







## S矩阵与正规乘积

### S矩阵与场的产生湮灭算符展开

$$\Phi_\alpha = a^\dagger \dots b^\dagger \Phi_0$$

$$\Phi_\beta = d^\dagger \dots f^\dagger \Phi_0$$

$$S_{\beta\alpha} = (\Phi_\beta, S\Phi_\alpha) = (\Phi_\beta, \mathbf{T} e^{-i \int_{-\infty}^{\infty} dt V(\psi, \pi)} \Phi_\alpha)$$

$$\psi(x) = \sum_{\sigma} \int \frac{d^3p}{(2\pi)^{3/2}} \left[ \underbrace{e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma)}_{\psi^+} + \underbrace{e^{ip \cdot x} v(\vec{p}, \sigma) a^\dagger(\vec{p}, \sigma)}_{\psi^-} \right] \quad \pi(x) = \pi^+(x) + \pi^-(x)$$

需要处理很多个产生湮灭算符的真空期望值的方法！

正规乘积： $\mathbf{N}(UVW \dots Z) = \delta_P U'V'W' \dots Z'$

其中  $U'V'W' \dots Z'$  中的算符排列顺序是在原来  $UVW \dots Z$  的算符排列顺序基础之上，将所有的消灭算符排在所有产生算符的左边 **面对大家**，而消灭算符之间的相对顺序和产生算符之间的相对顺序则保持不变。

从  $UVW \dots Z$  调换到  $U'V'W' \dots Z'$ ，费米子交换偶数次  $\delta_P = 1$ ，否则  $\delta_P = -1$ 。



算符的收缩:  $\psi_l(x)\psi_{l'}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

$$\underline{\psi_l(x)\psi_{l'}(x')} = (\Phi_0, \mathbf{T}(\psi_l(x)\psi_{l'}(x'))\Phi_0)$$

证明: 不妨设  $t > t'$

$$\begin{aligned} \mathbf{T}(\psi_l(x)\psi_{l'}(x')) &= \mathbf{T}(\psi_l(x))\psi_{l'}(x') = \mathbf{N}(\psi_l(x))(\psi_{l'}^+(x') + \psi_{l'}^-(x')) \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) + \psi_l^+(x)\psi_{l'}^-(x') \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) \pm \psi_{l'}^-(x')\psi_l^+(x) + [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp} \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l^-(x)\psi_{l'}^-(x')) + \mathbf{N}(\psi_l^+(x)\psi_{l'}^-(x')) + [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp} \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}^+(x')) + \mathbf{N}(\psi_l(x)\psi_{l'}^-(x')) + (\Phi_0, [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp}\Phi_0) \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \psi_l^+(x)\psi_{l'}^-(x')\Phi_0) \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \psi_l(x)\psi_{l'}(x')\Phi_0) \\ &= \mathbf{N}(\psi_l(x)\psi_{l'}(x')) + (\Phi_0, \mathbf{T}\psi_l(x)\psi_{l'}(x')\Phi_0) \end{aligned}$$



算符的收缩:  $\underline{\psi_l(x)\psi_{l'}(x')} \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

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推论: 相互之间的对易或反对易性质导致

- ▶ 相邻的不同场的场算符之间的收缩等于零
- ▶ 相邻的消灭算符之间,产生算符之间的收缩等于零



## Wick定理

算符的收缩:  $\psi_l(x)\psi_{l'}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

相邻的不同场之间的收缩等于零; 消灭算符之间, 产生算符之间的收缩等于零

定义不相邻的场算符的收缩:  $\mathbf{N}(U \underline{VW} \cdots \underline{YZ}) \equiv \delta_p \underline{VZ} \mathbf{N}(UW \cdots Y)$

其中的下划线是指将两端的算符  $V$  和  $Z$  进行收缩操作, 对下划线中间的算符  $W \cdots Y$  不发生作用, 只是考虑因子  $\delta_p = (-1)^n$ ,  $n$  是将  $V$  和  $Z$  调到一起所交换的费米子算符的数目.

推论:  $\mathbf{N}(U \underline{V} \underline{W} \cdots \underline{XY} \underline{Z}) \equiv \delta_p \underline{VZ} \underline{WY} \mathbf{N}(U \cdots X)$

预备定理: 如果算符  $Z$  的时间值比所有算符都早, 则有

$$\mathbf{N}(UV \cdots XY)Z = \mathbf{N}(UV \cdots XYZ) + \mathbf{N}(\underline{UV} \cdots XYZ) + \mathbf{N}(U \underline{V} \cdots XYZ) + \cdots + \mathbf{N}(UV \cdots X\underline{YZ})$$

证明: 如果  $Z$  是湮灭算符

$$\underline{UZ} = \mathbf{T}(UZ) - \mathbf{N}(UZ) = UZ - UZ = 0 \Rightarrow \text{预备定理恒成立, 只需讨论 } Z \text{ 是产生算符的情况!}$$



算符的收缩:  $\psi_l(x)\psi_{l'}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

相邻的双线性场算符的收缩等于运算后的真空期望值

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推论:  $\mathbf{N}(U \underline{VW} \cdots \underline{XYZ}) \equiv \delta_P \underline{VZ} \underline{WY} \mathbf{N}(U \cdots X)$

预备定理: 如果算符 $Z$ 的时间值比所有算符都早, 则有

$$\mathbf{N}(UV \cdots XY)Z = \mathbf{N}(UV \cdots XYZ) + \mathbf{N}(\underline{UV} \cdots XYZ) + \mathbf{N}(UV \cdots \underline{XYZ}) + \cdots + \mathbf{N}(UV \cdots XY\underline{Z})$$

证明:  $Z$ 是产生算符; 若 $UV \cdots XY$ 是湮灭算符的情形成立, 则对一族产生算符 $P$

$$P\mathbf{N}(UV \cdots XY)Z = P\mathbf{N}(UV \cdots XYZ) + P\mathbf{N}(\underline{UV} \cdots XYZ) + P\mathbf{N}(UV \cdots \underline{XYZ}) + \cdots + P\mathbf{N}(UV \cdots XY\underline{Z})$$

$$\text{利用 } \underline{PZ} = 0 \quad \Rightarrow \quad \mathbf{N}(P\underline{UV} \cdots XYZ) = 0$$

$$\mathbf{N}(P\underline{UV} \cdots XY)Z = \mathbf{N}(P\underline{UV} \cdots XYZ) + \mathbf{N}(P\underline{UV} \cdots \underline{XYZ}) + \mathbf{N}(P\underline{UV} \cdots XY\underline{Z}) + \cdots + \mathbf{N}(P\underline{UV} \cdots XY\underline{Z})$$

$\Rightarrow$  预备定理恒成立, 只需讨论 $UV \cdots XY$ 是湮灭算符的情况!



## Wick定理

算符的收缩:  $\psi_l(x)\psi_{l'}(x') \equiv \mathbf{T}(\psi_l(x)\psi_{l'}(x')) - \mathbf{N}(\psi_l(x)\psi_{l'}(x'))$

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相邻的不同场之间的收缩等于零; 消灭算符之间, 产生算符之间的收缩等于零

定义不相邻的场算符的收缩:  $\mathbf{N}(U \underline{VW} \cdots \underline{YZ}) \equiv \delta_p \underline{VZ} \mathbf{N}(UW \cdots Y)$

$\delta_p = (-1)^n$ ,  $n$ 是将 $V$ 和 $Z$ 调到一起所交换的费米子算符的数目。

推论:  $\mathbf{N}(U \underline{V} \underline{W} \cdots \underline{XY} \underline{Z}) \equiv \delta_p \underline{VZ} \underline{WY} \mathbf{N}(U \cdots X)$

预备定理: 如果算符 $Z$ 的时间值比所有算符都早, 则有

$$\mathbf{N}(UV \cdots XY)Z = \mathbf{N}(UV \cdots XYZ) + \mathbf{N}(\underline{UV} \cdots XYZ) + \mathbf{N}(U \underline{V} \cdots XYZ) + \cdots + \mathbf{N}(UV \cdots \underline{XYZ})$$

证明:  $Z$ 是产生算符;  $UV \cdots XY$ 湮灭算符: 归纳法

$$n = 1: \quad \mathbf{N}(Y)Z = YZ = \mathbf{T}(YZ) = \mathbf{N}(YZ) + \underline{YZ}$$

$\Rightarrow$  预备定理成立, 假设对 $n$ 成立, 只需推出对 $n+1$ 也成立! 对湮灭算符 $R$ :

$$R\mathbf{N}(UV \cdots XY)Z = \underbrace{R\mathbf{N}(UV \cdots XYZ) + R\mathbf{N}(\underline{UV} \cdots XYZ) + R\mathbf{N}(U \underline{V} \cdots XYZ) + \cdots + R\mathbf{N}(UV \cdots \underline{XYZ})}_{\text{计算见下行}}$$

$$R\mathbf{N}(UV \cdots XYZ) = \delta_p \mathbf{T}(RZ)UV \cdots XY = \delta_p (\mathbf{N}(RZ) + \underline{RZ})UV \cdots XY = \mathbf{N}(RUV \cdots XYZ) + \mathbf{N}(\underline{RUV} \cdots XYZ)$$

$$\mathbf{N}(RUV \cdots XY)Z = \mathbf{N}(RUV \cdots XYZ) + \mathbf{N}(\underline{RUV} \cdots XYZ) + \mathbf{N}(R \underline{UV} \cdots XYZ) + \cdots + \mathbf{N}(RUV \cdots \underline{XYZ})$$







## Wick定理

## Wick定理:

## 定理1:

$$\begin{aligned}
 \mathbf{T}(UV \cdots XYZ) &= \mathbf{N}(UV \cdots XYZ) && \text{无收缩项} \\
 &+ \mathbf{N}(\underline{UV} \cdots XYZ) + \cdots + \mathbf{N}(UV \cdots \underline{XYZ}) + && \text{一次收缩项} \\
 &+ \mathbf{N}(\underline{UV} \cdots \underline{XYZ}) + \cdots && \text{二次收缩项} \\
 &+ \cdots \\
 &+ \mathbf{N}(\underline{UVW} \cdots \underline{XYZ}) + \cdots && \text{全部收缩项}
 \end{aligned}$$

## 定理2:

若  $T$  乘积中含  $N$  乘积, 展开仍成立, 但需将各  $N$  乘积内部各算符之间的收缩略去。

$$\Phi_\alpha = a^\dagger \cdots b^\dagger \Phi_0 \qquad \Phi_\beta = d^\dagger \cdots f^\dagger \Phi_0$$

$$S_{\beta\alpha} = (\Phi_\beta, S\Phi_\alpha) = (\Phi_\beta, \mathbf{T} e^{-i \int_{-\infty}^{\infty} dt V(\psi, \pi)} \Phi_\alpha)$$

$$\psi(x) = \sum_{\sigma} \int \frac{d^3 p}{(2\pi)^{3/2}} \underbrace{[e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma)]}_{\psi^+} + \underbrace{[e^{ip \cdot x} v(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]}_{\psi^-} \qquad \pi(x) = \pi^+(x) + \pi^-(x)$$



## Wick定理

## 场的收缩:

$$\psi(x) = \sum_{\sigma} \int \frac{d^3 p}{(2\pi)^{3/2}} [e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]$$

$$\underline{\psi_l(x) \psi_{l'}(x')} = (\Phi_0, \mathbf{T}(\psi_l(x) \psi_{l'}(x')) \Phi_0) = \theta(t - t') [\psi_l^+(x), \psi_{l'}^-(x')] \mp \theta(t' - t) [\psi_{l'}^+(x'), \psi_l^-(x)] \mp$$

$$\text{标量场: } u(\vec{p}) = v(\vec{p}) = \frac{1}{\sqrt{2(\vec{p}^2 + M^2)^{1/4}}} \quad f(z) = \frac{1}{2\pi i} \oint_{\text{逆时针}} dw \frac{f(w)}{w-z}$$

$$\begin{aligned} \underline{\phi(x) \phi^{\dagger}(y)} &= \int \frac{d^3 p d^3 p'}{(2\pi)^3 2(\vec{p}^2 + M^2)^{1/4} (\vec{p}'^2 + M^2)^{1/4}} \{ [e^{-ip \cdot x} a(\vec{p}), e^{ip' \cdot y} a^{\dagger}(\vec{p}')] \theta(x^0 - y^0) + [e^{-ip' \cdot y} a^c(\vec{p}'), e^{ip \cdot x} a^{\dagger}(\vec{p})] \theta(y^0 - x^0) \} \\ &= \int \frac{d^3 p}{2(2\pi)^3 \sqrt{\vec{p}^2 + M^2}} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} [e^{-i\sqrt{\vec{p}^2 + M^2}(x^0 - y^0)} \theta(x^0 - y^0) + e^{i\sqrt{\vec{p}^2 + M^2}(x^0 - y^0)} \theta(y^0 - x^0)] \\ &= \int \frac{d^3 p}{2(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \int_{-\infty}^{\infty} \frac{dp^0}{2i\pi \sqrt{\vec{p}^2 + M^2}} \left[ -\frac{1}{p^0 - \sqrt{\vec{p}^2 + M^2} + i0^{\dagger}} + \frac{1}{p^0 + \sqrt{\vec{p}^2 + M^2} - i0^{\dagger}} \right] \\ &= \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \frac{i}{p^2 - M^2 + i0^{\dagger}} = \underline{\quad x \quad} \underline{\quad y \quad} \end{aligned}$$



## Wick定理

## 场的收缩:

$$\psi(x) = \sum_{\sigma} \int \frac{d^3p}{(2\pi)^{3/2}} [e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]$$

$$\langle \psi_l(x) \psi_{l'}(x') \rangle = (\Phi_0, \mathbf{T}(\psi_l(x) \psi_{l'}(x')) \Phi_0) = \theta(t - t') [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp} \pm \theta(t' - t) [\psi_{l'}^+(x'), \psi_l^-(x)]_{\mp}$$

$$\text{旋量场: } u(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) u(0, \sigma) \quad v(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) v(0, \sigma)$$

$$\begin{aligned} \langle \psi(x) \bar{\psi}(y) \rangle &= \int \frac{d^3p d^3p'}{(2\pi)^3} \sum_{\sigma\sigma'} \{ [e^{-ip \cdot x} u_l(\vec{p}, \sigma) a(\vec{p}, \sigma), e^{ip' \cdot y} \bar{u}_{l'}(\vec{p}', \sigma') a^{\dagger}(\vec{p}', \sigma')]_{+} \theta(x^0 - y^0) \\ &\quad - [e^{-ip' \cdot y} \bar{v}_{l'}(\vec{p}', \sigma') a^c(\vec{p}', \sigma'), e^{ip \cdot x} v_l(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]_{+} \theta(y^0 - x^0) \} \\ &= \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \sum_{\sigma} [e^{-i\sqrt{\vec{p}^2 + M^2}(x^0 - y^0)} u_l(\vec{p}, \sigma) \bar{u}_{l'}(\vec{p}, \sigma) \theta(x^0 - y^0) - e^{i\sqrt{\vec{p}^2 + M^2}(x^0 - y^0)} \bar{v}_{l'}(-\vec{p}, \sigma) v_l(-\vec{p}, \sigma) \theta(y^0 - x^0)] \\ &= \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \int_{-\infty}^{\infty} \frac{dp^0}{2i\pi 2\sqrt{\vec{p}^2 + M^2}} \left[ -\frac{p^0 \gamma^0 - \vec{p} \cdot \vec{\gamma} + M}{p^0 - \sqrt{\vec{p}^2 + M^2} + i0^{\dagger}} - \frac{-p^0 \gamma^0 + \vec{p} \cdot \vec{\gamma} - M}{p^0 + \sqrt{\vec{p}^2 + M^2} - i0^{\dagger}} \right]_{ll'} \\ &= \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-y)} \left[ \frac{i(p_{\mu} \gamma^{\mu} + M)}{p^2 - M^2 + i0^{\dagger}} \right]_{ll'} = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-y)} \left[ \frac{i}{p_{\mu} \gamma^{\mu} - M + i0^{\dagger}} \right]_{ll'} \quad \xrightarrow{x \quad y} \end{aligned}$$



## Wick定理

## 场的收缩:

$$\psi(x) = \sum_{\sigma} \int \frac{d^3p}{(2\pi)^{3/2}} [e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]$$

$$\langle \psi_l(x) \psi_{l'}(x') \rangle = (\Phi_0, \mathbf{T}(\psi_l(x) \psi_{l'}(x')) \Phi_0) = \theta(t - t') [\psi_l^+(x), \psi_{l'}^-(x')]_{\mp} \pm \theta(t' - t) [\psi_{l'}^+(x'), \psi_l^-(x)]_{\mp}$$

$$\text{有质矢量场: } u^{\mu}(\vec{p}, \sigma) = v^{\mu*}(\vec{p}, \sigma) = (2p^0)^{-1/2} e^{\mu}(\vec{p}, \sigma) \quad e^{\mu}(\vec{p}, \sigma) \equiv L_{\nu}^{\mu}(\vec{p}) e^{\nu}(0, \sigma)$$

$$\begin{aligned} \langle v^{\mu}(x) v^{\nu\dagger}(y) \rangle &= \int \frac{d^3p d^3p'}{2(2\pi)^3 (\vec{p}^2 + M^2)^{1/4} (\vec{p}'^2 + M^2)^{1/4}} \sum_{\sigma\sigma'} \{ [e^{-ip \cdot x} e^{\mu}(\vec{p}, \sigma) a(\vec{p}, \sigma), e^{ip' \cdot y} e^{\nu*}(\vec{p}', \sigma') a^{\dagger}(\vec{p}', \sigma')] \\ &\quad + [e^{-ip' \cdot y} e^{\nu}(\vec{p}', \sigma') a^{\dagger}(\vec{p}', \sigma'), e^{ip \cdot x} e^{\mu}(\vec{p}, \sigma) a(\vec{p}, \sigma)] \theta(y^0 - x^0) \} \end{aligned}$$

$$= \int \frac{d^3p e^{i\vec{p} \cdot (\vec{x} - \vec{y})}}{2(2\pi)^3 \sqrt{\vec{p}^2 + M^2}} \sum_{\sigma} [e^{-i\sqrt{\vec{p}^2 + M^2}(x^0 - y^0)} e^{\mu}(\vec{p}, \sigma) e^{\nu*}(\vec{p}, \sigma) \theta(x^0 - y^0) + e^{i\sqrt{\vec{p}^2 + M^2}(x^0 - y^0)} e^{\nu}(-\vec{p}, \sigma) e^{\mu*}(-\vec{p}, \sigma) \theta(y^0 - x^0)]$$

$$= \int \frac{d^3p e^{i\vec{p} \cdot (\vec{x} - \vec{y})}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dp^0 e^{-ip^0(x^0 - y^0)}}{2i\pi 2\sqrt{\vec{p}^2 + M^2}} \left[ -\frac{-g^{\mu\nu} + p^{\mu} p^{\nu} / M^2}{p^0 - \sqrt{\vec{p}^2 + M^2} + i0^{\dagger}} + \frac{-g^{\mu\nu} + p^{\mu} p^{\nu} / M^2}{p^0 + \sqrt{\vec{p}^2 + M^2} - i0^{\dagger}} \right]$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - y)} \frac{i(-g^{\mu\nu} + p^{\mu} p^{\nu} / M^2)}{p^2 - M^2 + i0^{\dagger}} = \begin{matrix} x & & y \\ & \text{~~~~~} & \\ & \text{~~~~~} & \end{matrix}$$



态之间的编时乘积的格林函数  $(\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+})$

$$U(t, t') = \Omega^{\dagger}(t) \Omega(t') \quad \psi_H(x) = \Omega(t) \psi(x) \Omega^{\dagger}(t) \quad \Psi_{\alpha}^{\pm} = \Omega(\mp\infty) \Phi_{\alpha}$$

$$S_{\beta\alpha} = (\Phi_{\beta}, U(\infty, -\infty) \Phi_{\alpha}) = (\Phi_{\beta}, \mathbf{T} e^{-i \int d^4x V(\psi, \pi)} \Phi_{\alpha})$$

$$S_{\beta\alpha}[J] \equiv (\Phi_{\beta}, \mathbf{T} e^{-i \int d^4x V_J(\psi, \pi)} \Phi_{\alpha}) \Leftarrow V_J(\psi, \pi) \equiv V(\psi, \pi) + \psi_l(x) J_l(x)$$

$$\left. \frac{\delta S_{\beta\alpha}[J]}{\delta J_{l_1}(x_1) \delta J_{l_2}(x_2) \cdots \delta J_{l_n}(x_n)} \right|_{J_l(x)=0} = (-i)^n (\Phi_{\beta}, \mathbf{T} \psi_{l_1}(x_1) \psi_{l_2}(x_2) \cdots \psi_{l_n}(x_n) e^{-i \int d^4x V(\psi, \pi)} \Phi_{\alpha})$$

$$= (-i)^n (\Phi_{\beta}, \mathbf{T} \psi_{l_1}(x_1) \psi_{l_2}(x_2) \cdots \psi_{l_n}(x_n) U(\infty, -\infty) \Phi_{\alpha}) \quad \text{Wick定理} \quad \underline{\text{所有可能的全部收缩项之和}}$$

$$\stackrel{t_1 \geq t_2 \geq \cdots \geq t_n}{=} (-i)^n (\Phi_{\beta}, U(\infty, t_1) \psi_{l_1}(x_1) U(t_1, t_2) \psi_{l_2}(x_2) U(t_2, t_3) \cdots U(t_{n-1}, t_n) \psi_{l_n}(x_n) U(t_n, -\infty) \Phi_{\alpha})$$

$$= (-i)^n (\Phi_{\beta}, \Omega^{\dagger}(\infty) \Omega(t_1) \psi_{l_1}(x_1) \Omega^{\dagger}(t_1) \Omega(t_2) \psi_{l_2}(x_2) \Omega^{\dagger}(t_2) \cdots \Omega(t_n) \psi_{l_n}(x_n) \Omega^{\dagger}(t_n) \Omega(-\infty) \Phi_{\alpha})$$

$$= (-i)^n (\Psi_{\beta}^{-}, \psi_{H,l_1}(x_1) \psi_{H,l_2}(x_2) \cdots \psi_{H,l_n}(x_n) \Psi_{\alpha}^{+})$$

$$= (-i)^n (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,l_1}(x_1) \psi_{H,l_2}(x_2) \cdots \psi_{H,l_n}(x_n) \Psi_{\alpha}^{+})$$



编时乘积的格林函数  $(\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$

以后会证明: 若已知所有的格林函数, 则就可以得到体系的作用量!

对前  $n$  个粒子在无穷将来  $t_1, \dots, t_n \rightarrow \infty$ , 后  $l$  个粒子在无穷过去  $t_{n+1}, \dots, t_{n+l} \rightarrow -\infty$  的情况:

$$\begin{aligned}
 & (\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) | \text{前}n\text{个粒子在无穷将来, 后}l\text{个粒子在无穷过去的情况} \\
 & = \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0) \\
 & \times \sum_{\mathcal{P}_{1, \dots, n}} \sum_{\mathcal{P}_{n+1, \dots, n+l}} \theta(x_1^0 - x_2^0) \cdots \theta(x_{n-1}^0 - x_n^0) \theta(x_{n+1}^0 - x_{n+2}^0) \cdots \theta(x_{n+l-1}^0 - x_{n+l}^0) \\
 & \times (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)
 \end{aligned}$$

▶ 编时因子  $\theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0)$  是 不可以被去掉 的!

▶  $(\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) =$   
 $(\Psi_0^-, \psi_{H,1}(x_1 + a) \cdots \psi_{H,n}(x_n + a) \psi_{H,n+1}(x_{n+1} + a) \cdots \psi_{H,n+l}(x_{n+l} + a) \Psi_0^+)$



编时乘积的格林函数  $(\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$

讨论 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去 的情况:

$(\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$  | 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$= \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0)$$

$$\times \sum_{\mathcal{P}_{1, \dots, n}} \sum_{\mathcal{P}_{n+1, \dots, n+l}} \theta(x_1^0 - x_2^0) \cdots \theta(x_{n-1}^0 - x_n^0) \theta(x_{n+1}^0 - x_{n+2}^0) \cdots \theta(x_{n+l-1}^0 - x_{n+l}^0)$$

$$\times (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$$

$$= \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0)$$

$$\times \sum_{\mathcal{P}_{1, \dots, n}} \sum_{\mathcal{P}_{n+1, \dots, n+l}} \theta(x_1^0 - x_2^0) \cdots \theta(x_{n-1}^0 - x_n^0) \theta(x_{n+1}^0 - x_{n+2}^0) \cdots \theta(x_{n+l-1}^0 - x_{n+l}^0)$$

$$\times \int d\gamma d\gamma' (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_\gamma^-) (\Psi_\gamma^-, \Psi_{\gamma'}^+) (\Psi_{\gamma'}^+, \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$$





## 约化公式

$$\begin{aligned}
 & (\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) | \text{前}n\text{个粒子在无穷将来, 后}l\text{个粒子在无穷过去的情况} \\
 & = \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0) \\
 & \quad \times \sum_{\mathcal{P}} \sum_{1, \dots, n} \sum_{\mathcal{P}} \sum_{n+1, \dots, n+l} \theta(x_1^0 - x_2^0) \cdots \theta(x_{n-1}^0 - x_n^0) \theta(x_{n+1}^0 - x_{n+2}^0) \cdots \theta(x_{n+l-1}^0 - x_{n+l}^0) \\
 & \quad \times \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d\vec{p}_1 \cdots d\vec{p}_{n+l} (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-) \\
 & \quad \times (\Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+)(\Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)
 \end{aligned}$$

$$\Psi_0^\pm = \Psi_0 \quad \Psi_{\vec{p}, \sigma}^\pm = \Psi_{\vec{p}, \sigma} \quad \text{进态和出态的粒子无穷分离} \quad \psi_{H,i}(x_i) = e^{iP \cdot x_i} \psi_H(0) e^{-iP \cdot x_i}$$

$$\begin{aligned}
 & (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-) = (\Psi_0, \psi_{H,1}(x_1) \Psi_{\vec{p}_1, \sigma_1}^-) \cdots (\Psi_0, \psi_{H,n}(x_n) \Psi_{\vec{p}_n, \sigma_n}^-) \\
 & = e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}^-) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n}^-) \\
 & (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) \\
 & = (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \psi_{H,n+1}(x_{n+1}) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+l}(x_{n+l}) \Psi_0) \\
 & = e^{ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+l}(0) \Psi_0)
 \end{aligned}$$



## 为什么只考虑 $n$ 和 $l$ 粒子的中间态?

$$\begin{aligned} & \int d\gamma d\gamma' (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_\gamma^-) (\Psi_\gamma^-, \Psi_{\gamma'}^+) (\Psi_{\gamma'}^+, \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) \\ &= \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d\vec{p}_1 \cdots d\vec{p}_{n+l} (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-) \\ & \times (\Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) \\ & (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-) = (\Psi_0, \psi_{H,1}(x_1) \Psi_{\vec{p}_1, \sigma_1}^-) \cdots (\Psi_0, \psi_{H,n}(x_n) \Psi_{\vec{p}_n, \sigma_n}^-) \\ & (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+1}(x_{n+1}) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) \\ &= (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \psi_{H,n+1}(x_{n+1}) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}^+, \psi_{H,n+l}(x_{n+l}) \Psi_0) \end{aligned}$$

♣ 如果考虑  $m > n$  粒子的中间态: 因子化后会出现真空态与单粒子态的内积, 因而为零

♠ 如果考虑  $m < n$  粒子的中间态: 无穷分离时因子化后的会出现单个场的真空期望值, 因而为零



**多粒子态的归一化:** 记  $\Phi_{p_1, \sigma_1, n_1; p_2, \sigma_2, n_2; \dots} = \Phi_{p_1, p_2, \dots}$

真空态  $\Phi_0$  和单粒子态  $\Phi_q$

$$(\Phi_0, \Phi_0) = 1 \quad (\Phi_{q'}, \Phi_q) = \delta(q' - q) \equiv \delta^3(\vec{q}' - \vec{q}) \delta_{\sigma' \sigma} \delta_{n' n}$$

**对两粒子态**  $\Phi_{q_1 q_2}$   $(\Phi_{q'_1 q'_2}, \Phi_{q_1 q_2}) = \delta(q'_1 - q_1) \delta(q'_2 - q_2) \pm \delta(q'_2 - q_1) \delta(q'_1 - q_2)$

负号对两个粒子都是费米子的情形, 正号对其它情形(两个粒子都是玻色子或一个是玻色子一个是费米子)。

**一般情况:**  $(\Phi_{q'_1 q'_2 \dots q'_M}, \Phi_{q_1 q_2 \dots q_N}) = \delta_{MN} \sum_{\mathcal{P}} \delta_{\mathcal{P}} \prod_i \delta(q_i - q'_{\mathcal{P}i})$

求和对所有可能的对指标  $1, 2, \dots, N$  的交换排序  $\mathcal{P}$  实行. 对交换排序中涉及奇数次费米子交换时,  $\delta_{\mathcal{P}} = -1$ , 其它情况的交换排序  $\delta_{\mathcal{P}} = 1$ .

**约定:**  $(\Phi_{\alpha'}, \Phi_{\alpha}) = \delta(\alpha' - \alpha) \quad \int d\alpha \dots \equiv \sum_{n_1 \sigma_1 n_2 \sigma_2 \dots} \int d\vec{p}_1 \int d\vec{p}_2 \dots$

$$\Phi = \int d\alpha \Phi_{\alpha} (\Phi_{\alpha}, \Phi) \quad 1 = \int d\alpha \Phi_{\alpha} (\Phi_{\alpha}, \Phi) \quad \underline{\text{多粒子态构成完备集!}}$$



## 约化公式

$$\begin{aligned}
 & (\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) | \text{前}n\text{个粒子在无穷将来, 后}l\text{个粒子在无穷过去的情况} \\
 & = \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0) \\
 & \quad \times \sum_{\mathcal{P}_{1,\dots,n}} \sum_{\mathcal{P}_{n+1,\dots,n+l}} \theta(x_1^0 - x_2^0) \cdots \theta(x_{n-1}^0 - x_n^0) \theta(x_{n+1}^0 - x_{n+2}^0) \cdots \theta(x_{n+l-1}^0 - x_{n+l}^0) \\
 & \quad \times \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d\vec{p}_1 \cdots d\vec{p}_{n+l} e^{-ip_1 x_1 - \cdots - ip_n x_n + ip_{n+1} x_{n+1} + \cdots + ip_{n+l} x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n}) \\
 & \quad \times (\Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0) \\
 & \quad \sum_{\mathcal{P}_{1,\dots,n}} \theta(x_1^0 - x_2^0) \cdots \theta(x_{n-1}^0 - x_n^0) = 1 \quad \sum_{\mathcal{P}_{n+1,\dots,n+l}} \theta(x_{n+1}^0 - x_{n+2}^0) \cdots \theta(x_{n+l-1}^0 - x_{n+l}^0) = 1 \\
 & = \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0) \\
 & \quad \times \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d\vec{p}_1 \cdots d\vec{p}_{n+l} e^{-ip_1 x_1 - \cdots - ip_n x_n + ip_{n+1} x_{n+1} + \cdots + ip_{n+l} x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n}) \\
 & \quad \times (\Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)
 \end{aligned}$$



## 约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$  | 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$= \theta(x_1^0) \cdots \theta(x_n^0) \theta(-x_{n+1}^0) \cdots \theta(-x_{n+l}^0)$$

$$\times \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d\vec{p}_1 \cdots d\vec{p}_{n+l} e^{-ip_1 x_1 - \cdots - ip_n x_n + ip_{n+1} x_{n+1} + \cdots + ip_{n+l} x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n})$$

$$\times (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \Psi_{\vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \Psi_{\vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)$$

$$\theta(x_i^0 - x_{i+1}^0) = -\frac{1}{2i\pi} \int_{-\infty}^{\infty} d\omega_i \frac{e^{-i\omega_i(x_i^0 - x_{i+1}^0)}}{\omega_i + i0^+}$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \int_{-\infty}^{\infty} d\omega_1 \cdots d\omega_{n+l} \frac{e^{-i\omega_1 x_1^0 - \cdots - i\omega_n x_n^0 + i\omega_{n+1} x_{n+1}^0 + \cdots + i\omega_{n+l} x_{n+l}^0}}{(\omega_1 + i0^+) \cdots (\omega_{n+l} + i0^+)}$$

$$\times \int d\vec{p}_1 \cdots d\vec{p}_{n+l} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n})$$

$$\times (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \Psi_{\vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \Psi_{\vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)$$



## 约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l})\Psi_0^+)$  | 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \int_{-\infty}^{\infty} \frac{d\omega_1 \cdots d\omega_{n+l} e^{-i\omega_1 x_1^0 \cdots -i\omega_n x_n^0 + i\omega_{n+1} x_{n+1}^0 \cdots + i\omega_{n+l} x_{n+l}^0}}{(\omega_1 + i0^+) \cdots (\omega_{n+l} + i0^+)}$$

$$\times \int d\vec{p}_1 \cdots d\vec{p}_{n+l} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n})$$

$$\times (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \int_{-\infty}^{\infty} \frac{d\omega_1 \cdots d\omega_{n+l}}{(\omega_1 + i0^+) \cdots (\omega_{n+l} + i0^+)}$$

$$\times \int d\vec{p}'_1 \cdots d\vec{p}'_{n+l} e^{-ip'_1 \cdot x_1 - \cdots - ip'_n \cdot x_n + ip'_{n+1} \cdot x_{n+1} + \cdots + ip'_{n+l} \cdot x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}'_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}'_n, \sigma_n})$$

$$\times (\Psi_{\vec{p}'_1, \sigma_1, \cdots, \vec{p}'_n, \sigma_n}^-, \Psi_{\vec{p}'_{n+1}, \sigma_{n+1}, \cdots, \vec{p}'_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}'_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}'_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)$$

$$\vec{p}'_i = \vec{p}_i \quad i = 1, 2, \cdots, n+l$$

$$p_i^{0'} = \sqrt{\vec{p}_i^2 + M_i^2} + \omega_i \quad i = 1, 2, \cdots, n+l$$



## 约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l})\Psi_0^+)$  | 前 $n$ 个粒子在无穷将来, 后 $l$ 个粒子在无穷过去的情况

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \int_{-\infty}^{\infty} \frac{d\omega_1 \cdots d\omega_{n+l}}{(\omega_1 + i0^+) \cdots (\omega_{n+l} + i0^+)}$$

$$\times \int d\vec{p}_1 \cdots d\vec{p}_{n+l} e^{-ip'_1 \cdot x_1 - \cdots - ip'_n \cdot x_n + ip'_{n+1} \cdot x_{n+1} + \cdots + ip'_{n+l} \cdot x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n})$$

$$\times (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)$$

$$\vec{p}'_i = \vec{p}_i \quad i = 1, 2, \cdots, n+l$$

$$p_i^0 = \sqrt{\vec{p}_i^2 + M_i^2} + \omega_i \quad i = 1, 2, \cdots, n+l$$

$$= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d^4 p_1 \cdots d^4 p_{n+l} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1})$$

$$\cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n}) (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0)$$

$$\cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0) (p_1^0 - \sqrt{\vec{p}_1^2 + M_1^2} + i0^+)^{-1} \cdots (p_{n+l}^0 - \sqrt{\vec{p}_{n+l}^2 + M_{n+l}^2} + i0^+)^{-1}$$



## 约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l})\Psi_0^+)$  | 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$\begin{aligned}
 &= \frac{(-1)^{n+l}}{(2i\pi)^{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \int d^4 p_1 \cdots d^4 p_{n+l} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1}) \\
 &\cdots (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n}) (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \vec{p}_n, \sigma_n, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \vec{p}_{n+l}, \sigma_{n+l}) (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0) \\
 &\cdots (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0) (p_1^0 - \sqrt{\vec{p}_1^2 + M_1^2 + i0^+})^{-1} \cdots (p_{n+l}^0 - \sqrt{\vec{p}_{n+l}^2 + M_{n+l}^2 + i0^+})^{-1} \\
 &\quad \frac{1}{(p^0 - \sqrt{\vec{p}^2 + M^2 + i0^+})} = \frac{(p^0 + \sqrt{\vec{p}^2 + M^2 - i0^+})}{(p^0)^2 - (\vec{p}^2 + M^2 - i0^+)} = \frac{2\sqrt{\vec{p}^2 + M^2}}{p^2 - M^2 + i0^+} + \text{非极点项} \\
 &= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_{n+l}}{(2\pi)^4} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ \frac{2i(2\pi)^3 p_1^0 (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1})}{p_1^2 - M_1^2 + i0^+} \right. \\
 &\quad \cdots \frac{2i(2\pi)^3 p_n^0 (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \vec{p}_n, \sigma_n, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \vec{p}_{n+l}, \sigma_{n+l}) \\
 &\quad \left. \times \frac{2i(2\pi)^3 p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}} \psi_{H,n+1}(0) \Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}} \psi_{H,n+l}(0) \Psi_0)}{p_{n+l}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \right\}
 \end{aligned}$$





## 约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l})\Psi_0^+)$  | 前 $n$ 个粒子在无穷将来, 后 $l$ 个粒子在无穷过去的情况

$$\begin{aligned}
 &= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_{n+l}}{(2\pi)^4} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ \frac{2i(2\pi)^3 p_1^0 (\Psi_0, \psi_{H,1}(0) \Psi_{\vec{p}_1, \sigma_1})}{p_1^2 - M_1^2 + i0^+} \right. \\
 &\quad \cdots \frac{2i(2\pi)^3 p_n^0 (\Psi_0, \psi_{H,n}(0) \Psi_{\vec{p}_n, \sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+) \\
 &\quad \left. \times \frac{2i(2\pi)^3 p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{H,n+1}(0) \Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{H,n+l}(0) \Psi_0)}{p_{n+l}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \right\} \\
 &= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_{n+l}}{(2\pi)^4} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ \frac{2i(2\pi)^3 p_1^0 (\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1})}{p_1^2 - M_1^2 + i0^+} \right. \\
 &\quad \cdots \frac{2i(2\pi)^3 p_n^0 (\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi_{\vec{p}_1, \sigma_1, \dots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \dots, \vec{p}_{n+l}, \sigma_{n+l}}^+) \quad \text{去掉了下标H!} \\
 &\quad \left. \times \frac{2i(2\pi)^3 p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0)}{p_{n+l}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \right\}
 \end{aligned}$$



## 约化公式

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$  | 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_{n+l}}{(2\pi)^4} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ \frac{2i(2\pi)^3 p_1^0 (\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1})}{p_1^2 - M_1^2 + i0^+} \right. \\ \cdots \frac{2i(2\pi)^3 p_n^0 (\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n})}{p_n^2 - M_n^2 + i0^+} (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+) \\ \left. \times \frac{2i(2\pi)^3 p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0)}{p_{n+1}^2 - M_{n+1}^2 + i0^+} \cdots \frac{2i(2\pi)^3 p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0)}{p_{n+l}^2 - M_{n+l}^2 + i0^+} + \text{非物理极点项} \right\}$$

$(\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$  | 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$= \int \frac{d^4 p_1 \cdots d^4 p_{n+l}}{(i\pi)^{n+l}} e^{-ip_1 \cdot x_1 - \cdots - ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} + \cdots + ip_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ p_1^0 (\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots p_n^0 (\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n}) \right. \\ \left. \times (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^-, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+) p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0) \cdots p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0) \right. \\ \left. + \text{物理零点项} \right\}$$



## 约化公式

$(\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) |$  前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去的情况

$$= \int \frac{d^4 p_1 \cdots d^4 p_{n+l}}{(i\pi)^{n+l}} e^{-ip_1 \cdot x_1 \cdots -ip_n \cdot x_n + ip_{n+1} \cdot x_{n+1} \cdots + ip_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ p_1^0 (\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots p_n^0 (\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n}) \right. \\ \left. \times (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \vec{p}_n, \sigma_n, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \vec{p}_{n+l}, \sigma_{n+l}) p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0) \cdots p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0) \right. \\ \left. + \text{物理零点项} \right\}$$

$(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$

不再要求 前  $n$  个粒子在无穷将来, 后  $l$  个粒子在无穷过去

$$\equiv \int \frac{d^4 p_1 \cdots d^4 p_{n+l}}{(2\pi)^{4n+4l}} e^{-ip_1 \cdot x_1 \cdots -ip_n \cdot x_n - ip_{n+1} \cdot x_{n+1} \cdots -ip_{n+l} \cdot x_{n+l}} G_{00}(p_1, \cdots, p_n, p_{n+1}, \cdots, p_{n+l})$$

$$(-p_1^2 + M_1^2) \cdots (-p_{n+l}^2 + M_{n+l}^2) G_{00}(p_1, \cdots, p_n, -p_{n+1}, \cdots, -p_{n+l})$$

$$= (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ p_1^0 (\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots p_n^0 (\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n}) (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \vec{p}_n, \sigma_n, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \vec{p}_{n+l}, \sigma_{n+l}) \right.$$

$$\left. \times p_{n+1}^0 (\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0) \cdots p_{n+l}^0 (\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0) + \text{物理零点项} \right\}$$



## 约化公式

$$\begin{aligned}
 & (-p_1^2 + M_1^2) \cdots (-p_{n+l}^2 + M_{n+l}^2) G_{00}(p_1, \cdots, p_n, -p_{n+1}, \cdots, -p_{n+l}) \\
 &= (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_{n+l}} \left\{ p_1^0(\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots p_n^0(\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n}) (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^- , \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+ ) \right. \\
 &\quad \left. \times p_{n+1}^0(\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0) \cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0) + \text{物理零点项} \right\} \\
 &= \int d^4x_1 \cdots d^4x_{n+l} e^{ip_1 \cdot x_1 \cdots + ip_n \cdot x_n - ip_{n+1} \cdot x_{n+1} \cdots - ip_{n+l} \cdot x_{n+l}} \times (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) \\
 &\quad \times (\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) \\
 &\lim_{p_i^2 \rightarrow M_i^2} \int d^4x_1 \cdots d^4x_{n+l} e^{ip_1 \cdot x_1 \cdots + ip_n \cdot x_n - ip_{n+1} \cdot x_{n+1} \cdots - ip_{n+l} \cdot x_{n+l}} \times (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) \\
 &\quad \times (\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+) \\
 &= (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_{n+l}} p_1^0(\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots p_n^0(\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n}) \\
 &\quad \times (\Psi_{\vec{p}_1, \sigma_1, \cdots, \vec{p}_n, \sigma_n}^- , \Psi_{\vec{p}_{n+1}, \sigma_{n+1}, \cdots, \vec{p}_{n+l}, \sigma_{n+l}}^+ ) p_{n+1}^0(\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0) \cdots p_{n+l}^0(\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0)
 \end{aligned}$$



## 约化公式

$$\int d^4x_1 \cdots d^4x_{n+l} e^{ip_1 \cdot x_1 \cdots + ip_n \cdot x_n - ip_{n+1} \cdot x_{n+1} \cdots - ip_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2)$$

$$\times (\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$$

$$= (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_{n+l}} P_1^0(\Psi_0, \psi_1(0) \Psi_{\vec{p}_1, \sigma_1}) \cdots P_n^0(\Psi_0, \psi_n(0) \Psi_{\vec{p}_n, \sigma_n})$$

$$\times (\Psi_{\vec{p}_1, \sigma_1}^-, \cdots, \Psi_{\vec{p}_n, \sigma_n}^+, \Psi_{\vec{p}_{n+1}, \sigma_{n+1}}^+, \cdots, \Psi_{\vec{p}_{n+l}, \sigma_{n+l}}^+) P_{n+1}^0(\Psi_{\vec{p}_{n+1}, \sigma_{n+1}}, \psi_{n+1}(0) \Psi_0) \cdots P_{n+l}^0(\Psi_{\vec{p}_{n+l}, \sigma_{n+l}}, \psi_{n+l}(0) \Psi_0)$$

**实标量场:**  $\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2} \sqrt{2p^0}} [e^{-ip \cdot x} a(\vec{p}) + e^{ip \cdot x} a^\dagger(\vec{p})] \quad a(\vec{p}) \Phi_{\vec{q}} = \delta(p - q) \Phi_0$

$$a(\vec{p}) \Psi_{\vec{q}} = Z_\phi \delta(p - q) \Psi_0 \quad (2\pi)^{3/2} \sqrt{2q^0} (\Psi_0, \phi(0) \Psi_{\vec{q}}) = Z_\phi \quad Z_\phi = 0 \text{ 束缚态}$$

$$(\Psi_{\vec{q}_1}^-, \cdots, \Psi_{\vec{q}_n}^+, \Psi_{\vec{q}_{n+1}}^+, \cdots, \Psi_{\vec{q}_{n+l}}^+)$$

$$= \frac{i^{n+l}}{(2\pi)^{3(n+l)/2} Z_\phi^{n+l} \sqrt{2q_1^0 \cdots 2q_{n+l}^0}} \int d^4x_1 \cdots d^4x_{n+l} e^{iq_1 \cdot x_1 \cdots + iq_n \cdot x_n - iq_{n+1} \cdot x_{n+1} \cdots - iq_{n+l} \cdot x_{n+l}}$$

$$\times (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}\phi_{H,1}(x_1) \cdots \phi_{H,n+l}(x_{n+l}) \Psi_0^+)$$



## 约化公式

$$\text{复标量场: } \phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p^0}} [e^{-ip \cdot x} a(\vec{p}) + e^{ip \cdot x} a^\dagger(\vec{p})]$$

$$\int d^4x_1 \cdots d^4x_{n+l} e^{ip_1 \cdot x_1 \cdots + ip_n^c \cdot x_n - ip_{n+1}^c \cdot x_{n+1} \cdots - ip_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}\phi_{H,1}(x_1) \cdots \phi_{H,n'}(x_{n'}) \phi_{H,n'+1}^\dagger(x_{n'+1}) \cdots \phi_{H,n}^\dagger(x_n) \phi_{H,n+1}(x_{n+1}) \cdots \phi_{H,n+l'}(x_{n+l'}) \phi_{H,n+l'+1}^\dagger(x_{n+l'+1}) \cdots \phi_{H,n+l}^\dagger(x_{n+l}) \Psi_0^+) \\ = (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_{n+l}} P_1^0(\Psi_0, \phi_1(0) \Psi_{\vec{p}_1}) \cdots P_{n'}^0(\Psi_0, \phi_{n'}(0) \Psi_{\vec{p}_{n'}}) P_{n'+1}^{c0}(\Psi_0, \phi_{n'+1}^\dagger(0) \Psi_{\vec{p}_{n'+1}^c}) \cdots P_{n+l}^{c0}(\Psi_0, \phi_{n+l}^\dagger(0) \Psi_{\vec{p}_{n+l}^c}) \\ \times (\Psi_{\vec{p}_1, \dots, \vec{p}_{n'}, \vec{p}_{n'+1}^c, \dots, \vec{p}_n^c}^-, \Psi_{\vec{p}_{n+1}^c, \dots, \vec{p}_{n+l'}^c, \vec{p}_{n+l'+1}^c, \dots, \vec{p}_{n+l}}^+)^{c0}(\Psi_{\vec{p}_{n+l}^c}, \phi_{n+l}(0) \Psi_0) \cdots P_{n+l'}^{c0}(\Psi_{\vec{p}_{n+l'}^c}, \phi_{n+l'}(0) \Psi_0) \\ \times P_{n+l'+1}^0(\Psi_{\vec{p}_{n+l'+1}}, \phi_{n+l'+1}^\dagger(0) \Psi_0) \cdots P_{n+l}^0(\Psi_{\vec{p}_{n+l} \sigma_{n+l}}, \phi_{n+l}^\dagger(0) \Psi_0)$$

$$a(\vec{p}) \Psi_{\vec{q}} = Z_\phi \delta(p - q) \Psi_0 \quad (2\pi)^{3/2} \sqrt{2q^0} (\Psi_0, \phi(0) \Psi_{\vec{q}}) = (2\pi)^{3/2} \sqrt{2q^0} (\Psi_{\vec{q}}, \phi^\dagger(0) \Psi_0) = Z_\phi \\ a^c(\vec{p}) \Psi_{\vec{q}^c} = Z_\phi \delta(p - q^c) \Psi_0 \quad (2\pi)^{3/2} \sqrt{2q^{c0}} (\Psi_0, \phi^\dagger(0) \Psi_{\vec{q}^c}) = (2\pi)^{3/2} \sqrt{2q^{c0}} (\Psi_{\vec{q}^c}, \phi(0) \Psi_0) = Z_\phi$$

$$(\Psi_{\vec{q}_1, \dots, \vec{q}_{n'}, \vec{q}_{n'+1}^c, \dots, \vec{q}_n^c}^-, \Psi_{\vec{q}_{n+1}^c, \dots, \vec{q}_{n+l'}^c, \vec{q}_{n+l'+1}^c, \dots, \vec{q}_{n+l}}^+)^{c0} = \frac{[i/(2\pi)^{3/2} \sqrt{2}]^{n+l}}{Z_\phi^{n+l} \sqrt{q_1^0 \cdots q_n^{c0} q_{n+1}^{c0} \cdots q_{n+l}^0}} \int d^4x_1 \cdots d^4x_{n+l}$$

$$\times e^{iq_1 \cdot x_1 \cdots - iq_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}\phi_{H,1}(x_1) \cdots \phi_{H,n}^\dagger(x_n) \phi_{H,n+1}(x_{n+1}) \cdots \phi_{H,n+l}^\dagger(x_{n+l}) \Psi_0^+)$$



## 约化公式

$$\int d^4x_1 \cdots d^4x_{n+l} e^{iq_1 \cdot x_1 + \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} \cdots - iq_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_{n'}) \bar{\psi}_{H,l_{n'+1}}(x_{n'+1}) \cdots \bar{\psi}_{H,l_n}(x_n) \psi_{H,l_{n+1}}(x_{n+1}) \cdots \psi_{H,l_{n+l'}}(x_{n+l'}) \bar{\psi}_{H,l_{n+l'+1}}(x_{n+l'+1}) \cdots \bar{\psi}_{H,l_{n+l}}(x_{n+l}) \Psi_0^+) \\ = (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_n^c \sigma_{n+1}^c \cdots \sigma_{n+l}} q_1^0(\Psi_0, \psi_{l_1}(0) \Psi_{\vec{q}_1, \sigma_1}) \cdots q_n^0(\Psi_0, \bar{\psi}_{l_n}(0) \Psi_{\vec{q}_n^c, \sigma_n^c}) \\ \times (\Psi_{\vec{q}_1, \sigma_1}^-, \cdots, \bar{\psi}_{l_n}^+, \Psi_{\vec{q}_{n+1}^c, \sigma_{n+1}^c}^+, \cdots, \bar{\psi}_{l_{n+l}}(0) \Psi_0)$$

**旋量场:**  $\psi_l(x) = (2\pi)^{-3/2} \sum_{\sigma} \int d\vec{p} [u_l(\vec{p}, \sigma) e^{-ip \cdot x} a(\vec{p}, \sigma) + v_l(\vec{p}, \sigma) e^{ip \cdot x} a^{\dagger}(\vec{p}, \sigma)]$

$$\bar{\psi}_l(x) = (2\pi)^{-3/2} \sum_{\sigma} \int d\vec{p} [\bar{u}_l(\vec{p}, \sigma) e^{ip \cdot x} a^{\dagger}(\vec{p}, \sigma) + \bar{v}_l(\vec{p}, \sigma) e^{-ip \cdot x} a^c(\vec{p}, \sigma)]$$

$$a(\vec{p}) \Psi_{\vec{q}} = Z_{\psi} \delta(p - q) \Psi_0$$

$$a^c(\vec{p}) \Psi_{\vec{q}^c} = Z_{\psi} \delta(p - q^c) \Psi_0$$

$$(2\pi)^{3/2} (\Psi_0, \psi_l(0) \Psi_{\vec{q}, \sigma}) = Z_{\psi} u_l(\vec{q}, \sigma)$$

$$(2\pi)^{3/2} (\Psi_{\vec{q}, \sigma}, \bar{\psi}_l(0) \Psi_0) = Z_{\psi} \bar{u}_l(\vec{q}, \sigma)$$

$$(2\pi)^{3/2} (\Psi_0, \bar{\psi}_l(0) \Psi_{\vec{q}^c, \sigma^c}) = Z_{\psi} \bar{v}_l(\vec{q}^c, \sigma^c)$$

$$(2\pi)^{3/2} (\Psi_{\vec{q}^c, \sigma^c}, \psi_l(0) \Psi_0) = Z_{\psi} v_l(\vec{q}^c, \sigma^c)$$



## 约化公式

$$\begin{aligned}
 & \int d^4x_1 \cdots d^4x_{n+l} e^{iq_1 \cdot x_1 + \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} - \cdots - iq_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) \\
 & \times (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \bar{\psi}_{H,l_n}(x_n) \psi_{H,l_{n+1}}(x_{n+1}) \cdots \bar{\psi}_{H,l_{n+l}}(x_{n+l}) \Psi_0^+) \\
 & = (-2^{5/2} i\pi^{3/2})^{n+l} Z_\psi^{n+l} q_1^0 \cdots q_n^0 q_{n+1}^0 \cdots q_{n+l}^0 \sum_{\sigma_1 \cdots \sigma_n^c \sigma_{n+1}^c \cdots \sigma_{n+l}} u_{l_1}(\vec{q}_1, \sigma_1) \cdots \bar{v}_{l_n}(\vec{q}_n^c, \sigma_n^c) \\
 & \times (\Psi_{\vec{q}_1, \sigma_1, \dots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}^c, \sigma_{n+1}^c, \dots, \vec{q}_{n+l}, \sigma_{n+l}}^+) v_{l_{n+1}}(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots \bar{u}_{l_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l})
 \end{aligned}$$

$$u(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) u(0, \sigma) \quad v(\vec{p}, \sigma) = \sqrt{M/p^0} D(L(p)) v(0, \sigma) \quad D(L(p)) \beta D^\dagger(L(p)) = \beta$$

$$\beta = \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad u(0, \frac{1}{2}) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \quad u(0, -\frac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \quad v(0, \frac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad v(0, -\frac{1}{2}) = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\bar{u}(\vec{p}, \sigma) u(\vec{p}, \sigma') = \frac{M}{p^0} \bar{u}(0, \sigma) \beta D^\dagger(L(p)) \beta D(L(p)) u(0, \sigma') = \frac{M}{p^0} \bar{u}(0, \sigma) u(0, \sigma') = \frac{M}{p^0} \delta_{\sigma\sigma'}$$

$$\bar{v}(\vec{p}, \sigma) v(\vec{p}, \sigma') = \frac{M}{p^0} \bar{v}(0, \sigma) \beta D^\dagger(L(p)) \beta D(L(p)) v(0, \sigma') = \frac{M}{p^0} \bar{v}(0, \sigma) v(0, \sigma') = -\frac{M}{p^0} \delta_{\sigma\sigma'}$$





$$\begin{aligned}
 & (\Psi_{\vec{q}_1, \sigma_1, \dots, \vec{q}_n, \sigma_n}^-, \Psi_{\vec{q}_{n+1}, \sigma_{n+1}, \dots, \vec{q}_{n+l}, \sigma_{n+l}}^+) \\
 &= (-1)^{-n'+l'-l} (2^{5/2} i \pi^{3/2})^{-n-l} Z_\psi^{-n-l} (M_1 \cdots M_{n+l})^{-1} \bar{u}_{l_1}(\vec{q}_1, \sigma_1) \cdots v_{l_n}(\vec{q}_n, \sigma_n) \\
 & \quad \times \bar{v}_{l_{n+1}}(\vec{q}_{n+1}, \sigma_{n+1}) \cdots u_{l_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l}) \int d^4 x_1 \cdots d^4 x_{n+l} e^{i q_1 \cdot x_1 + \cdots + i q_n^c \cdot x_n - i q_{n+1}^c \cdot x_{n+1} \cdots - i q_{n+l} \cdot x_{n+l}} \\
 & \quad \times (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T} \psi_{H, l_1}(x_1) \cdots \bar{\psi}_{H, l_n}(x_n) \psi_{H, l_{n+1}}(x_{n+1}) \cdots \bar{\psi}_{H, l_{n+l}}(x_{n+l}) \Psi_0^+) \\
 &= (-1)^{-n'+l'-l} (2^{5/2} i \pi^{3/2})^{-n-l} Z_\psi^{-n-l} (M_1 \cdots M_{n+l})^{-1} \bar{u}_{l_1}(\vec{q}_1, \sigma_1) \cdots v_{l_n}(\vec{q}_n, \sigma_n) \\
 & \quad \times \bar{v}_{l_{n+1}}(\vec{q}_{n+1}, \sigma_{n+1}) \cdots u_{l_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l}) \int d^4 x_1 \cdots d^4 x_{n+l} e^{i q_1 \cdot x_1 + \cdots + i q_n^c \cdot x_n - i q_{n+1}^c \cdot x_{n+1} \cdots - i q_{n+l} \cdot x_{n+l}} \\
 & \quad \times [(-i \not{\partial}_{x_1} - M_1)(i \not{\partial}_{x_1} - M_1)]_{l_1 \bar{l}_1} \cdots [(-i \not{\partial}_{x_n} - M_n)(i \not{\partial}_{x_n} - M_n)]_{\bar{l}_n l_n} [(-i \not{\partial}_{x_{n+1}} - M_{n+1})(i \not{\partial}_{x_{n+1}} - M_{n+1})]_{l_{n+1} \bar{l}_{n+1}} \\
 & \quad \cdots [(-i \not{\partial}_{x_{n+l}} - M_{n+l})(i \not{\partial}_{x_{n+l}} - M_{n+l})]_{\bar{l}_{n+l} l_{n+l}} (\Psi_0^-, \mathbf{T} \psi_{H, \bar{l}_1}(x_1) \cdots \bar{\psi}_{H, \bar{l}_n}(x_n) \psi_{H, \bar{l}_{n+1}}(x_{n+1}) \cdots \psi_{H, \bar{l}_{n+l}}(x_{n+l}) \Psi_0^+)
 \end{aligned}$$



## 约化公式

$$\begin{aligned}
 & (\Psi_{\vec{q}_1, \sigma_1, \dots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}^c, \sigma_{n+1}^c, \dots, \vec{q}_{n+l}, \sigma_{n+l}}^+) \\
 &= (-1)^{-n'+l'-l} (2^{5/2} i \pi^{3/2})^{-n-l} Z_{\psi}^{-n-l} (M_1 \cdots M_{n+l})^{-1} \bar{u}_{l_1}(\vec{q}_1, \sigma_1) \cdots v_{l_n}(\vec{q}_n^c, \sigma_n^c) \\
 & \quad \times \bar{v}_{l_{n+1}}(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots u_{l_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l}) \int d^4 x_1 \cdots d^4 x_{n+l} e^{i q_1 \cdot x_1 + \cdots + i q_n^c \cdot x_n - i q_{n+1}^c \cdot x_{n+1} - \cdots - i q_{n+l} \cdot x_{n+l}} \\
 & \quad \times [(-i\partial_{x_1} - M_1)(i\partial_{x_1} - M_1)]_{l_1 \bar{l}_1} \cdots [(-i\partial_{x_n} - M_n)(i\partial_{x_n} - M_n)]_{\bar{l}_n l_n} [(-i\partial_{x_{n+1}} - M_{n+1})(i\partial_{x_{n+1}} - M_{n+1})]_{l_{n+1} \bar{l}_{n+1}} \\
 & \quad \cdots [(-i\partial_{x_{n+l}} - M_{n+l})(i\partial_{x_{n+l}} - M_{n+l})]_{\bar{l}_{n+l} l_{n+l}} (\Psi_0^-, \mathbf{T} \psi_{H, \bar{l}_1}(x_1) \cdots \bar{\psi}_{H, \bar{l}_n}(x_n) \psi_{H, \bar{l}_{n+1}}(x_{n+1}) \cdots \psi_{H, \bar{l}_{n+l}}(x_{n+l}) \Psi_0^+) \\
 &= (-1)^{-n'+l'-l} (2^{5/2} i \pi^{3/2})^{-n-l} Z_{\psi}^{-n-l} (M_1 \cdots M_{n+l})^{-1} \bar{u}_{l_1}(\vec{q}_1, \sigma_1) \cdots v_{l_n}(\vec{q}_n^c, \sigma_n^c) \\
 & \quad \times \bar{v}_{l_{n+1}}(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots u_{l_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l}) \int d^4 x_1 \cdots d^4 x_{n+l} e^{i q_1 \cdot x_1 + \cdots + i q_n^c \cdot x_n - i q_{n+1}^c \cdot x_{n+1} - \cdots - i q_{n+l} \cdot x_{n+l}} \\
 & \quad \times [(-\not{q}_1 - M_1)(i\partial_{x_1} - M_1)]_{l_1 \bar{l}_1} \cdots [(-i\partial_{x_n} - M_n)(\not{q}_n^c - M_n)]_{\bar{l}_n l_n} [(\not{q}_{n+1}^c - M_{n+1})(i\partial_{x_{n+1}} - M_{n+1})]_{l_{n+1} \bar{l}_{n+1}} \\
 & \quad \cdots [(-i\partial_{x_{n+l}} - M_{n+l})(-\not{q}_{n+l} - M_{n+l})]_{\bar{l}_{n+l} l_{n+l}} (\Psi_0^-, \mathbf{T} \psi_{H, \bar{l}_1}(x_1) \cdots \bar{\psi}_{H, \bar{l}_n}(x_n) \psi_{H, \bar{l}_{n+1}}(x_{n+1}) \cdots \psi_{H, \bar{l}_{n+l}}(x_{n+l}) \Psi_0^+) \\
 & (\not{q}_{n+l} - M_{n+l}) u(\vec{q}_{n+l}, \sigma_{n+l}) = 0 \quad \bar{u}(\vec{q}_1, \sigma) (\not{q}_1 - M_1) = 0 \quad (\not{q}_n^c + M_n) v(\vec{q}_n^c, \sigma_n^c) = 0 \quad \bar{v}(\vec{q}_{n+1}^c, \sigma_{n+1}^c) (\not{q}_{n+1}^c + M_{n+1}) = 0
 \end{aligned}$$



$$\begin{aligned} & (\Psi_{\vec{q}_1, \sigma_1, \dots, \vec{q}_{n'}, \sigma_{n'}, \vec{q}_{n'+1}^c, \sigma_{n'+1}^c, \dots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}^c, \sigma_{n+1}^c, \dots, \vec{q}_{n+l'}^c, \sigma_{n+l'}^c, \vec{q}_{n+l'+1}, \sigma_{n+l'+1}, \dots, \vec{q}_{n+l}, \sigma_{n+l}}^+ ) \\ &= (-1)^{-n'+l'-l} [(2\pi)^{\frac{3}{2}} i]^{-n-l} Z_\psi^{-n-l} \bar{u}_{l_1}(\vec{q}_1, \sigma_1) \cdots \bar{u}_{l_{n'}}(\vec{q}_{n'}, \sigma_{n'}) v_{l_{n'+1}}(\vec{q}_{n'+1}^c, \sigma_{n'+1}^c) \cdots v_{l_n}(\vec{q}_n^c, \sigma_n^c) \\ & \quad \times \bar{v}_{l_{n+1}}(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots \bar{v}_{l_{n+l'}}(\vec{q}_{n+l'}^c, \sigma_{n+l'}^c) u_{l_{n+l'+1}}(\vec{q}_{n+l'+1}, \sigma_{n+l'+1}) \cdots u_{l_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l}) \\ & \times \int d^4 x_1 \cdots d^4 x_{n+l} e^{iq_{1'} \cdot x_1 + \cdots + iq_{n'} \cdot x_{n'} + iq_{n'+1}^c \cdot x_{n'+1} + \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} - \cdots - iq_{n+l'}^c \cdot x_{n+l'} - iq_{n+l'+1} \cdot x_{n+l'+1} - \cdots - iq_{n+l} \cdot x_{n+l}} \\ & \quad \times (i\partial_{x_1} - M_1)_{l_1 \bar{l}_1} \cdots (i\partial_{x_{n'}} - M_{n'})_{l_{n'} \bar{l}_{n'}} (-i\partial_{x_{n'+1}} - M_{n'+1})_{\bar{l}_{n'+1} l_{n'+1}} \cdots (-i\partial_{x_n} - M_n)_{\bar{l}_n l_n} \\ & \quad \times (i\partial_{x_{n+1}} - M_{n+1})_{l_{n+1} \bar{l}_{n+1}} \cdots (i\partial_{x_{n+l'}} - M_{n+l'})_{l_{n+l'} \bar{l}_{n+l'}} (-i\partial_{x_{n+l'+1}} - M_{n+l'+1})_{\bar{l}_{n+l'+1} l_{n+l'+1}} \cdots (-i\partial_{x_{n+l}} - M_{n+l})_{\bar{l}_{n+l} l_{n+l}} \\ & \quad \times (\Psi_0^-, \mathbf{T} \psi_{H, \bar{l}_1}(x_1) \cdots \bar{\psi}_{H, \bar{l}_n}(x_n) \psi_{H, \bar{l}_{n+1}}(x_{n+1}) \cdots \bar{\psi}_{H, \bar{l}_{n+l}}(x_{n+l}) \Psi_0^+) \end{aligned}$$



## 约化公式

$$\begin{aligned}
 & \int d^4x_1 \cdots d^4x_{n+l} e^{iq_1 \cdot x_1 \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} \cdots - iq_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) \\
 & \times (\Psi_0^-, \mathbf{T}v_{H, \mu_1}(x_1) \cdots v_{H, \mu_n}^\dagger(x_n) v_{H, \mu_{n+1}}(x_{n+1}) \cdots v_{H, \mu_{n+l}}^\dagger(x_{n+l}) \Psi_0^+) \\
 & = (-16i\pi^3)^{n+l} \sum_{\sigma_1 \cdots \sigma_n^c \sigma_{n+1} \cdots \sigma_{n+l}} q_1^0(\Psi_0, v_{\mu_1}(0) \Psi_{\vec{q}_1, \sigma_1}) \cdots q_n^0(\Psi_0, v_{\mu_n}(0) \Psi_{\vec{q}_n^c, \sigma_n^c}) \\
 & \quad \times (\Psi_{\vec{q}_1, \sigma_1, \dots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}, \sigma_{n+1}, \dots, \vec{q}_{n+l}, \sigma_{n+l}}^+) q_{n+1}^0(\Psi_{\vec{q}_{n+1}, \sigma_{n+1}, v_{\mu_{n+1}}(0)} \Psi_0) \cdots q_{n+l}^0(\Psi_{\vec{q}_{n+l}, \sigma_{n+l}, v_{\mu_{n+l}}(0)} \Psi_0)
 \end{aligned}$$

**矢量场:** 
$$v^\mu(x) = (2\pi)^{-3/2} \sum_{\sigma=\pm\Gamma} \int \frac{d\vec{p}}{\sqrt{2p^0}} [e^{\mu}(\vec{p}, \sigma) a(\vec{p}, \sigma) e^{-ip \cdot x} + e^{\mu*}(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma) e^{ip \cdot x}]$$

$$(2\pi)^{3/2} \sqrt{2q^0} (\Psi_0, v^\mu(0) \Psi_{\vec{q}, \sigma}) = Z_v e^{\mu}(\vec{q}, \sigma) \quad (2\pi)^{3/2} \sqrt{2q^0} (\Psi_{\vec{q}^c, \sigma^c}, v^\mu(0) \Psi_0) = Z_v e^{\mu*}(\vec{q}^c, \sigma^c)$$

$$(2\pi)^{3/2} \sqrt{2q^0} (\Psi_0, v^{\mu\dagger}(0) \Psi_{\vec{q}^c, \sigma^c}) = Z_v e^{\mu}(\vec{q}^c, \sigma^c) \quad (2\pi)^{3/2} \sqrt{2q^0} (\Psi_{\vec{q}, \sigma}, v^{\mu\dagger}(0) \Psi_0) = Z_v e^{\mu*}(\vec{q}, \sigma)$$

$$\int d^4x_1 \cdots d^4x_{n+l} e^{iq_1 \cdot x_1 \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} \cdots - iq_{n+l} \cdot x_{n+l}} \times (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}v_{H, \mu_1}(x_1) \cdots v_{H, \mu_{n+l}}^\dagger(x_{n+l}) \Psi_0^+)$$

$$= (-4i\pi^{3/2})^{n+l} Z_v^{n+l} \sqrt{q_1^0 \cdots q_{n+l}^0} \sum_{\sigma_1 \cdots \sigma_{n+l}} e_{\mu_1}(\vec{q}_1, \sigma_1) \cdots e_{\mu_n}(\vec{q}_n^c, \sigma_n^c) (\Psi_{\vec{q}_1, \sigma_1, \dots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}, \sigma_{n+1}, \dots, \vec{q}_{n+l}, \sigma_{n+l}}^+)$$

$$\times e_{\mu_{n+1}}^*(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots e_{\mu_{n+l}}^*(\vec{q}_{n+l}, \sigma_{n+l})$$



## 约化公式

$$\int d^4x_1 \cdots d^4x_{n+l} e^{iq_1 \cdot x_1 \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} \cdots - iq_{n+l} \cdot x_{n+l}} \times (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}v_{H,\mu_1}(x_1) \cdots v_{H,\mu_{n+l}}^\dagger(x_{n+l}) \Psi_0^+)$$

$$= (-4i\pi^{3/2})^{n+l} Z_v^{n+l} \sum_{\sigma_1 \cdots \sigma_{n+l}^c} \sqrt{q_1^0} e_{\mu_1}(\vec{q}_1, \sigma_1) \cdots \sqrt{q_n^{c0}} e_{\mu_n}(\vec{q}_n^c, \sigma_n^c) (\Psi_{\vec{q}_1, \sigma_1, \cdots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}^c, \sigma_{n+1}^c, \cdots, \vec{q}_{n+l}, \sigma_{n+l}}^+)$$

$$\times \sqrt{q_{n+1}^{c0}} e_{\mu_{n+1}}^*(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots \sqrt{q_{n+l}^0} e_{\mu_{n+l}}^*(\vec{q}_{n+l}, \sigma_{n+l})$$

$$(\Psi_{\vec{q}_1, \sigma_1, \cdots, \vec{q}_n^c, \sigma_n^c}^-, \Psi_{\vec{q}_{n+1}^c, \sigma_{n+1}^c, \cdots, \vec{q}_{n+l}, \sigma_{n+l}}^+) e^\mu(\vec{p}, \sigma) e_\mu(\vec{p}, \sigma') = -\delta_{\sigma\sigma'} \quad \text{作业28}$$

$$= \frac{e^{\mu_1}(\vec{q}_1, \sigma_1) \cdots e^{\mu_n}(\vec{q}_n^c, \sigma_n^c) e^{*\mu_{n+1}}(\vec{q}_{n+1}^c, \sigma_{n+1}^c) \cdots e^{*\mu_{n+l}}(\vec{q}_{n+l}, \sigma_{n+l})}{(4i\pi^{3/2})^{n+l} Z_v^{n+l} \sqrt{q_1^0} \cdots \sqrt{q_n^{c0}} \sqrt{q_{n+1}^{c0}} \cdots \sqrt{q_{n+l}^0}} \int d^4x_1 \cdots d^4x_{n+l}$$

$$\times e^{iq_1 \cdot x_1 \cdots + iq_n^c \cdot x_n - iq_{n+1}^c \cdot x_{n+1} \cdots - iq_{n+l} \cdot x_{n+l}} (\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T}v_{H,\mu_1}(x_1) \cdots v_{H,\mu_{n+l}}^\dagger(x_{n+l}) \Psi_0^+)$$



## 格林函数的极点

态之间的编时乘积的格林函数  $(\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1)\psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n)\Psi_{\alpha}^{+})$

$$\begin{aligned} \text{Wick定理: } \mathbf{T}(UV \cdots XYZ) &= \mathbf{N}(UV \cdots XYZ) && \text{无收缩项} \\ &+ \mathbf{N}(\underline{UV} \cdots XYZ) + \cdots + \mathbf{N}(UV \cdots \underline{XYZ}) + && \text{一次收缩项} \\ &+ \mathbf{N}(\underline{UV} \cdots \underline{XYZ}) + \cdots && \text{二次收缩项} \\ &+ \cdots \\ &+ \mathbf{N}(\underline{\underline{UVW}} \cdots \underline{\underline{XYZ}}) + \cdots && \text{全部收缩项} \end{aligned}$$

若  $T$  乘积中含  $N$  乘积, 展开仍成立, 但需将各  $N$  乘积内部各算符之间的收缩略去.

应用Wick定理:  $U(t, t') = \Omega^{\dagger}(t)\Omega(t')$   $\Psi_{\alpha}^{\pm} = \Omega(\mp\infty)\Phi_{\alpha}$

$$\begin{aligned} &(\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1)\psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n)\Psi_{\alpha}^{+}) \\ &= (\Omega(\infty)\Phi_{\beta}, \mathbf{T} \Omega(t_1)\psi_1(x_1)U(t_1, t_2)\psi_2(x_2)U(t_2, t_3) \cdots U(t_{n-1}, t_n)\psi_n(x_n)\Omega^{\dagger}(t_n)\Omega(-\infty)\Phi_{\alpha}) \\ &= (\Phi_{\beta}, \mathbf{T} \psi_1(x_1)\psi_2(x_2) \cdots \psi_n(x_n)U(\infty, -\infty)\Phi_{\alpha}) = \underline{\text{所有可能的特别收缩项之和}} \end{aligned}$$

保留下来的正规乘积中的湮灭算符必须正好湮灭  $\Phi_{\alpha}$  态到某个态, 产生算符的厄米共扼必须正好湮灭  $\Phi_{\beta}$  态到同样的态!



## 格林函数的极点

编时乘积的格林函数  $(\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+})$

考虑  $x_1^0, \cdots, x_r^0 > 0, x_{r+1}^0, \cdots, x_n^0 < 0$

$$(\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+})$$

$$= \theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0]) \\ \times (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(x_1) \cdots \psi_{H,r}(x_r)\} \mathbf{T}\{\psi_{H,r+1}(x_{r+1}) \cdots \psi_{H,n}(x_n)\} \Psi_{\alpha}^{+})$$

$$= \theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0]) \\ \times \int d\gamma (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(x_1) \cdots \psi_{H,r}(x_r)\} \Psi_{\gamma}) (\Psi_{\gamma}, \mathbf{T}\{\psi_{H,r+1}(x_{r+1}) \cdots \psi_{H,n}(x_n)\} \Psi_{\alpha}^{+})$$

= 单粒子中间态项 + 多粒子中间态项

$\Psi_{\gamma}$  是一组多粒子态完备集；是  $H$  的本征态



## 格林函数的极点

单粒子中间态项:  $x_1^0, \dots, x_r^0 > 0, x_{r+1}^0, \dots, x_n < 0$

$$\theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0])$$

$$\times \sum_{\sigma} \int d\vec{p} (\Psi_{\vec{\beta}}^{-}, \mathbf{T}\{\psi_{H,1}(x_1) \cdots \psi_{H,r}(x_r)\} \Psi_{\vec{\beta},\sigma}) (\Psi_{\vec{\beta},\sigma} \mathbf{T}\{\psi_{H,r+1}(x_{r+1}) \cdots \psi_{H,n}(x_n)\} \Psi_{\alpha}^{+})$$

$$\psi_{H,i}(x_i) = e^{iP \cdot x_i} \psi_{H,i}(0) e^{-iP \cdot x_i}$$

$$\psi_{H,i}(x_i - x) = e^{-iP \cdot x} \psi_{H,i}(x_i) e^{iP \cdot x}$$

$$\psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,r}(x_r) = e^{iP \cdot x_1} \psi_{H,1}(0) \psi_{H,2}(x_2 - x_1) \cdots \psi_{H,r}(x_r - x_1) e^{-iP \cdot x_1}$$

$$\psi_{H,r+1}(x_{r+1}) \psi_{H,r+2}(x_{r+2}) \cdots \psi_{H,n}(x_n) = e^{iP \cdot x_{r+1}} \psi_{H,r+1}(0) \psi_{H,r+2}(x_{r+2} - x_{r+1}) \cdots \psi_{H,n}(x_n - x_{r+1}) e^{-iP \cdot x_{r+1}}$$

$$e^{-iP \cdot x_1} \Psi_{\vec{\beta},\sigma} (\Psi_{\vec{\beta},\sigma} e^{iP \cdot x_{r+1}} = e^{-iP \cdot x_1} \Psi_{\vec{\beta},\sigma}) (e^{-iP \cdot x_{r+1}} \Psi_{\vec{\beta},\sigma} = e^{-iP \cdot (x_1 - x_{r+1})} \Psi_{\vec{\beta},\sigma}) (\Psi_{\vec{\beta},\sigma}$$

$$= \theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0])$$

$$\times \sum_{\sigma} \int d\vec{p} e^{i(p_{\beta} - p) x_1 + i(p - p_{\alpha}) x_{r+1}} (\Psi_{\vec{\beta}}^{-}, \mathbf{T}\{\psi_{H,1}(0) \psi_{H,2}(x_2 - x_1) \cdots \psi_{H,r}(x_r - x_1)\} \Psi_{\vec{\beta},\sigma})$$

$$\times (\Psi_{\vec{\beta},\sigma} \mathbf{T}\{\psi_{H,r+1}(0) \psi_{H,r+2}(x_{r+2} - x_{r+1}) \cdots \psi_{H,n}(x_n - x_{r+1})\} \Psi_{\alpha}^{+})$$





## 格林函数的极点

单粒子中间态项:  $x_i = x_1 + y_i$  ( $i = 2, 3, \dots, r$ )  $x_i = x_{r+1} + y_i$  ( $i = r+2, \dots, n$ )

$$\theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0])$$

$$\times \sum_{\sigma} \int d\vec{p} (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(x_1) \cdots \psi_{H,r}(x_r)\} \Psi_{\vec{p},\sigma}^{-}) (\Psi_{\vec{p},\sigma}^{-}, \mathbf{T}\{\psi_{H,r+1}(x_{r+1}) \cdots \psi_{H,n}(x_n)\} \Psi_{\alpha}^{+})$$

$$= \theta(\min[x_1^0 \cdots x_r^0] - \max[x_{r+1}^0 \cdots x_n^0])$$

$$\times \sum_{\sigma} \int d\vec{p} e^{i(p_{\beta}-p)x_1 + i(p-p_{\alpha})x_{r+1}} (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(0)\psi_{H,2}(x_2-x_1) \cdots \psi_{H,r}(x_r-x_1)\} \Psi_{\vec{p},\sigma}^{-})$$

$$\times (\Psi_{\vec{p},\sigma}^{-}, \mathbf{T}\{\psi_{H,r+1}(0)\psi_{H,r+2}(x_{r+2}-x_{r+1}) \cdots \psi_{H,n}(x_n-x_{r+1})\} \Psi_{\alpha}^{+})$$

$$\min[x_1^0, \dots, x_r^0] - \max[x_{r+1}^0, \dots, x_n^0] = x_1^0 - x_{r+1}^0 + \min[0, y_2^0, \dots, y_r^0] - \max[0, y_{r+1}^0, \dots, y_n^0]$$

$$= \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \dots, y_r^0] - \max[0, y_{r+1}^0, \dots, y_n^0])}$$

$$\theta(\tau) = - \int_{-\infty}^{\infty} \frac{d\omega}{2i\pi} \frac{e^{-i\omega\tau}}{\omega + i0^+}$$

$$\times \sum_{\sigma} \int d\vec{p} (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(0)\psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r)\} \Psi_{\vec{p},\sigma}^{-}) (\Psi_{\vec{p},\sigma}^{-}, \mathbf{T}\{\psi_{H,r+1}(0)\psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n)\} \Psi_{\alpha}^{+})$$

$$\times e^{i(p_{\beta}-p)x_1 + i(p-p_{\alpha})x_{r+1} - i\omega(x_1^0 - x_{r+1}^0)}$$



## 格林函数的极点

$$\begin{aligned}
 & (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) \quad x_1^0, \cdots, x_r^0 > 0, \quad x_{r+1}^0, \cdots, x_n^0 < 0 \\
 & = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \cdots, y_r^0] - \max[0, y_{r+1}^0, \cdots, y_n^0])} e^{i(p_{\beta} - p)x_1 + i(p - p_{\alpha})x_{r+1} - i\omega(x_1^0 - x_{r+1}^0)} \\
 & \quad \times \sum_{\sigma} \int d\vec{p} (\Psi_{\beta}^{-} \mathbf{T} \{\psi_{H,1}(0) \psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r)\} \Psi_{\vec{p},\sigma}) (\Psi_{\vec{p},\sigma} \mathbf{T} \{\psi_{H,r+1}(0) \psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n)\} \Psi_{\alpha}^{+}) \\
 & \quad x_i = x_1 + y_i \quad (i = 2, 3, \cdots, r) \quad x_i = x_{r+1} + y_i \quad (i = r + 2, \cdots, n) \\
 & (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) = \int \frac{d^4 q_1 \cdots d^4 q_n}{(2\pi)^{4n}} e^{-iq_1 \cdot x_1 - \cdots - iq_n \cdot x_n} G_{\beta\alpha}(q_1, \cdots, q_n) \\
 & - i(q_1 \cdot x_1 + \cdots + q_n \cdot x_n) \\
 & = -i[(q_1 + \cdots + q_r) \cdot x_1 + (q_{r+1} + \cdots + q_n) \cdot x_{r+1} + q_2 \cdot y_2 + \cdots + q_r \cdot y_r + q_{r+2} \cdot y_{r+2} + \cdots + q_n \cdot y_n] \\
 & (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) \quad \tilde{q}_1 \equiv q_1 + \cdots + q_r \quad \tilde{q}_{r+1} \equiv q_{r+1} + \cdots + q_n \\
 & = \int \frac{d^4 \tilde{q}_1 d^4 \tilde{q}_{r+1} d^4 q_2 \cdots d^4 q_r d^4 q_{r+2} \cdots d^4 q_n}{(2\pi)^{4n}} e^{-i\tilde{q}_1 \cdot x_1 - i\tilde{q}_{r+1} \cdot x_{r+1} - iq_2 \cdot y_2 - \cdots - iq_r \cdot y_r - iq_{r+2} \cdot y_{r+2} - \cdots - iq_n \cdot y_n} \\
 & \quad \times G_{\beta\alpha}(q_1, \cdots, q_n)
 \end{aligned}$$



## 格林函数的极点

$$\begin{aligned}
 & (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) | \text{单粒子中间态} & x_1^0, \cdots, x_r^0 > 0, \quad x_{r+1}^0, \cdots, x_n^0 < 0 \\
 & = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^{+}} e^{-i\omega(\min[0, y_2^0, \cdots, y_r^0] - \max[0, y_{r+1}^0, \cdots, y_n^0])} \int d\vec{p} e^{i(p_{\beta} - p)x_1 + i(p - p_{\alpha})x_{r+1} - i\omega(x_1^0 - x_{r+1}^0)} \\
 & \quad \times \sum (\Psi_{\beta}^{-} \mathbf{T} \{ \psi_{H,1}(0) \psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r) \} \Psi_{\vec{p}, \sigma}^{-}) (\Psi_{\vec{p}, \sigma}^{-} \mathbf{T} \{ \psi_{H,r+1}(0) \psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n) \} \Psi_{\alpha}^{+}) \\
 & (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) & \tilde{q}_1 \equiv q_1 + \cdots + q_r \quad \tilde{q}_{r+1} \equiv q_{r+1} + \cdots + q_n \\
 & = \int \frac{d^4 \tilde{q}_1 d^4 \tilde{q}_{r+1} d^4 q_2 \cdots d^4 q_r d^4 q_{r+2} \cdots d^4 q_n}{(2\pi)^{4n}} e^{-i\tilde{q}_1 \cdot x_1 - i\tilde{q}_{r+1} \cdot x_{r+1} - iq_2 \cdot y_2 - \cdots - iq_r \cdot y_r - iq_{r+2} \cdot y_{r+2} - \cdots - iq_n \cdot y_n} \\
 & \quad \times G_{\beta\alpha}(q_1, \cdots, q_n) \quad y_i = x_i - y_1 \quad (i = 2, 3, \cdots, r) \quad y_i = x_i - x_{r+1} \quad (i = r+2, \cdots, n)
 \end{aligned}$$

$$\begin{aligned}
 G_{\beta\alpha}(q_1, \cdots, q_n) &= \int d^4 x_1 \cdots d^4 x_n e^{i\tilde{q}_1 \cdot x_1 + i\tilde{q}_{r+1} \cdot x_{r+1} + iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r + iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n} \\
 & \quad \times (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) \\
 &= \int d^4 x_1 d^4 x_{r+1} d^4 y_2 \cdots d^4 y_r d^4 y_{r+2} \cdots d^4 y_n (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) \\
 & \quad \times e^{i\tilde{q}_1 \cdot x_1 + i\tilde{q}_{r+1} \cdot x_{r+1} + iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r + iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n}
 \end{aligned}$$



## 格林函数的极点

$$(\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) | \text{单粒子中间态} \quad x_1^0, \cdots, x_r^0 > 0, \quad x_{r+1}^0, \cdots, x_n^0 < 0$$

$$= \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \cdots, y_r^0] - \max[0, y_{r+1}^0, \cdots, y_n^0])} \int d\vec{p} e^{i(p_{\beta} - p)x_1 + i(p - p_{\alpha})x_{r+1} - i\omega(x_1^0 - x_{r+1}^0)}$$

$$\times \sum_{\sigma} (\Psi_{\beta}^{-} \mathbf{T} \{ \psi_{H,1}(0) \psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r) \} \Psi_{\bar{p},\sigma}) (\Psi_{\bar{p},\sigma} \mathbf{T} \{ \psi_{H,r+1}(0) \psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n) \} \Psi_{\alpha}^{+})$$

$$\tilde{q}_1 \equiv q_1 + \cdots + q_r \quad \tilde{q}_{r+1} \equiv q_{r+1} + \cdots + q_n \quad y_i = x_i - y_1 \quad (i=2, \cdots, r) \quad y_i = x_i - x_{r+1} \quad (i=r+2, \cdots, n)$$

$$G_{\beta\alpha}(q_1, \cdots, q_n) = \int d^4x_1 d^4x_{r+1} d^4y_2 \cdots d^4y_r d^4y_{r+2} \cdots d^4y_n (\Psi_{\beta}^{-}, \mathbf{T} \psi_{H,1}(x_1) \psi_{H,2}(x_2) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+})$$

$$\times e^{i\tilde{q}_1 \cdot x_1 + i\tilde{q}_{r+1} \cdot x_{r+1} + iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r + iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n}$$

$$G_{\beta\alpha}(q_1, \cdots, q_n) | \text{单粒子中间态} = \int d^4y_2 \cdots d^4y_r d^4y_{r+2} \cdots d^4y_n e^{iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r + iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n}$$

$$\times \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \cdots, y_r^0] - \max[0, y_{r+1}^0, \cdots, y_n^0])} \int d\vec{p} (2\pi)^8 \delta(\vec{p} - \vec{p}_{\beta} - \vec{q}_1)$$

$$\times \delta(-\sqrt{\vec{p}^2 + M^2} - \omega + p_{\beta}^0 + \tilde{q}_1^0) \delta(-\vec{q}_{r+1} + \vec{p}_{\alpha} - \vec{p}) \delta(\tilde{q}_{r+1}^0 - p_{\alpha}^0 + \sqrt{\vec{p}^2 + M^2} + \omega)$$

$$\times \sum_{\sigma} (\Psi_{\beta}^{-} \mathbf{T} \{ \psi_{H,1}(0) \psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r) \} \Psi_{\bar{p},\sigma}) (\Psi_{\bar{p},\sigma} \mathbf{T} \{ \psi_{H,r+1}(0) \psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n) \} \Psi_{\alpha}^{+})$$



## 格林函数的极点

 $G_{\alpha\beta}(q_1, \dots, q_n)$  | 单粒子中间态

$$= \int d^4 y_2 \cdots d^4 y_r d^4 y_{r+2} \cdots d^4 y_n e^{iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r + iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n} \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \dots, y_r^0] - \max[0, y_{r+1}^0, \dots, y_n^0])}$$

$$\times \sum_{\sigma} \int d\vec{p} (\Psi_{\beta}^{-} \mathbf{T} \{ \psi_{H,1}(0) \psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r) \} \Psi_{\vec{p},\sigma}) (\Psi_{\vec{p},\sigma} \mathbf{T} \{ \psi_{H,r+1}(0) \psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n) \} \Psi_{\alpha}^{+})$$

$$\times (2\pi)^8 \delta(\vec{p} - \vec{p}_{\beta} - \vec{q}_1 - \cdots - \vec{q}_r) \delta(-\sqrt{\vec{p}^2 + M^2} - \omega + p_{\beta}^0 + q_1^0 + \cdots + q_r^0)$$

$$\times \delta(-\vec{q}_{r+1} - \cdots - \vec{q}_n + \vec{p}_{\alpha} - \vec{p}) \delta(q_{r+1}^0 + \cdots + q_n^0 - p_{\alpha}^0 + \sqrt{\vec{p}^2 + M^2} + \omega)$$

$$q \equiv q_1 + \cdots + q_r + p_{\beta} = -q_{r+1} - \cdots - q_n + p_{\alpha}$$

$$= i(2\pi)^7 \delta^4(q_1 + \cdots + q_n + p_{\beta} - p_{\alpha}) \frac{\sum_{\sigma} M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n)}{q^0 - \sqrt{\vec{q}^2 + M^2} + i0^+}$$

$$\frac{1}{q^0 - \sqrt{\vec{q}^2 + M^2} + i0^+} = \frac{q^0 + \sqrt{\vec{q}^2 + M^2} - i0^+}{(q^0)^2 - (\vec{q}^2 + M^2 - i0^+)} = \frac{2\sqrt{\vec{q}^2 + M^2}}{q^2 - M^2 + i0^+} + \text{非极点项}$$

$$= i(2\pi)^7 \delta^4(q_1 + \cdots + q_n + p_{\beta} - p_{\alpha}) \frac{2q^0 \sum_{\sigma} M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n)}{q^2 - M^2 + i0^+}$$



## 格林函数的极点

$$G_{\beta\alpha}(q_1, \dots, q_n) = \int d^4x_1 \cdots d^4x_n e^{iq_1 \cdot x_1 + \cdots + iq_n \cdot x_n} (\Psi_{\beta}^{-}, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_{\alpha}^{+})$$

$G_{\beta\alpha}(q_1, \dots, q_n)$  | 单粒子中间态

$$= \int d^4y_2 \cdots d^4y_r d^4y_{r+2} \cdots d^4y_n e^{iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r + iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n} \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i0^+} e^{-i\omega(\min[0, y_2^0, \dots, y_r^0] - \max[0, y_{r+1}^0, \dots, y_n^0])}$$

$$\times \sum_{\sigma} \int d\vec{p} (\Psi_{\beta}^{-} \mathbf{T}\{\psi_{H,1}(0)\psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r)\} \Psi_{\vec{p},\sigma}) (\Psi_{\vec{p},\sigma} \mathbf{T}\{\psi_{H,r+1}(0)\psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n)\} \Psi_{\alpha}^{+})$$

$$\times (2\pi)^8 \delta(\vec{p} - \vec{p}_{\beta} - \vec{q}_1 - \cdots - \vec{q}_r) \delta(-\sqrt{\vec{p}^2 + M^2} - \omega + p_{\beta}^0 + q_1^0 + \cdots + q_r^0)$$

$$\times \delta(-\vec{q}_{r+1} - \cdots - \vec{q}_n + \vec{p}_{\alpha} - \vec{p}) \delta(q_{r+1}^0 + \cdots + q_n^0 - p_{\alpha}^0 + \sqrt{\vec{p}^2 + M^2} + \omega)$$

$$q \equiv q_1 + \cdots + q_r + p_{\beta} = -q_{r+1} - \cdots - q_n + p_{\alpha}$$

$$2q^0 \sum_{\sigma} M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n)$$

$$= i(2\pi)^7 \delta^4(q_1 + \cdots + q_n + p_{\beta} - p_{\alpha}) \frac{\sigma}{q^2 - M^2 + i0^+} + \text{非极点项}$$

$$M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) = \int d^4y_2 \cdots d^4y_r e^{iq_2 \cdot y_2 + \cdots + iq_r \cdot y_r} (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(0)\psi_{H,2}(y_2) \cdots \psi_{H,r}(y_r)\} \Psi_{\vec{q},\sigma})$$

$$M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n) = \int d^4y_{r+2} \cdots d^4y_n e^{iq_{r+2} \cdot y_{r+2} + \cdots + iq_n \cdot y_n} (\Psi_{\vec{q},\sigma}, \mathbf{T}\{\psi_{H,r+1}(0)\psi_{H,r+2}(y_{r+2}) \cdots \psi_{H,n}(y_n)\} \Psi_{\alpha}^{+})$$



格林函数的极点

$$G_{\beta\alpha}(q_1, \dots, q_n) = \int d^4x_1 \dots d^4x_n e^{iq_1 \cdot x_1 + \dots + iq_n \cdot x_n} (\Psi_{\beta}^-, \mathbf{T}\psi_{H,1}(x_1) \dots \psi_{H,n}(x_n) \Psi_{\alpha}^+)$$

$$M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) = \int d^4y_2 \dots d^4y_r e^{iq_2 \cdot y_2 + \dots + iq_r \cdot y_r} (\Psi_{\beta}^-, \mathbf{T}\{\psi_{H,1}(0)\psi_{H,2}(y_2) \dots \psi_{H,r}(y_r)\} \Psi_{\vec{q},\sigma}^+)$$

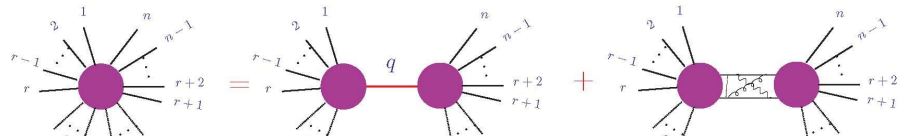
$$M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n) = \int d^4y_{r+2} \dots d^4y_n e^{iq_{r+2} \cdot y_{r+2} + \dots + iq_n \cdot y_n} (\Psi_{\vec{q},\sigma}^+, \mathbf{T}\{\psi_{H,r+1}(0)\psi_{H,r+2}(y_{r+2}) \dots \psi_{H,n}(y_n)\} \Psi_{\alpha}^+)$$

$$G_{\beta\alpha}(q_1, \dots, q_n) = \sum_{\sigma} \int \frac{d^4q}{(2\pi)^4} \left[ (2\pi)^4 \delta(q_1 + \dots + q_r + p_{\beta} - q) (2\pi)^{\frac{3}{2}} (2q^0)^{\frac{1}{2}} M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) \right]$$

$\times \frac{i}{q^2 - M^2 + i0^+}$  通常单粒子态的贡献是最主要的!  $\Rightarrow$  单粒子为主 有效拉氏量

$\times \left[ (2\pi)^4 \delta(q + q_{r+1} + \dots + q_n - p_{\alpha}) (2\pi)^{\frac{3}{2}} (2q^0)^{\frac{1}{2}} M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n) \right]$

+ 多粒子中间态项 + 非极点项





## 格林函数的极点

$$\begin{aligned}
 G_{\beta\alpha}(q_1, \dots, q_n) &= \int d^4x_1 \dots d^4x_n e^{iq_1 \cdot x_1 + \dots + iq_n \cdot x_n} (\Psi_{\beta}^{-}, \mathbf{T}\psi_{H,1}(x_1) \dots \psi_{H,n}(x_n) \Psi_{\alpha}^{+}) \\
 &= i(2\pi)^7 \delta^4(q_1 + \dots + q_n + p_{\beta} - p_{\alpha}) (q^2 - M^2 + i0^+)^{-1} 2q^0 \sum_{\sigma} M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n) \\
 &\quad + \text{多粒子中间态项} + \text{非极点项} \\
 M_{\beta|\vec{q},\sigma}(q_2, \dots, q_r) &= \int d^4y_2 \dots d^4y_r e^{iq_2 \cdot y_2 + \dots + iq_r \cdot y_r} (\Psi_{\beta}^{-}, \mathbf{T}\{\psi_{H,1}(0)\psi_{H,2}(y_2) \dots \psi_{H,r}(y_r)\} \Psi_{\vec{q},\sigma}) \\
 M_{-\vec{q},\sigma|\alpha}(q_{r+2}, \dots, q_n) &= \int d^4y_{r+2} \dots d^4y_n e^{iq_{r+2} \cdot y_{r+2} + \dots + iq_n \cdot y_n} (\Psi_{\vec{q},\sigma}^{-}, \mathbf{T}\{\psi_{H,r+1}(0)\psi_{H,r+2}(y_{r+2}) \dots \psi_{H,n}(y_n)\} \Psi_{\alpha}^{+}) \\
 \int d^4x d^4y e^{iq \cdot x + iq' \cdot y} (\Psi_0^{-}, \mathbf{T}\psi_{H,1}(x)\psi_{H,2}(y) \Psi_0^{+}) &= \int d^4x d^4y e^{iq \cdot (x-y) + i(q+q') \cdot y} (\Psi_0^{-}, \mathbf{T}\psi_{H,1}(x)\psi_{H,2}(y) \Psi_0^{+}) \\
 &= i(2\pi)^7 \delta^4(q + q') \frac{2q^0 \sum_{\sigma} (\Psi_0^{-}, \psi_{H,1}(0) \Psi_{\vec{q},\sigma}) (\Psi_{\vec{q},\sigma}, \psi_{H,2}(0) \Psi_0^{+})}{q^2 - M^2 + i0^+} + \text{其它项} \\
 (\Psi_0^{-}, \mathbf{T}\psi_{H,1}(x)\psi_{H,2}(y) \Psi_0^{+}) &= \int \frac{d^4q}{2\pi} e^{-iq \cdot (x-y)} \frac{2iq^0 \sum_{\sigma} (\Psi_0^{-}, \psi_{H,1}(0) \Psi_{\vec{q},\sigma}) (\Psi_{\vec{q},\sigma}, \psi_{H,2}(0) \Psi_0^{+})}{q^2 - M^2 + i0^+} + \dots
 \end{aligned}$$





## 格林函数的极点

两点函数的谱函数:  $X$ :标量  $\psi(x) = e^{iP \cdot x} \psi(0) e^{-iP \cdot x}$   $\Psi_0$ 完全可替换为  $\Psi_\alpha$

$$\begin{aligned}
 (\Psi_0, X(x) X^\dagger(y) \Psi_0) &= \sum_n (\Psi_0, X(x) \Psi_n) (\Psi_n, X^\dagger(y) \Psi_0) = \sum_n e^{-ip_n \cdot (x-y)} |(\Psi_0, X(0) \Psi_n)|^2 \\
 &= \int d^4 p e^{-ip \cdot (x-y)} \underbrace{\sum_n \delta^4(p-p_n) |(\Psi_0, X(0) \Psi_n)|^2}_{(2\pi)^{-3} \theta(p^0) \rho(p^2) \quad p_n^0 \geq 0} = \int_0^\infty d\mu^2 \rho(\mu^2) \underbrace{\int \frac{d^4 p}{(2\pi)^3} e^{-ip \cdot (x-y)} \theta(p^0) \delta(p^2 - \mu^2)}_{\Delta_+(x-y, \mu^2)}
 \end{aligned}$$

$$\begin{aligned}
 (\Psi_0, X^\dagger(y) X(x) \Psi_0) &= \sum_n (\Psi_0, X^\dagger(y) \Psi_n) (\Psi_n, X(x) \Psi_0) = \sum_n e^{ip_n \cdot (x-y)} |(\Psi_n, X(0) \Psi_0)|^2 \\
 &= \int d^4 p e^{ip \cdot (x-y)} \underbrace{\sum_n \delta^4(p-p_n) |(\Psi_n, X(0) \Psi_0)|^2}_{(2\pi)^{-3} \theta(p^0) \tilde{\rho}(p^2) \quad p_n^0 \geq 0} = \int_0^\infty d\mu^2 \tilde{\rho}(\mu^2) \underbrace{\int \frac{d^4 p}{(2\pi)^3} e^{ip \cdot (x-y)} \theta(p^0) \delta(p^2 - \mu^2)}_{\Delta_+(y-x, \mu^2)}
 \end{aligned}$$

$$(\Psi_0, [X(x), X^\dagger(y)] \Psi_0) = \int_0^\infty d\mu^2 [\rho(\mu^2) \Delta_+(x-y, \mu^2) - \tilde{\rho}(\mu^2) \Delta_+(y-x, \mu^2)] \xrightarrow{\text{类空; 求导}} \rho(\mu^2) = \tilde{\rho}(\mu^2)$$

$$(\Psi_0, \mathbf{T} X(x) X^\dagger(y) \Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2) \underbrace{[\theta(x^0 - y^0) \Delta_+(x-y, \mu^2) + \theta(y^0 - x^0) \Delta_+(y-x, \mu^2)]}_{\Delta_F(x-y, \mu^2)}$$



## 格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_n, X(0)\Psi_0)|^2$$

$$(\Psi_0, \mathbf{T} X(x) X^\dagger(y) \Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2) \Delta_F(x-y, \mu^2) \quad \Delta_+(x-y, \mu^2) \equiv \int \frac{d^4 p}{(2\pi)^3} e^{-ip \cdot (x-y)} \theta(p^0) \delta(p^2 - \mu^2)$$

$$(\Psi_0, [X(x), X^\dagger(y)] \Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2) [\Delta_+(x-y, \mu^2) - \Delta_+(y-x, \mu^2)]$$

$$\left. \frac{\partial}{\partial x^0} \Delta_+(x-y, \mu^2) \right|_{x^0=y^0} = \int \frac{d^4 p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x}-\vec{y})} \theta(p^0) (-ip^0) \delta((p^0)^2 - \vec{p}^2 - \mu^2) = -\frac{i}{2} \delta(\vec{x}-\vec{y})$$

$$\left. \frac{\partial}{\partial x^0} \Delta_+(y-x, \mu^2) \right|_{x^0=y^0} = \int \frac{d^4 p}{(2\pi)^3} e^{-i\vec{p} \cdot (\vec{x}-\vec{y})} \theta(p^0) (ip^0) \delta((p^0)^2 - \vec{p}^2 - \mu^2) = \frac{i}{2} \delta(\vec{x}-\vec{y})$$

$$(\Psi_0, [X(x), X^\dagger(y)] \Psi_0) = -i\delta(\vec{x}-\vec{y}) \int_0^\infty d\mu^2 \rho(\mu^2)$$

如果  $\dot{X}\dot{X}^\dagger = -\ddot{X}X^\dagger$  是动能项,  $\dot{X}$  对应的广义动量就是  $-X^\dagger$

$$\left. [\dot{X}(x), X^\dagger(y)] \right|_{x^0=y^0} = -i\delta(\vec{x}-\vec{y}) \Rightarrow \int_0^\infty d\mu^2 \rho(\mu^2) = 1 \quad \text{只对基本场成立!}$$



## 格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_n, X(0)\Psi_0)|^2$$

$$(\Psi_0, \mathbf{T} X(x)X^\dagger(y)\Psi_0) = \int_0^\infty d\mu^2 \rho(\mu^2) \Delta_F(x-y, \mu^2) \quad \Delta_+(x-y, \mu^2) \equiv \int \frac{d^4 p}{(2\pi)^3} e^{-ip \cdot (x-y)} \theta(p^0) \delta(p^2 - \mu^2)$$

$$\Delta_F(x-y, \mu^2) \equiv [\theta(x^0 - y^0) \Delta_+(x-y, \mu^2) + \theta(y^0 - x^0) \Delta_+(y-x, \mu^2)]$$

$$= \int \frac{d^4 p}{(2\pi)^3} \delta((p^0)^2 - (\vec{p})^2 - \mu^2) \theta(p^0) [e^{-ip \cdot (x-y)} \theta(x^0 - y^0) + e^{ip \cdot (x-y)} \theta(y^0 - x^0)]$$

$$= \int \frac{d^4 p}{2p^0 (2\pi)^3} \delta(p^0 - \sqrt{\vec{p}^2 + \mu^2}) \theta(p^0) [e^{-ip \cdot (x-y)} \theta(x^0 - y^0) + e^{ip \cdot (x-y)} \theta(y^0 - x^0)]$$

$$= \int \frac{d\vec{p}}{2\sqrt{(\vec{p})^2 + \mu^2} (2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} [e^{-i\sqrt{\vec{p}^2 + \mu^2}(x^0 - y^0)} \theta(x^0 - y^0) + e^{i\sqrt{\vec{p}^2 + \mu^2}(x^0 - y^0)} \theta(y^0 - x^0)]$$

$$= \int \frac{d\vec{p}}{2\sqrt{(\vec{p})^2 + \mu^2} (2\pi)^3} \int_{-\infty}^{\infty} \frac{dp^0}{2i\pi} e^{-ip^0(x^0 - y^0) + i\vec{p} \cdot (\vec{x} - \vec{y})} \left[ -\frac{1}{p^0 - \sqrt{\vec{p}^2 + \mu^2} + i0^+} + \frac{1}{p^0 + \sqrt{\vec{p}^2 + \mu^2} - i0^+} \right]$$

$$= \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \frac{i}{p^2 - \mu^2 + i0^+} \quad (\Psi_0, \mathbf{T} X(x)X^\dagger(y)\Psi_0) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \int_0^\infty d\mu^2 \frac{i\rho(\mu^2)}{p^2 - \mu^2 + i0^+}$$



## 格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T} X(x) X^\dagger(y) \Psi_0) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \Delta_X(p^2 + i0^+) \quad \Delta_X(z) = \int_0^\infty d\mu^2 \frac{i\rho(\mu^2)}{z - \mu^2}$$

若中间态谱中最低质量  $M$  的单粒子态  $\Psi_{\vec{p}_\alpha}$  ( $\Psi_0, X(0)\Psi_{\vec{p}_\alpha}) \neq 0$ , 其对谱函数贡献:

$$\begin{aligned} & (2\pi)^3 \int d\vec{p}_\alpha \delta(p^0 - \sqrt{\vec{p}_\alpha^2 + M^2}) \delta(\vec{p} - \vec{p}_\alpha) |(\Psi_0, X(0)\Psi_{\vec{p}_\alpha})|^2 \\ &= (2\pi)^3 \delta(p^0 - \sqrt{\vec{p}^2 + M^2}) |(\Psi_0, X(0)\Psi_{\vec{p}})|^2 = (2\pi)^3 \theta(p^0) \frac{1}{2p^0} \delta((p^0)^2 - \vec{p}^2 - M^2) |(\Psi_0, X(0)\Psi_{\vec{p}})|^2 \\ &= Z_\alpha \delta(p^2 - M^2) \quad Z_\alpha \equiv (2\pi)^3 \theta(p^0) \frac{1}{2p^0} |(\Psi_0, X(0)\Psi_{\vec{p}})|^2 \geq 0 \end{aligned}$$

$$\rho(\mu^2) = Z_\alpha \delta(p^2 - M^2) + \sigma(\mu^2) \quad 1 = Z_\alpha + \int_{M_0^2}^\infty d\mu^2 \sigma(\mu^2) \quad M_0 \geq M \text{ 为其它粒子或连续谱出现的下限} \quad Z_\alpha \leq 1$$

$$\Delta_X(z) = \frac{iZ_\alpha}{z - M^2} + \int_{M_0^2}^\infty d\mu^2 \frac{i\rho(\mu^2)}{z - \mu^2} \quad \text{可能有割线!} \quad -i\Delta_X(M^2 \pm i0^+) = \pm\infty \quad -i\Delta_X(M^2 \pm i0^+) = \mp i\infty$$



## 格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T} X(x) X^\dagger(y) \Psi_0) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \Delta_X(p^2 + i0^+) \quad \Delta_X(z) = \int_0^\infty d\mu^2 \frac{i\rho(\mu^2)}{z - \mu^2}$$

$$\text{对复数 } z, \text{ 引入 } \Pi(z) \equiv z - i\Delta_X^{-1}(z) \quad \Delta_X(z) = \frac{i}{z - \Pi(z)}$$

$\Delta_X(z), \Pi(z)$  在除正实轴外的 全复  $z$  平面 解析

在正实轴上下两边, 对实  $x \geq 0$

$$\frac{i}{y + i0^+} = \mathbf{P} \frac{i}{y} + \pi\delta(y) \quad \frac{i}{y - i0^+} = \frac{-i}{-y + i0^+} = - \left[ \mathbf{P} \frac{i}{-y} + \pi\delta(-y) \right] = \mathbf{P} \frac{i}{y} - \pi\delta(y)$$

$$\Delta_X(x + i0^+) - \Delta(x - i0^+) = \int_0^\infty d\mu^2 \rho(\mu^2) \left[ \frac{i}{x + i0^+ - \mu^2} - \frac{i}{x - i0^+ - \mu^2} \right] = 2\pi\rho(x)$$

$$\Pi(x \pm i0^+) \equiv u(x) \mp iv(x) \quad \text{若有单粒子态极点: } \Pi(M^2) = M^2 \Rightarrow u(M^2) = M^2 \quad v(M^2) = 0$$

$$v(x) = \frac{i}{2} [\Pi(x + i0^+) - \Pi(x - i0^+)] = \frac{i}{2} [-i\Delta_X^{-1}(x + i0^+) + i\Delta_X^{-1}(x - i0^+)]$$

$$= \frac{\Delta_X(x - i0^+) - \Delta_X(x + i0^+)}{2\Delta_X(x + i0^+)\Delta_X(x - i0^+)} = \frac{-\pi\rho(x)}{\Delta_X(x + i0^+)\Delta_X(x - i0^+)} = \pi\rho(x) \left[ [x - u(x)]^2 + v^2(x) \right] \geq 0$$



## 格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T} X(x) X^\dagger(y) \Psi_0) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \Delta_X(p^2 + i0^+) \quad \Pi(x \pm i0^+) \equiv u(x) \mp iv(x) \quad v(x) \geq 0$$

$$\Delta_X(z) = \frac{i}{z - \Pi(z)} = \int_0^\infty d\mu^2 \frac{i\rho(\mu^2)}{z - \mu^2} \quad \Delta_X(x+i0^+) - \Delta_X(x-i0^+) = 2\pi\rho(x)$$

$$\text{定义复平面上的 } \Delta_X^{\text{II}}(z) \equiv \frac{i}{z - u(z) + iv(z)} \quad \text{若 } u(x \pm i0^+) = u(x), v(x \pm i0^+) = v(x)$$

$$\Delta_X^{\text{II}}(x+i0^+) = \Delta_X(x+i0^+) \Rightarrow \Delta_X^{\text{II}}(z) \text{ 可看成是 } \Delta_X(z) \text{ 在第二叶黎曼面上的解析延拓}$$

第二叶黎曼面上的极点  $z_0$  满足:  $\underline{z_0 - u(z_0) + iv(z_0) = 0}$  除  $z_0 = M^2$  外, 可能有或无其它解!

若有解:  $z_0 = re^{-i\theta} = (\tilde{M} - \frac{i}{2}\Gamma)^2$  总可以这样选!  $\tilde{M} \equiv \sqrt{r} \cos \frac{\theta}{2}, \Gamma \equiv 2\sqrt{r} \sin \frac{\theta}{2} \geq 0$

$$\tilde{M}^2 - \frac{\Gamma^2}{4} - i\tilde{M}\Gamma - u(z_0) + iv(z_0) = 0 \quad \tilde{M}^2 = \frac{\Gamma^2}{4} + \text{Re}u(z_0) + \text{Im}v(z_0) \quad \tilde{M}\Gamma = \text{Re}v(z_0) - \text{Im}u(z_0)$$

$$\tilde{M}\Gamma \stackrel{\text{Im}z_0 \ll \text{Re}z_0}{\Rightarrow} \underbrace{\text{Re}v(\text{Re}z_0) - \text{Im}u(\text{Re}z_0)}_{=0} + \underbrace{\text{Re}[v'(\text{Re}z_0)i\text{Im}z_0] - \text{Im}[u'(\text{Re}z_0)i\text{Im}z_0]}_{=0}$$

$$= v(\text{Re}z_0) + u'(\text{Re}z_0)\tilde{M}\Gamma \Rightarrow \tilde{M} = \frac{v(\text{Re}z_0)}{\Gamma[1 - u'(\text{Re}z_0)]} \stackrel{u'(\text{Re}z_0) < 1}{\Rightarrow} \geq 0$$



## 格林函数的极点

$$\rho(p^2) \equiv (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_0, X(0)\Psi_n)|^2 = (2\pi)^3 \sum_n \delta^4(p-p_n) |(\Psi_n, X(0)\Psi_0)|^2 \geq 0$$

$$(\Psi_0, \mathbf{T} X(x) X^\dagger(y) \Psi_0) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \Delta_X(p^2 + i0^+) \quad \Pi(x \pm i0^+) \equiv u(x) \mp iv(x) \quad v(x) \geq 0$$

$$\Delta_X(z) = \frac{i}{z - \Pi(z)} = \int_0^\infty d\mu^2 \frac{i\rho(\mu^2)}{z - \mu^2} \quad \Delta_X(x+i0^+) - \Delta_X(x-i0^+) = 2\pi\rho(x)$$

$\Delta_X^{\text{II}}(z) \equiv \frac{i}{z-u(z)+iv(z)}$  可看成是  $\Delta_X(z)$  在第二叶黎曼面上的解析延拓

第二叶黎曼面上的极点  $z_0$  满足:  $\underline{z_0 - u(z_0) + iv(z_0) = 0}$  除  $z_0 = M^2$  外, 可能有或无其它解!

如果有解: 可选  $z_0 = (\tilde{M} - \frac{i}{2}\Gamma)^2$

极点附近:  $\Delta_X^{\text{II}}(z) \equiv \frac{i}{z-u(z)+iv(z)} \approx \frac{i}{z-u(z_0)+iv(z_0)} \approx \frac{i}{z - (\tilde{M} - \frac{i}{2}\Gamma)^2}$  **Breit-Wigner公式**

$\Gamma \geq 0$   $\tilde{M} \Big|_{\text{Im}z_0 \ll \text{Re}z_0, u'(\text{Re}z_0) < 1} \geq 0$   $e^{-iHt} \Rightarrow e^{-iMt} \Rightarrow e^{-i\tilde{M}t - \frac{\Gamma}{2}t} \Rightarrow e^{-\Gamma t}$  在实轴上, 极点被 **共振峰** 取代!

$$\left| \frac{i}{x - (\tilde{M} - \frac{i}{2}\Gamma)^2} \right|^2 = \frac{1}{(x - \tilde{M}^2 + \frac{\Gamma^2}{4})^2 + \tilde{M}^2\Gamma^2} \xleftarrow{\text{最高 } x = \tilde{M}^2} \frac{1}{\Gamma^2(\tilde{M}^2 + \frac{\Gamma^2}{16})} \xleftarrow{\text{半高 } x \simeq (\tilde{M} \pm \frac{\Gamma}{2})^2} \frac{1}{2\Gamma^2(\tilde{M}^2 + \frac{\Gamma^2}{8} \pm \tilde{M}\frac{\Gamma}{2})}$$

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## 格林函数的极点

关于格林函数对于动量变量的依赖关系:

♣ 极点位置是粒子的物理质量! 极点位于实轴! 实轴上还可能会有割线!

◇ 除实轴外, 格林函数在首叶动量全平面 解析!

♡ 解析函数实部与虚部存在强烈的关联 色散关系!

♠ 还存在其它动量叶, 在其上可能会在非实轴上存在极点! 导致动量依赖多值

¶ 第二叶上极点可表为  $\tilde{M} - i\Gamma/2$  沿第一叶割线上方的积分通过围道可包括进其贡献

✕  $\Gamma$  产生态演化几率的衰减  $e^{-\Gamma t}$ ; 它是共振峰的 半宽度!





## 格林函数的极点

$$\mathbf{S} \text{ 矩阵与格林函数} \quad \psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t} \quad \pi_S(x) = e^{-iH_0 t} \pi(x) e^{iH_0 t}$$

$$\psi_H(\vec{x}, t) \equiv \Omega(t) \psi(\vec{x}, t) \Omega^\dagger(t) = e^{iHt} \psi_S(\vec{x}, t) e^{-iHt} \quad \Omega(t) = e^{iHt} e^{-iH_0 t}$$

$$(\partial_1^2 + M_1^2) \cdots (\partial_{n+l}^2 + M_{n+l}^2) (\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n+l}(x_{n+l}) \Psi_0^+)$$

$$= \int \frac{d^4 q_1 \cdots d^4 q_{n+l}}{(i\pi)^{n+l}} e^{-iq_1 \cdot x_1 \cdots -iq_n \cdot x_n + iq_{n+1} \cdot x_{n+1} \cdots + iq_{n+l} \cdot x_{n+l}} \sum_{\sigma_1 \cdots \sigma_{n+l}} q_1^0 (\Psi_0^-, \psi_1(0) \Psi_{\vec{q}_1, \sigma_1}^-) \cdots q_n^0 (\Psi_0^-, \psi_n(0) \Psi_{\vec{q}_n, \sigma_n}^-)$$

$$\times (\Psi_{\vec{q}_1, \sigma_1}^-, \cdots \Psi_{\vec{q}_n, \sigma_n}^-, \Psi_{\vec{q}_{n+1}, \sigma_{n+1}}^+, \cdots \Psi_{\vec{q}_{n+l}, \sigma_{n+l}}^+) q_{n+1}^0 (\Psi_{\vec{q}_{n+1}, \sigma_{n+1}}^+, \psi_{n+1}(0) \Psi_0^+) \cdots q_{n+l}^0 (\Psi_{\vec{q}_{n+l}, \sigma_{n+l}}^+, \psi_{n+l}(0) \Psi_0^+)$$

$$(\Psi_0^-, \mathbf{T} \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_0^+) \stackrel{t_1 > \cdots > t_n}{=} (\Psi_0^-, \psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_0^+)$$

$$= (\Psi_0^-, e^{iHt_1} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iHt_n} \Psi_0^+)$$

$$= (\Psi_0^-, e^{-iH(t'-t_1)} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iH(t_n-t')} \Psi_0^+)$$

## 需要研究:

$$(\Psi_0^-, e^{-iH(t'-t_1)} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iH(t_n-t')} \Psi_0^+)$$



## 量子场算符本征态的矩阵元

## 量子场

$$\psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t} \quad \psi(x) = \sum_{\sigma} \int \frac{d^3 p}{(2\pi)^{3/2}} [e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]$$

$$\dot{\psi}_S(x) = \dot{\pi}_S(x) = 0 \quad [\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)]_{\mp} = [\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = 0 \quad [\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = i\delta(\vec{x} - \vec{y})$$

$$(\Psi_0^-, e^{-iH(t' - t_1)} \psi_{S,1}(x_1) e^{-iH(t_1 - t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1} - t_n)} \psi_{S,n}(x_n) e^{-iH(t_n - t)} \Psi_0^+)$$

## 固定空间点量子场算符的本征态: 费米子对应grassman数

$$\psi_{S,l}(x) |\psi(\vec{x})\rangle = \psi_l(\vec{x}) |\psi(\vec{x})\rangle \quad \int d\psi(\vec{x}) |\psi(\vec{x})\rangle \langle \psi(\vec{x})| = I \quad \langle \psi(\vec{x}) | \psi'(\vec{x}) \rangle = \delta(\psi(\vec{x}) - \psi'(\vec{x}))$$

$$\pi_{S,l}(x) |\pi(\vec{x})\rangle = \pi_l(\vec{x}) |\pi(\vec{x})\rangle \quad \int d\pi(\vec{x}) |\pi(\vec{x})\rangle \langle \pi(\vec{x})| = I \quad \langle \pi(\vec{x}) | \pi'(\vec{x}) \rangle = \delta(\pi(\vec{x}) - \pi'(\vec{x}))$$

- ▶ 在有限维空间中,厄米算符的全部本征态矢量构成完备集
- ▶ 任意一个算符总可以分解为一个厄米算符和一个反厄米算符的叠加

所有空间点量子场算符本征态:  $|\psi\rangle \equiv \prod_{\vec{x}} |\psi(\vec{x})\rangle \quad |\pi\rangle \equiv \prod_{\vec{x}} |\pi(\vec{x})\rangle$

$$\int \mathcal{D}_s \psi |\psi\rangle \langle \psi| = I \quad \langle \psi | \psi' \rangle = \delta(\psi - \psi') \equiv \prod_{\vec{x}} \delta(\psi(\vec{x}) - \psi'(\vec{x})) \quad \mathcal{D}_s \psi \equiv \prod_{\vec{x}} d\psi(\vec{x})$$



## 量子场算符本征态的矩阵元

**Grassmann数**  $\Leftrightarrow$  反对易数:  $\theta\eta = -\eta\theta$

单变量:  $\theta^2 = 0 \quad f(\theta) = A + B\theta \quad \frac{\partial f(\theta)}{\partial \theta} = B \quad \frac{\partial^2}{\partial \theta^2} = 0$

$$\int d\theta f(\theta) = \int d\theta (A + B\theta) = A \int d\theta + B \int d\theta \theta$$

要求平移不变性:  $\int d\theta f(\theta) = \int d\theta f(\theta + \eta) = (A - B\eta) \int d\theta + B \int d\theta \theta \Rightarrow \eta \int d\theta = 0$

$$\int d\theta = 0 \quad \text{约定: } \int d\theta \theta = 1 \Rightarrow \int d\theta f(\theta) = \frac{\partial f(\theta)}{\partial \theta} = B$$

$$1 = \int d\theta \delta(\theta - \eta) = \int d\theta \delta(\theta) \Rightarrow \delta(\theta) = \theta \quad \text{但是: } \delta(-\theta) = -\delta(\theta) \neq \delta(\theta)$$

多变量:  $\theta_i \theta_j + \theta_j \theta_i = 0$

$$\frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} + \frac{\partial}{\partial \theta_j} \frac{\partial}{\partial \theta_i} = 0 \quad \text{左微商}$$

$$f(\theta) = \sum_{k=0}^n \sum_{i_1, \dots, i_k} C_{i_1, \dots, i_k}^{(k)} \theta_{i_1} \theta_{i_2} \cdots \theta_{i_k} \quad \delta(\theta) = \theta_1 \cdots \theta_n$$



## 量子场算符本征态的矩阵元

## 构造实标量场本征态:

$$\phi_S(x) = e^{-iH_0 t} \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2p^0}} [e^{-ip \cdot x} a(\vec{p}) + e^{ip \cdot x} a^\dagger(\vec{p})] e^{iH_0 t}$$

$$\phi_S(x) |\phi(\vec{x})\rangle = \phi(\vec{x}) |\phi(\vec{x})\rangle \quad \int d\phi(\vec{x}) |\phi(\vec{x})\rangle \langle \phi(\vec{x})| = I \quad \langle \phi(\vec{x}) | \phi'(\vec{x}') \rangle = \delta(\phi(\vec{x}) - \phi'(\vec{x}'))$$

$$|\phi(\vec{x})\rangle = \int d^3 q f(\vec{q}) e^{-iH_0 t} \exp\left\{ (2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}') \right\} \Phi_0$$

$$\begin{aligned} \phi_S(\vec{x}) |\phi(\vec{x})\rangle &= \int d^3 q f(\vec{q}) e^{-iH_0 t} \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2p^0}} [e^{-ip \cdot x} a(\vec{p}) + e^{ip \cdot x} a^\dagger(\vec{p})] \\ &\quad \times e^{(2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}')} \Phi_0 \\ &= \int d^3 q f(\vec{q}) e^{-iH_0 t} \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2p^0}} e^{(2\pi)^{3/2} \sqrt{2q^0} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}')} \\ &\quad \times \left[ e^{ip \cdot x} a^\dagger(\vec{p}) - \left[ \left\{ (2\pi)^{3/2} \sqrt{2q^0} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}') \right\}, e^{-ip \cdot x} a(\vec{p}) \right] \right] \Phi_0 \\ &= \int d^3 q f(\vec{q}) e^{-iH_0 t} \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2p^0}} e^{(2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}')} \\ &\quad \times \left[ e^{ip \cdot x} a^\dagger(\vec{p}) + e^{-ip \cdot x} \left\{ (2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) \delta(q-p) - \int d^3 q' e^{2iq' \cdot x} a^\dagger(\vec{q}') \delta(q'-p) \right\} \right] \Phi_0 \end{aligned}$$



## 量子场算符本征态的矩阵元

构造实标量场本征态:

$$\phi_S(x) = e^{-iH_0 t} \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2p^0}} [e^{-ip \cdot x} a(\vec{p}) + e^{ip \cdot x} a^\dagger(\vec{p})] e^{iH_0 t}$$

$$\phi_S(x) |\phi(\vec{x})\rangle = \phi(\vec{x}) |\phi(\vec{x})\rangle \quad \int d\phi(\vec{x}) |\phi(\vec{x})\rangle \langle\phi(\vec{x})| = I \quad \langle\phi(\vec{x})|\phi'(\vec{x}')\rangle = \delta(\phi(\vec{x}) - \phi'(\vec{x}'))$$

$$|\phi(\vec{x})\rangle = \int d^3 q f(\vec{q}) e^{-iH_0 t} \exp\left\{(2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}')\right\} \Phi_0$$

$$\begin{aligned} \phi_S(\vec{x}) |\phi(\vec{x})\rangle &= \int d^3 q f(\vec{q}) e^{-iH_0 t} \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2p^0}} e^{(2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}')} \\ &\quad \times \left[ e^{ip \cdot x} a^\dagger(\vec{p}) + e^{-ip \cdot x} \left\{ (2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) \delta(q-p) - \int d^3 q' e^{2iq' \cdot x} a^\dagger(\vec{q}') \delta(q'-p) \right\} \right] \Phi_0 \\ &= \phi(\vec{x}) \int d^3 q f(\vec{q}) e^{-iH_0 t} e^{(2\pi)^{3/2} \sqrt{2q^0} e^{iq \cdot x} \phi(\vec{x}) a^\dagger(\vec{q}) - \frac{1}{2} \int d^3 q' e^{2iq' \cdot x} a^{\dagger 2}(\vec{q}')} \Phi_0 \end{aligned}$$

研究性问题: 如何调节  $f(\vec{q})$ , 使得

$$\int d\phi(\vec{x}) |\phi(\vec{x})\rangle \langle\phi(\vec{x})| = I \quad \langle\phi(\vec{x})|\phi'(\vec{x}')\rangle = \delta(\phi(\vec{x}) - \phi'(\vec{x}'))$$



## 量子场算符本征态的矩阵元

## 场及其动量

$$[\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)]_{\mp} = [\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = 0 \quad [\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = i\delta(\vec{x} - \vec{y})$$

$$\pi_{S,l}(x) e^{i \int d\vec{y} \xi(\vec{y}) \psi_S(y)} |\pi\rangle = e^{i \int d\vec{y} \xi(\vec{y}) \psi_S(y)} \{ \pi_{S,l}(x) + (-i) \int d\vec{z} \xi(\vec{z}) [\psi_S(z), \pi_{S,l}(x)]_{\mp} \} |\pi\rangle \stackrel{e^A B e^{-A} = e^{Ad(A)} B}{=} |\pi + \xi\rangle$$

$$= (\pi_l(\vec{x}) + \xi_l(\vec{x})) e^{i \int d\vec{y} \xi(\vec{y}) \psi_S(y)} |\pi\rangle \Rightarrow e^{i \int d\vec{y} \xi(\vec{y}) \psi_S(y)} |\pi\rangle = |\pi + \xi\rangle \Rightarrow |\pi\rangle = e^{i \int d\vec{y} \pi(\vec{y}) \psi_S(y)} |0\rangle_{\pi}$$

$$\psi_{S,l}(x) e^{-i \int d\vec{y} \zeta(\vec{y}) \pi_S(y)} |\psi\rangle = e^{-i \int d\vec{y} \zeta(\vec{y}) \pi_S(y)} \{ \psi_{S,l}(x) + i \int d\vec{z} \zeta(\vec{z}) [\pi_S(z), \psi_{S,l}(x)]_{\mp} \} |\psi\rangle \stackrel{e^A B e^{-A} = e^{Ad(A)} B}{=} |\psi + \zeta\rangle$$

$$= (\psi_l(\vec{x}) + \zeta_l(\vec{x})) e^{-i \int d\vec{y} \zeta(\vec{y}) \pi_S(y)} |\psi\rangle \Rightarrow e^{-i \int d\vec{y} \zeta(\vec{y}) \pi_S(y)} |\psi\rangle = |\psi + \zeta\rangle \Rightarrow |\psi\rangle = e^{-i \int d\vec{y} \psi(\vec{y}) \pi_S(y)} |0\rangle_{\psi}$$

$$\langle \psi | \pi \rangle = \langle \psi | e^{i \int d\vec{y} \pi(\vec{y}) \psi_S(y)} |0\rangle_{\pi} = e^{i \int d\vec{y} \pi(\vec{y}) \psi(\vec{y})} \langle \psi | 0 \rangle_{\pi} = e^{i \int d\vec{y} \pi(\vec{y}) \psi(\vec{y})} \langle \psi | 0 \rangle_{\pi} e^{i \int d\vec{z} \psi(\vec{z}) \pi_S(z)} |0\rangle_{\pi}$$

$$= e^{i \int d\vec{y} \pi(\vec{y}) \psi(\vec{y})} \langle \psi | 0 \rangle_{\pi}$$

$$\delta(\pi' - \pi) = \int \mathcal{D}_s \psi \langle \pi' | \psi \rangle \langle \psi | \pi \rangle = \int \mathcal{D}_s \psi e^{i \int d\vec{y} [-\pi'(\vec{y}) + \pi(\vec{y})] \psi(\vec{y})} |\langle \psi | 0 \rangle_{\pi}|^2 = \delta(\pi' - \pi) |\langle \psi | 0 \rangle_{\pi}|^2 \prod_{\vec{x}} (2\pi)$$

$$\langle \psi | \pi \rangle = e^{i \int d\vec{y} \pi(\vec{y}) \psi(\vec{y})} / \sqrt{\prod_{\vec{x}} (2\pi)} \quad \text{如果是费米场 } \pi, \psi \text{ 是 Grassman 变量}$$

$$\pi_{S,l}(\vec{x}, t) = -i \frac{\delta}{\delta \psi_{S,l}(\vec{x}, t)}$$



## 泛函微商

$$[\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)]_{\mp} = [\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = 0 \quad [\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = i\delta(\vec{x} - \vec{y})$$

$$\langle \psi | \pi \rangle = e^{i \int d\vec{y} \pi(\vec{y}) \psi(\vec{y})} / \prod_{\vec{x}} \sqrt{2\pi} \quad \pi_{S,l}(\vec{x}, t) = -i \frac{\delta}{\delta \psi_{S,l}(\vec{x}, t)}$$

三维空间:  $\frac{\delta}{\delta \psi(\vec{x}, t)} \psi(\vec{y}, t) = \delta(\vec{x} - \vec{y})$

$$\frac{\delta}{\delta \psi(\vec{x}, t)} f[\psi(\vec{y}, t)] \equiv \lim_{\Delta \rightarrow 0} \frac{f[\psi(\vec{y}, t) + \delta(\vec{x} - \vec{y})\Delta] - f[\psi(\vec{y}, t)]}{\Delta} = f'[\psi(\vec{y}, t)]\delta(\vec{x} - \vec{y})$$

$$\frac{\delta}{\delta \psi(\vec{x}, t)} \int d^3y f[\psi(\vec{y}, t)] = \int d^3y f'[\psi(\vec{y}, t)] \frac{\delta}{\delta \psi(\vec{x}, t)} \psi(\vec{y}, t) = f'[\psi(\vec{x}, t)]$$



## 量子场算符本征态的矩阵元

量子场  $\psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t}$   $\psi(x) = \sum_{\sigma} \int \frac{d^3 p}{(2\pi)^{3/2}} [e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^\dagger(\vec{p}, \sigma)]$

$$\dot{\psi}_S(x) = \dot{\pi}_S(x) = 0 \quad [\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)]_{\mp} = [\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = 0 \quad [\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = i\delta(\vec{x} - \vec{y})$$

$$(\Psi_0^-, e^{-iH(t'-t_1)} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iH(t_n-t)} \Psi_0^+)$$

所有空间点量子场算符本征态:  $|\psi\rangle \equiv \prod_{\vec{x}} |\psi(\vec{x})\rangle$   $|\pi\rangle \equiv \prod_{\vec{x}} |\pi(\vec{x})\rangle$

$$\psi_{S,l}(x) |\psi\rangle = \psi_l(\vec{x}) |\psi\rangle \quad \int \mathcal{D}_s \psi |\psi\rangle \langle \psi| = I \quad \langle \psi | \psi' \rangle = \delta(\psi - \psi') \quad \mathcal{D}_s \psi \equiv \prod_{\vec{x}} d\psi(\vec{x})$$

$$\pi_{S,l}(x) |\pi\rangle = \pi_l(\vec{x}) |\pi\rangle \quad \int \mathcal{D}_s \pi |\pi\rangle \langle \pi| = I \quad \langle \pi | \pi' \rangle = \delta(\pi - \pi') \quad \mathcal{D}_s \pi \equiv \prod d\pi(\vec{x})$$

$$(\Psi_0^-, e^{-iH(t'-t_1)} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iH(t_n-t)} \Psi_0^+)$$

$$= \int \mathcal{D}_s \psi' \mathcal{D}_s \psi_1 \cdots \mathcal{D}_s \psi_n \mathcal{D}_s \psi (\Psi_0^- |\psi'\rangle \langle \psi'| e^{-iH(t'-t_1)} \psi_{S,1}(x_1) |\psi_1\rangle \langle \psi_1| e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) |\psi_2\rangle \langle \psi_2| \cdots \psi_{S,n-1}(x_{n-1}) |\psi_{n-1}\rangle \langle \psi_{n-1}| e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) |\psi_n\rangle \langle \psi_n| e^{-iH(t_n-t)} |\psi\rangle \langle \psi| \Psi_0^+)$$

$$= \int \mathcal{D}_s \psi' \cdots \mathcal{D}_s \psi \psi_1(\vec{x}_1) \psi_2(\vec{x}_2) \cdots \psi_{n-1}(\vec{x}_{n-1}) \psi_n(\vec{x}_n) (\Psi_0^- |\psi'\rangle \langle \psi'| e^{-iH(t'-t_1)} |\psi_1\rangle \langle \psi_1| e^{-iH(t_1-t_2)} |\psi_2\rangle \langle \psi_2| \cdots \langle \psi_{n-1}| e^{-iH(t_{n-1}-t_n)} |\psi_n\rangle \langle \psi_n| e^{-iH(t_n-t)} |\psi\rangle \langle \psi| \Psi_0^+)$$

$$\Rightarrow \text{需要计算 } \langle \psi' | e^{-iH(t'-t)} | \psi \rangle$$





## 时间演化算符的矩阵元

量子场  $\psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t}$   $\psi(x) = \sum_{\sigma} \int \frac{d^3 p}{(2\pi)^{3/2}} [e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^{\dagger}(\vec{p}, \sigma)]$

$$\dot{\psi}_S(x) = \dot{\pi}_S(x) = 0 \quad [\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)]_{\mp} = [\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = 0 \quad [\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)]_{\mp} = i\delta(\vec{x} - \vec{y})$$

$$(\Psi_0^-, e^{-iH(t'-t_1)} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iH(t_n-t)} \Psi_0^+)$$

所有空间点量子场算符本征态:  $|\psi\rangle \equiv \prod_{\vec{x}} |\psi(\vec{x})\rangle$   $|\pi\rangle \equiv \prod_{\vec{x}} |\pi(\vec{x})\rangle$

$$\psi_{S,l}(x) |\psi\rangle = \psi_l(\vec{x}) |\psi\rangle \quad \int \mathcal{D}_s \psi |\psi\rangle \langle \psi| = I \quad \langle \psi | \psi' \rangle = \delta(\psi - \psi') \quad \mathcal{D}_s \psi \equiv \prod_{\vec{x}} d\psi(\vec{x})$$

$$\pi_{S,l}(x) |\pi\rangle = \pi_l(\vec{x}) |\pi\rangle \quad \int \mathcal{D}_s \pi |\pi\rangle \langle \pi| = I \quad \langle \pi | \pi' \rangle = \delta(\pi - \pi') \quad \mathcal{D}_s \pi \equiv \prod_{\vec{x}} d\pi(\vec{x})$$

需要计算  $\langle \psi' | e^{-iH(t'-t)} | \psi \rangle$

将  $t', t$  中等间隔地分为  $m+1$  份, 从左向右分别标记为  $t_m, t_{m-1}, \dots, t_2, t_1$   
 $m$  选择的足够大  $\Delta t = t_{i+1} - t_i = (t' - t)/(m+1) \xrightarrow{m \rightarrow \infty} 0$

$$\langle \psi' | e^{-iH(t'-t)} | \psi \rangle = \int \mathcal{D}_s \psi_m \cdots \mathcal{D}_s \psi_1 \langle \psi' | e^{-iH(t'-t_m)} | \psi_m \rangle \langle \psi_m | e^{-iH(t_m-t_{m-1})} | \psi_{m-1} \rangle \cdots$$

$$\cdots \langle \psi_2 | e^{-iH(t_2-t_1)} | \psi_1 \rangle \langle \psi_1 | e^{-iH(t_1-t)} | \psi \rangle \quad \Rightarrow \quad \underline{\text{需要计算}} \quad \langle \psi_{i+1} | e^{-iH\Delta t} | \psi_i \rangle$$



无穷小演化算符:  $e^{-iH\Delta t}$   $\Delta t = (t' - t)/(m + 1) \rightarrow 0$

$$\langle \psi | \pi \rangle = e^{i \int d\vec{y} \pi(\vec{y}) \psi(\vec{y})} / \prod_{\vec{x}} \sqrt{2\pi}$$

$$\begin{aligned} \langle \psi_{i+1} | e^{-iH\Delta t} | \psi_i \rangle &= \langle \psi_{i+1} | e^{-i\Delta t H(\psi_S, \pi_S)} | \psi_i \rangle \\ &= \delta(\psi_{i+1} - \psi_i) - i\Delta t \langle \psi_{i+1} | H(\psi_S, \pi_S) | \psi_i \rangle \\ &= \delta(\psi_{i+1} - \psi_i) - i\Delta t \int [\mathcal{D}_S \pi_{i+1}] \langle \psi_{i+1} | \pi_{i+1} \rangle \langle \pi_{i+1} | H(\psi_S, \pi_S) | \psi_i \rangle \\ &= \delta(\psi_{i+1} - \psi_i) - i\Delta t \int [\mathcal{D}_S \pi_{i+1}] H(\psi_i, \pi_{i+1}) \langle \psi_{i+1} | \pi_{i+1} \rangle \langle \pi_{i+1} | \psi_i \rangle \\ &= \delta(\psi_{i+1} - \psi_i) - i\Delta t \int [\mathcal{D}_S \pi_{i+1}] H(\psi_i, \pi_{i+1}) e^{i \int d\vec{y} \pi_{i+1}(\vec{y}) [\psi_{i+1}(\vec{y}) - \psi_i(\vec{y})]} / \prod_{\vec{x}} (2\pi) \\ &= \int [\mathcal{D}_S \pi_{i+1}] [1 - i\Delta t H(\psi_i, \pi_{i+1})] e^{i \int d\vec{y} \pi_{i+1}(\vec{y}) [\psi_{i+1}(\vec{y}) - \psi_i(\vec{y})]} / \prod_{\vec{x}} (2\pi) \\ &= \int [\mathcal{D}_S \pi_{i+1}] e^{i \int d\vec{y} \pi_{i+1}(\vec{y}) [\psi_{i+1}(\vec{y}) - \psi_i(\vec{y})] - i\Delta t H(\psi_i, \pi_{i+1})} / \prod_{\vec{x}} (2\pi) \end{aligned}$$



## 时间演化算符的矩阵元

无穷小演化算符:  $e^{-iH\Delta t} \quad \Delta t = (t' - t)/(m + 1) \rightarrow 0$

$$\langle \psi_{i+1} | e^{-iH\Delta t} | \psi_i \rangle = \int [\mathcal{D}_s \pi_{i+1}] e^{i \int d\vec{y} \pi_{i+1}(\vec{y}) [\psi_{i+1}(\vec{y}) - \psi_i(\vec{y})] - i\Delta t H(\psi_i, \pi_{i+1})} / \prod_{\vec{x}} (2\pi)$$

$$\langle \psi' | e^{-iH(t' - t)} | \psi \rangle = \int \mathcal{D}_s \psi_m \cdots \mathcal{D}_s \psi_1 \langle \psi' | e^{-iH(t' - t_m)} | \psi_m \rangle \langle \psi_m | e^{-iH(t_m - t_{m-1})} | \psi_{m-1} \rangle \cdots \\ \cdots \langle \psi_2 | e^{-iH(t_2 - t_1)} | \psi_1 \rangle \langle \psi_1 | e^{-iH(t_1 - t)} | \psi \rangle$$

$$\psi_{m+1} = \psi' \quad \psi_0 = \psi \\ = \left[ \prod_{\vec{x}} (2\pi) \right]^{-(m+1)} \int \left[ \prod_{i=1}^m \mathcal{D}_s \psi_i \right] \int \left[ \prod_{i=0}^m \mathcal{D}_s \pi_{i+1} \right] \\ \times e^{\sum_{i=0}^m \{ i \int d\vec{x} \pi_{i+1}(\vec{x}) [\psi_{i+1}(\vec{x}) - \psi_i(\vec{x})] - i\Delta t H(\psi_i, \pi_{i+1}) \}} \quad \psi_i \Rightarrow \psi_{i+1}$$

$$H = \int d\vec{x} \mathcal{H} \\ = \lim_{m \rightarrow \infty} \left[ \prod_{\vec{x}} (2\pi) \right]^{-(m+1)} \int_{\psi}^{\psi'} \mathcal{D}\psi \int \mathcal{D}\pi e^{i \int_{t'}^{t} d^4x \{ \pi(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] \}}$$

其中我们定义  $\psi_i(\vec{x}) \equiv \psi(\vec{x}, t_i)$ ,  $\pi_i(\vec{x}) \equiv \pi(\vec{x}, t_i)$   $\int_{\psi}^{\psi'} \mathcal{D}\psi = \int \left[ \prod_{i=1}^m \mathcal{D}_s \psi_i \right]$

因此  $\psi_{i+1}(\vec{x}) - \psi_i(\vec{x}) = \psi(\vec{x}, t_{i+1}) - \psi(\vec{x}, t_i) = \dot{\psi}(\vec{x}, t_i) \Delta t$



## 时间演化算符的矩阵元

$$\langle \psi' | e^{-iH(t'-t)} | \psi \rangle = \lim_{n \rightarrow \infty} [\prod_{\bar{x}} (2\pi)]^{-(n+1)} \int_{\psi}^{\psi'} \mathcal{D}\psi \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ \pi(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] \}}$$

$$\text{双线性}\pi: \mathcal{H}[\psi(x), \pi(x)] = c_{lm} \pi_l(x) \pi_m(x) + \mathcal{V}[\psi(x)]$$

$$\int \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ \pi^T(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] \}} = \int \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ \pi^T(x) \dot{\psi}(x) - \pi^T(x) C \pi(x) - \mathcal{V}[\psi(x)] \}}$$

$$= \int \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ -[\pi^T(x) - \frac{1}{2} \dot{\psi}^T(x) C^{-1}] C [\pi(x) - \frac{1}{2} C^{-1} \dot{\psi}(x)] + \frac{1}{4} \dot{\psi}^T(x) C^{-1} \dot{\psi}(x) - \mathcal{V}[\psi(x)] \}}$$

$$= \text{Const} \times e^{i \int_t^{t'} d^4x \{ \frac{1}{4} \dot{\psi}^T(x) C^{-1} \dot{\psi}(x) - \mathcal{V}[\psi(x)] \}}$$

$$\text{Const} \equiv \int \mathcal{D}\pi e^{-i \int_t^{t'} d^4x \pi^T(x) C \pi(x)}$$

将  $\mathcal{H}[\psi(x), \pi(x)]$  看成是一个经典场体系的哈密顿量密度

$$\dot{\psi}_l(x) = \frac{\partial \mathcal{H}[\psi(x), \pi(x)]}{\partial \pi_l(x)} = 2c_{lm} \pi_m(x)$$

$$\pi(x) = \frac{1}{2} C^{-1} \dot{\psi}(x)$$

$$\mathcal{L}[\psi(x), \dot{\psi}(x)] = \pi(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] = \frac{1}{4} \dot{\psi}^T(x) C^{-1} \dot{\psi}(x) - \mathcal{V}[\psi(x)]$$

$$\int \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ \pi(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] \}} = \text{Const} \times e^{i \int_t^{t'} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]}$$



将  $\mathcal{H}[\psi(x), \pi(x)]$  看成是一个经典场体系的哈密顿量密度

$$\dot{\psi}_l(x) = \frac{\partial \mathcal{H}[\psi(x), \pi(x)]}{\partial \pi_l(x)} = 2c_{lm}\pi_m(x) \quad \pi(x) = \frac{1}{2}C^{-1}\dot{\psi}(x)$$

$$\mathcal{L}[\psi(x), \dot{\psi}(x)] = \pi(x)\dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] = \frac{1}{4}\dot{\psi}^T(x)C^{-1}\dot{\psi}(x) - \mathcal{V}[\psi(x)]$$

$$\int \mathcal{D}\pi e^{i\int_t' d^4x \{\pi(x)\dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)]\}} = \text{Const} \times e^{i\int_t' d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]}$$

正规乘积  $\stackrel{\text{算符} \rightarrow \text{路径积分}}{=} \rightarrow$  普通乘积

$$\mathcal{L}_0 = \begin{cases} \frac{1}{2}[(\partial_\mu \phi(x))^2 - M^2 \phi^2(x)] & \text{实自由标量场} \\ \text{Re } \bar{\psi}(x)(i\gamma^\mu \partial_\mu - M)\psi(x) & \text{自由旋量场} \\ -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M^2 v^2(x) & \text{实自由矢量场} \end{cases}$$



## 时间演化算符的矩阵元

$$\langle \psi' | e^{-iH(t'-t)} | \psi \rangle = \lim_{m \rightarrow \infty} [\prod_{\vec{x}} (2\pi)]^{-(m+1)} \int_{\psi}^{\psi'} \mathcal{D}\psi \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ \pi(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] \}}$$

将  $\mathcal{H}[\psi(x), \pi(x)]$  看成是一个经典场体系的哈密顿量密度

$$\int \mathcal{D}\pi e^{i \int_t^{t'} d^4x \{ \pi(x) \dot{\psi}(x) - \mathcal{H}[\psi(x), \pi(x)] \}} = \text{Const} \times e^{i \int_t^{t'} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]}$$

$$\langle \psi' | e^{-iH(t'-t)} | \psi \rangle = \text{const} \times \int_{\psi}^{\psi'} \mathcal{D}\psi e^{i \int_t^{t'} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]} \quad \text{const} = \text{Const} \times \lim_{m \rightarrow \infty} [\prod_{\vec{x}} (2\pi)]^{-(m+1)}$$

$$\langle \Psi_0^- | e^{-iH(t'-t_1)} \psi_{S,1}(x_1) e^{-iH(t_1-t_2)} \psi_{S,2}(x_2) \cdots \psi_{S,n-1}(x_{n-1}) e^{-iH(t_{n-1}-t_n)} \psi_{S,n}(x_n) e^{-iH(t_n-t)} \Psi_0^+ \rangle$$

$$= \int \mathcal{D}_s \psi' \cdots \mathcal{D}_s \psi \psi_1(\vec{x}_1) \psi_2(\vec{x}_2) \cdots \psi_{n-1}(\vec{x}_{n-1}) \psi_n(\vec{x}_n) (\Psi_0^- | \psi') \langle \psi' | e^{-iH(t'-t_1)} | \psi_1 \rangle \langle \psi_1 | e^{-iH(t_1-t_2)} | \psi_2 \rangle \cdots \langle \psi_{n-1} | e^{-iH(t_{n-1}-t_n)} | \psi_n \rangle \langle \psi_n | e^{-iH(t_n-t)} | \psi \rangle \langle \psi | \Psi_0^+ \rangle$$

$$= \int \mathcal{D}\psi \psi_1(x_1) \psi_2(x_2) \cdots \psi_{n-1}(x_{n-1}) \psi_n(x_n) (\Psi_0^- | \psi') e^{i \int_t^{t'} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]} \langle \psi | \Psi_0^+ \rangle$$

$$\int \mathcal{D}\psi \equiv \int \mathcal{D}_s \psi' \mathcal{D}_s \psi \int_{\psi}^{\psi'} \mathcal{D}\psi$$



实标量场的波函数: 取  $\psi_{H,i} = \psi_{H,l_i}$ ,  $\psi_i = \psi_{l_i}$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) = \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]} (\Psi_0^- | \psi') \langle \psi | \Psi_0^+ \rangle$$

$$\psi(x) = \int \frac{d^3p}{\sqrt{2E}(2\pi)^3} [e^{-ip \cdot x} a(\vec{p}) + e^{ip \cdot x} a^\dagger(\vec{p})] \quad \int d^3x e^{-i\vec{p} \cdot \vec{x}} \psi(x) = \frac{1}{\sqrt{2E}} [e^{-iEt} a(\vec{p}) + e^{iEt} a^\dagger(-\vec{p})]$$

$$\pi(x) = \dot{\psi}(x) = -i \int \frac{d^3p}{(2\pi)^3} \sqrt{\frac{E}{2}} [e^{-ip \cdot x} a(\vec{p}) - e^{ip \cdot x} a^\dagger(\vec{p})] \quad \int d^3x e^{-i\vec{p} \cdot \vec{x}} \pi(x) = -i \sqrt{\frac{E}{2}} [e^{-iEt} a(\vec{p}) - e^{iEt} a^\dagger(-\vec{p})]$$

$$\int d^3x e^{-i\vec{p} \cdot \vec{x} + iEt} \left[ \sqrt{\frac{E}{2}} \psi(\vec{x}, t) + \frac{i}{\sqrt{2E}} \pi(\vec{x}, t) \right] = a(\vec{p}) \quad a(\vec{p}) \Phi_0 = 0$$

$$\Phi_0 = \Omega^\dagger(\mp\infty) \Psi_0^\pm \quad \Omega(t) = e^{iHt} e^{-iH_0 t} \quad \psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t} \quad \pi_S(x) = e^{-iH_0 t} \pi(x) e^{iH_0 t}$$

$$\int d^3x e^{-i\vec{p} \cdot \vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} \psi(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi(\vec{x}, \tau) \right] \Omega^\dagger(\mp\infty) \Psi_0^\pm = 0$$

$$\Rightarrow \int d^3x e^{-i\vec{p} \cdot \vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} \psi(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi(\vec{x}, \tau) \right] e^{iH_0 \tau} \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$

$$\Rightarrow \int d^3x e^{-i\vec{p} \cdot \vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} \psi_S(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi_S(\vec{x}, \tau) \right] \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$



## 实标量场的波函数1

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) = \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]} (\Psi_0^- | \psi' \rangle \langle \psi | \Psi_0^+)$$

$$\Phi_0 = \Omega^\dagger(\mp\infty) \Psi_0^\pm \quad \Omega(t) = e^{iHt} e^{-iH_0t} \quad \psi_S(x) = e^{-iH_0t} \psi(x) e^{iH_0t} \quad \pi_S(x) = e^{-iH_0t} \pi(x) e^{iH_0t}$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} \psi_S(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi_S(\vec{x}, \tau) \right] \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x}} \left[ \sqrt{\frac{E}{2}} \psi_S(x) + \frac{i}{\sqrt{2E}} \pi_S(x) \right] \Psi_0^\pm = 0$$

$$[\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)] = [\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)] = 0 \quad [\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)] = i\delta(\vec{x} - \vec{y}) \quad \pi_{S,l}(\vec{x}, t) = \frac{-i\delta}{\delta\psi_{S,l}(\vec{x}, t)}$$

$$\psi_{S,l}(x) |\psi\rangle = \psi_l(\vec{x}) |\psi\rangle \quad \int d^3x e^{-i\vec{p}\cdot\vec{x}} \left[ \sqrt{\frac{E}{2}} \psi(\vec{x}) + \frac{1}{\sqrt{2E}} \frac{\delta}{\delta\psi(\vec{x})} \right] \langle \psi | \Psi_0^\pm \rangle = 0$$

$$\langle \psi | \Psi_0^\pm \rangle = \mathcal{N} e^{-\frac{1}{2} \int d^3x d^3y \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}) \psi(\vec{y})} \stackrel{\mathcal{E}(\vec{x}, \vec{y}) = \mathcal{E}(\vec{y}, \vec{x})}{=} \int d^3x e^{-i\vec{p}\cdot\vec{x}} [E\psi(\vec{x}) - \int d^3y \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y})] = 0$$

$$e^{-i\vec{p}\cdot\vec{x}} E = \int d^3y e^{-i\vec{p}\cdot\vec{y}} \mathcal{E}(\vec{y}, \vec{x}) \stackrel{E = \sqrt{\vec{p}^2 + M^2}}{=} \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \sqrt{\vec{p}^2 + M^2}$$





## 实标量场的波函数2

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) = \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]} (\Psi_0^- | \psi' \rangle \langle \psi | \Psi_0^+)$$

$$\langle \psi | \Psi_0^\pm \rangle = \mathcal{N} e^{-\frac{1}{2} \int d^3x d^3y \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}) \psi(\vec{y})} \xrightarrow{\mathcal{E}(\vec{x}, \vec{y}) = \mathcal{E}(\vec{y}, \vec{x})} \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \sqrt{\vec{p}^2 + M^2}$$

$$t = -\infty, \psi(\vec{x}) \equiv \psi(\vec{x}, -\infty) \quad t' = \infty, \psi'(\vec{x}) \equiv \psi(\vec{x}, \infty)$$

$$\begin{aligned} (\Psi_0^- | \psi' \rangle \langle \psi | \Psi_0^+) &= |\mathcal{N}|^2 e^{-\frac{1}{2} \int d^3x d^3y \mathcal{E}(\vec{x}, \vec{y}) [\psi(\vec{x}, \infty) \psi(\vec{y}, \infty) + \psi(\vec{x}, -\infty) \psi(\vec{y}, -\infty)]} \\ &= \lim_{\epsilon \rightarrow 0^+} |\mathcal{N}|^2 e^{-\frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}, \tau) \psi(\vec{y}, \tau) e^{-\epsilon |\tau|}} \end{aligned}$$

$$\begin{aligned} f(\infty) + f(-\infty) &= \int_{-\infty}^{\infty} d\left[\frac{\tau}{|\tau|} f(\tau)\right] \lim_{\epsilon \rightarrow 0^+} e^{-\epsilon |\tau|} = \lim_{\epsilon \rightarrow 0^+} \int_{-\infty}^{\infty} [d\left\{\frac{\tau}{|\tau|} f(\tau) e^{-\epsilon |\tau|}\right\} - \frac{\tau}{|\tau|} f(\tau) d e^{-\epsilon |\tau|}] \\ &= \lim_{\epsilon \rightarrow 0^+} \epsilon \int_{-\infty}^{\infty} \frac{\tau}{|\tau|} f(\tau) e^{-\epsilon |\tau|} d|\tau| = \lim_{\epsilon \rightarrow 0^+} \epsilon \int_{-\infty}^{\infty} d\tau f(\tau) e^{-\epsilon |\tau|} \end{aligned}$$

$$\begin{aligned} (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) \\ &\quad \times \exp \left\{ i \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] - \frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}, \tau) \psi(\vec{y}, \tau) e^{-\epsilon |\tau|} \right\} \end{aligned}$$



## 实标量场的波函数3

$$\begin{aligned}
 (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) \\
 &\quad \times \exp \left\{ i \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] - \frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}, \tau) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\} \\
 (\Psi_0^-, \Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}\psi \exp \left\{ i \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] - \frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}, \tau) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\}
 \end{aligned}$$

$$\mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \sqrt{\vec{p}^2 + M^2}$$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi(x_1) \cdots \psi(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}}$$

$$S[\psi] \equiv \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] + \lim_{\epsilon \rightarrow 0^+} \frac{i\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{x}, \tau) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|}$$



旋量场的波函数: 取  $\psi_{H,i} = \psi_{H,i}$ ,  $\psi_i = \psi_i$

$$\psi(x) = (2\pi)^{-\frac{3}{2}} \sum_{\sigma} \int d\vec{p} [u(\vec{p}, \sigma) e^{-ip \cdot x} a(\vec{p}, \sigma) + v(\vec{p}, \sigma) e^{ip \cdot x} a^{\dagger}(\vec{p}, \sigma)] \quad \pi(x) = i\psi^*(x)$$

$$(p_{\mu} \gamma^{\mu} - M)u(\vec{p}, \sigma) = 0 \Rightarrow \gamma^0 \frac{\vec{p} \cdot \vec{\gamma} + M}{E} u(\vec{p}, \sigma) = u(\vec{p}, \sigma)$$

$$(p_{\mu} \gamma^{\mu} + M)v(\vec{p}, \sigma) = 0 \Rightarrow \gamma^0 \frac{\vec{p} \cdot \vec{\gamma} + M}{E} v(-\vec{p}, \sigma) = -v(-\vec{p}, \sigma)$$

$$\int d^3x e^{-i\vec{p} \cdot \vec{x}} \gamma^0 \frac{\vec{p} \cdot \vec{\gamma} + M}{E} \psi(x) = (2\pi)^{\frac{3}{2}} \sum_{\sigma} [e^{-iEt} u(\vec{p}, \sigma) a(\vec{p}, \sigma) - e^{iEt} v(-\vec{p}, \sigma) a^{\dagger}(-\vec{p}, \sigma)]$$

$$i \int d^3x e^{-i\vec{p} \cdot \vec{x}} \pi^*(x) = \int d^3x e^{-i\vec{p} \cdot \vec{x}} \psi(x) = (2\pi)^{\frac{3}{2}} \sum_{\sigma} [e^{-iEt} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{iEt} v(-\vec{p}, \sigma) a^{\dagger}(-\vec{p}, \sigma)]$$

$$\Phi_0 = \Omega^{\dagger}(\mp\infty) \Psi_0^{\pm} \quad \Omega(t) = e^{iHt} e^{-iH_0 t} \quad \psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t} \quad \pi_S(x) = e^{-iH_0 t} \pi(x) e^{iH_0 t}$$

$$\int d^3x e^{-i\vec{p} \cdot \vec{x} + iE\tau} [\gamma^0 \frac{\vec{p} \cdot \vec{\gamma} + M}{E} \psi(\vec{x}, \tau) + i\pi^*(\vec{x}, \tau)] \Omega^{\dagger}(\mp\infty) \Psi_0^{\pm} = 0$$

$$\Rightarrow \int d^3x e^{-i\vec{p} \cdot \vec{x} + iE\tau} [\gamma^0 \frac{\vec{p} \cdot \vec{\gamma} + M}{E} \psi_S(\vec{x}, \tau) + i\pi_S^*(\vec{x}, \tau)] \Psi_0^{\pm} \stackrel{\tau \rightarrow \mp\infty}{=} 0$$



## 旋量场的波函数1

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) = \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[\psi(x), \bar{\psi}(x)]} (\Psi_0^- | \psi' \rangle \langle \psi | \Psi_0^+)$$

$$\Phi_0 = \Omega^\dagger(\mp\infty) \Psi_0^\pm \quad \Omega(t) = e^{iHt} e^{-iH_0 t} \quad \psi_S(x) = e^{-iH_0 t} \psi(x) e^{iH_0 t} \quad \pi_S(x) = e^{-iH_0 t} \pi(x) e^{iH_0 t}$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x} + iE\tau} [\gamma^0 \frac{\vec{p}\cdot\vec{\gamma} + M}{E} \psi_S(\vec{x}, \tau) + i\pi_S^*(\vec{x}, \tau)] \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x}} [\gamma^0 \frac{\vec{p}\cdot\vec{\gamma} + M}{E} \psi_S(x) + i\pi_S^*(x)] \Psi_0^\pm = 0$$

$$\{\psi_{S,l}(\vec{x}, t), \psi_{S,l'}(\vec{y}, t)\} = \{\pi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)\} = 0 \quad \{\psi_{S,l}(\vec{x}, t), \pi_{S,l'}(\vec{y}, t)\} = i\delta(\vec{x} - \vec{y}) \quad \pi_{S,l}(\vec{x}, t) = i \frac{\delta}{\delta\psi_l(\vec{x}, t)}$$

$$\psi_{S,l}(x) |\psi\rangle = \psi_l(\vec{x}) |\psi\rangle \quad \int d^3x e^{-i\vec{p}\cdot\vec{x}} [\gamma^0 \frac{\vec{p}\cdot\vec{\gamma} + M}{E} \psi(\vec{x}) + \frac{\delta}{\delta\psi^*(\vec{x})}] \langle \psi | \Psi_0^\pm \rangle = 0$$

$$\langle \psi | \Psi_0^\pm \rangle = \mathcal{N} e^{\int d^3x d^3y \bar{\psi}(\vec{x}) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y})} \Rightarrow \int d^3x e^{-i\vec{p}\cdot\vec{x}} \gamma^0 [\frac{\vec{p}\cdot\vec{\gamma} + M}{E} \psi(\vec{x}) + \int d^3y \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y})] = 0$$

$$e^{-i\vec{p}\cdot\vec{x}} \frac{\vec{p}\cdot\vec{\gamma} + M}{E} = - \int d^3y e^{-i\vec{p}\cdot\vec{y}} \mathcal{E}(\vec{y}, \vec{x}) \stackrel{E = \sqrt{\vec{p}^2 + M^2}}{\Rightarrow} \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \frac{-\vec{p}\cdot\vec{\gamma} - M}{\sqrt{\vec{p}^2 + M^2}}$$



## 旋量场的波函数2

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) = \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)]} (\Psi_0^- | \psi' \rangle \langle \psi | \Psi_0^+)$$

$$\langle \psi | \Psi_0^\pm \rangle = \mathcal{N} e^{\int d^3x d^3y \bar{\psi}(\vec{x}) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y})} \Rightarrow \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \frac{-\vec{p} \cdot \vec{\gamma} - M}{\sqrt{\vec{p}^2 + M^2}}$$

$$t = -\infty, \psi(\vec{x}) \equiv \psi(\vec{x}, -\infty) \quad t' = \infty, \psi'(\vec{x}) \equiv \psi(\vec{x}, \infty)$$

$$(\Psi_0^- | \psi' \rangle \langle \psi | \Psi_0^+) = |\mathcal{N}|^2 e^{\int d^3x d^3y [\bar{\psi}(\vec{x}, \infty) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, \infty) + \bar{\psi}(\vec{x}, -\infty) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, -\infty)]}$$

$$= \lim_{\epsilon \rightarrow 0^+} |\mathcal{N}|^2 e^{\int d^3x d^3y \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|}}$$

$$\begin{aligned} f(\infty) + f(-\infty) &= \int_{-\infty}^{\infty} d\left[\frac{\tau}{|\tau|} f(\tau)\right] \lim_{\epsilon \rightarrow 0^+} e^{-\epsilon|\tau|} = \lim_{\epsilon \rightarrow 0^+} \int_{-\infty}^{\infty} [d\left\{\frac{\tau}{|\tau|} f(\tau) e^{-\epsilon|\tau|}\right\} - \frac{\tau}{|\tau|} f(\tau) d e^{-\epsilon|\tau|}] \\ &= \lim_{\epsilon \rightarrow 0^+} \epsilon \int_{-\infty}^{\infty} \frac{\tau}{|\tau|} f(\tau) e^{-\epsilon|\tau|} d|\tau| = \lim_{\epsilon \rightarrow 0^+} \epsilon \int_{-\infty}^{\infty} d\tau f(\tau) e^{-\epsilon|\tau|} \end{aligned}$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) = |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n)$$

$$\times \exp \left\{ i \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] + \epsilon \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\}$$



## 旋量场的波函数3

$$\begin{aligned}
 (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) \\
 &\quad \times \exp \left\{ i \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] + \epsilon \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\} \\
 (\Psi_0^-, \Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}\psi \exp \left\{ i \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] + \epsilon \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\}
 \end{aligned}$$

$$\mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \frac{-\vec{p} \cdot \vec{\gamma} - M}{\sqrt{\vec{p}^2 + M^2}}$$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,1}(x_1) \cdots \psi_{H,n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi(x_1) \cdots \psi(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}}$$

$$S[\psi] \equiv \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] - \lim_{\epsilon \rightarrow 0^+} i\epsilon \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{y}) \psi(\vec{y}, \tau) e^{-\epsilon|\tau|}$$



取无质量矢量场的波函数: 取  $\psi_{H,i} = v_{H,\mu_i}$ ,  $\psi_i = v_{\mu_i}$

$$(\Psi_0^-, \mathbf{T}v_{H,\mu_1}(x_1) \cdots v_{H,\mu_n}(x_n) \Psi_0^+) = \int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[v(x), \dot{v}(x)]} (\Psi_0^- | v' \rangle \langle v | \Psi_0^+)$$

$$v^\mu(x) = \sum_{\sigma=\pm 1} \int \frac{d^3p}{\sqrt{2E}(2\pi)^{3/2}} [e^{-ip \cdot x} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} e^{\mu*}(\vec{p}, \sigma) a^\dagger(\vec{p}, \sigma)] \quad e^0(\vec{p}, \sigma) = 0$$

$$\int d^3x e^{-i\vec{p} \cdot \vec{x}} v^\mu(x) = \frac{(2\pi)^{3/2}}{\sqrt{2E}} \sum_{\sigma=\pm 1} [e^{-iEt} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{iEt} e^{\mu*}(-\vec{p}, \sigma) a^\dagger(-\vec{p}, \sigma)]$$

$$\pi^\mu(x) = \dot{v}^\mu(x) = -i \sum_{\sigma=\pm 1} \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{E}{2}} [e^{-ip \cdot x} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) - e^{ip \cdot x} e^{\mu*}(\vec{p}, \sigma) a^\dagger(\vec{p}, \sigma)] \quad \pi^0 = 0$$

$$\int d^3x e^{-i\vec{p} \cdot \vec{x}} \pi^\mu(x) = -i(2\pi)^{3/2} \sqrt{\frac{E}{2}} \sum_{\sigma=\pm 1} [e^{-iEt} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) - e^{iEt} e^{\mu*}(-\vec{p}, \sigma) a^\dagger(-\vec{p}, \sigma)]$$

$$(2\pi)^{-3/2} \int d^3x e^{-i\vec{p} \cdot \vec{x} + iEt} \left[ \sqrt{\frac{E}{2}} v^\mu(\vec{x}, t) + \frac{i}{\sqrt{2E}} \pi^\mu(\vec{x}, t) \right] = \sum_{\sigma=\pm 1} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) \quad a(\vec{p}, \sigma) \Phi_0 = 0$$



实无质量矢量场的波函数1: 取  $\psi_{H,i} = v_{H,\mu_i}$ ,  $\psi_i = v_{\mu_i}$

$$(\Psi_0^-, \mathbf{T} v_{H,\mu_1}(x_1) \cdots v_{H,\mu_n}(x_n) \Psi_0^+) = \int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[v(x), \dot{v}(x)]} (\Psi_0^- | v' \rangle \langle v | \Psi_0^+)$$

$$(2\pi)^{-3/2} \int d^3x e^{-i\vec{p}\cdot\vec{x} + iEt} \left[ \sqrt{\frac{E}{2}} v^\mu(\vec{x}, t) + \frac{i}{\sqrt{2E}} \pi^\mu(\vec{x}, t) \right] = \sum_{\sigma=\pm 1} e^\mu(\vec{p}, \sigma) a(\vec{p}, \sigma) \quad a(\vec{p}, \sigma) \Phi_0 = 0$$

$$\Phi_0 = \Omega^\dagger(\mp\infty) \Psi_0^\pm \quad \Omega(t) = e^{iHt} e^{-iH_0 t} \quad v_S^\mu(x) = e^{-iH_0 t} v^\mu(x) e^{iH_0 t} \quad \pi_S^i(x) = e^{-iH_0 t} \pi^i(x) e^{iH_0 t}$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} v^\mu(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi^\mu(\vec{x}, \tau) \right] \Omega^\dagger(\mp\infty) \Psi_0^\pm = 0$$

$$\Rightarrow \int d^3x e^{-i\vec{p}\cdot\vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} v^\mu(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi^\mu(\vec{x}, \tau) \right] e^{iH_0 \tau} \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$

$$\Rightarrow \int d^3x e^{-i\vec{p}\cdot\vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} v_S^\mu(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi_S^\mu(\vec{x}, \tau) \right] \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$





## 实无质量矢量场的波函数2

$$(\Psi_0^-, \mathbf{T}v_{H,\mu_1}(x_1) \cdots v_{H,\mu_n}(x_n) \Psi_0^+) = \int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[v(x), \dot{v}(x)]} (\Psi_0^- | v' \rangle \langle v | \Psi_0^+)$$

$$\Phi_0 = \Omega^\dagger(\mp\infty) \Psi_0^\pm \quad \Omega(t) = e^{iHt} e^{-iH_0 t} \quad v_S(x) = e^{-iH_0 t} v(x) e^{iH_0 t} \quad \pi_S(x) = e^{-iH_0 t} \pi(x) e^{iH_0 t}$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x} + iE\tau} \left[ \sqrt{\frac{E}{2}} v_S^\mu(\vec{x}, \tau) + \frac{i}{\sqrt{2E}} \pi_S^\mu(\vec{x}, \tau) \right] \Psi_0^\pm \stackrel{\tau \rightarrow \mp\infty}{=} 0$$

$$\int d^3x e^{-i\vec{p}\cdot\vec{x}} \left[ \sqrt{\frac{E}{2}} v_S^\mu(x) + \frac{i}{\sqrt{2E}} \pi_S^\mu(x) \right] \Psi_0^\pm = 0$$

$$[v_S^i(\vec{x}, t), v_S^j(\vec{y}, t)] = [\pi_S^i(\vec{x}, t), \pi_S^j(\vec{y}, t)] = 0 \quad [\psi_S^i(\vec{x}, t), \pi_S^j(\vec{y}, t)] = i\delta^{ij} \delta(\vec{x} - \vec{y}) \quad \pi_S^i(\vec{x}, t) = -i \frac{\delta}{\delta v^i(\vec{x}, t)}$$

$$v_S^\mu(x) |v\rangle = v^\mu(\vec{x}) |v\rangle \quad v^0(\vec{x}) = 0 \quad \int d^3x e^{-i\vec{p}\cdot\vec{x}} \left[ \sqrt{\frac{E}{2}} v^j(\vec{x}) + \frac{1}{\sqrt{2E}} \frac{\delta}{\delta v^i(\vec{x})} \right] \langle v | \Psi_0^\pm \rangle = 0$$

$$\langle v | \Psi_0^\pm \rangle = \mathcal{N} e^{\frac{1}{2} \int d^3x d^3y \mathcal{E}(\vec{x}, \vec{y}) v_\mu(\vec{x}) v^\mu(\vec{y})} \stackrel{\mathcal{E}(\vec{x}, \vec{y}) = \mathcal{E}(\vec{y}, \vec{x})}{=} \int d^3x e^{-i\vec{p}\cdot\vec{x}} \left[ E v^\mu(\vec{x}) - \int d^3y \mathcal{E}(\vec{x}, \vec{y}) v^\mu(\vec{y}) \right] = 0$$

$$e^{-i\vec{p}\cdot\vec{x}} E = \int d^3y e^{-i\vec{p}\cdot\vec{y}} \mathcal{E}(\vec{y}, \vec{x}) \stackrel{E = \sqrt{\vec{p}^2}}{=} \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \sqrt{\vec{p}^2}$$



## 实无质量矢量场的波函数3

$$\langle \Psi_0^-, \mathbf{T} v_{H, \mu_1}(x_1) \cdots v_{H, \mu_n}(x_n) \Psi_0^+ \rangle = \int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n) e^{i \int_{t'}^{t''} d^4x \mathcal{L}[v(x), \dot{v}(x)]} \langle \Psi_0^- | v' \rangle \langle v | \Psi_0^+ \rangle$$

$$\langle v | \Psi_0^\pm \rangle = \mathcal{N} e^{\frac{1}{2} \int d^3x d^3y \mathcal{E}(\vec{x}, \vec{y}) v_\mu(\vec{x}) v^\mu(\vec{y})} \stackrel{\mathcal{E}(\vec{x}, \vec{y}) = \mathcal{E}(\vec{y}, \vec{x})}{\implies} \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \sqrt{p^2}$$

$$t = -\infty, v(\vec{x}) \equiv v(\vec{x}, -\infty) \quad t' = \infty, v'(\vec{x}) \equiv v(\vec{x}, \infty)$$

$$\langle \Psi_0^- | v' \rangle \langle v | \Psi_0^+ \rangle = |\mathcal{N}|^2 e^{\frac{1}{2} \int d^3x d^3y \mathcal{E}(\vec{x}, \vec{y}) [v_\mu(\vec{x}, \infty) v^\mu(\vec{y}, \infty) + v_\mu(\vec{x}, -\infty) v^\mu(\vec{y}, -\infty)]}$$

$$= \lim_{\epsilon \rightarrow 0^+} |\mathcal{N}|^2 e^{\frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) v_\mu(\vec{x}, \tau) v^\mu(\vec{y}, \tau) e^{-\epsilon|\tau|}}$$

$$\begin{aligned} f(\infty) + f(-\infty) &= \int_{-\infty}^{\infty} d\left[\frac{\tau}{|\tau|} f(\tau)\right] \lim_{\epsilon \rightarrow 0^+} e^{-\epsilon|\tau|} = \lim_{\epsilon \rightarrow 0^+} \int_{-\infty}^{\infty} [d\left\{\frac{\tau}{|\tau|} f(\tau) e^{-\epsilon|\tau|}\right\} - \frac{\tau}{|\tau|} f(\tau) d e^{-\epsilon|\tau|}] \\ &= \lim_{\epsilon \rightarrow 0^+} \epsilon \int_{-\infty}^{\infty} \frac{\tau}{|\tau|} f(\tau) e^{-\epsilon|\tau|} d|\tau| = \lim_{\epsilon \rightarrow 0^+} \epsilon \int_{-\infty}^{\infty} d\tau f(\tau) e^{-\epsilon|\tau|} \end{aligned}$$

$$\langle \Psi_0^-, \mathbf{T} v_{H, \mu_1}(x_1) \cdots v_{H, \mu_n}(x_n) \Psi_0^+ \rangle = |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n)$$

$$\times \exp \left\{ i \int d^4x \mathcal{L}[v(x), \dot{v}(x)] + \frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) v_\mu(\vec{x}, \tau) v^\mu(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\}$$



## 实无质量矢量场的波函数4

$$\begin{aligned}
 (\Psi_0^-, \mathbf{T}v_{H,\mu_1}(x_1) \cdots v_{H,\mu_n}(x_n) \Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n) \\
 &\quad \times \exp \left\{ i \int d^4x \mathcal{L}[v(x), \dot{v}(x)] + \frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) v_{\mu}(\vec{x}, \tau) v^{\mu}(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\} \\
 (\Psi_0^-, \Psi_0^+) &= |\mathcal{N}|^2 \lim_{\epsilon \rightarrow 0^+} \int \mathcal{D}v \exp \left\{ i \int d^4x \mathcal{L}[v(x), \dot{v}(x)] + \frac{\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) v_{\mu}(\vec{x}, \tau) v^{\mu}(\vec{y}, \tau) e^{-\epsilon|\tau|} \right\}
 \end{aligned}$$

$$\mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \sqrt{p^2}$$

$$\frac{(\Psi_0^-, \mathbf{T}v_{H,\mu_1}(x_1) \cdots v_{H,\mu_n}(x_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}v v_{\mu_1}(x_1) \cdots v_{\mu_n}(x_n) e^{iS[v]}}{\int \mathcal{D}v e^{iS[v]}}$$

$$S[v] \equiv \int d^4x \mathcal{L}[v(x), \dot{v}(x)] - \lim_{\epsilon \rightarrow 0^+} \frac{i\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) v_{\mu}(\vec{x}, \tau) v^{\mu}(\vec{y}, \tau) e^{-\epsilon|\tau|}$$

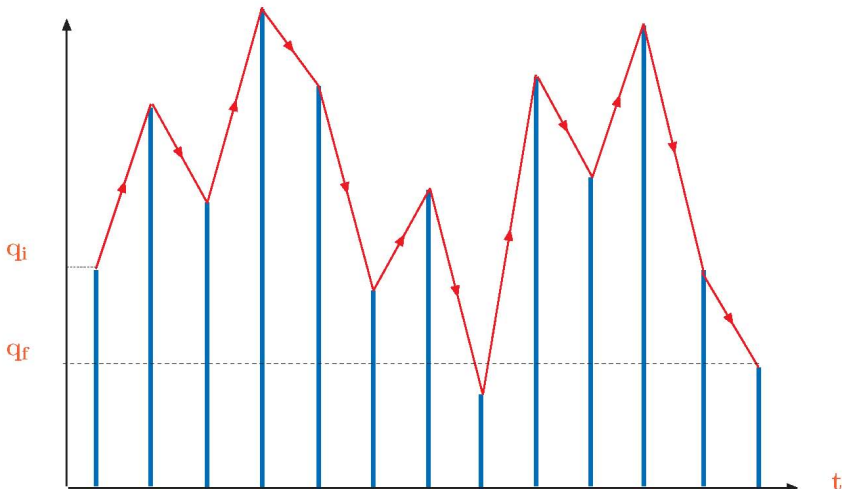




波函数

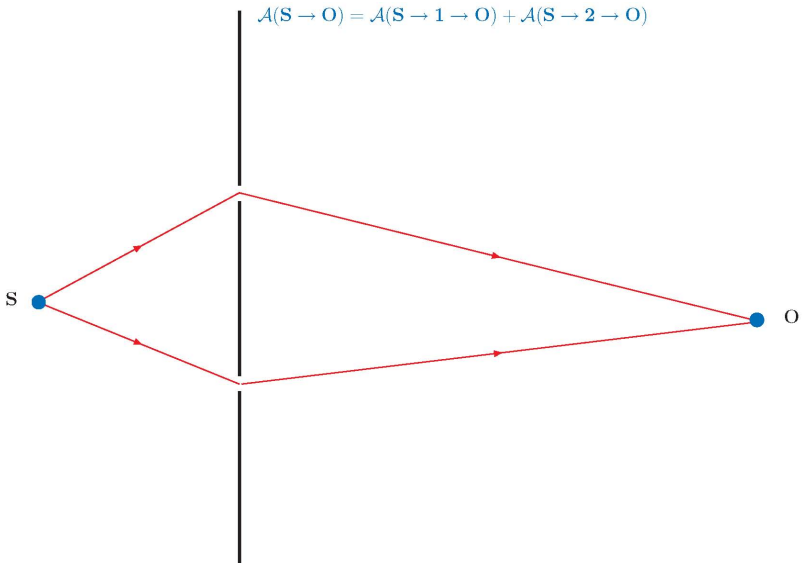
$$\mathcal{A}(q_i \rightarrow q_f) = \sum_{q(t_i)} \mathcal{A}(q_i \rightarrow q(t_1) \rightarrow \dots \rightarrow q(t_n) \rightarrow q_f)$$

$q(t)$





$$\mathcal{A}(S \rightarrow O) = \mathcal{A}(S \rightarrow 1 \rightarrow O) + \mathcal{A}(S \rightarrow 2 \rightarrow O)$$

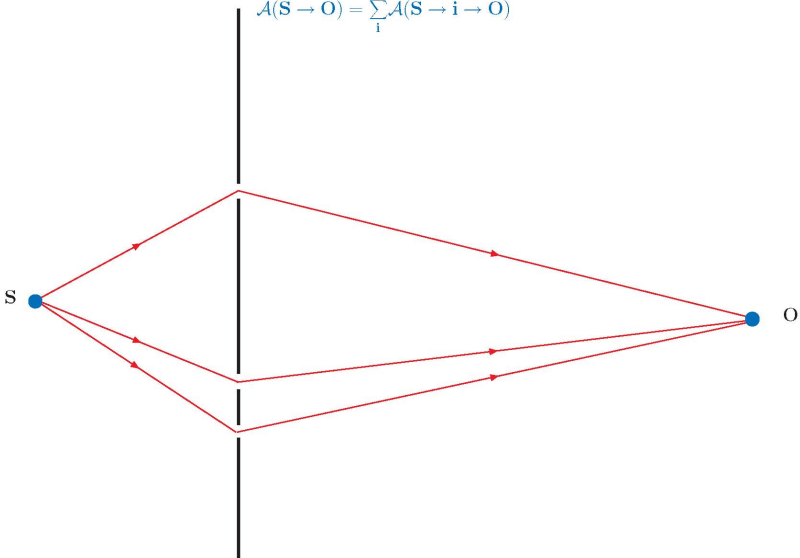


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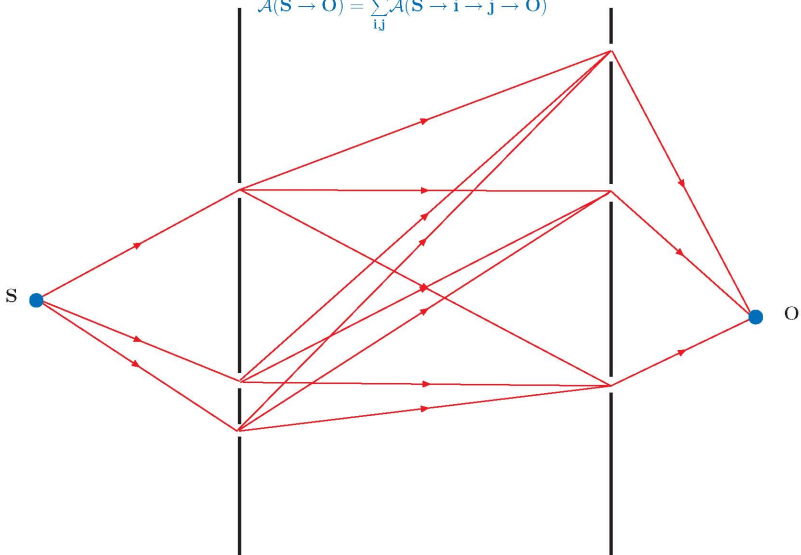
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$$\mathcal{A}(S \rightarrow O) = \sum_i \mathcal{A}(S \rightarrow i \rightarrow O)$$





$$\mathcal{A}(S \rightarrow O) = \sum_{i,j} \mathcal{A}(S \rightarrow i \rightarrow j \rightarrow O)$$









## 连接图的生成泛函与作用量

$$e^{iW[J]} = \int \mathcal{D}\phi e^{iS[\phi]} e^{i \int d^4x J(x)\phi(x)}$$

泛函傅立叶反变换:

$$\begin{aligned} e^{iS[\phi]} &= \int \mathcal{D}J e^{iW[J]} e^{-i \int d^4x J(x)\phi(x)} \\ &= \int \mathcal{D}J \int \mathcal{D}\phi' e^{iS[\phi']} e^{i \int d^4x J(x)[\phi'(x) - \phi(x)]} = \mathbf{const} \int \mathcal{D}\phi' e^{iS[\phi']} \delta(\phi' - \phi) \end{aligned}$$

作用量和联接图的生成泛函互为泛函傅立叶变换!

知道了所有连接格林函数,也就知道了作用量,反之亦然!



## 玻色场的生成泛函

## 有效作用量与顶角生成泛函

$$\text{有效作用量: } \psi_{c,l}(x) \equiv \frac{\delta W[J]}{\delta J_l(x)} \quad \Gamma[\psi_c] \equiv W[J] - \int d^4x J_l(x) \psi_{c,l}(x)$$

$$\frac{\delta \Gamma[\psi_c]}{\delta \psi_{c,l}(x)} = \int d^4y \frac{\delta W[J]}{\delta J_{l'}(y)} \frac{\delta J_{l'}(y)}{\delta \psi_{c,l}(x)} - \int d^4y \frac{\delta J_{l'}(y)}{\delta \psi_{c,l}(x)} \psi_{c,l'}(y) - J_l(x) = -J_l(x)$$

$$S_J[\psi] \equiv S[\psi] + \int d^4x J_l(x) \psi_l(x) \quad \frac{\delta S_J[\psi]}{\delta \psi_l(x)} = 0 \quad \Rightarrow \quad \frac{\delta S[\psi]}{\delta \psi_l(x)} = -J_l(x)$$

## 顶角生成泛函

$$\Gamma_{l_1 \dots l_n}(x_1, \dots, x_n) \equiv \left. \frac{\delta^n \Gamma[\psi_c]}{\delta \psi_{c,l_1}(x_1) \dots \delta \psi_{c,l_n}(x_n)} \right|_{\psi_c = \psi_0} \quad \psi_{0,l}(x) = \left. \frac{\delta W[J]}{\delta J_l(x)} \right|_{J=0}$$

$$\Gamma[\psi_c] = \Gamma[\psi_0] + \sum_{n=1} \frac{1}{n!} \int d^4x_1 \dots d^4x_n \Gamma_{l_1 \dots l_n}(x_1, \dots, x_n) [\psi_{c,l_1}(x_1) - \psi_{0,l_1}(x_1)] \dots [\psi_{c,l_n}(x_n) - \psi_{0,l_n}(x_n)]$$

$$\delta(x-x') \delta_{ll'} = \frac{\delta \psi_{c,l}(x)}{\delta \psi_{c,l'}(x')} = \int d^4y \frac{\delta \psi_{c,l}(x)}{\delta J_{l'}(y)} \frac{\delta J_{l'}(y)}{\delta \psi_{c,l'}(x')} = - \int d^4y \frac{\delta^2 W[J]}{\delta J_l(x) \delta J_{l'}(y)} \frac{\delta^2 \Gamma[\psi_c]}{\delta \psi_{c,l'}(y) \delta \psi_{c,l}(x')}$$

$$\int d^4y (\Psi_0^-, \psi_{H,l}(x) \psi_{H,l'}(y) \Psi_0^+)_C \Gamma_{l'l'}(y, x') = i \delta(x-x') \delta_{ll'}$$



## 玻色场的生成泛函

## 两线顶角:

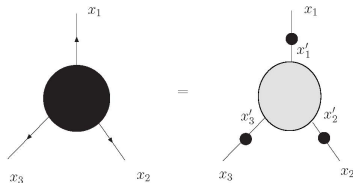
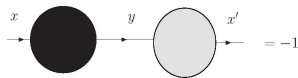
$$\delta(x-x')\delta_{ll'} = \frac{\delta\psi_{c,l}(x)}{\delta\psi_{c,l'}(x')} = \int d^4y \frac{\delta\psi_{c,l}(x)}{\delta J_{l'}(y)} \frac{\delta J_{l'}(y)}{\delta\psi_{c,l'}(x')} = - \int d^4y \frac{\delta^2 W[J]}{\delta J_l(x)\delta J_{l'}(y)} \frac{\delta^2 \Gamma[\psi_c]}{\delta\psi_{c,l'}(y)\delta\psi_{c,l}(x')}$$

## 三线顶角:

$$0 = \frac{\delta}{\delta J(x_3)} \int d^4x_2 \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)} \frac{\delta\Gamma[\psi_c]}{\delta\psi_c(x_2)\delta\psi_c(x')}$$

$$= \int d^4x_2 \left[ \frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)} \frac{\delta^2 \Gamma[\psi_c]}{\delta\psi_c(x_2)\delta\psi_c(x')} + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)} \int d^4y \frac{\delta^3 \Gamma[\psi_c]}{\delta\psi_c(y)\delta\psi_c(x_2)\delta\psi_c(x')} \frac{\delta\psi_c(y)}{\delta J(x_3)} \right]$$

$$\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)} = \int d^4x'_1 d^4x'_2 d^4x'_3 \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x'_1)} \frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x'_2)} \frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x'_3)} \frac{\delta^3 \Gamma[\psi_c]}{\delta\psi_c(x'_1)\delta\psi_c(x'_2)\delta\psi_c(x'_3)}$$





玻色场的生成泛函

$$\text{四线顶角: } \frac{\delta^4 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)}$$

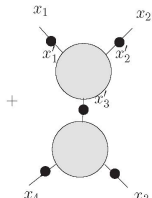
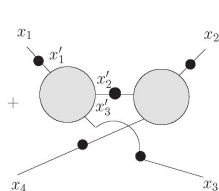
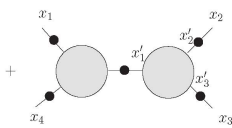
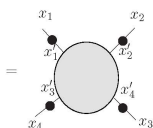
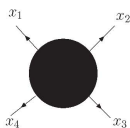
$$= \frac{\delta}{\delta J(x_4)} \left[ \int d^4 x'_1 d^4 x'_2 d^4 x'_3 \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x'_1)} \frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x'_2)} \frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x'_3)} \frac{\delta^3 \Gamma[\psi_c]}{\delta \psi_c(x'_1)\delta \psi_c(x'_2)\delta \psi_c(x'_3)} \right]$$

$$= \int d^4 x'_1 d^4 x'_2 d^4 x'_3 d^4 x'_4 \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x'_1)} \frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x'_2)} \frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x'_3)} \frac{\delta^2 W[J]}{\delta J(x_4)\delta J(x'_4)}$$

$$\times \frac{\delta^4 \Gamma[\psi_c]}{\delta \psi_c(x'_1)\delta \psi_c(x'_2)\delta \psi_c(x'_3)\delta \psi_c(x'_4)} + \int d^4 x'_1 d^4 x'_2 d^4 x'_3 \frac{\delta^3 \Gamma[\psi_c]}{\delta \psi_c(x'_1)\delta \psi_c(x'_2)\delta \psi_c(x'_3)}$$

$$\times \left[ \frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x'_1)\delta J(x_4)} \frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x'_2)} \frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x'_3)} + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x'_1)} \frac{\delta^3 W[J]}{\delta J(x_2)\delta J(x'_2)\delta J(x_4)} \frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x'_3)} \right.$$

$$\left. + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x'_1)} \frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x'_2)} \frac{\delta^3 W[J]}{\delta J(x_3)\delta J(x'_3)\delta J(x_4)} \right]$$





## 玻色场的生成泛函

## 自由玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi]}$$

$$S[\psi] \equiv \int d^4x \mathcal{L}[\psi(x), \dot{\psi}(x)] \pm \lim_{\epsilon \rightarrow 0^+} \frac{i\epsilon}{2} \int d^3x d^3y \int_{-\infty}^{\infty} d\tau \mathcal{E}(\vec{x}, \vec{y}) \psi_l(\vec{x}, \tau) \psi_l(\vec{y}, \tau) e^{-\epsilon|\tau|} \begin{array}{l} + \text{标量} \\ - \text{矢量} \end{array}$$

$$Z[J] \equiv \int \mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_J \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x) \psi_l(x)$$

$$S_0 = \frac{1}{2} \int d^4x d^4x' [iD_{ll'}^{-1}(x, x') \psi_l(x) \psi_{l'}(x') \pm \lim_{\epsilon \rightarrow 0^+} i\epsilon \mathcal{E}(\vec{x}, \vec{x}') \psi_l(x) \psi_{l'}(x') \delta(t-t')] e^{-\epsilon|t+t'|/2}$$

$$\begin{aligned} Z_0[J] &= \int \mathcal{D}\psi \exp i\left\{ \frac{1}{2} \int d^4x d^4x' iD_{ll'}^{-1}(x, x') \psi_l(x) \psi_{l'}(x') + \int d^4x J_l(x) \psi_l(x) \right\} \\ &= \int \mathcal{D}\psi \exp i\text{Tr}\left[ \frac{1}{2} \psi iD^{-1} \psi + J\psi \right] = \int \mathcal{D}\psi \exp i\left\{ \frac{i}{2} \text{Tr}[(\psi - iJD)D^{-1}(\psi - iDJ) + JDJ] \right\} \\ &= C \times \exp\left[ \frac{-1}{2} \int d^4x d^4x' D_{ll'}^{-1}(x, x') J_l(x) J_{l'}(x') \right] \quad C = \int \mathcal{D}\psi \exp\left[ \frac{-1}{2} \int d^4x d^4x' D_{ll'}^{-1}(x, x') \psi_l(x) \psi_{l'}(x') \right] \end{aligned}$$

$$W_0[J] = \frac{i}{2} \int d^4x d^4x' D_{ll'}^{-1}(x, x') J_l(x) J_{l'}(x') \quad D_{ll'}^{-1}(x, x') = D_{ll'}^{-1}(x, x') \pm \lim_{\epsilon \rightarrow 0^+} \epsilon \mathcal{E}(\vec{x}, \vec{x}') \delta(t-t') e^{-\epsilon|t+t'|/2}$$



## 玻色场的生成泛函

## 自由玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi]}$$

$$S_0 = \frac{1}{2} \int d^4x d^4x' [iD_{ll'}^{-1}(x, x')\psi_l(x)\psi_{l'}(x') \pm \lim_{\epsilon \rightarrow 0^+} i\epsilon \mathcal{E}(\vec{x}, \vec{x}')\psi_l(x)\psi_{l'}(x')\delta(t-t')] e^{-\epsilon|t+t'|/2}$$

$$Z_0[J] = C \times \exp\left[-\frac{1}{2} \int d^4x d^4x' D'_{ll'}(x, x') J_l(x) J_{l'}(x')\right] \quad C = \int \mathcal{D}\psi \exp\left[-\frac{1}{2} \int d^4x d^4x' D'_{ll'}{}^{-1}(x, x') \psi_l(x) \psi_{l'}(x')\right]$$

$$W_0[J] = \frac{i}{2} \int d^4x d^4x' D'_{ll'}(x, x') J_l(x) J_{l'}(x') \quad D'_{ll'}{}^{-1}(x, x') = D_{ll'}^{-1}(x, x') \pm \lim_{\epsilon \rightarrow 0^+} \epsilon \mathcal{E}(\vec{x}, \vec{x}') \delta(t-t') e^{-\epsilon|t+t'|/2}$$

$$\psi_{c,l}(x) = \frac{\delta W_0[J]}{\delta J_l(x)} = \int d^4x' iD'_{ll'}(x, x') J_{l'}(x') \quad J_l(x) = -i \int d^4x' D'_{ll'}{}^{-1}(x, x') \psi_{c,l'}(x')$$

$$\Gamma_0[\psi_c] = W_0[J] - \int d^4x J_l(x) \psi_{c,l}(x) = \frac{i}{2} \int d^4x d^4x' D'_{ll'}{}^{-1}(x, x') \psi_{c,l}(x) \psi_{c,l'}(x') = S_0[\psi_c]$$

$$\text{自由玻色场只有两线顶角: } \Gamma_{0,ll'}(x, x') = \frac{\delta^2 \Gamma_0[\phi_c]}{\delta \psi_{c,l}(x) \delta \psi_{c,l'}(x')} = iD'_{ll'}{}^{-1}(x, x')$$



## 玻色场的生成泛函

## 玻色场传播子 (两线顶角的逆)

$$S_0 = \frac{1}{2} \int d^4x d^4x' [iD_{ll'}^{-1}(x, x')\psi_l(x)\psi_{l'}(x') \pm \lim_{\epsilon \rightarrow 0^+} i\epsilon \mathcal{E}(\vec{x}, \vec{x}')\psi_l(x)\psi_{l'}(x')\delta(t-t')] e^{-\epsilon|t+t'|/2}$$

$$W_0[J] = \frac{i}{2} \int d^4x d^4x' D'_{ll'}(x, x') J_l(x) J_{l'}(x') \quad D'_{ll'}(x, x') = D_{ll'}^{-1}(x, x') \pm \lim_{\epsilon \rightarrow 0^+} \epsilon \mathcal{E}(\vec{x}, \vec{x}')\delta(t-t') e^{-\epsilon|t+t'|/2}$$

$$W[J] = W[0] + \sum_{n=1}^{\infty} \frac{i^{n-1}}{n!} \int d^4x_1 \cdots d^4x_n (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+)_C J_{l_1}(x_1) \cdots J_{l_n}(x_n)$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+)_C \stackrel{\text{自由场}}{=} \begin{cases} 0 & n \neq 2 \\ D'_{l_1 l_2}(x_1, x_2) = \underline{\psi_{l_1}(x_1)\psi_{l_2}(x_2)} & n = 2 \end{cases}$$

实自由标量场:  $S_0[\phi] = \int d^4x \frac{1}{2} [(\partial_\mu \phi(x))^2 - M^2 \phi^2(x)]$

$$iD^{-1}(x, x') = -(\partial_x^2 + M^2)\delta(x-x') = \int \frac{d^4p}{(2\pi)^4} (p^2 - M^2) e^{-ip \cdot (x-x')} \quad \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x}-\vec{y})} \sqrt{\vec{p}^2 + M^2}$$

$$D'^{-1}(x, x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} [-i(p^2 - M^2) + \epsilon \sqrt{\vec{p}^2 + M^2} e^{-\epsilon \frac{|t+t'|}{2}}] = -i \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} (p^2 - M^2 + i0^+)$$

$$D'(x, x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} \frac{i}{p^2 - M^2 + i0^+}$$





## 玻色场的生成泛函

## 玻色场传播子 (两线顶角的逆)

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+)_C \stackrel{\text{自由场}}{=} \begin{cases} 0 & n \neq 2 \\ \mathcal{D}'_{l_1 l_2}(x_1, x_2) = \underline{\psi_{l_1}(x_1)\psi_{l_2}(x_2)} & n = 2 \end{cases}$$

$$\text{对完全格林函数: } \left. \frac{\delta^2 W[J]}{\delta J_l(x)\delta J_{l'}(x')} \right|_{J_l=0} \stackrel{\text{自由场}}{=} i\mathcal{D}'_{ll'}(x, x')$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+) = \left. \frac{\delta^n Z[J]}{i^n \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \right|_{J_l=0} = e^{-iW[J]} \left. \frac{\delta^n e^{iW[J]}}{i^n \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \right|_{J_l=0}$$

$$\stackrel{\text{自由场}}{=} \begin{cases} 0 & n = \text{奇数} \\ \sum_{\mathcal{P} 2,3,\dots,n} \underbrace{\mathcal{D}'_{l_1 l_2}(x_1, x_2) \cdots \mathcal{D}'_{l_{n-1} l_n}(x_{n-1}, x_n)}_{\text{不连接图}} & n = \text{偶数} \end{cases}$$

不计传播子内部的顺序差别

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\psi_{H,l_2}(x_2)\psi_{H,l_3}(x_3)\psi_{H,l_4}(x_4)\Psi_0^+) \\ = \mathcal{D}'_{l_1 l_2}(x_1, x_2)\mathcal{D}'_{l_3 l_4}(x_3, x_4) + \mathcal{D}'_{l_1 l_3}(x_1, x_3)\mathcal{D}'_{l_2 l_4}(x_2, x_4) + \mathcal{D}'_{l_1 l_4}(x_1, x_4)\mathcal{D}'_{l_2 l_3}(x_2, x_3)$$



## 玻色场的生成泛函

## 实自由标量场的粒子与相互作用力诠释

$$S_0[\phi] = \int d^4x \frac{1}{2} [(\partial_\mu \phi(x))^2 - M^2 \phi^2(x)]$$

$$W_0[J] = \frac{i}{2} \int d^4x d^4x' \mathcal{D}'(x, x') J(x) J(x') \quad \mathcal{D}'(x, x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} \frac{i}{p^2 - M^2 + i0^+}$$

$$J(p) \equiv \int d^4x e^{-ip \cdot x} J(x) \quad W_0[J] = -\frac{1}{2} \int \frac{d^4p}{(2\pi)^4} J^*(p) \frac{1}{p^2 - M^2 + i0^+} J(p)$$

两个相互分离的局域源:  $J(x) = J_1(x) + J_2(x)$

$$W_{0,\text{int}}[J] = -\frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \left[ J_1^*(p) \frac{1}{p^2 - M^2 + i0^+} J_2(p) + J_2^*(p) \frac{1}{p^2 - M^2 + i0^+} J_1(p) \right]$$

相互作用在  $p^2 = M^2$  处最强!

$$J_i(x) = \delta(\vec{x} - \vec{x}_i) \quad J_i(p) = 2\pi \delta(p^0) e^{i\vec{p} \cdot \vec{x}_i} \quad \left. 2\pi \delta(p^0) \right|_{p^0=0} = T \quad W_{0,\text{int}}[J] = -E_J T$$

$$E_J = - \int \frac{d\vec{p}}{(2\pi)^3} \frac{e^{i\vec{p} \cdot (\vec{x}_1 - \vec{x}_2)}}{p^2 + M^2} = \frac{-1}{(2\pi)^2} \int_0^\infty dp p^2 \int_{-1}^1 d \cos \theta \frac{e^{ip|\vec{x}_1 - \vec{x}_2| \cos \theta}}{p^2 + M^2} = \frac{-1}{4\pi |\vec{x}_1 - \vec{x}_2|} e^{-m|\vec{x}_1 - \vec{x}_2|}$$

$$\frac{dE_J}{d|\vec{x}_1 - \vec{x}_2|} > 0 \Rightarrow \text{吸引势! 库伦定律, 比萨定律(见后); 万有引力定律}$$



## 玻色场的生成泛函

## 玻色场传播子 (两线顶角的逆)

$$S_0 = \frac{1}{2} \int d^4x d^4x' [iD_{ll'}^{-1}(x, x') \psi_l(x) \psi_{l'}(x') \pm \lim_{\epsilon \rightarrow 0^+} i\epsilon \mathcal{E}(\vec{x}, \vec{x}') \psi_l(x) \psi_{l'}(x') \delta(t - t')] e^{-\epsilon|t+t'|/2}$$

$$W_0[J] = \frac{i}{2} \int d^4x d^4x' D_{ll'}^{-1}(x, x') J_l(x) J_{l'}(x') \quad D_{ll'}^{-1}(x, x') = D_{ll'}(x, x') \pm \lim_{\epsilon \rightarrow 0^+} \epsilon \mathcal{E}(\vec{x}, \vec{x}') \delta(t - t') e^{-\epsilon|t+t'|/2}$$

$$W[J] = W[0] + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int d^4x_1 \cdots d^4x_n (\Psi_0^-, \mathbf{T} \psi_{H, l_1}(x_1) \cdots \psi_{H, l_n}(x_n) \Psi_0^+) c J_{l_1}(x_1) \cdots J_{l_n}(x_n)$$

实零质量自由矢量场:  $S_0[\phi] = \int d^4x \frac{1}{2} \{-[\partial_\mu v_\nu(x)][\partial^\mu v^\nu(x)]\}$

$$iD^{\mu\nu, -1}(x, x') = g^{\mu\nu} \partial_x^2 \delta(x - x') = \int \frac{d^4p}{(2\pi)^4} (-p^2) g^{\mu\nu} e^{-ip \cdot (x - x')} \quad \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \sqrt{\vec{p}^2}$$

$$D^{\mu\nu, -1}(x, x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - x')} g^{\mu\nu} (ip^2 - \epsilon \sqrt{\vec{p}^2} e^{-\epsilon|t+t'|/2}) = i \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - x')} g^{\mu\nu} (p^2 + i0^+)$$

$$D^{\mu\nu'}(x, x') = \int \frac{d^4p}{(2\pi)^4} \frac{-ig^{\mu\nu} e^{-ip \cdot (x - x')}}{p^2 + i0^+} \Rightarrow W_0[J] = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - x')} \frac{J_\mu^*(p) J^\mu(p)}{p^2 + i0^+}$$

$$\frac{J^\mu(x) = \delta_0^\mu [\delta(\vec{x} + \vec{x}_1) + \delta(\vec{x} - \vec{x}_2)]}{\Rightarrow} E_J = \frac{1}{4\pi |\vec{x}_1 - \vec{x}_2|} \quad \text{平方反比: } \underline{\text{同号电荷相斥, 同向电流相吸!}}$$



## 玻色场的生成泛函

## 相互作用玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} = \left. \frac{\delta^n Z[J]}{i^n Z[J] \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \right|_{J=0}$$

$$Z[J] \equiv \int \mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_J \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS_J[\psi]} \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x) \psi_l(x)$$

$$S_{0,J} = S_0[\psi] + \int d^4x J_l(x) \psi_l(x) \quad S = S_0 + \int d^4x \mathcal{L}_I[\psi(x)] \quad S_0 = \frac{1}{2} \int d^4x d^4x' i\mathcal{D}'_{ll'}{}^{-1}(x, x') \psi_l(x) \psi_{l'}(x')$$

$$Z_0[J] \equiv \int \mathcal{D}\psi e^{iS_{0,J}[\psi]} = C \times \exp\left[\frac{-1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}(x, x') J_l(x) J_{l'}(x')\right] \quad V = -\int d\vec{x} \mathcal{L}_I \quad \mathcal{H} = -\mathcal{L}_I[\psi]$$

## 微扰展开:

$$\frac{\delta \int d^4x J_l(x) \psi_l(x)}{\delta J_l(y)} = \psi_l(y) \quad \frac{\delta e^{i \int d^4x J_l(x) \psi_l(x)}}{i \delta J_l(y)} = e^{i \int d^4x J_l(x) \psi_l(x)} \frac{\delta i \int d^4x J_l(x) \psi_l(x)}{i \delta J_l(y)} = \psi_l(y) e^{i \int d^4x J_l(x) \psi_l(x)}$$

$$Z[J] = \int \mathcal{D}\psi e^{i \int d^4x \mathcal{L}_I[\psi(x)]} e^{iS_{0,J}[\psi]} = \int \mathcal{D}\psi e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]} e^{iS_{0,J}[\psi]} = e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]} \int \mathcal{D}\psi e^{iS_{0,J}[\psi]} \\ = e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]} Z_0[J] = C \times e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]} \exp\left[\frac{-1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}(x, x') J_l(x) J_{l'}(x')\right]$$

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## 玻色场的生成泛函

## 相互作用玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} = \left. \frac{\delta^n Z[J]}{i^n Z[J] \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \right|_{J=0}$$

$$Z[J] \equiv \int \mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_J \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi]} \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x) \psi_l(x)$$

$$S_{0,J} = S[\psi] + \int d^4x J_l(x) \psi_l(x) \quad S = S_0 + \int d^4x \mathcal{L}_I[\psi(x)] \quad S_0 = \frac{1}{2} \int d^4x d^4x' i\mathcal{D}'_{ll'}{}^{-1}(x, x') \psi_l(x) \psi_{l'}(x')$$

$$Z_0[J] \equiv \int \mathcal{D}\psi e^{iS_{0,J}[\psi]} = C \times \exp\left[\frac{-1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}(x, x') J_l(x) J_{l'}(x')\right] \quad V = - \int d\vec{x} \mathcal{L}_I \quad \mathcal{H} = -\mathcal{L}_I[\psi]$$

微扰展开(变形):

$$Z[J] = C \times e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]} e^{-\frac{1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}(x, x') J_l(x) J_{l'}(x')} e^{i \int d^4x J_l(x) \psi_l(x)} \Big|_{\psi(x)=0}$$

$$= C \times e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta J(x)}]} e^{-\frac{1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}(x, x') \frac{\delta^2}{i^2 \delta \psi_l(x) \delta \psi_{l'}(x')}} e^{i \int d^4x J_l(x) \psi_l(x)} \Big|_{\psi(x)=0}$$

$$= C \times e^{\frac{1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}(x, x') \frac{\delta^2}{\delta \psi_l(x) \delta \psi_{l'}(x')}} e^{i \int d^4x [\mathcal{L}_I[\psi] + J_l(x) \psi_l(x)]} \Big|_{\psi(x)=0}$$



## 玻色场的生成泛函

## 相互作用玻色场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} = \frac{\delta^n Z[J]}{i^n Z[J] \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \Big|_{J=0}$$

$$Z[J] \equiv \int \mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_J \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi]} \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x) \psi_l(x)$$

$$S_{0,J} = S[\psi] + \int d^4x J_l(x) \psi_l(x) \quad S = S_0 + \int d^4x \mathcal{L}_I[\psi(x)] \quad S_0 = \frac{1}{2} \int d^4x d^4x' i\mathcal{D}'_{ll'}{}^{-1}(x, x') \psi_l(x) \psi_{l'}(x')$$

微扰展开(变形):

$$Z[J] = C \times e^{\frac{1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}{}^{-1}(x, x') \frac{\delta^2}{\delta \psi_l(x) \delta \psi_{l'}(x')} e^{i \int d^4x [\mathcal{L}_I[\psi] + J_l(x) \psi_l(x)]} \Big|_{\psi(x)=0}$$

$$(\Psi_0^-, \Psi_0^+) = C' \times e^{\frac{1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}{}^{-1}(x, x') \frac{\delta^2}{\delta \psi_l(x) \delta \psi_{l'}(x')} e^{i \int d^4x \mathcal{L}_I[\psi]} \Big|_{\psi(x)=0}$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n)\Psi_0^+) = C' e^{\frac{1}{2} \int d^4x d^4x' \mathcal{D}'_{ll'}{}^{-1}(x, x') \frac{\delta^2}{\delta \psi_l(x) \delta \psi_{l'}(x')} \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i \int d^4x \mathcal{L}_I[\psi]} \Big|_{\psi(x)=0}$$



## 玻色场的生成泛函

## 玻色场的费曼图与费曼规则

$$\begin{aligned}
 (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+) &= (\Phi_0^-, \mathbf{T}\psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i\int d^4x \mathcal{L}_I[\psi]} \Phi_0) \quad \text{所有可能的收缩} \\
 &= C' e^{\frac{1}{2}\int d^4x d^4x' \mathcal{D}'_{ll'}(x,x')} \frac{\delta^2}{\delta\psi_i(x)\delta\psi_{l'}(x')} \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{i\int d^4x \mathcal{L}_I[\psi]} \Big|_{\psi(x)=0}
 \end{aligned}$$

- ▶ 每一个传播子必须微掉两个场(两个场的收缩得到传播子) 微商产生两个 $\delta$ 函数抵消传播子的两个 $\int d^4x$ 积分, 并将传播子的坐标取为被微掉的两个场的坐标  $\Rightarrow$  每一个传播子 $D'(x, x')$ 连接两个场  $\frac{1}{2}$ 用与抵消两个场的交换,  $x, x'$ 分别是这两个场的坐标 如果总共有 $l$ 个传播子, 本应有因子 $\frac{1}{l!}$ , 但考虑到这 $l$ 个传播子共有 $l!$ 个安排方式, 它们给出同样结果, 因此可略去因子 $\frac{1}{l!}$ , 同时不再计入不同的安排方式
- ▶ 相互作用顶角贡献位于同一时空点的若干个场 $i\int d^4x \mathcal{L}_I[\psi(x)]$ , 外线贡献位于不同时空点的 $n$ 个场 $\psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n)$
- ▶ 所有场都必须被全部微掉(收缩掉) $\Rightarrow$ 传播子止于顶角或外线
- ▶ 微商对所有的场是平权的, 任何一个微商对每个场都要操作一次
- ▶ 如果总共有 $k$ 个顶角, 应该有因子 $\frac{1}{k!}$

存在一个用图形方式(顶角, 传播子, 外线)按一定规则表达结果的方式: 费曼图



## 费米场的生成泛函

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\bar{\psi}_{H,l'_1}(x'_1)\cdots\psi_{H,l_n}(x_n)\bar{\psi}_{H,l'_n}(x'_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \psi_{l_1}(x_1)\bar{\psi}_{l'_1}(x'_1)\cdots\psi_{l_n}(x_n)\bar{\psi}_{l'_n}(x'_n) e^{iS[\psi,\bar{\psi}]}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS[\psi,\bar{\psi}]}}$$

$$(\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi,\bar{\psi}]} \quad S[\psi,\bar{\psi}] \equiv \int d^4x \mathcal{L}[\psi(x), \bar{\psi}(x)] + \epsilon \text{ 项}$$

$$Z[I,\bar{I}] \equiv \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS[\psi,\bar{\psi}]} = e^{iW[I,\bar{I}]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+) \quad S_I[\psi,\bar{\psi}] = S[\psi,\bar{\psi}] + \int d^4x [\bar{I}_l(x)\psi_l(x) + \bar{\psi}_l(x)I_l(x)]$$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\bar{\psi}_{H,l'_1}(x'_1)\cdots\psi_{H,l_n}(x_n)\bar{\psi}_{H,l'_n}(x'_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\delta^{2n} Z[J]}{Z[0]\delta\bar{I}_{l_1}(x_1)\delta I'_{l'_1}(x'_1)\cdots\delta\bar{I}_{l_n}(x_n)\delta I'_{l'_n}(x'_n)} \Big|_{I=\bar{I}=0}$$

$$\frac{Z[I,\bar{I}]}{Z[0]} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1\cdots d^4x'_n \frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\bar{\psi}_{H,l'_1}(x'_1)\cdots\psi_{H,l_n}(x_n)\bar{\psi}_{H,l'_n}(x'_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} \bar{I}_{l_1}(x_1)I'_{l'_1}(x'_1)\cdots\bar{I}_{l_n}(x_n)I'_{l'_n}(x'_n)$$

$$(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\bar{\psi}_{H,l'_1}(x'_1)\cdots\psi_{H,l_n}(x_n)\bar{\psi}_{H,l'_n}(x'_n)\Psi_0^+)_C \equiv \frac{i\delta^{2n} W[I,\bar{I}]}{\delta\bar{I}_{l_1}(x_1)\delta I'_{l'_1}(x'_1)\cdots\delta\bar{I}_{l_n}(x_n)\delta I'_{l'_n}(x'_n)} \Big|_{I=\bar{I}=0}$$

$$W[I,\bar{I}] = \sum_{n=1}^{\infty} \frac{-i}{n!} \int d^4x_1\cdots d^4x'_n (\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\bar{\psi}_{H,l'_1}(x'_1)\cdots\psi_{H,l_n}(x_n)\bar{\psi}_{H,l'_n}(x'_n)\Psi_0^+)_C \bar{I}_{l_1}(x_1)I'_{l'_1}(x'_1)\cdots\bar{I}_{l_n}(x_n)I'_{l'_n}(x'_n)$$





## 费米场的生成泛函

## 有效作用量

$$\psi_{c,l}(x) \equiv \frac{\delta W[I, \bar{I}]}{\delta \bar{I}_l(x)} \quad \bar{\psi}_{c,l}(x) \equiv -\frac{\delta W[I, \bar{I}]}{\delta I_l(x)}$$

$$\Gamma[\psi_c, \bar{\psi}_c] \equiv W[I, \bar{I}] - \int d^4x [\bar{I}_l(x)\psi_{c,l}(x) + \bar{\psi}_{c,l}(x)I_l(x)]$$

$$\begin{aligned} \frac{\delta \Gamma[\psi_c, \bar{\psi}_c]}{\delta \psi_{c,l}(x)} &= \int d^4y \left[ \frac{\delta \bar{I}_{l'}(y)}{\delta \psi_{c,l}(x)} \frac{\delta W[I, \bar{I}]}{\delta \bar{I}_{l'}(y)} + \frac{\delta I_{l'}(y)}{\delta \psi_{c,l}(x)} \frac{\delta W[I, \bar{I}]}{\delta I_{l'}(y)} - \frac{\delta \bar{I}_{l'}(y)}{\delta \psi_{c,l}(x)} \psi_{c,l'}(y) + \bar{\psi}_{c,l'}(y) \frac{\delta I_{l'}(y)}{\delta \psi_{c,l}(x)} + \bar{I}_l(x) \right] \\ &= \bar{I}_l(x) \end{aligned}$$

$$\begin{aligned} \frac{\delta \Gamma[\psi_c, \bar{\psi}_c]}{\delta \bar{\psi}_{c,l}(x)} &= \int d^4y \left[ \frac{\delta \bar{I}_{l'}(y)}{\delta \bar{\psi}_{c,l}(x)} \frac{\delta W[I, \bar{I}]}{\delta \bar{I}_{l'}(y)} + \frac{\delta I_{l'}(y)}{\delta \bar{\psi}_{c,l}(x)} \frac{\delta W[I, \bar{I}]}{\delta I_{l'}(y)} + \bar{\psi}_{c,l'}(y) \frac{\delta I_{l'}(y)}{\delta \bar{\psi}_{c,l}(x)} - \frac{\delta \bar{I}_{l'}(y)}{\delta \bar{\psi}_{c,l}(x)} \psi_{c,l'}(y) - I_l(x) \right] \\ &= -I_l(x) \end{aligned}$$

$$S_I[\psi, \bar{\psi}] = S[\psi, \bar{\psi}] + \int d^4x [\bar{I}_l(x)\psi_l(x) + \bar{\psi}_l(x)I_l(x)]$$

$$\frac{\delta S_I[\psi, \bar{\psi}]}{\delta \psi_l(x)} = \frac{\delta S[\psi, \bar{\psi}]}{\delta \psi_l(x)} = 0 \quad \Rightarrow \quad \frac{\delta S[\psi, \bar{\psi}]}{\delta \psi_l(x)} = \bar{I}_l(x) \quad \frac{\delta S[\psi, \bar{\psi}]}{\delta \bar{\psi}_l(x)} = -I_l(x)$$



## 顶角生成泛函

$$\psi_{c,l}(x) \equiv \frac{\delta W[I, \bar{I}]}{\delta \bar{I}_l(x)} \quad \bar{\psi}_{c,l}(x) \equiv -\frac{\delta W[I, \bar{I}]}{\delta I_l(x)} \quad \Gamma[\psi_c, \bar{\psi}_c] \equiv W[I, \bar{I}] - \int d^4x [\bar{I}_l(x) \psi_{c,l}(x) + \bar{\psi}_{c,l}(x) I_l(x)]$$

$$\psi_{0,l}(x) = \left. \frac{\delta W[I, \bar{I}]}{\delta \bar{I}_l(x)} \right|_{I=\bar{I}=0} \quad \bar{\psi}_{0,l}(x) = -\left. \frac{\delta W[I, \bar{I}]}{\delta I_l(x)} \right|_{I=\bar{I}=0} \quad \frac{\delta \Gamma[\psi_c, \bar{\psi}_c]}{\delta \psi_{c,l}(x)} = \bar{I}_l(x) \quad \frac{\delta \Gamma[\psi_c, \bar{\psi}_c]}{\delta \bar{\psi}_{c,l}(x)} = -I_l(x)$$

$$\Gamma_{l_1 l'_1 \dots l_n l'_n}(x_1, x'_1, \dots, x_n, x'_n) \equiv \left. \frac{\delta^{2n} \Gamma[\psi_c, \bar{\psi}_c]}{\delta \bar{\psi}_{c,l_1}(x_1) \delta \psi_{c,l'_1}(x'_1) \dots \delta \bar{\psi}_{c,l_n}(x_n) \delta \psi_{c,l'_n}(x'_n)} \right|_{\psi_c=0, \bar{\psi}_c=0}$$

$$\Gamma[\psi_c, \bar{\psi}_c] = \Gamma[0, 0] + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \int d^4x_1 d^4x'_1 \dots d^4x_n d^4x'_n \Gamma_{l_1 l'_1 \dots l_n l'_n}(x_1, x'_1, \dots, x_n, x'_n) \bar{\psi}_{c,l_1}(x_1) \psi_{c,l'_1}(x'_1) \dots \bar{\psi}_{c,l_n}(x_n) \psi_{c,l'_n}(x'_n)$$

$$\begin{aligned} \delta(x-x') \delta_{ll'} &= \frac{\delta \psi_{c,l}(x)}{\delta \psi_{c,l'}(x')} = \int d^4y \left[ \frac{\delta I_{l'}(y)}{\delta \psi_{c,l'}(x')} \frac{\delta \psi_{c,l}(x)}{\delta I_{l'}(y)} + \frac{\delta \bar{I}_{l'}(y)}{\delta \psi_{c,l'}(x')} \frac{\delta \psi_{c,l}(x)}{\delta \bar{I}_{l'}(y)} \right] \\ &= \int d^4y \left[ -\frac{\delta^2 \Gamma[\psi_c, \bar{\psi}_c]}{\delta \psi_{c,l'}(x') \delta \bar{\psi}_{c,l'}(y)} \frac{\delta^2 W[I, \bar{I}]}{\delta I_{l'}(y) \delta \bar{I}_l(x)} + \frac{\delta^2 \Gamma[\psi_c, \bar{\psi}_c]}{\delta \psi_{c,l'}(x') \delta \psi_{c,l'}(y)} \frac{\delta^2 W[I, \bar{I}]}{\delta \bar{I}_{l'}(y) \delta \bar{I}_l(x)} \right] \end{aligned}$$

$$\int d^4y (\Psi_0^-, \psi_{H,l}(x) \psi_{H,l'}(y) \Psi_0^+) \Gamma_{l'l'}(y, x') = -i \delta(x-x') \delta_{ll'}$$



## 自由费米场

$$S_0 = \int d^4x d^4x' iS_{ff}^{-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x') - \lim_{\epsilon \rightarrow 0^+} i\epsilon \int d^3x d^3x' \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{x}') \psi(\vec{x}', \tau) e^{-\epsilon|\tau|}$$

$$= \int d^4x d^4x' iS_{ff}^{\prime-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x') \quad iS_{ff}^{\prime-1}(x, x') = (i\gamma^\mu \partial_{\mu, x} - M) \delta(x - x')$$

$$S_{ff}^{\prime-1}(x, x') = -i(i\gamma^\mu \partial_{\mu, x} - M) \delta(x - x') - \epsilon \mathcal{E}(\vec{x}, \vec{x}') e^{-\epsilon \frac{|t-t'|}{2}} \delta(t-t') \quad \mathcal{E}(\vec{x}, \vec{y}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \frac{-\vec{p} \cdot \vec{\gamma} - M}{\sqrt{\vec{p}^2 + M^2}}$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} [-i(\not{p} - M) + \epsilon e^{-\epsilon \frac{|t-t'|}{2}} \frac{\vec{p} \cdot \vec{\gamma} + M}{\sqrt{\vec{p}^2 + M^2}}] = -i \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} [\not{p} - M + i0^+]$$

注意  $\vec{p}$  中的微小移动不影响  $p^0$  的极点:  $(p^0)^2 - \vec{p}^2 (1 - i0^+)^2 - (M - i0^+)^2 = (p^0)^2 - \vec{p}^2 - M^2 + i0^+ = p^2 + i0^+$

$$\frac{(\Psi_0^-, \mathbf{T} \psi_{H, l_1}(x_1) \bar{\psi}_{H, l'_1}(x'_1) \cdots \psi_{H, l_n}(x_n) \bar{\psi}_{H, l'_n}(x'_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi_{l_1}(x_1) \bar{\psi}_{l'_1}(x'_1) \cdots \psi_{l_n}(x_n) \bar{\psi}_{l'_n}(x'_n) e^{iS[\psi, \bar{\psi}]}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]}}$$

$$(\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]}$$

$$S[\psi, \bar{\psi}] \equiv \int d^4x \mathcal{L}[\psi(x), \bar{\psi}(x)] - \lim_{\epsilon \rightarrow 0^+} i\epsilon \int d^3x d^3x' \int_{-\infty}^{\infty} d\tau \bar{\psi}(\vec{x}, \tau) \mathcal{E}(\vec{x}, \vec{x}') \psi(\vec{x}', \tau) e^{-\epsilon|\tau|}$$



## 费米场的生成泛函

$$\text{自由费米场 } S_0 = \int d^4x d^4x' iS_{ll'}^{-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x')$$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1)\bar{\psi}_{H,l'_1}(x'_1) \cdots \psi_{H,l_n}(x_n)\bar{\psi}_{H,l'_n}(x'_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi_{l_1}(x_1)\bar{\psi}_{l'_1}(x'_1) \cdots \psi_{l_n}(x_n)\bar{\psi}_{l'_n}(x'_n) e^{iS[\psi, \bar{\psi}]}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]}}$$

$$\begin{aligned} (\Psi_0^-, \Psi_0^+) &= |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]} & S[\psi, \bar{\psi}] &\equiv \int d^4x \mathcal{L}[\psi(x), \bar{\psi}(x)] \Big|_{\mathcal{S} \rightarrow \mathcal{S}'} \\ Z_0[I, \bar{I}] &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp i \left\{ \int d^4x d^4x' iS_{ll'}^{-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x') + \int d^4x [\bar{I}_l(x) \psi_l(x) + \bar{\psi}_l(x) I_l(x)] \right\} \\ &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp i \text{Tr}[\bar{\psi} iS'^{-1} \psi + \bar{I} \psi + \bar{\psi} I] \\ &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp i [\text{Tr}[(\bar{\psi} - \bar{i}S') iS'^{-1} (\psi - iS'I) + \bar{I} iS'I]] \\ &= C \times \exp \left[ - \int d^4x d^4x' S'_{ll'}(x, x') \bar{I}_l(x) I_{l'}(x') \right] \end{aligned}$$

$$W_0[I, \bar{I}] = i \int d^4x d^4x' S'_{ll'}(x, x') \bar{I}_l(x) I_{l'}(x') \quad C = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp \left[ - \int d^4x d^4x' S'_{ll'}^{-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x') \right]$$

$$S'^{-1}(x, x') = -i(i\gamma^\mu \partial_{\mu,x} - M + i0^+) \delta(x - x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} (-i)(\gamma^\mu p_\mu - M + i0^+)$$



自由费米场  $S_0 = \int d^4x d^4x' iS'_{ll'}^{-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x')$

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \bar{\psi}_{H,l'_1}(x'_1) \cdots \psi_{H,l_n}(x_n) \bar{\psi}_{H,l'_n}(x'_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi_{l_1}(x_1) \bar{\psi}_{l'_1}(x'_1) \cdots \psi_{l_n}(x_n) \bar{\psi}_{l'_n}(x'_n) e^{iS[\psi, \bar{\psi}]}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]}}$$

$$(\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]} \quad S[\psi, \bar{\psi}] \equiv \int d^4x \mathcal{L}[\psi(x), \bar{\psi}(x)] \Big|_{S \rightarrow S'}$$

$$Z_0[I, \bar{I}] = C \times \exp\left[-\int d^4x d^4x' S'_{ll'}(x, x') \bar{I}_l(x) I_{l'}(x')\right] \quad S'^{-1}(x, x') = -i(i\gamma^\mu \partial_{\mu,x} - M + i0^+) \delta(x - x')$$

$$W_0[I, \bar{I}] = i \int d^4x d^4x' S'_{ll'}(x, x') \bar{I}_l(x) I_{l'}(x') \quad C = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\left[-\int d^4x d^4x' S'_{ll'}^{-1}(x, x') \bar{\psi}_l(x) \psi_{l'}(x')\right]$$

$$\psi_{c,l}(x) \equiv \frac{\delta W_0[I, \bar{I}]}{\delta \bar{I}_l(x)} = i \int d^4x' S'_{ll'}(x, x') I_{l'}(x') \quad \bar{\psi}_{c,l}(x) \equiv -\frac{\delta W_0[I, \bar{I}]}{\delta I_l(x)} = i \int d^4x' \bar{I}_{l'}(x') S'_{l'l}(x', x)$$

$$\Gamma_0[\psi_c, \bar{\psi}_d] \equiv W_0[I, \bar{I}] - \int d^4x [\bar{I}_l(x) \psi_{c,l}(x) + \bar{\psi}_{c,l}(x) I_l(x)] = \int d^4x d^4x' iS'_{ll'}^{-1}(x, x') \bar{\psi}_{c,l}(x) \psi_{c,l'}(x') = S_0[\psi_c, \bar{\psi}_d]$$

自由费米场只有两线顶角:  $\Gamma_{0,ll'}(x, x') = \frac{\delta^2 \Gamma_0[\phi_c]}{\delta \bar{\psi}_{c,l}(x) \delta \psi_{c,l'}(x')} = -iS'_{ll'}^{-1}(x, x')$



## 相互作用费米场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H, I_1}(x_1) \cdots \bar{\psi}_{H, I'_n}(x'_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi_{I_1}(x_1) \cdots \bar{\psi}_{I'_n}(x'_n) e^{iS[\psi, \bar{\psi}]}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]}} = \left. \frac{\delta^n Z[I, \bar{I}]}{Z[I, \bar{I}] \delta \bar{I}_1(x_1) \cdots \delta I'_n(x'_n)} \right|_{I=\bar{I}=0}$$

$$Z[I, \bar{I}] \equiv \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_I[\psi, \bar{\psi}]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_I \quad S_I[\psi, \bar{\psi}] = S[\psi, \bar{\psi}] + \int d^4x [\bar{I}_I(x) \psi_I(x) + \bar{\psi}_I(x) I_I(x)]$$

$$S_{0, I} = S[\psi, \bar{\psi}] + \int d^4x (\bar{I} \psi + \bar{\psi} I) \quad S = S_0 + \int d^4x \mathcal{L}_I[\psi(x), \bar{\psi}] \quad S_0 = \int d^4x d^4x' iS'_{II'}^{-1}(x, x') \bar{\psi}_I(x) \psi_{I'}(x')$$

$$Z_0[I, \bar{I}] \equiv \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_{0, I}[\psi, \bar{\psi}]} = C \times \exp\left[-\int d^4x d^4x' S'_{II'}(x, x') \bar{I}_I(x) I_{I'}(x')\right]$$

## 微扰展开:

$$Z[I, \bar{I}] = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{i \int d^4x \mathcal{L}_I[\psi(x), \bar{\psi}(x)]} e^{iS_{0, I}[\psi, \bar{\psi}]} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta \bar{I}(x)}, -\frac{\delta}{i\delta I(x)}]} e^{iS_{0, I}[\psi, \bar{\psi}]}$$

$$= e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta \bar{I}(x)}, -\frac{\delta}{i\delta I(x)}]} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_{0, I}[\psi, \bar{\psi}]} = e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta \bar{I}(x)}, -\frac{\delta}{i\delta I(x)}]} Z_0[I, \bar{I}]$$

$$= C \times e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta \bar{I}(x)}, -\frac{\delta}{i\delta I(x)}]} \exp\left[-\int d^4x d^4x' S'_{II'}(x, x') \bar{I}_I(x) I_{I'}(x')\right]$$



## 相互作用费米场

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,I_1}(x_1) \cdots \bar{\psi}_{H,I'_n}(x'_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi_{I_1}(x_1) \cdots \bar{\psi}_{I'_n}(x'_n) e^{iS[\psi, \bar{\psi}]}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS[\psi, \bar{\psi}]}} = \frac{\delta^n Z[I, \bar{I}]}{Z[I, \bar{I}] \delta \bar{I}_1(x_1) \cdots \delta I'_n(x'_n)} \Big|_{\bar{I}=\mathbf{0}}$$

$$Z[I, \bar{I}] \equiv \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_I[\psi, \bar{\psi}]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_I \quad S_I[\psi, \bar{\psi}] = S[\psi, \bar{\psi}] + \int d^4x [\bar{I}_I(x) \psi_I(x) + \bar{\psi}_I(x) I_I(x)]$$

$$S_{0,I} = S[\psi, \bar{\psi}] + \int d^4x (\bar{I} \psi + \bar{\psi} I) \quad S = S_0 + \int d^4x \mathcal{L}_I[\psi(x), \bar{\psi}] \quad S_0 = \int d^4x d^4x' iS_{II'}^{-1}(x, x') \bar{\psi}_I(x) \psi_{I'}(x')$$

$$Z_0[I, \bar{I}] \equiv \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_{0,I}[\psi, \bar{\psi}]} = C \times \exp\left[-\int d^4x d^4x' S'_{II'}(x, x') \bar{I}_I(x) I_{I'}(x')\right]$$

## 微扰展开(变形):

$$Z[I, \bar{I}] = C \times e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta \bar{I}(x)}, -\frac{\delta}{i\delta I(x)}]} \exp\left[-\int d^4x d^4x' S'_{II'}(x, x') \bar{I}_I(x) I_{I'}(x')\right]$$

$$= C \times e^{i \int d^4x \mathcal{L}_I[\frac{\delta}{i\delta \bar{I}(x)}, -\frac{\delta}{i\delta I(x)}]} e^{-\int d^4x d^4x' S'_{II'}(x, x') \frac{\delta^2}{\delta \bar{\psi}_I(x) \delta \bar{\psi}_{I'}(x')}} e^{i \int d^4x [\bar{I}_I(x) \psi_I(x) + \bar{\psi}_I(x) I_I(x)]} \Big|_{\bar{\psi}=\psi=0}$$

$$= C \times e^{\int d^4x d^4x' S'_{II'}(x, x') \frac{\delta^2}{\delta \bar{\psi}_{I'}(x') \delta \bar{\psi}_I(x)}} e^{i \int d^4x [\mathcal{L}_I[\psi, \bar{\psi}] + \bar{I}_I(x) \psi_I(x) + \bar{\psi}_I(x) I_I(x)]} \Big|_{\bar{\psi}=\psi=0}$$







## 费米场的费曼图与费曼规则

$$(\Psi_0^-, \mathbf{T} \psi_{H, l_1}(x_1) \cdots \bar{\psi}_{H, l'_n}(x'_n) \Psi_0^+) = (\Phi_0^-, \psi_{l_1}(x_1) \cdots \bar{\psi}_{l'_n}(x'_n) e^{i \int d^4 x \mathcal{L}_I[\psi, \bar{\psi}]} \Phi_0^+) \quad \text{所有可能的收缩}$$

$$= C \times e^{\int d^4 x d^4 x' S'_{ll'}(x, x') \frac{\delta^2}{\delta \bar{\psi}_{l'}(x') \delta \psi_l(x)}} \psi_{l_1}(x_1) \cdots \bar{\psi}_{l'_n}(x'_n) e^{i \int d^4 x \mathcal{L}_I[\psi, \bar{\psi}]} \Big|_{\bar{\psi}=\psi=0}$$

- ▶ 每一个传播子必须微掉  $\psi_l$  和  $\bar{\psi}_{l'}$  场各一个 (**两个场的收缩得到传播子**) 微商产生两  $\delta$  函数抵消传播子的两个  $\int d^4 x$  积分, 并将传播子的坐标取为被微掉的两个场的坐标  $\Rightarrow$  每一个传播子  $S'_{ll'}(x, x')$  连接两个场,  $x, x'$  分别是  $\psi$  和  $\bar{\psi}$  两个场的坐标
- ▶ 相互作用顶角贡献位于同一时空点的若干个场  $i \int d^4 x \mathcal{L}_I[\psi(x), \bar{\psi}(x)]$ , 外线贡献位于不同时空点的  $2n$  个场  $\psi_{l_1}(x_1) \bar{\psi}_{l'_1}(x'_1) \cdots \psi_{l_n}(x_n) \bar{\psi}_{l'_n}(x'_n)$
- ▶ 所有场都必须被全部微掉 (**收缩掉**)  $\Rightarrow$  传播子止于顶角或外线
- ▶ 微商对所有的场是平权的
- ▶ 如果总共有  $k$  个顶角, 应该有因子  $\frac{1}{k!}$
- ▶ 封闭的费米子圈产生额外的  $-1$   $\text{tr}[\psi_1 \bar{\psi}_2 \psi_2 \bar{\psi}_3 \cdots \psi_{n-1} \bar{\psi}_n \psi_n \bar{\psi}_1]$

存在一个用图形方式(顶角, 传播子, 外线)按一定规则表达结果的方式: **费曼图**



## 拉格朗日体系

## 量子场论与经典拉格朗日量

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} = \frac{\delta^n Z[J]}{i^n Z[J] \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \Big|_{J=0}$$

$$Z[J] \equiv \int \mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_J \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi]} \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x) \psi_l(x)$$

$$S_J = S[\psi] + \int d^4x J_l(x) \psi_l(x) \quad S = S_0 + \int d^4x \mathcal{L}_I[\psi(x)] \quad S_0 = \frac{1}{2} \int d^4x d^4x' i\mathcal{D}'_{ll'}^{-1}(x, x') \psi_l(x) \psi_{l'}(x')$$

## 经典拉格朗日体系:

$$S_0 = \int d^4x \mathcal{L}_0 \quad \mathcal{L}_0 = \pi_l \dot{\psi}_l - \mathcal{H}_0 \quad H_0 = \int d\vec{x} \mathcal{H}_0 \quad \pi_l = \frac{\partial \mathcal{L}_0}{\partial \dot{\psi}_l}$$

$$S = \int d^4x \mathcal{L} \quad \mathcal{L} = \mathcal{L}_0 - \mathcal{H} \quad H = H_0 + V = \int d\vec{x} [\pi_l \dot{\psi}_l - \mathcal{L}] \quad V = \int d\vec{x} \mathcal{H} \quad \pi_l \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}_l} = \frac{\partial \mathcal{L}_0}{\partial \dot{\psi}_l}$$

$$0 = \delta S = \int d^4x \left[ \frac{\partial \mathcal{L}}{\partial \psi_l(x)} \delta \psi_l(x) + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \delta \partial_\mu \psi_l(x) \right] = \int d^4x \left[ \frac{\partial \mathcal{L}}{\partial \psi_l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \right] \delta \psi_l(x)$$

$$\frac{\partial \mathcal{L}}{\partial \psi_l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} = 0$$

经典拉格朗日场方程



## 量子场论与经典拉格朗日量

$$\frac{(\Psi_0^-, \mathbf{T}\psi_{H,l_1}(x_1) \cdots \psi_{H,l_n}(x_n) \Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} = \frac{\int \mathcal{D}\psi \psi_{l_1}(x_1) \cdots \psi_{l_n}(x_n) e^{iS[\psi]}}{\int \mathcal{D}\psi e^{iS[\psi]}} = \frac{\delta^n Z[J]}{i^n Z[J] \delta J_{l_1}(x_1) \cdots \delta J_{l_n}(x_n)} \Big|_{J=0}$$

$$Z[J] \equiv \int \mathcal{D}\psi e^{iS_J[\psi]} = |\mathcal{N}|^{-2} (\Psi_0^-, \Psi_0^+)_J \quad (\Psi_0^-, \Psi_0^+) = |\mathcal{N}|^2 \int \mathcal{D}\psi e^{iS[\psi]} \quad S_J[\psi] = S[\psi] + \int d^4x J_l(x) \psi_l(x)$$

$$0 = \delta S \Rightarrow \frac{\partial \mathcal{L}}{\partial \psi_l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} = 0 \quad \text{经典拉格朗日场方程}$$

## 量子拉格朗日场方程:

$$0 = \int \mathcal{D}\psi e^{iS_J[\psi + \delta\psi]} - \int \mathcal{D}\psi e^{iS_J[\psi]} = i \int \mathcal{D}\psi \left[ \delta S[\psi] + \int d^4x J_l(x) \delta\psi_l(x) \right] e^{iS_J[\psi]}$$

$$\delta S[\psi] = \int d^4x \left[ \frac{\partial \mathcal{L}}{\partial \psi_l(x)} \delta\psi_l(x) + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \delta \partial_\mu \psi_l(x) \right] = \int d^4x \delta\psi_l(x) \left[ \frac{\partial \mathcal{L}}{\partial \psi_l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \right]$$

$$\int \mathcal{D}\psi \left[ \frac{\partial \mathcal{L}}{\partial \psi_l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} + J_l(x) \right] e^{iS_J[\psi]} = 0 \quad \text{Schwinger-Dyson 方程}$$

$$\int \mathcal{D}\psi O[\psi_l(y)] \left[ \frac{\partial \mathcal{L}}{\partial \psi_l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \right] e^{iS[\psi]} = \frac{[O(\frac{\partial}{\partial x}), x] = O'(\frac{\partial}{\partial x})}{=} - \int \mathcal{D}\psi O'[\psi_l(y)] \delta(x-y) e^{iS[\psi]}$$



## 经典场的Noether定理:

拉格朗日量密度具有整体连续对称性是指它在场量的如下变换下保持不变:

$$\psi_l(x) \rightarrow \psi'_l(x) = \psi_l(x) + i\epsilon\mathcal{F}_l(x) \quad \epsilon \text{ 是无穷小的常数}$$

$$\Rightarrow \frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu\mathcal{F}_l(x) = 0$$

将变换参数取为依赖于时空坐标的参数  $\epsilon(x)$ 

$$\delta S = \int d^4x \left[ \frac{\partial\mathcal{L}}{\partial\psi_l(x)}\mathcal{F}_l(x)i\epsilon(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\partial_\mu(\mathcal{F}_l(x)i\epsilon(x)) \right] = -\int d^4x \mathcal{J}^\mu(x)\partial_\mu\epsilon(x) = \int d^4x \epsilon(x)\partial_\mu\mathcal{J}^\mu(x)$$

$$\mathcal{J}^\mu(x) \equiv -i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)}\mathcal{F}_l(x) \quad \text{整体连续对称性变换对应的“流”} \quad \frac{\delta S}{\delta\partial_\mu\epsilon(x)} = -\mathcal{J}^\mu(x)$$

若场满足拉格朗日方程

$$\frac{\partial\mathcal{L}}{\partial\psi_l(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_l(x)} = 0 \quad \Rightarrow \quad \partial_\mu\mathcal{J}^\mu(x) = 0$$



## 经典场的Noether定理:

拉格朗日量密度具有整体连续对称性是指它在场量的如下变换下保持不变:

$$\psi_i(x) \rightarrow \psi'_i(x) = \psi_i(x) + i\epsilon\mathcal{F}_i(x) \quad \epsilon \text{ 是无穷小的常数}$$

$$\Rightarrow \frac{\partial\mathcal{L}}{\partial\psi_i(x)}\mathcal{F}_i(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_i(x)}\partial_\mu\mathcal{F}_i(x) = 0 \quad \text{若变换前后的差不为零, 而是等于时空的全微商, 则称 反常}$$

$$\mathcal{J}^\mu(x) \equiv -i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_i(x)}\mathcal{F}_i(x) \quad \text{整体连续对称性变换对应的"流"} \quad \frac{\delta S}{\delta\partial_\mu\epsilon(x)} = -\mathcal{J}^\mu(x)$$

$$\text{若场满足拉格朗日方程: } \frac{\partial\mathcal{L}}{\partial\psi_i(x)} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_i(x)} = 0 \Rightarrow \partial_\mu\mathcal{J}^\mu(x) = 0$$

拉格朗日量密度若不具有下面整体连续对称性但场满足拉格朗日方程

$$\psi_i(x) \rightarrow \psi'_i(x) = \psi_i(x) + i\epsilon\mathcal{F}_i(x) \quad \text{仍可定义相应的流: } \mathcal{J}^\mu(x) \equiv -i\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_i(x)}\mathcal{F}_i(x)$$

$$\delta\mathcal{L} = \epsilon\left[\frac{\partial\mathcal{L}}{\partial\psi_i(x)}\mathcal{F}_i(x) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_i(x)}\partial_\mu\mathcal{F}_i(x)\right] = \epsilon\partial_\mu\mathcal{J}^\mu(x)$$



## 拉格朗日体系

## 量子场的Noether定理:

拉格朗日量密度具有整体连续对称性是指它在场量的如下变换下保持不变:

$$\psi_l(x) \rightarrow \psi'_l(x) = \psi_l(x) + i\epsilon \mathcal{F}_l(x) \quad \epsilon \text{ 是无穷小的常数}$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial \psi_l(x)} \mathcal{F}_l(x) + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \partial_\mu \mathcal{F}_l(x) = 0 \quad \text{不要有对称性, } \epsilon \text{ 依赖于 } x \Rightarrow \delta S = \int d^4x \epsilon(x) \partial_\mu \mathcal{J}^\mu(x)$$

$$\mathcal{J}^\mu(x) \equiv -i \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \mathcal{F}_l(x) \quad \text{整体连续对称性变换对应的“流”} \quad \frac{\delta S}{\delta \partial_\mu \epsilon(x)} = -\mathcal{J}^\mu(x)$$

$$0 = \int \mathcal{D}\psi [\delta S + \int d^4x J_l(x) \delta \psi_l(x)] e^{iS_J[\psi]} = \int \mathcal{D}\psi \int d^4x [\epsilon(x) \partial_\mu \mathcal{J}^\mu + J_l(x) i\epsilon(x) \mathcal{F}_l(x)] e^{iS_J[\psi]}$$

$$\Rightarrow \int \mathcal{D}\psi [\partial_\mu \mathcal{J}^\mu + iJ_l(x) \mathcal{F}_l(x)] e^{iS_J[\psi]} = 0 \quad \text{Ward-Takahashi-Taylor 恒等式}$$

$$\text{对称性“荷”}: \quad Q(t) \equiv \int d\vec{x} J^0(x) = -i \int d\vec{x} \pi_l(\vec{x}, t) \mathcal{F}_l(\vec{x}, t) \quad \pi_l \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}_l}$$

$$\text{经典: } \partial_\mu \mathcal{J}^\mu = 0 \Rightarrow \dot{Q}(t) = 0 \quad \text{量子: } \int \mathcal{D}\psi [\dot{Q}(t) + i \int d^3x J_l(\vec{x}, t) \mathcal{F}_l(\vec{x}, t)] e^{iS_J[\psi]} = 0$$

$$[\psi_l(\vec{x}, t), \pi_l(\vec{y}, t)] = i\delta(\vec{x} - \vec{y}) \delta_{ll} \Rightarrow [Q(t), \psi_l(\vec{x}, t)] = -\mathcal{F}_l(\vec{x}, t) \quad \text{起场变换生成元的作用}$$



$$\begin{aligned} \psi_l(x) &\rightarrow \psi'_l(x') = \psi_l(x) & \psi'_l(x) &= \psi_l(x - \epsilon) = \psi_l(x) - \epsilon^\mu \partial_\mu \psi_l(x) \\ \text{能动量守恒与时空平移} \quad x &\rightarrow x' = x + \epsilon \\ \delta S &= \int d^4x \mathcal{L}[\psi'(x')] - \int d^4x \mathcal{L}[\psi(x)] = \int d^4x \{ \mathcal{L}[\psi'(x')] - \mathcal{L}[\psi'(x)] + \mathcal{L}[\psi'(x)] - \mathcal{L}[\psi(x)] \} \end{aligned}$$

$$= \int d^4x \left[ \frac{\partial \mathcal{L}}{\partial x^\mu} \epsilon^\mu - \frac{\partial \mathcal{L}}{\partial \psi_l(x)} \epsilon^\mu \partial_\mu \psi_l(x) - \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \epsilon^\mu \partial_\nu \partial_\mu \psi_l(x) \right]$$

$$= \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial x^\mu} \epsilon^\mu - \partial_\nu \left[ \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \right] \epsilon^\mu \partial_\mu \psi_l(x) - \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \epsilon^\mu \partial_\nu \partial_\mu \psi_l(x) \right\}$$

$$= -\epsilon^\mu \int d^4x \partial_\nu \left[ -g_\mu^\nu \mathcal{L} + \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \partial_\mu \psi_l(x) \right] \quad T_\nu^\mu \equiv -\mathcal{L} g_\nu^\mu + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \partial_\nu \psi_l(x)$$

$$P_\mu(t) = \int d\vec{x} T^0_\mu = \int d\vec{x} [-\mathcal{L} g_\mu^0 + \pi_l(x) \partial_\mu \psi_l(x)]$$

$$H \equiv P^0 = \int d\vec{x} [-\mathcal{L} + \pi_l(x) \dot{\psi}_l(x)] \quad \vec{P} \equiv \int d\vec{x} \pi_l(x) \nabla \psi_l(x) \quad \text{平移生成元的场算符表示} \quad \mathcal{F}_{l,\mu}(x) = i\partial_\mu \psi_l(x)$$

$$\Rightarrow [P^\mu(t), \psi_l(\vec{x}, t)] = -i\partial^\mu \psi_l(\vec{x}, t) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} P^\mu [-e^{-ip \cdot x} u(\vec{p}, \sigma) a(\vec{p}, \sigma) + e^{ip \cdot x} v(\vec{p}, \sigma) a^\dagger(\vec{p}, \sigma)]$$

$$\int \mathcal{D}\psi [P^\mu(t) - \int d^3x J_l(\vec{x}, t) \partial_\mu \psi_l(\vec{x}, t)] e^{iS_J[\psi]} = 0$$

角动量守恒与时空转动  $x^\mu \rightarrow x^{\mu'} = \Lambda^\mu_{\nu'} x^\nu$ 

$$\Lambda^\mu_{\nu'} = g^\mu_{\nu'} + \omega^\mu_{\nu'} \quad \omega_{\mu\nu} = -\omega_{\nu\mu} \quad \psi^l(x) \rightarrow \psi^l(x') = \psi_l(x) + \frac{i}{2} \omega^{\mu\nu} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x)$$

$$\psi^l(x) = \psi_l(\Lambda^{-1}x) + \frac{i}{2} \omega^{\mu\nu} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x) = \psi_l(x) - \omega^{\mu\nu} x_\nu \partial_\mu \psi_l(x) + \frac{i}{2} \omega^{\mu\nu} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x)$$

$$\delta S = \int d^4x \mathcal{L}[\psi'(x')] - \int d^4x \mathcal{L}[\psi(x)] = \int d^4x \{ \mathcal{L}[\psi'(x')] - \mathcal{L}[\psi'(x)] + \mathcal{L}[\psi'(x)] - \mathcal{L}[\psi(x)] \}$$

$$= \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial x^\mu} \omega^{\mu\nu} x_\nu + \frac{\partial \mathcal{L}}{\partial \psi_l(x)} \omega^{\sigma\rho} [-x_\rho \partial_\sigma \psi_l(x) + \frac{i}{2} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x)] \right.$$

$$\left. + \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \omega^{\sigma\rho} \partial_\nu [-x_\rho \partial_\sigma \psi_l(x) + \frac{i}{2} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x)] \right\}$$

$$= \int d^4x \left\{ \partial_\mu [x_\nu \mathcal{L}] \omega^{\mu\nu} + \partial_\nu \left[ \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \omega^{\sigma\rho} [-x_\rho \partial_\sigma \psi_l(x) + \frac{i}{2} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x)] \right. \right.$$

$$\left. + \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} \omega^{\sigma\rho} \partial_\nu [-x_\rho \partial_\sigma \psi_l(x) + \frac{i}{2} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x)] \right\}$$

$$= \omega^{\sigma\rho} \int d^4x \partial_\nu \{ x_\rho \mathcal{L} g^\nu_\sigma + \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} [-x_\rho \partial_\sigma \psi_l(x) + \frac{i}{2} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x)] \}$$





## 拉格朗日体系

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu \quad \Lambda^\mu_\nu = g^\mu_\nu + \omega^\mu_\nu \quad \omega_{\mu\nu} = -\omega_{\nu\mu} \quad \psi_l(x) \rightarrow \psi'_l(x') = \psi_l(x) + \frac{i}{2} \omega^{\mu\nu} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x)$$

$$\psi'_l(x) = \psi_l(\Lambda^{-1}x) + \frac{i}{2} \omega^{\mu\nu} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x) = \psi_l(x) - \omega^{\mu\nu} x_\nu \partial_\mu \psi_l(x) + \frac{i}{2} \omega^{\mu\nu} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x)$$

$$\delta S = \omega^{\sigma\rho} \int d^4x \partial_\nu \{ x_\rho \mathcal{L} g^\nu_\sigma + \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} [-x_\rho \partial_\sigma \psi_l(x) + \frac{i}{2} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x)] \}$$

$$= \frac{1}{2} \omega^{\sigma\rho} \int d^4x \partial_\nu \{ x_\sigma T^\nu_\rho - x_\rho T^\nu_\sigma + i \frac{\partial \mathcal{L}}{\partial \partial_\nu \psi_l(x)} (\mathcal{J}_{\sigma\rho})_l^m \psi_m(x) \}$$

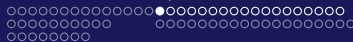
$$\mathcal{M}^{\mu\sigma\rho} \equiv x^\sigma T^{\mu\rho} - x^\rho T^{\mu\sigma} + i \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} (\mathcal{J}^{\sigma\rho})_l^m \psi_m(x) \quad T^\mu_\nu \equiv -\mathcal{L} g^\mu_\nu + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_l(x)} \partial_\nu \psi_l(x)$$

$$J^{\mu\nu}(t) = \int d\vec{x} \mathcal{M}^{0\mu\nu} = \int d\vec{x} [x^\mu T^{0\nu} - x^\nu T^{0\mu} + i\pi_l(x) (\mathcal{J}^{\mu\nu})_l^m \psi_m(x)] \quad \text{转动生成元的场算符表示}$$

$$J^{\mu\nu} = -J^{\nu\mu} \quad J^\mu_\mu = 0 \quad J^{ij} = L^{ij} + S^{ij} \quad L^{ij} = \int d\vec{x} (x^i P^j - x^j P^i) \quad S^{ij} = i \int d\vec{x} \pi_l(x) (\mathcal{J}^{ij})_l^m \psi_m(x)$$

$$\text{无穷小独立变换参数: } 1/2 \omega^{\mu\nu} \Rightarrow \mathcal{F}_{l,\mu\nu}(x) = \frac{i}{2} (x_\nu \partial_\mu - x_\mu \partial_\nu) \psi_l(x) + \frac{1}{2} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x)$$

$$[J_{\mu\nu}(t), \psi_l(\vec{x}, t)] = \frac{i}{2} (x_\nu \partial_\mu - x_\mu \partial_\nu) \psi_l(x) + \frac{1}{2} (\mathcal{J}_{\mu\nu})_l^m \psi_m(x)$$



## $\lambda\phi^4$ 理论概况

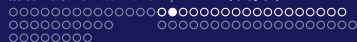
### 经典理论:

$$\mathcal{L}[\phi(x)] = \frac{1}{2}[\partial_\mu\phi(x)][\partial^\mu\phi(x)] - \frac{1}{2}M^2\phi^2(x) - \frac{\lambda}{4!}\phi^4(x)$$

- ▶ 正常:  $M$ 为实数;  $\lambda \geq 0$
- ▶  $M$ 为虚数  $\Rightarrow$  非稳定系统 自发破缺!
- ▶  $\lambda = 0 \Rightarrow$  自由场!
- ▶  $\lambda < 0 \Rightarrow$  系统无定义(能量无下界)!

### 量子理论:

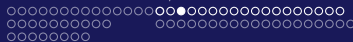
- ▶ 理论上最简单的3 + 1维量子场理论
- ▶ 现实中最不清楚的3 + 1维量子场理论
- ▶ 高于和等于3 + 1维的理论是平庸的(有限大 $\lambda$ 导致零观测耦合强度)
- ▶ 低于3 + 1维的理论是非平庸的. 构造场论的研究对象!

 $\lambda\phi^4$  理论的费曼图 $\lambda\phi^4$  理论的强耦合极限格点化:  $D$  维欧氏空间;  $N$  个分量

$$\begin{aligned} \mathcal{L}_{\lambda\phi^4}[\vec{\phi}(x)] &= -\frac{1}{2}[\partial_\mu\vec{\phi}(x)] \cdot [\partial^\mu\vec{\phi}(x)] - \frac{1}{2}M^2\vec{\phi}^2(x) - \frac{\lambda}{4!}[\vec{\phi}^2(x)]^2 \\ &= -\frac{1}{2}\lim_{\Delta\rightarrow 0}\sum_\mu\left[\frac{\vec{\phi}(x_\mu+\Delta)-\vec{\phi}(x_\mu)}{\Delta}\right]^2 - \frac{1}{2}M^2\vec{\phi}^2(x) - \frac{\lambda}{4!}[\vec{\phi}^2(x)]^2 \\ &= -\frac{1}{2}\lim_{\Delta\rightarrow 0}\sum_\mu\frac{\vec{\phi}^2(x_\mu+\Delta)+\vec{\phi}^2(x_\mu)-2\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2} - \frac{1}{2}M^2\vec{\phi}^2(x) - \frac{\lambda}{4!}[\vec{\phi}^2(x)]^2 \end{aligned}$$

## 强耦合极限:

$$\begin{aligned} e^{-W_{\lambda\phi^4}[\vec{J}]} &\equiv \int [\mathcal{D}\vec{\phi}] e^{-\int d^Dx\{\mathcal{L}_{\lambda\phi^4}[\vec{\phi}(x)]+\vec{J}(x)\cdot\vec{\phi}(x)\}} \\ &= \int [\mathcal{D}\vec{\phi}] e^{-\int d^Dx\{-\frac{1}{2}\sum_\mu\frac{\vec{\phi}^2(x_\mu+\Delta)+\vec{\phi}^2(x_\mu)-2\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2}-\frac{1}{2}M^2\vec{\phi}^2(x)-\frac{\lambda}{4!}[\vec{\phi}^2(x)]^2+\vec{J}(x)\cdot\vec{\phi}(x)\}} \\ &= \int [\mathcal{D}\vec{\phi}] e^{-\int d^Dx\{\sum_\mu\frac{\vec{\phi}(x_\mu+\Delta)\cdot\vec{\phi}(x_\mu)}{\Delta^2}-\frac{1}{2}(\frac{2}{\Delta^2}+M^2)\vec{\phi}^2(x)-\frac{\lambda}{4!}[\vec{\phi}^2(x)]^2+\vec{J}(x)\cdot\vec{\phi}(x)\}} \end{aligned}$$

 $\lambda\phi^4$  理论的费曼图

## 强耦合极限:

$$e^{-W_{\lambda\phi^4}[\vec{J}]} = \int [\mathcal{D}\vec{\phi}] e^{-\int d^Dx \left\{ \sum_{\mu} \frac{\vec{\phi}(x_{\mu}+\Delta) \cdot \vec{\phi}(x_{\mu})}{\Delta^2} - \frac{1}{2} \left( \frac{2}{\Delta^2} + M^2 \right) \vec{\phi}^2(x) - \frac{\lambda}{4!} [\vec{\phi}^2(x)]^2 + \vec{J}(x) \cdot \vec{\phi}(x) \right\}}$$

$$= \int [\mathcal{D}\vec{\phi}] e^{-\sum_i \left\{ \sum_{i' \text{ nearest}} \Delta^{D-2} \vec{\phi}_{i'} \cdot \vec{\phi}_i - \frac{1}{2} (2\Delta^{D-2} + \Delta^D M^2) \vec{\phi}_i^2 - \Delta^D \frac{\lambda}{4!} [\vec{\phi}_i^2]^2 + \Delta^D \vec{J}_i \cdot \vec{\phi}_i \right\}}$$

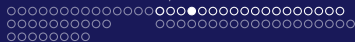
$$\stackrel{\vec{S}_i \equiv \Delta^{D/2-1} \vec{\phi}_i}{=} \int [\mathcal{D}\vec{S}] e^{-\sum_i \left\{ \sum_{i' \text{ nearest}} \vec{S}_{i'} \cdot \vec{S}_i - \frac{1}{2} (2 + \Delta^2 M^2) \vec{S}_i^2 - \Delta^{4-D} \frac{\lambda}{4!} [\vec{S}_i^2]^2 + \Delta^{1+D/2} \vec{J}_i \cdot \vec{S}_i \right\}}$$

$$= \int [\mathcal{D}\vec{S}] e^{-\sum_i \left\{ \sum_{i' \text{ nearest}} \vec{S}_{i'} \cdot \vec{S}_i - \Delta^{4-D} \frac{\lambda}{4!} [\vec{S}_i^2 + \frac{6}{\lambda} \Delta^{D-4} (2 + \Delta^2 M^2)]^2 + \Delta^{1+D/2} \vec{J}_i \cdot \vec{S}_i \right\}}$$

$$\delta(x) = \lim_{\lambda \rightarrow \infty} \sqrt{\frac{\lambda}{4! \pi}} e^{-\frac{\lambda}{4!} x^2} \stackrel{\lambda \rightarrow \infty}{=} \int [\mathcal{D}\vec{S}] \left[ \prod_i \delta[\vec{S}_i^2 + \frac{6}{\lambda} \Delta^{D-4} (2 + \Delta^2 M^2)] \right] e^{-\sum_i \left\{ \sum_{i' \text{ nearest}} \vec{S}_{i'} \cdot \vec{S}_i + \Delta^{1+D/2} \vec{J}_i \cdot \vec{S}_i \right\}}$$

$$\stackrel{N=2s+1, M^2 = -\frac{\lambda}{6} s(s-1) \Delta^{2-D} - 2\Delta^{-2}}{=} \int [\mathcal{D}\vec{S}] \left[ \prod_i \delta[\vec{S}_i^2 - s(s+1)] \right] e^{-\sum_i \left\{ \sum_{i' \text{ nearest}} \vec{S}_{i'} \cdot \vec{S}_i + \Delta^{1+D/2} \vec{J}_i \cdot \vec{S}_i \right\}}$$

D维Ising模型

 $\lambda\phi^4$ 理论的费曼图 $\lambda\phi^4$ 理论的费曼规则

$$\mathcal{L}[\phi(x)] = \frac{1}{2}[\partial_\mu \phi(x)][\partial^\mu \phi(x)] - \frac{1}{2}M^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x) \quad \mathcal{D}'(x, x') = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-x')} \frac{i}{p^2 - M^2 + i0^+}$$

$$\begin{aligned} (\Psi_0^-, \mathbf{T} \phi_H(x_1) \cdots \phi_H(x_n) \Psi_0^+) &= C' e^{\frac{1}{2} \int d^4 x d^4 x' \mathcal{D}'(x, x') \frac{\delta^2}{\delta \phi(x) \delta \phi(x')}} \phi(x_1) \cdots \phi(x_n) e^{\frac{-i\lambda}{4!} \int d^4 x \phi^4(x)} \Big|_{\phi(x)=0} \\ &= (\Phi_0, \mathbf{T}[\phi(x_1) \cdots \phi(x_n) e^{\frac{-i\lambda}{4!} \int d^4 x \phi^4(x)}] \Phi_0) \end{aligned}$$

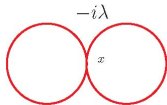
## 坐标空间:

- ▶ 对  $n$  点连通格林函数  $(\Psi_0^-, \mathbf{T} \phi_H(x_1) \cdots \phi_H(x_n) \Psi_0^+)_C$ , 标出  $n$  个时空点  $x_1, \dots, x_n$ , 每个点引出一条线
- ▶ 对  $m$  阶相互作用, 分别在  $m$  个时空点  $y_1, \dots, y_m$  引入相互作用顶点, 每个顶点引出四条线
- ▶ 将引出的  $n + 4m$  条线两两相连, 得到所有可能的拓扑不等价的连接图
- ▶ 每条连线代表  $\mathcal{D}'(x, x') = \underline{\phi(x)\phi(x')}$ , 每个顶点代表  $-i\lambda$ , 需要对所有顶点的时空坐标求积分
- ▶ 若图中出现对称连线, 必须乘以对称因子  $1/S$

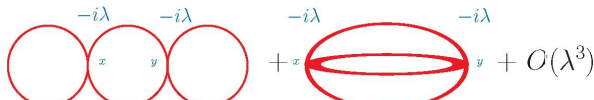
( $S$ : 每个单个圈有因子 2; 连接两点的  $n$  条线有因子  $n!$ ; 真空图有镜像对称加因子 2)

 $\lambda\phi^4$  理论的费曼图坐标空间  $\lambda\phi^4$  理论的费曼图——连接真空图

$$\begin{aligned} & \frac{1}{2!} \int d^4x d^4x' d^4y d^4y' \frac{1}{2} \mathcal{D}'(x, x') \frac{1}{2} \mathcal{D}'(y, y') \frac{\delta^2}{\delta\phi(x)\delta\phi(x')} \frac{\delta^2}{\delta\phi(y)\delta\phi(y')} \frac{-i\lambda}{4!} \int d^4z \phi^4(z) \\ &= (\Phi_0, \mathbf{T} \left[ \frac{-i\lambda}{4!} \int d^4x \phi^4(x) \right] \Phi_0) = 3 \frac{-i\lambda}{4!} \int d^4x \phi(x)\phi(x)\phi(x)\phi(x) \\ &= \frac{-i\lambda}{8} \int d^4x \Delta^2(x, x) \end{aligned}$$

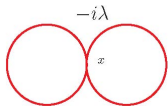


$$\begin{aligned} & \frac{1}{4!} \int d^4x_1 d^4x'_1 d^4x_2 d^4x'_2 d^4x_3 d^4x'_3 d^4x_4 d^4x'_4 \frac{1}{2} \mathcal{D}'(x_1, x'_1) \frac{1}{2} \mathcal{D}'(x_2, x'_2) \frac{1}{2} \mathcal{D}'(x_3, x'_3) \frac{1}{2} \mathcal{D}'(x_4, x'_4) \\ & \times \frac{\delta^2}{\delta\phi(x_1)\delta\phi(x'_1)} \frac{\delta^2}{\delta\phi(x_2)\delta\phi(x'_2)} \frac{\delta^2}{\delta\phi(x_3)\delta\phi(x'_3)} \frac{\delta^2}{\delta\phi(x_4)\delta\phi(x'_4)} \frac{1}{2!} \left( \frac{-i\lambda}{4!} \right)^2 \int d^4z d^4z' \phi^4(z)\phi^4(z') \\ &= \frac{(-i\lambda)^2}{16} \int d^4x d^4y \Delta(x, x)\Delta^2(x, y)\Delta(y, y) + \frac{(-i\lambda)^2}{48} \int d^4x d^4y \Delta^4(x, y) + O(\lambda^3) \end{aligned}$$



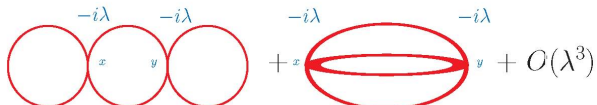
 $\lambda\phi^4$  理论的费曼图坐标空间  $\lambda\phi^4$  理论的费曼图——连接真空图

$$\begin{aligned} & \frac{1}{2!} \int d^4x d^4x' d^4y d^4y' \frac{1}{2} \mathcal{D}'(x, x') \frac{1}{2} \mathcal{D}'(y, y') \frac{\delta^2}{\delta\phi(x)\delta\phi(x')} \frac{\delta^2}{\delta\phi(y)\delta\phi(y')} \frac{-i\lambda}{4!} \int d^4z \phi^4(z) \\ &= (\Phi_0, \mathbf{T} \left[ \frac{-i\lambda}{4!} \int d^4x \phi^4(x) \right] \Phi_0) = 3 \frac{-i\lambda}{4!} \int d^4x \underbrace{\phi(x)\phi(x)}_{\text{loop}} \underbrace{\phi(x)\phi(x)}_{\text{loop}} \\ &= \frac{-i\lambda}{8} \int d^4x \Delta^2(x, x) \end{aligned}$$



$$(\Phi_0, \mathbf{T} \left[ \frac{1}{2!} \left( \frac{-i\lambda}{4!} \right)^2 \int d^4x d^4y \phi^4(x) \phi^4(y) \right] \Phi_0) + O(\lambda^3)$$

$$\begin{aligned} &= \frac{1}{2!} \left( \frac{-i\lambda}{4!} \right)^2 \int d^4x d^4y \left[ (2 * 3)^2 * 2 \underbrace{\phi(x)\phi(x)}_{\text{loop}} \underbrace{[\phi(x)\phi(y)]^2}_{\text{loop}} \underbrace{\phi(y)\phi(y)}_{\text{loop}} + 4! [\phi(x)\phi(y)]^4 \right] + O(\lambda^3) \\ &= \frac{(-i\lambda)^2}{16} \int d^4x d^4y \Delta(x, x) \Delta^2(x, y) \Delta(y, y) + \frac{(-i\lambda)^2}{48} \int d^4x d^4y \Delta^4(x, y) + O(\lambda^3) \end{aligned}$$

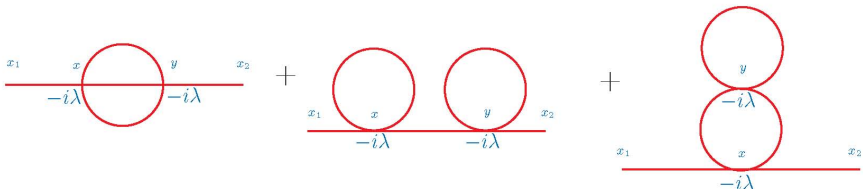






 $\lambda\phi^4$  理论的费曼图坐标空间  $\lambda\phi^4$  理论的费曼图——连接2点格林函数图

$$\begin{aligned} & \frac{1}{5!} \int d^4 y_1 d^4 y'_1 d^4 y_2 d^4 y'_2 d^4 y_3 d^4 y'_3 d^4 y_4 d^4 y'_4 d^4 y_5 d^4 y'_5 \frac{1}{2} \mathcal{D}'(y_1, y'_1) \frac{1}{2} \mathcal{D}'(y_2, y'_2) \frac{1}{2} \mathcal{D}'(y_3, y'_3) \frac{1}{2} \mathcal{D}'(y_4, y'_4) \\ & \times \frac{1}{2} \mathcal{D}'(y_5, y'_5) \frac{\delta^2}{\delta\phi(y_1)\delta\phi(y'_1)} \frac{\delta^2}{\delta\phi(y_2)\delta\phi(y'_2)} \frac{\delta^2}{\delta\phi(y_3)\delta\phi(y'_3)} \frac{\delta^2}{\delta\phi(y_4)\delta\phi(y'_4)} \frac{\delta^2}{\delta\phi(y_5)\delta\phi(y'_5)} \\ & \times \frac{1}{2!} \left(\frac{-i\lambda}{4!}\right)^2 \int d^4 z_1 d^4 z_2 \phi(x_1)\phi(x_2)\phi^4(z_1)\phi^4(z_2) \\ & = \int d^4 x d^4 y \left[ \frac{(-i\lambda)^2}{6} \Delta(x_1, x) \Delta^3(x, y) \Delta(y, x_2) + \frac{(-i\lambda)^2}{4} \Delta(x_1, x) \Delta(x, x) \Delta(x, y) \Delta(y, y) \Delta(y, x_2) \right. \\ & \quad \left. + \frac{(-i\lambda)^2}{4} \Delta(x_1, x) \Delta(x, y) \Delta(y, y) \Delta(y, x) \Delta(x, x_2) \right] + O(\lambda^3) \end{aligned}$$

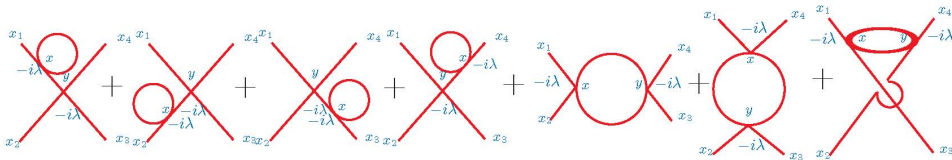






 $\lambda\phi^4$  理论的费曼图坐标空间  $\lambda\phi^4$  理论的费曼图——连接4点格林函数图

$$\begin{aligned} & \frac{1}{6!} \int d^4 y_1 d^4 y_1' d^4 y_2 d^4 y_2' d^4 y_3 d^4 y_3' d^4 y_4 d^4 y_4' d^4 y_5 d^4 y_5' d^4 y_6 d^4 y_6' \frac{1}{2} D'(y_1, y_1') \frac{1}{2} D'(y_2, y_2') \frac{1}{2} D'(y_3, y_3') \\ & \times \frac{1}{2} D'(y_4, y_4') \frac{1}{2} D'(y_5, y_5') \frac{1}{2} D'(y_6, y_6') \frac{\delta^2}{\delta\phi(y_1)\delta\phi(y_1')} \frac{\delta^2}{\delta\phi(y_2)\delta\phi(y_2')} \frac{\delta^2}{\delta\phi(y_3)\delta\phi(y_3')} \frac{\delta^2}{\delta\phi(y_4)\delta\phi(y_4')} \\ & \times \frac{\delta^2}{\delta\phi(y_5)\delta\phi(y_5')} \frac{\delta^2}{\delta\phi(y_6)\delta\phi(y_6')} \frac{1}{2!} \left(\frac{-i\lambda}{4!}\right)^2 \int d^4 z d^4 z' \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\phi^4(z)\phi^4(z') + O(\lambda^3) \\ & = \frac{(-i\lambda)^2}{2} \int d^4 x d^4 y \Delta(x_1, x)\Delta(x, x)\Delta(x, y)\Delta(x_2, y)\Delta(y, x_3)\Delta(y, x_4) + x_1 \Leftrightarrow x_2, x_3, x_4 \\ & + \frac{(-i\lambda)^2}{2} \int d^4 x d^4 y \Delta(x_1, x)\Delta(x_2, x)\Delta^2(x, y)\Delta(y, x_3)\Delta(y, x_4) + x_2 \Leftrightarrow x_3, x_4 + O(\lambda^3) \end{aligned}$$





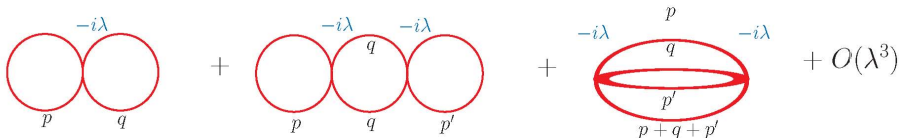


 $\lambda\phi^4$ 理论的费曼图动量空间  $\lambda\phi^4$  理论的费曼图——连接真空图

$$\Delta(p) \equiv i/(p^2 - M^2 + i0^+)$$

$$(2\pi)^4 \delta(p) \stackrel{p=0}{=} \int d^4x$$

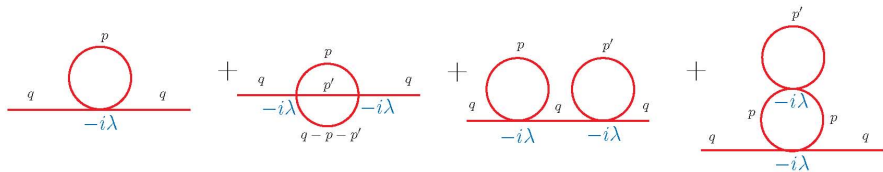
$$\int d^4x \left[ \frac{-i\lambda}{8} \int \frac{d^4p d^4q}{(2\pi)^8} \Delta(p)\Delta(q) + \frac{(-i\lambda)^2}{16} \int \frac{d^4p d^4q d^4p'}{(2\pi)^{12}} \Delta(p)\Delta^2(q)\Delta(p') \right. \\ \left. + \frac{(-i\lambda)^2}{48} \int \frac{d^4p d^4q d^4p'}{(2\pi)^{12}} \Delta(p)\Delta(q)\Delta(p')\Delta(p+q+p') \right] + O(\lambda^3)$$



动量空间  $\lambda\phi^4$  理论的费曼图——连接2点格林函数图

$$\Delta(p) \equiv i/(p^2 - M^2 + i0^+) \quad (2\pi)^4 \delta(p) \stackrel{p=0}{=} \int d^4x$$

$$\begin{aligned} & \frac{-i\lambda}{2} \int \frac{d^4p}{(2\pi)^4} \Delta(q)\Delta(p)\Delta(q) + \frac{(-i\lambda)^2}{6} \int \frac{d^4pd^4p'}{(2\pi)^8} \Delta(q)\Delta(p)\Delta(p')\Delta(q-p-p')\Delta(q) \\ & + \frac{(-i\lambda)^2}{4} \int \frac{d^4pd^4p'}{(2\pi)^8} \Delta(q)\Delta(p)\Delta(q)\Delta(p')\Delta(q) \\ & + \frac{(-i\lambda)^2}{4} \int \frac{d^4pd^4p'}{(2\pi)^8} \Delta(q)\Delta^2(p)\Delta(p')\Delta(q) + O(\lambda^3) \end{aligned}$$

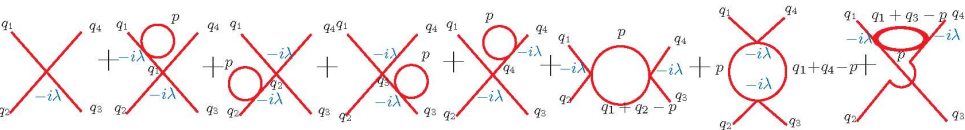




 $\lambda\phi^4$  理论的费曼图动量空间  $\lambda\phi^4$  理论的费曼图——连接4点格林函数图

$$\Delta(p) \equiv i/(p^2 - M^2 + i0^+) \quad (2\pi)^4 \delta(p) \stackrel{p=0}{=} \int d^4x$$

$$\begin{aligned}
 & -i\lambda \Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(q_4) \quad q_1 + q_2 + q_3 + q_4 = 0 \\
 & + \frac{(-i\lambda)^2}{2} \int \frac{d^4p}{(2\pi)^4} [\Delta(q_1)\Delta(p)\Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(q_4) + \Delta(q_1)\Delta(q_2)\Delta(p)\Delta(q_2)\Delta(q_3)\Delta(q_4) \\
 & + \Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(p)\Delta(q_3)\Delta(q_4) + \Delta(q_1)\Delta(q_2)\Delta(q_3)\Delta(q_4)\Delta(p)\Delta(q_4)] \\
 & + \frac{(-i\lambda)^2}{2} \int \frac{d^4p}{(2\pi)^4} \Delta(q_1)\Delta(q_2)\Delta(p)[\Delta(q_1+q_2-p) + \Delta(q_1+q_4-p) + \Delta(q_1+q_3-p)]\Delta(q_3)\Delta(q_4)
 \end{aligned}$$



 $\lambda\phi^4$ 理论的费曼图

在 $\lambda\phi^4$ 理论中用连接费曼图构造完全的费曼图

**n点格林函数:**  $G_n(x_1, \dots, x_n) \equiv (\Psi_0^-, \mathbf{T}\phi_H(x_1) \cdots \phi_H(x_n)\Psi_0^+)_C$

$$\frac{(\Psi_0^-, \mathbf{T}\phi_H(x_1) \cdots \phi_H(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)}$$

$$= \sum_{\nu_1 + \dots + n\nu_n = n} \frac{n!}{\nu_1! \cdots \nu_n! (1!)^{\nu_1} \cdots (n!)^{\nu_n}} \underbrace{G_1(x_1) \cdots G_1(x_{\nu_1})}_{\nu_1 \text{ terms}} \underbrace{G_2(x_{\nu_1+1}, x_{\nu_1+2}) \cdots G_2(x_{\nu_1+2\nu_2-1}, x_{\nu_1+2\nu_2})}_{\nu_2 \text{ terms}} \cdots$$

完全格林函数的生成泛函:

$$\frac{Z[J]}{Z[0]} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \cdots d^4x_n \frac{(\Psi_0^-, \mathbf{T}\phi_H(x_1) \cdots \phi_H(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} J(x_1) \cdots J(x_n)$$

$$= \sum_{n=0}^{\infty} \sum_{\nu_1 + \dots + n\nu_n = n} \frac{i^n}{n!} \frac{n!}{\nu_1! \cdots \nu_n! (1!)^{\nu_1} \cdots (n!)^{\nu_n}} \int d^4x_1 \cdots d^4x_n J(x_1) \cdots J(x_n)$$

$$\times \underbrace{G_1(x_1) \cdots G_1(x_{\nu_1})}_{\nu_1 \text{ terms}} \underbrace{G_2(x_{\nu_1+1}, x_{\nu_1+2}) \cdots G_2(x_{\nu_1+2\nu_2-1}, x_{\nu_1+2\nu_2})}_{\nu_2 \text{ terms}} \cdots$$



在 $\lambda\phi^4$ 理论中用连接费曼图构造完全的费曼图

完全格林函数的生成泛函:

$$\begin{aligned}
 \frac{Z[J]}{Z[0]} &= \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \cdots d^4x_n \frac{(\Psi_0^-, \mathbf{T}\psi_H(x_1) \cdots \psi_H(x_n)\Psi_0^+)}{(\Psi_0^-, \Psi_0^+)} J(x_1) \cdots J(x_n) \\
 &= \sum_{n=0}^{\infty} \sum_{\nu_1 + \cdots + n\nu_n = n} \frac{i^{\nu_1 + \cdots + n\nu_n}}{n!} \frac{n!}{\nu_1! \cdots \nu_n! (1!)^{\nu_1} \cdots (n!)^{\nu_n}} \int d^4x_1 \cdots d^4x_n J(x_1) \cdots J(x_n) \\
 &\quad \times \underbrace{G_1(x_1) \cdots G_1(x_{\nu_1})}_{\nu_1 \text{ terms}} \underbrace{G_2(x_{\nu_1+1}, x_{\nu_1+2}) \cdots G_2(x_{\nu_1+2\nu_2-1}, x_{\nu_1+2\nu_2})}_{\nu_2 \text{ terms}} \cdots \\
 &= \sum_{\nu_1=0}^{\infty} \frac{i^{\nu_1}}{\nu_1! (1!)^{\nu_1}} \left[ \int d^4x_1 J(x_1) G_1(x_1) \right]^{\nu_1} \sum_{\nu_2=0}^{\infty} \frac{i^{\nu_2}}{\nu_2! (2!)^{\nu_2}} \left[ \int d^4x_2 d^4x_3 J(x_2) J(x_3) G_2(x_2, x_3) \right]^{\nu_2} \times \cdots \\
 &= \exp \left[ \frac{i}{1!} \int d^4x_1 J(x_1) G_1(x_1) + \frac{i}{2!} \int d^4x_2 d^4x_3 J(x_2) J(x_3) G_2(x_2, x_3) + \cdots \right] = e^{iW[J] - iW[0]}
 \end{aligned}$$



## 4费米理论的费曼图

## 4费米理论概况

## 经典理论:

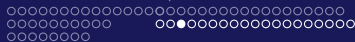
$$\mathcal{L}[\psi(x)] = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - M)\psi(x) - \frac{g}{4}[\bar{\psi}(x)\Gamma\psi(x)]^2$$

- ▶ 正常:  $M$ 为实数;  $g \geq 0$ ;  $\Gamma \subset \gamma$ 矩阵
- ▶  $M = 0 \Rightarrow$  有可能动力学产生质量依赖 $\Gamma$ !
- ▶  $g = 0 \Rightarrow$  自由场!
- ▶  $g < 0 \Rightarrow$  系统无定义(能量无下界)!

## 量子理论:

- ▶ 理论上最低阶的3 + 1维纯费米子相互作用量子场理论
- ▶ 现实中最实用的的3 + 1维有效纯费米子量子场理论
- ▶ 3 + 1维的理论是不可重整的
- ▶ 1 + 1维和2 + 1的理论是可重整的





## 4 费米理论的费曼图

## 4 费米理论的费曼规则

$$\mathcal{L}[\psi(x), \bar{\psi}(x)] = \bar{\psi}(x)(i\partial - M)\psi(x) - \frac{g}{4}[\bar{\psi}(x)\Gamma\psi(x)]^2 \quad \mathcal{S}'(x, x') = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} i(\not{p} - M + i0^+)^{-1}$$

$$(\Psi_0^-, \mathbf{T}\psi_{H, l_1}(x_1) \cdots \bar{\psi}_{H, l'_n}(x'_n) \Psi_0^+) = C \times e^{\int d^4 x d^4 x' S'_{ll'}(x, x') \frac{\delta^2}{\delta\psi_{l'}(x') \delta\bar{\psi}_l(x)}} \psi_{l_1}(x_1) \cdots \bar{\psi}_{l'_n}(x'_n) e^{-\frac{ig}{4} \int d^4 x (\bar{\psi}\Gamma\psi)^2} \Big|_{\bar{\psi}=\psi=0}$$

坐标空间:

$$= (\Phi_0, \mathbf{T}\{\psi_{l_1}(x_1) \cdots \bar{\psi}_{l'_n}(x'_n) e^{-\frac{ig}{4} \int d^4 x (\bar{\psi}\Gamma\psi)^2}\} \Psi_0)$$

- ▶ 对  $2n$  点连通格林函数  $(\Psi_0^-, \mathbf{T}\psi_{H, l_1}(x_1) \bar{\psi}_{H, l'_1}(x'_1) \cdots \psi_{H, l_n}(x_n) \bar{\psi}_{H, l'_n}(x'_n) \Psi_0^+)_C$ , 标出  $n$  个时空点  $x_1, \cdots, x_n$ , 每个点引出一条出线, 另外  $n$  个时空点  $x'_1, \cdots, x'_n$ , 每个点引出一条进线,
- ▶ 对  $m$  阶相互作用, 分别在  $m$  个时空点  $y_1, \cdots, y_m$  引入相互作用顶点, 每个顶点引出两条出线、两条进线
- ▶ 将引出的  $n + 2m$  条出线与  $n + 2m$  条进线两两相连, 得到所有可能的拓扑不等价的连接图
- ▶ 每条连线代表  $S'(x, x') = \underline{\psi(x)\bar{\psi}(x')}$ , 每个顶点代表  $-ig\Gamma_{l_1 l'_1} \Gamma_{l_2 l'_2}$ , 需要对所有顶点的时空坐标求积分



## 4费米理论的费曼图

## 坐标空间4费米理论的费曼图——连接真空图

$$\begin{aligned}
 & \frac{1}{2!} \int d^4x d^4x' d^4y d^4y' S_{l_1 l_1'}(x, x') S_{l_2 l_2'}(y, y') \frac{\delta^2}{\delta \bar{\psi}_{l_1'}(x') \delta \psi_{l_1}(x)} \frac{\delta^2}{\delta \bar{\psi}_{l_2'}(y') \delta \psi_{l_2}(y)} \left(-\frac{ig}{4}\right) \int d^4x (\bar{\psi} \Gamma \psi)^2 \\
 &= (\Phi_0, \mathbf{T} \left[ \left(-\frac{ig}{4}\right) \int d^4x (\bar{\psi} \Gamma \psi)^2 \right] \Psi_0) \\
 &= \left(-\frac{ig}{4}\right) \int d^4x \left[ \underline{\bar{\psi}(x) \Gamma \psi(x)} \underline{\bar{\psi}(x) \Gamma \psi(x)} - \text{tr}[\underline{\Gamma \psi(x) \bar{\psi}(x)} \underline{\Gamma \psi(x) \bar{\psi}(x)}] \right] \\
 &= \frac{-ig}{4} \int d^4x \{ [\text{tr}(S(x, x) \Gamma)]^2 - \text{tr}[S(x, x) \Gamma]^2 \}
 \end{aligned}$$





## 4费米理论的费曼图

## 坐标空间4费米理论的费曼图——连接真空图

$$\frac{1}{4!} \int d^4x_1 d^4x'_1 d^4x_2 d^4x'_2 d^4x_3 d^4x'_3 d^4x_4 d^4x'_4 S_{l_1 l'_1}(x_1, x'_1) S_{l_2 l'_2}(x_2, x'_2) S_{l_3 l'_3}(x_3, x'_3) S_{l_4 l'_4}(x_4, x'_4)$$

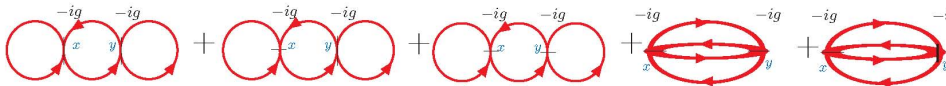
$$\times \frac{\delta^2}{\delta \bar{\psi}_{l'_1}(x'_1) \delta \psi_{l_1}(x_1)} \frac{\delta^2}{\delta \bar{\psi}_{l'_2}(x'_2) \delta \psi_{l_2}(x_2)} \frac{\delta^2}{\delta \bar{\psi}_{l'_3}(x'_3) \delta \psi_{l_3}(x_3)} \frac{\delta^2}{\delta \bar{\psi}_{l'_4}(x'_4) \delta \psi_{l_4}(x_4)}$$

$$\times \frac{1}{2!} \left(-\frac{ig}{4}\right)^2 \int d^4x d^4y [\bar{\psi}(x) \Gamma \psi(x)]^2 [\bar{\psi}(y) \Gamma \psi(y)]^2$$

$$= \frac{(-ig)^2}{8} \int d^4x d^4y \{ -\text{tr}[\Gamma S(x, x)] \text{tr}[\Gamma S(x, y) \Gamma S(y, x)] \text{tr}[\Gamma S(y, y)]$$

$$+ 2\text{tr}[\Gamma S(x, x) \Gamma S(x, y) \Gamma S(y, x)] \text{tr}[\Gamma S(y, y)] - \text{tr}[\Gamma S(x, x) \Gamma S(x, y) \Gamma S(y, y) \Gamma S(y, x)] \}$$

$$+ \frac{(-ig)^2}{16} \int d^4x d^4y \left[ \{\text{tr}[\Gamma S(x, y) \Gamma S(y, x)]\}^2 - \text{tr}[\Gamma S(x, y) \Gamma S(y, x) \Gamma S(x, y) \Gamma S(y, x)] \right] + O(g^3)$$







## 4费米理论的费曼图

## 坐标空间4费米理论的费曼图——连接真空图

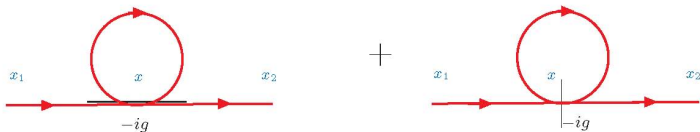
$$\begin{aligned}
 &= \langle \Phi_0, \mathbf{T} \left\{ \frac{1}{2!} \left(-\frac{ig}{4}\right)^2 \int d^4x d^4y [\bar{\psi}(x)\Gamma\psi(x)]^2 [\bar{\psi}(y)\Gamma\psi(y)]^2 \right\} \Psi_0 \rangle \\
 &= \frac{1}{2!} \left(-\frac{ig}{4}\right)^2 \int d^4x \left[ -2 * 2 \underline{\bar{\psi}(x)\Gamma\psi(x)} \underline{\text{tr}[\Gamma\psi(x)\bar{\psi}(y)\Gamma\psi(y)\bar{\psi}(x)]} \underline{\bar{\psi}(y)\Gamma\psi(y)} \right. \\
 &\quad -4 * 2 \underline{\text{tr}[\Gamma\psi(x)\bar{\psi}(x)\Gamma\psi(x)\bar{\psi}(y)\Gamma\psi(y)\bar{\psi}(x)]} \underline{\bar{\psi}(y)\Gamma\psi(y)} \\
 &\quad -2 * 2 \underline{\text{tr}[\Gamma\psi(x)\bar{\psi}(x)\Gamma\psi(x)\bar{\psi}(y)\Gamma\psi(y)\bar{\psi}(y)\Gamma\psi(y)\bar{\psi}(x)]} \\
 &\quad \left. + 2 \underline{\text{tr}[\Gamma\psi(x)\bar{\psi}(y)\Gamma\psi(y)\bar{\psi}(x)]} \underline{\text{tr}[\Gamma\psi(x)\bar{\psi}(y)\Gamma\psi(y)\bar{\psi}(x)]} \right] \\
 &= \frac{(-ig)^2}{8} \int d^4x d^4y \{ -\text{tr}[\Gamma S(x,x)] \text{tr}[\Gamma S(x,y)\Gamma S(y,x)] \text{tr}[\Gamma S(y,y)] \\
 &\quad + 2\text{tr}[\Gamma S(x,x)\Gamma S(x,y)\Gamma S(y,x)] \text{tr}[\Gamma S(y,y)] - \text{tr}[\Gamma S(x,x)\Gamma S(x,y)\Gamma S(y,y)\Gamma S(y,x)] \} \\
 &\quad + \frac{(-ig)^2}{16} \int d^4x d^4y \left[ \{\text{tr}[\Gamma S(x,y)\Gamma S(x,y)]\}^2 - \text{tr}[\Gamma S(x,y)\Gamma S(y,x)\Gamma S(x,y)\Gamma S(y,x)] \right] + O(g^3)
 \end{aligned}$$



## 4费米理论的费曼图

## 坐标空间4费米理论的费曼图——连接2点格林函数图

$$\begin{aligned}
 & \frac{1}{2!} \int d^4x d^4x' d^4y d^4y' S_{l_1 l_1'}(x, x') S_{l_2 l_2'}(y, y') \frac{\delta^2}{\delta \bar{\psi}_{l_1'}(x') \delta \psi_{l_1}(x)} \frac{\delta^2}{\delta \bar{\psi}_{l_2'}(y') \delta \psi_{l_2}(y)} \psi_{l_1}(x_1) \bar{\psi}_{l_1'}(x_2) \\
 & \times \left(-\frac{ig}{4}\right) \int d^4z [\bar{\psi}(z) \Gamma \psi(z)]^2 \\
 & = (\Phi_0, \mathbf{T} \{ \psi_{l_1}(x_1) \bar{\psi}_{l_1'}(x_2) \left(-\frac{ig}{4}\right) \int d^4x [\bar{\psi}(x) \Gamma \psi(x)]^2 \} \Psi_0) \\
 & = -\frac{ig}{4} \int d^4x [ \underline{2\psi_{l_1}(x_1) \bar{\psi}(x) \bar{\psi}(x) \Gamma \psi(x) \Gamma \psi(x) \bar{\psi}_{l_1'}(x_2)} - \underline{2\psi_{l_1}(x_1) \bar{\psi}(x) \Gamma \psi(x) \bar{\psi}(x) \Gamma \psi(x) \bar{\psi}_{l_1'}(x_2)} ] \\
 & = \frac{ig}{2} \int d^4x \{ S_{ll_1}(x_1, x) \Gamma_{l_1 l_2} \text{tr}[S(x, x) \Gamma] S_{l_2 l_1'}(x, x_2) - S_{ll_1}(x_1, x) \Gamma_{l_1 l_2} S_{l_2 l_3}(x, x) \Gamma_{l_3 l_4} S_{l_4 l_1'}(x, x_2) \}
 \end{aligned}$$

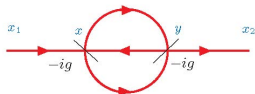




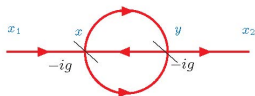
## 4费米理论的费曼图

坐标空间4费米理论的费曼图——连接2点格林函数图

$$-\frac{(-ig)^2}{4} \int d^4x d^4y S_{ll_1}(x_1, x) \Gamma_{l_1 l_2} \text{tr}[S(x, y) \Gamma]^2 S_{l_2 l_3}(x, y) \Gamma_{l_3 l_4} S_{l_4 l'}(y, x_2)$$



$$+\frac{(-ig)^2}{4} \int d^4x d^4y S_{ll_1}(x_1, x) \Gamma_{l_1 l_2} S_{l_2 l_3}(x, y) \Gamma_{l_3 l_4} S_{l_3 l_4}(y, x) \Gamma_{l_4 l_5} S_{l_4 l_5}(x, y) \Gamma_{l_5 l_6} S_{l_6 l'}(y, x_2)$$



$$+\frac{(-ig)^2}{4} \int d^4x d^4y S(x_1, x) \Gamma \{ \text{tr}[S(x, x) \Gamma] S(x, y) \Gamma \text{tr}[S(y, y) \Gamma] - \text{tr}[S(x, x) \Gamma] S(x, y) \Gamma S(y, y) \Gamma - S(x, x) \Gamma S(x, y) \Gamma \text{tr}[S(y, y) \Gamma] + S(x, x) \Gamma S(x, y) \Gamma S(y, y) \Gamma \} S(y, x_2)$$

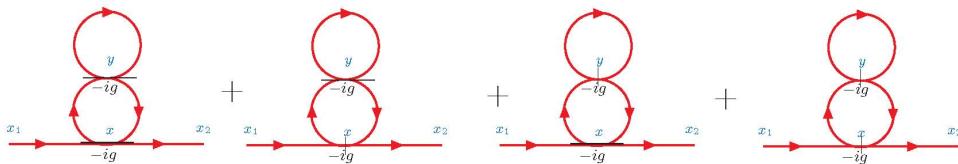




## 4费米理论的费曼图

## 坐标空间4费米理论的费曼图——连接2点格林函数图

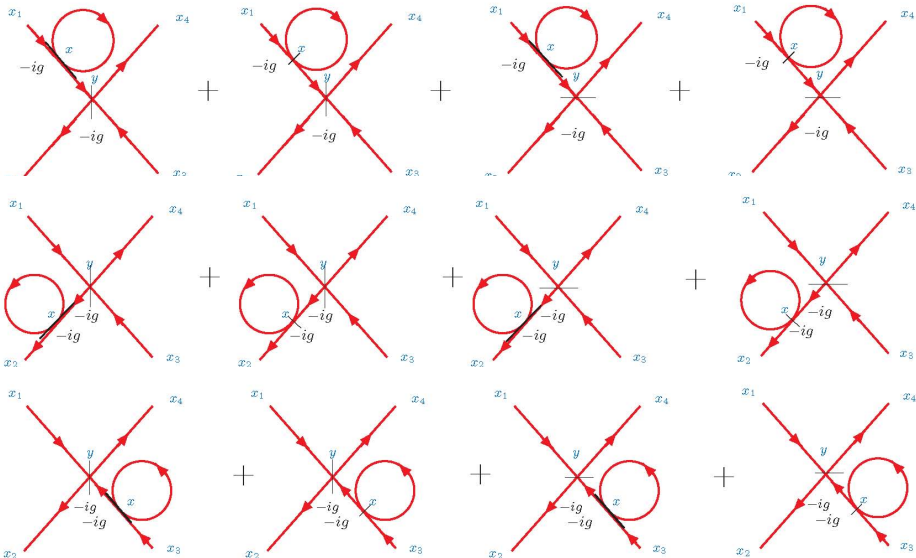
$$+\frac{(-ig)^2}{4} \int d^4x d^4y S(x_1, x) \Gamma \left[ \text{tr}\{\Gamma S(x, y) \text{tr}[\Gamma S(y, y)] \Gamma S(y, x)\} - \Gamma \text{tr}[S(x, y) \Gamma S(y, y) \Gamma S(y, x)] \right. \\ \left. - \Gamma S(x, y) \text{tr}[\Gamma S(y, y)] \Gamma S(y, x) + \Gamma S(x, y) \Gamma S(y, y) \Gamma S(y, x) \right] S(x, x_1) + O(g^3)$$





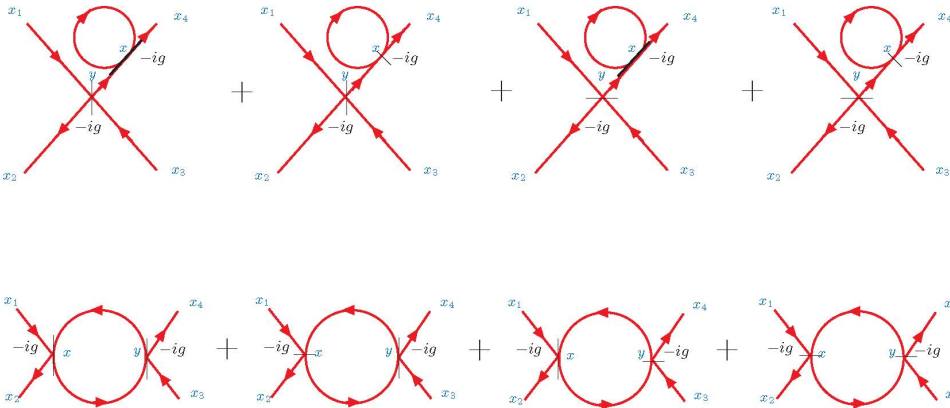


4费米理论的费曼图



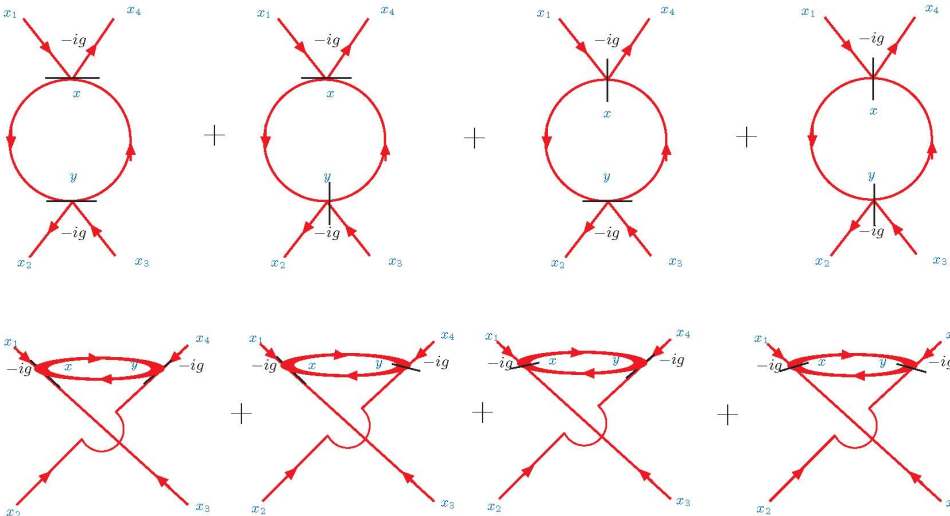


4费米理论的费曼图





4费米理论的费曼图

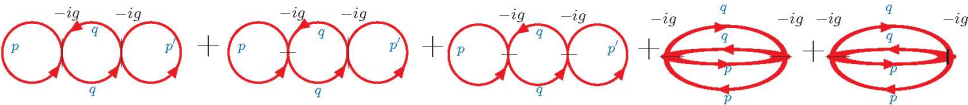
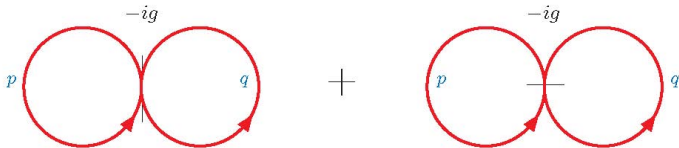






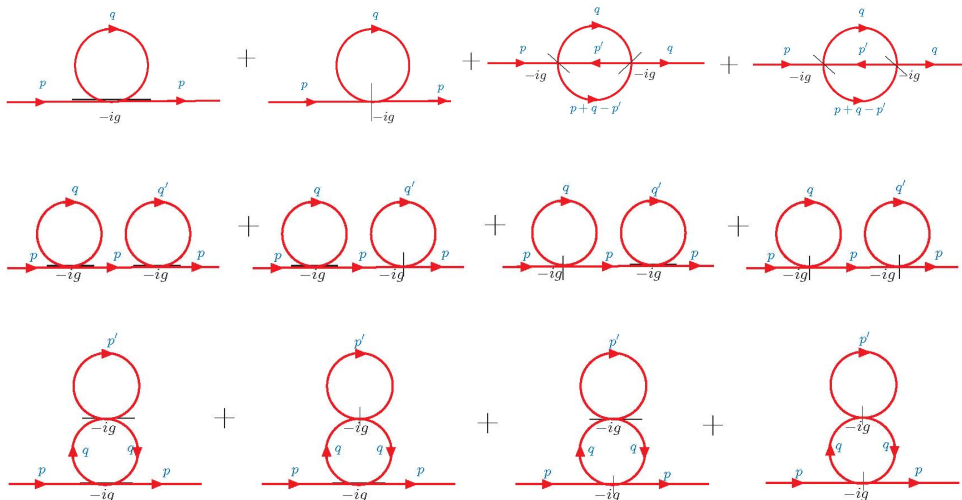


4费米理论的费曼图





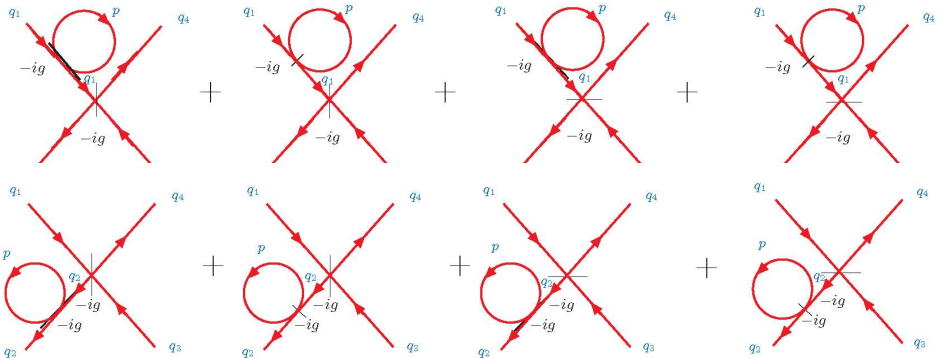
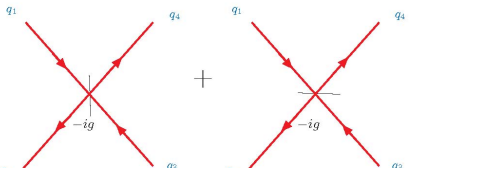
## 4费米理论的费曼图





4费米理论的费曼图

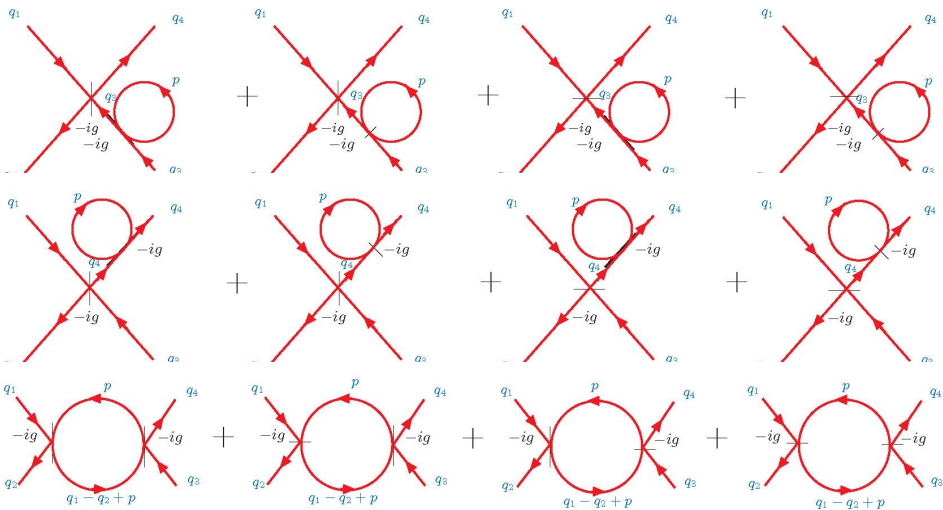
$$q_1 - q_2 + q_3 - q_4 = 0$$





4费米理论的费曼图

$$q_1 - q_2 + q_3 - q_4 = 0$$





## 4费米理论的费曼图

$$q_1 - q_2 + q_3 - q_4 = 0$$

