量子场论　量子电动力学

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经典电动力学

$$\mathcal{L}_{\text{Maxwell}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - J_\mu(x)A^\mu(x) \qquad -J_\mu A^\mu = \vec{J}\cdot\vec{A} - \rho\phi$$

$$\mathcal{L}_{\text{free vector}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M^2A^2(x) \qquad F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

- 电磁场是零质量的矢量场
- 物质场以电磁流的方式进入电磁场的拉格朗日量

规范对称性

- 零质量矢量量子场具有规范对称性: $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\Omega(x)$
- 规范对称性导致流守恒： $\partial^\mu J_\mu = 0$

加入费米子　最小耦合： $J_\mu(x) = e\overline{\psi}(x)\gamma_\mu\psi(x) = J^\dagger_\mu(x)$

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)$$

费米子的规范变换： $\psi(x) \to \psi'(x) = e^{-ie\Omega(x)}\psi(x) \qquad [Q,\psi(x)] = -e\psi(x)$

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$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

量子化 $A_0(x) = 0$ 略去$i0^+$项

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ \delta[A_0] e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \int \mathcal{D}[\partial_0\Omega]\ e^{-i\int d^4x\ \frac{\lambda}{2}[\partial_0\Omega]^2}\delta[A_0+\partial_0\Omega] e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}A_0^2+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

不同规范的选择 $F[A_\mu(x)] = 0$

- 洛伦兹规范 **(Lorentz gauge):** $F[A_\mu(x)] = \partial_\mu A^\mu(x)$
- 库伦规范 **(Coulomb gauge):** $F[A_\mu(x)] = \nabla\cdot\vec{A}(x)$
- 瞬时规范 **(Temporal gauge):** $F[A_\mu(x)] = A^0(x)$
- 轴规范 **(Axial gauge):** $F[A_\mu(x)] = A^3(x)$
- 幺正规范 **(Unitary gauge):** $F[A_\mu(x)] = \phi(x) - \phi^\dagger(x)$

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$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

量子化准备: $\delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|}\delta(x-x_i) \Rightarrow \int_{-\infty}^{\infty} dx\, \delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|}$

$$\Delta(A) \equiv \int \mathcal{D}\Omega\, \delta[F(A_\mu + \partial_\mu\Omega)] = {\rm Det}^{-1}\mathcal{M}(A) \qquad F[A_\mu(x)] = 0$$

只考虑 $F[A_\mu + B_\mu] = F[A_\mu] + F[B_\mu]$

$$\mathcal{M}(A;x,y) \equiv \frac{\delta F[A_\mu(x) + \partial_\mu\Omega(x)]}{\delta\Omega(y)} = \frac{\delta F[\partial_\mu\Omega(x)]}{\delta\Omega(y)} = \text{与}A_\mu\text{无关!} \qquad \Delta^{-1}\int \mathcal{D}\Omega\, \delta[F(A_\mu + \partial_\mu\Omega)] = 1$$

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

略去$i0^+$项

$$\overset{A'_\mu = A_\mu + \partial_\mu\Omega}{=\!=\!=\!=\!=\!=\!=} \Delta^{-1}\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \int \mathcal{D}\Omega\, \delta\{F[A'_\mu]\}\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

$$\overset{\psi' = e^{-ie\Omega}\psi}{=\!=\!=\!=\!=\!=} \Delta^{-1}\int \mathcal{D}A'_\mu \mathcal{D}\overline{\psi}'\mathcal{D}\psi' \int \mathcal{D}\Omega\, \delta\{F[A'_\mu]\}\, e^{i\int d^4x\, \{-\frac{1}{4}F'_{\mu\nu}(x)F^{\mu\nu\prime}(x) + \overline{\psi}'(x)[i\partial\!\!\!/ - eA\!\!\!/'(x) - M]\psi'(x)\}}$$

$$= \Delta^{-1}\int \mathcal{D}\Omega \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\, \delta\{F[A_\mu]\} e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

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$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

量子化 $F[A_\mu(x)] = 0$ 略去$i0^+$项

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ \delta\{F[A_\mu]\}e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

$$=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \int \mathcal{D}[F[\partial_\mu\Omega]]e^{-i\int d^4x\frac{\lambda}{2}F^2[\partial_\mu\Omega]}\delta\{F[A_\mu]+F[\partial_\mu\Omega]\}e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)\}}$$

$$=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}F^2[A_\mu(x)]+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

不同规范的选择 $F[A_\mu(x)] = 0$

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量子电动力学的变形： 纯旋量场理论

略去$i0^+$项

$$\int d^4x\,\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)\}$$

$$=\int d^4x\,\{-\frac{1}{2}(\partial_\mu A_\nu-\partial_\nu A_\mu)\partial^\mu A^\nu-\frac{\lambda}{2}[\partial^\mu A_\mu]^2-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}[i\partial\!\!\!/-M]\psi\}$$

$$=\int d^4x\,\{\frac{1}{2}A_\mu[g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu]A_\nu-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}[i\partial\!\!\!/-M]\psi\}$$

$$=\int d^4x\,\{\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu-A_\mu J^\mu+\overline{\psi}[i\partial\!\!\!/-M]\psi\}\quad iD_0^{-1,\mu\nu}=g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu\quad J^\mu=e\overline{\psi}\gamma^\mu\psi$$

$$\int \mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\,\{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)\}}$$

$$=\int \mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\,\{\frac{1}{2}(A+iJD_0)_\mu iD_0^{-1,\mu\nu}(A+iD_0J)_\nu+\frac{i}{2}J_\mu D_0^{\mu\nu}J_\nu+\overline{\psi}[i\partial\!\!\!/-M]\psi\}}$$

$$=C\times\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{-\frac{e^2}{2}\int d^4x d^4y\,[\overline{\psi}(x)\gamma_\mu\psi(x)]D_0^{\mu\nu}(x,y)[\overline{\psi}(y)\gamma_\nu\psi(y)]+i\int d^4x\,\overline{\psi}(x)[i\partial\!\!\!/-M]\psi(x)}$$

$$D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)\qquad C=\int \mathcal{D}A_\mu\; e^{\frac{i}{2}\int d^4x\,A_\mu iD_0^{-1,\mu\nu}A_\nu}$$

量子电动力学的变形： 纯旋量场理论

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= C\times\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{-\frac{e^2}{2}\int d^4x d^4y\ [\overline{\psi}(x)\gamma_\mu\psi(x)]D_0^{\mu\nu}(x,y)[\overline{\psi}(y)\gamma_\nu\psi(y)]\ +i\int d^4x\ \overline{\psi}(x)[i\partial\!\!\!/ - M]\psi(x)}$$

$$D_0^{\mu\nu}(x,y) = \frac{i}{\partial_x^2}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y) \qquad C=\int\mathcal{D}A_\mu\ e^{\frac{i}{2}\int d^4x\ A_\mu iD_0^{-1,\mu\nu}A_\nu}$$

Fierz 变换

$$\delta^{\xi\xi'}\delta^{\zeta\zeta'} = a^{\zeta\xi}\delta^{\xi'\zeta'} + b^{\zeta\xi}(\gamma_5)^{\xi'\zeta'} + c_\mu^{\zeta\xi}(\gamma^\mu)^{\xi'\zeta'} + d_\mu^{\zeta\xi}(\gamma^\mu\gamma_5)^{\xi'\zeta'} + f_{\mu\nu}^{\zeta\xi}(\sigma^{\mu\nu})^{\xi'\zeta'}$$

两边同时乘$\delta^{\zeta'\xi'}$, $(\gamma_5)^{\zeta'\xi'}$, $(\gamma_{\mu'})^{\zeta'\xi'}$ $(\gamma_{\mu'}\gamma_5)^{\zeta'\xi'}$ $(\sigma_{\mu'\nu'})^{\zeta'\xi'} = \frac{i}{2}[\gamma_{\mu'},\gamma_{\nu'}]^{\zeta'\xi'}$

$$\delta^{\zeta\xi}=4a^{\zeta\xi} \quad (\gamma_5)^{\zeta\xi}=4b^{\zeta\xi} \quad (\gamma_{\mu'})^{\zeta\xi}=4c_{\mu'}^{\zeta\xi} \quad (\gamma_{\mu'}\gamma_5)^{\zeta\xi}=-4d_{\mu'}^{\zeta\xi} \quad (\sigma_{\mu'\nu'})^{\zeta\xi}=-8f_{\mu'\nu'}^{\zeta\xi}$$

$$\delta^{\xi\xi'}\delta^{\zeta\zeta'}=\frac{1}{4}\left[\delta^{\zeta\xi}\delta^{\xi'\zeta'}+(\gamma_5)^{\zeta\xi}(\gamma_5)^{\xi'\zeta'}+(\gamma_\mu)^{\zeta\xi}(\gamma^\mu)^{\xi'\zeta'}-(\gamma_\mu\gamma_5)^{\zeta\xi}(\gamma^\mu\gamma_5)^{\xi'\zeta'}+\frac{1}{2}(\sigma_{\mu\nu})^{\zeta\xi}(\sigma^{\mu\nu})^{\xi'\zeta'}\right]$$

量子电动力学的变形： 纯旋量场理论

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)-\frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2+\overline{\psi}(x)[i\partial\!\!\!/ -eA\!\!\!/(x)-M]\psi(x)\}}$$

$$= C\times\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{-\frac{e^2}{2}\int d^4x d^4y\ [\overline{\psi}(x)\gamma_\mu\psi(x)]D_0^{\mu\nu}(x,y)[\overline{\psi}(y)\gamma_\nu\psi(y)]\ +i\int d^4x\ \overline{\psi}(x)[i\partial\!\!\!/ -M]\psi(x)}$$

$$D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)\qquad C=\int \mathcal{D}A_\mu\ e^{\frac{i}{2}\int d^4x\ A_\mu iD_0^{-1,\mu\nu}A_\nu}$$

$$\delta^{\xi\xi'}\delta^{\zeta\zeta'}=\frac{1}{4}\left[\delta^{\zeta\xi}\delta^{\xi'\zeta'}+(\gamma_5)^{\zeta\xi}(\gamma_5)^{\xi'\zeta'}+(\gamma_\mu)^{\zeta\xi}(\gamma^\mu)^{\xi'\zeta'}-(\gamma_\mu\gamma_5)^{\zeta\xi}(\gamma^\mu\gamma_5)^{\xi'\zeta'}+\frac{1}{2}(\sigma_{\mu\nu})^{\zeta\xi}(\sigma^{\mu\nu})^{\xi'\zeta'}\right]$$

$$[\overline{\psi}(x)\gamma_\mu\psi(x)][\overline{\psi}(y)\gamma_\nu\psi(y)]=\overline{\psi}_{\xi'}(x)[\gamma_\mu\psi(x)]_\xi\overline{\psi}_\zeta(y)[\gamma_\nu\psi(y)]_{\zeta'}\delta^{\xi\xi'}\delta^{\zeta\zeta'}$$

$$=\frac{1}{4}\Big[[\overline{\psi}(x)\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma_\mu\psi(x)]+[\overline{\psi}(x)\gamma_5\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma_5\gamma_\mu\psi(x)]+[\overline{\psi}(x)\gamma_\sigma\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma^\sigma\gamma_\mu\psi(x)]$$

$$-[\overline{\psi}(x)\gamma_5\gamma_\sigma\gamma_\nu\psi(y)][\overline{\psi}(y)\gamma_5\gamma^\sigma\gamma_\mu\psi(x)]+\frac{1}{2}[\overline{\psi}(x)\sigma_{\sigma\rho}\gamma_\nu\psi(y)][\overline{\psi}(y)\sigma^{\sigma\rho}\gamma_\mu\psi(x)]\Big]$$

$$=\cdots\cdots$$

量子电动力学的变形： 纯矢量场理论

略去$i0^+$项

$$\int \prod_i^n [d\bar\theta_i d\theta_i]\, e^{\sum_{ij}^n \bar\theta_i \mathcal{M}_{ij}\theta_j} = \int \prod_i^n [d\bar\theta_i d\theta_i]\, e^{\sum_{ijk}^n \bar\theta_i U_{ik}^* a_k U_{kj}\theta_j} \qquad \mathcal{M}_{ij} = \sum_k^n U_{ik}^* a_k U_{kj}$$

$$= \int \prod_i^n [d\bar\theta_i' d\theta_i']\, e^{\sum_k^n \bar\theta_k' a_k \theta_k'} \qquad \theta_k' = \sum_j^n U_{kj}\theta_j \quad \bar\theta_k' = \sum_i^n U_{ik}^* \bar\theta_i \quad \prod_i^n [d\bar\theta_i d\theta_i] = \prod_i^n [d\bar\theta_i' d\theta_i']$$

$$= \int \prod_i^n [d\bar\theta_i' d\theta_i'] \prod_k^n e^{\bar\theta_k' a_k \theta_k'} = \int \prod_i^n [d\bar\theta_i' d\theta_i'] \prod_k^n [1 + \bar\theta_k' a_k \theta_k'] = \int \prod_i^n [d\bar\theta_i' d\theta_i'] \prod_k^n [\bar\theta_k' a_k \theta_k']$$

$$= \int \prod_i^n [-d\bar\theta_i'\, \bar\theta_i'\, d\theta_i'\, \theta_i'\, a_i] = \prod_i (-a_i) = \mathrm{Det}(-\mathcal{M}) = e^{\mathrm{Tr}\,\ln(-\mathcal{M})}$$

$$\int \mathcal{D}A_\mu \mathcal{D}\bar\psi \mathcal{D}\psi\, e^{i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2 + \bar\psi(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)\}}$$

$$= \int \mathcal{D}A_\mu\, e^{\mathrm{Tr}\,\ln[i\partial\!\!\!/ - eA\!\!\!/(x) - M] + i\int d^4x\, \{-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu(x)]^2\}}$$

需发展计算行列式的方法！

1+1维QED（Schwinger模型）精确可解就是因为其行列式可以严格求出，并且结果对光子场是二次依赖！

格林函数生成泛函

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu[g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu-i0^+]A_\nu-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}[i\partial\!\!\!/-M+i0^+]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu-eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}iS_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$iD_0^{-1,\mu\nu}=g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu-i0^+ \qquad iS_0^{-1}=i\partial\!\!\!/-M+i0^+$$

$$=e^{-i\int d^4x\ e\frac{\delta}{-i\delta J^\mu(x)}\frac{\delta}{-i\delta I(x)}\gamma^\mu\frac{\delta}{i\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=e^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{\frac{1}{2}(A+iJD_0)_\mu iD_0^{-1,\mu\nu}(A+iD_0J)_\nu+(\overline{\psi}-i\bar{I}S_0)iS_0^{-1}(\psi-iS_0I)+\frac{i}{2}J_\mu D_0^{\mu\nu}J_\nu+i\bar{I}S_0I\}}$$

$$=C\times e^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\ e^{i\int d^4yd^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]}$$

$$C=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ [\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0^{-1}\psi]}$$

$$D_0^{\mu\nu}(y,z)=\frac{i}{\partial_y^2-i0^+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z) \qquad S_0(y,z)=\frac{i}{i\partial\!\!\!/_y-M+i0^+}\delta(y-z)$$

顶角生成泛函

$$\frac{\delta W[J,I,\bar{I}]}{\delta J_\mu(x)} = -A_c^\mu(x) \qquad \frac{\delta W[J,I,\bar{I}]}{\delta \bar{I}_l(x)} = \psi_{c,l}(x) \qquad \frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)} = -\overline{\psi}_{c,l}(x)$$

$$\Gamma[A_c,\psi_c,\overline{\psi}_c] \equiv W[J,I,\bar{I}] - \int d^4x\, [-J_\mu(x)A_c^\mu(x) + \bar{I}_l(x)\psi_{c,l}(x) + \overline{\psi}_{c,l}(x)I_l(x)]$$

$$\frac{\delta\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta A_c^\mu(x)} = J_\mu(x) \qquad \frac{\delta\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta\psi_{c,l}(x)} = \bar{I}_l(x) \qquad \frac{\delta\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta\overline{\psi}_{c,l}(x)} = -I_l(x)$$

$$\int d^4y\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta J_{\nu'}(y)}\bigg|_{J=I=\bar{I}=0} \frac{\delta^2\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta A_c^{\nu'}(y)\delta A_c^{\nu}(x')}\bigg|_{A_c=A_0,\ \psi_c=\psi_0,\ \overline{\psi}_c=\overline{\psi}_0} = -\delta(x-x')g^\mu_{\ \nu}$$

$$\int d^4y\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_l(x)\delta I_{l''}(y)}\bigg|_{J=I=\bar{I}=0} \frac{\delta^2\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta\overline{\psi}_{c,l''}(y)\delta\psi_{c,l'}(x')}\bigg|_{A_c=A_0,\ \psi_c=\psi_0,\ \overline{\psi}_c=\overline{\psi}_0} = -\delta(x-x')\delta_{ll'}$$

$$\frac{\delta^3 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J=I=\bar{I}=0} = -\int d^4x'd^4y'd^4z'\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta J_{\mu'}(x')}\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l_1}(y')}\, \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(z)\delta I_{l_2}(z')}\bigg|_{J=I=\bar{I}=0}$$

$$\times \frac{\delta^3\Gamma[A_c,\psi_c,\overline{\psi}_c]}{\delta A_c^{\mu'}(x')\delta\overline{\psi}_{c,l_1}(y')\delta\psi_{c,l_2}(z')}\bigg|_{A_c=A_0,\ \psi_c=\psi_0,\ \overline{\psi}_c=\overline{\psi}_0}$$

格林函数与顶角函数

$$Z[J,I,\bar I] = e^{iW[J,I,\bar I]} = C \times e^{e\int d^4x \frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta I(x)}\gamma^\mu \frac{\delta}{\delta \bar I(x)}}\ e^{i\int d^4y d^4z [\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z) + i\bar I(y)S_0(y,z)I(z)]}$$

$$D_0^{\mu\nu}(x,y) = \frac{i}{\partial_x^2 - i0+}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y) \qquad S_0(x,y) = \frac{i}{i\partial\!\!\!/_x - M + i0+}\delta(x-y)$$

格林函数:

$$\frac{\delta^2 W[J,I,\bar I]}{\delta J_\mu(x)\delta J_\nu(y)}\Big|_{J=I=\bar I=0} \equiv iD^{\mu\nu}(x,y) = iD_0^{\mu\nu}(x,y) + \text{高阶修正}$$

$$\frac{\delta^2 W[J,I,\bar I]}{\delta \bar I_l(x)\delta I_{l'}(y)}\Big|_{J=I=\bar I=0} \equiv -iS(x,y) = -iS_0(x,y) + \text{高阶修正}$$

$$\frac{\delta^3 W[J,I,\bar I]}{\delta J^\mu(x)\delta \bar I_l(y)\delta I_{l'}(z)}\Big|_{J=I=\bar I=0} \equiv G_\mu(x,y,z) = \int d^4x' D_{0,\mu\nu}(x,x')[S_0(y,x')ie\gamma^\nu S_0(x',z)]_{ll'} + \text{高阶修正}$$

$$\int d^4z \frac{\delta^2\Gamma[A,\psi,\overline{\psi}]}{\delta\psi_{l''}(x')\delta\overline{\psi}_{l'}(z)} \frac{\delta^2 W[J,I,\bar I]}{\delta I_{l'}(z)\delta \bar I_l(y)}\Big|_{I=\bar I=0} = -\delta_{l''l}\delta(x'-y)$$

$$\int d^4z \left[\frac{\delta^2\Gamma[A,\psi,\overline{\psi}]}{\delta\psi_{l''}(x')\delta\overline{\psi}_{l'}(z)} \frac{\delta^3 W[J,I,\bar I]}{\delta J^\mu(x)\delta I_{l'}(z)\delta \bar I_l(y)} - \int d^4y' \frac{\delta W[J,I,\bar I]}{\delta J^\mu(x)\delta J^\nu(y')} \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A_\nu(y')\delta\psi_{l''}(x')\delta\overline{\psi}_{l'}(z)} \frac{\delta^2 W[J,I,\bar I]}{\delta I_{l'}(z)\delta \bar I_l(y)}\right]_{I=\bar I=J^\mu=0} = 0$$

$$\frac{\delta^3 W[J,I,\bar I]}{\delta J^\mu(x)\delta \bar I_l(y)\delta I_{l'}(z)}\Big|_{J=I=\bar I=0} = -\int d^4x' d^4y' d^4z' \frac{\delta W[J,I,\bar I]}{\delta J^\mu(x)\delta J^\nu(x')} \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A_\nu(x')\delta\psi_{l_1'}(z')\delta\overline{\psi}_{l_1}(y')} \frac{\delta^2 W[J,I,\bar I]}{\delta I_{l_1}(y')\delta \bar I_l(y)} \frac{\delta^2 W[J,I,\bar I]}{\delta I_{l'}(z)\delta \bar I_{l_1'}(z')}\Big|_{J=I=\bar I=0}$$

格林函数与顶角函数

$$Z[J,I,\bar{I}] = e^{iW[J,I,\bar{I}]} = C \times e^{e\int d^4x \frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta I(x)} \gamma^\mu \frac{\delta}{\delta \bar{I}(x)}} e^{i\int d^4y d^4z [\frac{i}{2} J_\mu(y) D_0^{\mu\nu}(y,z) J_\nu(z) + i\bar{I}(y) S_0(y,z) I(z)]}$$

$$D_0^{\mu\nu}(x,y) = \frac{i}{\partial_x^2 - i0^+}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_x^\mu \partial_x^\nu}{\partial_x^2}]\delta(x-y) \qquad S_0(x,y) = \frac{i}{i\partial\!\!\!/_x - M + i0^+}\delta(x-y)$$

格林函数:

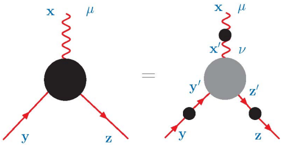
$$\frac{\delta^2 W[J,I,\bar{I}]}{\delta J_\mu(x)\delta J_\nu(y)}\Big|_{J=I=\bar{I}=0} \equiv iD^{\mu\nu}(x,y) = iD_0^{\mu\nu}(x,y) + \text{高阶修正}$$

$$\frac{\delta^2 W[J,I,\bar{I}]}{\delta \bar{I}_l(x)\delta I_{l'}(y)}\Big|_{J=I=\bar{I}=0} \equiv -iS(x,y) = -iS_0(x,y) + \text{高阶修正}$$

$$\frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta \bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J=I=\bar{I}=0} \equiv G_\mu(x,y,z) = \int d^4x' D_{0,\mu\nu}(x,x')[S_0(y,x') ie\gamma^\nu S_0(x',z)]_{ll'} + \text{高阶修正}$$

$$G_\mu(x,y,z) = \int d^4x' d^4y' d^4z' D_{\mu\nu}(x,x')[S(y,y') i\Gamma^\nu(x',y',z') S(z',z)]_{ll'}$$

$$\frac{\delta^3 \Gamma[A,\psi,\bar{\psi}]}{\delta A_\mu(x)\delta\bar{\psi}_l(y)\delta\psi_{l'}(z)} \equiv -\Gamma^\mu_{ll'}(x,y,z) = -e\gamma^\mu_{ll'}\delta(z-x)\delta(y-x) + \text{高阶修正}$$



$$\Gamma[A,\psi,\bar{\psi}] = \int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}[\partial^\mu A_\mu]^2 + \bar{\psi}[i\partial\!\!\!/ - eA\!\!\!/ - M]\psi\} + \text{高阶修正}$$

微扰展开

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\text{ term}\}}$$

$$= C\times e^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\; e^{i\int d^4yd^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]}$$

$$C=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\,[\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0\psi]}$$

$$D_0^{\mu\nu}(y,z)=\frac{i}{\partial_y^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)\qquad S_0(y,z)=\frac{i}{i\partial\!\!\!/_y-M+i0+}\delta(y-z)$$

$$Z[J,I,\bar{I}]=Ce^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}e^{i\int d^4yd^4z[\frac{i}{2}J_\mu(y)D_0^{\mu\nu}(y,z)J_\nu(z)+i\bar{I}(y)S_0(y,z)I(z)]+i\int d^4x[-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$=Ce^{e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}e^{i\int d^4yd^4z[-\frac{i}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+i\frac{\delta}{\delta\psi(y)}S_0(y,z)\frac{\delta}{\delta\overline{\psi}(z)}]}e^{i\int d^4x[-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$=Ce^{i\int d^4yd^4z[-\frac{i}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+i\frac{\delta}{\delta\psi(y)}S_0(y,z)\frac{\delta}{\delta\overline{\psi}(z)}]}e^{i\int d^4x[-eA_\mu\overline{\psi}\gamma^\mu\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

微扰展开(续)

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$= Ce^{i\int d^4yd^4z[-\frac{i}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+i\frac{\delta}{\delta\psi(y)}S_0(y,z)\frac{\delta}{\delta\overline{\psi}(z)}]}e^{i\int d^4x[-eA_\mu\overline{\psi}\gamma^\mu\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I]}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$C=\int \mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ [\frac{1}{2}A_\mu iD_0^{-1,\mu\nu}A_\nu+\overline{\psi}iS_0^{-1}\psi]}$$

$$D_0^{\mu\nu}(y,z)=\frac{i}{\partial_y^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)\qquad S_0(y,z)=\frac{i}{i\partial\!\!\!/_y-M+i0^+}\delta(y-z)$$

$$(\Psi_0^-,\Psi_0^+)=C'\times e^{\int d^4yd^4z\ [\frac{1}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$(\Psi_0^-,\mathbf{T}\ [\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ \Psi_0^+)$$

$$=C\times e^{\int d^4yd^4z\ [\frac{1}{2}\frac{\delta}{\delta A^\mu(y)}D_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)}+S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ [\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')$$

$$\times A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

坐标空间费曼规则

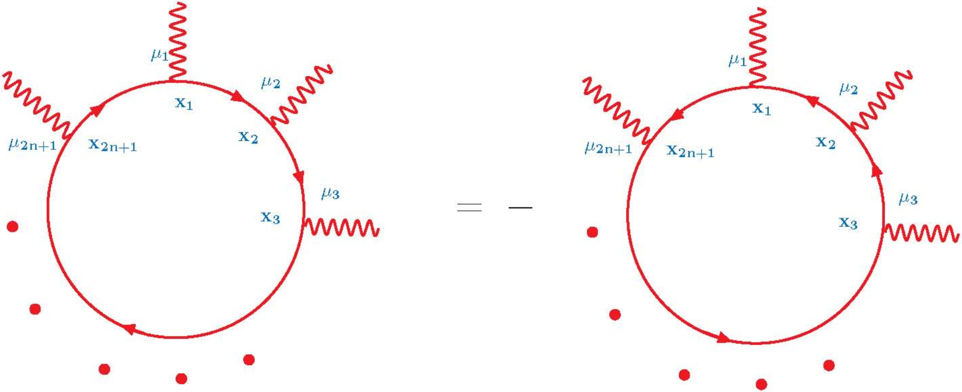
$$(\Psi_0^-, \mathbf{T}\left[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)\right]\Psi_0^+)$$

$$= C \times e^{\int d^4y d^4z \left[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}\right]}\left[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')\right.$$

$$\left.\times A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)\right] e^{-i\int d^4x\, eA_\mu\overline{\psi}\gamma^\mu\psi}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

- 对$(\Psi_0^-, \mathbf{T}\left[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)\right]\Psi_0^+)_C$,标出$n$个时空点$x_1\cdots x_n$,每点引出一条出费米线, 另外$n$个时空点$x_1'\cdots x_n'$, 每点引出一条进费米线，$m$个时空点$y_1\cdots y_m$,每点引出一条光子线
- 对k阶相互作用,分别在k个时空点$z_1,\cdots,z_k$引入相互作用顶点,每个顶点引出一条光子线、一条出费米线和一条进费米线
- 将引出的$n+k$条出费米线与$n+k$条进费米线两两相连,并将$m+k$条光子线两两相连，得到所有可能的拓扑不等价的连接图
- 每对费米子连线代表$S_{0,ll'}(x,x')$,每对光子连线代表$D_0^{\mu\nu}(x,x')$, 每个顶点代表$-ie\gamma^\mu_{ll'}$, 需要对所有顶点的时空坐标求积分
- 如图中出现对称的连线,须乘以对称因子. 封闭费米圈产生额外-1因子

法雷(Furry)定理



应用：QED中奇数个光子场的格林函数为零！

$$(\Psi_0^-, \mathbf{T}\, A_{\mu_1}(x_1) A_{\mu_2}(x_2) \cdots A_{\mu_{2n+1}}(x_{2n+1}) \Psi_0^+)_C = 0$$

法雷(Furry)定理

格林函数中包含奇数个顶角的封闭费米子圈图相互抵消。

证明:

$$\mathcal{C}=i\gamma^2\gamma^0=-\mathcal{C}^{-1}\qquad \mathcal{C}\gamma_\mu\mathcal{C}^{-1}=-\gamma_\mu^T\qquad S_0(x,y)=\frac{i}{i\partial\!\!\!/_x-M+i0^+}\delta(x-y)$$

$$\mathcal{C}S_0(x,y)\mathcal{C}^{-1}=\frac{i}{-i\partial\!\!\!/_x^T-M+i0^+}\delta(x-y)=\frac{i}{i\partial\!\!\!/_y^T-M+i0^+}\delta(x-y)=S_0^T(y,x)$$

$$\begin{aligned}&\mathrm{tr}[\gamma_{\mu_1}S_0(x_1,x_2)\gamma_{\mu_2}S_0(x_2,x_3)\gamma_{\mu_3}\cdots\gamma_{\mu_n}S_0(x_n,x_1)]\\
&=\mathrm{tr}[\mathcal{C}\gamma_{\mu_1}\mathcal{C}^{-1}\mathcal{C}S_0(x_1,x_2)\mathcal{C}^{-1}\mathcal{C}\gamma_{\mu_2}\mathcal{C}^{-1}\mathcal{C}S_0(x_2,x_3)\mathcal{C}^{-1}\mathcal{C}\gamma_{\mu_3}\cdots\gamma_{\mu_n}\mathcal{C}^{-1}\mathcal{C}S_0(x_n,x_1)\mathcal{C}^{-1}]\\
&=(-1)^n\mathrm{tr}[\gamma_{\mu_1}^TS_0^T(x_2,x_1)\gamma_{\mu_2}^TS_0^T(x_3,x_2)\gamma_{\mu_3}^T\cdots\gamma_{\mu_n}^TS_0^T(x_1,x_n)]\\
&=(-1)^n\mathrm{tr}[S_0(x_1,x_n)\gamma_{\mu_n}\cdots\gamma_{\mu_3}S_0(x_3,x_2)\gamma_{\mu_2}S_0(x_2,x_1)\gamma_{\mu_1}]\end{aligned}$$

$\mathrm{tr}[\gamma_{\mu_1}S_0(x_1,x_2)\gamma_{\mu_2}S_0(x_2,x_3)\gamma_{\mu_3}\cdots\gamma_{\mu_{2n+1}}S_0(x_{2n+1},x_1)]$和
$\mathrm{tr}[S_0(x_1,x_{2n+1})\gamma_{\mu_{2n+1}}\cdots\gamma_{\mu_3}S_0(x_3,x_2)\gamma_{\mu_2}S_0(x_2,x_1)\gamma_{\mu_1}]$

大小相等，符号相反，相互抵消！

Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

费米子场的平移

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\;\{[i\partial\!\!\!/-eA\!\!\!/(x)-M]\psi(x)+I(x)\}\;e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\left[[i\partial\!\!\!/_x-ie\gamma^\mu\frac{\delta}{\delta J^\mu(x)}-M]\frac{\delta}{\delta i\bar{I}(x)}+I(x)\right]Z[J,I,\bar{I}]=0$$

$$i(i\partial\!\!\!/_x-M)\frac{\delta Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta\bar{I}(x)}+e\gamma^\mu\frac{\delta^2 Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta J^\mu(x)\delta\bar{I}(x)}-I(x)=0$$

$$(i\partial\!\!\!/_x-M)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}-ie\gamma^\mu\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}(x)}+e\gamma^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}+I(x)=0$$

$$(i\partial\!\!\!/_x-M)_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}+\delta_{ll''}\delta(x-y)=0$$

费米子的Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\bar{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\bar{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\bar{\psi}I+i0^+\ \text{term}\}}$$

$$(i\partial\!\!\!/_x-M)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}-ie\gamma^\mu\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}(x)}+e\gamma^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}+I(x)=0$$

$$(i\partial\!\!\!/_x-M)_{ll'}\frac{\delta^2W[J,I,\bar{I}]}{\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\Big|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta I_{l''}(y)\delta\bar{I}_{l'}(x)}\Big|_{J^\mu=\bar{I}=I=0}+\delta_{ll''}\delta(x-y)=0$$

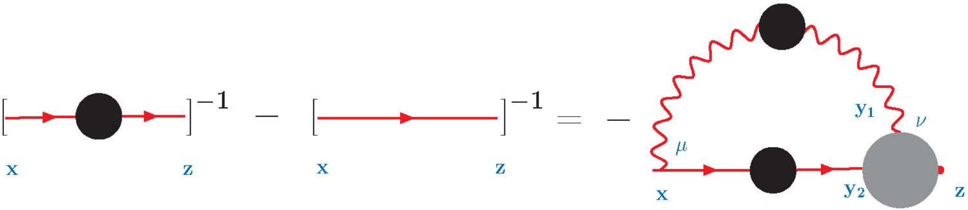
$$W_0=i\int d^4xd^4y\,\bar{I}(x)S_0(x,y)I(y)\qquad S_0(x,y)=i(i\partial\!\!\!/_x-M+i0^+)^{-1}\delta(x-y)$$

$$\int d^4z\ (i\partial\!\!\!/_x-M)_{ll'}\delta(x-z)\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(z)\delta I_{l''}(y)}\Big|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_{l'}(x)\delta I_{l''}(y)}\Big|_{J^\mu=\bar{I}=I=0}-\delta_{ll''}\delta(x-y)=0$$

$$\int d^4z\ S^{-1}_{0,ll'}(x,z)S_{l'l''}(z,y)-ie\gamma^\mu_{ll'}G_{\mu,l'l''}(x,x,y)-\delta_{ll''}\delta(x-y)=0$$

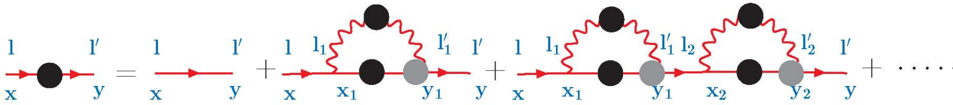
$$S^{-1}(x,z)-S_0^{-1}(x,z)=-ie\gamma^\mu\int d^4y\ G_\mu(x,x,y)S^{-1}(y,z)=e\gamma^\mu\int d^4y_1d^4y_2D_{\mu\nu}(x,y_1)S(x,y_2)\Gamma^\nu(y_1,y_2,z)$$

费米子的Schwinger-Dyson方程



$$S^{-1}(x,z) - S_0^{-1}(x,z) = e\gamma^{\mu} \int d^4y_1 d^4y_2 D_{\mu\nu}(x,y_1) S(x,y_2) \Gamma^{\nu}(y_1,y_2,z) \equiv \Sigma(x,z)$$

$$S = (S_0^{-1} + \Sigma)^{-1} = S_0(1 + \Sigma S_0)^{-1} = S_0 - S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$$



Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

光子场的平移

$$\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\{[g^{\mu\nu}\partial^2-(1-\lambda)\partial^\mu\partial^\nu]A_\nu-e\overline{\psi}\gamma^\mu\psi-J^\mu\}\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\{-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta}{\delta iJ^\nu(x)}-e\gamma^\mu_{ll'}\frac{\delta^2}{\delta I_l(x)\delta\bar{I}_{l'}(x)}-J^\mu(x)\}Z[J,I,\bar{I}]=0$$

$$i[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta J^\nu(x)}-e\gamma^\mu_{ll'}\frac{\delta^2 Z[J,I,\bar{I}]}{Z[J,I,\bar{I}]\delta I_l(x)\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta W[J,I,\bar{I}]}{\delta J^\nu(x)}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x)\delta\bar{I}_{l'}(x)}+e\gamma^\mu_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(x)\delta J^\sigma(y)}\bigg|_{J^\mu=\bar{I}=I=0}+ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\sigma(y)\delta I_l(x)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}+g^\mu_\sigma\delta(x-y)=0$$

光子的Schwinger-Dyson方程

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta W[J,I,\bar{I}]}{\delta J^\nu(x)}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x)\delta\bar{I}_{l'}(x)}+e\gamma^\mu_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(x)\delta J^\sigma(y)}\bigg|_{J^\mu=\bar{I}=I=0}+ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\sigma(y)\delta I_l(x)\delta\bar{I}_{l'}(x)}\bigg|_{J^\mu=\bar{I}=I=0}+g^\mu_\sigma\delta(x-y)=0$$

$$W_0=\frac{1}{2}\int d^4xd^4y\ J_\mu(x)iD_0^{\mu\nu}(x,y)J_\nu(y)\qquad iD_0^{-1,\mu\nu}(x,z)=[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-z)$$

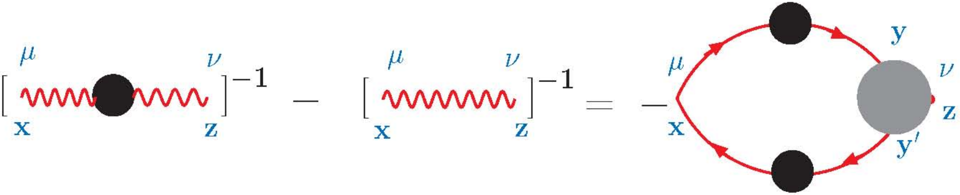
$$\int d^4z[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-z)\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(z)\delta J^\sigma(y)}\bigg|_{J^\mu=\bar{I}=I=0}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\sigma(y)\delta\bar{I}_{l'}(x)\delta I_l(x)}\bigg|_{J^\mu=\bar{I}=I=0}+g^\mu_\sigma\delta(x-y)=0$$

$$-\int d^4z\ D_0^{-1,\mu\nu}(x,z)D_{\nu\sigma}(z,y)-ie\gamma^\mu_{ll'}G_{\sigma,l'l}(y,x,x)+g^\mu_\sigma\delta(x-y)=0$$

$$D^{-1,\mu\nu}(x,z)-D_0^{-1,\mu\nu}(x,z)=ie\int d^4y\ \mathrm{tr}[\gamma^\mu G_\sigma(y,x,x)]D^{-1,\sigma\nu}(y,z)$$

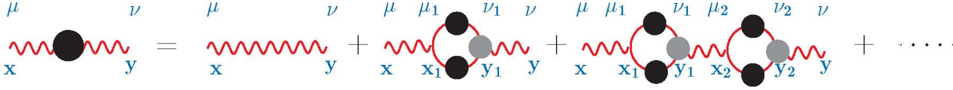
$$=-e\int d^4yd^4y'\ \mathrm{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)]$$

光子的Schwinger-Dyson方程



$$D^{-1,\mu\nu}(x,z) - D_0^{-1,\mu\nu}(x,z) = -e\int d^4y d^4y' \underbrace{\mathrm{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)]}_{\text{费米子圈和顶角产生的两负号相互抵消了！}} = \Pi^{\mu\nu}(x,z)$$

$$D = (D_0^{-1} + \Pi)^{-1} = D_0(1 + \Pi D_0)^{-1} = D_0 - D_0 \Pi D_0 + D_0 \Pi D_0 \Pi D_0 + \cdots$$



光子的Schwinger-Dyson方程

$$Z[J,I,\bar{I}] = e^{iW[J,I,\bar{I}]} = \int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi \; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}[\partial^\mu A_\mu]^2 + \overline{\psi}[i\partial\!\!\!/ - eA\!\!\!/ - M]\psi - J^\mu A_\mu + \bar{I}\psi + \overline{\psi}I + i0^+ \text{ term}\}}$$

$$D^{-1,\mu\nu}(x,z) - D_0^{-1,\mu\nu}(x,z) = -e\int d^4y d^4y' \,\text{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)] = \Pi^{\mu\nu}(x,z)$$

真空极化张量： 只考虑自能（oblique）

$$S(x,y) = \langle\psi(x)\overline{\psi}(y)\rangle \qquad \Gamma^\mu(z,y,y') = e\gamma^\mu\delta(y-z)\delta(y'-z)$$

$$\Pi^{\mu\nu}(x,z) \equiv -e\int d^4y \,\text{tr}[\gamma^\mu S(x,y)\Gamma^\nu(z,y,y')S(y',x)]$$

$$\xRightarrow{\text{oblique}} -e^2\text{tr}[\gamma^\mu\langle\psi(x)\overline{\psi}(z)\rangle\gamma^\nu\langle\psi(z)\overline{\psi}(x)\rangle] = \langle\, \overline{\psi}(x)e\gamma^\mu\psi(x)\overline{\psi}(z)e\gamma^\nu\psi(z)\,\rangle$$

$$= \langle\, J^\mu_{\text{em}}(x)J^\nu_{\text{em}}(z)\,\rangle \qquad J^\mu_{\text{em}}(x) \equiv -\overline{\psi}(x)e\gamma^\mu\psi(x)$$

光子场与费米场的联系

$$-[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\frac{\delta W[J,I,\bar{I}]}{\delta J^\nu(x)}-ie\gamma^\mu_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x)\delta\bar{I}_{l'}(x)}+e\gamma^\mu_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x)}-J^\mu(x)=0$$

$$-\int d^4z\, iD_0^{-1,\mu'\nu}(x',z)\frac{\delta W[J,I,\bar{I}]}{\delta J^\nu(z)}-ie\gamma^{\mu'}_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x')\delta\bar{I}_{l'}(x')}+e\gamma^{\mu'}_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x')}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x')}-J^{\mu'}(x')=0$$

$$W_0=\frac{1}{2}\int d^4x d^4y\, J_\mu(x)iD_0^{\mu\nu}(x,y)J_\nu(y) \qquad iD_0^{-1,\mu\nu}(x,z)=[g^{\mu\nu}\partial_x^2-(1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-z)$$

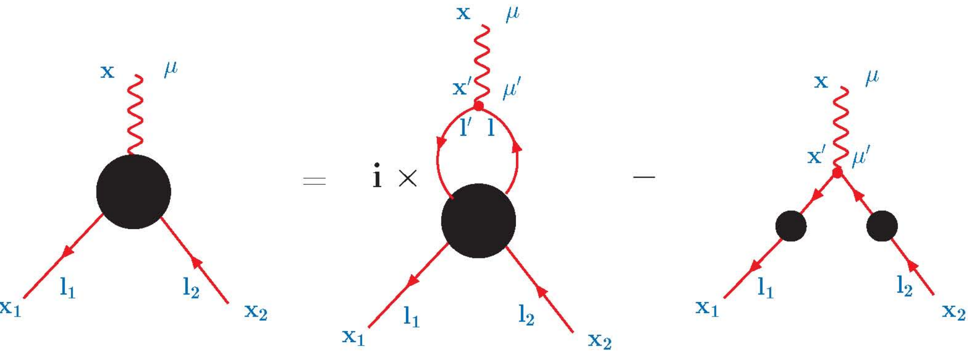
$$\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\int d^4z\, D_{0,\mu\mu'}(x,x')\left[e\gamma^{\mu'}_{ll'}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_l(x')\delta\bar{I}_{l'}(x')}+ie\gamma^{\mu'}_{ll'}\frac{\delta W[J,I,\bar{I}]}{\delta I_l(x')}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l'}(x')}-iJ^{\mu'}(x')\right]=0$$

$$\left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\nu(y)\delta J^\mu(x)}\right|_{J^\mu=I=\bar{I}=0}+e\int d^4z\, D_{0,\mu\mu'}(x,x')\gamma^{\mu'}_{ll'}\left.\frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\nu(y)\delta I_l(x')\delta\bar{I}_{l'}(x')}\right|_{J^\mu=I=\bar{I}=0}-iD_{0,\mu\nu}(x,y)=0$$

$$\frac{\delta^3 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta J^\mu(x)}+e\gamma^{\mu'}_{ll'}\int d^4z D_{0,\mu\mu'}(x,x')\left[\frac{\delta^4 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x')\delta\bar{I}_{l'}(x')}+\frac{i\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x')}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x')}\right]$$

$$\xlongequal{J^\mu=I=\bar{I}=0} 0$$

光子场与费米场的联系



$$\frac{\delta^3 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta J^{\mu}(x)} = ie\gamma^{\mu'}_{ll'}\int d^4z\, D_{0,\mu\mu'}(x,x')\left[i\frac{\delta^4 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x')\delta\bar{I}_{l'}(x')} - \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x')}\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x')}\right]$$

Ward-Takahashi-Taylor恒等式

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\text{ term}\}}$$

规范变换: $A_\mu\to A_\mu+\partial_\mu\Omega\quad \psi\to e^{-ie\Omega}\psi$

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\{-\lambda[\partial^2\Omega][\partial^\mu A_\mu]-J^\mu[\partial_\mu\Omega]+ie\Omega(\overline{\psi}I-\bar{I}\psi)\}e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\{-\lambda\partial^2\partial^\mu A_\mu+\partial_\mu J^\mu+ie(\overline{\psi}I-\bar{I}\psi)\}e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi+\cdots\}}=0$$

$$\{\lambda\partial_x^2\partial_x^\mu\frac{\delta}{i\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta}{i\delta I(x)}-\bar{I}(x)\frac{\delta}{i\delta\bar{I}(x)}]\}Z[J,I,\bar{I}]=0$$

$$\lambda\partial_x^2\partial_x^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta W[J,I,\bar{I}]}{\delta I(x)}-\bar{I}(x)\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}]=0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0}=\partial_{\nu,x}\delta(x-y)\;\Rightarrow\;\partial_x^2\partial_x^\mu\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^{\mu_1}(x_1)\cdots\delta J^{\mu_n}(x_n)}\bigg|_{J^\mu=\bar{I}=I=0}\overset{n\geq2}{=\!=\!=\!=}0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^3W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}=ie\frac{\delta^2W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

Ward-Takahashi-Taylor恒等式

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$\lambda\partial_x^2\partial_x^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta W[J,I,\bar{I}]}{\delta I(x)}-\bar{I}(x)\frac{\delta W[J,I,\bar{I}]}{\delta \bar{I}(x)}]=0$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\Big|_{J^\mu=\bar{I}=I=0}=\partial_{\nu,x}\delta(x-y)$$

$$-\lambda\partial_x^2\partial_x^\mu\frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J^\mu=\bar{I}=I=0}=ie\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

光子场纵向分量无量子修正: $D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)$

$$\partial_{\mu,x}iD^{\mu\nu}(x,y)=-\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y)=\partial_{\mu,x}iD_0^{\mu\nu}(x,y)$$

相互作用顶角:

$$-\lambda\int d^4x'd^4y'd^4z'\partial_x^2\partial_x^\mu D_{\mu\nu}(x,x')S(y,y')i\Gamma^\nu(x',y',z')S(z',z)=eS(y,z)[\delta(x-z)-\delta(x-y)]$$

$$\partial_{\mu,x}\Gamma^\mu(x,y,z)=eS^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$$

Ward-Takahashi-Taylor恒等式

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int \mathcal{D}A_\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/-M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\ \text{term}\}}$$

$$\lambda\partial_x^2\partial_x^\mu\frac{\delta W[J,I,\bar{I}]}{\delta J^\mu(x)}+\partial_x^\mu J_\mu(x)+ie[I(x)\frac{\delta W[J,I,\bar{I}]}{\delta I(x)}-\bar{I}\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}(x)}]=0$$

光子场纵向分量无量子修正: $D_0^{\mu\nu}(x,y)=\frac{i}{\partial_x^2-i0+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]\delta(x-y)$

$$\partial_{\mu,x}iD^{\mu\nu}(x,y)=-\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y)=\partial_{\mu,x}iD_0^{\mu\nu}(x,y)$$

相互作用顶角: $\partial_{\mu,x}\Gamma^\mu(x,y,z)=eS^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$

$\Gamma_0^\mu(x,y,z)=e\gamma^\mu\delta(y-x)\delta(x-z)$ $\qquad iS_0^{-1}(y,z)=(i\partial\!\!\!/_y-M+i0^+)\delta(y-z)$

$$eS_0^{-1}(y,z)[\delta(y-x)-\delta(z-x)]=-ie[\delta(y-x)-\delta(z-x)](i\partial\!\!\!/_y-M+i0^+)\delta(y-z)$$
$$=e[\delta(y-x)-\delta(z-x)]\partial\!\!\!/_y\delta(y-z)=-e\partial\!\!\!/_z\delta(y-x)\delta(y-z)-e\partial\!\!\!/_y\delta(z-x)\delta(y-z)$$
$$=-e\delta(y-x)\partial\!\!\!/_z\delta(x-z)-e\delta(z-x)\partial\!\!\!/_y\delta(y-x)=e\delta(y-x)\partial\!\!\!/_x\delta(x-z)+e\delta(z-x)\partial\!\!\!/_x\delta(y-x)$$
$$=e\partial\!\!\!/_x[\delta(y-x)\delta(x-z)]=\partial_{\mu,x}\Gamma_0^\mu(x,y,z)$$

三点顶角Ward-Takahashi-Taylor恒等式的变形表达

光子场纵向分量无量子修正: $D_0^{\mu\nu}(x,y) = \frac{i}{\partial_x^2 - i0+}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_x^\mu \partial_x^\nu}{\partial_x^2}]\delta(x-y)$

$$\partial_{\mu,x} iD^{\mu\nu}(x,y) = -\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y) = \partial_{\mu,x} iD_0^{\mu\nu}(x,y)$$

$$\frac{-\delta W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta J^\mu(x)} + e\gamma_{ll'}^{\mu'}\int d^4z D^{-1}_{0,\mu\mu'}(x,x')\left[\frac{i\delta^2 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x')\delta\bar{I}_{l'}(x')} + \frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x')}\frac{\delta W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x')}\right]$$

$$\overset{J^\mu=I=\bar{I}=0}{=\!=\!=\!=\!=\!=\!=} 0$$

$$-\lambda\partial_x^2\partial_x^\mu \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0} = ie\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

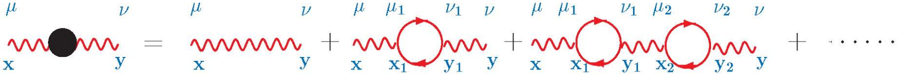
$$e\gamma_{ll'}^\mu\partial_\mu\left[-\frac{\delta^2 W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l_2}(x_2)\delta I_l(x)\delta\bar{I}_{l'}(x)} + i\frac{\delta W[J,I,\bar{I}]}{\delta\bar{I}_{l_2}(x_2)\delta I_l(x)}\frac{\delta W[J,I,\bar{I}]}{\delta I_{l_1}(x_1)\delta\bar{I}_{l'}(x)}\right]_{J^\mu=\bar{I}=I=0}$$

$$= e\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0}[\delta(x-z)-\delta(x-y)]$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y)$$

$$= i\frac{e^{\int d^4y d^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ A_\mu(x)A_\nu(y)\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}\Bigg|_{A_\mu=\overline{\psi}=\psi=0}$$



$$= iD_{0,\mu\nu}(x,y) + i\int d^4x_1 d^4y_1 D_{0,\mu\mu_1}(x,x_1)(-)\Pi^{\mu_1\nu_1}(x_1,y_1)D_{0,\nu_1\nu}(y_1,y)$$

$$+i\int d^4x_1 d^4y_1 d^4x_2 d^4y_2 D_{0,\mu\mu_1}(x,x_1)\Pi^{\mu_1\nu_1}(x_1,y_1)D_{0,\nu_1\mu_2}(y_1,x_2)\Pi^{\mu_2\nu_2}(x_2,y_2)D_{0,\nu_2\nu}(y_2,y) + \cdots$$

$$= i[D_0(1 - \Pi D_0 + \Pi D_0\Pi D_0 + \cdots)]_{\mu\nu}(x,y) = i[D_0(1+\Pi D_0)^{-1}]_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

SD方程： $D^{-1} - D_0^{-1} = \Pi$

$$\Pi^{\mu\nu}(x,y) = (-1)\mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$= -e^2\mathrm{tr}[\gamma^\mu(i\partial\!\!\!/_x - M + i0^+)^{-1}\delta(x-y)\gamma^\nu(i\partial\!\!\!/_y - M + i0^+)\delta(y-x)] + O(e^4)$$

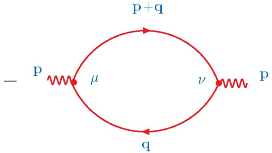
光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\right|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$= -e^2\mathrm{tr}[\gamma^\mu(i\partial\!\!\!/_x - M + i0^+)^{-1}\delta(x-y)\gamma^\nu(i\partial\!\!\!/_y - M + i0^+)\delta(y-x)] + O(e^4)$$

$$= -e^2\int \frac{d^4pd^4q}{(2\pi)^8} e^{-i(p-q)\cdot(x-y)}\mathrm{tr}[\gamma^\mu(p\!\!\!/ - M + i0^+)^{-1}\gamma^\nu(q\!\!\!/ - M + i0^+)] + O(e^4)$$



$$\Pi^{\mu\nu}(p) = -e^2\int \frac{d^4q}{(2\pi)^4}\ \mathrm{tr}[\gamma^\mu(p\!\!\!/ + q\!\!\!/ - M + i0^+)^{-1}\gamma^\nu(q\!\!\!/ - M + i0^+)] + O(e^4)$$

$$= -e^2\int \frac{d^4q}{(2\pi)^4}\ \mathrm{tr}[\gamma^\mu \frac{p\!\!\!/ + q\!\!\!/ + M}{(p+q)^2 - M^2 + i0^+}\gamma^\nu \frac{q\!\!\!/ + M}{q^2 - M^2 + i0^+}] + O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\Big|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = -e^2\int\frac{d^4q}{(2\pi)^4}\,\mathrm{tr}[\gamma^\mu\frac{\not p+\not q+M}{(p+q)^2-M^2+i0^+}\gamma^\nu\frac{\not q+M}{q^2-M^2+i0^+}] + O(e^4)$$

$$\frac{1}{AB} = \int_0^1\frac{dx}{[(1-x)A+xB]^2} \qquad \mathrm{tr}\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^\rho = 4(g^{\mu\sigma}g^{\nu\rho}-g^{\mu\nu}g^{\sigma\rho}+g^{\mu\rho}g^{\nu\sigma})$$

$$\frac{1}{[(p+q)^2-M^2+i0^+](q^2-M^2+i0^+)} = \int_0^1\frac{dx}{\{x[(p+q)^2-M^2+i0^+]+(1-x)(q^2-M^2+i0^+)\}^2}$$

$$= \int_0^1\frac{dx}{[(q+xp)^2-M^2+i0^+ + p^2x(1-x)]^2}$$

$$\mathrm{tr}[\gamma^\mu[\not p+\not q+M]\gamma^\nu(\not q+M)] = 4[(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu - g^{\mu\nu}(p+q)\cdot q+M^2g^{\mu\nu}]$$

$$\Pi^{\mu\nu}(p) = -4e^2\int_0^1 dx\int\frac{d^4q}{(2\pi)^4}\frac{(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu-g^{\mu\nu}(p+q)\cdot q+M^2g^{\mu\nu}}{[(q+xp)^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

光子传播子

$$i(\Psi_0^-,\mathbf{T}\,A_\mu(x)A_\nu(y)\;\Psi_0^+)_C=\left.\frac{\delta^2W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\right|_{J^\mu=\bar{I}=I=0}=D^{-1}_{\mu\nu}(x,y)=[i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y)=\mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)]+O(e^4)=\int\frac{d^4p}{(2\pi)^4}\;e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p)=-4e^2\int_0^1dx\int\frac{d^4q}{(2\pi)^4}\frac{(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu-g^{\mu\nu}(p+q)\cdot q+M^2g^{\mu\nu}}{[(q+xp)^2-M^2+i0^++p^2x(1-x)]^2}+O(e^4)$$

$$\overset{\text{维数正规化}}{=\!=\!=\!=\!=}-4e^2\int_0^1dx\;\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{(p^\mu+q^\mu)q^\nu+(p^\nu+q^\nu)q^\mu-g^{\mu\nu}(p+q)\cdot q+M^2g^{\mu\nu}}{[(q+xp)^2-M^2+i0^++p^2x(1-x)]^2}+O(e^4)$$

$$\overset{q\to q-xp}{=\!=\!=\!=\!=}-4e^2\int_0^1dx\;\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\;[q^2-M^2+i0^++p^2x(1-x)]^{-2}\{[(1-x)p^\mu+q^\mu](q^\nu-xp^\nu)$$

讨论见后！ $$+[(1-x)p^\nu+q^\nu](q^\mu-xp^\mu)-g^{\mu\nu}[p(1-x)+q]\cdot(q-xp)+M^2g^{\mu\nu}\}+O(e^4)$$

$$=-4e^2\int_0^1dx\;\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{2[(x-1)xp^\mu p^\nu+q^\mu q^\nu]-g^{\mu\nu}[p^2(x-1)x+q^2]+M^2g^{\mu\nu}}{[q^2-M^2+i0^++p^2x(1-x)]^2}+O(e^4)$$

$$\overset{q^\mu q^\nu\to\frac{g^{\mu\nu}}{D}q^2}{=\!=\!=\!=\!=\!=}4e^2\int_0^1dx\;\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu-p^2g^{\mu\nu})+(1-\frac{2}{D})g^{\mu\nu}q^2-M^2g^{\mu\nu}}{[q^2-M^2+i0^++p^2x(1-x)]^2}+O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\right|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = 4e^2\int_0^1 dx\ \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[q^2-M^2+i0^+ + p^2x(1-x)]^2} + O(e^4)$$

Wick转动： $q^0 = iq_E^D \quad \vec{q} = \vec{q}_E$

$$\Pi^{\mu\nu}(p) = 4e^2\int_0^1 dx\mu^{4-D}\int \frac{d\vec{q}}{(2\pi)^D}\int_{-\infty}^{\infty} dq^0\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[(q^0)^2 - \vec{q}\cdot\vec{q} - M^2 + i0^+ + p^2x(1-x)]^2}$$

$$= -4e^2\int_0^1 dx\mu^{4-D}\int \frac{d\vec{q}}{(2\pi)^D}\int_{i\infty}^{-i\infty} dq^0\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) + (1-\frac{2}{D})g^{\mu\nu}q^2 - M^2g^{\mu\nu}}{[(q^0)^2 - \vec{q}\cdot\vec{q} - M^2 + i0^+ + p^2x(1-x)]^2}$$

$$= 4ie^2\int_0^1 dx\mu^{4-D}\int \frac{d\vec{q}_E}{(2\pi)^D}\int_{-\infty}^{\infty} dq_E^D\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2g^{\mu\nu}}{[-q_E^2 - M^2 + i0^+ + p^2x(1-x)]^2}$$

$$= 4ie^2\int_0^1 dx\ \mu^{4-D}\int \frac{d^Dq_E}{(2\pi)^D}\ \frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2g^{\mu\nu}}{[q_E^2 + M^2 - i0^+ - p^2x(1-x)]^2} + O(e^4)$$

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\right|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = 4ie^2\int_0^1 dx\ \mu^{4-D}\int\frac{d^Dq_E}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2g^{\mu\nu}}{[q_E^2+M^2-i0^+ - p^2x(1-x)]^2} + O(e^4)$$

$$\int d^Dq_E = \Omega_D\int_0^\infty \kappa^{D-1}d\kappa \qquad \int_0^1 dx\ x^{\alpha-1}(1-x)^{\beta-1} \equiv B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$(\sqrt{\pi})^D = [\int_{-\infty}^{\infty} dx e^{-x^2}]^D = \int d^Dq_E\ e^{-q_E^2} = \Omega_D\int_0^\infty d\kappa\ \kappa^{D-1}e^{-\kappa^2} = \frac{\Omega_D}{2}\int_0^\infty d\kappa^2\ (\kappa^2)^{\frac{D}{2}-1}e^{-\kappa^2} = \frac{1}{2}\Omega_D\Gamma(D/2)$$

$$\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}\int_0^\infty d\kappa^2\frac{(\kappa^2)^{\frac{D}{2}-1}}{(\kappa^2+\nu^2)^2} \overset{x=\frac{\nu^2}{\kappa^2+\nu^2}}{=\!=\!=\!=\!=} \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\int_0^1 dx\ x^{1-\frac{D}{2}}(1-x)^{\frac{D}{2}-1} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\int_0^\infty d\kappa\frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}\int_0^\infty d\kappa^2\frac{(\kappa^2)^{\frac{D}{2}}}{(\kappa^2+\nu^2)^2} \overset{x=\frac{\nu^2}{\kappa^2+\nu^2}}{=\!=\!=\!=\!=} \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\int_0^1 dx\ x^{-\frac{D}{2}}(1-x)^{\frac{D}{2}} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})$$

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\right|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = 4ie^2\int_0^1 dx\ \mu^{4-D}\int \frac{d^Dq_E}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}q_E^2 - M^2 g^{\mu\nu}}{[q_E^2 + M^2 - i0^+ - p^2x(1-x)]^2} + O(e^4)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})$$

$$\Pi^{\mu\nu}(p) = 4ie^2\int_0^1 dx\ \Omega_D\mu^{4-D}\int_0^\infty \frac{\kappa^{D-1}d\kappa}{(2\pi)^D}\frac{(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - (1-\frac{2}{D})g^{\mu\nu}\kappa^2 - M^2 g^{\mu\nu}}{[\kappa^2 + M^2 - i0^+ - p^2x(1-x)]^2}$$

$$\overset{\Omega_D=\frac{2\pi^{\frac{D}{2}}}{\Gamma(D/2)}}{=\!=\!=\!=\!=\!=} \frac{4ie^2\pi^{\frac{D}{2}}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\int_0^1 dx\{[(1-x)x(2p^\mu p^\nu - p^2g^{\mu\nu}) - M^2g^{\mu\nu}][M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2}$$
$$\times\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2}) - g^{\mu\nu}(1-\frac{2}{D})[M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})\}$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = \frac{4ie^2\pi^{D/2}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\int_0^1 dx\ \{[(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - M^2 g^{\mu\nu}][M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2}$$
$$\times\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2}) - g^{\mu\nu}(1-\frac{2}{D})[M^2-i0^+-p^2x(1-x)]^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})\} + O(e^4)$$

$$(1-\tfrac{2}{D})\Gamma(1+\tfrac{D}{2})\Gamma(1-\tfrac{D}{2}) = -\Gamma(\tfrac{D}{2})\Gamma(2-\tfrac{D}{2})$$

$$= \frac{4ie^2\pi^{D/2}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})\int_0^1 dx\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2}$$
$$\times\{[(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - M^2 g^{\mu\nu}] + g^{\mu\nu}[M^2 - i0^+ - p^2x(1-x)]\} + O(e^4)$$
$$= (p^2 g^{\mu\nu} - p^\mu p^\nu)\frac{(-i)e^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\ (1-x)x\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2} + O(e^4)$$

光子传播子

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(D_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$\Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\frac{(-i)e^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\ (1-x)x\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2} + O(e^4)$$

$$\Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\pi(p^2) \qquad p_\mu\Pi^{\mu\nu}(p) = 0$$

$$\pi(p^2) = -\frac{ie^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\ (1-x)x\ [M^2 - i0^+ - p^2x(1-x)]^{\frac{D}{2}-2} + O(e^4)$$

$$\Pi^{\mu\nu}(x,y) = (-g^{\mu\nu}\partial_x^2 + \partial_x^\mu\partial_x^\nu)\pi(-\partial_x^2)\delta(x-y) \qquad iD_0^{-1,\mu\nu}(x,y) = [g^{\mu\nu}\partial_x^2 - (1-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-y)$$

$$(D_0^{-1}+\Pi)^{\mu\nu}(x,y) = -i\{(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu)[1 - i\pi(-\partial_x^2)] + \lambda\partial_x^\mu\partial_x^\nu\}\delta(x-y)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = D_{\mu\nu}(x,y) = \{\frac{i}{\partial_x^2[1-i\pi(-\partial_x^2)]}[g^{\mu\nu} - \frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}] + \frac{1}{\lambda}\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^4}\}\delta(x-y)$$

$$(\Psi_0^-, \mathbf{T}A_\mu(x)A_\nu(y)\Psi_0^+)_C = -i\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\Big|_{J^\mu=\bar{I}=I=0} = \{\frac{i}{\partial_x^2[1-i\pi(-\partial_x^2)]}[g^{\mu\nu}-\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}]+\frac{1}{\lambda}\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^4}\}\delta(x-y)$$

$$\pi(p^2) = -\frac{ie^2\mu^{4-D}}{2^{D-3}\pi^{D/2}}\Gamma(2-\frac{D}{2})\int_0^1 dx\,(1-x)x\,[M^2-i0^+-p^2x(1-x)]^{\frac{D}{2}-2}+O(e^4)$$

- $\Gamma(2-\frac{D}{2}) \xRightarrow{D\to 4} \frac{1}{2-D/2}-\gamma+O(2-D/2) \qquad \gamma = 0.5772157$

  $\pi(p^2)$在$1+3$维时空产生发散!
- $\pi(p^2)$无极点 $\Rightarrow$ 辐射修正不影响光子质量 1+1维：$M=0$使光子获得质量！
- 辐射修正使光子的波函数前多了一个因子 $\frac{1}{1+i\pi(0)}$

$$A^{\frac{D}{2}-2} = e^{(\frac{D}{2}-2)\ln A} \xRightarrow{D\to 4} 1+(\frac{D}{2}-2)\ln A+O((\frac{D}{2}-2)^2)$$

$$\pi(p^2) = -\frac{ie^2\mu^{4-D}}{2\pi^2}[\frac{1}{2-D/2}-\gamma+\cdots]\int_0^1 dx\,(1-x)x\left[\frac{[M^2-i0^+-p^2x(1-x)]}{4\pi}\right]^{\frac{D}{2}-2}+O(e^4)$$

$$= -\frac{ie^2}{12\pi^2}\left[\frac{1}{2-D/2}-\gamma-6\int_0^1 dx\,(1-x)x\,\ln\frac{[M^2-i0^+-p^2x(1-x)]}{4\pi\mu^2}\right]+O(e^4)$$

常用的正规化

维数正规化 1999年诺贝尔物理奖

$$\int \frac{d^4q}{(2\pi)^4} \Rightarrow \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}$$ 协变, 但只停留在圈积分的层次, 若扩充到γ矩阵, γ_5有问题

维数正规化中无幂次发散!

$$\int d^Dq_E = \Omega_D \int_0^\infty \kappa^{D-1} d\kappa \qquad \int_0^1 dx\, x^{\alpha-1}(1-x)^{\beta-1} \equiv B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\int_0^\infty d\kappa^2 \frac{(\kappa^2)^{\frac{D}{2}-1+n}}{\kappa^2+\nu^2} \overset{x=\frac{\nu^2}{\kappa^2+\nu^2}}{=\!=\!=\!=\!=} -(\nu^2)^{\frac{D}{2}-1+n}\int_1^0 dx\, x^{-\frac{D}{2}-n}(1-x)^{\frac{D}{2}-1+n} = (\nu^2)^{\frac{D}{2}-1+n}\Gamma(\frac{D}{2}+n)\Gamma(1-\frac{D}{2}-n)$$

$$d\kappa^2 = -\frac{\nu^2}{x^2}dx \qquad \frac{1}{\kappa^2+\nu^2} = \frac{x}{\nu^2} \qquad \kappa^2 = \nu^2 x^{-1}(1-x)$$

$$\int_0^\infty d\kappa^2\, (\kappa^2)^{\frac{D}{2}-2+n} = \lim_{\nu\to 0}\int_0^\infty d\kappa^2 \frac{(\kappa^2)^{\frac{D}{2}-1+n}}{\kappa^2+\nu^2} = \lim_{\nu\to 0}(\nu^2)^{\frac{D}{2}-1+n}\Gamma(\frac{D}{2}+n)\Gamma(1-\frac{D}{2}-n) \overset{\frac{D}{2}-2+n>-1}{=\!=\!=\!=\!=\!=} 0$$

常用的正规化

维数正规化 1999年诺贝尔物理奖

$$\int \frac{d^4q}{(2\pi)^4} \Rightarrow \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}$$ 协变, 但只停留在圈积分的层次, 若扩充到γ矩阵, γ_5有问题

动量截断正规化

$$\int \frac{d^4q_E}{(2\pi)^4} \Rightarrow \frac{\Omega_4}{(2\pi)^4}\int_0^\Lambda dq_E\, q_E^3$$ 不协变

Pauli-Villars正规化

$$\frac{1}{(p+q)^2-M^2+i0^+} \Rightarrow \frac{1}{(p+q)^2-M^2+i0^+} - \frac{1}{(p+q)^2-\Lambda^2+i0^+}$$

维数正规化与动量截断的互换: $\frac{1}{2-D/2}-\gamma+\ln 4\pi\mu^2 \Leftrightarrow \ln\Lambda^2$

$$\frac{2\pi^{D/2}\mu^{4-D}}{(2\pi)^D\Gamma(D/2)}\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{16\pi^2}\left(\frac{\nu^2}{4\pi\mu^2}\right)^{\frac{D}{2}-2}\Gamma(2-\frac{D}{2}) = \frac{1}{16\pi^2}\left[\frac{1}{2-D/2}-\gamma+\ln\frac{4\pi\mu^2}{\nu^2}\right]$$

$$\frac{2\pi^2}{(2\pi)^4}\int_0^\Lambda d\kappa\frac{\kappa^3}{(\kappa^2+\nu^2)^2} = \frac{1}{16\pi^2}\int_0^{\Lambda^2} d\kappa^2\frac{\kappa^2}{(\kappa^2+\nu^2)^2} = \frac{1}{16\pi^2}[\ln\frac{\Lambda^2}{\nu^2}+1]$$

关于圈动量积分中的动量平移

- 收敛积分和对数发散积分可以进行动量平移
- 线性发散和二次发散积分动量平移后相差一有限项
- 三次以上发散动量平移后相差低至少二阶的偶次发散项

对数发散： $\frac{1}{A^2B^2} = 6\int_0^1 dx \frac{x(1-x)}{[xA+(1-x)B]^4}$

$$\int d^4q\left[\frac{1}{[(q+p)^2-m^2]^2}-\frac{1}{[q^2-m^2]^2}\right]=\int d^4q\frac{[q^2-m^2]^2-[(q+p)^2-m^2]^2}{[(q+p)^2-m^2]^2[q^2-m^2]^2}$$

$$=6\int d^4q\int_0^1 dx\, x(1-x)\frac{-2(q^2-m^2)(p^2+2q\cdot p)-(p^2+2q\cdot p)^2}{[(xp+q)^2-m^2+p^2x(1-x)]^4}$$

$$=6\int d^4q\int_0^1 dx\, x(1-x)\frac{-2[(q-xp)^2-m^2][p^2+2(q-xp)\cdot p]-[p^2+2(q-xp)\cdot p]^2}{[q^2-m^2+p^2x(1-x)]^4}$$

$\overset{x\leftrightarrow 1-x}{=\!=\!=\!=\!=\!=} 0$ 其中利用了 $x(1-x) \overset{x\leftrightarrow 1-x}{=\!=\!=\!=\!=\!=} x(1-x)$ $\quad 2x-1 \overset{x\leftrightarrow 1-x}{=\!=\!=\!=\!=\!=} 1-2x$

被积函数分子 $= -2q^2(1-2x)p^2+(8x-4)(q\cdot p)^2-2(x^2p^2-m^2)(1-2x)p^2-p^4(1-2x)^2$

$\sim [-2(1-2x)x^2-(1-2x)^2]p^4=(2x-1)(2x^2-2x+1)p^4=(2x-1)[2x(x-1)-1]p^4$

线性发散： $\frac{1}{A^2B^2} = 6\int_0^1 dx \frac{x(1-x)}{[xA+(1-x)B]^4}$

$$\int d^4q\left[\frac{q\cdot p}{[(q+p)^2-m^2]^2} - \frac{(q-p)\cdot p}{[q^2-m^2]^2}\right] = \int d^4q\left[\frac{q\cdot p\{[q^2-m^2]^2-[(q+p)^2-m^2]^2\}}{[(q+p)^2-m^2]^2[q^2-m^2]^2} + \frac{p^2}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx \frac{6x(1-x)q\cdot p[-2(q^2-m^2)(p^2+2q\cdot p)-(p^2+2q\cdot p)^2]}{[(xp+q)^2-m^2+p^2x(1-x)]^4} + \frac{p^2}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx \frac{6x(1-x)(q-xp)\cdot p\{-2[(q-xp)^2-m^2][p^2+2(q-xp)\cdot p]-[p^2+2(q-xp)\cdot p]^2\}}{[q^2-m^2+p^2x(1-x)]^4} + \right.$$

被积函数分子 $= 6x(1-x)q\cdot p[-4q^2 q\cdot p + 4q\cdot p(1-2x)p^2 - 4(x^2p^2-m^2)q\cdot p - 4q\cdot p(1-2x)p^2]$
$-6(1-x)x^2p^2\{-2q^2(1-2x)p^2+(8x-4)(q\cdot p)^2-2(x^2p^2-m^2)(1-2x)p^2-p^4(1-2x)^2\}$

$\sim 6x(1-x)[-q^4p^2-(x^2p^2-m^2)q^2p^2]-6(1-x)x^2p^2\{3q^2p^2-2m^2p^2-p^4[x^2(1-2x)+(1-2x)^2]\}$

对数发散项 $= \int d^4q\left[\int_0^1 dx \frac{-6x(1-x)[q^2-m^2+p^2x(1-x)+m^2-p^2x(1-x)]^2p^2}{[q^2-m^2+p^2x(1-x)]^4} + \frac{p^2}{[q^2-m^2]^2}\right]$

$$\xRightarrow{\text{准到发散项}} \int d^4q\left[\int_0^1 dx \frac{-p^2}{[q^2-m^2+p^2x(1-x)]^2} + \frac{p^2}{[q^2-m^2]^2}\right] = 0$$

二次发散： $\frac{1}{A^2B^2} = 6\int_0^1 dx \frac{x(1-x)}{[xA+(1-x)B]^4}$

$$\int d^4q\left[\frac{q^\mu q^\nu}{[(q+p)^2-m^2]^2} - \frac{(q^\mu-p^\mu)(q^\nu-p^\nu)}{[q^2-m^2]^2}\right] = \int d^4q\left[\frac{q^\mu q^\nu}{[(q+p)^2-m^2]^2} - \frac{q^\mu q^\nu + p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx\, 6x(1-x)\frac{q^\mu q^\nu[-2(q^2-m^2)(p^2+2q\cdot p)-(p^2+2q\cdot p)^2]}{[(xp+q)^2-m^2+p^2x(1-x)]^4} - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx\, \frac{\{-2[(q-xp)^2-m^2][p^2+2(q-xp)\cdot p]-[p^2+2(q-xp)\cdot p]^2\}}{[q^2-m^2+p^2x(1-x)]^4}\right.$$
$$\left.\times 6x(1-x)(q^\mu q^\nu - xq^\mu p^\nu - xp^\mu q^\nu + x^2p^\mu p^\nu) - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$\xrightarrow{\text{准到发散项}} \int d^4q\left[\int_0^1 dx\, \frac{24q^2 q\cdot px^2(1-x)(q^\mu p^\nu + p^\mu q^\nu)}{[q^2-m^2+p^2x(1-x)]^4} - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= \int d^4q\left[\int_0^1 dx\, \frac{12q^4x^2(1-x)p^\mu p^\nu}{[q^2-m^2+p^2x(1-x)]^4} - \frac{p^\mu p^\nu}{[q^2-m^2]^2}\right]$$

$$= p^\mu p^\nu \int d^4q\left[\int_0^1 dx\, \frac{6q^4x(1-x)}{[q^2-m^2+p^2x(1-x)]^4} - \frac{1}{[q^2-m^2]^2}\right] \xrightarrow{\text{准到发散项}} 0$$

二次发散与规范对称性

$$\Pi^{\mu\nu}(p) = -2ie^2 \int_0^1 dx\, \Omega_4 \int_0^{\Lambda^2} \frac{\kappa^2 d\kappa^2}{(2\pi)^4} \frac{(1-x)x(2p^\mu p^\nu - p^2 g^{\mu\nu}) - \frac{1}{2} g^{\mu\nu}\kappa^2 - M^2 g^{\mu\nu}}{[\kappa^2 + M^2 - i0^+ - p^2 x(1-x)]^2}$$

$$= -\frac{2ie^2\Omega_4}{(2\pi)^4}\int_0^1 dx \Big[-\frac{1}{2} g^{\mu\nu}\Lambda^2 + 2(1-x)x(p^\mu p^\nu - p^2 g^{\mu\nu}) \ln[\frac{\Lambda^2}{M^2 - p^2 x(1-x)} + 1]$$

$$+ [\frac{3}{2} M^2 g^{\mu\nu} - (1-x)x(2p^\mu p^\nu - \frac{1}{2} p^2 g^{\mu\nu}) \frac{\Lambda^2}{\Lambda^2 + M^2 - p^2 x(1-x)}]\Big]$$

Ward-Takahashi-Taylor恒等式: $\partial_{\mu,x} D^{\mu\nu}(x,y) = \partial_{\mu,x} D_0^{\mu\nu}(x,y)$

$$D_0^{\mu\nu}(x,y) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip\cdot(x-y)} D_0^{\mu\nu}(p) \qquad D_0^{\mu\nu}(p) = \frac{-i}{p^2}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{p^\mu p^\nu}{p^2}]$$

$$D^{\mu\nu}(p) = [D_0^{-1}(p) + \Pi(p)]^{-1\mu\nu}(p) \qquad \Pi^{\mu\nu}(p) = p^2 g^{\mu\nu}\pi(p^2) - p^\mu p^\nu \pi_1(p^2)$$

$$iD^{-1,\mu\nu}(p) = -p^2[1 - i\pi(p^2)]g^{\mu\nu} + [1 - \lambda - i\pi_1(p^2)]p^\mu p^\nu \overset{\pi=\pi_1=0}{=\!=\!=\!=\!=\!=} iD_0^{-1,\mu\nu}(p)$$

$$D^{\mu\nu}(p) = \frac{-i}{p^2[1 - i\pi(p^2)]}\left[g^{\mu\nu} + \frac{p^\mu p^\nu[1 - \lambda - i\pi_1(p^2)]}{p^2[\lambda - i\pi(p^2) + i\pi_1(p^2)]}\right]$$

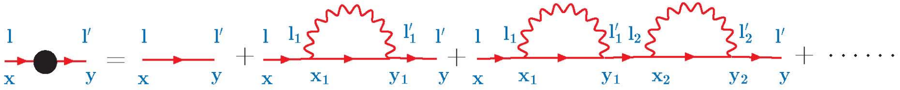
规范不变性禁戒二次发散!

$$p_\mu D^{\mu\nu}(p) = p_\mu D_0^{\mu\nu}(p) = -i\frac{p^\nu}{\lambda p^2} \Rightarrow \pi_1(p^2) = \pi(p^2) \qquad \Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\pi(p^2)$$

费米子传播子

$$-i(\Psi_0^-, \mathbf{T}\,\psi_l(x)\overline{\psi}_{l'}(y)\;\Psi_0^+)_C = \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\right|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y)$$

$$= -i\left.\frac{e^{\int d^4y d^4z\,[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\;\psi_l(x)\overline{\psi}_{l'}(y)\;e^{-i\int d^4x\, eA_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\,[\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\;e^{-i\int d^4x\, eA_\mu\overline{\psi}\gamma^\mu\psi}}\right|_{A_\mu=\overline{\psi}=\psi=0}$$



$$= -iS_{0,ll'}(x,y) - i\int d^4x_1 d^4y_1 S_{0,ll_1}(x,x_1)(-1)\Sigma_{l_1l_1'}(x_1,y_1)S_{0,l_1'l}(y_1,y)$$

$$-i\int d^4x_1 d^4y_1 d^4x_2 d^4y_2 S_{0,ll_1}(x,x_1)\Sigma_{l_1l_1'}(x_1,y_1)S_{0,l_1'l_2}(y_1,x_2)\Sigma_{l_2l_2'}(x_2,y_2)S_{0,l_2'l'}(y_2,y) + \cdots$$

$$= -i[S_0(1-\Sigma S_0+\Sigma S_0\Sigma S_0+\cdots)]_{ll'}(x,y) = -i[S_0(1+\Sigma S_0)^{-1}]_{ll'}(x,y) = -i[(S_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

SD方程： $S^{-1} - S_0^{-1} = \Sigma$

$$\Sigma_{ll'}(x,y) = [(-i)e\gamma^\mu S_0(x,y)D_{0,\mu\nu}(x,y)(-i)e\gamma^\nu]_{ll'} + O(e^4) = \int\frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$= e^2\{\gamma^\mu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-y)\gamma^\nu\frac{1}{\partial_y^2 - i0^+}[g_{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_{\mu,y}\partial_{\nu,y}}{\partial_y^2}]\}_{ll'}\delta(y-x) + O(e^4)$$

费米子传播子

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\Big|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y) = -i[(S_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

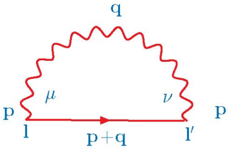
$$\Sigma_{ll'}(x,y) = [(-i)e\gamma^\mu S_0(x,y)D_{0,\mu\nu}(x,y)(-i)e\gamma^\nu]_{ll'} + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$= e^2\gamma^\mu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-y)\gamma^\nu\frac{1}{\partial_y^2 - i0^+}[g^{\mu\nu} - (1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-x) + O(e^4)$$

$$= e^2\int\frac{d^4pd^4q}{(2\pi)^8}e^{-i(p-q)\cdot(x-y)}\{\gamma^\mu\frac{1}{p\!\!\!/-M-i0^+}\gamma^\nu\frac{-1}{q^2+i0^+}[g_{\mu\nu}-(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}]\}_{ll'}+O(e^4)$$

费曼规范: $\lambda = 1$ $\qquad \frac{1}{AB} = \int_0^1 \frac{dx}{[(1-x)A+xB]^2}$

$$\Sigma(p) = -e^2\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\frac{p\!\!\!/+q\!\!\!/+M}{(p+q)^2-M^2+i0^+}\gamma_\mu\frac{1}{q^2+i0^+} + O(e^4)$$



$$= -e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{\gamma^\mu(p\!\!\!/+q\!\!\!/+M)\gamma_\mu}{\{x[(p+q)^2-M^2+i0^+]+(1-x)(q^2+i0^+)\}^2} + O(e^4)$$

$$= 2e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{p\!\!\!/+q\!\!\!/-2M}{\{x[(p+q)^2-M^2+i0^+]+(1-x)(q^2+i0^+)\}^2} + O(e^4)$$

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\Big|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y) = -i[(S_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

$$\Sigma_{ll'}(x,y) = [(-i)e\gamma^\mu S_0(x,y)D_0(x,y)(-i)e\gamma^\nu]_{ll'} + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$\Sigma(p) = 2e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{\not{p}+\not{q}-2M}{\{x[(p+q)^2-M^2+i0^+]+(1-x)(q^2+i0^+)\}^2} + O(e^4)$$

$$= 2e^2\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{\not{p}+\not{q}-2M}{[(q+xp)^2+x(1-x)p^2-xM^2+i0^+]^2} + O(e^4)$$

$$= 2e^2\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\int_0^1 dx\frac{\not{p}+\not{q}-2M}{[(q+xp)^2+x(1-x)p^2-xM^2+i0^+]^2} + O(e^4)$$

$$= 2e^2\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\int_0^1 dx\frac{(1-x)\not{p}+\not{q}-2M}{[q^2+x(1-x)p^2-xM^2+i0^+]^2} + O(e^4)$$

Wick转动： $\int d^Dq \to i\int d^Dq_E = i\Omega_D\int_0^\infty \kappa^{D-1}d\kappa \quad q^2 \to -\kappa^2 \quad \Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$

$$\Sigma(p) = \frac{4i\mu^{4-D}e^2\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^\infty \kappa^{D-1}d\kappa\int_0^1 dx\frac{(1-x)\not{p}-2M}{[-\kappa^2+x(1-x)p^2-xM^2+i0^+]^2} + O(e^4)$$

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = [i\partial\!\!\!/_x - M - i0^+ + \Sigma(i\partial_x)]^{-1}\delta(x-y)$$

$$\Sigma(p) = \frac{4i\mu^{4-D}e^2\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^\infty \kappa^{D-1}d\kappa \int_0^1 dx \frac{(1-x)p\!\!\!/ - 2M}{[\kappa^2 - x(1-x)p^2 + xM^2 - i0^+]^2} + O(e^4)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Sigma(p) = \frac{2ie^2\mu^{4-D}\Gamma(2-\frac{D}{2})}{(4\pi)^{D/2}}\int_0^1 dx[(1-x)p\!\!\!/ - 2M][-x(1-x)p^2 + xM^2 - i0^+]^{\frac{D}{2}-2} + O(e^4)$$

$$\Gamma(2-\frac{D}{2}) \xRightarrow{D\to 4} \frac{1}{2-D/2} - \gamma + O(2-D/2) \qquad A^{\frac{D}{2}-2} \xRightarrow{D\to 4} 1 + (\frac{D}{2}-2)\ln A + O((\frac{D}{2}-2)^2)$$

$$\Sigma(p) = \frac{ie^2}{8\pi^2}\left[(\frac{1}{2-D/2} - \gamma)(\frac{1}{2}p\!\!\!/ - 2M) - \int_0^1 dx\ [(1-x)p\!\!\!/ - 2M]\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta \bar{I}_l(y)\delta I_{l'}(z)}\Bigg|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \frac{e^{\int d^4y d^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]} A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z) e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\ [\frac{1}{2}D_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + S_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ eA_\mu\overline{\psi}\gamma^\mu\psi}}\Bigg|_{A_\mu=\overline{\psi}=\psi=0}$$

$$= \int d^4x'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,x')ie\gamma^\nu_{l_1l_2}S_{0,l_2l'}(x',z) + \int d^4x'd^4y'd^4z'\ D_{0,\mu\nu}(x,x')$$

$$\times S_{0,ll_1}(y,y')ie\gamma^{\mu'}_{l_1l_2}S_{0,l_2l_3}(y',x')(-i)e\gamma^\nu_{l_3l_4}S_{0,l_4l_5}(x',z')(-i)e\gamma^{\nu'}_{l_5l_6}S_{0,l_5l'}(z',z)D_{0,\mu'\nu'}(y',z') + O(e^5)$$

$$= \int d^4x'd^4y'd^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

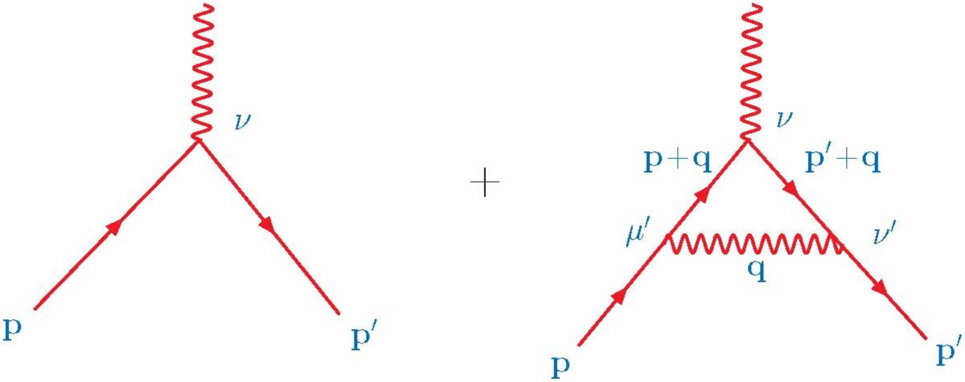
三线顶角

$$\Gamma^\nu(x,y,z) = e\gamma^\nu\delta(y-x)\delta(x-z) - e^3\gamma^{\mu'}S_0(y,x)\gamma^\nu S_0(x,z)\gamma^{\nu'}D_{0,\mu'\nu'}(y,z) + O(e^5)$$

$$= e\gamma^\nu\delta(y-x)\delta(x-z) + ie^3\gamma^{\mu'}(i\partial\!\!\!/_y - M - i0^+)^{-1}\delta(y-x)\gamma^\nu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-z)\gamma^{\nu'}$$

$$\times\frac{1}{\partial_y^2 - i0^+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{\partial_{\mu',y}\partial_{\nu',y}}{\partial_y^2}]\delta(y-z) + O(e^5)$$

三线顶角



相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \int d^4x' d^4y' d^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

$$\Gamma^\nu(x,y,z) = e\gamma^\nu\delta(y-x)\delta(x-z) + ie^3\gamma^{\mu'}(i\partial\!\!\!/_y - M - i0^+)^{-1}\delta(y-x)\gamma^\nu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-z)\gamma^{\nu'}$$
$$\times\frac{1}{\partial_y^2 - i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{\partial_{\mu',y}\partial_{\nu',y}}{\partial_y^2}]\delta(y-z) + O(e^5)$$
$$= e\gamma^\nu\int\frac{d^4pd^4p'}{(2\pi)^8}e^{-ip\cdot(y-x)-ip'\cdot(x-z)} + ie^3\int\frac{d^4pd^4p'd^4q}{(2\pi)^{12}}e^{-ip\cdot(y-x)-ip'\cdot(x-z)-iq\cdot(y-z)}$$
$$\times\gamma^{\mu'}(p\!\!\!/ - M - i0^+)^{-1}\gamma^\nu(p\!\!\!/' - M - i0^+)^{-1}\gamma^{\nu'}\frac{-1}{q^2+i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{q_{\mu'}q_{\nu'}}{q^2}] + O(e^5)$$
$$= \int\frac{d^4pd^4p'}{(2\pi)^8}e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + ie^3\int\frac{d^4q}{(2\pi)^4}\gamma^{\mu'}\frac{1}{p\!\!\!/ + q\!\!\!/ - M - i0+}\gamma^\nu\frac{1}{p\!\!\!/' + q\!\!\!/ - M - i0+}\gamma^{\nu'}\frac{-1}{q^2+i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{q_{\mu'}q_{\nu'}}{q^2}] + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \int d^4x' d^4y' d^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + ie^3\int\frac{d^4q}{(2\pi)^4}\gamma^{\mu'}\frac{1}{\not p+\not q-M-i0^+}\gamma^\nu\frac{1}{\not p'+\not q-M-i0^+}\gamma^{\nu'}\frac{-1}{q^2+i0^+}[g_{\mu'\nu'}-(1-\frac{1}{\lambda})\frac{q_{\mu'}q_{\nu'}}{q^2}]+O(e^5)$$

费曼规范： $\lambda = 1$ $\qquad \frac{1}{ABC} = 2\int_0^1 dx\int_0^x dy \frac{1}{[Ay+B(x-y)+C(1-x)]^3}$

$$= e\gamma^\nu - ie^3\int\frac{d^4q}{(2\pi)^4}\frac{\gamma^\mu(\not p+\not q+M)\gamma^\nu(\not p'+\not q+M)\gamma_\mu}{[(p+q)^2-M^2+i0^+][(p'+q)^2-M^2+i0^+](q^2+i0^+)} + O(e^5)$$

$$= e\gamma^\nu - 2ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{\gamma^\mu[(\not p+\not q)\gamma^\nu(\not p'+\not q)+M(\not p+\not q)\gamma^\nu+M\gamma^\nu(\not p'+\not q)+M^2\gamma^\nu]\gamma_\mu}{\{y[(p+q)^2-M^2+i0^+]+(x-y)[(p'+q)^2-M^2+i0^+]+(1-x)(q^2+i0^+)\}^3}$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{(\not p'+\not q)\gamma^\nu(\not p+\not q)-2M(p^\nu+q^\nu)-2M(p'^\nu+q^\nu)+M^2\gamma^\nu}{[q^2+2yp\cdot q+2(x-y)p'\cdot q+yp^2+(x-y)p'^2-xM^2+i0^+]^3} + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta \bar{I}_l(y)\delta I_{l'}(z)}\bigg|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= \int d^4x' d^4y' d^4z'\ D_{0,\mu\nu}(x,x')S_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')S_{0,l_2l'}(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4pd^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{(\not p'+\not q)\gamma^\nu(\not p+\not q) - 2M(p^\nu+q^\nu) - 2M(p'^\nu+q^\nu) + M^2\gamma^\nu}{[q^2+2yp\cdot q+2(x-y)p'\cdot q+yp^2+(x-y)p'^2-xM^2+i0+]^3} + O(e^5)$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^4q}{(2\pi)^4}\frac{(\not p'+\not q)\gamma^\nu(\not p+\not q) - 2M(p^\nu+p'^\nu+2q^\nu) + M^2\gamma^\nu}{\{[q+yp+(x-y)p']^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0+\}^3} + O(e^5)$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\int\frac{d^Dq}{(2\pi)^D}\frac{(\not p'+\not q)\gamma^\nu(\not p+\not q) - 2M(p^\nu+p'^\nu+2q^\nu) + M^2\gamma^\nu}{\{[q+yp+(x-y)p']^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0+\}^3} + O(e^5)$$

$$= e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\Big[[(1-x+y)\not p' - y\not p + \not q]\gamma^\nu[(1-y)\not p + (y-x)\not p' + \not q] - 2M[(1-2y)$$

$$\times p^\nu + (1-2x+2y)p'^\nu + 2q^\nu] + M^2\gamma^\nu\Big]\{q^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^{-3} + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p') \qquad \Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + 4ie^3\int_0^1 dx\int_0^x dy\ \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\Big[[(1-x+y)\not{p}'-y\not{p}+\not{q}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'+\not{q}]+M^2\gamma^\nu$$
$$-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu+2q^\nu]\Big]\{q^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^{-3}+O(e^5)$$

$$= e\gamma^\nu - \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']+(M^2+\frac{2}{D}\kappa^2)\gamma^\nu$$
$$-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]\Big]\{-\kappa^2-[yp+(x-y)p']^2+yp^2+(x-y)p'^2-xM^2+i0^+\}^{-3}+O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']+(M^2+\frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]\Big]\{\kappa^2+[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+\}^{-3}+O(e^5)$$

$$\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2}) \qquad \int_0^\infty d\kappa\frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^2} = \frac{1}{2}(\nu^2)^{\frac{D}{2}-1}\Gamma(1+\frac{D}{2})\Gamma(1-\frac{D}{2})$$

$$\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa\frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-2}\Gamma(1+\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p') \qquad \Delta^2 \equiv [yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+$$

$$\Gamma^\nu_{\rm div}(p,p') = \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2+1)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D+1}d\kappa\gamma^\nu[\kappa^2+\Delta^2]^{-3}+O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x') S_0(y,y') i\Gamma^\nu(x',y',z') S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)} \Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{\frac{D}{2}}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + (M^2 + \frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-2}\Gamma(1+\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\text{div}}(p,p') + \Gamma^\nu_{\text{fin}}(p,p') \qquad \Delta^2 \equiv [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+$$

$$\Gamma^\nu_{\text{div}}(p,p') = \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2+1)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D+1}d\kappa\gamma^\nu[\kappa^2+\Delta^2]^{-3} + O(e^5)$$

$$= \gamma^\nu \frac{e^3\Gamma(2-\frac{D}{2})}{8\pi^2}\int_0^1 dx\int_0^x dy\left[\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right]^{\frac{D}{2}-2} + O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x') S_0(y,y') i\Gamma^\nu(x',y',z') S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{\frac{D}{2}}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']+(M^2+\frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]\Big]\{\kappa^2+[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+\}^{-3}+O(e^5)$$

$$\int_0^\infty d\kappa\frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa\frac{\kappa^{D+1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-2}\Gamma(1+\frac{D}{2})\Gamma(2-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p') \qquad \Gamma(2-\frac{D}{2}) \xRightarrow{D\to 4} \frac{1}{2-D/2}-\gamma+O(2-D/2)$$

$$\Gamma^\nu_{\rm div}(p,p') = \gamma^\nu\frac{e^3\Gamma(2-\frac{D}{2})}{8\pi^2}\int_0^1 dx\int_0^x dy\left[\frac{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}{4\pi\mu^2}\right]^{\frac{D}{2}-2}+O(e^5)$$

$$= \gamma^\nu\frac{e^3}{8\pi^2}\left[\frac{1/2}{2-D/2}-\frac{\gamma}{2}-\int_0^1 dx\int_0^x dy\ \ln\frac{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}{4\pi\mu^2}\right]+O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p\, d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{\frac{D}{2}}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + (M^2+\frac{2}{D}\kappa^2)\gamma^\nu - 2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]\Big]\{\kappa^2+[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+\}^{-3}+O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2})$$

$$\Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p')$$

$$\Gamma^\nu_{\rm div}(p,p') = \gamma^\nu\frac{e^3}{8\pi^2}\left[\frac{1/2}{2-D/2}-\frac{\gamma}{2}-\int_0^1 dx\int_0^x dy\ \ln\frac{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}{4\pi\mu^2}\right]+O(e^5)$$

$$\Gamma^\nu_{\rm fin}(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]\Big]\{\kappa^2+[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+\}^{-3}+O(e^5)$$

相互作用顶角

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p') \qquad \Gamma^\nu(p,p') = \Gamma^\nu_{\rm div}(p,p') + \Gamma^\nu_{\rm fin}(p,p')$$

$$\Gamma^\nu_{\rm div}(p,p') = \gamma^\nu \frac{e^3}{8\pi^2}\left[\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx \int_0^x dy\ \ln \frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^5)$$

$$\Gamma^\nu_{\rm fin}(p,p') = e\gamma^\nu + \frac{8\mu^{4-D}e^3\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^1 dx\int_0^x dy\int_0^\infty \kappa^{D-1}d\kappa\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']$$

$$+M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{\kappa^2 + [yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-3} + O(e^5)$$

$$\int_0^\infty d\kappa \frac{\kappa^{D-1}}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{\frac{D}{2}-3}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2}) \qquad \int_0^\infty d\kappa \frac{\kappa^3}{(\kappa^2+\nu^2)^3} = \frac{1}{4}(\nu^2)^{-1}$$

$$\Gamma^\nu_{\rm fin}(p,p') = e\gamma^\nu + \frac{2e^3\pi^2}{(2\pi)^4}\int_0^1 dx\int_0^x dy\Big[[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']$$

$$+M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]\Big]\{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+\}^{-1} + O(e^5)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' D_{0,\mu\nu}(x,x')S_0(y,y')i\Gamma^\nu(x',y',z')S_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{e^3}{8\pi^2}\Bigg[\gamma^\nu\Big\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\Big\}$$

$$+\int_0^1 dx\int_0^x dy\frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\Bigg] + O(e^5)$$

$$y \to x - y \qquad \int_0^x dy \to \int_0^x dy$$

$$\Gamma^\nu(p,p') = e\gamma^\nu + \frac{e^3}{8\pi^2}\Bigg[\gamma^\nu\Big\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy\ \ln\frac{[(x-y)p+yp']^2 - (x-y)p^2 - yp'^2 + xM^2 - i0^+}{4\pi\mu^2}\Big\}$$

$$+\int_0^1 dx\int_0^x dy\frac{[(1-y)\not{p}' + (y-x)\not{p}]\gamma^\nu[(1-x+y)\not{p} - y\not{p}'] + M^2\gamma^\nu - 2M[(1-2x+2y)p^\nu + (1-2y)p'^\nu]}{[(x-y)p+yp']^2 - (x-y)p^2 - yp'^2 + xM^2 - i0^+}\Bigg] + O(e^5)$$

面临的问题与对策

问题：

- 光子和费米子传播子及光子与费米子的相互作用顶角的一圈辐射修正有紫外发散
- 紫外发散通过时空维数的延拓变成在$D=4$时的极点
- 理论是否已经无意义了？

对策：

- 格林函数的发散并不意味着物理可观测量发散，它只意味用拉格朗日量中的量表达的物理量发散
- 实验上观测的是物理量与物理量之间的关系，只要这种关系中没有发散，理论就是有意义的！
- 拉格朗日量中有限的量导致发散的物理量
$\Rightarrow$探索拉格朗日量中发散的量能否导致有限的物理量
- 拉格朗日量中发散的量能否导致有限的物理量$\Rightarrow$物理量与物理量之间的关系没有发散

重整拉格朗日量

拉格朗日量中发散的量？

- 拉格朗日量中发散的量叫“裸量” 加下标B：
  裸场， 裸参数（质量、耦合常数、. . .）
- 裸量是发散的！ 用有限的量“重整化量”表达裸量

$$\psi_B = Z_2^{1/2}\psi \qquad A_B^\mu = Z_3^{1/2}A^\mu \qquad e_B Z_2 Z_3^{1/2} = Z_1 e \qquad M_B = M + \delta M \qquad \lambda_B = Z_\lambda \lambda$$

用重整化量表达的拉格朗日量与抵消项

$$\begin{aligned}\mathcal{L}_{\text{QED}} &= -\frac{1}{4}F_{B,\mu\nu}(x)F_B^{\mu\nu}(x) - \frac{\lambda_B}{2}[\partial_\mu A_B^\mu(x)]^2 + \overline{\psi}_B(x)[i\not\partial - e_B \not{A}_B(x) - M_B]\psi_B(x) \\ &= -\frac{Z_3}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - Z_3 Z_\lambda \frac{\lambda}{2}[\partial_\mu A^\mu(x)]^2 + Z_2\overline{\psi}(x)[i\not\partial - Z_1 Z_2^{-1} e\not{A}(x) - M - \delta M]\psi(x) \\ &= -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial_\mu A^\mu(x)]^2 + \overline{\psi}(x)[i\not\partial - e\not{A}(x) - M]\psi(x) - \frac{1}{4}(Z_3 - 1)F_{\mu\nu}(x)F^{\mu\nu}(x) \\ &\quad - \frac{\lambda}{2}(Z_3 Z_\lambda - 1)[\partial_\mu A^\mu(x)]^2 + \overline{\psi}(x)[(Z_2 - 1)(i\not\partial - M) - (Z_1 - 1)e\not{A}(x) - Z_2\delta M]\psi(x)\end{aligned}$$

对数发散、二次发散的重整相消与理论的自然性

以粒子质量为例

$$M_{\text{phy}}^2 = (u_0 + u_1 + u_2 + \cdots)\Lambda^2 + (l_0 + l_1 + l_2 + \cdots)M_r^2 \ln(\Lambda^2/M_r^2) + (f_0 + f_1 + f_2 + \cdots)M_r^2$$

- 二次发散对高能区物理依赖十分明显
- 对数发散对高能区物理依赖不太明显

三种典型的物理能量尺度： $M_{\text{phy}}^2 \sim M_r^2 \sim (100GeV)^2$

$\Lambda = 10^3\text{GeV}$	$u_0 + u_1 + u_2 + \cdots \sim 10^{-2}$	$l_0 + l_1 + l_2 + \cdots \sim 1$
$\Lambda = 10^{16}\text{GeV}$	$u_0 + u_1 + u_2 + \cdots \sim 10^{-28}$	$l_0 + l_1 + l_2 + \cdots \sim 10^{-1}$
$\Lambda = 10^{19}\text{GeV}$	$u_0 + u_1 + u_2 + \cdots \sim 10^{-36}$	$l_0 + l_1 + l_2 + \cdots \sim 10^{-1}$

自然性：

- 二次发散的存在要求计算的精细调节，对数发散不需要
- 精细调节对计算是不稳定的，不自然！
- 在标量场、矢量场和旋量场中只有标量场和有质量的矢量场具有二次发散，因而它们是不自然的！

重整格林函数和顶角函数

$$\psi_B = Z_2^{1/2}\psi \qquad A_B^\mu = Z_3^{1/2}A^\mu \qquad e_B Z_2 Z_3^{1/2} = Z_1 e \qquad M_B = M + \delta M \qquad \lambda_B = Z_\lambda \lambda$$

重整化的格林函数

$$(\Psi_0^-, \mathbf{T}\,[\psi_{B,l_1}(x_1)\overline{\psi}_{B,l_1'}(x_1')\cdots\psi_{B,l_n}(x_n)\overline{\psi}_{B,l_n'}(x_n')A_{B,\mu_1}(y_1)\cdots A_{B,\mu_m}(y_m)]\ \Psi_0^+)_C$$

$$= Z_2^n Z_3^{m/2}(\Psi_0^-, \mathbf{T}\,[\psi_{l_1}(x_1)\overline{\psi}_{l_1'}(x_1')\cdots\psi_{l_n}(x_n)\overline{\psi}_{l_n'}(x_n')A_{\mu_1}(y_1)\cdots A_{\mu_m}(y_m)]\ \Psi_0^+)$$

重整化的顶角函数

$$\frac{\delta^{2n+m}\Gamma[A_B,\psi_B,\overline{\psi}_B]}{\delta\overline{\psi}_{B,l_1}(x_1)\delta\psi_{B,l_1'}(x_1')\cdots\delta\overline{\psi}_{B,l_n}(x_n)\delta\psi_{B,l_n'}(x_n')\delta A_B^{\mu_1}(y_1)\cdots\delta A_B^{\mu_m}(y_m)}\Bigg|_{A_B=A_0,\ \psi_B=\psi_0,\ \overline{\psi}_B=\overline{\psi}_0}$$

$$= Z_2^{-n}Z_3^{-m/2}\frac{\delta^{2n+m}\Gamma[A_B,\psi_B,\overline{\psi}_B]}{\delta\overline{\psi}_{l_1}(x_1)\delta\psi_{l_1'}(x_1')\cdots\delta\overline{\psi}_{l_n}(x_n)\delta\psi_{l_n'}(x_n')\delta A^{\mu_1}(y_1)\cdots\delta A^{\mu_m}(y_m)}\Bigg|_{A_B=A_0,\ \psi_B=\psi_0,\ \overline{\psi}_B=\overline{\psi}_0}$$

Ward-Takahashi-Taylor恒等式

$$\partial_{\mu,x} iD_B^{\mu\nu}(x,y) = -\frac{\partial_x^\nu}{\lambda_B\partial_x^2}\delta(x-y) \qquad \partial_{\mu,x}\Gamma_B^\mu(x,y,z) = e_B S_B^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$$

$$Z_3\partial_{\mu,x} iD^{\mu\nu}(x,y) = -Z_\lambda^{-1}\frac{\partial_x^\nu}{\lambda\partial_x^2}\delta(x-y) \qquad Z_2^{-1}Z_3^{-\frac{1}{2}}\partial_{\mu,x}\Gamma^\mu(x,y,z) = iZ_1Z_2^{-1}Z_3^{-\frac{1}{2}}eZ_2^{-1}S^{-1}(y,z)[\delta(y-x)-\delta(z-x)]$$

$Z_\lambda = Z_3^{-1}$ $\qquad$ $Z_1 = Z_2$ 顶角重整化完全由光子的重整化决定；**Abel**近似

重整化格林函数的生成泛函

$$Z[J,I,\bar{I}]=e^{iW[J,I,\bar{I}]}=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{-\frac{Z_3}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda}{2}[\partial^\mu A_\mu]^2+Z_2\overline{\psi}[i\partial\!\!\!/-eA\!\!\!/(x)-M-\delta M]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I+i0^+\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{Z_3}{2}A_\mu[g^{\mu\nu}\partial^2-(1-Z_3^{-1}\lambda)\partial^\mu\partial^\nu-i0^+]A_\nu-Z_2eA_\mu\overline{\psi}\gamma^\mu\psi+Z_2\overline{\psi}[i\partial\!\!\!/-M-\delta M-i0^+]\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu i\tilde{D}_0^{-1,\mu\nu}A_\nu-Z_2eA_\mu\overline{\psi}\gamma^\mu\psi+\overline{\psi}i\tilde{S}_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$i\tilde{D}_0^{-1,\mu\nu}=Z_3g^{\mu\nu}\partial^2-(Z_3-\lambda)\partial^\mu\partial^\nu-i0^+ \qquad i\tilde{S}_0^{-1}=Z_2[i\partial\!\!\!/-M-\delta M-i0^+]$$

$$=e^{i\int d^4x\ Z_2e\frac{\delta}{-i\delta J^\mu(x)}\frac{\delta}{-i\delta I(x)}\gamma^\mu\frac{\delta}{i\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ \{\frac{1}{2}A_\mu i\tilde{D}_0^{-1,\mu\nu}A_\nu+\overline{\psi}i\tilde{S}_0^{-1}\psi-J^\mu A_\mu+\bar{I}\psi+\overline{\psi}I\}}$$

$$=e^{-Z_2e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\{\frac{1}{2}(A+iJ\tilde{D}_0)_\mu i\tilde{D}_0^{-1,\mu\nu}(A+i\tilde{D}_0J)_\nu+(\overline{\psi}-i\bar{I}S_0)iS_0^{-1}(\psi-iS_0I)+\frac{i}{2}J_\mu\tilde{D}_0^{\mu\nu}J_\nu+i\bar{I}\tilde{S}_0I\}}$$

$$=C\times e^{-Z_2e\int d^4x\frac{\delta}{\delta J^\mu(x)}\frac{\delta}{\delta I(x)}\gamma^\mu\frac{\delta}{\delta\bar{I}(x)}}\ e^{\int d^4yd^4z[-\frac{1}{2}J_\mu(y)\tilde{D}_0^{\mu\nu}(y,z)J_\nu(z)-\bar{I}(y)\tilde{S}_0(y,z)I(z)]}$$

$$C=\int\mathcal{D}A_\mu\mathcal{D}\overline{\psi}\mathcal{D}\psi\ e^{i\int d^4x\ [\frac{1}{2}A_\mu i\tilde{D}_0^{-1,\mu\nu}A_\nu+\overline{\psi}\ i\tilde{S}_0^{-1}\psi]}$$

$$D_0\to\tilde{D}_0 \qquad S_0\to\tilde{S}_0 \qquad e\to Z_2e$$

$$\tilde{D}_0^{\mu\nu}(y,z)=\frac{i}{Z_3\partial_y^2-i0^+}[g^{\mu\nu}-(1-\frac{Z_3}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z) \qquad \tilde{S}_0(y,z)=\frac{iZ_2^{-1}}{i\partial\!\!\!/_y-M-\delta M-i0^+}\delta(y-z)$$

S矩阵的重整化不变性

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0,\mathbf{T}\psi_{H,B,1}(x_1)\cdots\psi_{H,B,n+l}(x_{n+l})\Psi_0)$$

$$=\int\frac{d^4q_1\cdots d^4q_{n+l}}{(i\pi)^{n+l}}e^{-iq_1x_1\cdots-iq_nx_n+iq_{n+1}x_{n+1}\cdots+iq_{n+l}x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}q_1^0(\Psi_0^-,\psi_{B,1}(0)\Psi_{\vec{q}_1,\sigma_1}^-)\cdots q_n^0(\Psi_0^-,\psi_{B,n}(0)\Psi_{\vec{q}_n\sigma_n}^-)$$

$$\times(\Psi_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n}^-,\Psi_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}}^+)_B\,q_{n+1}^0(\Psi_{\vec{q}_{n+1},\sigma_{n+1}}^+,\psi_{B,n+1}(0)\Psi_0^+)\cdots q_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}}^+,\psi_{B,n+l}(0)\Psi_0^+)$$

$$\psi_{H,B,i}(x)=Z_i^{1/2}\psi_{H,i}(x)\qquad \psi_{B,i}=Z_i^{1/2}\psi_i(x)$$

$$(\partial_1^2+M_1^2)\cdots(\partial_{n+l}^2+M_{n+l}^2)(\Psi_0,\mathbf{T}\psi_{H,1}(x_1)\cdots\psi_{H,n+l}(x_{n+l})\Psi_0)$$

$$=\int\frac{d^4q_1\cdots d^4q_{n+l}}{(i\pi)^{n+l}}e^{-iq_1\cdot x_1\cdots-iq_n\cdot x_n+iq_{n+1}\cdot x_{n+1}\cdots+iq_{n+l}\cdot x_{n+l}}\sum_{\sigma_1\cdots\sigma_{n+l}}q_1^0(\Psi_0^-,\psi_1(0)\Psi_{\vec{q}_1,\sigma_1}^-)\cdots q_n^0(\Psi_0^-,\psi_n(0)\Psi_{\vec{q}_n\sigma_n}^-)$$

$$\times(\Psi_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n}^-,\Psi_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}}^+)\,q_{n+1}^0(\Psi_{\vec{q}_{n+1},\sigma_{n+1}}^+,\psi_{n+1}(0)\Psi_0^+)\cdots q_{n+l}^0(\Psi_{\vec{q}_{n+l},\sigma_{n+l}}^+,\psi_{n+l}(0)\Psi_0^+)$$

$$(\Psi_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n}^-,\Psi_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}}^+)_B=(\Psi_{\vec{q}_1,\sigma_1,\cdots,\vec{q}_n,\sigma_n}^-,\Psi_{\vec{q}_{n+1}\sigma_{n+1},\cdots\vec{q}_{n+l}\sigma_{n+l}}^+)$$

发散度

考虑一个有L个圈的费曼图，假设其分别有E_f, E_b条费米子和玻色子外线，I_f, I_b条费米子和玻色子内线，v_i个含d_i个微商，f_i条费米子线和b_i玻色子线的第i种类型顶角。考虑这个费曼图的发散度ω.则

$$\omega = 4L - 2I_b - I_f + \sum_i d_i v_i$$

$$L = I_f + I_b + 1 - \sum_i v_i \qquad E_f + 2I_f = \sum_i f_i v_i \qquad E_b + 2I_b = \sum_i b_i v_i$$

$$\omega = 4 - \frac{3}{2}E_f - E_b + \sum_i (\frac{3}{2}f_i + b_i + d_i - 4)v_i$$

设第i型顶角的耦合常数的量纲按质量量纲计为g_i

$$g_i + \frac{3}{2}f_i + b_i + d_i = 4 \quad \Rightarrow \quad \omega = 4 - \frac{3}{2}E_f - E_b - \sum_i g_i v_i$$

理论按可重整性分类

考虑一个有L个圈的费曼图，假设其分别有E_f, E_b条费米子和玻色子外线，v_i个含 d_i个微商，f_i条费米子线 和 b_i玻色子线的第i种类型顶角，第i型顶角的耦合常数的量纲按质量量纲计为g_i。此费曼图的发散度ω.

$$\omega = 4 - \frac{3}{2}E_f - E_b - \sum_i g_i v_i \qquad g_i = 4 - \frac{3}{2}f_i - b_i - d_i$$

理论和算符分类

- $g_i < 0$对应的顶角叫**irrelevant**算符。它的存在导致随着v_i的增加将出现无穷多发散类型的费曼图，因而理论是不可重整的。
- $g_i = 0$对应的顶角叫**marginal**算符；$g_i > 0$对应的顶角叫**relevant**算符。它们的存在导致理论有望可重整。满足此条件的相互作用顶角类型为:

Marginal		$\bar\psi\partial\psi$	$\bar\psi\phi\psi, \bar\psi A\!\!\!/\psi$				$\phi\partial^2\phi, \partial^2A^2$		$\partial A\phi^2, \partial A^3$	ϕ^4, A^4, ϕ^2A^2
Relevant	$\bar\psi\psi$			ϕ	ϕ^2, A^2	$\phi\partial A$		$\phi^3, A^2\phi$		
f_i	2	2	2	0	0	0	0	0	0	0
b_i	0	0	1	1	2	2	2	3	3	4
d_i	0	1	0	0	0	1	2	0	1	0

其中考虑了协变性的要求。

QED的可重整性 $\omega_{\rm QED} = 4 - \frac{3}{2}E_f - E_b$

$\omega_{\rm QED}$	1	0	2	1	0
E_f	2	2	0	0	0
E_b	0	1	2	3	4
	费米子传播子	费米光子三线顶角	光子传播子	光子三线顶角 法雷定理$=0$	光子四线顶角 不发散!

证明光子四线顶角不发散

$$0 \overset{\rm WTI}{=\!=\!=\!=\!=} \lambda\partial_x^2\partial_x^\mu \frac{\delta^2 W[J,I,\bar I]}{\delta J^\mu(x)\delta J^{\mu_1}(x_1)\delta J^{\mu_2}(x_2)\delta J^{\mu_3}(x_3)}\Big|_{J^\mu=\bar I=I=0}$$

$$= \lambda\partial_x^2\partial_x^\mu i^4\int d^4y d^4y_1 d^4y_2 d^4y_3\, D_{\mu\nu}(x,y)D_{\mu_1\nu_1}(x_1,y_1)D_{\mu_2\nu_2}(x_2,y_2)D_{\mu_3\nu_3}(x_3,y_3)\Gamma^{\nu\nu_1\nu_2\nu_3}(y,y_1,y_2,y_3)$$

$$\overset{\partial_x^\mu iD_{\mu\nu}(x,y)=-\frac{\partial_{\nu,x}}{\lambda\partial_x^2}\delta(x-y)}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=} -\partial_{\nu,x}i^3\int d^4y_1 d^4y_2 d^4y_3\, D_{\mu_1\nu_1}(x_1,y_1)D_{\mu_2\nu_2}(x_2,y_2)D_{\mu_3\nu_3}(x_3,y_3)\Gamma^{\nu\nu_1\nu_2\nu_3}(x,y_1,y_2,y_3)$$

$$p_\nu\Gamma^{\nu\nu_1\nu_2\nu_3}(p,p_1,p_2,p_3) = 0$$

$${\rm tr}[\not q\gamma^\mu\not q\gamma^\nu\not q\gamma^\sigma\not q\gamma^\rho] = \frac{q^4}{24}(g_{\mu'\nu'}g_{\sigma'\rho'} + g_{\mu'\sigma'}g_{\nu'\rho'} + g_{\mu'\rho'}g_{\nu'\sigma'}){\rm tr}[\gamma^{\mu'}\gamma^\mu\gamma^{\nu'}\gamma^\nu\gamma^{\sigma'}\gamma^\sigma\gamma^{\rho'}\gamma^\rho]$$

$$= \frac{q^4}{24}{\rm tr}[4\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho - 8g^{\mu\sigma}g^{\nu\rho} + 4\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho] = \frac{2q^4}{3}(2g^{\mu\nu}g^{\sigma\rho} + 2g^{\mu\rho}g^{\nu\sigma} - 4g^{\mu\sigma}g^{\nu\rho})$$

$${\rm tr}[\not q\gamma^\mu\not q\gamma^\nu\not q\gamma^\sigma\not q\gamma^\rho + \text{all permutations}] = 0$$

可重整与不可重整相互作用的物理诠释

重定义时空坐标 $x \equiv sx'$, 固定x'变s, 并保证作用量中动能项不变

$$\text{动能} = \int d^4x \left\{ \frac{1}{2}[\partial_\mu \phi(x)]^2 + \overline{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) - \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \right\}$$

$$= \int d^4x' \left\{ \frac{1}{2}[\partial'_\mu \phi'(x')]^2 + \overline{\psi}'(x') i\gamma^\mu \partial'_\mu \psi'(x') - \frac{1}{4} F'_{\mu\nu}(x') F'^{\mu\nu}(x') \right\}$$

$\phi(x) \equiv s^{-1}\phi'(x')$ $\psi \equiv s^{-3/2}\psi'(x')$ $A_\mu(x) \equiv s^{-1} A_\mu(x')$

若理论中有非动能算符O_i 其中含d_i个微商,f_i个费米场,b_i个玻色场 耦合常数c_i

$$\int d^4x \, c_i O_i(x) = \int d^4x' \, c_i s^{4-\frac{3}{2}f_i - b_i - d_i} O'_i(x') = \int d^4x' \, c_i s^{g_i} O'_i(x')$$

对固定的x',增大s的效果等价于使耦合常数c_i增大s^{g_i}倍!

- 相关算符可重整 $g_i > 0$ 在长距离低能起重要所用, 在短距离可忽略
- 临界算符可重整 $g_i = 0$ 在所有距离所有能量区间都起重要所用
- 不相关算符不可重整 $g_i < 0$ 在短距离高能起重要所用, 在长距离可忽略

理论可重整性的再讨论

普通意义的可重整

- 拉格朗日量中只有有限数目的幂次型的可重整相互作用项
- 理论的发散可以通过重新定义拉格朗日量中的场和耦合常数被吸收掉
- 如果出现了有限个拉格朗日量中没有的发散类型项零质量标量场的质量项, 在拉格朗日量中引入相应的抵消项,而重整化项取为零
- 一圈图可重整并不意味完全可重整有质量非阿贝尔理论! 高圈图有交缠发散

推广意义的可重整

- 拉格朗日量中有无限数目的幂次型相互作用项: 有效拉氏量
  - 写下理论对称性允许的所有可能的幂次型相互作用项
  - 制定数幂规则,安排计算次序.到计算的任一固定阶,考虑所有被允许的幂次型相互作用项, 数幂规则和对称性限制只能由有限个幂次型相互作用项产生贡献
- 到计算的任一固定阶,由于由于已考虑了所有被数幂规则和对称性所允许的幂次型相互作用项, 理论所出现的发散因此都可以通过重新定义相应拉格朗日量中的场和耦合常数被吸收掉.

光子传播子 $D_0 \to \tilde{D}_0 \quad S_0 \to \tilde{S}_0 \quad e \to Z_2 e$

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left.\frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\right|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y)$$

$$= i\left.\frac{e^{\int d^4y d^4z\ [\frac{1}{2}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + \tilde{S}_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ A_\mu(x)A_\nu(y)\ e^{-i\int d^4x\ Z_2 e A_\mu \overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\ [\frac{1}{2}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + \tilde{S}_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ Z_2 e A_\mu \overline{\psi}\gamma^\mu\psi}}\right|_{A_\mu=\overline{\psi}=\psi=0}$$

$$= i\tilde{D}_{0,\mu\nu}(x,y) + i\int d^4x_1 d^4y_1 \tilde{D}_{0,\mu\mu_1}(x,x_1)(-1)\Pi^{\mu_1\nu_1}(x_1,y_1)\tilde{D}_{0,\nu_1\nu}(y_1,y)$$

$$+i\int d^4x_1 d^4y_1 d^4x_2 d^4y_2 \tilde{D}_{0,\mu\mu_1}(x,x_1)\Pi^{\mu_1\nu_1}(x_1,y_1) i\tilde{D}_{0,\nu_1\mu_2}(y_1,x_2)\Pi^{\mu_2\nu_2}(x_2,y_2) i\tilde{D}_{0,\nu_2\nu}(y_2,y) + \cdots$$

$$= [i\tilde{D}_0(1-\Pi\tilde{D}_0+\Pi\tilde{D}_0\Pi\tilde{D}_0+\cdots)]_{\mu\nu}(x,y) = [i\tilde{D}_0(1+\Pi\tilde{D}_0)^{-1}]_{\mu\nu}(x,y) = [(\tilde{D}_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[Z_2 e\gamma^\mu \tilde{S}_0(x,y) Z_2 e\gamma^\nu \tilde{S}_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Pi^{\mu\nu}(p)$$

$$= -e^2 \mathrm{tr}[\gamma^\mu (i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-y)\gamma^\nu(i\partial\!\!\!/_y - M - i0^+)\delta(y-x)] + O(e^4)$$

一圈图的重整化

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(\tilde{D}_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}(p^2g^{\mu\nu} - p^\mu p^\nu)\pi(p^2)$$

$$\pi(p^2) = -\frac{ie^2}{12\pi^2}\left[\frac{1}{2-D/2} - \gamma - 6\int_0^1 dx\,(1-x)x\,\ln\frac{[M^2 - i0^+ - p^2x(1-x)]}{4\pi\mu^2}\right] + O(e^4)$$

$$\Pi^{\mu\nu}(x,y) = (-g^{\mu\nu}\partial_x^2 + \partial_x^\mu\partial_x^\nu)\pi(-\partial_x^2)\delta(x-y) \qquad i\tilde{D}_0^{-1,\mu\nu}(x,y) = [Z_3 g^{\mu\nu}\partial_x^2 - (Z_3-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-y)$$

$$(\tilde{D}_0^{-1}+\Pi)^{\mu\nu}(x,y) = -i\{(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu)[Z_3 - i\pi(-\partial_x^2)] + \lambda\partial_x^\mu\partial_x^\nu\}\delta(x-y)$$

$\overline{MS}$重整化

$$Z_3^{\overline{MS}}(\mu) = 1 + \frac{e^2}{12\pi^2}\left[\frac{1}{2-\frac{D}{2}} - \gamma + \ln\frac{4\pi\mu^2}{M^2}\right] + O(e^4)$$

$$\tilde{\pi}^{\overline{MS}}(p^2) \equiv i\pi(p^2) - Z_3^{\overline{MS}}(\mu) + 1 = -\frac{e^2}{2\pi^2}\int_0^1 dx\,(1-x)x\,\ln\frac{M^2 - i0^+ - p^2x(1-x)}{M^2} + O(e^4)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left\{\frac{i}{\partial_x^2[1-\tilde{\pi}^{\overline{MS}}(-\partial_x^2)]}\left[g^{\mu\nu} - \frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}\right] + \frac{1}{\lambda}\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^4}\right\}\delta(x-y)$$

一圈图的重整化

$$i(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta J^\nu(y)}\bigg|_{J^\mu=\bar{I}=I=0} = iD_{\mu\nu}(x,y) = [i(\tilde{D}_0^{-1}+\Pi)^{-1}]_{\mu\nu}(x,y)$$

$$\Pi^{\mu\nu}(x,y) = \mathrm{tr}[e\gamma^\mu S_0(x,y)e\gamma^\nu S_0(y,x)] + O(e^4) = \int \frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}(p^\mu p^\nu - p^2 g^{\mu\nu})\pi(p^2)$$

$$\pi(p^2) = -\frac{ie^2}{12\pi^2}\left[\frac{1}{2-D/2} - \gamma - 6\int_0^1 dx\ (1-x)x\ \ln\frac{[M^2 - i0^+ - p^2x(1-x)]}{4\pi\mu^2}\right] + O(e^4)$$

$$\Pi^{\mu\nu}(x,y) = (-g^{\mu\nu}\partial_x^2 + \partial_x^\mu\partial_x^\nu)\pi(-\partial_x^2)\delta(x-y) \qquad i\tilde{D}_0^{-1,\mu\nu}(x,y) = [Z_3 g^{\mu\nu}\partial_x^2 - (Z_3-\lambda)\partial_x^\mu\partial_x^\nu]\delta(x-y)$$

$$(\tilde{D}_0^{-1}+\Pi)^{\mu\nu}(x,y) = -i\{(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu)[Z_3 - i\pi(-\partial_x^2)] + \lambda\partial_x^\mu\partial_x^\nu\}\delta(x-y)$$

要求重整化点$p^2 = -\overline{\mu}^2$处的留数与自由场相同　$\mu = 0$: 在壳重整化

$$Z_3(\overline{\mu}) - i\pi(-\overline{\mu}^2) = 1 \Rightarrow Z_3(\overline{\mu}) = 1 + \frac{e^2}{12\pi^2}\left[\frac{1}{2-\frac{D}{2}} - \gamma - 6\int_0^1 dx\ (1-x)x\ln\frac{[M^2+\overline{\mu}^2x(1-x)]}{4\pi\mu^2}\right] + O(e^4)$$

$$\tilde{\pi}(p^2,\overline{\mu}) \equiv i\pi(p^2) - i\pi(-\overline{\mu}^2) = -\frac{e^2}{2\pi^2}\int_0^1 dx\ (1-x)x\ \ln\frac{M^2 - i0^+ - p^2x(1-x)}{M^2+\overline{\mu}^2x(1-x)} + O(e^4)$$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)A_\nu(y)\ \Psi_0^+)_C = \left\{\frac{i}{\partial_x^2[1-\tilde{\pi}(-\partial_x^2,\overline{\mu})]}\left[g^{\mu\nu} - \frac{\partial_x^\mu\partial_x^\nu}{\partial_x^2}\right] + \frac{1}{\lambda}\frac{\partial_x^\mu\partial_x^\nu}{\partial_x^4}\right\}\delta(x-y)$$

费米子传播子 $D_0 \to \tilde{D}_0 \quad S_0 \to \tilde{S}_0 \quad e \to Z_2 e$

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J, I, \bar{I}]}{\delta \bar{I}_l(x)\delta I_{l'}(y)}\bigg|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y)$$

$$= -i\frac{e^{\int d^4y d^4z\ [\frac{1}{2}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + \tilde{S}_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ \psi_l(x)\overline{\psi}_{l'}(y)\ e^{-i\int d^4x\ Z_2 e A_\mu \overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\ [\frac{1}{2}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)} + \tilde{S}_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ Z_2 e A_\mu \overline{\psi}\gamma^\mu\psi}}\bigg|_{A_\mu=\overline{\psi}=\psi=0}$$

$$= -i\tilde{S}_{0,ll'}(x,y) - i\int d^4x_1 d^4y_1 \tilde{S}_{0,ll_1}(x,x_1)(-1)\Sigma_{l_1 l_1'}(x_1,y_1)i\tilde{S}_{0,l_1' l}(y_1,y)$$

$$-i\int d^4x_1 d^4y_1 d^4x_2 d^4y_2 \tilde{S}_{0,ll_1}(x,x_1)\Sigma_{l_1 l_1'}(x_1,y_1)\tilde{S}_{0,l_1' l_2}(y_1,x_2)\Sigma_{l_2 l_2'}(x_2,y_2)\tilde{S}_{0,l_2' l'}(y_2,y) + \cdots$$

$$= -i[\tilde{S}_0(1-\Sigma\tilde{S}_0+\Sigma\tilde{S}_0\Sigma\tilde{S}_0+\cdots)]_{ll'}(x,y) = [\tilde{S}_0(1+\Sigma\tilde{S}_0)^{-1}]_{ll'}(x,y) = [(\tilde{S}_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

$$\Sigma_{ll'}(x,y) = Z_2^2[(-i)e\gamma^\mu\tilde{S}_0(x,y)\tilde{D}_{0,\mu\nu}(x,y)(-i)e\gamma^\nu]_{ll'} + O(e^4) = \int\frac{d^4p}{(2\pi)^4}\ e^{-ip\cdot(x-y)}\Sigma_{ll'}(p)$$

$$= Z_2 Z_3^{-1} e^2\{\gamma^\mu(i\partial\!\!\!/_x - M - \delta M - i0^+)^{-1}\delta(x-y)\gamma^\nu\frac{1}{\partial_y^2 - i0+}[g_{\mu\nu} - (1-\frac{Z_3}{\lambda})\frac{\partial_{\mu,y}\partial_{\nu,y}}{\partial_y^2}]\}_{ll'}\delta(y-x) + O(e^4)$$

式中的Z_2, Z_3可以取为1, δM可以略去，差别是高阶项！ 仍可取费曼规范$\lambda = 1$

一圈图的重整化

$$-i(\Psi_0^-, \mathbf{T}\ \psi_l(x)\overline{\psi}_{l'}(y)\ \Psi_0^+)_C = \frac{\delta^2 W[J,I,\bar{I}]}{\delta\bar{I}_l(x)\delta I_{l'}(y)}\bigg|_{J^\mu=\bar{I}=I=0} = -iS_{ll'}(x,y) = -i[(\tilde{S}_0^{-1}+\Sigma)^{-1}]_{ll'}(x,y)$$

$$\Sigma(x,y)=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}\Sigma(p) \qquad \Sigma(p)=\not{p}\Sigma_A(p^2)-M\Sigma_B(p^2) \qquad i\tilde{S}_0^{-1}=Z_2[i\not\partial-M-\delta M-i0^+]$$

$$\Sigma_A(p^2) = \frac{ie^2}{16\pi^2}\left[(\frac{1}{2-D/2}-\gamma)-2\int_0^1 dx\,(1-x)\ln\frac{x(x-1)p^2+xM^2-i0^+}{4\pi\mu^2}\right]+O(e^4)$$

$$\Sigma_B(p^2) = \frac{ie^2}{4\pi^2}\left[(\frac{1}{2-D/2}-\gamma)-\int_0^1 dx\,\ln\frac{x(x-1)p^2+xM^2-i0^+}{4\pi\mu^2}\right]+O(e^4)$$

$$Z_2 = 1+\delta Z_2$$

$$-i(\Psi_0^-, \mathbf{T}\psi_l(x)\overline{\psi}_{l'}(y)\Psi_0^+)_C = Z_2^{-1}(i\not\partial - M - \delta M - i0^+ - i\Sigma)^{-1}_{ll'}(x,y)$$

$$=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}\{\not{p}[1-i\Sigma_A(p^2)-\delta Z_2]-M[1-i\Sigma_B(p^2)-\delta Z_2]-\delta M\}^{-1}$$

$$\tilde{\Sigma}_A(p^2)\equiv i\Sigma_A(p^2)+\delta Z_2 \qquad \tilde{\Sigma}_B(p^2)=i\Sigma_B(p^2)+\delta Z_2-\delta M/M$$

$$(\Psi_0^-, \mathbf{T}\psi_l(x)\overline{\psi}_{l'}(y)\Psi_0^+)_C = \int\frac{d^4p}{(2\pi)^4}\frac{i\,e^{-ip\cdot(x-y)}}{\not{p}[1-\tilde{\Sigma}_A(p^2)]-M[1-\tilde{\Sigma}_B(p^2)]}$$

$$(\Psi_0^-, \mathbf{T}\psi_l(x)\overline{\psi}_{l'}(y)\Psi_0^+)_C = \int \frac{d^4p}{(2\pi)^4} \frac{i\, e^{-ip\cdot(x-y)}}{\not p[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]}$$

$$\tilde{\Sigma}_A(p^2) \equiv i\Sigma_A(p^2) + \delta Z_2 \qquad \tilde{\Sigma}_B(p^2) = i\Sigma_B(p^2) + \delta Z_2 - \delta M/M \qquad Z_2 = 1 + \delta Z_2$$

$$\Sigma_A(p^2) = \frac{ie^2}{16\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - 2\int_0^1 dx\,(1-x)\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

$$\Sigma_B(p^2) = \frac{ie^2}{4\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - \int_0^1 dx\,\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

$\overline{MS}$重整化

$$\delta Z_2^{\overline{MS}} = \frac{e^2}{16\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - \ln\frac{4\pi\mu^2}{M^2}\right] + O(e^4)$$

$$\delta Z_2^{\overline{MS}} - \delta M^{\overline{MS}}/M = \frac{e^2}{4\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - \ln\frac{4\pi\mu^2}{M^2}\right] + O(e^4)$$

$$\tilde{\Sigma}_A^{\overline{MS}}(p^2) = \frac{e^2}{8\pi^2}\int_0^1 dx\,(1-x)\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{M^2} + O(e^4)$$

$$\tilde{\Sigma}_B^{\overline{MS}}(p^2) = \frac{e^2}{4\pi^2}\int_0^1 dx\,\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{M^2} + O(e^4)$$

一圈图的重整化

$$(\Psi_0^-, \mathbf{T}\psi_l(x)\overline{\psi}_{l'}(y)\Psi_0^+)_C = \int \frac{d^4p}{(2\pi)^4} \frac{i\, e^{-ip\cdot(x-y)}}{\not p[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]}$$

$$\tilde{\Sigma}_A(p^2) \equiv i\Sigma_A(p^2) + \delta Z_2 \qquad \tilde{\Sigma}_B(p^2) = i\Sigma_B(p^2) + \delta Z_2 - \delta M/M \qquad Z_2 = 1 + \delta Z_2$$

$$\Sigma_A(p^2) = \frac{ie^2}{16\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - 2\int_0^1 dx\,(1-x)\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

$$\Sigma_B(p^2) = \frac{ie^2}{4\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - \int_0^1 dx\,\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

在壳重整化： $\tilde{\Sigma}_A(M^2) = \tilde{\Sigma}_B(M^2) = 0$

$$Z_2^{\text{OnShell}} = 1 - i\Sigma_A(M^2) = 1 + \frac{e^2}{16\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - 2\int_0^1 dx\,(1-x)\ln\frac{x^2M^2}{4\pi\mu^2}\right] + O(e^4)$$

$$-\delta M^{\text{OnShell}} = M[-i\Sigma_B(M^2) - \delta Z_2] = iM\Sigma_A(M^2) + \frac{Me^2}{4\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - \int_0^1 dx\,\ln\frac{x^2M^2}{4\pi\mu^2}\right] + O(e^4)$$

$$\tilde{\Sigma}_A^{\text{OnShell}}(p^2) = \frac{e^2}{8\pi^2}\int_0^1 dx\,(1-x)\ln\frac{(x-1)p^2 + M^2 - i0^+}{xM^2} + O(e^4)$$

$$\tilde{\Sigma}_B^{\text{OnShell}}(p^2) = \frac{e^2}{4\pi^2}\int_0^1 dx\,\ln\frac{(x-1)p^2 + M^2 - i0^+}{xM^2} + O(e^4)$$

一圈图的重整化

$$(\Psi_0^-, \mathbf{T}\psi_l(x)\overline{\psi}_{l'}(y)\Psi_0^+)_C = \int \frac{d^4p}{(2\pi)^4}\frac{i\, e^{-ip\cdot(x-y)}}{\not p[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]}$$

$$\tilde{\Sigma}_A(p^2) \equiv i\Sigma_A(p^2) + \delta Z_2 \qquad \tilde{\Sigma}_B(p^2) = i\Sigma_B(p^2) + \delta Z_2 - \delta M/M \qquad Z_2 = 1 + \delta Z_2$$

$$\Sigma_A(p^2) = \frac{ie^2}{16\pi^2}\left[(\frac{1}{2-D/2} - \gamma) - 2\int_0^1 dx\,(1-x)\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

$$\Sigma_B(p^2) = \frac{ie^2}{4\pi^2}\left[(\frac{1}{2-D/2} - \gamma) - \int_0^1 dx\,\ln\frac{x(x-1)p^2 + xM^2 - i0^+}{4\pi\mu^2}\right] + O(e^4)$$

在类空点$p^2 = -\overline{\mu}^2$处重整化: $\tilde{\Sigma}_A(-\overline{\mu}^2) = \tilde{\Sigma}_B(-\overline{\mu}^2) = 0$

$$Z_2(\overline{\mu}) = 1 - i\Sigma_A(-\overline{\mu}^2) = 1 + \frac{e^2}{16\pi^2}\left[\frac{1}{2-D/2} - \gamma - 2\int_0^1 dx\,(1-x)\ln\frac{x(1-x)\overline{\mu}^2 + xM^2}{4\pi\mu^2}\right] + O(e^4)$$

$$-\delta M(\overline{\mu}) = M[-i\Sigma_B(-\overline{\mu}^2) - \delta Z_2] = iM\Sigma_A(-\overline{\mu}^2) + \frac{Me^2}{4\pi^2}\left[\frac{1}{2-D/2} - \gamma - \int_0^1 dx\ln\frac{x(1-x)\overline{\mu}^2 + xM^2}{4\pi\mu^2}\right] + O(e^4)$$

$$\tilde{\Sigma}_A(p^2, \overline{\mu}) = \frac{e^2}{8\pi^2}\int_0^1 dx\,(1-x)\ln\frac{(x-1)p^2 + M^2 - i0^+}{(1-x)\overline{\mu}^2 + M^2} + O(e^4)$$

$$\tilde{\Sigma}_B(p^2, \overline{\mu}) = \frac{e^2}{4\pi^2}\int_0^1 dx\,\ln\frac{(x-1)p^2 + M^2 - i0^+}{(1-x)\overline{\mu}^2 + M^2} + O(e^4)$$

费米子与光子相互作用顶角 $D_0 \to \tilde{D}_0 \quad S_0 \to \tilde{S}_0 \quad e \to Z_2 e$

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)\ \Psi_0^+)_C = \frac{\delta^3 W[J,I,\bar{I}]}{\delta J^\mu(x)\delta\bar{I}_l(y)\delta I_{l'}(z)}\Big|_{J^\mu=\bar{I}=I=0} = G_{\mu,ll'}(x,y,z)$$

$$= -\frac{e^{\int d^4y d^4z[\frac{1}{2}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)}+\tilde{S}_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]} A_\mu(x)\psi_l(y)\overline{\psi}_{l'}(z)e^{-i\int d^4x Z_2 e A_\mu\overline{\psi}\gamma^\mu\psi}}{e^{\int d^4y d^4z\ [\frac{1}{2}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta^2}{\delta A^\mu(y)\delta A^\nu(z)}+\tilde{S}_0(y,z)\frac{\delta^2}{\delta\overline{\psi}(z)\delta\psi(y)}]}\ e^{-i\int d^4x\ Z_2 e A_\mu\overline{\psi}\gamma^\mu\psi}}\Big|_{A_\mu=\overline{\psi}=\psi=0}$$

$$= \int d^4x'\ \tilde{D}_{0,\mu\nu}(x,x')\tilde{S}_{0,ll_1}(y,x')iZ_2e\gamma^\nu_{l_1l_2}\tilde{S}_{0,l_2l'}(x',z) + \int d^4x'd^4y'd^4z'\ \tilde{D}_{0,\mu\nu}(x,x')$$

$$\times\tilde{S}_{0,ll_1}(y,y')iZ_2e\gamma^{\mu'}_{l_1l_2}\tilde{S}_{0,l_2l_3}(y',x')(-i)Z_2e\gamma^\nu_{l_3l_4}\tilde{S}_{0,l_4l_5}(x',z')(-i)Z_2e\gamma^{\nu'}_{l_5l_6}S_{0,l_5l'}(z',z)\tilde{D}_{0,\mu'\nu'}(y',z') + O$$

$$= \int d^4x'd^4y'd^4z'\ \tilde{D}_{0,\mu\nu}(x,x')\tilde{S}_{0,ll_1}(y,y')i\Gamma^\nu_{l_1l_2}(x',y',z')\tilde{S}_{0,l_2l'}(z',z)$$

三线顶角： 将修正项中的重整化常数略去，它们属于高阶效应

$$\Gamma^\nu(x,y,z) = Z_2e\gamma^\nu\delta(y-x)\delta(x-z) - e^3\gamma^{\mu'}S_0(y,x)\gamma^\nu S_0(x,z)\gamma^{\nu'}D_{0,\mu'\nu'}(y,z) + O(e^5)$$

$$= Z_2e\gamma^\nu\delta(y-x)\delta(x-z) + ie^3\gamma^{\mu'}(i\partial\!\!\!/_y - M - i0^+)^{-1}\delta(y-x)\gamma^\nu(i\partial\!\!\!/_x - M - i0^+)^{-1}\delta(x-z)\gamma^{\nu'}$$

$$\times\frac{1}{\partial_y^2 - i0+}[g_{\mu'\nu'} - (1-\frac{1}{\lambda})\frac{\partial_{\mu',y}\partial_{\nu',y}}{\partial_y^2}]\delta(y-z) + O(e^5)$$

一圈图的重整化

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' \tilde{D}_{0,\mu\nu}(x,x')\tilde{S}_0(y,y') i\Gamma^\nu(x',y',z')\tilde{S}_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = Z_2 e\gamma^\nu - \frac{e^3}{8\pi^2}\Big[\gamma^\nu\{\frac{-1/2}{2-D/2}+\frac{\gamma}{2}+\int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}{4\pi\mu^2}\}$$

$$+\int_0^1 dx\int_0^x dy \frac{[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']+M^2\gamma^\nu-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}\Big]+O(e^5)$$

$\overline{MS}$ 重整化

$$Z_2^{\overline{MS}} = 1+\frac{e^2}{16\pi^2}\Big[(\frac{1}{2-D/2}-\gamma)-\ln 4\pi\Big]+O(e^4)$$

$$\Gamma^{\overline{MS},\nu}(p,p') = e\gamma^\nu - \frac{e^3}{8\pi^2}\Big[\gamma^\nu\int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}{\mu^2}$$

$$-\int_0^1 dx\int_0^x dy \frac{[(1-x+y)\not{p}'-y\not{p}]\gamma^\nu[(1-y)\not{p}+(y-x)\not{p}']+M^2\gamma^\nu-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}\Big]+O(e^5)$$

一圈图的重整化

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' \tilde{D}_{0,\mu\nu}(x,x')\tilde{S}_0(y,y') i\Gamma^\nu(x',y',z')\tilde{S}_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = Z_2 e\gamma^\nu + \frac{e^3}{8\pi^2}\left[\gamma^\nu\left\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx \int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\right\}\right.$$

$$\left. + \int_0^1 dx \int_0^x dy \frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\right] + O(e^5)$$

在壳重整化

$$Z_2^{\rm OnShell} = 1 + \frac{e^2}{16\pi^2}\left[\left(\frac{1}{2-D/2} - \gamma\right) - 2\int_0^1 dx\,(1-x)\ln\frac{x^2M^2}{4\pi\mu^2}\right] + O(e^4)$$

$$\Gamma^{{\rm OnShell},\nu}(p,p') = e\gamma^\nu - \frac{e^3}{8\pi^2}\left[\gamma^\nu \int_0^1 dx \int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{(1-x)^2M^2}\right.$$

$$\left. + \int_0^1 dx \int_0^x dy \frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\right] + O(e^5)$$

一圈图的重整化

$$(\Psi_0^-, \mathbf{T}\, A_\mu(x)\psi(y)\overline{\psi}(z)\ \Psi_0^+)_C = \int d^4x' d^4y' d^4z' \tilde{D}_{0,\mu\nu}(x,x')\tilde{S}_0(y,y') i\Gamma^\nu(x',y',z')\tilde{S}_0(z',z)$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')$$

$$\Gamma^\nu(p,p') = Z_2 e\gamma^\nu + \frac{e^3}{8\pi^2}\Big[\gamma^\nu\Big\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi\mu^2}\Big\}$$

$$+\int_0^1 dx\int_0^x dy \frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\Big] + O(e^5)$$

在类空点$p^2 = \overline{\mu}^2$重整化

$$Z_2(\mu) = 1 + \frac{e^2}{16\pi^2}\Big[\frac{1}{2-D/2} - \gamma - 2\int_0^1 dx\ (1-x)\ln\frac{x(1-x)\mu^2 + xM^2}{4\pi\mu^2}\Big] + O(e^4)$$

$$\Gamma^\nu(p,p',\mu) = e\gamma^\nu - \frac{e^3}{8\pi^2}\Big[\gamma^\nu\int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{x(1-x)\mu^2 + xM^2}$$

$$+\int_0^1 dx\int_0^x dy \frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\Big] + O(e^5)$$

重整化耦合常数的尺度依赖 $e_B Z_2 Z_3^{1/2} = Z_1 e \quad Z_1 = Z_2 \Rightarrow \alpha_B \equiv \frac{e_B^2}{4\pi} = Z_3^{\overline{MS},-1}(\mu)\alpha_{\overline{MS}}$

Z_3一圈修正与λ无关

$$Z_3^{\overline{MS}}(\mu) = 1 + \frac{e^2}{12\pi^2}\left[\frac{1}{2-\frac{D}{2}} - \gamma + \ln\frac{4\pi\mu^2}{M^2}\right] + O(e^4)$$

$$\alpha_B = \frac{\alpha_{\overline{MS}}(\mu)}{1+\frac{\alpha_{\overline{MS}}(\mu)}{3\pi}\left[\frac{1}{2-\frac{D}{2}}-\gamma+\ln\frac{4\pi\mu^2}{M^2}\right]+O(\alpha^2)} \qquad \frac{1}{\alpha_B} = \frac{1}{\alpha_{\overline{MS}}(\mu)} + \frac{1}{3\pi}\left[\frac{1}{2-\frac{D}{2}}-\gamma+\ln\frac{4\pi\mu^2}{M^2}\right] + O(\alpha)$$

$$\alpha_{\overline{MS}}(\mu)\Big|_{4\pi\mu^2=M^2 e^{\gamma-\frac{1}{2-\frac{D}{2}}}} = \alpha_B$$

领头阶对数$\alpha^n \ln^n$求和： 发现在Z_3中的$O(e^4)$项不含ln项，只包含$\ln^2$项

$$\alpha_B = \alpha_{\overline{MS}}(\mu_0)[1 + c + c^2 + c^3 + c^4 + \cdots] \qquad c \equiv -\frac{1}{3\pi}\left[\frac{1}{2-\frac{D}{2}}-\gamma+\ln\frac{4\pi\mu^2}{M^2}\right]$$

次领头阶：$\alpha^n \ln^{n-1}$

$$\frac{1}{\alpha_{\overline{MS}}(\mu)} = \frac{1}{\alpha_{\overline{MS}}(\mu_0)} - \frac{1}{3\pi}\ln\frac{\mu^2}{\mu_0^2}$$

$$\alpha_{\overline{MS}}(\mu) = \alpha_{\overline{MS}}(\mu_0)[1 + c_0 + c_0^2 + c_0^3 + c_0^4 + \cdots] \qquad c_0 \equiv -\frac{\alpha_{\overline{MS}}(\mu_0)}{3\pi}\ln\frac{\mu^2}{\mu_0^2}$$

重整化耦合常数的尺度依赖性

$$\alpha_B = Z_3^{\overline{MS},-1}\alpha_{\overline{MS}}(\mu) \quad Z_3^{\overline{MS}}(\mu) = 1 + \frac{\alpha_{\overline{MS}}}{3\pi}[\frac{1}{2-\frac{D}{2}} - \gamma + \frac{4\pi\mu^2}{M^2}] + O(e^4)$$

$$\alpha_{\overline{MS}}(\mu) = \frac{\alpha_{\overline{MS}}(\mu_0)}{1 - \frac{1}{3\pi}\alpha_{\overline{MS}}(\mu_0)\ln\frac{\mu^2}{\mu_0^2}}$$

$$\frac{1}{\alpha_{\overline{MS}}(\mu)} + \frac{1}{3\pi}[\frac{1}{2-\frac{D}{2}} - \gamma + \frac{4\pi\mu^2}{M^2}] = \frac{1}{\alpha_{\overline{MS}}(\mu_0)} + \frac{1}{3\pi}[\frac{1}{2-\frac{D}{2}} - \gamma + \frac{4\pi\mu_0^2}{M^2}]$$

尺度依赖性:

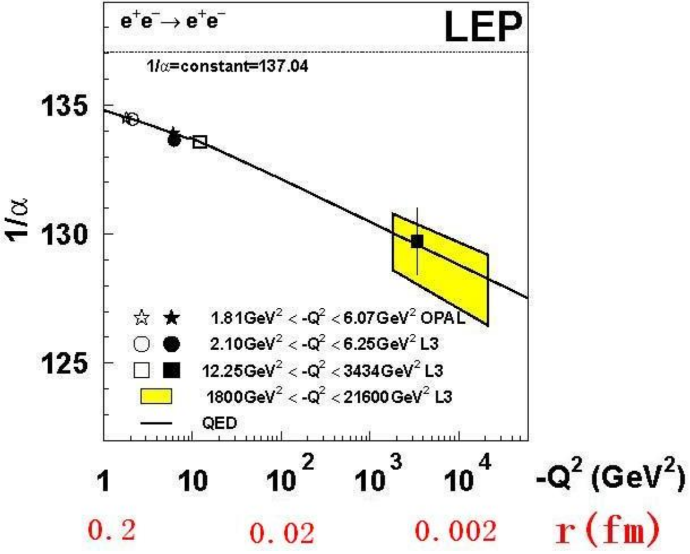
- 小尺度: $\alpha_{\overline{MS}}(\mu) \xRightarrow{\mu\to 0\text{Thomson极限}} 0 \xRightarrow{\text{计及有限项}} \frac{1}{137}$
- 大尺度: $\alpha_{\overline{MS}}(\mu) \xRightarrow{\mu\to\mu_0 e^{\frac{3\pi}{\alpha_{\overline{MS}}(\mu_0)}}} \infty$ 朗道极点！ 预言不可靠！

平庸性:

$$\alpha_{\overline{MS}}(\mu) = \frac{\alpha_{\overline{MS}}(\mu_0)}{1 - \frac{1}{3\pi}\alpha_{\overline{MS}}(\mu_0)\ln\frac{\mu^2}{\mu_0^2}} \xRightarrow{\mu_0\to\infty} 0$$

若理论适用于所有尺度, 则重整化的耦合常数为零！

跑动耦合常数



$e^+e^- \to e^+e^-$
LEP
1/α=constant=137.04
1/α
135
130
125
1.81GeV^2 < -Q^2 < 6.07GeV^2 OPAL
2.10GeV^2 < -Q^2 < 6.25GeV^2 L3
12.25GeV^2 < -Q^2 < 3434GeV^2 L3
1800GeV^2 < -Q^2 < 21600GeV^2 L3
QED
1
10
10^2
10^3
10^4
-Q^2 (GeV^2)
0.2
0.02
0.002
r (fm)

理论上的困惑: 零荷问题

- 如果要求理论适用到无穷大能标⇒重整化的电荷为零
- 理论适用到无穷大能标导致无电磁相互作用
- 量子电动力学不能适用到所有能量区间
- 当时几乎所有场的量子理论都具有此性质
- 引起对量子场论的极大不信任

1955: 弱耦合量子电动力学虽是一个理论,它本质上和逻辑上是不完备的!

量子场理论的高能不可靠!

零荷问题

Landau, L.D. and I.Pomeranchuk, Dokl. Akad. Nauk SSSR 102,489(1955)

We reach the conclusion that within the limits of formal electrodynamics a point interaction is equivalent, for any intensity whatever, to no interaction at all

QED预言高能区作用增强,理论不自洽 ⇒ 位于高能区的强作用更应不自洽!

Landau, L.D., Fundamental Problems, in Pauli Memorial Volume, pg.245, (Interscience, New York, 1960)

We are driven to the conclusion that the Hamiltonian method for strong interaction is dead and must be buried, although of course with deserved honor.

It is well known that theoretical physics is at present almost helpless in dealing with the problem of strong interactions.... By now the nullification of the theory is tacitly accepted even by theoretical physicists who profess to dispute it. This is evident from the almost complete disappearance of papers on meson theory and particularly from Dyson's assertion that the correct theory will not be found in the next hundred years.

零荷问题

Why discovery of zero charge problem did not inspire a search for asymptotically free theories?

David J. Gross, hep-th/9809060

- *many other theories were explored, in each case they behaved as QED*
- *Landau and Pomeranchuk concluded that this problem was inherent in any quantum field theory, that an asymptotically free theory could not exist.*

Goldberger, M. in The Quantum theory of Fields: The 12th Solvay Conference, 1961 (Interscience, New York)

My own feeling is that we have learned a great deal from field theory....that I am quite happy to discard it as an old, but rather friendly, mistress who I would be willing to recognize on the street if I should encounter her again. From a philosophical point of view and certainly from a practical one the S-matrix approach at the moment seems to me by far the most attractive.

维数转移

$$\alpha_B = Z_3^{\overline{MS},-1}\alpha_{\overline{MS}}(\mu) \quad Z_3^{\overline{MS}}(\mu) = 1 + \frac{\alpha}{3\pi}[\frac{1}{2-\frac{D}{2}} - \gamma + \frac{4\pi\mu^2}{M^2}] + O(e^4)$$

$$\alpha_{\overline{MS}}(\mu) = \frac{\alpha_{\overline{MS}}(\mu_0)}{1 - \frac{1}{3\pi}\alpha_{\overline{MS}}(\mu_0)\ln\frac{\mu^2}{\mu_0^2}}$$

存在不随尺度变化的标度 Λ_{QED}

$$\frac{1}{\alpha_{\overline{MS}}(\mu)} - \frac{1}{\alpha_{\overline{MS}}(\mu_0)} = -\frac{1}{3\pi}\ln\frac{\mu^2}{\mu_0^2}$$

$$\frac{1}{\alpha_{\overline{MS}}(\mu)} + \frac{1}{3\pi}\ln\mu^2 = \frac{1}{\alpha_{\overline{MS}}(\mu_0)} + \frac{1}{3\pi}\ln\mu_0^2 \equiv \frac{1}{3\pi}\ln\Lambda_{\text{QED}}^2 \qquad \alpha_{\overline{MS}}(\mu) = \frac{3\pi}{\ln\frac{\Lambda_{\text{QED}}^2}{\mu^2}}$$

- 随尺度变化的无量纲参数$\alpha(\mu)$ $\Leftrightarrow$ 不随尺度变化的有量纲参数Λ_{QED}
- $\mu^2 \approx \Lambda_{\text{QED}}^2 \Rightarrow \alpha_{\overline{MS}}(\Lambda_{\text{QED}}) \approx \infty$.

  $\Lambda_{\text{QED}} = \mu e^{\frac{3\pi}{2\alpha_{\overline{MS}}(\mu)}}$ 近似是朗道鬼的尺度！

可以用无量纲的参数产生理论的量纲参数！

顶角函数的尺度依赖性　　考虑2n点费米m点光子的相互作用顶角

$$\Gamma_{B,2n+m}[p_i,\alpha_B,M_B,\lambda_B]=Z_2^{\overline{MS},-n}(\mu)Z_3^{\overline{MS},-m/2}(\mu)\Gamma[p_i,\alpha_{\overline{MS}}(\mu),M(\mu),\lambda(\mu),\mu]$$

$\Gamma[p_i,\alpha_{\overline{MS}}(\mu),M,\lambda(\mu),\mu]$的质量量纲为 $-3n-m+4$

$$\Gamma_{2n+m}[e^tp_i,\alpha_{\overline{MS}}(\mu),e^tM,\lambda(\mu),e^t\mu]=e^{(4-3n-m)t}\Gamma_{2n+m}[p_i,\alpha_{\overline{MS}}(\mu),M,\lambda(\mu),\mu]$$

$$\begin{aligned}\Gamma_{B,2n+m}(e^tp_i,\alpha_B,M_B,\lambda_B)&=Z_2^{\overline{MS},-n}(\mu)Z_3^{\overline{MS},-m/2}(\mu)\Gamma_{2n+m}[e^tp_i,\alpha_{\overline{MS}}(\mu),M(\mu),\lambda(\mu),\mu]\\&=Z_2^{\overline{MS},-n}(e^t\mu)Z_3^{\overline{MS},-m/2}(e^t\mu)\Gamma_{2n+m}[e^tp_i,\alpha_{\overline{MS}}(e^t\mu),M(e^t\mu),\lambda(e^t\mu),e^t\mu]\\&=e^{t(4-3n-m)}Z_2^{\overline{MS},-n}(e^t\mu)Z_3^{\overline{MS},-m/2}(e^t\mu)\Gamma_{2n+m}[p_i,\alpha_{\overline{MS}}(e^t\mu),e^{-t}M(e^t\mu),\lambda(e^t\mu),\end{aligned}$$

$$\Gamma_{2n+m}[e^tp_i,\alpha_{\overline{MS}}(\mu),M(\mu),\lambda(\mu),\mu]=\frac{e^{t(4-3n-m)}Z_2^{\overline{MS},-n}(e^t\mu)Z_3^{\overline{MS},-m/2}(e^t\mu)}{Z_2^{\overline{MS},-n}(\mu)Z_3^{\overline{MS},-m/2}(\mu)}\times\Gamma_{2n+m}[p_i,\alpha(t),e^{-t}M(t),\lambda(t),\mu]$$

$$\alpha(t)\equiv\alpha_{\overline{MS}}(e^t\mu)=\frac{\alpha_{\overline{MS}}(\mu)}{1-\frac{1}{3\pi}\alpha_{\overline{MS}}(\mu)t}$$

$M(t)\equiv M(e^t\mu)\quad\lambda(t)\equiv\lambda(e^t\mu)$　顶角函数尺度变化可用跑动耦合常数反映！

大质量影响,小质量忽略

关于跑动行为的讨论

QED

$$\alpha(t) = \frac{\alpha_{\overline{MS}}(\mu)}{1-\frac{1}{3\pi}\alpha_{\overline{MS}}(\mu)t}$$

- $\overline{MS}$重整化方案导致红外渐进自由
- $\alpha(\Lambda_{理论}) = \infty$定义了理论适用尺度的上限$\Lambda_{理论}$：郎道鬼
- 只适用低能，否则平庸 $\Rightarrow$ 存在其它的高能理论 电弱统一
- 可在低能区进行微扰计算
- 绝大多数量子场论都是此类理论

紫外渐进自由理论

- $\alpha(\Lambda_{理论}) = \infty$定义了微扰理论适用尺度的下极限$\Lambda_{理论}$：红外奴役
- 高能区变成近似的自由场理论 $\Rightarrow$ 适用尺度可以任意地推高 $\Rightarrow$ 是统一理论的候选者
- 可在高能区进行微扰计算
- 只有非阿贝尔规范理论可以具有紫外渐进自由 2004年诺贝尔物理奖

A.Zee Phys.Rev.D7, June 15, 1973 (received March 12)

"Study of the Renormalization Group for Small Coupling Constants"

A renormalizable quantum field theory is said to be stagnant if it is asymptotically free. ....We show that Cartan's four families A,B,C,D and exceptional algebra G_2 possess no stagnant representation. On the basis of this result we conjecture that there are no assymptotically free quantum field theories in four dimensions.

D.J.Gross and F.Wilczek PRL, 30, June 25 1973 (Received April 27)

"Ultraviolet Behavior of Non-Abelian Gauge Theories"

It is shown that a wide class of non-abelian gauge theories have, up to calculate logarithmic corrections, free-field-theory asymptotic behavior.

David Politzer Phys.Rev.Lett, 30, 25 June 1973 (Received 3 May)

"Reliable Purturbative Results for Strong Interactions?"

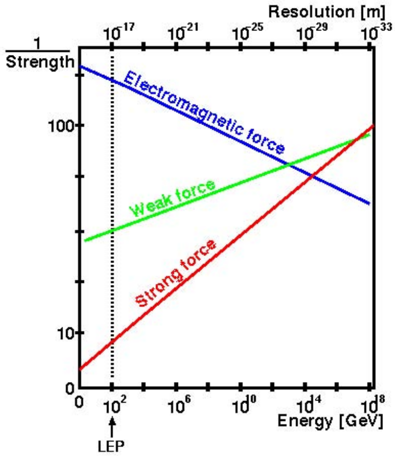
An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions.

跑动耦合常数


2008诺贝尔奖获得者北京论坛
戴维·格罗斯教授清华大学名誉教授聘请仪式暨学术演讲会
Address to Tsinghua University by Prof.David J. Gross
清华海外名师讲堂
TSINGHUA GLOBAL VISION LECTURES
清华大学
清华海外名师
TSINGHUA GLOBAL VISION

关于耦合常数的跑动行为与统一场论

- ♣ 强、弱和电磁作用都由规范场理论描述为统一提供了线索和基础！
- ♢ 但不同作用的强度不同，怎么统一？
- ♡ 作用强度 随能量尺度跑动 提供可能性！
- ♠ 弱和电磁作用重组强度在250GeV尺度上变成一样电弱统一！
- ¶ 强、弱和电磁作用的强度在$10^{15}GeV$的尺度上近似变成一样大统一！



- ✠ 超对称可在一圈图水平使相互作用强度在$10^{15}GeV$尺度上变成一样！

关于反常量纲的讨论: 考虑$2n$点费米m点光子的相互作用顶角

$$\Gamma[p_i,\alpha_{\overline{MS}}(\mu),M,\lambda(\mu),\mu]\text{的质量量纲为}4-3n-m$$

$$\Gamma_{2n+m}[e^t p_i,\alpha_{\overline{MS}}(\mu),M(\mu),\lambda(\mu),\mu]=\frac{e^{t(4-3n-m)}Z_2^{\overline{MS},-n}(e^t\mu)Z_3^{\overline{MS},-m/2}(e^t\mu)}{Z_2^{\overline{MS},-n}(\mu)Z_3^{\overline{MS},-m/2}(\mu)}\times\Gamma_{2n+m}[p_i,\alpha(t),e^{-t}M(t),\lambda(t),\mu]$$

$$\frac{Z_2^{\overline{MS},-n}(e^t\mu)Z_3^{\overline{MS},-m/2}(e^t\mu)}{Z_2^{\overline{MS},-n}(\mu)Z_3^{\overline{MS},-m/2}(\mu)}\equiv e^{tD_\Gamma(t)}\qquad D_\Gamma(t):\quad\text{反常量纲}$$

相互作用会改变顶角函数的标度变换行为!

- 相互作用导致反常量纲γ_i, $c_i'=c_i s^{g_i-\gamma_i}$ 使顶角偏离原始的标度行为
- 强（弱）的相互作用⇔强（弱）的标度行为破坏 QCD的建立
- 紫外（红外）渐进自由 ⇔ 紫外（红外）区域近似的原始标度行为
- 反常量纲可改变"相关","临界","不相关"性质 $\overline{\psi}\psi$在walking理论中从3变到1

为什么描述基本相互作用的量子场论一定是

非阿贝尔规范理论?

因为只有它是 自洽的量子场论！

- 能观测到的非零的耦合 不平庸！
- 有效的相互作用随尺度增长不断衰减，直到为零 渐进自由！
- 不必须要求在高能区出现新的相互作用 基本理论！
- 不同相互作用的耦合常数在高能区碰到一起 相互作用的统一！

课程对照

- 《规范场》70439024（物理系）； 《规范场论》70320264(工物系)
- 《粒子物理》70430064（物理系）; 70320244(工物系)

微扰QED

$$\mathcal{L}_{\rm QED}=-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\overline{\psi}(x)[i\partial\!\!\!/-eA\!\!\!/(x)-M_e-\delta M]\psi(x) \qquad \mathcal{L}_I(\psi,\pi)=\overline{\psi}(x)[-eA\!\!\!/(x)-\delta M]\psi(x)$$

$$S=\mathbf{T}\,e^{-i\int_{-\infty}^{\infty}dt\ V(\psi,\pi)}=\mathbf{T}\,e^{i\int d^4x\ \mathcal{L}_I(\psi,\pi)} \qquad V(\psi,\pi)=-\int d^3x\,\mathcal{L}_I(\psi,\pi)$$

$$\psi(x)=\sum_\sigma\int\frac{d\vec{p}}{(2\pi)^{3/2}}[e^{-ip\cdot x}u(\vec{p},\sigma)a(\vec{p},\sigma)+e^{ip\cdot x}v(\vec{p},\sigma)a^{c\dagger}(\vec{p},\sigma)]$$

$$\overline{\psi}(x)=\sum_\sigma\int\frac{d\vec{p}}{(2\pi)^{3/2}}[e^{-ip\cdot x}\overline{v}(\vec{p},\sigma)a^{c}(\vec{p},\sigma)+e^{ip\cdot x}\overline{u}(\vec{p},\sigma)a^{\dagger}(\vec{p},\sigma)]$$

$$A^\mu(x)=(2\pi)^{-3/2}\sum_{\sigma=\pm1}\int\frac{d\vec{p}}{\sqrt{2p^0}}[e^\mu(\vec{p},\sigma)a(\vec{p},\sigma)e^{-ip\cdot x}+e^{\mu*}(\vec{p},\sigma)a^\dagger(\vec{p},\sigma)e^{ip\cdot x}]$$

微扰展开

准到二阶波函数的抵消项没有贡献!

$$S=1+S_1+S_2+\cdots \qquad S_1=-ie\int d^4x\mathbf{T}[\overline{\psi}(x)A\!\!\!/(x)\psi(x)]$$

$$S_2=-\frac{e^2}{2}\int d^4xd^4y\mathbf{T}[\overline{\psi}(x)A\!\!\!/(x)\psi(x)\overline{\psi}(y)A\!\!\!/(y)\psi(y)]-i\delta M\int d^4x\mathbf{T}[\overline{\psi}(x)\psi(x)]$$

QED一阶S矩阵

$$S_1=-ie\int d^4x\,\mathbf{T}[\overline{\psi}(x)\not{A}(x)\psi(x)]=-ie\int d^4x\,\{\mathbf{N}[\overline{\psi}(x)\not{A}(x)\psi(x)]+\mathbf{N}[\underline{\overline{\psi}(x)\not{A}(x)\psi(x)}]\}$$

$$=-ie\int d^4x\,\{\mathbf{N}[\overline{\psi}(x)\not{A}(x)\psi(x)]+\underline{\overline{\psi}(x)\gamma_\mu\psi(x)}A^\mu(x)\}$$

$$\underline{\overline{\psi}(x)\gamma_\mu\psi(x)}=-\mathrm{tr}[\gamma_\mu\underline{\psi(x)\overline{\psi}(x)}]=\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}[\gamma_\mu\frac{i}{\not{p}-M+i0^\dagger}]=0$$

$$=-ie\int d^4x\,\{\underbrace{\overline{\psi}^+(x)\gamma^\mu\psi^+(x)A^+_\mu(x)}_{0\leftarrow e^+\,e\,\gamma}+\underbrace{\overline{\psi}^-(x)\gamma^\mu\psi^+(x)A^+_\mu(x)}_{e\leftarrow e\,\gamma}+\underbrace{\overline{\psi}^-(x)\gamma^\mu\psi^-(x)A^+_\mu(x)}_{e\,e^+\leftarrow\gamma}$$

$$-\underbrace{\mathrm{tr}[\psi^-(x)\overline{\psi}^+(x)\gamma^\mu]A^+_\mu(x)}_{e^+\leftarrow e^+\,\gamma}+\underbrace{A^-_\mu(x)\overline{\psi}^+(x)\gamma^\mu\psi^+(x)}_{\gamma\leftarrow e^+\,e}+\underbrace{\overline{\psi}^-(x)\gamma^\mu A^-_\mu(x)\psi^+(x)}_{e\,\gamma\leftarrow e}$$

$$+\underbrace{\overline{\psi}^-(x)\gamma^\mu\psi^-(x)A^-_\mu(x)}_{e\,e^+\,\gamma\leftarrow 0}-\underbrace{A^-_\mu(x)\mathrm{tr}[\gamma^\mu\psi^-(x)\overline{\psi}^+(x)]}_{\gamma\,e^+\leftarrow e^+}\}=0$$

S_1对矩阵元没有贡献！ 或者质心系能量、动量守恒无法实现

或条件$p_e^\mu+p_{e^+}^\mu=k^\mu$和$p_e^2=p_{e^+}^2=M_e^2$，$k^2=0$相冲突！

QED二阶S矩阵

$$S_2=-\frac{e^2}{2}\int d^4xd^4y\,\mathbf{T}[\overline{\psi}(x)\not{A}(x)\psi(x)\overline{\psi}(y)\not{A}(y)\psi(y)]-i\delta M\int d^4x\mathbf{T}[\overline{\psi}(x)\psi(x)]$$

$$=-\frac{e^2}{2}\int d^4xd^4y\{\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)]\underline{A_\mu(x)A_\nu(y)}$$

$$+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]+\mathbf{N}[\overline{\psi}(y)\gamma^\nu\underline{\psi(y)\overline{\psi}(x)}\gamma^\mu\psi(x)A_\mu(x)A_\nu(y)]$$

$$+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\psi(y)\underline{A_\mu(x)A_\nu(y)}]+\mathbf{N}[\overline{\psi}(y)\gamma^\nu\underline{\psi(y)\overline{\psi}(x)}\gamma^\mu\psi(x)\underline{A_\mu(x)A_\nu(y)}]$$

$$+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]\}+\ldots$$

$$=-\frac{e^2}{2}\int d^4xd^4y\{\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]$$ 因能动量守恒与质壳条件冲突对矩阵元无贡献!

$$+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)]\underline{A_\mu(x)A_\nu(y)}+2\mathbf{N}[\overline{\psi}(x)\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]$$

$$+2\mathbf{N}[\overline{\psi}(x)\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\psi(y)\underline{A_\mu(x)A_\nu(y)}]+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]$$

$$+\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]\}-i\delta M\int d^4x\{\mathbf{N}[\overline{\psi}(x)\psi(x)]-\mathrm{tr}[\underline{\psi(x)\overline{\psi}(x)}]\}$$

QED二阶S矩阵

$S_2\big|_{\text{对矩阵元有贡献的部份}}$

$$=-i\delta M\int d^4x\{\underbrace{\mathbf{N}[\overline{\psi}(x)\psi(x)]}_{\text{电子自能}}-\underbrace{\mathrm{tr}[\psi(x)\overline{\psi}(x)]}_{\text{真空}\to\text{真空}}\}-\frac{e^2}{2}\int d^4xd^4y\{\underbrace{\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]}_{\text{真空}\to\text{真空}}$$

$$+\underbrace{2\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]}_{\text{电子自能}}+\underbrace{\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]}_{\text{光子自能}}$$

$$+\underbrace{\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)]A_\mu(x)A_\nu(y)}_{\text{4电子过程}}+\underbrace{2\mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)]}_{\text{2电子2光子过程}}\}$$

QED二阶S矩阵中真空真空跃迁项

$$S_2|_{真空\to真空} = -\frac{e^2}{2}\int d^4x d^4y\, \mathrm{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)] + i\delta M\int d^4x \mathrm{tr}[\psi(x)\overline{\psi}(x)]$$

$$= \frac{e^2}{2}\int d^4x d^4y\, \mathrm{tr}[\psi(y)\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu]A_\mu(x)A_\nu(y) + i\delta M\int d^4x \mathrm{tr}[\psi(x)\overline{\psi}(x)]$$

$$= -\frac{ie^2}{2}\int d^4x d^4y \int \frac{d^4p d^4p' d^4q}{(2\pi)^{12}} e^{-i(p-p'+q)\cdot(x-y)} \mathrm{tr}[\frac{1}{\not{p}'-M_e+i0^\dagger}\gamma^\mu \frac{1}{\not{p}-M_e+i0^\dagger}\gamma^\nu] \frac{-g_{\mu\nu}+(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}}{q^2+i0^\dagger}$$

$$= -\frac{ie^2}{2}\int d^4x \int \frac{d^4p d^4q}{(2\pi)^8} \mathrm{tr}[\frac{1}{\not{p}+\not{q}-M_e+i0^\dagger}\gamma^\mu \frac{1}{\not{p}-M_e+i0^\dagger}\gamma^\nu] \frac{-g_{\mu\nu}+(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}}{q^2+i0^\dagger} + \ldots$$

$$(\Phi_0, S\Phi_0) = 1 - \frac{ie^2}{2}\int d^4x \int \frac{d^4p d^4q}{(2\pi)^8} \mathrm{tr}[\frac{1}{\not{p}+\not{q}-M_e+i0^\dagger}\gamma^\mu \frac{1}{\not{p}-M_e+i0^\dagger}\gamma^\nu] \frac{-g_{\mu\nu}+(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}}{q^2+i0^\dagger}$$

$$-\delta M\int d^4x \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\frac{1}{\not{p}-M_e+i0^\dagger} + O(e^4)$$

应该只贡献纯相因子！

四次和二次发散： 去掉发散需要在拉氏量中加入与场量无关的纯真空能抵消项!

QED二阶S矩阵中电子自能跃迁项

$$S_2|_{电子自能} = -e^2\int d^4x d^4y\, \mathbf{N}[\overline{\psi}(x)\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\psi(y)]\underline{A_\mu(x)A_\nu(y)} - i\delta M\int d^4x \mathbf{N}[\overline{\psi}(x)\psi(x)]$$

$$= -e^2\int d^4x d^4y\, \mathrm{tr}\{\mathbf{N}[\psi(y)\overline{\psi}(x)]\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\}\underline{A_\mu(x)A_\nu(y)} - i\delta M\int d^4x \mathbf{N}[\overline{\psi}(x)\psi(x)]$$

$$= -e^2\int d^4x d^4y\, \mathrm{tr}\{[\underbrace{\psi^+(y)\overline{\psi}^+(x)}_{0\leftarrow e\,e^+\ \text{被能量守恒禁戒}} + \underbrace{\psi^-(y)\overline{\psi}^-(x)}_{e\,e^+\leftarrow 0\ \text{被能量守恒禁戒}} + \underbrace{\psi^-(y)\overline{\psi}^+(x)}_{e^+\leftarrow e^+}$$

$$-\underbrace{\overline{\psi}^-(x)\psi^+(y)}_{e\leftarrow e}]\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\}\underline{A_\mu(x)A_\nu(y)} - i\delta M\int d^4x \mathbf{N}[\overline{\psi}(x)\psi(x)]$$

$$= -e^2\int d^4x d^4y \int \frac{d\vec{p}d\vec{p}'}{(2\pi)^3}\frac{d^4q d^4q'}{(2\pi)^8} e^{-i(q+q')\cdot(x-y)} \sum_{\sigma\sigma'} \mathrm{tr}\{[e^{-ip\cdot x+ip'\cdot y} v(\vec{p}',\sigma')\overline{v}(\vec{p},\sigma)a^{c\dagger}(\vec{p}',\sigma')a^c(p,\sigma)$$

$$-e^{ip\cdot x-ip'\cdot y}\overline{u}(\vec{p},\sigma)u(\vec{p}',\sigma')a^\dagger(\vec{p},\sigma)a(\vec{p}',\sigma')]\gamma^\mu\frac{1}{\not{q}'-M_e+i0^\dagger}\gamma^\nu\}\frac{g_{\mu\nu}-(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}}{q^2+i0^\dagger}+\ldots$$

QED二阶S矩阵中电子自能跃迁项

$S_2|_{电子自能}$

$$= -e^2\int d^4xd^4y\int \frac{d\vec{p}d\vec{p}'}{(2\pi)^3}\frac{d^4qd^4q'}{(2\pi)^8}e^{-i(q+q')\cdot(x-y)}\sum_{\sigma\sigma'}\text{tr}\{[e^{-ip\cdot x+ip'\cdot y}v(\vec{p}',\sigma')\overline{v}(\vec{p},\sigma)a^{c\dagger}(\vec{p}',\sigma')a^c(p,\sigma)$$

$$-e^{ip\cdot x-ip'\cdot y}\overline{u}(\vec{p},\sigma)u(\vec{p}',\sigma')a^\dagger(\vec{p},\sigma)a(\vec{p}',\sigma')]\gamma^\mu\frac{1}{\not{q}'-M_e+i0^\dagger}\gamma^\nu\}\frac{g_{\mu\nu}-(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}}{q^2+i0^\dagger}+\ldots$$

$$\overset{q'\to q'-q}{=\!=\!=\!=\!=}-2\pi\int d\vec{p}d\vec{p}'d^4q'\sum_{\sigma\sigma'}\text{tr}\{[\delta^4(p+q')\delta^4(p'+q')v(\vec{p}',\sigma')\overline{v}(\vec{p},\sigma)a^{c\dagger}(\vec{p}',\sigma')a^c(\vec{p},\sigma)$$

$$-\delta^4(-p+q')\delta^4(-p'+q')\overline{u}(\vec{p},\sigma)u(\vec{p}',\sigma')a^\dagger(\vec{p},\sigma)a(\vec{p}',\sigma')]\Sigma(q')\}-i\delta M\int d^4x\mathbf{N}[\overline{\psi}(x)\psi(x)]$$

$$\Sigma(q')=e^2\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\frac{1}{\not{q}'-\not{q}-M_e+i0^\dagger}\gamma^\nu\frac{g_{\mu\nu}-(1-\frac{1}{\lambda})\frac{q_\mu q_\nu}{q^2}}{q^2+i0^\dagger}$$

$$=-2\pi\int d\vec{p}d\vec{p}'\ \delta^4(p-p')\sum_{\sigma\sigma'}\text{tr}\{\overline{v}(\vec{p},\sigma)[\Sigma(-p)-i\delta M]v(\vec{p},\sigma')a^{c\dagger}(\vec{p},\sigma')a^c(\vec{p},\sigma)$$

$$-\overline{u}(\vec{p},\sigma)[\Sigma(p)-i\delta M]u(\vec{p},\sigma')a^\dagger(\vec{p},\sigma)a(\vec{p},\sigma')\}$$

QED二阶S矩阵中电子自能跃迁项

$$S_2\big|_{电子自能} = -2\pi \int d\vec{p} d\vec{p}' \; \delta^4(p-p') \sum_{\sigma\sigma'} \mathrm{tr}\{\bar{v}(\vec{p},\sigma)[\Sigma(-p) - i\delta M] v(\vec{p},\sigma') a^{c\dagger}(\vec{p},\sigma') a^c(\vec{p},\sigma)$$
$$-\bar{u}(\vec{p},\sigma)[\Sigma(p) - i\delta M] u(\vec{p},\sigma') a^\dagger(\vec{p},\sigma) a(\vec{p},\sigma')\}$$

在壳重整化 $p^2 = p^{c2} = M_e^2 \quad (\not{p} - M_e)u(\vec{p},\sigma) = (\not{p} + M_e)v(\vec{p},\sigma) = 0$

$$\Sigma(p) = \not{p}\Sigma_A(p^2) - M_e\Sigma_B(p^2) \qquad 0 = \tilde{\Sigma}_A(M_e^2) = i\Sigma_A(M_e^2) + \delta Z_2 \qquad \Sigma_A(M_e^2) = i\delta Z_2$$

$$0 = \tilde{\Sigma}_B(M_e^2) = i\Sigma_B(M_e^2) + \delta Z_2 - \delta M/M_e \qquad \Sigma_B(M_e^2) = i\delta Z_2 - i\delta M/M_e$$

$$\bar{u}(\vec{p},\sigma)[\Sigma(p) - i\delta M]u(\vec{p},\sigma') = \bar{u}(\vec{p},\sigma)[M_e\Sigma_A(M_e^2) - M_e\Sigma_B(M_e^2) - i\delta M]u(\vec{p},\sigma') = 0$$

$$\bar{v}(\vec{p},\sigma)[\Sigma(-p) - i\delta M]v(\vec{p},\sigma') = \bar{v}(\vec{p},\sigma)[M_e\Sigma_A(M_e^2) - M_e\Sigma_B(M_e^2) - i\delta M]v(\vec{p},\sigma') = 0$$

电子是稳定的!

$$(\Phi_{\vec{p}_e',\sigma'}, S\Phi_{\vec{p}_e,\sigma})\Big|_{准到二阶} = \delta^4(p_e - p_e')\{\delta_{\sigma\sigma'} + 2\pi\bar{u}(\vec{p}_e,\sigma)[\Sigma(p_e) - i\delta M]u(\vec{p}_e,\sigma')\} = \delta^4(p_e - p_e')\delta_{\sigma\sigma'}$$

$$(\Phi_{\vec{p}_{e+}',\sigma'}, S\Phi_{\vec{p}_{e+},\sigma})\Big|_{准到二阶} = \delta^4(p_{e+}' - p_{e+})\{\delta_{\sigma\sigma'} - 2\pi\bar{v}(\vec{p}_{e+},\sigma)[\Sigma(-p_{e+}) - i\delta M]v(\vec{p}_{e+},\sigma')\} = \delta^4(p_{e+} - p_{e+}')\delta_{\sigma\sigma'}$$

QED二阶S矩阵中光子自能跃迁项

$$S_2|_{\text{光子自能}} = -\frac{e^2}{2}\int d^4x d^4y\, \mathbf{N}[\underline{\overline{\psi}(x)\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu\psi(y)}A_\mu(x)A_\nu(y)]$$

$$= \frac{e^2}{2}\int d^4x d^4y\, \mathrm{tr}[\underline{\psi(y)\overline{\psi}(x)}\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]\mathbf{N}[A_\mu(x)A_\nu(y)]$$

$$= \frac{e^2}{2}\int d^4x d^4y\, \mathrm{tr}[\underline{\psi(y)\overline{\psi}(x)}\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu][A_\mu^-(x)A_\nu^+(y)+A_\nu^-(y)A_\mu^+(x)]$$

$$= e^2\int d^4x d^4y\, \mathrm{tr}[\underline{\psi(y)\overline{\psi}(x)}\gamma^\mu\underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]A_\mu^-(x)A_\nu^+(y)$$

$$= -e^2\int d^4x d^4y \int \frac{d\vec{p}d\vec{p}'}{2\sqrt{p^0p'^0}(2\pi)^3}\frac{d^4q'd^4q}{(2\pi)^8}e^{-i(q'-q)\cdot(x-y)}\mathrm{tr}[\frac{1}{\not{q}-M_e+i0^\dagger}\gamma^\mu\frac{1}{\not{q}'-M_e+i0^\dagger}\gamma^\nu]$$

$$\times\sum_{\sigma\sigma'}e^{-ip\cdot y+ip'\cdot x}e_\mu^*(\vec{p}',\sigma')e_\nu(\vec{p},\sigma)a^\dagger(\vec{p}',\sigma')a(\vec{p},\sigma)$$

$$= -e^2\int\frac{d\vec{p}d\vec{p}'}{2\sqrt{p^0p'^0}(2\pi)^3}d^4q'd^4q\,\mathrm{tr}[\frac{1}{\not{q}-M_e+i0^\dagger}\gamma^\mu\frac{1}{\not{q}'-M_e+i0^\dagger}\gamma^\nu]$$

$$\times\delta(q'-q-p')\delta(q'-q-p)\sum e_\mu^*(\vec{p}',\sigma')e_\nu(\vec{p},\sigma)a^\dagger(\vec{p}',\sigma')a(\vec{p},\sigma)$$

QED二阶S矩阵中光子自能跃迁项

$$S_2|_{光子自能} = e^2 \int d^4x d^4y \,\mathrm{tr}[\underline{\psi(y)\overline{\psi}(x)}\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu] A_\mu^-(x) A_\nu^+(y)$$

$$= -e^2 \int \frac{d\vec{p}d\vec{p}'}{2\sqrt{p^0 p'^0}(2\pi)^3} d^4q' d^4q \,\mathrm{tr}[\frac{1}{\not{q} - M_e + i0^\dagger}\gamma^\mu \frac{1}{\not{q}' - M_e + i0^\dagger}\gamma^\nu]$$

$$\times \delta(q' - q - p')\delta(q' - q - p) \sum_{\sigma\sigma'} e_\mu^*(\vec{p}', \sigma')e_\nu(\vec{p}, \sigma)a^\dagger(\vec{p}', \sigma')a(\vec{p}, \sigma)$$

$$= -\pi \int \frac{d\vec{p}d\vec{p}'}{\sqrt{p^0 p'^0}} \delta(p' - p)\; e^2 \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}[\frac{1}{\not{q} - M_e + i0^\dagger}\gamma^\mu \frac{1}{\not{q} + \not{p} - M_e + i0^\dagger}\gamma^\nu]$$

$$\times \sum_{\sigma\sigma'} e_\mu^*(\vec{p}', \sigma')e_\nu(\vec{p}, \sigma)a^\dagger(\vec{p}', \sigma')a(\vec{p}, \sigma)$$

$$= -\pi \int \frac{d\vec{p}d\vec{p}'}{\sqrt{p^0 p'^0}} \delta^4(p - p')\Pi^{\mu\nu}(p) \sum_{\sigma\sigma'} e_\mu^*(\vec{p}', \sigma')e_\nu(\vec{p}, \sigma)a^\dagger(\vec{p}', \sigma')a(\vec{p}, \sigma)$$

QED二阶S矩阵中光子自能跃迁项

$$S_2|_{\text{光子自能}} = -\pi \int \frac{d\vec{p}d\vec{p}'}{\sqrt{p^0 p'^0}} \delta^4(p-p')\Pi^{\mu\nu}(p) \sum_{\sigma\sigma'} e^*_\mu(\vec{p}',\sigma')e_\nu(\vec{p},\sigma)a^\dagger(\vec{p}',\sigma')a(\vec{p},\sigma)$$

$$\overset{p^2=0,\ p^\nu e_\nu(\vec{p},\sigma)=0}{=\!=\!=\!=\!=\!=\!=\!=\!=} \pi \int \frac{d\vec{p}d\vec{p}'}{\sqrt{p^0 p'^0}} \delta^4(p-p')[p^\mu p^\nu - p^2 g^{\mu\nu}]\pi(p^2) \sum_{\sigma\sigma'} e^*_\mu(\vec{p}',\sigma')e_\nu(\vec{p},\sigma)a^\dagger(\vec{p}',\sigma')a(\vec{p},\sigma)$$

$$= 0$$

光子是稳定的!

$$(\Phi_{\vec{p}'_\gamma,\sigma'}, S\Phi_{\vec{p}_\gamma,\sigma})\Big|_{\text{准到二阶}}$$

$$= \delta^4(p_\gamma - p'_\gamma)\{\delta_{\sigma\sigma'} + \frac{\pi}{\sqrt{p^0_\gamma p'^0_\gamma}}[p^\mu_\gamma p^\nu_\gamma - p^2_\gamma g^{\mu\nu}]\pi(p^2_\gamma) \sum_{\sigma\sigma'} e^*_\mu(\vec{p}'_\gamma,\sigma')e_\nu(\vec{p}_\gamma,\sigma)a^\dagger(\vec{p}'_\gamma,\sigma')a(\vec{p}_\gamma,\sigma)\}$$

$$= \delta^4(p_\gamma - p'_\gamma)\delta_{\sigma\sigma'}$$

QED二阶S矩阵中4电子过程项

分析各种可能的项

- $0 \to 4$和$4 \to 0$过程不满足能量守恒，被禁戒
- $1 \to 3$和$3 \to 1$在1的静止系看不能满足能量守恒，被禁戒
- 只有$2 \to 2$的过程是被允许的

所有可能的$2 \to 2$过程

$$S_2|_{\textbf{4电子过程项}} = -\frac{e^2}{2}\int d^4x d^4y\, \mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)]\underline{A_\mu(x)A_\nu(y)}$$

$$= -\frac{e^2}{2}\int d^4x d^4y\, \{\overline{\psi}^-(y)\gamma^\nu\psi^-(y)\overline{\psi}^+(x)\gamma^\mu\psi^+(x) + \overline{\psi}^-(x)\gamma^\mu\psi^-(x)\overline{\psi}^+(y)\gamma^\nu\psi^+(y)$$

$$-\gamma^\mu_{l_1l_2}\psi^-_{l_2}(x)\gamma^\nu_{l_3l_4}\psi^-_{l_4}(y)\overline{\psi}^+_{l_1}(x)\overline{\psi}^+_{l_3}(y) - \overline{\psi}^-_{l_1}(x)\gamma^\mu_{l_1l_2}\overline{\psi}^-_{l_3}(y)\gamma^\nu_{l_3l_4}\psi^+_{l_2}(x)\psi^+_{l_4}(y)$$

$$+\mathrm{tr}[\gamma^\mu\psi^-(x)\overline{\psi}^-(y)\gamma^\nu\psi^+(y)\overline{\psi}^+(x)] + \mathrm{tr}[\gamma^\nu\psi^-(y)\overline{\psi}^-(x)\gamma^\mu\psi^+(x)\overline{\psi}^+(y)]\}\underline{A_\mu(x)A_\nu(y)}$$

QED二阶S矩阵中4电子过程项

所有可能的$2\to 2$过程

$$S_2|_{4电子过程项} = -\frac{e^2}{2}\int d^4x d^4y\, \mathbf{N}[\overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(y)]A_\mu(x)A_\nu(y)$$

$$= -\frac{e^2}{2}\int d^4x d^4y\, \{\overline{\psi}^-(y)\gamma^\nu\psi^-(y)\overline{\psi}^+(x)\gamma^\mu\psi^+(x) + \overline{\psi}^-(x)\gamma^\mu\psi^-(x)\overline{\psi}^+(y)\gamma^\nu\psi^+(y)$$
$$-\gamma^\mu_{l_1l_2}\psi^-_{l_2}(x)\gamma^\nu_{l_3l_4}\psi^-_{l_4}(y)\overline{\psi}^+_{l_1}(x)\overline{\psi}^+_{l_3}(y) - \overline{\psi}^-_{l_1}(x)\gamma^\mu_{l_1l_2}\overline{\psi}^-_{l_3}(y)\gamma^\nu_{l_3l_4}\psi^+_{l_2}(x)\psi^+_{l_4}(y)$$
$$-\mathrm{tr}[\gamma^\mu\psi^-(x)\overline{\psi}^-(y)\gamma^\nu\psi^+(y)\overline{\psi}^+(x)] - \mathrm{tr}[\gamma^\nu\psi^-(y)\overline{\psi}^-(x)\gamma^\mu\psi^+(x)\overline{\psi}^+(y)]\}A_\mu(x)A_\nu(y)$$

$$= -\frac{e^2}{2}\int d^4x d^4y\, \{\underbrace{2\overline{\psi}^-(y)\gamma^\nu\psi^-(y)\overline{\psi}^+(x)\gamma^\mu\psi^+(x) - 2\mathrm{tr}[\gamma^\mu\psi^-(x)\overline{\psi}^-(y)\gamma^\nu\psi^+(y)\overline{\psi}^+(x)]}$$

虚湮灭和再产生　电子正电子散射(Bhabha散射)：$e\ e^+ \leftarrow e\ e^+$　交换虚光子

$$-\underbrace{\gamma^\mu_{l_1l_2}\psi^-_{l_2}(x)\gamma^\nu_{l_3l_4}\psi^-_{l_4}(y)\overline{\psi}^+_{l_1}(x)\overline{\psi}^+_{l_3}(y)} - \underbrace{\overline{\psi}^-_{l_1}(x)\gamma^\mu_{l_1l_2}\overline{\psi}^-_{l_3}(y)\gamma^\nu_{l_3l_4}\psi^+_{l_2}(x)\psi^+_{l_4}(y)}\}\, A_\mu(x)A_\nu(y)$$

正电子正电子散射：$e^+\ e^+ \leftarrow e^+\ e^+$　电子电子散射(M∅ller散射)：$e\ e \leftarrow e\ e$

基本物理过程

S_2 **Bhabha散射**

$$= -e^2\int d^4x d^4y \{\overline{\psi}^-(y)\gamma^\nu\psi^-(y)\overline{\psi}^+(x)\gamma^\mu\psi^+(x) - \mathrm{tr}[\gamma^\mu\psi^-(x)\overline{\psi}^-(y)\gamma^\nu\psi^+(y)\overline{\psi}^+(x)]\ A_\mu(x)A_\nu(y)$$

虚湮灭和再产生　电子正电子散射(Bhabha散射)： $e\ e^+ \leftarrow e\ e^+$　交换虚光子

Feynman gauge

$$========= ie^2\int d^4xd^4y\int\frac{d^3p_1d^3p_2d^3p_3d^3p_4}{(2\pi)^6}\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\Big[e^{ip_1\cdot y+ip_2\cdot y-ip_3\cdot x-ip_4\cdot x}\overline{u}(\vec{p}_1,\sigma_1)\gamma_\mu v(\vec{p}_2,\sigma_2)$$

$$\times\overline{v}(\vec{p}_3,\sigma_3)\gamma^\mu u(\vec{p}_4,\sigma_4)\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq\cdot(x-y)}}{q^2+i0+}a^\dagger(\vec{p}_1,\sigma_1)a^{c\dagger}(\vec{p}_2,\sigma_2)a^c(\vec{p}_3,\sigma_3)a(\vec{p}_4,\sigma_4) - e^{ip_1\cdot x+ip_2\cdot y-ip_3\cdot y-ip_4\cdot x}$$

$$\times\mathrm{tr}[\gamma^\mu v(\vec{p}_1,\sigma_1)\overline{u}(\vec{p}_2,\sigma_2)\gamma_\mu u(\vec{p}_3,\sigma_3)\overline{v}(\vec{p}_4,\sigma_4)]\int\frac{d^4q'}{(2\pi)^4}\frac{e^{iq'\cdot(x-y)}}{q'^2+i0+}a^{\dagger,c}(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)a(\vec{p}_3,\sigma_3)a^c(\vec{p}_4,\sigma_4)$$

$$= ie^2(2\pi)^{-2}\int d^3p_1d^3p_2d^3p_3d^3p_4\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\Big[\delta(p_1+p_2-p_3-p_4)\overline{u}(\vec{p}_1,\sigma_1)\gamma_\mu v(\vec{p}_2,\sigma_2)\overline{v}(\vec{p}_3,\sigma_3)\gamma^\mu u(\vec{p}_4,\sigma_4)$$

$$\times\frac{1}{(p_1+p_2)^2+i0+}a^\dagger(\vec{p}_1,\sigma_1)a^{c\dagger}(\vec{p}_2,\sigma_2)a^c(\vec{p}_3,\sigma_3)a(\vec{p}_4,\sigma_4) - \delta(p_2-p_3-p_4+p_1)$$

$$\times\mathrm{tr}[\gamma^\mu v(\vec{p}_1,\sigma_1)\overline{u}(\vec{p}_2,\sigma_2)\gamma_\mu u(\vec{p}_3,\sigma_3)\overline{v}(\vec{p}_4,\sigma_4)]\frac{1}{(p_2-p_3)^2+i0+}a^{c\dagger}(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)a(\vec{p}_3,\sigma_3)a^c(\vec{p}_4,\sigma_4)\Big]$$

基本物理过程

$$S_2\big|_{\text{正电子正电子散射}} = e^2/2 \int d^4x d^4y\, \gamma^{\mu}_{l_1 l_2}\psi^-_{l_2}(x)\gamma^{\nu}_{l_3 l_4}\psi^-_{l_4}(y)\overline{\psi}^+_{l_1}(x)\overline{\psi}^+_{l_3}(y)\, \underline{A_\mu(x)A_\nu(y)}$$

$$\overset{\text{Feynman gauge}}{=\!=\!=\!=\!=\!=\!=\!=} -ie^2\int d^4x d^4y \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^6} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^{ip_1\cdot x+ip_2\cdot y-ip_3\cdot x-ip_4\cdot y}\gamma^{\mu}_{l_1 l_2} v_{l_2}(\vec{p}_1,\sigma_1)$$
$$\times\gamma^{\nu}_{l_3 l_4} v_{l_4}(\vec{p}_2,\sigma_2)\bar{v}_{l_1}(\vec{p}_3,\sigma_3)\bar{v}_{l_3}(\vec{p}_4,\sigma_4)\int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)}\frac{g_{\mu\nu}}{q^2+i0^+}$$
$$\times a^{c\dagger}(\vec{p}_1,\sigma_1)a^{c\dagger}(\vec{p}_2,\sigma_2)a^{c}(\vec{p}_3,\sigma_3)a^{c}(\vec{p}_4,\sigma_4)$$

$$= -ie^2\int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^2}\delta(p_1+p_2-p_3-p_4)\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\frac{\bar{v}(\vec{p}_3,\sigma_3)\gamma^\mu v(\vec{p}_1,\sigma_1)\bar{v}_{l_3}(\vec{p}_4,\sigma_4)\gamma_\mu v(\vec{p}_2,\sigma_2)}{(p_1-p_3)^2+i0^+}$$
$$\times\, a^{c\dagger}(\vec{p}_1,\sigma_1)a^{c\dagger}(\vec{p}_2,\sigma_2)a^{c}(\vec{p}_3,\sigma_3)a^{c}(\vec{p}_4,\sigma_4)$$

基本物理过程

$$S_2\big|_{\text{Møller散射}} = e^2/2\int d^4x d^4y\, \overline{\psi}^-_{l_1}(x)\gamma^\mu_{l_1l_2}\overline{\psi}^-_{l_3}(y)\gamma^\nu_{l_3l_4}\psi^+_{l_2}(x)\psi^+_{l_4}(y)\, \underline{A_\mu(x)A_\nu(y)}$$

$$\overset{\text{Feynman gauge}}{=\!=\!=\!=\!=\!=\!=\!=} -ie^2\int d^4x d^4y \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^6} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^{ip_1\cdot x+ip_2\cdot y-ip_3\cdot x-ip_4\cdot y}\overline{u}_{l_1}(\vec{p}_1,\sigma_1)$$
$$\times\gamma^\mu_{l_1l_2}\overline{u}_{l_3}(\vec{p}_2,\sigma_2)\gamma^\nu_{l_3l_4}u_{l_2}(\vec{p}_3,\sigma_3)u_{l_4}(\vec{p}_4,\sigma_4)\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\frac{g_{\mu\nu}}{q^2+i0^+}$$
$$\times\, a^\dagger(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)a(\vec{p}_3,\sigma_3)a(\vec{p}_4,\sigma_4)$$

$$= -ie^2\int\frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^2}\delta(p_1+p_2-p_3-p_4)\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\frac{\overline{u}(\vec{p}_1,\sigma_1)\gamma^\mu u(\vec{p}_3,\sigma_3)\overline{u}(\vec{p}_2,\sigma_2)\gamma_\mu u(\vec{p}_4,\sigma_4)}{(p_1-p_3)^2+i0^+}$$
$$\times\, a^\dagger(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)a(\vec{p}_3,\sigma_3)a(\vec{p}_4,\sigma_4)$$

QED二阶S矩阵中2电子2光子过程项

分析各种可能的项

- $0 \to 4$和$4 \to 0$过程不满足能量守恒，被禁戒
- 电子$\to 3$和$3 \to$电子过程在电子的静止系不能满足能量守恒，被禁戒
- 光子$\to 3$和$3 \to$光子过程在2电子质心系不能满足能量守恒，被禁戒
- 只有$2 \to 2$的过程是被允许的

所有可能的$2 \to 2$过程

$$S_2|_{\text{2电子2光子过程项}} = -\frac{e^2}{2}\int d^4x d^4y \, \mathrm{N}[\overline{\psi}(x)\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu \psi(y) A_\mu(x) A_\nu(y)]$$

$$= \frac{e^2}{2}\int d^4x d^4y \, [\psi_{l'}^-(y)\overline{\psi}_l^-(x)A_\mu^+(x)A_\nu^+(y) + A_\mu^-(x)A_\nu^-(y)\psi_{l'}^+(y)\overline{\psi}_l^+(x)$$
$$-\overline{\psi}_l^-(x)A_\mu^-(x)A_\nu^+(y)\psi_{l'}^+(y) + A_\nu^-(y)\psi_{l'}^-(y)\overline{\psi}_l^+(x)A_\mu^+(x)$$
$$+A_\mu^-(x)\psi_{l'}^-(y)\overline{\psi}_l^+(x)A_\nu^+(y) - A_\nu^-(y)\overline{\psi}_l^-(x)\psi_{l'}^+(y)A_\mu^+(x)][\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

QED二阶S矩阵中2电子2光子过程项

所有可能的$2 \to 2$过程

$$S_2|_{\text{2电子2光子过程项}} = -\frac{e^2}{2}\int d^4x d^4y\, \mathbf{N}[\overline{\psi}(x)\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu \psi(y) A_\mu(x) A_\nu(y)]$$

$$= \frac{e^2}{2}\int d^4x d^4y\, [\psi_{l'}^-(y)\overline{\psi}_l^-(x)A_\mu^+(x)A_\nu^+(y) + A_\mu^-(x)A_\nu^-(y)\psi_{l'}^+(y)\overline{\psi}_l^+(x)$$
$$-\overline{\psi}_l^-(x)A_\mu^-(x)A_\nu^+(y)\psi_{l'}^+(y) + A_\nu^-(y)\psi_{l'}^-(y)\overline{\psi}_l^+(x)A_\mu^+(x)$$
$$+A_\mu^-(x)\psi_{l'}^-(y)\overline{\psi}_l^+(x)A_\nu^+(y) - A_\nu^-(y)\overline{\psi}_l^-(x)\psi_{l'}^+(y)A_\mu^+(x)][\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

$$= \frac{e^2}{2}\int d^4x d^4y\, [\ \underbrace{\psi_{l'}^-(y)\overline{\psi}_l^-(x)A_\mu^+(x)A_\nu^+(y)} + \underbrace{A_\mu^-(x)A_\nu^-(y)\psi_{l'}^+(y)\overline{\psi}_l^+(x)}$$

光子对撞正负电子对产生：$e^+\ e \leftarrow \gamma\ \gamma$　　正负电子对湮灭成一对光子：$\gamma\ \gamma \leftarrow e^+\ e$

$$- \underbrace{\overline{\psi}_l^-(x)A_\mu^-(x)A_\nu^+(y)\psi_{l'}^+(y)} + \underbrace{A_\nu^-(y)\psi_{l'}^-(y)\overline{\psi}_l^+(x)A_\mu^+(x)}\ ]\ [\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

电子光子散射(Compton散射)：$e\ \gamma \leftarrow e\ \gamma$　　正电子光子散射：$e^+\ \gamma \leftarrow e^+\ \gamma$

基本物理过程

$$S_2|_{光子对撞正负电子对产生} = \frac{e^2}{2}\int d^4x d^4y \underbrace{\psi^-_{l'}(y)\overline{\psi}^-_l(x)A^+_\mu(x)A^+_\nu(y)}_{光子对撞正负电子对产生:\ e^+\ e \leftarrow \gamma\ \gamma} [\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

$$= -\frac{ie^2}{2}\int d^4x d^4y \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^6\sqrt{2(p^3)^0}\sqrt{2(p^4)^0}} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^{ip_1\cdot y+ip_2\cdot x-ip_3\cdot x-ip_4\cdot y} v_{l'}(\vec{p}_1,\sigma_1)\overline{u}_l(\vec{p}_2,\sigma_2)e_\mu(\vec{p}_3,\sigma_3)$$

$$\times e_\nu(\vec{p}_4,\sigma_4)\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\left[\gamma^\mu\frac{1}{\not{q}-M+i0^+}\gamma^\nu\right]_{ll'} a^{c\dagger}(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)a_\gamma(\vec{p}_3,\sigma_3)a_\gamma(\vec{p}_4,\sigma_4)$$

$$= -\frac{ie^2}{2}\int\frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^2\sqrt{2(p^3)^0}\sqrt{2(p^4)^0}}\delta(p_1+p_2-p_3-p_4)\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e_\mu(\vec{p}_3,\sigma_3)e_\nu(\vec{p}_4,\sigma_4)$$

$$\times\overline{u}(\vec{p}_2,\sigma_2)\left[\gamma^\mu\frac{\not{p}_2-\not{p}_3+M}{(p_2-p_3)^2-M^2+i0^+}\gamma^\nu\right]v(\vec{p}_1,\sigma_1)\ a^{c\dagger}(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)a_\gamma(\vec{p}_3,\sigma_3)a_\gamma(\vec{p}_4,\sigma_4)$$

基本物理过程

$$S_2|_{\text{正负电子对湮灭成一对光子}} = \frac{e^2}{2}\int d^4x d^4y \quad \underbrace{A^-_\mu(x)A^-_\nu(y)\psi^+_{l'}(y)\overline{\psi}^+_l(x)} \quad [\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

正负电子对湮灭成一对光子： $\gamma\ \gamma \leftarrow e^+\ e$

$$= -\frac{ie^2}{2}\int d^4x d^4y \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^6\sqrt{2(p^1)^0}\sqrt{2(p^2)^0}} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^{ip_1\cdot x+ip_2\cdot y-ip_3\cdot y-ip_4\cdot x} e^*_\mu(\vec{p}_1,\sigma_1)e^*_\nu(\vec{p}_2,\sigma_2)u_{l'}(\vec{p}_3,\sigma$$

$$\times\bar{v}_l(\vec{p}_4,\sigma_4)\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\left[\gamma^\mu\frac{1}{\not{q}-M+i0^+}\gamma^\nu\right]_{ll'} a^\dagger_\gamma(\vec{p}_1,\sigma_1)a^\dagger_\gamma(\vec{p}_2,\sigma_2)a(\vec{p}_3,\sigma_3)a^c(\vec{p}_4,\sigma_4)$$

$$= -\frac{ie^2}{2}\int\frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^2\sqrt{2(p^1)^0}\sqrt{2(p^2)^0}}\delta(p_1+p_2-p_3-p_4)\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^*_\mu(\vec{p}_1,\sigma_1)e^*_\nu(\vec{p}_2,\sigma_2)$$

$$\times\bar{v}(\vec{p}_4,\sigma_4)\left[\gamma^\mu\frac{\not{p}_1-\not{p}_4+M}{(p_1-p_4)^2-M^2+i0^+}\gamma^\nu\right]u(\vec{p}_3,\sigma_1)a^\dagger_\gamma(\vec{p}_1,\sigma_1)a^\dagger_\gamma(\vec{p}_2,\sigma_2)a(\vec{p}_3,\sigma_3)a^c(\vec{p}_4,\sigma_4)$$

基本物理过程

$$S_2|_{\text{Compton散射}} = -\frac{e^2}{2}\int d^4x d^4y \quad \underbrace{\overline{\psi}_l^-(x) A_\mu^-(x) A_\nu^+(y) \psi_{l'}^+(y)} \quad [\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

电子光子散射(Compton散射)： $e\ \gamma \leftarrow e\ \gamma$

$$= \frac{ie^2}{2}\int d^4x d^4y \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^6\sqrt{2(p^2)^0}\sqrt{2(p^3)^0}} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^{ip_1\cdot x + ip_2\cdot x - ip_3\cdot y - ip_4\cdot x} \overline{u}_l(\vec{p}_1,\sigma_1) e_\mu^*(\vec{p}_2,\sigma_2) e_\nu(\vec{p}_3,\sigma_3)$$

$$\times u_{l'}(\vec{p}_4,\sigma_4) \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \left[\gamma^\mu \frac{1}{\not{q} - M + i0+}\gamma^\nu\right]_{ll'} a^\dagger(\vec{p}_1,\sigma_1) a_\gamma^\dagger(\vec{p}_2,\sigma_2) a_\gamma(\vec{p}_3,\sigma_3) a(\vec{p}_4,\sigma_4)$$

$$= \frac{ie^2}{2}\int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^2\sqrt{2(p^2)^0}\sqrt{2(p^3)^0}} \delta(p_1 + p_2 - p_3 - p_4) \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e_\mu^*(\vec{p}_2,\sigma_2) e_\nu(\vec{p}_3,\sigma_3)$$

$$\times \overline{u}(\vec{p}_1,\sigma_1)\left[\gamma^\mu \frac{\not{p}_1 + \not{p}_2 + M}{(p_1+p_2)^2 - M^2 + i0+}\gamma^\nu\right] u(\vec{p}_4,\sigma_4)\ a^\dagger(\vec{p}_1,\sigma_1) a_\gamma^\dagger(\vec{p}_2,\sigma_2) a_\gamma(\vec{p}_3,\sigma_3) a(\vec{p}_4,\sigma_4)$$

基本物理过程

$$S_2\big|_{\text{正电子光子散射}} = \frac{e^2}{2}\int d^4x d^4y\ \underbrace{A^-_\nu(y)\psi^-_{l'}(y)\overline{\psi}^+_l(x)A^+_\mu(x)}_{\text{正电子光子散射：}\ e^+\ \gamma\ \leftarrow\ e^+\ \gamma}\ [\gamma^\mu \underline{\psi(x)\overline{\psi}(y)}\gamma^\nu]_{ll'}$$

$$= -\frac{ie^2}{2}\int d^4xd^4y\int\frac{d^3p_1d^3p_2d^3p_3d^3p_4}{(2\pi)^6\sqrt{2(p^1)^0}\sqrt{2(p^4)^0}}\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^{ip_1\cdot y+ip_2\cdot y-ip_3\cdot x-ip_4\cdot x}e^*_\nu(\vec{p}_1,\sigma_1)v_{l'}(\vec{p}_2,\sigma_2)\overline{v}_l(\vec{p}_3,\sigma_3)$$

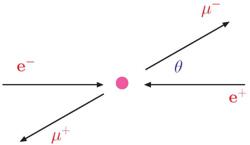
$$\times e_\mu(\vec{p}_4,\sigma_4)\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\left[\gamma^\mu\frac{1}{\not{q}-M+i0^+}\gamma^\nu\right]_{ll'} a^\dagger_\gamma(\vec{p}_1,\sigma_1)a^{c\dagger}(\vec{p}_2,\sigma_2)a^c(\vec{p}_3,\sigma_3)a_\gamma(\vec{p}_4,\sigma_4)$$

$$= \frac{ie^2}{2}\int\frac{d^3p_1d^3p_2d^3p_3d^3p_4}{(2\pi)^2\sqrt{2(p^1)^0}\sqrt{2(p^4)^0}}\delta(p_1+p_2-p_3-p_4)\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} e^*_\nu(\vec{p}_1,\sigma_1)e_\mu(\vec{p}_4,\sigma_4)$$

$$\times\overline{v}(\vec{p}_3,\sigma_3)\left[\gamma^\mu\frac{-\not{p}_1-\not{p}_2+M}{(p_1+p_2)^2-M^2+i0^+}\gamma^\nu\right]v(\vec{p}_2,\sigma_2)\ a^\dagger_\gamma(\vec{p}_1,\sigma_1)a^{c\dagger}(\vec{p}_2,\sigma_2)a^c(\vec{p}_3,\sigma_3)a_\gamma(\vec{p}_4,\sigma_4)$$

过程$e^+e^- \to \mu^+\mu^-$的基本情况

- 最简单的QED过程，也是高能物理最重要的过程
- e^+e^-对撞机用此过程对机器进行刻度
- 相关过程$e^+e^- \to q\bar{q}$ 对确定基本粒子性质极端有用



散射截面

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \frac{(2\pi)^4|\vec{p}_{\mu^-}|E_{\mu^-}E_{\mu^+}E_{e^-}E_{e^+}}{E^2|\vec{p}_{e^-}|}|M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}|^2$$

$$S_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'} = -2\pi i M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}\delta(p_{\mu^+}+p_{\mu^-}-p_{e^+}-p_{e^-})$$

质心系：能量守恒要求$E \geq 2M_\mu$

$$E_{e^-} = E_{e^+} = \frac{1}{2}E = \sqrt{|\vec{p}_{e^-}|^2 + M_e^2} \qquad \vec{p}_{e^-} = -\vec{p}_{e^+} = \sqrt{\frac{1}{4}E^2 - M_e^2}\,\vec{e}_z$$

$$\vec{p}_{\mu^-}\cdot\vec{e}_z = |p_{\mu^-}|\cos\theta \qquad \vec{p}_{\mu^-} + \vec{p}_{\mu^+} = 0 \qquad E_{\mu^-} = E_{\mu^+} = \frac{1}{2}E = \sqrt{|\vec{p}_{\mu^-}|^2 + M_\mu^2}$$

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \pi^4E^2\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}\,|M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}|^2$$

维克定理

$$S_2|_{e^+e^-\to\mu^+\mu^-} = -e^2\int d^4x d^4y\, \overline{\psi}^-_\mu(y)\gamma^\nu\psi^-_\mu(y)\overline{\psi}^+_e(x)\gamma^\mu\psi^+_e(x)\,\underline{A_\mu(x)A_\nu(y)}$$

$$\overset{\text{Feynman gauge}}{=\!=\!=\!=\!=\!=\!=\!=} ie^2\int d^4xd^4y\int\frac{d^3p_{\mu^-}d^3p_{\mu^+}d^3p_{e^+}d^3p_{e^-}}{(2\pi)^6}\sum_{\sigma_1,\sigma_2,\sigma_1',\sigma_2'} e^{ip_{\mu^-}\cdot y+ip_{\mu^+}\cdot y-ip_{e^+}\cdot x-ip_{e^-}\cdot x}$$

$$\times\overline{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\overline{v}(\vec{p}_{e^+},\sigma_1')\gamma^\mu u(\vec{p}_{e^-},\sigma_2')\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iq\cdot(x-y)}}{q^2+i0^+}$$

$$a^\dagger(\vec{p}_{\mu^-},\sigma_1)a^{c\dagger}(\vec{p}_{\mu^+},\sigma_2)a^c(\vec{p}_{e^+},\sigma_1')a(\vec{p}_{e^-},\sigma_2')$$

$$= ie^2(2\pi)^{-2}\int d^3p_{\mu^-}d^3p_{\mu^+}d^3p_{e^+}d^3p_{e^-}\sum_{\sigma_1,\sigma_2,\sigma_1',\sigma_2'}\delta(p_{\mu^-}+p_{\mu^+}-p_{e^+}-p_{e^-})\frac{1}{(p_{\mu^-}+p_{\mu^+})^2+i0^+}$$

$$\times\overline{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\overline{v}(\vec{p}_{e^+},\sigma_1')\gamma^\mu u(\vec{p}_{e^-},\sigma_2')a^\dagger(\vec{p}_{\mu^-},\sigma_1)a^{c\dagger}(\vec{p}_{\mu^+},\sigma_2)a^c(\vec{p}_{e^+},\sigma_1')a(\vec{p}_{e^-},\sigma_2')$$

$$S_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'} = -2\pi i M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}\delta(p_{\mu^+}+p_{\mu^-}-p_{e^+}-p_{e^-})$$

$$= ie^2(2\pi)^{-2}\delta(p_{\mu^-}+p_{\mu^+}-p_{e^+}-p_{e^-})\frac{\overline{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\overline{v}(\vec{p}_{e^+},\sigma_1')\gamma^\mu u(\vec{p}_{e^-},\sigma_2')}{(p_{\mu^-}+p_{\mu^+})^2+i0^+}$$

约化公式

$$\frac{d\sigma(e^+e^-\to\mu^+\mu^-)}{d\Omega} = \pi^4 E^2\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}\,|M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}|^2$$

$$S_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'} = -2\pi i M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}\delta(p_{\mu^+}+p_{\mu^-}-p_{e^+}-p_{e^-})$$

$$\begin{aligned}(\Psi^-_{\vec{p}_{\mu^-},\sigma_1,\vec{p}_{\mu^+},\sigma_2},\Psi^+_{\vec{p}_{e^+},\sigma_1',\vec{p}_{e^-},\sigma_2'}) &= (2\pi)^{-6}\bar{u}_{l_1}(\vec{p}_{\mu^-},\sigma_1)v_{l_2}(\vec{p}_{\mu^+},\sigma_2)\bar{v}_{l_1'}(\vec{p}_{e^+},\sigma_1')u_{l_2'}(\vec{p}_{e^-},\sigma_2')\\
&\times\int d^4x_1d^4x_2d^4y_1d^4y_2\; e^{ip_{\mu^-}\cdot x_1+ip_{\mu^+}\cdot x_2-ip_{e^+}\cdot y_1-ip_{e^-}\cdot y_2}(i\partial\!\!\!/_{x_1}-M_\mu)_{l_1\bar{l}_1}(-i\partial\!\!\!/_{x_2}-M_\mu)_{\bar{l}_2l_2}(i\partial\!\!\!/_{y_1}-M_e)_{l_1'\bar{l}_1'}\\
&\times(-i\partial\!\!\!/_{y_2}-M_e)_{\bar{l}_2'l_2'}(\Psi_0^-,\mathbf{T}\psi_{H,\mu,\bar{l}_1}(x_1)\overline{\psi}_{H,\mu,\bar{l}_2}(x_2)\psi_{H,e,\bar{l}_1'}(y_1)\overline{\psi}_{H,e,\bar{l}_2'}(y_2)\Psi_0^+)\end{aligned}$$

$$iS_0^{-1}(x,y,M) = (i\partial\!\!\!/_x-M-i0^+)\delta(x-y)$$

$$\begin{aligned}&= (2\pi)^{-6}\bar{u}_{l_1}(\vec{p}_{\mu^-},\sigma_1)v_{l_2}(\vec{p}_{\mu^+},\sigma_2)\bar{v}_{l_1'}(\vec{p}_{e^+},\sigma_1')u_{l_2'}(\vec{p}_{e^-},\sigma_2')\int d^4x_1d^4x_1'd^4x_2d^4x_2'd^4y_1d^4y_1'd^4y_2d^4y_2'\\
&\quad\times e^{ip_{\mu^-}\cdot x_1+ip_{\mu^+}\cdot x_2-ip_{e^+}\cdot y_1-ip_{e^-}\cdot y_2}S^{-1}_{0,l_1\bar{l}_1}(x_1,x_1',M_\mu)S^{-1}_{0,\bar{l}_2l_2}(x_2',x_2,M_\mu)S^{-1}_{0,l_1'\bar{l}_1'}(y_1,y_1',M_e)\\
&\quad\times S^{-1}_{0,\bar{l}_2'l_2'}(y_2',y_2,M_e)(\Psi_0^-,\mathbf{T}\psi_{H,\mu,\bar{l}_1}(x_1')\overline{\psi}_{H,\mu,\bar{l}_2}(x_2')\psi_{H,e,\bar{l}_1'}(y_1')\overline{\psi}_{H,e,\bar{l}_2'}(y_2')\Psi_0^+)\end{aligned}$$

基本公式

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \pi^4 E^2 \sqrt{\frac{E^2 - 4M_\mu^2}{E^2 - 4M_e^2}} |M_{\vec{p}_{\mu^-},\sigma_1,\, \vec{p}_{\mu^+},\sigma_2,\, \vec{p}_{e^+},\sigma_1',\, \vec{p}_{e^-},\sigma_2'}|^2$$

$$S_{\vec{p}_{\mu^-},\sigma_1,\, \vec{p}_{\mu^+},\sigma_2,\, \vec{p}_{e^+},\sigma_1',\, \vec{p}_{e^-},\sigma_2'} = -2\pi i M_{\vec{p}_{\mu^-},\sigma_1,\, \vec{p}_{\mu^+},\sigma_2,\, \vec{p}_{e^+},\sigma_1',\, \vec{p}_{e^-},\sigma_2'} \delta(p_{\mu^+} + p_{\mu^-} - p_{e^+} - p_{e^-})$$

$$= (2\pi)^{-6} \bar{u}_{l_1}(\vec{p}_{\mu^-},\sigma_1) v_{l_2}(\vec{p}_{\mu^+},\sigma_2) \bar{v}_{l_1'}(\vec{p}_{e^+},\sigma_1') u_{l_2'}(\vec{p}_{e^-},\sigma_2') \int d^4x_1 d^4x_1' d^4x_2 d^4x_2' d^4y_1 d^4y_1' d^4y_2 d^4y_2'$$

$$\times e^{ip_{\mu^-}\cdot x_1 + ip_{\mu^+}\cdot x_2 - ip_{e^+}\cdot y_1 - ip_{e^-}\cdot y_2} S^{-1}_{0,l_1\bar{l}_1}(x_1, x_1', M_\mu) S^{-1}_{0,\bar{l}_2 l_2}(x_2', x_2, M_\mu) S^{-1}_{0,l_1'\bar{l}_1'}(y_1, y_1', M_e) S^{-1}_{0,\bar{l}_2' l_2'}(y_2', y_2, M_e)$$

$$\times (\Psi_0^-, \mathbf{T} \psi_{H,\mu,\bar{l}_1}(x_1') \overline{\psi}_{H,\mu,\bar{l}_2}(x_2') \psi_{H,e,\bar{l}_1'}(y_1') \overline{\psi}_{H,e,\bar{l}_2'}(y_2') \Psi_0^+)$$

$$(\Psi_0^-, \mathbf{T}\, [\psi_{l_1}(x_1) \overline{\psi}_{l_2}(x_2) \psi_{l_1'}(y_1) \overline{\psi}_{l_2'}(y_2)]\ \Psi_0^+)$$

$$= C \times e^{\int d^4y d^4z\, [\frac{1}{2} \frac{\delta}{\delta A^\mu(y)} \tilde{D}_0^{\mu\nu}(y,z) \frac{\delta}{\delta A^\nu(z)} + \frac{\delta}{\delta \psi_e(y)} \tilde{S}_0(y,z,M_e) \frac{\delta}{\delta \overline{\psi}_e(z)} + \frac{\delta}{\delta \psi_\mu(y)} \tilde{S}_0(y,z,M_\mu) \frac{\delta}{\delta \overline{\psi}_\mu(z)}]}$$

$$\times\ [\psi_{\mu,l_1}(x_1) \overline{\psi}_{\mu,l_2}(x_2) \psi_{e,l_1'}(y_1) \overline{\psi}_{e,l_2'}(y_2)]\ e^{-i\int d^4x\ Z_2 e A_\mu (\overline{\psi}_\mu \gamma^\mu \psi_\mu + \overline{\psi}_e \gamma^\mu \psi_e)} \Big|_{A_\mu = \overline{\psi}_\mu = \psi_\mu = \overline{\psi}_e = \psi_e = 0}$$

数图计算

$$(\Psi_0^-, \mathbf{T}\,[\psi_{l_1}(x_1)\overline{\psi}_{l_2}(x_2)\psi_{l_1'}(y_1)\overline{\psi}_{l_2'}(y_2)]\ \Psi_0^+)_C$$

$$= e^{\int d^4y d^4z\,[\frac{1}{2}\frac{\delta}{\delta A^\mu(y)}\tilde{D}_0^{\mu\nu}(y,z)\frac{\delta}{\delta A^\nu(z)} + \frac{\delta}{\delta\psi_e(y)}\tilde{S}_0(y,z,M_e)\frac{\delta}{\delta\overline{\psi}_e(z)} + \frac{\delta}{\delta\psi_\mu(y)}\tilde{S}_0(y,z,M_\mu)\frac{\delta}{\delta\overline{\psi}_\mu(z)}]}$$

$$\times\ [\psi_{\mu,l_1}(x_1)\overline{\psi}_{\mu,l_2}(x_2)\psi_{e,l_1'}(y_1)\overline{\psi}_{e,l_2'}(y_2)]\ e^{-i\int d^4x\ Z_2 e A_\mu(\overline{\psi}_\mu\gamma^\mu\psi_\mu + \overline{\psi}_e\gamma^\mu\psi_e)}\Big|_{A_\mu=\overline{\psi}_\mu=\psi_\mu=\overline{\psi}_e=\psi_e=0,\ \text{connect}}$$

$$\stackrel{\text{数图}}{=\!=\!=} \int d^4y d^4z [S_0(x_1,y,M_\mu)ie\gamma_\mu S_0(y,x_2,M_\mu)]_{l_1 l_2} D_0^{\mu\nu}(y,z)[S_0(y_1,z,M_e)ie\gamma_\nu S_0(z,y_2,M_e)]_{l_1' l_2'}$$

$$M_{\vec{p}_{\mu^-},\sigma_1,\ \vec{p}_{\mu^+},\sigma_2,\ \vec{p}_{e^+},\sigma_1',\ \vec{p}_{e^-},\sigma_2'}\delta(p_{\mu^+} + p_{\mu^-} - p_{e^+} - p_{e^-})$$

$$= i(2\pi)^{-7}\overline{u}_{l_1}(\vec{p}_{\mu^-},\sigma_1)v_{l_2}(\vec{p}_{\mu^+},\sigma_2)\overline{v}_{l_1'}(\vec{p}_{e^+},\sigma_1')u_{l_2'}(\vec{p}_{e^-},\sigma_2')\int d^4x_1 d^4x_1' d^4x_2 d^4x_2' d^4y_1 d^4y_1' d^4y_2 d^4y_2'$$

$$\times e^{ip_{\mu^-}\cdot x_1 + ip_{\mu^+}\cdot x_2 - ip_{e^+}\cdot y_1 - ip_{e^-}\cdot y_2} S^{-1}_{0,l_1\bar{l}_1}(x_1,x_1',M_\mu)S^{-1}_{0,\bar{l}_2 l_2}(x_2',x_2,M_\mu)S^{-1}_{0,l_1'\bar{l}_1'}(y_1,y_1',M_e)S^{-1}_{0,\bar{l}_2' l_2'}(y_2',y_2,M_e)$$

$$\times\int d^4y d^4z [S_0(x_1',y,M_\mu)ie\gamma_\mu S_0(y,x_2',M_\mu)]_{\bar{l}_1\bar{l}_2} D_0^{\mu\nu}(y,z)[S_0(y_1',z,M_e)ie\gamma_\nu S_0(z,y_2',M_e)]_{\bar{l}_1'\bar{l}_2'}$$

$$M_{\vec{p}_{\mu^-},\sigma_1,\ \vec{p}_{\mu^+},\sigma_2,\ \vec{p}_{e^+},\sigma_1',\ \vec{p}_{e^-},\sigma_2'}\delta(p_{\mu^+}+p_{\mu^-}-p_{e^+}-p_{e^-})$$

$$= i(2\pi)^{-7}\bar{u}_{l_1}(\vec{p}_{\mu^-},\sigma_1)v_{l_2}(\vec{p}_{\mu^+},\sigma_2)\bar{v}_{l_1'}(\vec{p}_{e^+},\sigma_1')u_{l_2'}(\vec{p}_{e^-},\sigma_2')\int d^4x_1d^4x_1'd^4x_2d^4x_2'd^4y_1d^4y_1'd^4y_2d^4y_2'$$

$$\times e^{ip_{\mu^-}\cdot x_1+ip_{\mu^+}\cdot x_2-ip_{e^+}\cdot y_1-ip_{e^-}\cdot y_2}S^{-1}_{0,l_1\bar{l}_1}(x_1,x_1',M_\mu)S^{-1}_{0,\bar{l}_2l_2}(x_2',x_2,M_\mu)S^{-1}_{0,l_1'\bar{l}_1'}(y_1,y_1',M_e)S^{-1}_{0,\bar{l}_2'l_2'}(y_2',y_2,M_e)$$

$$\times\int d^4yd^4z[S_0(x_1',y,M_\mu)ie\gamma_\mu S_0(y,x_2',M_\mu)]_{\bar{l}_1\bar{l}_2}D_0^{-1\mu\nu}(y,z)[S_0(y_1',z,M_e)ie\gamma_\nu S_0(z,y_2',M_e)]_{\bar{l}_1'\bar{l}_2'}$$

$$= e^2(2\pi)^{-7}\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{e^+},\sigma_1')\gamma_\nu u(\vec{p}_{e^-},\sigma_2')\int d^4yd^4z$$

$$\times e^{i(p_{\mu^-}+p_{\mu^+})\cdot y-i(p_{e^+}+p_{e^-})\cdot z}\frac{1}{\partial_y^2-i0^+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{\partial_y^\mu\partial_y^\nu}{\partial_y^2}]\delta(y-z)$$

$$\overset{q=p_{e^+}+p_{e^-}}{=\!=\!=\!=\!=}-(2\pi)^{-3}e^2\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{e^+},\sigma_1')\gamma_\nu u(\vec{p}_{e^-},\sigma_2')\frac{\delta(p_{\mu^+}+p_{\mu^-}-p_{e^+}-p_{e^-})}{q^2+i0^+}$$

$$\times[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{q^\mu q^\nu}{q^2}]$$

$e^+e^- \to \mu^+\mu^-$

$$M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}$$

$$\overset{q=p_{e^+}+p_{e^-}}{=\!=\!=\!=\!=} -e^2(2\pi)^{-3}\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{e^+},\sigma_1')\gamma_\nu u(\vec{p}_{e^-},\sigma_2')\frac{1}{q^2+i0^+}[g^{\mu\nu}-(1-\frac{1}{\lambda})\frac{q^\mu q^\nu}{q^2}]$$

$$(\not{p}-M)u(\vec{p},\sigma)=0 \quad \bar{u}(-\vec{p},\sigma)(\not{p}-M)=0 \qquad (\not{p}+M)v(\vec{p},\sigma)=0 \quad \bar{v}(-\vec{p},\sigma)(\not{p}+M)=0$$

$$\bar{u}(-\vec{p},\sigma)\not{p}v(\vec{p},\sigma) = \frac{1}{2}\bar{u}(-\vec{p},\sigma)(\not{p}-M+\not{p}+M)v(\vec{p},\sigma) = 0$$

$$\bar{v}(-\vec{p},\sigma)\not{p}u(\vec{p},\sigma) = \frac{1}{2}\bar{v}(-\vec{p},\sigma)(\not{p}-M+\not{p}+M)u(\vec{p},\sigma) = 0$$

$$M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'} = -e^2(2\pi)^{-3}\frac{\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{e^+},\sigma_1')\gamma^\mu u(\vec{p}_{e^-},\sigma_2')}{(p_{e^-}+p_{e^+})^2}$$

$$|M_{\vec{p}_{\mu^-},\sigma_1,\,\vec{p}_{\mu^+},\sigma_2,\,\vec{p}_{e^+},\sigma_1',\,\vec{p}_{e^-},\sigma_2'}|^2 = \frac{e^4(2\pi)^{-6}}{(p_{e^-}+p_{e^+})^4}|\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{e^+},\sigma_1')\gamma^\mu u(\vec{p}_{e^-},\sigma_2')|^2$$

$$= \frac{e^4(2\pi)^{-6}}{(p_{e^-}+p_{e^+})^4}\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{\mu^+},\sigma_2)\gamma_\nu u(\vec{p}_{\mu^-},\sigma_1)\bar{v}(\vec{p}_{e^+},\sigma_1')\gamma^\mu u(\vec{p}_{e^-},\sigma_2)\bar{u}(\vec{p}_{e^-},\sigma_2)\gamma^\nu v(\vec{p}_{e^+},\sigma_1')$$

非极化 $e^+e^- \to \mu^+\mu^-$ 过程：入射态自旋平均，出射态自旋求和

$$\overline{|M_{\vec{p}_{\mu^-},\vec{p}_{\mu^+},\vec{p}_{e^+},\vec{p}'_{e^-}}|^2} = \frac{1}{4}\sum_{\sigma_1,\sigma_2}\sum_{\sigma'_1\sigma'_2}|M_{\vec{p}_{\mu^-},\sigma_1,\vec{p}_{\mu^+},\sigma_2,\vec{p}_{e^+},\sigma'_1,\vec{p}_{e^-},\sigma'_2}|^2$$

$$= \frac{e^4(2\pi)^{-6}}{4(p_{e^-}+p_{e^+})^4}\sum_{\sigma_1,\sigma_2}\sum_{\sigma'_1\sigma'_2}\bar{u}(\vec{p}_{\mu^-},\sigma_1)\gamma_\mu v(\vec{p}_{\mu^+},\sigma_2)\bar{v}(\vec{p}_{\mu^+},\sigma_2)\gamma_\nu u(\vec{p}_{\mu^-},\sigma_1)\bar{v}(\vec{p}_{e^+},\sigma'_1)\gamma^\mu u(\vec{p}_{e^-},\sigma_2)$$

$$\times\bar{u}(\vec{p}_{e^-},\sigma_2)\gamma^\nu v(\vec{p}_{e^+},\sigma'_1)$$

$$\sum_\sigma u(\vec{p},\sigma)\bar{u}(\vec{p},\sigma) = \frac{\not{p}+M}{2p^0} \qquad \sum_\sigma v(\vec{p},\sigma)\bar{v}(\vec{p},\sigma) = \frac{\not{p}-M}{2p^0}$$

$$= \frac{e^4(2\pi)^{-6}}{64p^0_{e^-}p^0_{e^+}p^0_{\mu^-}p^0_{\mu^+}(p_{e^-}+p_{e^+})^4}\mathrm{tr}[(\not{p}_{\mu^-}+M_\mu)\gamma_\mu(\not{p}_{\mu^+}-M_\mu)\gamma_\nu]\mathrm{tr}[(\not{p}_{e^+}-M_e)\gamma^\mu(\not{p}_{e^-}+M_e)\gamma^\nu]$$

$$= \frac{e^4(2\pi)^{-6}[p_{\mu^-,\mu}p_{\mu^+,\nu}+p_{\nu^-,\mu}p_{\mu^+,\mu}-g_{\mu\nu}(p_{\mu^-}\cdot p_{\mu^+}+M^2_\mu)][p^\mu_{e^-}p^\nu_{e^+}+p^\nu_{e^-}p^\mu_{e^+}-g^{\mu\nu}(p_{e^-}\cdot p_{e^+}+M^2_e)]}{4p^0_{e^-}p^0_{e^+}p^0_{\mu^-}p^0_{\mu^+}(p_{e^-}+p_{e^+})^4}$$

$$= e^4(2\pi)^{-6}[4p^0_{e^-}p^0_{e^+}p^0_{\mu^-}p^0_{\mu^+}(p_{e^-}+p_{e^+})^4]^{-1}[2p_{\mu^-}\cdot p_{e^-}p_{\mu^+}\cdot p_{e^+}+2p_{\mu^-}\cdot p_{e^+}p_{\mu^+}\cdot p_{e^-}$$

$$-2p_{\mu^-}\cdot p_{\mu^+}(p_{e^-}\cdot p_{e^+}+M^2_e)-2p_{e^-}\cdot p_{e^+}(p_{\mu^-}\cdot p_{\mu^+}+M^2_\mu)+4(p_{\mu^-}\cdot p_{\mu^+}+M^2_\mu)(p_{e^-}\cdot p_{e^+}+M^2_e)]$$

$$= \frac{e^4(2\pi)^{-6}[p_{\mu^-}\cdot p_{e^-}p_{\mu^+}\cdot p_{e^+}+p_{\mu^-}\cdot p_{e^+}p_{\mu^+}\cdot p_{e^-}+M^2_e p_{\mu^-}\cdot p_{\mu^+}+M^2_\mu p_{e^-}\cdot p_{e^+}+2M^2_\mu M^2_e]}{2p^0_{e^-}p^0_{e^+}p^0_{\mu^-}p^0_{\mu^+}(p_{e^-}+p_{e^+})^4}$$

非极化 $e^+e^- \to \mu^+\mu^-$ 过程

质心系 $\vec{p}_{e^-} = -\vec{p}_{e^+} = \sqrt{\frac{1}{4}E^2 + M_e^2}\vec{e}_z \qquad E_{e^-} = E_{e^+} = \frac{1}{2}E = \sqrt{|\vec{p}_{e^-}|^2 + M_e^2}$

$$\vec{p}_{\mu^-}\cdot\vec{e}_z = |p_{\mu^-}|\cos\theta \qquad \vec{p}_{\mu^-} + \vec{p}_{\mu^+} = 0 \qquad E_{\mu^-} = E_{\mu^+} = \frac{1}{2}E = \sqrt{|\vec{p}_{\mu^-}|^2 + M_\mu^2}$$

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \pi^4 E^2 \sqrt{\frac{E^2 - 4M_\mu^2}{E^2 - 4M_e^2}}\,\overline{|M_{\vec{p}_{\mu^-},\,\vec{p}_{\mu^+},\,\vec{p}_{e^+},\,\vec{p}_{e^-}}|^2}$$

$$\overline{|M_{\vec{p}_{\mu^-},\,\vec{p}_{\mu^+},\,\vec{p}_{e^+},\,\vec{p}'_{e^-}}|^2} = \frac{e^4[p_{\mu^-}\cdot p_{e^-}p_{\mu^+}\cdot p_{e^+} + p_{\mu^-}\cdot p_{e^+}p_{\mu^+}\cdot p_{e^-} + M_e^2 p_{\mu^-}\cdot p_{\mu^+} + M_\mu^2 p_{e^-}\cdot p_{e^+} + 2M_\mu^2 M_e^2]}{2(2\pi)^6 p^0_{e^-}p^0_{e^+}p^0_{\mu^-}p^0_{\mu^+}(p_{e^-}+p_{e^+})^4}$$

$$p_{\mu^-}\cdot p_{e^-} = p_{\mu^+}\cdot p_{e^+} = \frac{1}{4}E^2 - |p_{e^+}||p_{\mu^-}|\cos\theta \qquad p_{\mu^-}\cdot p_{\mu^+} = (p^0_{\mu^-})^2 + \vec{p}_{\mu^-}\cdot\vec{p}_{\mu^-} = \frac{1}{2}E^2 - M_\mu^2$$

$$p_{\mu^-}\cdot p_{e^+} = p_{\mu^+}\cdot p_{e^-} = \frac{1}{4}E^2 + |p_{e^-}||p_{\mu^-}|\cos\theta \quad p_{e^-}\cdot p_{e^+} = \frac{1}{2}E^2 - M_e^2 \quad (p_{e^-}+p_{e^+})^2 = 2M_e^2 + 2(\frac{1}{2}E^2 - M_e^2) = E^2$$

$$= \frac{8e^4}{E^8(2\pi)^6}[(\frac{E^2}{4} - |p_{\mu^-}||p_{e^-}|\cos\theta)^2 + (\frac{E^2}{4} + |p_{\mu^-}||p_{e^+}|\cos\theta)^2 + M_e^2(\frac{E^2}{2} - M_\mu^2) + M_\mu^2(\frac{E^2}{2} - M_e^2) + 2M_\mu^2 M_e^2]$$

$$= \frac{e^4}{E^6(2\pi)^6}[E^2 + \frac{(E^2 - 4M_\mu^2)(E^2 - 4M_e^2)}{E^2}\cos^2\theta + 4M_e^2 + 4M_\mu^2]$$

非极化 $e^+e^- \to \mu^+\mu^-$ 过程

质心系 $\vec{p}_{e^-} = -\vec{p}_{e^+} = \sqrt{\frac{1}{4}E^2 - M_e^2}\vec{e}_z \qquad E_{e^-} = E_{e^+} = \frac{1}{2}E = \sqrt{|\vec{p}_{e^-}|^2 + M_e^2}$

$$\vec{p}_{\mu^-}\cdot\vec{e}_z = |p_{e^-}|\cos\theta \qquad \vec{p}_{\mu^-} + \vec{p}_{\mu^+} = 0 \qquad E_{\mu^-} = E_{\mu^+} = \frac{1}{2}E = \sqrt{|\vec{p}_{\mu^-}|^2 + M_\mu^2}$$

$$p_{\mu^-}\cdot p_{e^-} = p_{\mu^+}\cdot p_{e^+} = \frac{1}{4}E^2 - |p_{e^+}||p_{\mu^-}|\cos\theta \qquad p_{\mu^-}\cdot p_{\mu^+} = (p^0_{\mu^-})^2 + \vec{p}_{\mu^-}\cdot\vec{p}_{\mu^-} = \frac{1}{2}E^2 - M_\mu^2$$

$$p_{\mu^-}\cdot p_{e^+} = p_{\mu^+}\cdot p_{e^-} = \frac{1}{4}E^2 + |p_{e^-}||p_{\mu^-}|\cos\theta \quad p_{e^-}\cdot p_{e^+} = \frac{1}{2}E^2 - M_e^2 \quad (p_{e^-}+p_{e^+})^2 = 2M_e^2 + 2(\frac{1}{2}E^2 - M_e^2) = E^2$$

$$\frac{d\sigma(e^+e^-\to\mu^+\mu^-)}{d\Omega} = \pi^4 E^2\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}\,\overline{|M_{\vec{p}_{\mu^-},\,\vec{p}_{\mu^+},\,\vec{p}_{e^+},\,\vec{p}_{e^-}}|^2}$$

$$\overline{|M_{\vec{p}_{\mu^-},\,\vec{p}_{\mu^+},\,\vec{p}_{e^+},\,\vec{p}'_{e^-}}|^2} = \frac{e^4}{E^6(2\pi)^6}[E^2 + \frac{(E^2-4M_\mu^2)(E^2-4M_e^2)}{E^2}\cos^2\theta + 4M_e^2 + 4M_\mu^2]$$

$$\frac{d\sigma(e^+e^-\to\mu^+\mu^-)}{d\Omega} = \frac{\alpha^2}{4E^2}\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}[1 + \frac{(E^2-4M_\mu^2)(E^2-4M_e^2)}{E^4}\cos^2\theta + \frac{4M_e^2+4M_\mu^2}{E^2}]$$

$$\sigma_{\text{total}}(e^+e^-\to\mu^+\mu^-) = \frac{\alpha^2\pi}{E^2}\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}[1 + \frac{(E^2-4M_\mu^2)(E^2-4M_e^2)}{3E^4} + \frac{4M_e^2+4M_\mu^2}{E^2}]$$

推广到非极化 $e^+e^-\to$ 强子的过程

非极化 $e^+e^-\to\ \mu^+\mu^-$ 过程

$$\frac{d\sigma(e^+e^-\to\mu^+\mu^-)}{d\Omega}=\frac{\alpha^2}{4E^2}\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}[1+\frac{(E^2-4M_\mu^2)(E^2-4M_e^2)}{E^4}\cos^2\theta+\frac{4M_e^2+4M_\mu^2}{E^2}]$$

$$\sigma_{\text{total}}(e^+e^-\to\mu^+\mu^-)=\frac{\alpha^2\pi}{E^2}\sqrt{\frac{E^2-4M_\mu^2}{E^2-4M_e^2}}[1+\frac{(E^2-4M_\mu^2)(E^2-4M_e^2)}{3E^4}+\frac{4M_e^2+4M_\mu^2}{E^2}]$$

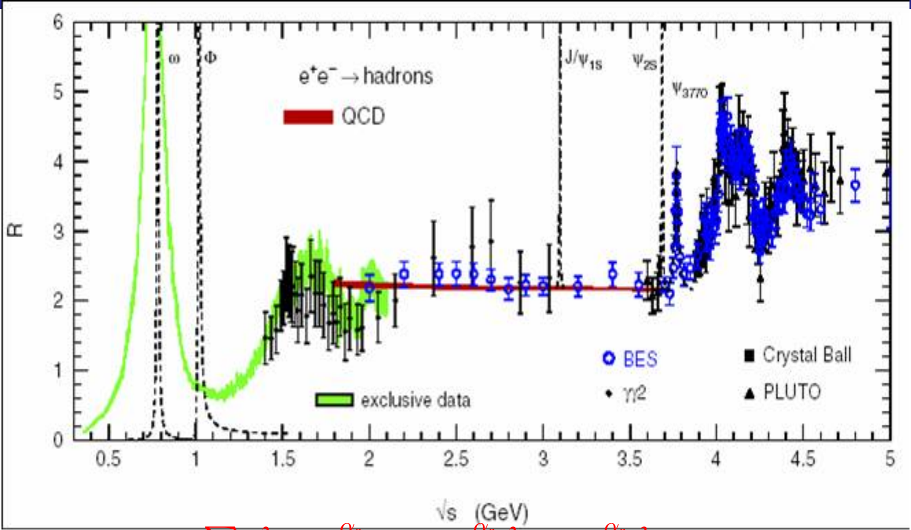
$e^+e^-\to$ 强子

- 可以用 $e^+e^-\to q\bar{q}$ 加强子化来实现,强子化效应在高能可忽略
- $e^+e^-\to q\bar{q}$ 与 $e^+e^-\to\ \mu^+\mu^-$的区别只在于：
  - 将μ子电荷e换为夸克电荷Qe.
    $Q_u=Q_c=Q_t=\frac{2}{3}\qquad Q_d=Q_s=Q_b=-\frac{1}{3}$
  - 将每种夸克重复计算N_c次（N_c种颜色）

$$R\equiv\frac{\sigma(e^+e^-\to\ q\bar{q})}{\sigma(e^+e^-\to\ \mu^+\mu^-)}=N_c\sum_i Q_i^2=\begin{cases}\frac{2}{3}N_c=2\\ \frac{10}{9}N_c=\frac{10}{3}\\ \frac{11}{9}N_c=\frac{11}{3}\end{cases}$$

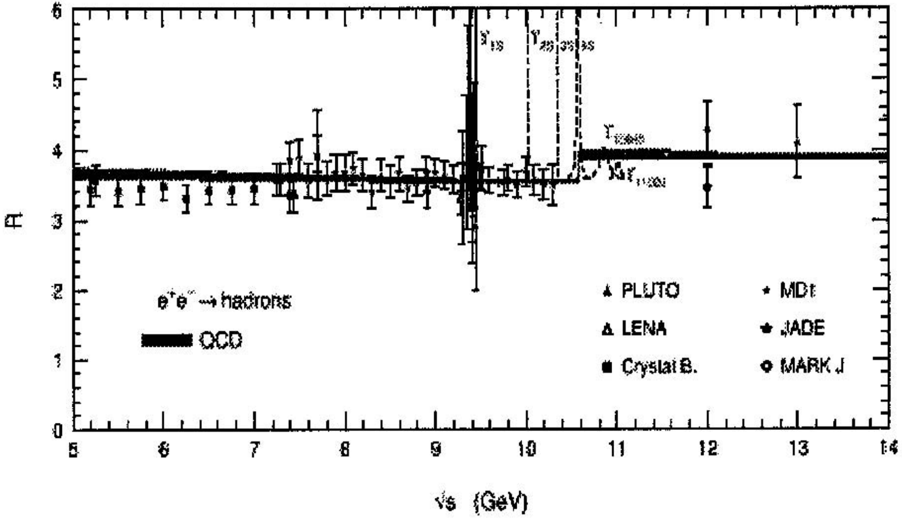
$N_f=3:\ u,d,s$
$N_f=4:\ u,d,s,c$
$N_f=5:\ u,d,s,c,b$

$e^+e^- \to \mu^+\mu^-$



$$\mathbf{R = N_c \sum_i Q_i^2 \left[1 + \left(\frac{\alpha_s}{\pi}\right) + 1.411\left(\frac{\alpha_s}{\pi}\right)^2 - 12.8\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right]}$$

$e^+e^- \to \mu^+\mu^-$



电流体系在外电磁场A^μ中的能量 $W = \int d^3x\, \vec{J}(x)\cdot\vec{A}(x)$

经典电动力学多极展开: $P^i_{经典} = \int d^3x\, x^i\rho$ $\quad Q^{ij}_{经典} = \frac{1}{3}\int d^3x\, x^ix^j\rho$ $\quad \mu^k_{经典} = \frac{1}{2}\int d^3x\, \epsilon^{kil}x^iJ^l$

$$\vec{A}(t,\vec{r}) = \vec{A}(t,0) + \sum_{n=1}^{\infty}\frac{1}{n!}\sum_{i_1i_2\dots i_n=1}^{3} x^{i_1}x^{i_2}\cdots x^{i_n}\left[\frac{\partial}{\partial x^{i_1}}\frac{\partial}{\partial x^{i_2}}\cdots\frac{\partial}{\partial x^{i_n}}\vec{A}(t,\vec{x})\right]_{\vec{x}=0}$$

$$W = \int d^3x\, \vec{J}(x)\cdot\left\{\vec{A}(t,0) + \sum_{n=1}^{\infty}\frac{1}{n!}\sum_{i_1i_2\dots i_n=1}^{3} x^{i_1}x^{i_2}\cdots x^{i_n}\left[\frac{\partial}{\partial x^{i_1}}\frac{\partial}{\partial x^{i_2}}\cdots\frac{\partial}{\partial x^{i_n}}\vec{A}(t,\vec{x})\right]_{\vec{x}=0}\right\} = \sum_{n=0} W_n$$

$$W_n = \sum_{i_1i_2\dots i_n=1}^{3}\int d^3x x^{i_1}x^{i_2}\cdots x^{i_n}\, \vec{J}(x)\cdot\frac{1}{n!}\left[\frac{\partial}{\partial x^{i_1}}\cdots\frac{\partial}{\partial x^{i_n}}\vec{A}(t,\vec{x})\right]_{\vec{x}=0}$$

$$W_0 = A^i(t,0)\int d^3x\, J^i(t,\vec{x}) = A^i(t,0)\int d^3x\left[\frac{\partial}{\partial x^l}[x^iJ^l(t,\vec{x})] + x^i\frac{\partial\rho(t,\vec{x})}{\partial t}\right] = \vec{A}(t,0)\cdot\dot{\vec{P}}_{经典}(t)$$

$$W_1 = \frac{1}{2}\frac{\partial A^i(t,\vec{x})}{\partial x^l}\Big|_{\vec{x}=0}\int d^3x[x^lJ^i - x^iJ^l + x^lJ^i + x^iJ^l]$$

稳恒电流体系在外电磁场中的能量磁矩起主要作用!

$$= \frac{1}{2}\frac{\partial A^i(t,\vec{x})}{\partial x^l}\Big|_{\vec{x}=0}\int d^3x\,[x^lJ^i - x^iJ^l + \frac{\partial}{\partial x^k}(x^ix^lJ^k) + x^ix^l\frac{\partial\rho}{\partial t}] = \vec{B}(t,0)\cdot\vec{\mu}_{经典}(t) + \frac{1}{6}\frac{\partial A^i(t,\vec{x})}{\partial x^l}\Big|_{\vec{x}=0}\dot{Q}^{il}_{经典}(t)$$

经典场论

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu]^2 + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)$$

$$J_\mu = e\overline{\psi}\gamma_\mu\psi \qquad \vec{\mu}_{经典} = \frac{1}{2}\int d^3x\, \vec{x}\times\vec{J} = \frac{e}{2}\int d^3x\, \vec{x}\times\overline{\psi}\vec{\gamma}\psi$$

对自由费米场: $[i\partial\!\!\!/ - M]\psi(x) = 0 \qquad \overline{\psi}(x)[i\overleftarrow{\partial\!\!\!/} + M] = 0 \qquad \epsilon^{ijk}[\gamma^i,\gamma^j] = -4i\sigma^k I$

$$\overline{\psi}(\gamma^\mu\partial\!\!\!/ - \overleftarrow{\partial\!\!\!/}\gamma^\mu + \overleftarrow{\partial}^\mu - \partial^\mu)\psi = \overline{\psi}(\partial_\nu\gamma^\mu\gamma^\nu - \overleftarrow{\partial}_\nu\gamma^\nu\gamma^\mu + \overleftarrow{\partial}^\mu - \partial^\mu)\psi$$
$$= \frac{1}{2}\overline{\psi}[\gamma^\mu,\gamma^\nu](\overleftarrow{\partial}_\nu + \partial_\nu)\psi = \frac{1}{2}\partial_\nu(\overline{\psi}[\gamma^\mu,\gamma^\nu]\psi)$$

$$\overline{\psi}\gamma^\mu\psi = \frac{1}{2M}\overline{\psi}(M\gamma^\mu + \gamma^\mu M)\psi$$
$$= \frac{1}{2M}\overline{\psi}[(i\overleftarrow{\partial\!\!\!/} + M)\gamma^\mu + \gamma^\mu(-i\partial\!\!\!/ + M) - i\overleftarrow{\partial}^\mu + i\partial^\mu]\psi + \frac{i}{4M}\partial_\nu(\overline{\psi}[\gamma^\mu,\gamma^\nu]\psi)$$
$$= \frac{1}{2M}\overline{\psi}[-i\overleftarrow{\partial}^\mu + i\partial^\mu]\psi + \frac{i}{4M}\partial_\nu(\overline{\psi}[\gamma^\mu,\gamma^\nu]\psi)$$

$$\mu^k_{经典} = \frac{e}{2}\epsilon^{ijk}\int d^3x\, x^i\overline{\psi}\gamma^j\psi = \frac{e}{2}\epsilon^{ijk}\int d^3x\, x^i\left[\frac{1}{2M}\overline{\psi}[-i\overleftarrow{\partial}^j + i\partial^j]\psi + \frac{i}{4M}\partial_\nu(\overline{\psi}[\gamma^j,\gamma^\nu]\psi)\right]$$
$$= \frac{ei}{2M}\epsilon^{ijk}\int d^3x\, [\overline{\psi}\, x^i\partial^j\, \psi - \frac{1}{4}(\overline{\psi}[\gamma^j,\gamma^i]\psi)] = \frac{e}{2M}\int d^3x\, \overline{\psi}\, [L^k + \sigma^k]\, \psi$$

经典场论

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu]^2 + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x)$$

$$\mathcal{L}_I = -J_\mu(x)A^\mu(x) = -\rho(x)A^0(x) + \vec{J}(x)\cdot\vec{A}(x) \qquad J_\mu(x) = e\overline{\psi}(x)\gamma_\mu\psi(x)$$

$$W = \int d^3x\, \vec{J}(x)\cdot\vec{A}(x) = \sum_{n=0} W_n$$

$$W_n = \sum_{i_1 i_2\ldots i_n=1}^{3}\int d^3x\, x^{i_1}x^{i_2}\cdots x^{i_n}\, \vec{J}(x)\cdot\frac{1}{n!}\left[\frac{\partial}{\partial x^{i_1}}\frac{\partial}{\partial x^{i_2}}\frac{\partial}{\partial x^{i_2}}\cdots\frac{\partial}{\partial x^{i_n}}\vec{A}(t,\vec{x})\right]_{\vec{x}=0}$$

$$W_0 = \vec{A}(t,0)\cdot\dot{\vec{P}}(t) \qquad W_1 = \vec{B}(t,0)\cdot\vec{\mu}(t) + \frac{1}{6}\frac{\partial A^i(t,\vec{x})}{\partial x^l}\bigg|_{\vec{x}=0}\dot{Q}^{il}(t)$$

$$P^i_{经典}(t) = \int d^3x\, x^i\rho(t,\vec{x}) \qquad Q^{ij}_{经典}(t) = \frac{1}{3}\int d^3x\, x^ix^j\rho(t,\vec{x}) \qquad \mu^k_{经典}(t) = \frac{1}{2}\int d^3x\, \epsilon^{kil}x^iJ^l(t,\vec{x})$$

$$\vec{\mu}_{经典}(t) = \frac{e}{2M}\int d^3x\, \overline{\psi}(t,\vec{x})\,[\vec{L}+\vec{\sigma}]\,\psi(t,\vec{x}) = \frac{e}{2M}\int d^3x\, \overline{\psi}(t,\vec{x})\,\vec{J}\,\psi(t,\vec{x})$$

$\vec{L}$是轨道角动量，σ是内禀角动量　　电子具有磁矩 $\mu_e = \frac{e}{2M}$

量子场论

$$\int d^4x\,\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu]^2 + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \ \Rightarrow\ \Gamma[A,\psi,\overline{\psi}]$$

$$\Gamma[A,\psi,\overline{\psi}] = \Gamma[0,0,0] + \int d^4x d^4y\ [\overline{\psi}(x)iS^{-1}(x,y)\psi(y) + A_\mu(x)iD^{-1,\mu\nu}(x,y)A_\nu(y)]$$

$$-\int d^4x d^4y d^4z\ A_\nu(x)\overline{\psi}(y)\Gamma^\nu(x,y,z)\psi(z) + \text{更多点的相互作用项}\ \text{1PI},\ O(e^5)$$

$$iD^{-1,\mu\nu}(x,y) = \frac{\delta^2\Gamma[A,\psi,\overline{\psi}]}{\delta A_\mu(x)\delta A_\nu(y)}\Big|_{A=\overline{\psi}=\psi=0} = \int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}iD^{-1,\mu\nu}(p)$$

$$-iS^{-1}_{ll'}(x,y) = \frac{\delta^2\Gamma[A,\psi,\overline{\psi}]}{\delta\overline{\psi}_l(x)\delta\psi_{l'}(y)}\Big|_{A=\overline{\psi}=\psi=0} = \int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}(-i)S^{-1}_{ll'}(p)$$

$$\Gamma^\nu_{ll'}(x,y,z) \equiv \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A^\nu(x)\delta\overline{\psi}_l(y)\delta\psi_{l'}(z)}\Big|_{A=\overline{\psi}=\psi=0} = \int \frac{d^4p d^4p'}{(2\pi)^8}e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu_{ll'}(p,p')$$

$$\Gamma[A,\psi,\overline{\psi}] = \Gamma[0,0,0] + \int d^4x\ \{\overline{\psi}(x)iS^{-1}(i\partial_x)\psi(x) + A_\mu(x)iD^{-1,\mu\nu}(i\partial_x)A_\nu(x)$$

$$-A_\nu(x)\ [\overline{\psi}(x)\Gamma^\nu(-i\overleftarrow{\partial}_x, i\overrightarrow{\partial}_x)\psi(x)]\ \} + \text{更多点的相互作用项}$$

量子场论

$$\int d^4x\, \mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{\lambda}{2}[\partial^\mu A_\mu]^2 + \overline{\psi}(x)[i\partial\!\!\!/ - eA\!\!\!/(x) - M]\psi(x) \Rightarrow \Gamma[A,\psi,\overline{\psi}]$$

自由费米场的场方程：$[i\partial\!\!\!/ - M]\psi(x) = 0 \qquad \overline{\psi}(x)[i\overleftarrow{\partial\!\!\!/} + M] = 0$

$$\Rightarrow \left.\frac{\delta\Gamma[A,\psi,\overline{\psi}]}{\delta\overline{\psi}(x)}\right|_{A=\overline{\psi}=\psi=0} = 0 \Rightarrow \int d^4y\, S^{-1}(x,y)\psi(y) = 0$$

$$\Rightarrow \left.\frac{\delta\Gamma[A,\psi,\overline{\psi}]}{\delta\psi(x)}\right|_{A=\overline{\psi}=\psi=0} = 0 \Rightarrow \int d^4y\, \overline{\psi}(y)S^{-1}(y,x) = 0$$

$$iS^{-1}(x,y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)}\{p\!\!\!/[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]\} \qquad \tilde{\Sigma}_A(M^2) = \tilde{\Sigma}_B(M^2) = 0$$

$$\{i\partial\!\!\!/_x[1-\tilde{\Sigma}_A(-\partial_x^2)] - M[1-\tilde{\Sigma}_B(-\partial_x^2)]\}\psi(x) = 0 \qquad \Rightarrow (i\partial\!\!\!/_x - M)\psi(x) = 0$$

$$\overline{\psi}(x)\{i\overleftarrow{\partial\!\!\!/}_x[1-\tilde{\Sigma}_A(-\overleftarrow{\partial}_x^2)] + M[1-\tilde{\Sigma}_B(-\overleftarrow{\partial}_x^2)]\} = 0 \Rightarrow \overline{\psi}(x)(i\overleftarrow{\partial\!\!\!/}_x - M) = 0$$

$$\Gamma[A,\psi,\overline{\psi}] = \Gamma[0,0,0] + \int d^4x\, \{\overline{\psi}(x) iS^{-1}(i\partial_x)\psi(x) + A_\mu(x) iD^{-1,\mu\nu}(i\partial_x)A_\nu(x)$$

$$-A_\nu(x)\,[\overline{\psi}(x)\Gamma^\nu(-i\overleftarrow{\partial}_x, i\overrightarrow{\partial}_x)\psi(x)]\,\} + \text{更多点的相互作用项}$$

$$iD^{-1,\mu\nu}(p) = (p^\mu p^\nu - p^2 g^{\mu\nu})[1-\tilde{\pi}(p^2)] - \lambda p^\mu p^\nu \qquad iS^{-1}(p) = \not{p}\,[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]$$

有限重整化

$$\overline{\psi}(x) \to \frac{1}{\sqrt{1-\tilde{\Sigma}_A(-\partial_x^2)}}\overline{\psi}(x) \qquad \psi(x) \to \frac{1}{\sqrt{1-\tilde{\Sigma}_A(-\partial_x^2)}}\psi(x) \qquad A_\mu(x) \to \frac{1}{\sqrt{1-\tilde{\pi}_A(-\partial_x^2)}}A_\mu(x)$$

$$\Gamma[A,\psi,\overline{\psi}] = \Gamma[0,0,0] + \int d^4x\, \{\overline{\psi}(x) i\tilde{S}^{-1}(i\partial_x)\psi(x) + A_\mu(x) i\tilde{D}^{-1,\mu\nu}(i\partial_x)A_\nu(x)$$

$$-A_\nu(x)\,[\overline{\psi}(x)\tilde{\Gamma}^\nu(-i\overleftarrow{\partial}_x, i\overrightarrow{\partial}_x)\psi(x)]\,\} + \text{更多点的相互作用项}$$

$$i\tilde{D}^{-1,\mu\nu}(p) = p^\mu p^\nu - p^2 g^{\mu\nu} - \frac{\lambda p^\mu p^\nu}{1-\tilde{\pi}(p^2)} \qquad iS^{-1}(p) = \not{p} - M\frac{1-\tilde{\Sigma}_B(p^2)}{1-\tilde{\Sigma}_A(p^2)}$$

$$\tilde{\Gamma}^\nu(p,p') = \frac{\Gamma^\nu(p,p')}{\sqrt{1-\tilde{\pi}((p-p')^2)}\sqrt{1-\tilde{\Sigma}_A(p^2)}\sqrt{1-\tilde{\Sigma}_A(p'^2)}} \to \frac{\Gamma^\nu(p,p')}{\sqrt{1-\tilde{\pi}((p-p')^2)}}$$

电子磁矩

$$\mathcal{L}_I = -e\int d^4x\, \overline{\psi}(x)\not{A}_c(x)\psi(x) \Rightarrow \int d^4x d^4y d^4z\, \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A^\mu(x)\delta\overline{\psi}_l(y)\delta\psi_{l'}(z)}\Big|_{A=\overline{\psi}=\psi=0} A_c^\mu(x)\overline{\psi}_l(y)\psi_{l'}(z) + \cdots$$

$$\tilde{\Gamma}^\nu_{ll'}(x,y,z) \equiv \frac{\delta^3\Gamma[A,\psi,\overline{\psi}]}{\delta A^\nu(x)\delta\overline{\psi}_l(y)\delta\psi_{l'}(z)}\Big|_{A=\overline{\psi}=\psi=0} = \int\frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\tilde{\Gamma}^\nu_{ll'}(p,p')$$

$$\tilde{\Gamma}^\nu(x,y,z) = \int\frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\tilde{\Gamma}^\nu(p,p') = \tilde{\Gamma}^\nu(i\partial_y, -i\partial_z)\delta(y-x)\delta(x-z)$$

$$\partial_{\nu,x}\Gamma^\nu(x,y,z) = eS^{-1}(y,z)[\delta(y-x)-\delta(z-x)] \qquad S^{-1}(y,z) = \int\frac{d^4p}{(2\pi)^4}e^{-i(y-z)\cdot p}S^{-1}(p)$$

$$\begin{aligned}\partial_{\nu,x}\Gamma^\nu(x,y,z) &= \int\frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)} i(p_\nu - p'_\nu)\Gamma^\nu(p,p') \\ &= e\int\frac{d^4p d^4p'}{(2\pi)^8}[S^{-1}(p')e^{-ip'\cdot(y-z)-ip\cdot(y-x)} - S^{-1}(p)e^{-ip\cdot(y-z)-ip'\cdot(x-z)}] \\ &= e\int\frac{d^4p d^4p'}{(2\pi)^8}[S^{-1}(p')e^{-ip'\cdot(x-z)-ip\cdot(y-x)} - S^{-1}(p)e^{-ip\cdot(y-x)-ip'\cdot(x-z)}]\end{aligned}$$

$$i(p_\nu - p'_\nu)\Gamma^\nu(p,p') = e[S^{-1}(p') - S^{-1}(p)] \qquad iS^{-1}(p) = \not{p}[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]$$

电子磁矩

$$\tilde{\Gamma}^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\tilde{\Gamma}^\nu(p,p') = \tilde{\Gamma}^\nu(i\partial_y, -i\partial_z)\delta(y-x)\delta(x-z)$$

$$\mathcal{L}_I \Rightarrow -\int d^4x d^4y d^4y\, \tilde{\Gamma}^\nu_{ll'}(x,y,z)A^\mu(x)\overline{\psi}_l(y)\psi_{l'}(z) + \cdots$$

$$= -\int d^4x d^4y d^4z\, A^\mu(x)\overline{\psi}_l(y)\psi_{l'}(z)\tilde{\Gamma}^\nu_{ll'}(i\partial_y, -i\partial_z)\delta(y-x)\delta(x-z) + \cdots$$

$$= -\int d^4x\, A^\mu(x)\,[\,\overline{\psi}_l(x)\tilde{\Gamma}^\nu_{ll'}(-i\overleftarrow{\partial}_x, i\overrightarrow{\partial}_x)\psi_{l'}(x)\,] + \cdots$$

$[i\partial\!\!\!/ - M]\psi(x) = 0 \quad \Rightarrow$ 最右边的$p\!\!\!/' = M$ 所有的的$p'^2 = M^2$

$\overline{\psi}(x)[-i\overleftarrow{\partial\!\!\!/} - M] = 0 \quad \Rightarrow$ 最左边的$p\!\!\!/ = M$ 所有的的$p^2 = M^2$

$$i(p_\nu - p'_\nu)\Gamma^\nu(p,p') = e\{p\!\!\!/'[1-\tilde{\Sigma}_A(p'^2)] + M[1-\tilde{\Sigma}_B(p'^2)] - p\!\!\!/[1-\tilde{\Sigma}_A(p^2)] - M[1-\tilde{\Sigma}_B(p^2)]\} = 0$$

$$(p_\nu - p'_\nu)\tilde{\Gamma}^\nu(p,p') = 0$$

$$\tilde{\Gamma}^\nu(p,p') = \frac{\Gamma^\nu(p,p')}{\sqrt{1-\tilde{\pi}((p-p')^2)}} = e[\gamma^\nu F_1(q^2) + \frac{p^\nu+p'^\nu}{2M}F_2(q^2)] \qquad q_\mu \equiv p_\mu - p'_\mu$$

$$\gamma^\nu = \frac{(-p\!\!\!/+M)\gamma^\nu + \gamma^\nu(-p\!\!\!/+M)+2p^\nu}{2M} \sim \frac{p^\nu}{M} \xRightarrow{\Gamma^\nu(p,p)=e\gamma^\nu} F_1(0)+F_2(0) = 1$$

$F_2(0)$是轨道角动量项

电子磁矩

$$\mathcal{L}_I = -\int d^4x\, A^\mu(x)\, [\, \overline{\psi}_l(x)\tilde{\Gamma}^\nu_{ll'}(-i\overleftarrow{\partial}_x, i\overrightarrow{\partial}_x)\psi_{l'}(x)\,] + \cdots$$

$$\tilde{\Gamma}^\nu(p,p') = \frac{\Gamma^\nu(p,p')}{\sqrt{1-\tilde{\pi}((p-p')^2)}} = e[\gamma^\nu F_1(q^2) + \frac{p^\nu + p'^\nu}{2M} F_2(q^2)] \qquad q_\mu \equiv p_\mu - p'_\mu \qquad F_1(0)+F_2(0) = 1$$

$$\mu^k_{经典} = \frac{e}{2}\epsilon^{ijk}\int d^3x\, x^i \overline{\psi}\gamma^j\psi = \frac{e}{2}\epsilon^{ijk}\int d^3x\, x^i\left[\frac{1}{2M}\overline{\psi}[-i\overleftarrow{\partial}^j + i\partial^j]\psi + \frac{i}{4M}\partial_\nu(\overline{\psi}[\gamma^j,\gamma^\nu]\psi)\right]$$

$$= \frac{ei}{2M}\epsilon^{ijk}\int d^3x\, [\overline{\psi}\, x^i\partial^j\, \psi - \frac{1}{4}(\overline{\psi}[\gamma^j,\gamma^i]\psi)] = \frac{e}{2M}\int d^3x\, \overline{\psi}\, [L^k + \sigma^k]\, \psi \qquad \mu_e = \frac{e}{2M}$$

$$J^\mu = \overline{\psi}\left[\gamma^\mu F_1(0) + \frac{-i\overleftarrow{\partial}^\mu + i\partial^\mu}{2M} F_2(0)\right]\psi$$

$$\mu^k_{量子} = \frac{e}{2}\epsilon^{ijk}\int d^3x\, x^i J^j = \frac{e}{2}\epsilon^{ijk}\int d^3x\, x^i\overline{\psi}\left[\gamma^j F_1(0) + \frac{-i\overleftarrow{\partial}^j + i\partial^j}{2M}F_2(0)\right]\psi$$

$$= \frac{ei}{2M}\epsilon^{ijk}\int d^3x\, \overline{\psi}\left[x^i\partial^j[F_1(0)+F_2(0)] - \frac{1}{4}[\gamma^j,\gamma^i]F_1(0)\right]\psi = \frac{e}{2M}\int d^3x\, \overline{\psi}[\vec{J} - \vec{\sigma}F_2(0)]\psi$$

$$= \mu^k_{经典} + \delta\mu^k_{量子} \qquad \delta\vec{\mu}_{量子} = \delta\mu_e\int d^3x\, \overline{\psi}\vec{\sigma}\psi \qquad \delta\mu_e = -F_2(0)\frac{e}{2M}$$

电子磁矩

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p') = \Gamma^\nu(i\partial_y, -i\partial_z)\delta(y-x)\delta(x-z)$$

$$\Gamma^\nu(p,p') = Z_2 e\gamma^\nu + \frac{e^3}{8\pi^2}\Bigg[\gamma^\nu\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi}\}$$

$$+\int_0^1 dx\int_0^x dy \frac{[(1-x+y)\not p' - y\not p]\gamma^\nu[(1-y)\not p + (y-x)\not p'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\Bigg] + O(e^5)$$

最右边的$\not p' = M$　最左边的$\not p = M$　所有的的$p^2 = p'^2 = M^2$

$$[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 = y^2M^2 + (x-y)^2M^2 + y(x-y)(2M^2 - q^2) = x^2M^2 + y(y-x)q^2$$

$$[(1-x+y)\not p' - y\not p]\gamma^\nu[(1-y)\not p + (y-x)\not p'] = [(1-x+y)\not p' - yM]\gamma^\nu[(1-y)\not p - (x-y)M]$$

$$= (1-x+y)\not p'\gamma^\nu(1-y)\not p - (1-x+y)\not p'\gamma^\nu(x-y)M - yM\gamma^\nu(1-y)\not p + yM\gamma^\nu(x-y)M$$

$$\not p'\gamma^\nu\not p = (2p'^\nu - \gamma^\nu\not p')\not p = 2Mp'^\nu - \gamma^\nu(2p'\cdot p - \not p\not p') = 2Mp'^\nu - \gamma^\nu 2p'\cdot p + (2p^\nu - M\gamma^\nu)\not p'$$

$$= 2Mp'^\nu + 2Mp^\nu + \gamma^\nu((p'-p)^2 - 3M^2)$$

$$= (1-x+y)(1-y)[2M(p'^\nu + p^\nu) + \gamma^\nu((p'-p)^2 - 3M^2)] - (1-x+y)(x-y)(2p'^\nu - \gamma^\nu M)M$$

$$-y(1-y)M(-M\gamma^\nu + 2p^\nu) + y(x-y)M^2\gamma^\nu$$

$$= -(1-x+y)(x-1)2Mp'^\nu - (1-y)(x-1)2Mp^\nu + \gamma^\nu\{(1-x+y)(1-y)(p'-p)^2$$

$$\Gamma^\nu(x,y,z) = \int \frac{d^4p d^4p'}{(2\pi)^8} e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p') = \Gamma^\nu(i\partial_y, -i\partial_z)\delta(y-x)\delta(x-z)$$

$$\Gamma^\nu(p,p') = Z_2 e\gamma^\nu + \frac{e^3}{8\pi^2}\left[\gamma^\nu\left\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy \ln\frac{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}{4\pi}\right\}\right.$$

$$\left. + \int_0^1 dx \int_0^x dy \frac{[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] + M^2\gamma^\nu - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 - i0^+}\right] + O(e^5)$$

最右边的$\not{p}' = M$ 最左边的$\not{p} = M$ 所有的的$p^2 = p'^2 = M^2$

$$[yp+(x-y)p']^2 - yp^2 - (x-y)p'^2 + xM^2 = x^2M^2 + y(y-x)q^2$$

$$[(1-x+y)\not{p}' - y\not{p}]\gamma^\nu[(1-y)\not{p} + (y-x)\not{p}'] - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]$$

$$= -(1-x+y)(x-1)2Mp'^\nu - (1-y)(x-1)2Mp^\nu + \gamma^\nu\{(1-x+y)(1-y)(p'-p)^2$$

$$+[(1-x+y)(x+2y-3) + y(1+x-2y)]M^2\} - 2M[(1-2y)p^\nu + (1-2x+2y)p'^\nu]$$

$$= -[(1-x+y)x + y - x]2Mp'^\nu - [(1-y)x - y]2Mp^\nu + \gamma^\nu\{(1-x+y)(1-y)(p'-p)^2$$

$$+[(1-x+y)(x+2y-3) + y(1+x-2y)]M^2\}$$

$$= -2M[(y - x^2 + xy)p'^\nu + (x - xy - y)p^\nu] + \gamma^\nu\{(1-x+y)(1-y)(p'-p)^2 + (1-x)(x-3)M^2\}$$

$$\Gamma^\nu(x,y,z)=\int\frac{d^4pd^4p'}{(2\pi)^8}e^{-ip\cdot(y-x)-ip'\cdot(x-z)}\Gamma^\nu(p,p')=\Gamma^\nu(i\partial_y,-i\partial_z)\delta(y-x)\delta(x-z)$$

$$\Gamma^\nu(p,p')=Z_2e\gamma^\nu+\frac{e^3}{8\pi^2}\left[\gamma^\nu\{\frac{1/2}{2-D/2}-\frac{\gamma}{2}-\int_0^1dx\int_0^xdy\ln\frac{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}{4\pi}\}\right.$$

$$\left.+\int_0^1dx\int_0^xdy\frac{[(1-x+y)\not p'-y\not p]\gamma^\nu[(1-y)\not p+(y-x)\not p']+M^2\gamma^\nu-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]}{[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2-i0^+}\right]+O(e^5)$$

$$[yp+(x-y)p']^2-yp^2-(x-y)p'^2+xM^2=x^2M^2+y(y-x)q^2$$

$$[(1-x+y)\not p'-y\not p]\gamma^\nu[(1-y)\not p+(y-x)\not p']-2M[(1-2y)p^\nu+(1-2x+2y)p'^\nu]$$

$$=-2M[(y-x^2+xy)p'^\nu+(x-xy-y)p^\nu]+\gamma^\nu\{(1-x+y)(1-y)q^2+(1-x)(x-3)M^2\}$$

$$\Gamma^\nu(p,p')=Z_2e\gamma^\nu+\frac{e^3}{8\pi^2}\left[\gamma^\nu\{\frac{1/2}{2-D/2}-\frac{\gamma}{2}-\int_0^1dx\int_0^xdy\ln\frac{x^2M^2+y(y-x)q^2-i0^+}{4\pi}\}\right.$$

$$\left.+\int_0^1dx\int_0^xdy\frac{-2M[(y-x^2+xy)p'^\nu+(x-xy-y)p^\nu]+\gamma^\nu\{(1-x+y)(1-y)q^2+(1-x)(x-3)M^2\}}{x^2M^2+y(y-x)q^2-i0^+}\right]+O(e^5)$$

$$\int_0^xdy\frac{y-x^2+xy}{x^2M^2+y(y-x)q^2-i0^+}=\int_{-x}^0dy'\frac{y'+x+xy'}{x^2M^2+(y'+x)y'q^2-i0^+}=\int_0^xdy\frac{-y+x-xy}{x^2M^2+(y-x)yq^2-i0^+}$$

$$\Gamma^\nu(p,p') = Z_2 e\gamma^\nu + \frac{e^3}{8\pi^2}\left[\gamma^\nu\left\{\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy \ln\frac{x^2M^2+y(y-x)q^2-i0^+}{4\pi}\right\}\right.$$

$$\left. + \int_0^1 dx\int_0^x dy \frac{-2M(p'^\nu+p^\nu)(x-xy-y)+\gamma^\nu\{(1-x+y)(1-y)q^2+(1-x)(x-3)M^2\}}{x^2M^2+y(y-x)q^2-i0^+}\right] + O(e^5)$$

$$= e[\gamma^\nu F_1(q^2) + \frac{p^\nu + p'^\nu}{2M}F_2(q^2)]$$

$$F_1(q^2) = Z_2 + \frac{e^2}{8\pi^2}\left[\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx\int_0^x dy \left[\ln\frac{x^2M^2+y(y-x)q^2-i0^+}{4\pi}\right.\right.$$

$$\left.\left. - \frac{(1-x+y)(1-y)q^2+(1-x)(x-3)M^2}{x^2M^2+y(y-x)q^2-i0^+}\right]\right] + O(e^4)$$

$$F_2(q^2) = -\frac{e^2}{8\pi^2}\int_0^1 dx\int_0^x dy\, \frac{4M^2(x-xy-y)}{x^2M^2+y(y-x)q^2-i0^+} + O(e^4)$$

$F_1(q^2)$依赖于重整化方案的选择，$F_2(q^2)$独立于重整化方案的选择

$$F_2(0) = -\frac{e^2}{8\pi^2}\int_0^1 dx\int_0^x dy\,\frac{4(x-xy-y)}{x^2} + O(e^4) = -\frac{e^2}{8\pi^2}\int_0^1 dx\int_0^x dy\,\frac{2(x^2-x^3)}{x^2} + O(e^4) = -\frac{e^2}{8\pi^2} + O(e^4)$$

$$\delta\mu_e = -\frac{e}{2M}F_2(0) = \frac{e}{2M}\frac{\alpha}{2\pi} = \frac{e}{2M}\times 0.0011617$$

电子磁矩

$$F_1(q^2) = Z_2 + \frac{e^2}{8\pi^2}\left[\frac{1/2}{2-D/2} - \frac{\gamma}{2} - \int_0^1 dx \int_0^x dy \, [\ln \frac{x^2M^2 + y(y-x)q^2 - i0^+}{4\pi} - \frac{(1-x+y)(1-y)q^2 + (1-x)(x-3)M^2}{x^2M^2 + y(y-x)q^2 - i0^+}]\right] + O(e^4)$$

$$F_2(q^2) = -\frac{e^2}{8\pi^2}\int_0^1 dx \int_0^x dy \, \frac{4M^2(x-xy-y)}{x^2M^2 + y(y-x)q^2 - i0^+} + O(e^4)$$

$F_1(q^2)$依赖于重整化方案的选择，$F_2(q^2)$独立于重整化方案的选择

$$F_2(0) = -\frac{e^2}{8\pi^2}\int_0^1 dx \int_0^x dy \, \frac{4(x-xy-y)}{x^2} + O(e^4) = -\frac{e^2}{8\pi^2}\int_0^1 dx \int_0^x dy \, \frac{2(x^2-x^3)}{x^2} + O(e^4) = -\frac{e^2}{8\pi^2} + O(e^4)$$

$$\delta\mu_e = -\frac{e}{2M}F_2(0) = \frac{e}{2M}\frac{\alpha}{2\pi} = \frac{e}{2M} \times 0.0011617$$

2006 PDG $\delta\mu_e = \frac{e}{2M}[0.00115965218590 \pm 0.00000000000380]$

2008 PDG $\delta\mu_e = \frac{e}{2M}[0.00115965218110 \pm 0.00000000000070]$

2010 PDG $\delta\mu_e = \frac{e}{2M}[0.00115965218073 \pm 0.00000000000028]$

准到一阶修正，对电子和μ介子的磁矩修正是一样的！ $F_1(0)$ 有红外发散！