Psi_{\bar{a}\bar{b}} \qquad u_{ab}(0,\sigma) = (2m)^{-\frac{1}{2}} C_{AB}(j\sigma; ab)$$

$$\sum_{\bar{\sigma}\bar{a}\bar{b}} \vec{\mathcal{J}}^{(j)}_{\sigma\bar{\sigma}} C_{AB}(j\bar{\sigma}; \bar{a}\bar{b})\Psi_{\bar{a}\bar{b}} = \sum_{\bar{\sigma}} \vec{\mathcal{J}}^{(j)}_{\sigma\bar{\sigma}}\Psi^j{}_{\bar{\sigma}} = \sum_{\bar{a}\bar{b}} C_{AB}(j\sigma; \bar{a}\bar{b})[\sum_a \vec{\mathcal{J}}^{(A)}_{\bar{a}a}\Psi_{a\bar{b}} + \sum_b \vec{\mathcal{J}}^{(B)}_{\bar{b}b}\Psi_{\bar{a}\bar{b}}] = \sum_{\bar{a}\bar{b}}[\sum_a \vec{\mathcal{J}}^{(A)}_{\bar{a}a} C_{AB}(j\sigma; a\bar{b}) + \sum_b \vec{\mathcal{J}}^{(B)}_{\bar{b}b} C_{AB}(j\sigma; \bar{a}b)]\Psi_{\bar{a}\bar{b}}$$

从矩阵元得出:　$-\vec{\mathcal{J}}^{(j)*}_{\sigma\sigma'} = (-1)^{-\sigma+\sigma'}\vec{\mathcal{J}}^{(j)}_{-\sigma,-\sigma'}$　$\Rightarrow$　$v_{ab}(0,\sigma) = (-1)^{j+\sigma} u_{ab}(0,-\sigma)$

推进变换:　$\cosh\theta = \sqrt{\vec{p}^2 + m^2}/m$　$\sinh\theta = |\vec{p}|/m$

$$L^i{}_k(\theta) = \delta_{ik} + (\cosh\theta - 1)\hat{p}_i\hat{p}_k \qquad L^i{}_0(\theta) = L^0{}_i(\theta) = \hat{p}_i\sinh\theta \qquad L^0{}_0(\theta) = \cosh\theta$$

无穷小θ: $[L(\theta)]^\mu{}_\nu = g^\mu{}_\nu + \omega^\mu{}_\nu$　$\Rightarrow$　$\omega^i{}_0 = \omega^0{}_i = \hat{p}_i\theta$　$\omega^i{}_j = \omega^0{}_0 = 0$

有质量的任意自旋量子场

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p [\kappa a(\vec{p}, \sigma) e^{ip \cdot x} u_{ab}(\vec{p}, \sigma) + \lambda a^\dagger(\vec{p}, \sigma) e^{-ip \cdot x} v_{ab}(\vec{p}, \sigma)]$$

$$\sum_a \vec{\mathcal{J}}_{\bar{a}a}^{(A)} u_{a\bar{b}}(0, \sigma) + \sum_b \vec{\mathcal{J}}_{\bar{b}b}^{(B)} u_{\bar{a}b}(0, \sigma) \stackrel{\vec{\mathcal{J}} = \vec{\mathcal{A}} + \vec{\mathcal{B}}}{=\!=\!=\!=\!=\!=} \sum_{a,b} \vec{\mathcal{J}}_{\bar{a}\bar{b},ab} u_{ab}(0, \sigma) = \sum_{\bar{\sigma}} u_{\bar{a}\bar{b}}(0, \bar{\sigma}) \vec{\mathcal{J}}_{\sigma\sigma}^{(j)}$$

$$\sum_a \vec{\mathcal{J}}_{\bar{a}a}^{(A)} v_{a\bar{b}}(0, \sigma) + \sum_b \vec{\mathcal{J}}_{\bar{b}b}^{(B)} v_{\bar{a}b}(0, \sigma) \stackrel{\vec{\mathcal{J}} = \vec{\mathcal{A}} + \vec{\mathcal{B}}}{=\!=\!=\!=\!=\!=} \sum_{a,b} \vec{\mathcal{J}}_{\bar{a}\bar{b},ab} v_{ab}(0, \sigma) = -\sum_{\bar{\sigma}} v_{\bar{a}\bar{b}}(0, \bar{\sigma}) \vec{\mathcal{J}}_{\bar{\sigma}\sigma}^{(j)*}$$

$$u_{ab}(0, \sigma) = (2m)^{-\frac{1}{2}} C_{AB}(j\sigma; ab)$$

推进变换： $\cosh\theta = \sqrt{\vec{p}^2 + m^2}/m \quad \sinh\theta = |\vec{p}|/m$

$$L^i_{\ k}(\theta) = \delta_{ik} + (\cosh\theta - 1)\hat{p}_i\hat{p}_k \qquad L^i_{\ 0}(\theta) = L^0_{\ i}(\theta) = \hat{p}_i \sinh\theta \qquad L^0_{\ 0}(\theta) = \cosh\theta$$

无穷小θ: $[L(\theta)]^\mu_{\ \nu} = g^\mu_{\ \nu} + \omega^\mu_{\ \nu} \implies \omega^i_{\ 0} = \omega^0_{\ i} = \hat{p}_i\theta \qquad \omega^i_{\ j} = \omega^0_{\ 0} = 0$

$$D(L(p)) = e^{-i\hat{p}\cdot\vec{\mathcal{K}}\theta} \stackrel{i\mathcal{K} = \mathcal{A} - \mathcal{B}}{=\!=\!=\!=\!=} e^{-\hat{p}\cdot\vec{\mathcal{J}}^{(A)}\theta} e^{\hat{p}\cdot\vec{\mathcal{J}}^{(B)}\theta} \implies D(L(p))_{a'b',ab} = \left(e^{-\hat{p}\cdot\vec{\mathcal{J}}^{(A)}\theta}\right)_{a'a} \left(e^{\hat{p}\cdot\vec{\mathcal{J}}^{(B)}\theta}\right)_{b'b}$$

$$u_{ab}(\vec{p}, \sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p}\cdot\vec{\mathcal{J}}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{\mathcal{J}}^{(B)}\theta}\right)_{bb'} C_{AB}(j\sigma; a'b') \qquad v_{ab}(\vec{p}, \sigma) = (-1)^{j+\sigma} u_{ab}(\vec{p}, -\sigma)$$

有质量的任意自旋量子场

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p [\kappa a(\vec{p},\sigma) e^{ip\cdot x} u_{ab}(\vec{p},\sigma) + \lambda a^{c\dagger}(\vec{p},\sigma) e^{-ip\cdot x} v_{ab}(\vec{p},\sigma)]$$

$$\textcolor{red}{u_{ab}(0,\sigma) = (2m)^{-1/2} C_{AB}(j\sigma;ab)}$$

$$u_{ab}(\vec{p},\sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} C_{AB}(j\sigma;a'b') \qquad v_{ab}(\vec{p},\sigma) = (-1)^{j+\sigma} u_{ab}(\vec{p},-\sigma)$$

$$[\psi_{ab}(x), \tilde{\psi}^\dagger_{ab}(y)]_{\mp} = (2\pi)^{-3} \int \frac{d^3p}{2p^0} \pi_{ab,\tilde{a}\tilde{b}}(\vec{p})[\kappa\tilde{\kappa}^* e^{ip\cdot(x-y)} \mp \lambda\tilde{\lambda}^* e^{-ip\cdot(x-y)}]$$

$$\pi_{ab,\tilde{a}\tilde{b}}(\vec{p}) \equiv 2p^0 \sum_\sigma u_{ab}(\vec{p},\sigma)\tilde{u}^*_{\tilde{a}\tilde{b}}(\vec{p},\sigma) = 2p^0 \sum_\sigma v_{ab}(\vec{p},\sigma)\tilde{v}^*_{\tilde{a}\tilde{b}}(\vec{p},\sigma) \qquad \tilde{j}=j, \ \ \tilde{\sigma}=\sigma$$

$$= \sum_{a'b'} \sum_{\tilde{a}'\tilde{b}'} \sum_\sigma C_{AB}(j\sigma;a'b') C_{\tilde{A}\tilde{B}}(j\sigma;\tilde{a}'\tilde{b}') \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)^*_{\tilde{a}\tilde{a}'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)^*_{\tilde{b}\tilde{b}'}$$

$$= \underbrace{P_{ab,\tilde{a}\tilde{b}}(\vec{p})}_{p^0偶幂次} + 2\sqrt{\vec{p}^2+m^2} \underbrace{Q_{ab,\tilde{a}\tilde{b}}(\vec{p})}_{\substack{p^0偶幂次 \\ x^0=y^0}} \quad \textcolor{red}{例子见后} \quad \textcolor{red}{P(-\vec{p}) = (-1)^{2A+2\tilde{B}}P(\vec{p}) \quad Q(-\vec{p}) = -(-1)^{2A+2\tilde{B}}Q(\vec{p})}$$

$$[\psi_{ab}(x),\tilde{\psi}^\dagger_{ab}(y)]_{\mp} \overset{x^0=y^0}{=\!=\!=} [\kappa\tilde{\kappa}^* \mp (-1)^{2A+2\tilde{B}}\lambda\tilde{\lambda}^*]P_{ab,\tilde{a}\tilde{b}}(-i\nabla)\Delta_+(\vec{x}-\vec{y},0)$$

$$\textcolor{red}{类空间隔只限于\vec{x}\neq\vec{y},\vec{x}=\vec{y}是类时间隔} \qquad + [\kappa\tilde{\kappa}^* \pm (-1)^{2A+2\tilde{B}}\lambda\tilde{\lambda}^*]Q_{ab,\tilde{a}\tilde{b}}(-i\nabla)\delta^3(\vec{x}-\vec{y})$$

有质量的任意自旋量子场

$$\pi_{ab,\tilde{a}\tilde{b}}(\vec{p}) \equiv 2p^0 \sum_\sigma u_{ab}(\vec{p},\sigma)\tilde{u}_{\tilde{a}\tilde{b}}^*(\vec{p},\sigma) = 2p^0 \sum_\sigma v_{ab}(\vec{p},\sigma)\tilde{v}_{\tilde{a}\tilde{b}}^*(\vec{p},\sigma)$$

$$= \sum_{a'b'} \sum_{\tilde{a}'\tilde{b}'} \sum_\sigma C_{AB}(j\sigma;a'b')C_{\tilde{A}\tilde{B}}(j\sigma;\tilde{a}'\tilde{b}') \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{\tilde{a}\tilde{a}'}^* \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{\tilde{b}\tilde{b}'}^*$$

$$= \underbrace{P_{ab,\tilde{a}\tilde{b}}(\vec{p})}_{p^0\text{偶幂次}} + 2\sqrt{\vec{p}^2+m^2}\, \underbrace{Q_{ab,\tilde{a}\tilde{b}}(\vec{p})}_{p^0\text{偶幂次}} \quad \text{例子见下} \quad P(-\vec{p})=(-1)^{2A+2\tilde{B}}P(\vec{p}) \quad Q(-\vec{p})=-(-1)^{2A+2\tilde{B}}Q(\vec{p})$$

考虑 $\vec{p}$ 沿 z 轴的情况： $\pi_{ab,\tilde{a}\tilde{b}}(\vec{p}) = \sum_\sigma C_{AB}(j\sigma;ab)C_{\tilde{A}\tilde{B}}(j\sigma;\tilde{a}\tilde{b})e^{(-a+b-\tilde{a}+\tilde{b})\theta}$

$$C_{AB}(j\sigma;ab)\bigg|_{\sigma\neq a+b} = C_{\tilde{A}\tilde{B}}(j\sigma;\tilde{a}\tilde{b})\bigg|_{\sigma\neq\tilde{a}+\tilde{b}} = 0 \quad\Rightarrow\quad -a+b-\tilde{a}+\tilde{b} = -2a+\sigma+2\tilde{b}-\sigma = 2\tilde{b}-2a$$

$$\cosh\theta = \frac{p^0}{m} \qquad \sinh\theta = \frac{p^3}{m} \quad\Rightarrow\quad e^{\pm\theta} = \cosh\theta \pm \sinh\theta = \frac{p^0 \pm p^3}{m}$$

$$\pi_{ab,\tilde{a}\tilde{b}}(\vec{p}) = \sum_\sigma C_{AB}(j\sigma;ab)C_{\tilde{A}\tilde{B}}(j\sigma;\tilde{a}\tilde{b}) \times \begin{cases} \left[\frac{p^0+p^3}{m}\right]^{2\tilde{b}-2a} & \tilde{b}\geq a \\[2mm] \left[\frac{p^0-p^3}{m}\right]^{2a-2\tilde{b}} & a\geq\tilde{b} \end{cases} \quad \underset{\overrightarrow{p}\to-\vec{p};\; p_0\to-p_0}{=\!=\!=\!=\!=\!=\!=\!=} (-1)^{2A+2\tilde{B}}\pi_{ab,\tilde{a}\tilde{b}}(\vec{p})$$

$(-1)^{2\tilde{b}-2a} = (-1)^{2A+2\tilde{B}}(-1)^{2\tilde{b}-2\tilde{B}-2a-2A} = (-1)^{2A+2\tilde{B}} \qquad (-1)^{2a-2\tilde{b}} = (-1)^{2A+2\tilde{B}}(-1)^{-2\tilde{b}-2\tilde{B}+2a-2A} = (-1)^{2A+2\tilde{B}}$

有质量的任意自旋量子场

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p [\kappa a(\vec{p}, \sigma) e^{ip \cdot x} u_{ab}(\vec{p}, \sigma) + \lambda a^{c\dagger}(\vec{p}, \sigma) e^{-ip \cdot x} v_{ab}(\vec{p}, \sigma)]$$

$$\textcolor{red}{u_{ab}(0, \sigma) = (2m)^{-1/2} C_{AB}(j\sigma; ab)}$$

$$u_{ab}(\vec{p}, \sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p} \cdot \vec{J}^{(A)} \theta} \right)_{aa'} \left(e^{\hat{p} \cdot \vec{J}^{(B)} \theta} \right)_{bb'} C_{AB}(j\sigma; a'b') \qquad v_{ab}(\vec{p}, \sigma) = (-1)^{j+\sigma} u_{ab}(\vec{p}, -\sigma)$$

$$[\psi_{ab}(x), \tilde{\psi}^\dagger_{\tilde{a}\tilde{b}}(y)]_\mp = (2\pi)^{-3} \int \frac{d^3p}{2p^0} \pi_{ab,\tilde{a}\tilde{b}}(\vec{p}) [\kappa\tilde{\kappa}^* e^{ip \cdot (x-y)} \mp \lambda\tilde{\lambda}^* e^{-ip \cdot (x-y)}]$$

$$\pi_{ab,\tilde{a}\tilde{b}}(\vec{p}) = P_{ab,\tilde{a}\tilde{b}}(\vec{p}) + 2\sqrt{\vec{p}^2 + m^2} Q_{ab,\tilde{a}\tilde{b}}(\vec{p}) \qquad \textcolor{red}{P(-\vec{p}) = (-1)^{2A+2\tilde{B}} P(\vec{p}) \qquad Q(-\vec{p}) = -(-1)^{2A+2\tilde{B}} Q(\vec{p})}$$

$$[\psi_{ab}(x), \tilde{\psi}^\dagger_{\tilde{a}\tilde{b}}(y)]_\mp \overset{x^0=y^0}{=\!=\!=} [\kappa\tilde{\kappa}^* \mp (-1)^{2A+2\tilde{B}} \lambda\tilde{\lambda}^*] P_{ab,\tilde{a}\tilde{b}}(-i\nabla) \Delta_+(\vec{x}-\vec{y}, 0)$$

$$+ [\kappa\tilde{\kappa}^* \pm (-1)^{2A+2\tilde{B}} \lambda\tilde{\lambda}^*] Q_{ab,\tilde{a}\tilde{b}}(-i\nabla) \delta^3(\vec{x}-\vec{y}) \qquad \textcolor{red}{\kappa\tilde{\kappa}^* = \pm(-1)^{2A+2\tilde{B}} \lambda\tilde{\lambda}^*}$$

<p style="text-align:center">自旋统计关系：j＝整数⇔玻色子；j＝半整数⇔费米子</p>

$$A = \tilde{A} \quad B = \tilde{B} \quad \Rightarrow \quad |\kappa|^2 = \pm(-1)^{2A+2B} |\lambda|^2 \overset{\textcolor{red}{2A+2B-2j=偶数}}{=\!=\!=\!=\!=\!=\!=\Rightarrow} \pm(-1)^{2j} = 1 \quad |\kappa|^2 = |\lambda|^2$$

$$|\tilde{\kappa}|^2 = |\tilde{\lambda}|^2 \quad \Rightarrow \quad \frac{\kappa}{\tilde{\kappa}} = (-1)^{-2\tilde{B}+2\tilde{B}} \frac{\lambda}{\tilde{\lambda}} \quad \Rightarrow \quad \lambda = (-1)^{2B} c\kappa \overset{\textcolor{red}{|\kappa|^2=|\lambda|^2}}{=\!=\!=\!=\Rightarrow} c = 1 \quad \Rightarrow \quad \textcolor{red}{\lambda = (-1)^{2B}\kappa}$$

无质量的任意自旋量子场

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p [\kappa a(\vec{p}, \sigma)e^{ip\cdot x}u_{ab}(\vec{p}, \sigma) + \lambda a^{c\dagger}(\vec{p}, \sigma)e^{-ip\cdot x}v_{ab}(\vec{p}, \sigma)]$$

$$u_{\bar{a}\bar{b}}(\vec{k}, \sigma)e^{i\theta\sigma} = \sum_{ab}D_{\bar{a}\bar{b},ab}(R(\theta))u_{ab}(\vec{k}, \sigma) \qquad u_{\bar{a}\bar{b}}(\vec{k}, \sigma) = \sum_{ab}D_{\bar{a}\bar{b},ab}(S(\alpha, \beta))u_{ab}(\vec{k}, \sigma)$$

$$v_{\bar{a}\bar{b}}(\vec{k}, \sigma)e^{-i\theta\sigma} = \sum_{ab}D_{\bar{a}\bar{b},ab}(R(\theta))v_{ab}(\vec{k}, \sigma) \qquad v_{\bar{a}\bar{b}}(\vec{k}, \sigma) = \sum_{ab}D_{\bar{a}\bar{b},ab}(S(\alpha, \beta))v_{ab}(\vec{k}, \sigma)$$

$$S^\mu_{\ \nu}(\alpha, \beta) = \begin{bmatrix} 1+\zeta & \alpha & \beta & -\zeta \\ \alpha & 1 & 0 & -\alpha \\ \beta & 0 & 1 & -\beta \\ \zeta & \alpha & \beta & 1-\zeta \end{bmatrix} \qquad R^\mu_{\ \nu}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

对无穷小θ: $D(R(\theta)) = 1 + i\theta\mathcal{J}^3$

$$\sigma u_{ab}(\vec{k}, \sigma) = (a+b)u_{ab}(\vec{k}, \sigma) \quad \Rightarrow \quad u_{ab}(\vec{k}, \sigma) \overset{\sigma \neq a+b}{=\!=\!=\!=\!=} 0$$

$$-\sigma v_{ab}(\vec{k}, \sigma) = (a+b)v_{ab}(\vec{k}, \sigma) \quad \Rightarrow \quad v_{ab}(\vec{k}, \sigma) \overset{\sigma \neq -a-b}{=\!=\!=\!=\!=} 0$$

从方程看可取 $v_{ab}(\vec{k}, \sigma) = u_{ab}(\vec{k}, -\sigma)$

无质量的任意自旋量子场

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_{\sigma} \int d^3p [\kappa a(\vec{p}, \sigma) e^{ip \cdot x} u_{ab}(\vec{p}, \sigma) + \lambda a^{c\dagger}(\vec{p}, \sigma) e^{-ip \cdot x} v_{ab}(\vec{p}, \sigma)]$$

$$u_{\bar{a}\bar{b}}(\vec{k}, \sigma) = \sum_{ab} D_{\bar{a}\bar{b}, ab}(S(\alpha, \beta)) u_{ab}(\vec{k}, \sigma) \qquad v_{\bar{a}\bar{b}}(\vec{k}, \sigma) = \sum_{ab} D_{\bar{a}\bar{b}, ab}(S(\alpha, \beta)) v_{ab}(\vec{k}, \sigma)$$

$$S^{\mu}_{\nu}(\alpha, \beta) = \begin{bmatrix} 1+\zeta & \alpha & \beta & -\zeta \\ \alpha & 1 & 0 & -\alpha \\ \beta & 0 & 1 & -\beta \\ \zeta & \alpha & \beta & 1-\zeta \end{bmatrix} \qquad u_{ab}(\vec{k}, \sigma) \stackrel{\sigma \neq a+b}{=\!=\!=\!=\!=} 0 \qquad v_{ab}(\vec{k}, \sigma) \stackrel{\sigma \neq -a-b}{=\!=\!=\!=\!=} 0$$

对无穷小 α, β: $D(S(\alpha, \beta)) = 1 - i\alpha(\mathcal{J}^{01} - \mathcal{J}^{13}) - i\beta(\mathcal{J}^{02} - \mathcal{J}^{23})$

$$\mathcal{J}^{01} - \mathcal{J}^{13} = \mathcal{K}^1 - \mathcal{J}^2 = -i\mathcal{A}^1 + i\mathcal{B}^1 - \mathcal{A}^2 - \mathcal{B}^2 \qquad \mathcal{J}^{02} - \mathcal{J}^{23} = \mathcal{K}^2 + \mathcal{J}^1 = -i\mathcal{A}^2 + i\mathcal{B}^2 + \mathcal{A}^1 + \mathcal{B}^1$$

$$0 = (J^{(A), 2} + iJ^{(A), 1})_{aa'} u_{a'b}(\vec{k}, \sigma) + (J^{(B), 2} - iJ^{(B), 1})_{bb'} u_{ab'}(\vec{k}, \sigma)$$

$$0 = (J^{(A), 1} - iJ^{(A), 2})_{aa'} u_{a'b}(\vec{k}, \sigma) + (J^{(B), 1} + iJ^{(B), 2})_{bb'} u_{ab'}(\vec{k}, \sigma)$$

$$(J^{(A), 1} - iJ^{(A), 2})_{aa'} u_{a'b}(\vec{k}, \sigma) = 0 \qquad (J^{(B), 1} + iJ^{(B), 2})_{bb'} u_{ab'}(\vec{k}, \sigma) = 0$$

$u_{ab}(\vec{k}, \sigma) \stackrel{a \neq -A, b \neq B}{=\!=\!=\!=\!=} 0$ $\sigma = B - A$ 类似地 $v_{ab}(\vec{k}, \sigma) \stackrel{a \neq -A, b \neq B}{=\!=\!=\!=\!=} 0$ $\sigma = A - B$ 反粒子具有相反的螺旋度！

无质量的任意自旋量子场

任意自旋的零质量量子场： (A, B)

$u_{ab}(\vec{k}, \sigma) \stackrel{a \neq -A,\ b \neq B}{=\!=\!=\!=\!=} 0$ $\quad \sigma = B - A = \pm j$ $\qquad$ 类似地 $v_{ab}(\vec{k}, \sigma) \stackrel{a \neq -A,\ b \neq B}{=\!=\!=\!=\!=} 0$ $\quad \sigma = A - B = \mp j$

$(A, B) \stackrel{\text{字称变换}}{\Longleftarrow\!=\!=\!=\!\Longrightarrow} (B, A)$ $\qquad \Leftrightarrow \qquad \beta \mathcal{A} \beta^{-1} = \mathcal{B} \qquad \beta \mathcal{B} \beta^{-1} = \mathcal{A}$

$\quad j = 0,\ \sigma = 0$ $\quad (0, 0)$场； $\quad (\frac{1}{2}, \frac{1}{2})$ 对应$\partial_\mu \phi$； $\quad (A, A)$场

$\quad j = \frac{1}{2}$ $\quad (\frac{1}{2}, 0)$或$(0, \frac{1}{2})$场 $\sigma = \pm \frac{1}{2}$ **Dirac**旋量$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$具有确定的宇称

$\quad j = 1$ $\quad (1, 0)$或$(0, 1)$场 $\sigma = \pm 1$ $(1, 0) \oplus (0, 1)$有确定宇称;对应$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\mu v_\mu$ v_μ**不显式出现**

$\quad j = A$ $\quad (A, 0)$或$(0, A)$场 $\sigma = \pm A$ $(A, 0) \oplus (0, A)$具有确定的宇称

可以证明：$(A, A + j)$是对$(0, j)$进行$2A$次微商；$(B + j, B)$是对$(j, 0)$进行$2B$次微商

只要讨论 $(j, 0)$ 只有$u_{-j,0}(\vec{k}, -j)$和$v_{-j,0}(\vec{k}, j)$非零 和 $(0, j)$ 只有$u_{0,j}(\vec{k}, j)$和$v_{0,j}(\vec{k}, -j)$非零 场 ！

目录 多粒子态 进态、出态与S矩阵 量子场 标量量子场 旋量量子场 矢量量子场 **任意自旋量子场** 绘景与CPT

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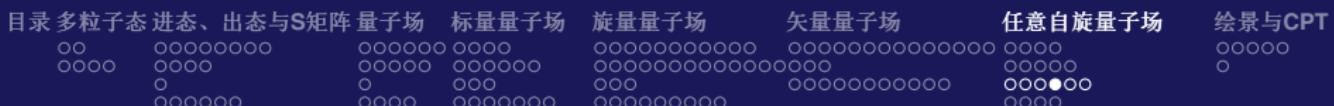
无质量的任意自旋量子场

$$\psi_{-j,0}(x) = (2\pi)^{-3/2} \int d^3p [\tilde{\kappa} a(\vec{p},-j) e^{ip\cdot x} u_{-j,0}(\vec{p},-j) + \tilde{\lambda} a^{c\dagger}(\vec{p},j) e^{-ip\cdot x} v_{-j,0}(\vec{p},j)]$$

$$\psi_{0,j}(x) = (2\pi)^{-3/2} \int d^3p [\kappa' a(\vec{p},j) e^{ip\cdot x} u_{0,j}(\vec{p},j) + \lambda' a^{c\dagger}(\vec{p},-j) e^{-ip\cdot x} v_{0,j}(\vec{p},-j)]$$

$$u_{\bar{a}\bar{b}}(\vec{p},\sigma) = \sqrt{\frac{\kappa}{p^0}} \sum_{a,b} D_{\bar{a}\bar{b}\,ab}(L(p)) u_{ab}(\vec{k},\sigma) \qquad v_{\bar{a}\bar{b}}(\vec{p},\sigma) = \sqrt{\frac{\kappa}{p^0}} \sum_{a,b} D_{\bar{a}\bar{b}\,ab}(L(p)) v_{ab}(\vec{k},\sigma)$$

$$L(p) = R(\hat{p}) B(\frac{|\vec{p}|}{\kappa}) \qquad D(L(p)) = D(R(\hat{p})) D(B(\frac{|\vec{p}|}{\kappa})) \qquad B(\frac{|\vec{p}|}{\kappa}) = e^{\omega}$$

$$B(u) = \begin{pmatrix} \frac{u^2+1}{2u} & 0 & 0 & \frac{u^2-1}{2u} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{u^2-1}{2u} & 0 & 0 & \frac{u^2+1}{2u} \end{pmatrix} = \exp \begin{pmatrix} 0 & 0 & 0 & \text{arc cosh}(\frac{u^2+1}{2u}) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \text{arc cosh}(\frac{u^2+1}{2u}) & 0 & 0 & 0 \end{pmatrix}$$

$$\omega_{03} = \text{arc cosh}(\frac{u^2+1}{2u}) \Big|_{u=\frac{|\vec{p}|}{\kappa}} = \ln u \Big|_{u=\frac{|\vec{p}|}{\kappa}} = \ln \frac{|\vec{p}|}{\kappa} \quad D(B(\frac{|\vec{p}|}{\kappa})) = e^{-i\omega_{03}\mathcal{K}^3} \underset{===}{i\mathcal{K}^3 = \mathcal{A}^3 = \mathcal{B}^3} \left(\frac{|\vec{p}|}{\kappa}\right)^{-\mathcal{A}^3 + \mathcal{B}^3} = \left(\frac{p^0}{\kappa}\right)$$

$$u_{\bar{a}\bar{b}}(\vec{p},\sigma) = \left(\frac{p^0}{\kappa}\right)^{j-1/2} \sum D_{\bar{a}\bar{b}\,ab}(R(\hat{p})) u_{ab}(\vec{k},\sigma) \qquad v_{\bar{a}\bar{b}}(\vec{p},\sigma) = \left(\frac{p^0}{\kappa}\right)^{j-1/2} \sum D_{\bar{a}\bar{b}\,ab}(R(\hat{p})) v_{ab}(\vec{k},\sigma)$$

无质量的任意自旋量子场

$$\psi_{-j,0}(x) = (2\pi)^{-\frac{3}{2}} \int d^3p \left(\frac{p^0}{\kappa}\right)^{j-1/2} \left[\tilde{\kappa}a(\vec{p},-j)e^{ip\cdot x}(Du)_{-j,0}(\vec{p},-j) + \tilde{\lambda}a^{c\dagger}(\vec{p},j)e^{-ip\cdot x}(Dv)_{-j,0}(\vec{p},j)\right]$$

$$\psi_{0,j}(x) = (2\pi)^{-\frac{3}{2}} \int d^3p \left(\frac{p^0}{\kappa}\right)^{j-1/2} \left[\kappa'a(\vec{p},j)e^{ip\cdot x}(Du)_{0,j}(\vec{p},j) + \lambda'a^{c\dagger}(\vec{p},-j)e^{-ip\cdot x}(Dv)_{0,j}(\vec{p},-j)\right]$$

$$(Du)_{\bar{a}\bar{b}}(\vec{p},\sigma) \equiv \sum_{a,b} D_{\bar{a}\bar{b}\,ab}(R(\hat{p}))u_{ab}(\vec{k},\sigma) \qquad (Dv)_{\bar{a}\bar{b}}(\vec{p},\sigma) \equiv \sum_{a,b} D_{\bar{a}\bar{b}\,ab}(R(\hat{p}))v_{ab}(\vec{k},\sigma)$$

$$[\psi_{-j,0}(x),\psi^\dagger_{-j,0}(y)]_\mp = \int \frac{d^3p}{(2\pi)^3} \frac{\pi(p)}{2p^0} \left[|\tilde{\kappa}|^2 e^{ip\cdot(x-y)} \mp |\tilde{\lambda}|^2 e^{-ip\cdot(x-y)}\right]$$

$$[\psi_{0,j}(x),\psi^\dagger_{0,j}(y)]_\mp = (2\pi)^{-3} \int \frac{d^3p}{2p^0} \pi'(p) \left[|\kappa'|^2 e^{ip\cdot(x-y)} \mp |\lambda'|^2 e^{-ip\cdot(x-y)}\right]$$

$$\pi(p) \xm1eq[\text{利用} v_{ab}(\vec{k},\sigma)=u_{ab}(\vec{k},-\sigma)]{} (p^0/\kappa)^{2j}(Du)_{-j,0}(\vec{p},-j)(Du)^*_{-j,0}(\vec{p},-j) \xmeq[p^0\to -p^0;\ \vec{p}\to-\vec{p}]{} (-1)^{2j}\pi(p)$$

$$\pi'(p) \xmeq[\text{利用} v_{ab}(\vec{k},\sigma)=u_{ab}(\vec{k},-\sigma)]{} (p^0/\kappa)^{2j}(Du)_{0,j}(\vec{p},j)(Du)^*_{0,j}(\vec{p},j) \xmeq[p^0\to -p^0;\ \vec{p}\to-\vec{p}]{} (-1)^{2j}\pi'(p) \quad \text{\color{magenta}后面证}$$

$$[\psi_{-j,0}(x),\psi^\dagger_{-j,0}(y)]_\mp = \pi(i\partial_x)[(-)^{2j}|\tilde{\kappa}|^2\Delta_+(y-x) \mp |\tilde{\lambda}|^2\Delta_+(x-y)] \xRightarrow{\text{\color{magenta}类似有质量}} \text{\color{red}自旋统计关系}$$

$$[\psi_{0,j}(x),\psi^\dagger_{0,j}(y)]_\mp = \pi'(i\partial_x)[(-)^{2j}|\kappa'|^2\Delta_+(y-x) \mp |\lambda'|^2\Delta_+(x-y)]$$

无质量的任意自旋量子场

$$\delta_{\bar{a},-j}\delta_{\bar{a}',-j} = \frac{1 \times 2 \times \cdots \times (2j-1) \times 2j}{(2j)!}\delta_{\bar{a},-j}\delta_{\bar{a}',-j} = \frac{1}{(2j)!}\left[\Pi^j_{\lambda=-j+1}(\lambda - J^{(A),3})\right]_{\bar{a}\bar{a}'}\underbrace{\delta_{\bar{a},-j}\delta_{\bar{a}',-j}}_{\text{可不要}}$$

$$\delta_{\bar{b},j}\delta_{\bar{b}',j} = \frac{2j \times (2j-1) \times \cdots \times 2 \times 1}{(2j)!}\delta_{\bar{b},j}\delta_{\bar{b}',j} = \frac{1}{(2j)!}\left[\Pi^{j-1}_{\lambda=-j}(J^{(B),3} - \lambda)\right]_{\bar{b}\bar{b}'}\delta_{\bar{b},j}\delta_{\bar{b}',j}$$

$$\pi(p) = \left(\frac{p^0}{\kappa}\right)^{2j}\left[u^\dagger(\vec{k},-j)D^\dagger(R(\hat{p}))\right]_{0,-j}\left[D(R(\hat{p}))u(\vec{k},-j)\right]_{-j,0}$$

$$= \left(\frac{p^0}{\kappa}\right)^{2j}\left[u^\dagger(\vec{k},-j)D^\dagger(R(\hat{p}))\right]_{0,\bar{a}}\delta_{\bar{a},-j}\delta_{\bar{a}',-j}\left[D(R(\hat{p}))u(\vec{k},-j)\right]_{\bar{a}',0}$$

$$= \frac{1}{(2j)!\kappa^{2j}}\left[u^\dagger(\vec{k},-j)D^\dagger(R(\hat{p}))\right]_{0,\bar{a}}\left[\Pi^j_{\lambda=-j+1}(p^0\lambda - J^{(A),3}p^0)\right]_{\bar{a},\bar{a}'}\left[D(R(\hat{p}))u(\vec{k},-j)\right]_{\bar{a}',0}$$

$$\underset{=====}{\color{red}D幺正} \frac{1}{(2j)!\kappa^{2j}}u^\dagger_{\bar{a},0}(\vec{k},-j)\left[\Pi^j_{\lambda=-j+1}({\color{red}p^0\lambda - \vec{J}^{(A)} \cdot \vec{p}})\right]_{\bar{a},\bar{a}'}u_{\bar{a}',0}(\vec{k},-j)$$

$$\pi'(p) = \left(\frac{p^0}{\kappa}\right)^{2j}\left[u^\dagger(\vec{k},j)D^\dagger(R(\hat{p}))\right]_{0,j}\left[D(R(\hat{p}))u(\vec{k},j)\right]_{0,j}$$

$$\underset{=====}{\color{red}D幺正} \frac{1}{(2j)!\kappa^{2j}}u^\dagger_{0,\bar{b}}(\vec{k},j)\left[\Pi^{j-1}_{\lambda=-j}({\color{red}\vec{J}^{(B)} \cdot \vec{p} - p^0\lambda})\right]_{\bar{b},\bar{b}'}u_{\bar{b}',0}(\vec{k},j)$$

分立对称性变换性质

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p [a(\vec{p},\sigma)e^{ip\cdot x}u_{ab}(\vec{p},\sigma) + (-1)^{2B}a^{c\dagger}(\vec{p},\sigma)e^{-ip\cdot x}v_{ab}(\vec{p},\sigma)]$$

$$u_{ab}(\vec{p},\sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} C_{AB}(j\sigma;a'b') \qquad v_{ab}(\vec{p},\sigma) = (-1)^{j+\sigma}u_{ab}(\vec{p},-\sigma)$$

$$Pa(\vec{p},\sigma)P^{-1} = \eta^* a(-\vec{p},\sigma) \qquad Pa^{c\dagger}(\vec{p},\sigma)P^{-1} = \eta^c a^{c\dagger}(-\vec{p},\sigma)$$

$$P\psi_{ab}^{AB}(x)P^{-1} = (2\pi)^{-3/2}\sum_\sigma \int d^3p[\eta^* a(-\vec{p},\sigma)e^{ip\cdot x}u_{ab}^{AB}(\vec{p},\sigma) + \eta^c(-1)^{2B}a^{c\dagger}(-\vec{p},\sigma)e^{-ip\cdot x}v_{ab}^{AB}(\vec{p},\sigma)]$$

$$C_{AB}(j\sigma;ab) = (-1)^{A+B-j}C_{BA}(j\sigma;ba)$$

$$\Rightarrow \quad u_{ab}^{AB}(-\vec{p},\sigma) = (-1)^{A+B-j}u_{ba}^{BA}(\vec{p},\sigma) \qquad v_{ab}^{AB}(-\vec{p},\sigma) = (-1)^{A+B-j}v_{ba}^{BA}(\vec{p},\sigma)$$

$$P\psi_{ab}^{AB}(x)P^{-1} = (2\pi)^{-3/2}\sum_\sigma \int d^3p(-1)^{A+B-j}[\eta^* a(\vec{p},\sigma)e^{ip\cdot\mathcal{P}x}u_{ba}^{BA}(\vec{p},\sigma)$$

$$+ \eta^c(-1)^{2B}a^{c\dagger}(\vec{p},\sigma)e^{-ip\cdot\mathcal{P}x}v_{ba}^{BA}(\vec{p},\sigma)]$$

$$\eta^c(-1)^{2B}/\eta^* = (-1)^{2A} \quad \Rightarrow \quad \eta^c = \eta^*(-1)^{2j} \qquad P\psi_{ab}^{AB}(x)P^{-1} = \eta^*(-1)^{A+B-j}\psi_{ba}^{BA}(-\vec{x},x^0)$$

分立对称性变换性质

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, u_{ab}(\vec{p},\sigma)[a(\vec{p},\sigma)e^{ip\cdot x} + (-1)^{2B}(-1)^{j-\sigma}a^{c\dagger}(\vec{p},-\sigma)e^{-ip\cdot x}]$$

$$u_{ab}(\vec{p},\sigma) = \frac{1}{\sqrt{2p^0}}\sum_{a'b'}\left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'}\left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'}C_{AB}(j\sigma;a'b') \qquad v_{ab}(\vec{p},\sigma) = (-1)^{j+\sigma}u_{ab}(\vec{p},-\sigma)$$

$$Ca(\vec{p},\sigma)C^{-1} = \xi^* a^c(\vec{p},\sigma) \qquad\qquad Ca^{c\dagger}(\vec{p},\sigma)C^{-1} = \xi^c a^\dagger(\vec{p},\sigma)$$

$$C\psi_{ab}^{AB}(x)C^{-1} = (2\pi)^{-3/2}\sum_\sigma \int d^3p \, u_{ab}^{AB}(\vec{p},\sigma)[\xi^* a^c(\vec{p},\sigma)e^{ip\cdot x} + \xi^c(-1)^{2B}a^\dagger(\vec{p},-\sigma)(-1)^{j-\sigma}e^{-ip\cdot x}]$$

$$\psi_{ba}^{BA\dagger}(x) = (2\pi)^{-3/2}\sum_\sigma \int d^3p \, u_{ba}^{BA*}(\vec{p},\sigma)[(-1)^{2A}(-1)^{j-\sigma}a^c(\vec{p},-\sigma)e^{ip\cdot x} + a^\dagger(\vec{p},\sigma)e^{-ip\cdot x}]$$

$$-\vec{J}_{\sigma\sigma'}^{(j)*} = (-1)^{-\sigma+\sigma'}\vec{J}_{-\sigma,-\sigma'}^{(j)} \quad\Rightarrow\quad \vec{J}^{(j)*} = -\mathcal{B}\vec{J}^{(j)}\mathcal{B}^{-1} \quad \mathcal{B}_{\bar{\sigma}\sigma} = (-1)^{j+\sigma}\delta_{\bar{\sigma},-\sigma} \quad \mathcal{B}_{\bar{\sigma}\sigma}^{-1} = (-1)^{-j+\sigma}\delta_{\bar{\sigma},-\sigma}$$

$$u_{ba}^{BA}(\vec{p},\sigma)^* = \frac{1}{\sqrt{2p^0}}\sum_{a'b'}\left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{-a,-a'}\left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{-b,-b'}(-1)^{a'-a}(-1)^{b'-b}C_{BA}(j\sigma;b'a')$$

$$C_{BA}(j,-\sigma;-b',-a') = C_{AB}(j\sigma;a'b')\delta_{a'+b',\sigma} \quad\Rightarrow\quad u_{-b,-a}^{BA}(\vec{p},-\sigma)^* = (-1)^{a+b-\sigma}u_{ab}^{AB}(\vec{p},\sigma)$$

$$\psi_{-b,-a}^{BA\dagger}(x) = (2\pi)^{-3/2}\sum_\sigma \int d^3p(-1)^{a+b-\sigma}u_{ab}^{AB}(\vec{p},\sigma)[(-1)^{2A}(-1)^{j+\sigma}a^c(\vec{p},-\sigma)e^{ip\cdot x} + a^\dagger(\vec{p},-\sigma)e^{-ip\cdot x}]$$

$$(-1)^{-2A-a-b-j}\psi_{-b,-a}^{BA\dagger}(x) = (2\pi)^{-3/2}\sum_\sigma \int d^3p \, u_{ab}^{AB}(\vec{p},\sigma)[a^c(\vec{p},\sigma)e^{ip\cdot x} + (-1)^{j-\sigma+2B}a^\dagger(\vec{p},-\sigma)e^{-ip\cdot x}]$$

分立对称性变换性质

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, u_{ab}(\vec{p},\sigma)[a(\vec{p},\sigma)e^{ip\cdot x} + (-1)^{2B}(-1)^{j-\sigma}a^{c\dagger}(\vec{p},-\sigma)e^{-ip\cdot x}]$$

$$u_{ab}(\vec{p},\sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} C_{AB}(j\sigma; a'b') \qquad v_{ab}(\vec{p},\sigma) = (-1)^{j+\sigma}u_{ab}(\vec{p},-\sigma)$$

$$Ca(\vec{p},\sigma)C^{-1} = \xi^* a^c(\vec{p},\sigma) \qquad Ca^{c\dagger}(\vec{p},\sigma)C^{-1} = \xi^c a^\dagger(\vec{p},\sigma)$$

$$C\psi_{ab}^{AB}(x)C^{-1} = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, u_{ab}^{AB}(\vec{p},\sigma)[\xi^* a^c(\vec{p},\sigma)e^{ip\cdot x} + \xi^c(-1)^{2B}a^\dagger(\vec{p},-\sigma)(-1)^{j-\sigma}e^{-ip\cdot x}]$$

$$\psi_{ba}^{BA\dagger}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, u_{ba}^{BA*}(\vec{p},\sigma)[(-1)^{2A}(-1)^{j-\sigma}a^c(\vec{p},-\sigma)e^{ip\cdot x} + a^\dagger(\vec{p},\sigma)e^{-ip\cdot x}]$$

$$\psi_{-b,-a}^{BA\dagger}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, (-1)^{a+b-\sigma} u_{ab}^{AB}(\vec{p},\sigma)[(-1)^{2A}(-1)^{j+\sigma}a^c(\vec{p},-\sigma)e^{ip\cdot x} + a^\dagger(\vec{p},\sigma)e^{-ip\cdot x}]$$

$$(-1)^{-2A-a-b-j}\psi_{-b,-a}^{BA\dagger}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, u_{ab}^{AB}(\vec{p},\sigma)[a^c(\vec{p},\sigma)e^{ip\cdot x} + (-1)^{j-\sigma+2B}a^\dagger(\vec{p},-\sigma)e^{-ip\cdot x}]$$

$$\xi^* = \xi^c \qquad C\psi_{ab}^{AB}(x)C^{-1} = \xi^*(-1)^{-2A-a-b-j}\psi_{-b,-a}^{BA\dagger}(x)$$

$$\xrightarrow[\text{粒子=反粒子}]{} \psi_{ab}^{AB}(x) = (-1)^{-2A-a-b-j}\psi_{-b,-a}^{BA\dagger}(x)$$

分立对称性变换性质

$$\psi_{ab}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3p \, u_{ab}(\vec{p},\sigma)[a(\vec{p},\sigma)e^{ip\cdot x} + (-1)^{2B}(-1)^{j-\sigma}a^{c\dagger}(\vec{p},-\sigma)e^{-ip\cdot x}]$$

$$u_{ab}(\vec{p},\sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} C_{AB}(j\sigma;a'b') \qquad v_{ab}(\vec{p},\sigma) = (-1)^{j+\sigma}u_{ab}(\vec{p},-\sigma)$$

$$Ta(\vec{p},\sigma)T^{-1} = \zeta^*(-1)^{j-\sigma}a(-\vec{p},-\sigma) \qquad\qquad Ta^{c\dagger}(\vec{p},\sigma)T^{-1} = \zeta^c(-1)^{j-\sigma}a^{c\dagger}(-\vec{p},-\sigma)$$

$$T\psi_{ab}^{AB}(x)T^{-1} = (2\pi)^{-\frac{3}{2}} \sum_\sigma \int d^3p \, u_{ab}^{AB}(\vec{p},\sigma)^*(-1)^{j-\sigma}[\zeta^*a(-\vec{p},-\sigma)e^{-ip\cdot x} + \zeta^c(-1)^{2B+j-\sigma}a^{c\dagger}(-\vec{p},\sigma)e^{ip\cdot x}]$$

$$C_{AB}(j\sigma;ab) = (-1)^{A+B-j}C_{AB}(j,-\sigma;-a,-b) \;\Rightarrow\; u_{ab}^{AB*}(-\vec{p},-\sigma) = (-1)^{a+b+\sigma+A+B-j}u_{-a,-b}^{AB}(\vec{p},\sigma)$$

$$\zeta^c = \zeta^* \qquad\qquad T\psi_{ab}^{AB}(x)T^{-1} = \zeta^*(-1)^{a+b+A+B-2j}\psi_{-a,-b}^{AB}(\vec{x},-x^0)$$

$$CPT\psi_{ab}^{AB}(x)[CPT]^{-1} = \xi^*\eta^*\zeta^*(-1)^{2A}\psi_{ab}^{AB\dagger}(-x)$$

量子场小结
$$\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [e^{-ip\cdot x} u(\vec{p},\sigma) a(\vec{p},\sigma) + e^{ip\cdot x} v(\vec{p},\sigma) a^{c\dagger}(\vec{p},\sigma)]$$

标量场: $\sigma = 0$ $\qquad u(\vec{p}) = v(\vec{p}) = 1/\sqrt{2(\vec{p}^2 + M^2)}$ $\qquad$ **自旋统计关系**

$$\mathcal{L}_{\text{自共轭}} = \frac{1}{2} : [(\partial_\mu \phi(x))^2 - M^2 \phi^2(x)] : \qquad (\partial^2 + M^2)\phi(x) = 0 \quad \text{没发现质量与对称性的联系}$$

旋量场: $\sigma = \pm 1/2$ $\quad u(\vec{p},\sigma) = \sqrt{M/p^0} D(L(p)) u(0,\sigma)$ $\qquad v(\vec{p},\sigma) = \sqrt{M/p^0} D(L(p)) v(0,\sigma)$

$$\mathcal{L}_0 =: \overline{\psi}(x)(i\gamma^\mu \partial_\mu - M)\psi(x) : \qquad (i\gamma^\mu \partial_\mu - M)\psi(x) = 0 \quad \text{旋量场质量与\underline{手征对称性}相联系}$$

矢量场: $\sigma = 0, \pm 1$ **or** ± 1 $\quad u(\vec{p},\sigma) = v^*(\vec{p},\sigma) = (2p^0)^{-1/2} e(\vec{p},\sigma)$ $\quad e^\mu(\vec{p},\sigma) \equiv L^\mu_\nu(\vec{p}) e^\nu(0,\sigma)$

$$\mathcal{L}_{\text{自共轭}} =: \{-\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2} M^2 v^2(x)\} : \qquad (\partial^2 + M^2)v^\mu(x) = 0 \quad \partial_\mu v^\mu(x) = 0$$

矢量场质量与规范对称性相联系

一般情况: $a = -A, \cdots, A; \ b = -B, \cdots, B$ $\quad u_{ab}(\vec{p},\sigma) = \frac{1}{\sqrt{2p^0}} \sum_{a'b'} \left(e^{-\hat{p}\cdot\vec{J}^{(A)}\theta}\right)_{aa'} \left(e^{\hat{p}\cdot\vec{J}^{(B)}\theta}\right)_{bb'} C_{AB}(j\sigma; a'b') = (-1)^{j+\sigma} v_{ab}(\vec{p},-\sigma)$

$$[\psi_l(\vec{x},t), \psi_{l'}(\vec{y},t)]_\mp = [\pi_l(\vec{x},t), \pi_{l'}(\vec{y},t)]_\mp = 0 \qquad [\psi_l(\vec{x},t), \pi_{l'}(\vec{y},t)]_\mp = i\delta(\vec{x} - \vec{y})$$

量子场小结续

$$\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [e^{-ip\cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip\cdot x} v(\vec{p}, \sigma) a^{c\dagger}(\vec{p}, \sigma)]$$

自旋统计关系

我们虽然建立了自由场理论,但还没讨论相互作用,因此还不能讨论S矩阵!

现在我们应该能够理解如下问题:

为什么所有具有同样自旋的自由粒子具有同样的波函数?

或说满足同样的场方程. 不管它是在什么物质层次上, 不管它有无内在结构.

因为相对性原理要求它们属于同一个洛伦兹群的表示

这个表示与体系的尺度无关, 完全将波函数确定下来, 因而导致确定的方程!

三种绘景

能动量算符

自由量子场: $\psi(x) = \sum_\sigma \int \dfrac{d^3p}{(2\pi)^{3/2}} [e^{-ip\cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip\cdot x} v(\vec{p}, \sigma) a^{c\dagger}(\vec{p}, \sigma)]$

$$\dot{\psi}(\vec{x}, t) = i[H_0, \psi(\vec{x}, t)] = \frac{\delta H_0}{\delta \pi(\vec{x}, t)} \qquad \dot{\pi}(\vec{x}, t) = i[H_0, \pi(\vec{x}, t)] = -\frac{\delta H_0}{\delta \psi(\vec{x}, t)}$$

$$[\psi_l(\vec{x}, t), \psi_{l'}(\vec{y}, t)]_\mp = [\pi_l(\vec{x}, t), \pi_{l'}(\vec{y}, t)]_\mp = 0 \qquad [\psi_l(\vec{x}, t), \pi_{l'}(\vec{y}, t)]_\mp = i\delta(\vec{x} - \vec{y})$$

自由场的能动量算符:

$$H_0 \equiv P_0^0 = \int d\vec{x} \ : [-\mathcal{L}_0 + \pi_l(x)\dot{\psi}_l(x)] : \qquad \vec{P}_0 \equiv \int d\vec{x} \ : \pi_l(x)\nabla\psi_l(x) : \quad \text{可直接算!}$$

$$\dot{\vec{P}}_0 = \int d\vec{x} \ : \{\dot{\pi}_l(x)\nabla\psi_l(x) + \pi_l(x)\nabla\dot{\psi}_l(x)\} := i\int d\vec{x}\{[H_0, \pi_l(x)]\nabla\psi_l(x) + \pi_l(x)\nabla[H_0, \psi_l(x)]\}$$

$$= i[H_0, \vec{P}_0] = 0$$

$$i[\vec{P}_0, \psi(\vec{x}, t)] = \pm i\int d\vec{y} \ [\pi_l(\vec{y}, t), \psi(\vec{x}, t)]_\mp \nabla_y \psi_l(\vec{y}, t) = \nabla\psi(\vec{x}, t)$$

$$i[\vec{P}_0, \pi(\vec{x}, t)] = i\int d\vec{y} \ \pi_l(\vec{y}, t)\nabla_y[\psi_l(\vec{y}, t), \pi(\vec{x}, t)]_\mp = \nabla\pi(\vec{x}, t)$$
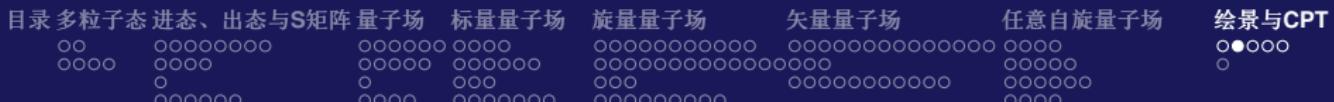
三种绘景

相互作用绘景和海森堡绘景

自由量子场: $\psi(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} [e^{-ip\cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip\cdot x} v(\vec{p}, \sigma) a^{c\dagger}(\vec{p}, \sigma)]$

$$\dot{\psi}(\vec{x}, t) = i[H_0, \psi(\vec{x}, t)] = \frac{\delta H_0}{\delta \pi(\vec{x}, t)} \qquad \dot{\pi}(\vec{x}, t) = i[H_0, \pi(\vec{x}, t)] = -\frac{\delta H_0}{\delta \psi(\vec{x}, t)}$$

$$[\psi_l(\vec{x}, t), \psi_{l'}(\vec{y}, t)]_\mp = [\pi_l(\vec{x}, t), \pi_{l'}(\vec{y}, t)]_\mp = 0 \qquad [\psi_l(\vec{x}, t), \pi_{l'}(\vec{y}, t)]_\mp = i\delta(\vec{x} - \vec{y})$$

相互作用: $\quad S = U(+\infty, -\infty) \qquad \Omega(\tau) = e^{iH\tau} e^{-iH_0\tau} \qquad \Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha$

$$U(\tau, \tau_0) = \Omega^\dagger(\tau)\Omega(\tau_0) = e^{iH_0\tau} e^{-iH(\tau-\tau_0)} e^{-iH_0\tau_0} = \mathbf{T}\, e^{-i\int_{\tau_0}^\tau dt\, V(t)}$$

$$V(t) \equiv e^{iH_0t} V e^{-iH_0t} = \int d\vec{x}\, \mathcal{H}(\vec{r}, t) \qquad V = H - H_0$$

海森堡绘景的场算符: $\quad \psi_H(\vec{x}, t) \equiv \Omega(t)\psi(\vec{x}, t)\Omega^\dagger(t) \qquad \pi_H(\vec{x}, t) \equiv \Omega(t)\pi(\vec{x}, t)\Omega^\dagger(t)$

$$\dot{\psi}_H(\vec{x}, t) = \dot{\Omega}(t)\psi(\vec{x}, t)\Omega^\dagger(t) + \Omega(t)\dot{\psi}(\vec{x}, t)\Omega^\dagger(t) + \Omega(t)\psi(\vec{x}, t)\dot{\Omega}^\dagger(t)$$

$$= i[H\Omega(t) - \Omega(t)H_0]\psi(\vec{x}, t)\Omega^\dagger(t) + i\Omega(t)[H_0, \psi(\vec{x}, t)]\Omega^\dagger(t) + i\Omega(t)\psi(\vec{x}, t)[H_0\Omega^\dagger(t) - \Omega^\dagger(t)H]$$

$$= i[H, \psi_H(\vec{x}, t)] \qquad\qquad \pi_H(\vec{x}, t) = i[H, \pi_H(\vec{x}, t)]$$

$$[\psi_{H,l}(\vec{x}, t), \psi_{H,l'}(\vec{y}, t)]_\mp = [\pi_{H,l}(\vec{x}, t), \pi_{H,l'}(\vec{y}, t)]_\mp = 0 \qquad [\psi_{H,l}(\vec{x}, t), \pi_{H,l'}(\vec{y}, t)]_\mp = i\delta(\vec{x} - \vec{y})$$

三种绘景

海森堡绘景算符时空平移

含相互作用的能动量算符：

$$H \equiv P^0 = \int d\vec{x} : [-\mathcal{L} + \pi_l(x)\dot{\psi}_l(x)] : \qquad \vec{P} \equiv \int d\vec{x} : \pi_l(x)\nabla\psi_l(x) :$$

$$\pi_l \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}_l} \quad \Rightarrow \quad \text{相互作用不含广义速度} \quad \Rightarrow \quad \vec{P} = \vec{P}_0$$

$$\Omega(\tau) = e^{iH\tau} e^{-iH_0\tau} \qquad \psi_H(\vec{x},t) \equiv \Omega(t)\psi(\vec{x},t)\Omega^{\dagger}(t) \quad \pi_H(\vec{x},t) \equiv \Omega(t)\pi(\vec{x},t)\Omega^{\dagger}(t)$$

$$U_0(a)\psi(x)U_0^{-1}(a) = \psi(x+a) \qquad U_0(a) = e^{ia_\mu P_0^\mu}$$

$$\vec{P} = \vec{P}_0 \qquad U(a) = e^{ia_\mu P^\mu}$$

$$\psi_H(x+a) = \Omega(t+a^0)\psi(x+a)\Omega^{\dagger}(t+a^0)$$

$$= e^{iH(t+a^0)} e^{-iH_0(t+a^0)} e^{i(a^0 H_0 - \vec{a}\cdot\vec{P}_0)}\psi(x) e^{-i(a^0 H_0 - \vec{a}\cdot\vec{P}_0)} e^{iH_0(t+a^0)} e^{-iH(t+a^0)}$$

$$= e^{iHt} e^{iHa^0} e^{-iH_0 t} e^{-i\vec{a}\cdot\vec{P}_0}\psi(x) e^{i\vec{a}\cdot\vec{P}_0} e^{iH_0 t} e^{-iHa^0} e^{-iHt}$$

$$= e^{i(Ha^0 - \vec{a}\cdot\vec{P})} e^{iHt} e^{-iH_0 t}\psi(x) e^{iH_0 t} e^{-iHt} e^{-i(Ha^0 - \vec{a}\cdot\vec{P})} = U(a)\psi_H(x)U^{-1}(a)$$
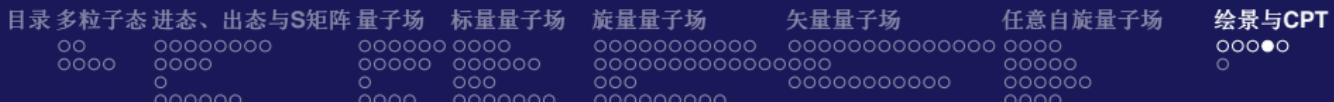
三种绘景

相互作用绘景和薛定谔绘景

自由量子场: $\psi(x) = \sum_\sigma \int \dfrac{d^3p}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma) + e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]$

$$\dot{\psi}(\vec{x},t) = i[H_0,\psi(\vec{x},t)] = \frac{\delta H_0}{\delta\pi(\vec{x},t)} \qquad\qquad \dot{\pi}(\vec{x},t) = i[H_0,\pi(\vec{x},t)] = -\frac{\delta H_0}{\delta\psi(\vec{x},t)}$$

$$[\psi_l(\vec{x},t),\psi_{l'}(\vec{y},t)]_\mp = [\pi_l(\vec{x},t),\pi_{l'}(\vec{y},t)]_\mp = 0 \qquad\qquad [\psi_l(\vec{x},t),\pi_{l'}(\vec{y},t)]_\mp = i\delta(\vec{x}-\vec{y})$$

相互作用: $\quad S = U(+\infty,-\infty) \qquad \Omega(\tau) = e^{iHt}e^{-iH_0\tau} \qquad \Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha$

$$U(\tau,\tau_0) = \Omega^\dagger(\tau)\Omega(\tau_0) = e^{iH_0\tau}e^{-iH(\tau-\tau_0)}e^{-iH_0\tau_0} = \mathbf{T}\,e^{-i\int_{\tau_0}^\tau dt\,V(t)}$$

$$V(t) \equiv e^{iH_0t}Ve^{-iH_0t} = \int d\vec{x}\,\mathcal{H}(\vec{r},t) \qquad\qquad V = H - H_0$$

薛定谔绘景的场算符: $\quad \psi_S(\vec{x},t) \equiv e^{-iH_0t}\psi(\vec{x},t)e^{iH_0t} \qquad \pi_S(\vec{x},t) \equiv e^{-iH_0t}\pi(\vec{x},t)e^{iH_0t}$

$$\dot{\psi}_S(\vec{x},t) = e^{-iH_0t}\psi(\vec{x},t)e^{iH_0t} + e^{-iH_0t}\dot{\psi}(\vec{x},t)e^{iH_0t} + e^{-iH_0t}\psi(\vec{x},t)e^{iH_0t}$$

$$= -ie^{-iH_0t}H_0\psi(\vec{x},t)e^{iH_0t} + ie^{-iH_0t}[H_0,\psi(\vec{x},t)]e^{iH_0t} + ie^{-iH_0t}\psi(\vec{x},t)H_0e^{iH_0t} = 0$$

$$\dot{\pi}_S(\vec{x},t) = 0$$

$$[\psi_{S,l}(\vec{x},t),\psi_{S,l'}(\vec{y},t)]_\mp = [\pi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp = 0 \qquad\qquad [\psi_{S,l}(\vec{x},t),\pi_{S,l'}(\vec{y},t)]_\mp = i\delta(\vec{x}-\vec{y})$$

三种绘景

关于用场量表达的哈密顿量

$\dot{H}_0 = 0$ 明显验证

$$\psi_S(\vec{x}, t) \equiv e^{-iH_0 t}\psi(\vec{x}, t)e^{iH_0 t} \qquad \pi_S(\vec{x}, t) \equiv e^{-iH_0 t}\pi(\vec{x}, t)e^{iH_0 t}$$

$$H \equiv H(\psi_S, \pi_S) = e^{-iH_0 t}H(\psi, \pi)e^{iH_0 t} \qquad H_0 = e^{-iH_0 t}H_0(\psi, \pi)e^{iH_0 t} \qquad V = e^{-iH_0 t}V(\psi, \pi)e^{iH_0 t}$$

相互作用: $\quad S = U(+\infty, -\infty) \qquad \Omega(\tau) = e^{iH\tau}e^{-iH_0\tau} \qquad \Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha$

$$U(\tau, \tau_0) = \Omega^\dagger(\tau)\Omega(\tau_0) = e^{iH_0\tau}e^{-iH(\tau-\tau_0)}e^{iH_0\tau_0} = \mathbf{T}\,e^{-i\int_{\tau_0}^{\tau} dt\, V(t)}$$

$$V(t) \equiv e^{iH_0 t}Ve^{-iH_0 t} = \int d\vec{x}\,\mathcal{H}(\vec{r}, t) \qquad V = H - H_0 \qquad [\mathcal{H}(x), \mathcal{H}(x')] \overset{\text{类空}}{=\!=\!=} 0$$

$$\int d\vec{x}\,\mathcal{H}(\vec{r}, t) = e^{iH_0 t}Ve^{-iH_0 t} = V(\psi, \pi) \qquad S = U(\infty, -\infty) = \mathbf{T}e^{-i\int_{-\infty}^{\infty} dt\, V(\psi,\pi)} = \mathbf{T}e^{-i\int d^4x\,\mathcal{H}(x)}$$

给出$V(\psi, \pi)$,利用ψ, π的产生湮灭算符展开即可计算S矩阵元$\Rightarrow$正则计算体系!

Wick定理给出进一步的简化!

CPT定理

$$\int d\vec{x}\, \mathcal{H}(\vec{r}, t) = e^{iH_0 t} V e^{-iH_0 t} = V(\psi, \pi) \qquad S = U(\infty, -\infty) = \mathbf{T}\, e^{-i\int d^4 x\, \mathcal{H}(x)}$$

$\mathcal{H}(\vec{r}, t)$是**厄米**的洛伦兹标量

$$CPT\psi_{ab}^{AB}(x)[CPT]^{-1} = \xi^* \eta^* \zeta^* (-1)^{2A} \psi_{ab}^{AB\dagger}(-x)$$

为使$\psi_{a_1 b_1}^{A_1 B_1}(x)\psi_{a_2 b_2}^{A_2 B_2}(x)\cdots$能耦合成标量$\mathcal{H}(\vec{r}, t)$, $A_1 + A_2 + \cdots = B_1 + B_2 + \cdots = 0$

沿z轴的无穷小转动不变导致$A_1+A_2+\cdots+B_1+B_2+\cdots=0$; 沿$z$轴的无穷小推进不变导致$A_1+A_2+\cdots-B_1-B_2-\cdots=0$

$$(-1)^{2A_1 + 2A_2 + \cdots} = 1$$

为使$\psi_{a_1 b_1}^{A_1 B_1}(x)\psi_{a_2 b_2}^{A_2 B_2}(x)\cdots$能保证$\mathcal{H}(\vec{r}, t)$厄米，$\psi_{a_1 b_1}^{A_1 B_1}(x)\psi_{a_2 b_2}^{A_2 B_2}(x)\cdots$必须与其共轭场同时出现

因而$\psi_{a_1 b_1}^{A_1 B_1}(x)\psi_{a_2 b_2}^{A_2 B_2}(x)\cdots$ 中**CPT**变换产生的复常数相角无贡献！

$$[CPT]\,\mathcal{H}(x)[CPT]^{-1} = \mathcal{H}(-x)$$

$$[CPT]\, S\,[CPT]^{-1} = \mathbf{T}\, e^{-i\int d^4 x\, CPT\mathcal{H}(x)[CPT]^{-1}} = \mathbf{T}\, e^{-i\int d^4 x\, \mathcal{H}(-x)} = \mathbf{T}\, e^{-i\int d^4 x\, \mathcal{H}(-x)} = S$$